Chapter 13
How Can a Father Be Supportive for the Mathematics Learning Process of a Child? – The Relationship Between Scaffolding and the Interactional Niche in the Development of Mathematical Learning in the Familial Context

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Abstract This chapter focuses on a father-child interaction during block play, which shapes the child's mathematical experiences and mathematics learning process. With the aim of analyzing and discussing such interaction process in detail, the negotiation of taken-as-shared meanings during block play is observed. For this, the concept of the interactional niche in the development of mathematical thinking is used. This concept sheds light on questions of how a father, as one of the main parts of family systems, uses some scaffolding functions and how such interaction process enables a child to learn mathematics in a play situation. The result demonstrates that the play with father takes place as a social act for the child, and the interaction process with father provides the child an effective mathematics learning process, and an interactional niche in the familial context emerges. It can be concluded that familial systems have crucial effects on the scaffolding process.

Keywords Family interactional niche · Father · Scaffolding · Block play · Early childhood · Familial context · Geometry · Interaction · Scaffold learning · Family systems · Leeway of participation · Negotiation of taken-as-shared meanings

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Introduction

NCTM reports that early experiences in mathematics have major importance on children's learning in the first 6 years of life, and young children in every setting experience mathematics through familial practices (NCTM, 2013, p. 1). In this regard, the activities, toys, materials, and social events introduced to children in their home environments shape their thought processes and performances in mathematics. So indeed, Connecticut State Board of Education suggests that family supports children's thinking and play in the emergence of their skills and abilities in each developmental domain (2007, p. viii). Thereby the familial environment gives children various opportunities to experience mathematical activities, which are potentially significant for learning mathematics. Furthermore mathematical thinking and learning come to be a "jigsaw" (Pound, 2006, p. 23) in which the child can make connections between things that are known and new information and experiences.

Research results reveal that early learning within play activities and with the participation of a family member turn out to be more productive and fruitful for the child than playing without an adult (Acar Bayraktar, 2014a, 2014b, 2016; Acar Bayraktar & Krummheuer, 2011). Regardless if the family member has adequate knowledge about mathematical issues, the interaction leads the child to learn something about mathematics (Acar Bayraktar, 2014a, 2016). In addition, emotional motivations of family members can suffice to provide different mathematics learning situations for the child (Acar Bayraktar, 2014a). Furthermore, while the network of family members links closely with everyday lives of children, playing with different family members is likely to provide various learning opportunities about mathematical ideas (Acar Bayraktar, 2014a, 2014b, 2016).

Similarly, Pound (2008) points out that children profit from discussing mathematical ideas with adults. Parents are the first adults in children's lives and the first and the most continuous provider of services and care for their children. They inform their children about any issue, from birth to death, while they also satisfy the emotional, physical, and motivational needs of their children. In this regard parents have a crucial role in the development of children in mathematics as well as in any other realities of life. Collins, Madsen, and Susman-Stillman (2002) point out that the education level of parents has an influence on the communication and parents' styles of interaction with their children. Their education level enters into their communication styles with their children, the children's social environment, and "daily informal and formal activities, which promote or discourage children's peer relationships" (Parke, 2004, p. 371). Parents with lower levels of education have less frequent interactions with their children in middle childhood, and when these children start school, the frequency of their interactions become less than half (Collins et al., 2002, p.79). In contrast, high parental guidance, often with advice-giving strategies, efforts to keep children from being influenced by peers, and talking to them about the future consequences of their behavior, lead children to low levels of antisocial behavior and higher levels of academic achievement (ibid.).

At the age of 5, children enter a wider social world and begin to "determine their own experiences including their contacts with particular others" (Collins et al., 2002, p. 73). Both fathers and mothers increase their attention to their children's school achievement and homework during middle childhood (Collins et al., 2002, p. 80). Furthermore they each make the uses of mathematics apparent so that children can benefit from them and learn complex mathematical meanings and understandings. The questions then arise on which roles fathers and mothers take in the mathematical development of their children and how and in which ways they separately provide and make possible such mathematical learning situations. While we already know something about children's engagements in mathematical situations with their mothers (e.g., Brandt & Tiedemann, 2010; Fisher, Hirsh-Pasek, Golinkoff, & Gryfe, 2008; Miller, Kelly, & Zhou, 2005; Tiedemann, 2013; Vandermaas-Peeler, 2008), we know only a little of children's engagements in mathematical situations with their fathers (e.g., Acar Bayraktar, 2014a, 2014b; Hawighorst, 2005). In many research cultures, father's role is issued less often than mothers' role (Parke, 2002, p. 62). Regarding this, this paper responds to this research need and focuses on the question of how fathers support the learning of early-year mathematics of their children.

According to traditional models of society, fathers are "financial providers" (Tamis-Lemonda, 2004, p. 220), and thus in western industrialized nations, they spend less time in direct one-to-one interaction with their children than mothers (Bornstein & Sawyer, 2008). Therefore usually they take less responsibility than mothers for child caring. According to family systems theory,[1] while mothers mostly attend to "the child's calm and comfort," fathers foster children's "openness to the world" (Tamis-LeMonda, 2004, p. 220). Fathers tend to encourage risk taking while simultaneously protecting their young from danger. During play activities with their fathers, children experience standing up for their own beliefs, while their fathers encourage them to face up to unfamiliar occurrences and their own mistakes, hence justifying themselves and taking risks in new sets of circumstances (ibid.). Such occasions lead children's social competences and functions to develop; they open children up to the outside world. Besides these, fathers encourage their children to complete tasks in the shortest amount of time, which is the primary goal in problem solving (Laakso, 1995, p. 447; see also Acar Bayraktar, 2014a, 2014b). Laakso points out that in the parent-child conversation, children experience more communicative breakdowns with their fathers than with their mothers, and thus there occur different communication styles between mother-child and father-child dyads (Laakso, 1995, p. 446). Moreover fathers ask questions more frequently than mothers, offer their children more information, use more elaborative labels, and come up with more imperative and short utterances in the interaction process with their

[1] Family system theory lays emphasis on the internal and external factors of a family and regards the family as a social system (for more, see Bornstein & Sawyer, 2008). This approach considers "the interdependence among the roles and functions of all family members" (Parke, 2004, p. 366) and helps me "to understand fully the nature of family relationships" and how family members deal with each other and these relationships affect the child's development.

children (Mullis & Mullis, 1986 see also Acar Bayraktar, 2014a, 2014b). Furthermore fathers give more responsibility to their children in completing their given tasks, while they pose more questions and vary the instructions given to their children more flexibly. They tend to make more requests for information, give more exact and elaborative descriptions in play situations, and use a greater proportion of verbalizations describing form, shape, and direction relations than mothers in course of interacting with their children (Laakso, 1995; see also Acar Bayraktar, 2014a, 2014b). Thereby they evoke the "activation function" during play interactions with their children, which involves an exploratory system whereby children experience novel issues in physical and social environments (Tamis-Lemonda, 2004, p. 222; see also Acar Bayraktar, 2014a, 2014b).

On the basis of theoretical aspects above, I observe a father-child dyad in game playing and try to answer some sub-questions:

1. How can a father make the uses of mathematics apparent so that his child can benefit from them and learn complex mathematical meanings and understandings?
2. How does a father in turn "scaffold" his child toward higher levels of mathematical development (Wood, Bruner, & Ross, 1976)?
3. How do education level of a parent and the role of father affect interaction and scaffolding process in the mathematical context?

I pursue these questions in an empirical and qualitatively laid out work, which is in line with the interactionistic research paradigm (Cobb & Bauersfeld, 1995). Thus, in line with abovementioned approaches to mathematical learning, I focus on the emergence of mutual understanding and coordination in discourses between a child and a father. This research has important implications for the fields of mathematics education research. Because the role of fathers in mathematics learning of their children is mostly overlooked or neglected in everyday practices of mathematics education research, this study can bring about any further questions and research themes in this field and maybe also in early childhood education research.

Specific Issues of the Theoretical Approach

The Theoretical Concept of NMT-Family

One of the central research purposes of this work is to examine the relationship between the participation of children and a family member in play situations and to find out how they interact with each other and how individual content-related learning occurs. In this regard, the concept of "interactional niche in the development of mathematical thinking in the familial context" (NMT-Family) (Acar Bayraktar & Krummheuer, 2011; see also Acar Bayraktar, 2014a, 2014b, 2016) is used.

The interactional niche in the development of mathematical thinking (NMT) is developed by Krummheuer (2014) and particularly based on "symbolic

Table 13.1 The structure of NMT-Family (Acar Bayraktar, 2016)

NMT-Family	Component: content	Component: cooperation	Component: pedagogy and education
Aspect: allocation	Mathematical issues, mathematical play	Play as a familial arrangements for cooperation	Developmental theories of mathematics education and proposals of activeness for parents on this theoretical basis
Aspect: situation	Interactive negotiation of the rules of play and the content	Leeway of participation	Folk theories of mathematics education, everyday routines in mathematics education
Aspect: child's contribution	Individual actions	Individual participation profile	Competence theories

interactionism (Blumer, 1969), the cultural historical approach of Vygotksky and Leont'ev, (see Bruner, 1996; Ernest, 2010; Wertsch & Tulviste, 1992) and the phenomenological sociology of Schütz (Schütz & Luckmann, 1979) and its expansion into ethnomethodology (Garfinkel, 1972)" (Krummheuer, 2014, p. 73). It comprises of "the aspect of the interactive local production" of mathematical developmental processes in "the micro-environment of the child" (Krummheuer & Schütte, 2016, p. 173) and answers the question, "How can the situationally emerging form of participation of a child in a social encounter be conceptualized as a moment in the child's development in mathematical thinking?" (Krummheuer, 2014, p. 72).

NMT-Family is the concept of an "interactional niche in the development of mathematical thinking in the familial context" (NMT-Family) (Acar Bayraktar & Krummheuer, 2011; see also Acar Bayraktar, 2014a, 2014b, 2016) and constructed as a sub-concept of NMT. Similar to the concept of NMT, it consists of the aspects of allocation, situation, and the child's contribution. This structure of NMT-Family (Acar Bayraktar, 2014a, 2014b, 2016) is shown in Table 13.1.

The aspect of *allocation* refers to the provided learning offerings of a group or a society, which specifically highlight cultural representations. The aspect of *situation* consists of the emerging performance occurring within the process of negotiating meaning. The aspect of the *child's contribution* involves the situational and individual contributions of the child in focus.

Scaffolding

Bruner (1983) highlights that parents elicit interactive play settings, which promotes child development to sophisticated levels. Furthermore he assumes scaffolding as one of geneses as parents' initiative for supporting children's learning. Thereby parents reflect on the child's perspective voluntarily and obviously, which enables the child an increasing or decreasing autonomy during play. According to Boekaerts, scaffolding refers to a metaphor which "captures the idea of an adaptable and temporary support that helps an individual during the initial period of gaining expertise" (1997, p. 171). Similarly, Brandt and Tiedemann (2010) define scaffolding as a kind of support, of which "key function is to arrange a situation, which allows the child to participate as a competent community member" (p. 430).

The term "scaffolding" extensively appeared in the work of Wood et al. (1976) about the role of tutoring in problem solving. They define scaffolding as an "adult controlling those elements of the task that are initially beyond the learner's capacity, thus permitting him to concentrate upon and complete only those elements that are within his range of competence" (Wood et al., 1976, p. 90). In the work of Wood and his colleagues, the adult person is referred to as an "expert," who "tutors" children during 3D structure building, and the "novice" or "tutee" is referred to as a person who is less adult or less expert and thus gets help from an "expert" (Wood et al., 1976; see also Bruner, 1996; Hammond & Müller, 2012; Nader-Grosbois, Normandeau, Ricard-Cossette, & Quintal, 2008). Their work aimed to examine "some of the major implications of [the] interactive, instructional relationship between the developing child and his elders for the study of skill acquisition and problem solving" (1976, p. 89). Wood and his colleagues define the usual type of tutoring as an "actual pattern of instruction," "in which one member *knows the answer* and the other does not, rather like a *practical* [situation] in which only the instructor *knows how*." (ibid). Thereby the tutor enables children to learn a subject through his or her instructions in the interaction process. For this, (s)he realizes six scaffolding functions called "recruitment, reduction in degrees of freedom, direction maintenance, marking critical features, frustration control, demonstration" (Wood et al., 1976, pp. 98). This process is called scaffolding, which is an "interactive system of exchange that tutors operate with an implicit theory of the learner's acts" (ibid, p. 99).

Bibok and his colleagues referring to Wood et al. define scaffolding as a process that an adult person "simultaneously aims to regulate both children's motivation (recruitment, frustration control) and cognition (reduction in degree of freedom, marking critical features, demonstration)" (Bibok, Carpendale, & Müller, 2009, p. 18). In addition to this, Anghileri (2006) points out scaffolding is not a teaching process but rather *flexible and dynamic* practice that an adult person is *responsive to individuals*, while they are learning independently and autonomously. In my study the focal medium is families; thus I think of scaffolding not as a teaching method but rather as a support that can also focus on the development of the child in a familial context (cf. Van de Pol, Volman, & Beishuizen, 2010). For me such kind of scaffolding differs from teacher scaffolding, which particularly aims at schooling or realizing school culture. Similarly Hammond and Müller (2012) consider parental scaffolding as "unique among potential forms of parental influence on children" at attempting to improve a child's problem solving (2012, p. 280). Tiedemann (2013) also perceives scaffolding as a support that adult and child realize and co-construct together in the situation of negotiation of meaning. In this regard, I perceive scaffolding as a kind of methodology of family members that they "intuitively and informally" realize scaffolding functions in order to support their children during play. For further aspects of scaffolding discussed in the literature, see also, e.g., Bakker, Smit, and Wegerif (2015), Belland, Walker, Olsen, and Leary (2015), and Van de Pol et al. (2010).

Block Play and the Baden Family

In this section, I present the empirical instrument that is embedded as a sub-project in the project of "early Steps in Mathematics Learning-Family Study" (erStMaL-FaSt) (for more, see Acar Bayraktar, 2014a, 2016). The example mathematical game selected from erStMaL-FaSt is the block play "Building 02." In the following sections, first, the game "Building 02" is analyzed. Subsequently, an empirical material is brought in, which comprises of a video recording and its transcription of the Baden family while playing game "Building 02."

A Block Play: "Building 02"

The game "Building 02" is based upon the game "Make 'n' Break" (Lawson & Lawson, 2008) and refers to a block play. It is constructed according to the specific design patterns of erStMaL-FaSt (Vogel, 2014), which means play situations focus on "one mathematical task or problem, which is presented in a playful or exploratory context according to the age of the child and represents the starting point of a common process of dispute" (Vogel, 2014, p. 225). One particular mathematical domain is addressed, and compatible materials, arrangement of space, and mathematical task are chosen. In a brief description, a specific design pattern contains (1) a definition of the play situation, (2) an application field, (3) an intended mathematical domain, (4) a mathematical context, (5) materials and playroom, and (6) an instruction manual (Acar Bayraktar, 2014a). Regarding all these facts (1) "Building 02" can be defined as a block play, which refers to the sum of all actions of building of three-dimensional versions of different geometrical shapes depicted on different playing cards with wooden blocks. (2) The application field is a familial context for the children ranging in age from 4 years upward. (3) The intended mathematical domain of this play situation is geometry, which includes two-dimensional (2D) and three-dimensional (3D) spaces. (5) Materials comprise of playing cards and wooden blocks.

The playing cards are scaled representations in four different levels of difficulty (Fig. 13.1). This means that the size of three-dimensional version of a geometrical shape does not match precisely the size of its two-dimensional version depicted on a card (Fig. 13.2).

(6) The instruction manual explains the rules: The cards are placed on the table face down. Players play five rounds in total by turns of each player in the game. In

Fig. 13.1 The wooden blocks and the game cards in different levels of "Building 02"

Fig. 13.2 The difference between sizes of the wooden block and the image on a playing card

Fig. 13.3 Recording position and the chosen card

each round, one player chooses a card from the deck and builds the figure depicted as a 2D representation. The aim of play is to build the figure shown on the chosen card. To check the compatibility between the built figure and the figure seen on the card, the other players give feedback. If it is correct, then the player is awarded the number of points shown on the card.

The Block Play of the Baden Family

Baden family is a German family who lives in a major German city. Conrad is the focus child who is aged 7 years and 1 month old. He has a younger sister, who is about 1 year old. His parents have higher education. His mother works as an architect, and his father is an engineer. While the parents are at work, a nanny looks after both children.

In the extract from the video recording to be discussed, Conrad is playing with his father. I first describe the beginning moves observed in this episode and then highlight and analyze key points of Conrad's turn at building this 3D object from its 2D image.

The extract comes from the first round of the play. The play begins with Conrad's turn. Conrad picks up the card from the deck. In other words, this is the first round of play, and this is the first card Conrad picked. The chosen card is shown in Fig. 13.3 and has the difficulty level 4. This means it is one of the hardest cards in the deck. The image on the card that Conrad chose technically comprises eight blocks. To be specific, eight blocks are set on top of each other, the frontal view of this structure is drawn as a picture, and then transitions between each block are made fluid. Thereby an image is produced which refers to a rectangle.

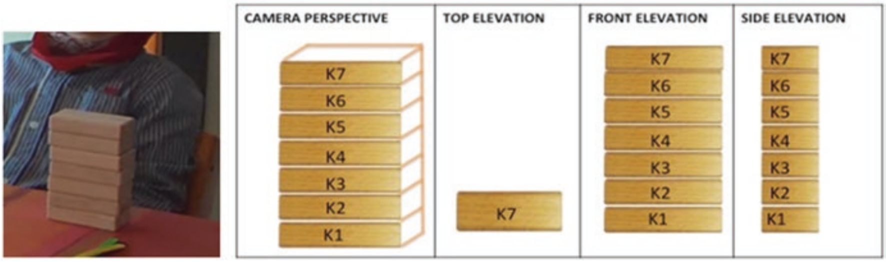

Fig. 13.4 The first block tower that Conrad built

Conrad looks at the chosen card very closely and scans the image on the card for about 15 s while moving his right point finger repeatedly from left to right along the length of the image on the card and keeping on moving his lips soundlessly. Then he says that he needs seven blocks and takes seven wooden blocks from the box. Thereupon he leaves the chosen card on the table and starts to build a block tower. He puts seven blocks (K1, K2, K3, K4, K5, K6, K7) on their X sides[2] horizontally on top of each other. As can be seen in Fig. 13.3, the card is located to Conrad's right, in front of the father, and the child is building the block tower in front of himself, to the left, but within reach, of the father. The block tower that Conrad built is shown in Fig. 13.4.

When the built block tower and the image on the chosen are compared, the front elevation of the built tower does match the image on the chosen card (see Figs. 13.3 and 13.4). Regarding the standard developmental phases of geometrical and educational issues (KMK, 2004; NCTM, 2000), it seems that Conrad is "parts of shapes identifier," "congruence determiner," and "3D shape composer" by building an identical block tower to the image on the chosen card (Clements & Sarama, 2014). By virtue of his visualization, he may be able to represent blocks at the detailed level of shapes to identify shapes in terms of their components. Moreover, he gives the impression of being very capable of coordinating both structures topologically and realizing that the built block tower and the image on the card ostensibly are the same frontal elevation. Furthermore he shows sufficient spatial abilities by composing shapes with anticipation, producing arches, corners, and crosses systematically. In this sense, he gives the impression of determining the congruence by comparing all attributes and all spatial relationships. Ultimately Conrad seems to achieve a vertical block tower identical to the image on the chosen card, although transitions between the various blocks in the image on the chosen card are fluid, and it is purposely complicated to predict how many blocks are needed and how they should be set to achieve an identical tower to the image on the card.

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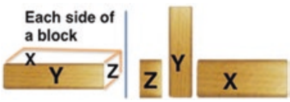
[2] Each side of wooden blocks 

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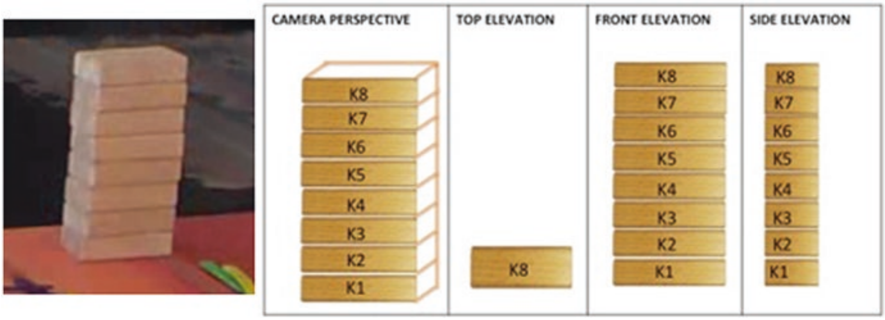

Fig. 13.5 The second block tower

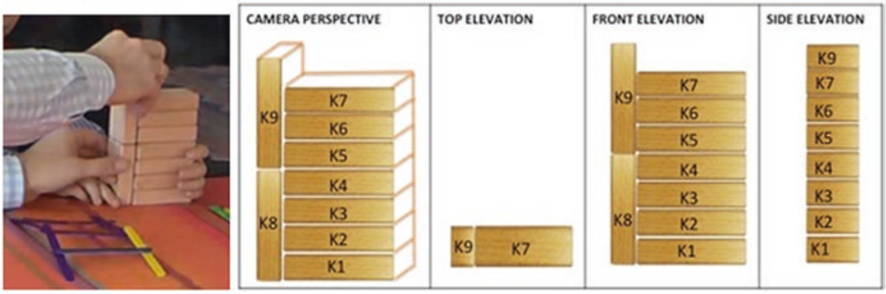

Fig. 13.6 The third block tower

Conrad then folds his arms, smiles, and looks at his father. After about 10 s, his father shakes his head from left to right and proceeds as described in the transcription[3] provided:

Transcript

1.	F:		*Takes a block (K8) from the box and sets it on its X side horizontally on K7* (see Fig. 13.5)
2.			Do you know, why?
3.	C:		No

[3] Rules of transcription

Column 1	Column 2	Column 3	Column 4
Serially numbered lines	Abbreviations for the names of the interacting people.(F, father; C, Conrad)	<: Indicates where people are talking or acting at the same time	Verbal (vocal) actions: regular fontNonverbal actions: italic font

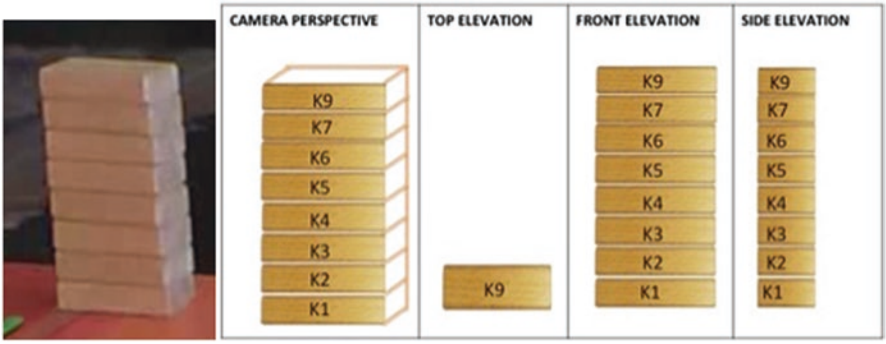

Fig. 13.7 The final block tower

4.	F:		*Takes K8 and sets it on its Z side vertically adjacent to the block tower*
5.		<	*Conrad had built and takes K9 from the box and puts it on top of K8* (see Fig. 13.6)
6.	C:	<	*Keeps both hands around and close to bottom of the tower he built* (see Fig. 13.6)
7.	F:		*Takes K9 away and places it on its X side horizontally on the chosen card on the table*
8.			Look (*shows the image on the card with his left index finger*)
9.	C:		*Looks at the card*
10.	F:		There are two quadrates. (*Shows the image on the card with K9*)
11.	C:		Yes
12.	F:		(*Holds K9 with his right hand.*) Thus another one comes upon it
13.		<	*Takes K8 away*
14.	C:		*Grasp K9 from his father's hand and puts it horizontally*
15.		<	*on its X side upon K7* (see Fig. 13.7)

Interaction Analysis

The father takes one more block (K8) from the box and sets it on its X side horizontally on top of K7 < 1>. Thereby he reconstructs the built block tower. But when the image on the card and the rebuilt tower are compared, the front elevation of the tower still matches the image on the chosen card. Perhaps the number of blocks "matters" for the father, and the block tower should exist just one more block. Conrad's father is an engineer, and, broadly speaking, in his job the mathematical exactness has crucial importance. Maybe, thus, he can predict how many blocks are exactly needed to achieve such an image as a block tower and tries to let Conrad experience such a block-building activity, in which built block tower matches exactly and successfully the image on the card. Maybe therefore he sets one more block upon K7. In this regard the father seems to realize one of *scaffolding* function, namely, either *demonstration* or *marking critical features*. He appears to *demonstrate* either how many blocks actually should be set more or how they set. Thereby he might also show how an ideal block tower can be built. In the sense of *demonstration*, the father seems to perform a perfection of building an ideal block tower and idealization of the act, which involves completion or even explication of the building action. By means of *marking critical features*, he seems to provide information either about Conrad's act, that he should have set one more block upon K7, or about the built block tower, that it was to comprise eight blocks. In both possibilities, he remarks on the critical feature of the built block tower in action that he sets "one more" block "upon" the built tower or "adds 8th block on 7th one" in the built block tower in order to achieve an ideal structure, which is completely identical to the image on the chosen card. Therewith he demonstrates this feature. By *marking critical features*, he accentuates geometrical and numerical features of the built block tower. In this regard his action seems to be made up of both geometrical and arithmetical approaches, which touch on folk psychology and folk pedagogy (Bruner, 1996).

Thereupon he asks whether Conrad knows why <2>. Most probably he asks Conrad whether he knows the reason why the father set one more block upon the block tower or why the block tower should exist eight blocks. Maybe he tries to keep Conrad partly in the field and to let Conrad think about the reason for setting one more block upon the block tower or why he reconstructs the block tower that Conrad built. In this sense the father gives the impression of realizing a scaffolding function called as *direction maintenance*. The father's reactions <1–2 > bring to the mind an aspect of family systems theory that fathers ask questions more than mothers, offer their children more information, use more elaborative labels, and come up with more imperative and short utterances in the interaction process with their children (see Mullis & Mullis, 1986). Furthermore, during play activities with their fathers, the fathers encourage their children to face up to unfamiliar occurrences and their own mistakes, hence justifying themselves and taking risks in new sets of circumstances (see Tamis-LeMonda, 2004). In this regard, Conrad's father might try either to demonstrate correct solution or to give definite instruction to Conrad about the way of building a right block tower and the reason for his action at the line <1>. Maybe he tries to let Conrad understand his point of view, and by posing such question <2>, he tries to encourage Conrad to think exactly about block tower and the image on the chosen card. Moreover he might try to encourage Conrad to face up to unfamiliar occurrence and his own mistake, hence justifying himself in this set of circumstance. In this regard he also seems to be an *activator*, who gives the impression of trying to activate Conrad's spatial knowledge by means of scaffolding. Moreover the family systems theory reinforces this idea that fathers evoke the "activation function" during play interactions with their children, which involves an exploratory system in which children experience novel issues in physical and social environments (see Tamis-LeMonda, 2004). By posing such question to Conrad, the father might try to offer Conrad such a situation that he can exchange his own ideas and so they can strive to reach an agreement with each other. Thereby the father seems to maintain the negotiation with Conrad using exploratory talk, and the interaction process gives the impression of rendering an expanded leeway of participation to Conrad.

Conrad replies him by saying no <3>. His reaction gives the sign of either not knowing why his father set one more block on the block tower or not understanding what his father really tries to do or show. So indeed, when the image on the card and the built tower are compared, the front elevation of the built tower still matches the image on the chosen card, and, thus, from Conrad's part, it seems to remain actually unclear why the father set one more block in the built tower.

The father takes K8 and K9 and sets them successively on their Z sides vertically adjacent to the block tower (see Fig. 13.6) <4–5>. Thereby he again rebuilds the block tower and somehow seems to highlight spatial relationships of 3D objects. When the image on the chosen card and the rebuilt tower are compared, the top-front-side elevations of the built tower do not match the image on the chosen card. Thus, it is unclear whether he tries to build a new block tower or to justify his argument or to show the reason for setting one more block in the tower that he did previously <1>. Wooden blocks are half unit blocks, sized 8 by 4 by 2 cm. The length of each unit is twice the width, which is twice the thickness. In this regard the length of each block is fourfold with the thickness. This means to reach the length of one block, one should set four blocks on top of each other. In this regard the father appears to show or emphasize the height or the length of the built block tower, which Conrad built. Maybe therefore he uses two blocks (K8, K9) in order to show or check in detail the extent of the block tower. In this sense, he might try to find a way to justify his argument at the line <1 > by setting both blocks on top of each other adjacent to the block tower. Regarding family systems theories, he seems to offer Conrad more information, vary the instruction given to his child and thus use more elaborative labels, give more exact and elaborative descriptions, and try to show direction relations in course of interacting with his child (Laakso, 1995; Mullis & Mullis, 1986).

At the same time, Conrad is keeping his both hands around the built block tower (see Fig. 13.6) <6>. Thereby he gives the impression of struggling to avoid hazard of the tower falling. Furthermore his reaction reinforces the idea that he is very capable of coordinating the 3D-structure topologically that he can predict the vertical built tower can fall down. In this regard the negotiation process between Conrad and his father seem collaborative that they build a block tower together collectively. Therefore, from a participatory point of view, they ascribe the role of collaborative game partner to each other. In this regard Conrad and his father seem to engage in the interaction process critically but collectively and constructively.

Thereafter the father takes K9 away and sets it on its X side horizontally on the chosen card which lays on the table and shows the image on the card with his left index finger while saying "look" <7–8 > (see Fig. 13.8). Most probably he tries to justify his argument either at the line <1 > or at the lines <4–5 > by showing the image on the chosen card. Bearing in mind the idea of family systems theory that fathers vary the instructions given to their children more flexibly and tend to make more requests for information, give more exact and elaborative descriptions in play situations, and show direction relations in course of interacting with their children (Laakso, 1995; Mullis & Mullis, 1986), he seems to vary the instruction about the built tower and to give his descriptions more precisely. Maybe thus his utterance is imperative and directive that Conrad should look at the card on the table so that he can "see" or "get" his point of view. Furthermore, by saying "look" to Conrad, he gives the impression of calling Conrad's attention to the image on the card. Regarding family systems theory, it does not seem to be surprising that he again comes up with an imperative and short utterance in the interaction process with his child as in the line <2 > (see Mullis & Mullis, 1986). By saying "look," by means of *scaffolding*, he seems to emphasize to Conrad that he should focus on the image on the chosen card and try keeping Conrad partly in the field. In this sense he gives the impression of realizing a type of scaffolding function, namely, *direction maintenance*. He might try to ensure that Conrad can exactly observe and explore the reason for setting one more block upon the block tower, which Conrad built. Thereby the father uses instant *directivity*, and the block-building activity of Conrad can be directed toward achieving particular outcomes that contribute to completion of the building of the matching tower. Hence looking from a participatory perspective, the father seems to be an *expert*, while he is reserving the role of *novice* for Conrad.

While Conrad is looking at the image on the card <9>, the father says that "there are two quadrates" by still showing the image on the card with his left index finger and keeping the block K9 on the card <10> (see Fig. 13.8). By looking at the image on the card, Conrad seems either to pay attention to his father's argument or to see the reason why one more block should come in the built tower. Thereby he gives the impression that he orients his father's utterances and actions by his reactions in the situation of negotiation of meaning.

By saying "there are two quadrates" <10> while still showing the image on the card, the father most likely emphasizes that the image on the card comprises of two quadrates. The term "quadrate" refers to the term "square," and two squares in equal measure put together make a new shape, a rectangle. So indeed, when the image on the chosen card is reviewed carefully, it is obvious that it is a rectangle and comprises two squares in equal measure (see Fig. 13.9). Considering the technical information about the structure of the chosen card, one should also emphasize that in each square fit exactly four blocks (see Fig. 13.9).

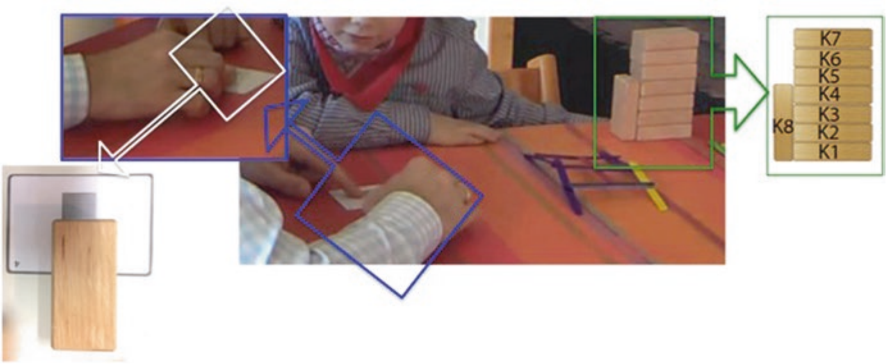

Fig. 13.8 The father shows the card with the help of the block K9

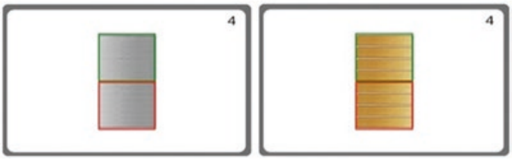

Fig. 13.9 The review of the image on the chosen card

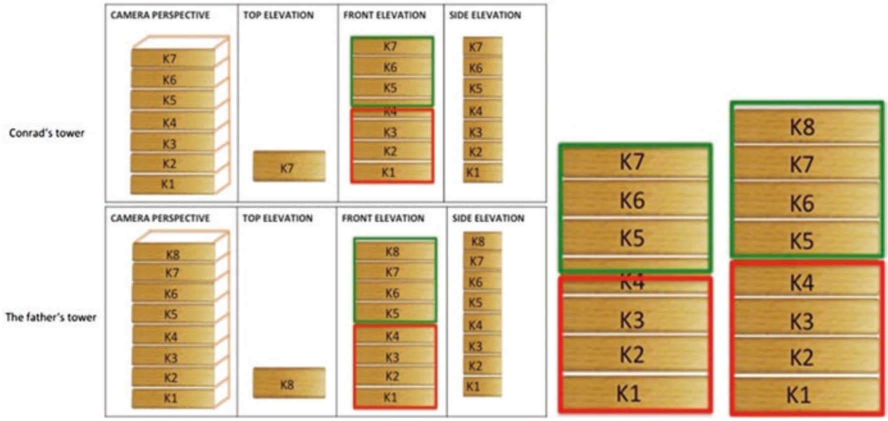

Fig. 13.10 Comparison from the frontal elevation between the first and the second built towers

Regarding this, when the first tower that Conrad built and the second tower that the father built (Fig. 13.10) are compared from the frontal elevation, their differences (see Fig. 13.10) can be scrutinized clearly that they both represent rectangles, but the rectangle of the father's tower can be divided into two squares in equal measure (outlined with red and green) easily, whereas the rectangle of Conrad's tower can be separated into another two rectangles in equal measure (outlined with red and green) only.

Furthermore, considering the idea that the image on the chosen card comprises two squares, one should also take into account the idea that 3D structures can only be divided into groups by computing the amount of the blocks. In this regard father's 3D tower can be divided into two equal groups, of which front elevations refer to squares and consist of four blocks (outlined with red and green), whereas Conrad's cannot (see Fig. 13.11).

As mentioned before <1>, Conrad's father is an engineer and presumably attaches great importance to the mathematical exactness in the game. The father's reaction <10> reinforces this idea and the interpretation in line <1 > that the number of blocks in the built tower "matters" for the father in order to let build a tower, which matches exactly and successfully the image on the card (see <1>). In respect of previous arguments of Conrad's father at lines <1, 4–5, 7–8>, he might still try to ensure the perfection of the built tower, of which frontal elevation is completely identical to the image on the card. Considering Figs. 13.6, 13.9, and 13.11, by setting K8 and K9 next to the built block tower (see <4–5>), the father might try to

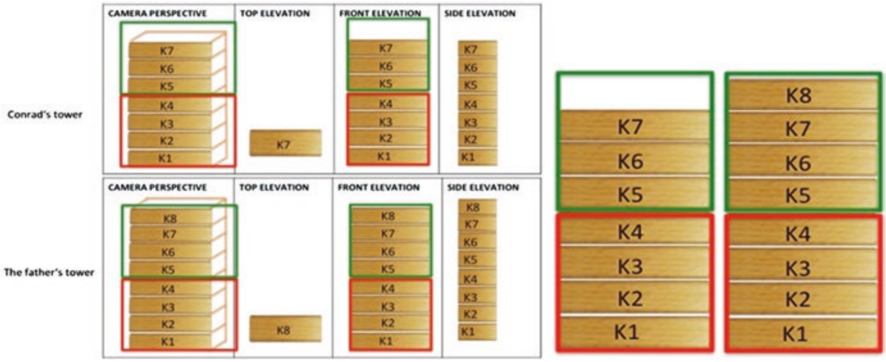

Fig. 13.11 Division of the first and the second built towers into two groups and their comparison from the frontal elevation

show that two quadrates (outlined with red and green) have the same length as these wooden blocks, and the length of each block is fourfold with the thickness (see Fig. 13.11). According to him, by setting four blocks on top of each other, one can reach the length of one block. Maybe therefore he tries to describe the distinctive nature of the image by diving in pieces. By setting eight blocks on top of each other <1>, one can reach two quadrates and thereby achieve the length of two blocks on top of each other that exactly matches the length of tower in the image on the chosen card. In this regard the father might try to suggest the ideal block tower should be built in the way of reaching two quadrates.

In any event the father seems to come up with the geometrical and numerical arguments that the image on the card consists of "two quadrates." Thereby he might emphasize that seven blocks cannot be equally divided into two quadrates (see Figs. 13.10 and 13.11). Maybe therefore he tries to call Conrad's attention to the point that the built tower should be made with eight blocks in order to get two quadrates perfectly. In this sense he gives the impression of coming up with the geometrical and numerical arguments together that the tower should comprise of two quadrates. "Two" represents the amount of the quadrates, and "two quadrates" represents one rectangle, namely, the image on the chosen card. From a developmental perspective, his reaction might activate both Conrad's geometrical and numerical skills in that he can consider his father's both geometrical and numerical arguments and produce 3D block tower properly with the image on the chosen card. Furthermore the geometrical argument of the father seems to enable Conrad to explore *composing and decomposing spatial fields unit by unit in both 2D and 3D spaces* to investigate and predict the results of combining, subdividing, and changing shapes, to understand the variety of ways in which geometric shapes and objects can be measured, and to explore and apply the concepts of congruence.

In addition, the father seems to realize a *scaffolding* function *marking critical features*. He appears to provide different information about the extent of the image on the chosen card. Thereby he gives the impression of accentuating a certain feature of the image on the chosen card, which is relevant. Furthermore his marking provides Conrad information about the way of building an exact block tower, which is totally identical to the image on a card. In this regard the father seems to be an *expert*, while he is ascribing the role of *novice* to Conrad**.** Furthermore, regarding some aspects of family system theory (see Mullis & Mullis, 1986; Tamis-LeMonda, 2004), one can say that Conrad's father gives exact and elaborative descriptions of the card and the block tower and uses a greater proportion of verbalizations describing form, shape, and direction relations in the course of interacting with his child. Moreover, he seems to encourage his child to face up to unfamiliar occurrence and his own mistake and hence enable Conrad in justifying himself.

Conrad gives an affirmative response <11>. Most probably he gets either the point of his father's view, or what he means, or what actually should be done to accomplish an ideal tower. Maybe the father's elaborated elucidation enabled Conrad to judge and justify his idea about the way of building the block tower. Maybe therefore he affirms his father and says "yes" in order to emphasize that he agreed with the necessity of setting 8th block in the block tower. Conrad's reaction gives the impression of accepting his father's argument and ascribes the role of *expert* to his father, while he takes the role of *novice*. Concordantly he seems to assign his father the role of *activator* who activates Conrad's knowledge about geometrical and spatial issues that he can judge the properties of the ideal block tower and the rightness of his father's assertion and make a decision – that the father is right. In this regard Conrad gives the impression of activating his spatial abilities through which he can recognize and operate geometric shapes and structures in the environment and specify their location (see KMK, 2004; NCTM, 2000).

The father holds K9 with his right hand and states "thus another one comes upon it" <12>. His utterance looks like a description of his action in line <1> that "another one block" should be physically added to the top surface of the block tower; in other words "another one block" should be set "onto" the block tower. His reaction reinforces the idea in line <1> that the number of blocks in the built tower "matters" for the father in order to achieve exact and successful match of the built tower and the image (see <1>). Furthermore this reaction of the father reinforces the idea at line <11> that the image on the card comprises of two quadrates, but seven blocks cannot be equally divided into two quadrates, and thus the built block tower should be made up with eight blocks in order to get these two quadrates and build a block tower that matches the image perfectly (see Figs. 13.9, 13.10, and 13.11). In this regard the utterance of "another one" might be also interpreted as a kind of elucidation of "one more block." In any event he obviously comes up with the numerical argument that "another one" block should be set upon the built block tower. By emphasizing that one block should come "upon" it, the father uses the vertical directionality term "upon" as "onto." In this way he does not only call attention to the point that another one block should come on top of the block tower but also verbalizes and namely expresses this action and, respectively, his action in line <1> vocally. In this regard the father seems to again highlight spatial relationships of 3D objects. Thus he gives the impression of coming up with the geometrical argument while continuing to provide numerical information. Therefore he seems to maintain fulfilling the scaffolding function *marking critical features* so that he interprets spatial relationships of the built block tower and the image on the chosen card. In this regard he seems to act as an *expert*, while he is reserving the role of *novice* to Conrad.

Conrad grasps K9 from his father's hand and puts it horizontally on its X side upon K7 (see Fig. 13.7), while the father is taking away K8 from the side of the block tower (see Fig. 13.6) <13–15>. By taking K8 away <13>, the father might try to help and leave Conrad a kind of block tower as the first block tower that Conrad already built with seven blocks (see Fig. 13.4). Thereby he seems to provide Conrad with an opportunity that he can go on his building action by setting "another one (block) upon" the first block tower built by Conrad (see line <12>). In this regard, considering family systems theories (Laakso, 1995; Tamis-LeMonda, 2004), the father seems to give more responsibility to Conrad in completing his given tasks – here it is building a block tower – while letting Conrad set the 8th block on to the block tower <15>. From another point of view, in the light of family systems theory, the father might try to complete the game in the shortest amount of time and might have not wish to waste time still with keeping on negotiating about the block tower. So indeed he does not enter into any further discussion about the built block tower and just takes the block K8 away.

At the same time, Conrad takes K9 from his father's hand and sets it on the top of built corpus <14–15>. Thereby Conrad gives the impression of understanding and performing the point of view of his father and what actually should be done to accomplish an ideal tower. Maybe therefore he sets "another one" block "onto" the built block tower (see line <12>). In this sense he appears to come to an agreement on the necessity of setting 8th block in the block tower. Thereby a *working consensus* between Conrad and his father seems to emerge about setting "another one" on the 7th block in order to reach ideal block tower (see lines <1, 12>), which perfectly matches the image on the card. Furthermore Conrad's reaction shows that the father's activities and responses work on Conrad that he executes the building activity in the same way as his father. By means of *scaffolding*, the father's *demonstration* in line <1 > seems to turn out well that Conrad got the idealized form of building an ideal block tower and imitated it back in a more appropriate form. Thus his reaction can be interpreted as a completion of a solution already partially executed. Additionally, the father's reactions (Tamis-LeMonda, 2004) evoke the "activation function" for Conrad so that he involves himself in such novel experiences in the block-building activity, through which an exploratory negotiation process can emerge. In that respect, Conrad's reaction gives the impression of accepting his father's argument and ascribes the role of *expert* to his father, while he takes the role of *novice*. Furthermore, from the developmental point of view, he acts as "units of shape composer" (Clements & Sarama, 2014, p. 182) that he seems to able to make adult-like structures with blocks from pictured models *unit by unit* perfectly and systematically.

Considering lines <1–15>, the negotiation of meaning between Conrad and his father is a collective argumentation process in that they engage collaboratively and communicatively in the block-building activity. They offer justifications and alternative hypotheses, while they are overcoming challenges. They perform collective argumentation in that they offer hypotheses, which can be made publicly accountable, and try to reach an agreement with each other. Conrad first offers his justification and hypothesis about building the block tower and then builds the first block tower. After that the father comes up with alternative hypotheses about the way of building an ideal block tower. Subsequently they reach an agreement with each other so that the father is taking one block away while Conrad is setting 8th block on the 7th block and building the last version of the block tower (see Fig. 13.7), and they achieve a perfectly built block tower, which is completely identical to the image on the chosen card. In this way Conrad succeeds in his turn. Ultimately Conrad's play turn in the first round ends.

The Relationship Between Scaffolding and NMT-Family

In the chosen sequence, from an allocative perspective, the father is the official game partner of Conrad, but he – situationally – sets about the scaffolding process. They realize a *collective argumentation* process in which the father uses and adopts *intuitively and informally* some scaffolding functions in the negotiation process with Conrad. Through his father's scaffolding and his referential verbal and nonverbal acts, Conrad explores and performs whole spatial consequences in the block-building activity. The negotiation process between Conrad and his father is accomplished in an exploratory way in that they are collaborating, reaching agreement with each other, and understanding each other's points of view. In this sense, the learning process for Conrad can emerge through his participation, in which he experiences to build an ideal and perfect matching block tower. Therefore his father takes the role of *activator*, who evokes Conrad's "activation function" so that Conrad exploratory experiences novel issues and the father's perfection and idealization about building an ideal block tower that enable Conrad a learning situation. In this sense the father takes on the role of an *expert*, while he is ascribing the role *novice* for Conrad. Within this context then, I argue, there can emerge a developmental niche for Conrad. According to the whole analysis, the three aspects of an interactional developmental niche in Conrad's familial context can be structured as follows:

The Aspect of Allocation

Content In the chosen scene, Conrad and his father are confronted with a spatial play situation. For more see the section "A Block Play: Building 02" in this paper.

Cooperation In the play situation Conrad and his father are game partners. Conrad's father is the adult person and his *official* conversation partner, who allocates the right to take the next play turn.

Pedagogy and Education Block building provides a view of children's initial abilities to compose 3D objects. In the chosen game, four goals are pursued: spatial structuring, operating on shapes and figures, static balancing between blocks, and identifying the faces of 3D shapes with 2D shapes. These competencies reflect an initial development of thinking at the level of relating parts and wholes.

The Aspect of Situation

Content The chosen play situation enables Conrad and his father to negotiate interactively about building a block tower, which perfectly and ideally matches the image on the card. A *dyadic* interaction process between Conrad and his father emerges as the father comes up with geometrical and numerical approaches to the building block tower. During block-building activity, Conrad and his father put forward their justifications, alternative hypotheses, and agreements. Moreover they share relevant information, strive to reach an agreement, and dedicate themselves to pursuit of the best solution. Thus they engage in the interaction process critically but constructively and collectively. In this respect the negotiation process between father and son emerges as an exploratory one. The father's perfection and his *geometrical* and *numerical arguments* enable Conrad to explore and build an ideal tower. Thus Conrad is exposed to examine spatial relations in great detail and experience of composing and decomposing spatial structures perfectly. In the course of the negotiation process, a working consensus occurs between Conrad and his father about the need of setting one more block upon the block tower.

Cooperation In this dyadic interaction process, Conrad and his father are collaborative game partners. They perform block-building activities *collaboratively* and mostly negotiate in an exploratory way so that Conrad actively experiences how to compose and decompose 2D and 3D shapes *unit by unit* and comprehensively. Thus the negotiation process generates for Conrad such a leeway of participation that he acts as *activated* to complete and achieve the ideal built block tower that matches the image on the card perfectly. In this regard the father acts an *activator*, who evokes Conrad's "activation function" so that he exploratory experiences novel issues and the father's perfection and idealization.

Pedagogy and Education In the chosen play situation, the father strikes a balance between playing with Conrad and at the same time realizing a scaffolding process. Regarding the six scaffolding functions, he exposes Conrad to three of them, namely, “demonstration, direction maintenance, and marking critical features,” whereas he does not draw on the other scaffolding functions called as “recruitment, frustration control, reduction in degree of freedom” (see Wood et al., 1976):

- Demonstration: The father models the idealized form of building a perfect matching block tower <1>. This means that he performs an idealization of the act and completes the Conrad’s *solution* <1> in order to reach perfect matching tower. Thus, the father provides Conrad with a position in which they become able to “imitate” it back in a more appropriate form. So indeed the father’s *demonstration* in line <1 > works on well that Conrad got the idealized form of building an ideal block tower and imitated it back in a more appropriate form in lines <14–15 >.
- Direction maintenance: The father tries to ensure that Conrad can exactly think about, observe, and explore the reason for setting one more block upon the first built block tower <2, 8>. Thereby the block-building activity of Conrad can be directed toward achieving particular outcomes that contribute to completion of building the perfect matching tower. Hence the father uses instant *directivity* and tries to keep Conrad in pursuit of a particular objective so that Conrad can be kept in the field, can directly maintain the building activity, and hereby become involved only in building an ideal block tower, which matches the image on the chosen card perfectly.
- Marking critical features: The father obviously emphasizes the geometrical and numerical features and different aspects of the building activity that are important or relevant for its completion <1–2, 4–5, 10, 12>. By approaching block-building activity from geometrical and numerical perspectives, the father accentuates certain features of the building of block tower and the image on the chosen card. In this regard, his markings let Conrad review spatial relationships of the built block tower and the image on the chosen card in great detail. Thereby they also provide Conrad information about the way of building an exact block tower, which is totally identical to the image on a card.

In this sense the father fulfills three scaffolding functions. Bearing in mind the idea of family systems theory (Laakso, 1995; Mullis & Mullis, 1986; Tamis-LeMonda, 2004), Conrad’s father varies instructions about the built block tower, offers Conrad more information, shows direction relations between block tower and the image, and gives more exact and elaborative descriptions of the card and the block tower <1–2, 4–5, 10, 7–8, 12>. He also ensures the mathematical exactness in the block-building activity too. Moreover, he encourages his child to face up to unfamiliar occurrence and his own mistake and hence enables Conrad in justifying himself <8>. Additionally, the father gives more responsibility to Conrad in completing block-building activity and thus encourages his son to face up to unfamiliar occurrences and their own mistakes, hence justifying themselves and taking risks in new sets of circumstances (see Tamis-LeMonda, 2004).

The Aspect of Child's Contribution

Content Conrad builds a vertical block tower identical to the image on the chosen card, although transitions between the various blocks in the image on the chosen card are fluid and it is purposely complicated to predict how many blocks are needed and how they should be set to achieve an identical tower to the image on the card. In this regard, Conrad acts as "parts of shapes identifier," "congruence determiner," and "3D shape composer" by building and matching block tower to the image on the chosen card (Clements & Sarama, 2014, pp. 164–175).

Cooperation Conrad collaborates with his father in the course of whole block-building activities in the play situation. In both turns Conrad apparently cares for his father's elaborative descriptions, demonstrations, verbal stimulations, and instructions. By accepting the geometrical and numerical arguments of his father, imitating the idealized form of building an ideal block tower shown by his father <1>, Conrad takes the role of *novice* while ascribing the roles of *expert* to the father. Furthermore he ensures himself such a leeway in which he participates in the play situation actively so that he effectively explores and experiences spatial features of building ideal block tower.

Pedagogy and Education Conrad has learning opportunities for building the perfect matching tower by exploring different spatial features and relations in great detail. Through his father's perfection and activation in the negotiation process, he can learn to compare, compose, and decompose 2D and 3D structures unit by unit comprehensively. The *collective* argumentation process with his father enables him to reconstruct *geometrical and numerical meanings*. Thereby Conrad accomplishes the perfect matching block tower. He represents 3D transformations, regulates their relations, links them with each other, and comes to conclusion in a short amount of time. He explores and examines directly the stability of the building towers and builds a sturdy tower. Through the father's usage of three scaffolding functions (demonstration, marking critical features, and direction maintenance), the father directs and maintains elaborations whereby Conrad's development is facilitated. Furthermore, by means of family systems theory (Tamis-LeMonda, 2004), Conrad is encouraged to face up to unfamiliar occurrence and to judge and justify his idea about the way of building the block tower. Thereby he gets the idealized form of building an ideal block tower and imitated it back in a more appropriate form so that he realizes a completion of a solution already partially executed. In this regard he acts as "units of units shape composer" (see Clements & Sarama, 2014) that he seems to become able to make adult-like structures with blocks from pictured models *unit by unit* perfectly and systematically, whereas at the beginning of his turn as a *3D shape composer*, he didn't. On a metacognitive level (Bruner, 1996), by providing explicit directions on how to build the ideal and perfect block tower, the father emphasizes crucial actions, guides at key points, and indicates alternatives as he leads Conrad to "internalisation of schemes, concepts and reasoning that are the

Table 13.2 The NMT-Family Baden

NMT-Family	Component: *content*	Component: *cooperation*	Component: *pedagogy and education*
Aspect: *allocation*	Geometry, spatial structuring, operating on shapes and figures	Playing with father	Development of spatial skills and transformational abilities in spatial thinking and learning
Aspect: *situation*	Negotiation between father and Conrad, geometrical and numerical arguments of Conrad's father; working consensus	Leeway of participation for Conrad; Activator (Father)-activated (Conrad)	The father's *idealization and perfection* of building block tower perfectly *Three Scaffolding functions* by father familial systems
Aspect: *child's contribution*	Operating on shapes and figures; "parts of shapes identifier"; "congruence determiner"; "3D shape composer"	Expert (Father)-novice (Conrad)	Building the perfect matching tower composing and decomposing 2D and 3D structures *unit by unit*; "units of units shape composer"

subject of intra-psychic regulations" (Boekaerts, 1997; Nader-Grosbois et al., 2008). Moreover through reaching, grasping, balancing, stacking, and moving blocks, Conrad gets an opportunity to learn hand-eye coordination. The negotiation process with his father thus inherently enables Conrad's temporal and representational cognitive developments (Bibok et al., 2009).

Regarding all these facts, interactional niche in the development of Conrad's geometrical thinking and learning occurs. Due to these three components, the interactional developmental niche in the Baden family is structured as follows (Table 13.2).

Conclusion

Mathematical play situations conducted in the familial context seem to be a possible contribution to the child's mathematical development. Conrad experiences mathematical learning opportunities during block play with his father. By profession as an engineer, the father has a higher education level. These facts seem to affect the quality of arguments about block-building activities, while Conrad and his father negotiate about mathematical meanings between each other. The father's perfections, directiveness, and usage of three scaffolding functions enable Conrad to become activated while acting as a novice. Furthermore the realizations of family functions offer Conrad's father such situation that he makes the uses of mathematics apparent so that he can evoke Conrad's activation functions during play. By virtue of the father's perfections of building an ideal and perfect block tower and realizations of some scaffolding functions, Conrad explores and reviews different spatial features in great detail. Thereby the father provides to Conrad a learning situation from spatial and numerical perspectives in terms of his folk psychology and pedagogy. Thus he has direct influence on Conrad's geometrical and numerical developments through which Conrad learns complex mathematical meanings affectively. In this manner, his father renders mindfulness of spatial features directly for Conrad. Therefore I argue that an interactional niche in the mathematics learning in the familial context emerges for Conrad.

Regarding the chosen example, it can be concluded that the usage of some scaffolding functions and realization of some family systems functions offer a child different opportunities to be exposed to different mathematical features and relations through which a mathematics learning situation can occur. It seems that the flux of this interaction process between child and family member underscores the developmental importance of coordination and dynamic match, i.e., reciprocity, mutuality, and synchrony of family member's and child's behaviors. Maybe therefore not all scaffolding functions have to be fulfilled while realizing some family system functions in order to achieve a learning situation for a child. The factors of the roles taken can change dynamically and mutually so that individuals can facilitate different types of learning and the way of negotiating can take place in different characters. But one factor stays stable that such mathematical play situations lead them to achieve different kinds of scaffolding processes in which one do not have to fulfill all scaffolding functions in order to offer a child a learning situation.

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