Chapter 11
Autonomous Underwater Vessels

Abstract The control of multi-DOF autonomous underwater vessels (AUVs) exhibits particular difficulties which are due to the complicated nonlinear model of the submersible vessels, the coupling between the systems control inputs and outputs, and the uncertainty about the values of their model's parameters. Moreover, the AUVs' dynamic model is subject to external perturbations which are caused by variable sea conditions and sea currents. Consequently, an efficient control scheme for AUVs should not only compensate for the nonlinearities of the associated dynamic model, but should also exhibit robustness to model parameter variations and to external disturbances. To this end, the present chapter provides results on robust control of AUVs, as well as on adaptive control of such submersible vessels. Thus the control problem for autonomous underwater vessels is treated with (i) global linearization methods (ii) approximate linearization methods and (iii) Lyapunov methods. The solution of the control problem requires a more elaborated procedure when the AUVs' dynamic model is underactuated. which means that the number of actuators included in its propulsion system is less than the number of its degrees of freedom.The methods developed in this chapter treat also the case of underactuated AUVs. Moreover, advanced estimation methods are used to identify in real time the unknown dynamics of the underwater vessels or disturbance forces and torques that affect them. This allows for the implementation of indirect adaptive control schemes for the AUVs. Additionally,for the precise localization of the AUVs and their safe navigation elaborated nonlinear filtering methods are developed. These permit to solve problems of multi-sensor fusion as well as problems of decentralized state estimation with the use of spatially distributed nonlinear filters that track the AUVs motion. In particular the chapter treats the following topics: (a) Global linearization-based control of autonomous underwater vessels, (b) Flatness-based adaptive fuzzy control of autonomous submarines, and (c) Nonlinear optimal control of autonomous submarines.

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11.1 Chapter Overview

The present chapter treats the following topics: (a) Global linearization-based control of autonomous underwater vessels, (b) Flatness-based adaptive fuzzy control of autonomous submarines, and (c) Nonlinear optimal control of autonomous submarines.

With reference to (a) the chapter solves the problem of control and navigation for Autonomous Underwater Vessels (AUVs) using differential flatness theory and the Derivative-free nonlinear Kalman Filter. First, differential flatness is proven for the 6-DOF dynamic model of the AUV. This allows for transforming the AUV model into the linear canonical (Brunovsky) form and for designing a state feedback controller. Uncertainty about the parameters of the AUV's dynamic model, as well external perturbations which affect its motion are issues that have to be taken into account in the controller's design. To compensate for model imprecision and disturbance terms, it is proposed to use a disturbance observer which relies on the previously analyzed the Derivative-free nonlinear Kalman Filter. The considered filtering method consists of the standard Kalman Filter recursion applied on the linearized model of the underwater vessel and of an inverse transformation based on differential flatness theory, which enables to obtain estimates of the state variables of the initial nonlinear model of the vessel. With the use of the Kalman Filter-based disturbance observer, simultaneous estimation of the non-measurable state variables of the AUV and of the perturbation terms that affect its dynamics is achieved. Moreover, after estimating such disturbances, their compensation is also accomplished.

With reference to (b) the chapter proposes adaptive fuzzy control based on differential flatness theory for autonomous submarines. It is proven that the dynamic model of the submarine, having as state variables the vessel's depth and its pitch angle, is a differentially flat one. This means that all its state variables and its control inputs can be written as differential functions of the flat output and its derivatives. By exploiting differential flatness properties the system's dynamic model is written in the multivariable linear canonical (Brunovsky) form, for which the design of a state feedback controller becomes possible. After this transformation, the new control inputs of the system contain unknown nonlinear parts, which are identified with the use of neurofuzzy approximators. The learning procedure for these estimators is determined by the requirement the first derivative of the closed-loop's Lyapunov function to be a negative one. Moreover, the Lyapunov stability analysis shows that H-infinity tracking performance is ascertained for the feedback control loop and this assures improved robustness to the aforementioned model uncertainty as well as to external perturbations.

With reference to (c) the chapter presents a nonlinear H-infinity (optimal) control approach for the problem of the control of the depth and heading angle of an autonomous submarine. This is a multi-variable nonlinear control problem and its solution allows for precise underwater navigation of the submarine. The submarine's dynamic model undergoes approximate linearization around a temporary equilibrium that is recomputed at each iteration of the control algorithm. The linearization procedure is based on Taylor series expansion and on the computation of the submarine's model Jacobian matrices. For the approximately linearized model, the optimal control problem is solved through the design of an H-infinity feedback controller. The computation of the controller's gain requires the solution of an algebraic Riccati equation, which is repetitively performed at each step of the control method. The stability of the control scheme is proven through Lyapunov analysis.

11.2 Global Linearization-Based Control of Autonomous Underwater Vessels

11.2.1 Outline

The control of 6-DOF autonomous underwater vessels (AUVs) exhibits particular difficulties which are due to the complicated nonlinear model of the vessel, the coupling between the system's control inputs and outputs, and the uncertainty about the values of the model's parameters. Moreover, the AUVs' dynamic model is subject to external perturbations which are due to variable sea conditions and sea currents [143, 144, 191, 411]. Consequently, an efficient control scheme for AUVs should not only compensate for the nonlinearities of the associated dynamic model, but should also exhibit robustness to model parameter variations and to external disturbances. To this end, during the last years, there have been several results on robust control of AUVs [251, 258, 288, 386, 453, 536, 635], as well as on adaptive control of such submersible vessels [253, 346, 462, 618].

In this section a new control method is proposed for the 6-DOF dynamic model of AUVs, based on differential flatness theory [450, 452, 457]. First it is proven, that the 6-DOF dynamic model of the AUV is a differentially flat one. This means that all its state variables and its control inputs can be expressed as differential functions of one single algebraic variable which is the so-called flat output [57, 145, 254, 267, 322, 472, 476, 519, 572]. By exploiting differential flatness properties, the AUVs' model is transformed into the linear canonical (Brunovsky) form. For the latter description of the AUVs the design of a state feedback controller is possible. Unlike approximate linearization methods the aforementioned transformation avoids numerical errors and truncation of nonlinear terms from the AUVs' dynamic model.

Another problem that has to be dealt with is that the control loop should compensate for modelling uncertainties and external perturbation terms affecting the AUVs. To this end, it is proposed to use the Derivative-free nonlinear Kalman Filter as a disturbance observer. This nonlinear filter consists of the Kalman Filter recursion applied on the equivalent linearized model of the AUVs together with an inverse transformation, based again on differential flatness theory, which enables to obtain estimates of the initial nonlinear AUVs' model. The aforementioned disturbance observer provides simultaneously estimates of non-measurable state variables of the AUV and of the external perturbation terms. By identifying external disturbance inputs their compensation becomes also possible.

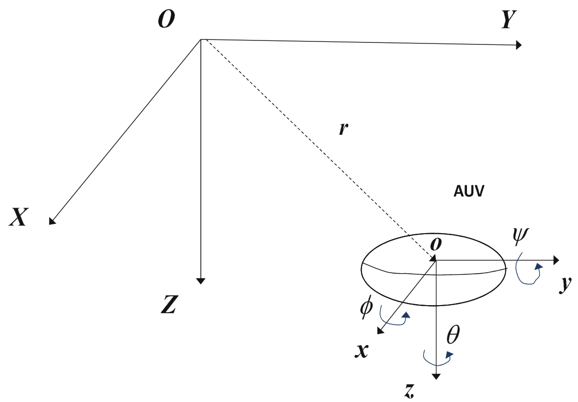

Fig. 11.1 Reference frames for the localization and navigation of the AUV

11.2.2 The 6-DOF Dynamic Model of the AUV

11.2.2.1 Kinematic Model of the AUV

Kinematic and dynamic modelling of AUVs and in general of marine vessels is needed for the development of efficient control for propulsion purposes [373, 388, 416]. In the modelling of AUVs an inertial and a body-fixed reference frame are usually defined. The inertial reference frame of the AUV denoted as $OXYZ$ and the body-fixed reference frame denoted as $Oxyz$, used for the localization and navigation of the underwater vessel are depicted in Fig. 11.1.

The state vector of the AUV in the inertial reference frame is defined as $x = [x_1, x_2]^T = [x, y, z, \phi, \theta, \psi]^T$, where $x_1 = [x, y, z]^T$ denotes linear displacement and $x_2 = [\psi, \theta, \psi]^T$ is the vector of Euler angles which denotes rotational displacement. The associated velocities vector is given by $\dot{x} = [\dot{x}_1, \dot{x}_2]^T = [\dot{x}, \dot{y}, \dot{z}, \dot{\phi}, \dot{\theta}, \dot{\psi}]^T$.

In the body-fixed reference frame the velocity vector of the AUV is denoted as $u = [u_1, u_2]^T = [u, v, w, p, q, r]^T$, where $u_1 = [u, v, w]^T$ is the vector of linear velocities and $u_2 = [p, q, r]^T$ is the vector of angular velocities.

The vector of external forces and torques which can be applied to the 6-DOF AUV is given by $\tau = [F_x, F_y, F_z, T_x, T_y, T_z]^T$. In this representation $\tau_1 = [F_x, F_y, F_z]^T$ is the vector of forces along the X, Y and Z axes respectively and $\tau_2 = [T_x, T_y, T_z]^T$ is the vector of torques causing rotation round the X, Y and Z axes.

The following transformation connects velocities expressed in the inertial reference frame $\dot{\eta}_1 = [\dot{x}, \dot{y}, \dot{z}]^T$ and velocities expressed in the body-fixed frame $v_1 = [u, v, w]^T$:

$\dot{\eta}_1 = J_1 v_1$ where

$$J_1 = \begin{pmatrix} \cos(\psi)cos(\theta) & -sin(\psi)cos(\phi) + cos(\psi)sin(\theta)sin(\phi) & sin(\psi)sin(\phi) + cos(\psi)cos(\phi)sin(\theta) \\ sin(\psi)cos(\theta) & cos(\psi)cos(\phi) + sin(\phi)sin(\theta)sin(\psi) & -cos(\psi)sin(\phi) + sin(\theta)sin(\psi)cos(\phi) \\ -sin(\theta) & cos(\theta)sin(\phi) & cos(\theta)cos(\phi) \end{pmatrix} \tag{11.1}$$

Moreover, the following transformation holds between angular velocities expressed in the inertial and in the body-fixed frame

$$\dot{\eta}_2 = J_2 v_2 \text{ where}$$

$$J_2 = \begin{pmatrix} 1 & sin(\phi)tan(\theta) & cos(\phi)tan(\theta) \\ 0 & cos(\phi) & -sin(\phi) \\ 0 & sin(\phi)/cos(\theta) & cos(\phi)/cos(\theta) \end{pmatrix} \tag{11.2}$$

Therefore, it holds

$$\begin{pmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \end{pmatrix} = \begin{pmatrix} J_1 & 0 \\ 0 & J_2 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \text{ or } \dot{\eta} = J \cdot v \tag{11.3}$$

11.2.2.2 Dynamic Model of the AUV

Dynamic models for AUVs have been extensively analyzed [373, 388]. The dynamic model of the AUV representing an equilibrium in forces and torques is

$$M_{RB}\dot{v} + C_{RB}(v) \cdot v = \tau_{RB} \tag{11.4}$$

where M_{RB} is the inertia matrix of the AUV, $C_{RB}(v)$ is the Coriolis and centrifugal forces matrix, $v = [u, v, w, p, q, r]^T$ is the velocities vector in the body-fixed reference frame and $\tau_{RB} = [F_x, F_y, F_z, T_x, T_y, T_z]^T = 0 \in R^{6\times 1}$ is the vector of external forces and torques exerted on the AUV when the latter is found at an equilibrium. All variables of Eq. (11.4) are expressed in the body-fixed frame.

The inertia matrix M_{RB} is given by

$$M_{RB} = \begin{pmatrix} m & 0 & 0 & 0 & mz_G & -my_G \\ 0 & m & 0 & -mz_G & 0 & mx_G \\ 0 & 0 & m & my_G & -mx_G & 0 \\ 0 & -mz_G & my_G & I_x & -I_{xy} & -I_{xz} \\ mz_G & 0 & -mx_G & -I_{xy} & I_y & -I_{yz} \\ -my_G & mx_G & 0 & -I_{xz} & -I_{yz} & I_z \end{pmatrix} \tag{11.5}$$

where I_x, I_y, I_z are inertia matrices, I_{xy}, I_{xz}, I_{yz} are inertia products and $r_G = [x_G, y_G, z_G]$ are the coordinates of the AUV's center of mass (in the body-fixed frame). The Coriolis matrix of the AUV is given by

$$C_{RB} = \begin{pmatrix} 0 & 0 & 0 & m(y_G q + z_G r) & -m(x_G q - w) & -m(x_G r + v) \\ 0 & 0 & 0 & -m(y_G p + w) & m(z_G r + x_G p) & -m(y_G r - u) \\ 0 & 0 & 0 & -m(z_G p - v) & -m(z_G q + u) & -m(x_G p + y_G q) \\ -m(y_G q + z_G r) & m(y_G p + w) & m(z_G p - v) & 0 & -I_{yz} q - I_{xz} p + I_z r & I_{yz} r + I_{xy} p - I_y q \\ m(x_G q - w) & -m(z_G r + x_G p) & m(z_G q + u) & I_{yx} q + I_{xz} p - I_z r & 0 & I_{xz} r + I_{xy} q + I_x p \\ m(x_G r + v) & m(y_G r - u) & -m(x_G p + y_G q) & -I_{yz} r - I_{xy} p - I_y q & I_{xz} r + I_{xy} q - I_x p & 0 \end{pmatrix} \tag{11.6}$$

The motion of the AUV is also affected by the inertia of the fluid that surrounds it. This is modeled as follows:

$$\tau_A = -M_A \dot{v} - C_A(v) v \tag{11.7}$$

This means that a force / torque is developed against the motion of the vessel and it varies proportionally to the vessel's acceleration. The new inertia matrix M_A is given by

$$M_A = \begin{pmatrix} A_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & A_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & A_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & A_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & A_{66} \end{pmatrix} \tag{11.8}$$

and the new Coriolis matrix is given by

$$C_A = \begin{pmatrix} 0 & 0 & 0 & 0 & A_{33} w & -A_{22} v \\ 0 & 0 & 0 & -A_{33} w & 0 & A_{11} u \\ 0 & 0 & 0 & A_{22} v & -A_{11} u & 0 \\ 0 & A_{33} w & -A_{22} v & 0 & A_{66} r & -A_{55} q \\ -A_{33} w & 0 & A_{11} u & -A_{66} r & 0 & A_{44} p \\ A_{22} v & -A_{11} u & 0 & A_{55} q & -A_{44} p & 0 \end{pmatrix} \tag{11.9}$$

The model is completed by the vector of a force / torque which resists to the motion of the underwater vessel and which is proportional to its velocity

$\tau_{DL} = -D(v) v$ where

$$D(v) = \begin{pmatrix} X_{|u|u}|u| & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_{|v|v}|v| & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_{|w|w}|w| & 0 & 0 & 0 \\ 0 & 0 & 0 & K_{|p|p}|p| & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{|q|q}|q| & 0 \\ 0 & 0 & 0 & 0 & 0 & N_{|r|r}|r| \end{pmatrix} \tag{11.10}$$

while the diagonal elements of matrix $D(v)$ are defined as follows:

$$\begin{array}{ll} X_{|u|u} = \frac{\rho}{2} V^{\frac{2}{3}} C_x(0^o, 0^o) & K_{|p|p} = \frac{\rho}{2} V^{\frac{5}{3}} C_p \\ Y_{|v|v} = \frac{\rho}{2} V^{\frac{2}{3}} C_y(90^o, 0^o) & M_{|q|q} = \frac{\rho}{2} V^{\frac{5}{3}} C_q \\ Z_{|w|w} = \frac{\rho}{2} V^{\frac{2}{3}} C_z(90^o, 90^o) & N_{|r|r} = \frac{\rho}{2} V^{\frac{5}{3}} C_r \end{array} \tag{11.11}$$

where ρ is the specific mass of the water, V is the volume of the submerged vessel and C_x, C_y, C_z, C_p, C_q, C_r are constants.

The weight of the AUV is $W = m{\cdot}g$, while the lift force exerted on the AUV is $B = \rho g V$, where ρ is the water's specific weight (both expressed in the inertial reference frame). These forces can be expressed in the body-fixed reference frame as follows: $f_W = J_1^{-1}[0, 0, W]^T$ and $f_B = -J_1^{-1}[0, 0, B]^T$. Moreover, there are torques which are generated due to these forces and these are given by $\tau_W = r_G \times f_W$ and $\tau_B = r_B \times f_B$, where $r_G = [x_G, y_G, z_G]^T$ and $r_B = [x_B, y_B, z_B]^T$. Thus, there is an additional vector of forces and torques applied on the AUV which is given by

$$\tau_{WB} = \begin{pmatrix} f_w + f_B \\ \tau_w + \tau_B \end{pmatrix} = \begin{pmatrix} (W - B)sin(\theta) \\ -(W - B)cos(\theta)sin(\phi) \\ -(W - B)cos(\theta)cos(\phi) \\ -(Y_G W - Y_B B)cos(\theta)cos(\phi) + (z_G W - z_B B)cos(\theta)sin(\psi) \\ (z_G W - z_B B)sin(\theta) + (x_G W - x_B B)cos(\theta)cos(\phi) \\ -(x_G W - x_B B)cos(\theta)sin(\phi) - (y_G W - y_B B)sin(\theta) \end{pmatrix} \tag{11.12}$$

By applying one more transformation on the aforementioned vector, the forces and torques due to the effects of weight and lift are finally expressed in the inertial reference frame. Thus, due to the effects of the resistive forces and torques which are generated by the surrounding fluid one has the dynamics

$$M_{RB}\dot{v} + C_{RB}(v)v = \tau_A + \tau_{DL} + \tau_{WB} + \tau \tag{11.13}$$

where $\tau_A = -M_A\dot{v} - C_A(v)v$, $\tau_{DL} = -D(v)v$ stands for forces and torques resisting the vessel's motion), $\tau_{WB} = -g_f$ represents forces and torques due to weight and lift effects, and τ is the vector of external torques and forces defining the vessel's propulsion. By combining Eqs. (11.4) and (11.7) one obtains the aggregate dynamics

$$(M_{RB} + M_A)\dot{v} + (C_{RB}(v) + C_A(v))v + D(v)v + g_f = \tau \tag{11.14}$$

The aggregate inertia matrix is $M = M_{RB} + M_A$, the aggregate Coriolis matrix is $C(v) = C_{RB}(v) + C_A(v)$. Thus, the dynamic and the kinematic model of the AUV are finally written as

$$M\dot{v} + Cv + D(v)v + g_f = \tau \tag{11.15}$$

$$\dot{\eta} = J(\eta)v \tag{11.16}$$

11.2.3 Differential Flatness of the AUV's Model

It will be proven that the dynamic model of the AUV is a differentially flat one, which means that all its state variables and its control inputs can be written as differential functions of the an algebraic variable (vector) which is the so-called flat output [57, 145, 254, 267, 322, 472, 476, 519, 572]. Using that $v = J^{-1}\dot{\eta}$ or $v = R\dot{\eta}$ Eq. (11.15) can be written equivalently as

$$\tilde{M}\ddot{\eta} + \tilde{C}\dot{\eta} + \tilde{D}(\dot{\eta})\dot{\eta} + g_f(\eta) = \tau \tag{11.17}$$

where η has been defined in the inertial reference frame as $\eta = [x, y, z, \phi, \theta, \psi]^T$, $\tilde{M} = MR$, $\tilde{C} = M\dot{R} + CR$ and $\tilde{D} = DR$. By denoting the inverse of the inertia matrix as $\tilde{M}^{-1} = N$ one obtains

$$\ddot{\eta} + N{\cdot}\tilde{C}\dot{\eta} + N{\cdot}\tilde{D}(\dot{\eta})\dot{\eta} + N{\cdot}g_f(\eta) = N{\cdot}\tau \tag{11.18}$$

Moreover, using the state vector elements notation $z_1 = x$, $z_2 = \dot{x}$, $z_3 = y$, $z_4 = \dot{y}$, $z_5 = z$, $z_6 = \dot{z}$, $z_7 = \phi$, $z_8 = \dot{\phi}$, $z_9 = \theta$, $z_{10} = \dot{\theta}$, $z_{11} = \psi$, $z_{12} = \dot{\psi}$ and by defining the state vector $Z = [z_1, z_2, z_3, z_4, z_5, z_6, z_7, z_8, z_9, z_{10}, z_{11}, z_{12}]^T$, the dynamic model of Eq. (11.18) becomes

$$\begin{array}{ll} \dot{z}_1 = z_2 & \dot{z}_2 + f_1(Z) = N_1(Z)\tau \\ \dot{z}_3 = z_4 & \dot{z}_4 + f_2(Z) = N_2(Z)\tau \\ \dot{z}_5 = z_6 & \dot{z}_6 + f_3(Z) = N_3(Z)\tau \\ \dot{z}_7 = z_8 & \dot{z}_8 + f_4(Z) = N_4(Z)\tau \\ \dot{z}_9 = z_{10} & \dot{z}_{10} + f_5(Z) = N_5(Z)\tau \\ \dot{z}_{11} = z_{12} & \dot{z}_{12} + f_6(Z) = N_6(Z)\tau \end{array} \tag{11.19}$$

where $\tau \in R^{6\times 1}$ is the vector of external forces and torques, $f_i(Z)$ $i = 1, \cdots, 6$ are the row elements of the vector $f = N{\cdot}\tilde{C}\dot{\eta} + N{\cdot}\tilde{D}(\dot{\eta})\dot{\eta} + N{\cdot}g_f(\eta)$, while $N_i(Z)$, $i = 1, \cdots, 6$ are the rows of matrix $N = M^{-1}$. The flat output of the system is taken to be the vector $Y = [z_1, z_3, z_5, z_7, z_9, z_{11}]$. From Eq. (11.19) it holds that $z_2 = \dot{z}_1$, $z_4 = \dot{z}_3$, $z_6 = \dot{z}_5$, $z_8 = \dot{z}_7$, $z_{10} = \dot{z}_9$ and $z_{12} = \dot{z}_{11}$. Therefore, it holds

$$\begin{array}{ll} z_2 = [1\;0\;0\;0\;0\;0]\dot{Y} & z_4 = [0\;1\;0\;0\;0\;0]\dot{Y} \\ z_6 = [0\;0\;1\;0\;0\;0]\dot{Y} & z_8 = [0\;0\;0\;1\;0\;0]\dot{Y} \\ z_{10} = [0\;0\;0\;0\;1\;0]\dot{Y} & z_2 = [0\;0\;0\;0\;0\;1]\dot{Y} \end{array} \tag{11.20}$$

Consequently the state vector elements given above can be written as functions of the flat output Y. Moreover, from Eq. (11.19) one has that

$$\begin{array}{ll} \ddot{z}_1 = v_1 = -f_1 + N_1\tau & \ddot{z}_3 = v_2 = -f_2 + N_2\tau \\ \ddot{z}_5 = v_3 = -f_3 + N_3\tau & \ddot{z}_7 = v_4 = -f_4 + N_4\tau \\ \ddot{z}_9 = v_5 = -f_5 + N_5\tau & \ddot{z}_{11} = v_6 = -f_6 + N_6\tau \end{array} \tag{11.21}$$

Therefore, one has

$$\begin{pmatrix} \ddot{z}_1 \\ \ddot{z}_3 \\ \ddot{z}_5 \\ \ddot{z}_7 \\ \ddot{z}_9 \\ \ddot{z}_{11} \end{pmatrix} = \begin{pmatrix} -f_1 \\ -f_2 \\ -f_3 \\ -f_4 \\ -f_5 \\ -f_6 \end{pmatrix} + \begin{pmatrix} N_1\tau \\ N_2\tau \\ N_3\tau \\ N_4\tau \\ N_5\tau \\ N_6\tau \end{pmatrix} \tag{11.22}$$

which is equivalently written as

$$\begin{matrix} \ddot{z}_a = -f_a(Z) + N\tau \Rightarrow \tau = N^{-1}(\ddot{z}_a + f_a(Z)) \\ \Rightarrow \tau = M(\ddot{z}_a + f_a(Z)) \end{matrix} \tag{11.23}$$

Consequently, the control inputs of the 6-DOF AUV model can be also written as functions of the flat output and its derivatives. Therefore, the AUV model is a differentially flat one.

11.2.4 Flatness-Based Control of the AUV

By exploiting the previously proven differential flatness properties of the AUV it will be shown that a stabilizing feedback controller can be designed for the AUV model. Using Eq. (11.19) the following control inputs are defined.

$$\begin{matrix} v_1 = -f_1 + N_1\tau & v_2 = -f_2 + N_2\tau \\ v_3 = -f_3 + N_3\tau & v_4 = -f_4 + N_4\tau \\ v_5 = -f_1 + N_1\tau & v_6 = -f_1 + N_1\tau \end{matrix} \tag{11.24}$$

or equivalently

$$v = -f_a + N\tau \Rightarrow \tau = N^{-1}(v + f_a) \Rightarrow \tau = M(v + f_a) \tag{11.25}$$

This means that if the transformed control inputs v are computed so as to assure asymptotic tracking of the AUV's reference setpoints, one can also find the real control inputs τ which should be exerted on the AUV for succeeding this objective. According to the above, the dynamic model of Eq. (11.19) can be written into the canonical (Brunovsky) form

$$\begin{matrix} \dot{z}_1 = z_2 & \dot{z}_2 = v_1 & \dot{z}_3 = z_4 & \dot{z}_4 = v_2 \\ \dot{z}_5 = z_6 & \dot{z}_6 = v_3 & \dot{z}_7 = z_8 & \dot{z}_8 = v_4 \\ \dot{z}_9 = z_{10} & \dot{z}_{10} = v_5 & \dot{z}_{11} = z_{12} & \dot{z}_{12} = v_6 \end{matrix} \tag{11.26}$$

which also takes the matrix form

$$\dot{Z} = AZ + BV \tag{11.27}$$

or equivalently one has the following state-space description for the system

$$\begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \\ \dot{z}_5 \\ \dot{z}_6 \\ \dot{z}_7 \\ \dot{z}_8 \\ \dot{z}_9 \\ \dot{z}_{10} \\ \dot{z}_{11} \\ \dot{z}_{12} \end{pmatrix} = \begin{pmatrix} 0\,1\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,1\,0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,1\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,1\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0\,0\,1\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,1 \\ 0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \\ z_7 \\ z_8 \\ z_9 \\ z_{10} \\ z_{11} \\ z_{12} \end{pmatrix} + \begin{pmatrix} 0\,0\,0\,0\,0\,0 \\ 1\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0 \\ 0\,1\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0 \\ 0\,0\,1\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,1\,0\,0 \\ 0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,1\,0 \\ 0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{pmatrix} \tag{11.28}$$

and the measurement equation for this system becomes

$$\begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \end{pmatrix} = \begin{pmatrix} 1\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,1\,0\,0\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,1\,0\,0\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,1\,0\,0\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0\,1\,0\,0\,0 \\ 0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,1\,0 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \\ z_5 \\ z_6 \\ z_7 \\ z_8 \\ z_9 \\ z_{10} \\ z_{11} \\ z_{12} \end{pmatrix} \tag{11.29}$$

Thus, using differential flatness theory the AUV's model has been written in a MIMO linear canonical (Brunovsky) form, which is both controllable and observable. After being written in the linear canonical form the AUV's state-space equation comprises 6 subsystems of the form

$$\ddot{y}_{f_i} = v_i,\ i = 1, \cdots, 6 \tag{11.30}$$

For each one of these subsystems a controller can be defined as follows

$$v_i = \ddot{y}_{f_i}^d - k_{d_i}(\dot{y}_{f_i} - \dot{y}_{f_i}^d) - k_{p_i}(y_{f_i} - y_{f_i}^d),\ i = 1, \cdots, 6 \tag{11.31}$$

Once the transformed control inputs vector $v \in R^{6\times 1}$ has been computed, one can use Eq. (11.25) to find also the torques and forces vector $\tau = M(v + f_a)$ that should be exerted on the AUV so as to achieve convergence to its reference setpoints.

11.2.5 Disturbances Compensation with the Derivative-Free Nonlinear Kalman Filter

It was shown that the initial nonlinear model of the AUV can be written in the MIMO canonical form of Eqs. (11.28) and (11.29). Next, it is assumed that the AUV's model is affected by additive input disturbances, thus one has

$$\begin{matrix} \ddot{z}_1 = v_1 + \tilde{d}_1 & \ddot{z}_2 = v_2 + \tilde{d}_2 \\ \ddot{z}_3 = v_3 + \tilde{d}_3 & \ddot{z}_4 = v_4 + \tilde{d}_4 \\ \ddot{z}_5 = v_5 + \tilde{d}_5 & \ddot{z}_6 = v_6 + \tilde{d}_6 \end{matrix} \tag{11.32}$$

The system's dynamics can be also written as $\dot{z}_1 = z_2$, $\dot{z}_2 = v_1 + \tilde{d}_1$, $\dot{z}_3 = z_4$, $\dot{z}_4 = v_2 + \tilde{d}_2$, $\dot{z}_5 = z_6$, $\dot{z}_6 = v_3 + \tilde{d}_3$, $\dot{z}_7 = z_8$, $\dot{z}_8 = v_4 + \tilde{d}_4$, $\dot{z}_9 = z_{10}$, $\dot{z}_{10} = v_5 + \tilde{d}_5$, $\dot{z}_{11} = z_{12}$, $\dot{z}_{12} = v_6 + \tilde{d}_6$.

Without loss of generality, it is assumed that the dynamics of the disturbances terms are described by their second order derivative, i.e. $\ddot{\tilde{d}}_i = f_{d_i}$, $i = 1, \cdots, 6$. Next, the extended state vector of the system is defined so as to include disturbance terms as well. Thus one has the additional state variables

$$\begin{matrix} z_{13} = \tilde{d}_1 & z_{14} = \dot{\tilde{d}}_1 & z_{15} = \ddot{\tilde{d}}_1 & z_{16} = \tilde{d}_2 & z_{17} = \dot{\tilde{d}}_2 & z_{18} = \ddot{\tilde{d}}_2 \\ z_{19} = \tilde{d}_3 & z_{20} = \dot{\tilde{d}}_3 & z_{21} = \ddot{\tilde{d}}_3 & z_{22} = \tilde{d}_4 & z_{23} = \dot{\tilde{d}}_4 & z_{24} = \ddot{\tilde{d}}_4 \\ z_{25} = \tilde{d}_5 & z_{26} = \dot{\tilde{d}}_6 & z_{27} = \ddot{\tilde{d}}_5 & z_{28} = \tilde{d}_6 & z_{29} = \dot{\tilde{d}}_6 & z_{30} = \ddot{\tilde{d}}_6 \end{matrix} \tag{11.33}$$

Thus, the disturbed system can be described by a state-space equation of the form

$$\begin{matrix} \dot{z}_f = A_f z_f + B_f v \\ z_f^{meas} = C_f z_f \end{matrix} \tag{11.34}$$

where $A_f \in R^{30\times 30}$, $B_f \in R^{30\times 6}$ and $C_f \in R^{6\times 30}$, with

$$A_f = \begin{pmatrix} 0_{1\times 1} & 1 & 0_{1\times 28} \\ 0_{1\times 12} & 1 & 0_{1\times 17} \\ 0_{1\times 3} & 1 & 0_{1\times 26} \\ 0_{1\times 15} & 1 & 0_{1\times 14} \\ 0_{1\times 5} & 1 & 0_{1\times 24} \\ 0_{1\times 18} & 1 & 0_{1\times 11} \\ 0_{1\times 7} & 1 & 0_{1\times 22} \\ 0_{1\times 21} & 1 & 0_{1\times 8} \\ 0_{1\times 9} & 1 & 0_{1\times 20} \\ 0_{1\times 24} & 1 & 0_{1\times 5} \\ 0_{1\times 11} & 1 & 0_{1\times 18} \\ 0_{1\times 27} & 1 & 0_{1\times 2} \\ 0_{1\times 13} & 1 & 0_{1\times 16} \\ 0_{1\times 14} & 1 & 0_{1\times 15} \\ 0_{1\times 30} & & \\ 0_{1\times 16} & 1 & 0_{1\times 13} \\ 0_{1\times 17} & 1 & 0_{1\times 12} \\ 0_{1\times 30} & & \\ 0_{1\times 19} & 1 & 0_{1\times 10} \\ 0_{1\times 20} & 1 & 0_{1\times 9} \\ 0_{1\times 30} & & \\ 0_{1\times 22} & 1 & 0_{1\times 7} \\ 0_{1\times 23} & 1 & 0_{1\times 6} \\ 0_{1\times 30} & & \\ 0_{1\times 25} & 1 & 0_{1\times 4} \\ 0_{1\times 26} & 1 & 0_{1\times 3} \\ 0_{1\times 30} & & \\ 0_{1\times 28} & 1 & 0_{1\times 1} \\ 0_{1\times 29} & 1 & \\ 0_{1\times 30} & & \end{pmatrix} \quad B_f = \begin{pmatrix} 0_{1\times 6} & & \\ 1 & 0_{1\times 5} & \\ 0_{1\times 1} & 1 & 0_{1\times 4} \\ 0_{1\times 6} & & \\ 0_{1\times 2} & 1 & 0_{1\times 3} \\ 0_{1\times 6} & & \\ 0_{1\times 3} & 1 & 0_{1\times 2} \\ 0_{1\times 6} & & \\ 0_{1\times 4} & 1 & 0_{1\times 1} \\ 0_{1\times 6} & & \\ 0_{1\times 5} & 1 & \\ 0_{18\times 6} & & \end{pmatrix} \quad C_f = \begin{pmatrix} 1 & 0_{1\times 29} & \\ 0_{1\times 2} & 1 & 0_{1\times 27} \\ 0_{1\times 4} & 1 & 0_{1\times 25} \\ 0_{1\times 6} & 1 & 0_{1\times 23} \\ 0_{1\times 8} & 1 & 0_{1\times 21} \\ 0_{1\times 10} & 1 & 0_{1\times 19} \end{pmatrix} \tag{11.35}$$

For the aforementioned model, and after carrying out discretization of matrices A_f, B_f and C_f with common discretization methods one can implement the standard Kalman Filter algorithm, consisting of a *time-update* and a *measurement update* stage [33, 431, 463]. As previously explained,this is Derivative-free nonlinear Kalman filtering for the model of the AUV which, unlike EKF, is performed without the need to compute Jacobian matrices and does not introduce numerical errors.

The dynamics of the disturbance terms $\tilde{d}_i,\ i = 1, \cdots, 6$ are taken to be unknown in the design of the associated disturbances' estimator. Defining as $\tilde{A}_d$, $\tilde{B}_d$, and $\tilde{C}_d$, the discrete-time equivalents of matrices $\tilde{A}_f$, $\tilde{B}_f$ and $\tilde{C}_f$ respectively, one has the following dynamics:

$$\dot{\hat{z}}_f = \tilde{A}_f \cdot \hat{z}_f + \tilde{B}_f \cdot \tilde{v} + K(z_f^{meas} - \tilde{C}_f \hat{z}_f) \tag{11.36}$$

where $K \in R^{30\times 6}$ is the state estimator's gain. The associated Kalman Filter-based disturbance estimator is given by [450, 452, 457]

measurement update:

$$\begin{aligned} K(k) &= P^-(k)\tilde{C}_d^T[\tilde{C}_d \cdot P^-(k)\tilde{C}_d^T + R]^{-1} \\ \hat{z}_f(k) &= \hat{z}_f^-(k) + K(k)[z_f^{meas}(k) - \tilde{C}_d\hat{z}_f^-(k)] \\ P(k) &= P^-(k) - K(k)\tilde{C}_d P^-(k) \end{aligned} \tag{11.37}$$

time update:

$$\begin{aligned} P^-(k+1) &= \tilde{A}_d(k)P(k)\tilde{A}_d^T(k) + Q(k) \\ \hat{z}_f^-(k+1) &= \tilde{A}_d(k)\hat{z}_f(k) + \tilde{B}_d(k)\tilde{v}(k) \end{aligned} \tag{11.38}$$

To compensate for the effects of the disturbance forces it suffices to use in the control loop the modified control input vector

$$v = \begin{pmatrix} v_1 - \hat{\hat{d}}_1 \\ v_2 - \hat{\hat{d}}_2 \\ v_3 - \hat{\hat{d}}_3 \\ v_4 - \hat{\hat{d}}_4 \\ v_5 - \hat{\hat{d}}_5 \\ v_6 - \hat{\hat{d}}_6 \end{pmatrix} \text{ or } v = \begin{pmatrix} v_1 - \hat{z}_{13} \\ v_2 - \hat{z}_{16} \\ v_3 - \hat{z}_{19} \\ v_4 - \hat{z}_{22} \\ v_5 - \hat{z}_{25} \\ v_6 - \hat{z}_{28} \end{pmatrix} \tag{11.39}$$

11.2.6 Simulation Tests

The efficiency of the proposed control scheme was tested through simulation experiments. First, results are given about tracking a 3D trajectory, having as projection in the xy-plane a circular path (Fig. 11.2). Additional simulation experiments for this first trajectory tracking problem are concerned with control of the AUV under disturbance forces and torques. The estimation of the disturbance forces and torques is shown in Fig. 11.3. Moreover, as shown in Figs. 11.4, 11.5 and 11.6, flatness-based control enabled accurate tracking of the reference trajectories for both the linear position and velocity variables and for the angular position and velocity variables (blue line: real value, green line estimated value, red line: setpoint).

Next, results are given about tracking a 3D trajectory, having as projection in the xy-plane an eight-shaped path (Fig. 11.7). Additional simulation experiments for this second trajectory tracking problem are concerned again with control of the AUV under disturbance forces and torques. The estimation of the disturbance forces and

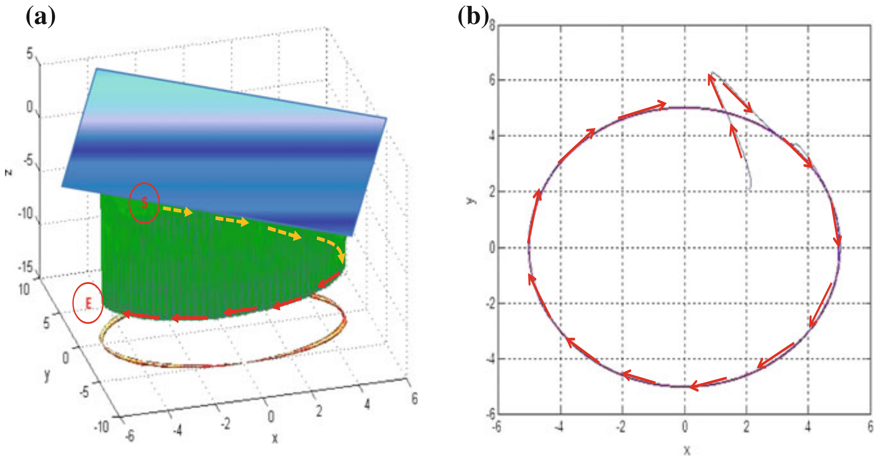

Fig. 11.2 Control of the 6-DOF AUV: **a** trajectory of the AUV in the cartesian space, **b** projection of the AUV's trajectory on the xy plane

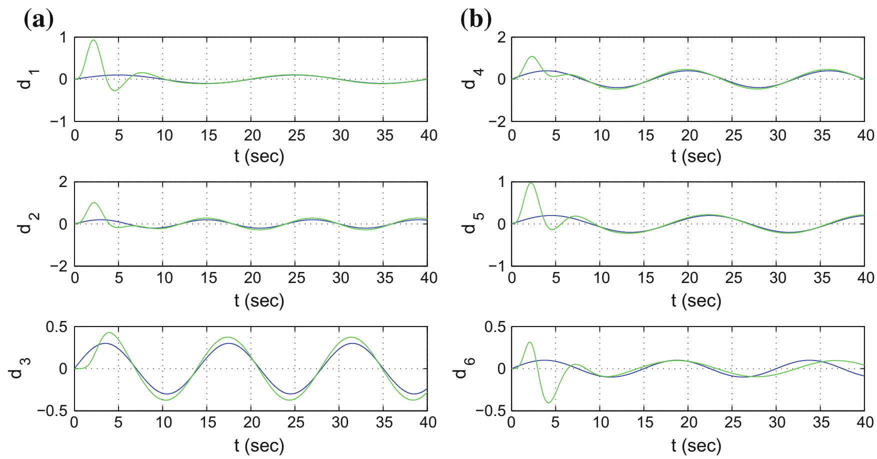

Fig. 11.3 Use of the Derivative-free nonlinear Kalman Filter in estimation of disturbances: **a** associated with linear motion, **b** associated with the rotational motion of the vehicle

torques is shown in Fig. 11.8. Moreover, as demonstrated in Figs. 11.9, 11.10 and 11.11, flatness-based control enabled accurate tracking of the reference trajectories for both the linear position and velocity variables and for the angular position and velocity variables (blue line: real value, green line estimated value, red line: setpoint).

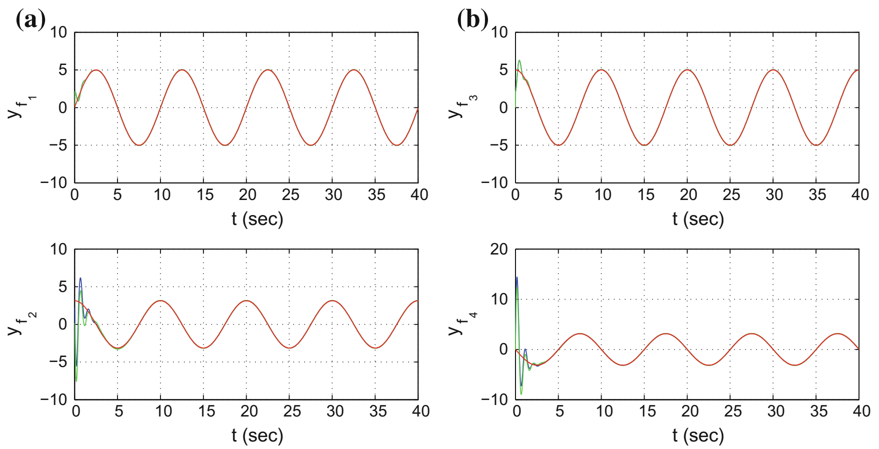

Fig. 11.4 Control of the AUV in the presence of external disturbances **a** position and velocity along the x axis, **b** position and velocity along the y axis

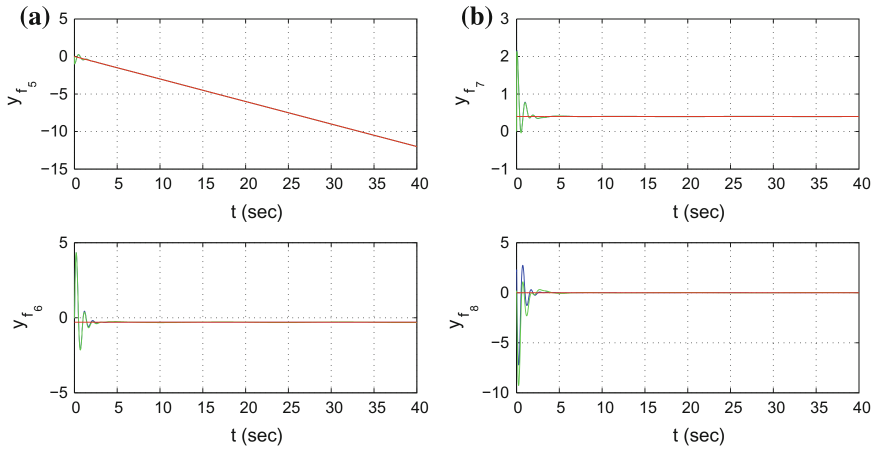

Fig. 11.5 Control of the AUV in the presence of external disturbances: **a** position and velocity along the z axis, **b** rotation angle ϕ and associated angular speed

11.3 Adaptive Fuzzy Control of Autonomous Submarines

11.3.1 Outline

Next, an adaptive control approach to the problem of control of Autonomous Underwater Vessels is presented, comprising both global linearization methods and Lyapunov stability analysis methods. The design of control systems for autonomous underwater vessels (AUVs) and submarines is a non-trivial problem because such

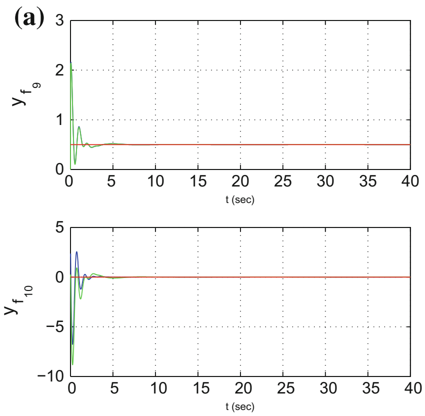

Fig. 11.6 Control of the AUV in the presence of disturbances: **a** rotation angle θ and associated angular speed, **b** rotation angle ψ and associated angular speed

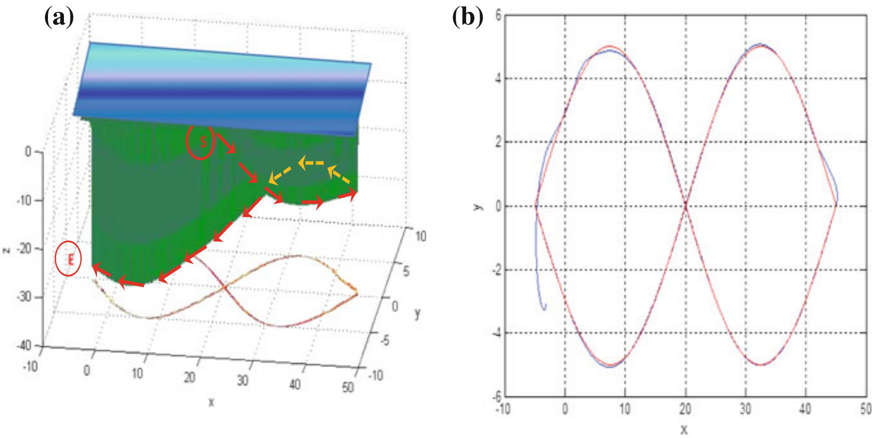

Fig. 11.7 Control of the 6-DOF AUV: **a** trajectory of the AUV in the cartesian space, **b** projection of the AUV's trajectory on the xy plane

systems exhibit a highly nonlinear multivariable dynamics with strong couplings between their inputs and outputs [128, 411, 516]. Besides, such systems function under variable conditions and thus their dynamic model is subject to parametric changes. Moreover, submersible autonomous robots and submarines are exposed to strong perturbations due to variable sea conditions and sea currents. Therefore, it is important to develop feedback control schemes for AUVs and submarines that will be little dependent on prior and exact knowledge of the associated dynamic model and will exhibit sufficient robustness to perturbation inputs [21, 143, 144, 191, 251, 386, 457, 522]. To this end, in the recent years several research results have been presented, in particular on robust control [253, 346, 518] and on adaptive control of AUVs [258, 375, 462, 635].

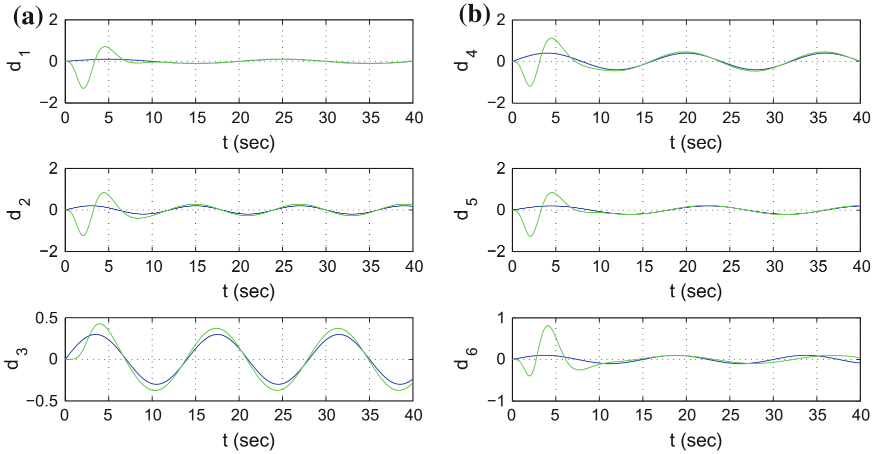

Fig. 11.8 Use of the Derivative-free nonlinear Kalman Filter in estimation of disturbances: **a** associated with linear motion, **b** associated with the rotational motion of the vehicle

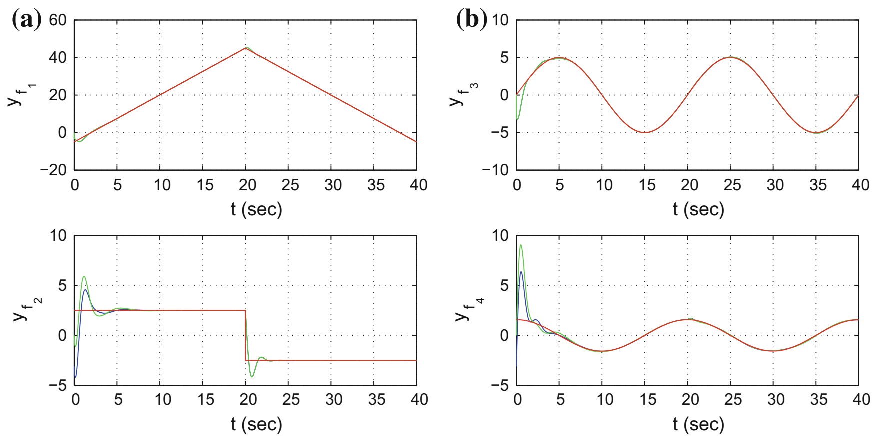

Fig. 11.9 Control of the AUV in the presence of external disturbances **a** position and velocity along the x axis, **b** position and velocity along the y axis

Adaptive fuzzy control methods can provide a solution to the problem of trajectory tracking and stabilization for autonomous submarines. As previously noted, adaptive fuzzy control schemes have been developed for unknown single-input single-output (SISO) and unknown multi-input multi-output (MIMO) dynamical systems. The capability of neurofuzzy controllers to compensate for model parametric uncertainties, external disturbances, as well as for incomplete measurement of the systems state vector has been analyzed in several studies [84, 89, 277, 524, 562]. Adaptive fuzzy control methods usually try to invert the systems dynamics, and thus to achieve convergence of its output to the desirable setpoints, starting from a description of the

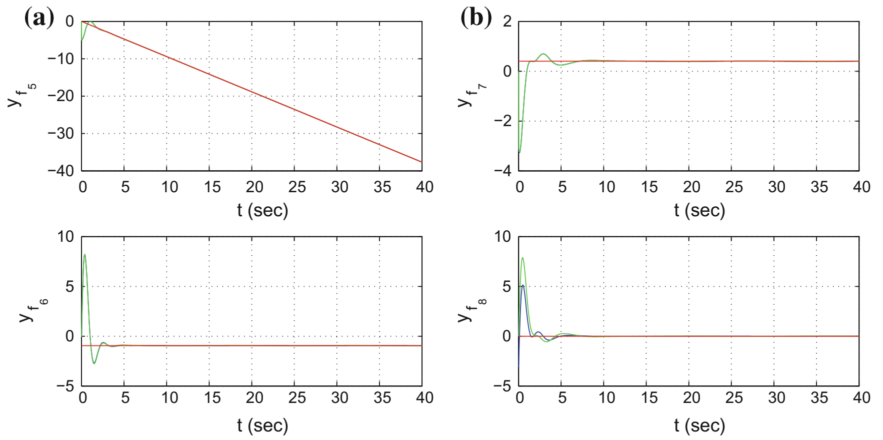

Fig. 11.10 Control of the AUV in the presence of external disturbances: **a** position and velocity along the z axis, **b** rotation angle ϕ and associated angular speed

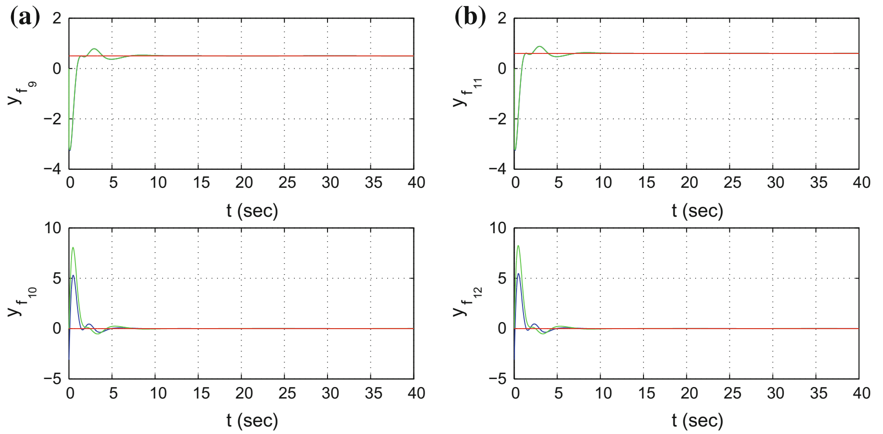

Fig. 11.11 Control of the AUV in the presence of disturbances: **a** rotation angle θ and associated angular speed, **b** rotation angle ψ and associated angular speed

system in the canonical form. On the other hand, differential flatness theory enables to transform the system's generic description $\dot{x} = f(x, u)$ into the canonical form and from that point on to develop adaptive control schemes. Consequently, differential flatness theory extends the class of nonlinear control systems to which adaptive neural / fuzzy control can be applied and this is a significant benefit for adaptive control theory [399, 457, 609, 617].

In this section, an adaptive control scheme is developed for autonomous submarines relying on differential flatness theory and on real-time identification of the unknown dynamics of the system with the use of neurofuzzy approximators

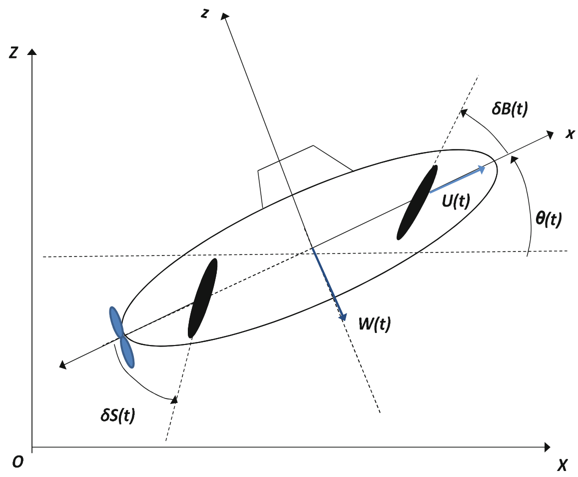

Fig. 11.12 Reference frames for the autonomous submarine

[450, 452]. First, it is proven that the dynamic model of the submersible vessel, comprising as state variables the submarine's depth and its pitch angle, is a differentially flat one. This means that all its state variables and its control inputs can be expressed as differential functions of a specific algebraic variable which is the so-called flat output. By exploiting the differential flatness properties of the submarine's model its transformation into the linear canonical (Brunovsky) form is accomplished. For the latter description of the system's dynamics the design of a MIMO state feedback controller becomes possible [57, 145, 254, 267, 322, 472, 476, 519]. In the transformed state-space model, the new control inputs of the submarine contain unknown nonlinear terms which are identified in real-time with the use of neuro-fuzzy approximators. The learning procedure for these estimators is determined by the requirement the first derivative of the closed-loop's Lyapunov function to be a negative one. Moreover, through Lyapunov stability analysis it is proven that the control system satisfies the H-infinity tracking conditions. This assures the control loop's robustness against model uncertainties and external perturbations. Finally, the efficiency of the submarine's control scheme is confirmed through simulation experiments.

11.3.2 The Dynamic Model of the Autonomous Submarine

The dive-plane nonlinear time-varying dynamic model of the submarine is considered (see. Fig. 11.12). The primary variables of this model are: (i) the diving speed along the vessel's z-axis (in a body-fixed frame), (ii) the pitch angle θ formed between the horizontal reference axis (in an inertial reference frame) and the x-axis of the vessel (in the body-fixed frame) [258].

The equations of motion of the vessel are:

Table 11.1 Parameters of the submarine's dynamic model

Parameter value	Parameter value	Parameter value
$Z_w^{'} = -0.0110$	$Z_{\dot{w}}^{'} = -0.0075$	$Z_{\theta}^{'} = -0.0045$
$Z_{\theta}^{'} = -0.0002$	$Z_{\delta B}^{'} = -0.0025$	$Z_{\delta S}^{'} = -0.0050$
$M_w^{'} = 0.0030$	$M_{\dot{w}}^{'} = -0.0002$	$M_{\theta}^{'} = -0.0025$
$M_{\dot{\theta}}^{'} = -0.0004$	$M_{\delta B}^{'} = 0.0005$	$M_{\delta S}^{'} = -0.0025$
$I_y^{'} = 5.6867{\cdot}10e^{-4}$	$L = 286\text{ft}$	$m = 1.52{\cdot}10^5\text{slug}$
$Z_g - Z_B = -1.5\text{ft}$	$U = 8.43\text{ft/s}$	$\rho = 2.0\ \text{slug/ft}^3$
$I_2^{'} = I_y^{'} - M_B^{'}$	$m = 2m/(\rho L^3))$	$m_3^{'} = m^{'} - Z_w^{'}$

$$\begin{aligned}\dot{w}(t) = \tfrac{Z_w^{'}U}{Lm_3^{'}}w(t) + \tfrac{1}{m_3^{'}}\dot{Z}_{\dot{\theta}}^{'} + m^{'})U\dot{\theta}(t) + \tfrac{Z_{\dot{Q}}^{'}L}{m_s^{'}}\dot{Q}(t) + \\ + \tfrac{Z_{\delta B}^{'}U^2}{m_3^{'}L}\delta B(t) + \tfrac{Z_{\delta S}^{'}U^2}{m_3^{'}L}\delta S(t) + \tfrac{Z_d(t)}{0.5\rho L^3 m_3^{'}} + Z_{\eta}(w,q)\end{aligned} \quad (11.40)$$

$$\begin{aligned}\dot{Q}(t) = \tfrac{M_{\dot{w}}^{'}}{LI_2^{'}}\dot{w}(t) + \tfrac{M_v^{'}U}{L^2I_2^{'}}w(t) + \tfrac{M_{\dot{\theta}}^{'}U}{LI_2^{'}}\dot{\theta}(t) + \\ + \tfrac{M_{\delta B}^{'}U^2}{L^2I_2^{'}}\delta B(t) + \tfrac{M_{\delta S}^{'}U^2}{L^2I_2^{'}}\delta S(t) + \tfrac{2mg(z_G - z_B)}{\rho L^5 I_2^{'}}\theta(t) + \tfrac{M_d(t)}{0.5\rho L^5 I_2^{'}} + M_{\eta}(w,q)\end{aligned} \quad (11.41)$$

In the above dynamic model of the submarine w is the velocity along the z-axis, h is the depth of the vessel measured in the inertial coordinates system, θ is the pitch angle, $Q = \dot{\theta}$ is the rate of change of the pitch angle, δB is the hydroplane deflection in the bow plane, δS is the hydroplane deflection in the stern plane, and Z_d, M_d are bounded disturbance inputs due to sea currents. Moreover, $Z_{\eta}(w,q)$, $M_{\eta}(w,q)$ are disturbance inputs representing the vessel's cross-flow drag (the latter is a function that contains the terms $w|w|$ and $Q|Q|$, as well as higher-order terms of w and Q).

Actually, for the computation of the mathematical model of the vessel the precise knowledge of the terms $Z_{\eta}(w,Q)$ and $M_{\eta}(w,Q)$ is not necessary since they can be treated by the adaptive control scheme as disturbances. The term $U = U_0$ denotes the x-axis (forward) velocity of the vessel (Table 11.1).

The dynamic model of the submarine is completed by the following coefficients, given in Table I [258]:

The control input of the submarine's model is described by the vector

$$u = [\delta B(t)\ \delta S(t)]^T \quad (11.42)$$

that is the control input consists of the hydroplane deflections in the bow and stern planes. A first description of the vessel's dynamics in matrix form is given by

$$\begin{pmatrix}\dot{w}\\ \dot{Q}\end{pmatrix} = \begin{pmatrix}f_W(w,\theta,Q,t)\\ f_{\theta}(w,\theta,Q,t)\end{pmatrix} + B_o u \quad (11.43)$$

where

$$\begin{pmatrix} f_w(w,\theta,Q,t) \\ f_\theta(f_W(w,\theta,Q,t) \end{pmatrix} = M^{-1}\cdot$$

$$\begin{pmatrix} \frac{Z_w^{'}U}{Lm_3^{'}}w(t) + \frac{1}{m_3^{'}}\dot{Z}_{\dot{\theta}}^{'} + m^{'})U\dot{\theta}(t) + \frac{Z_{\dot{Q}}^{'}L}{m_s^{'}}\dot{Q}(t) + \frac{Z_d(t)}{0.5\rho L^3 m_3^{'}} + Z_\eta(w,q) \\ \frac{M_{\dot{w}}^{'}}{LI_2^{'}}\dot{w}(t) + \frac{M_v^{'}U}{L^2 I_2^{'}}w(t) + \frac{M_{\dot{\theta}}^{'}U}{LI_2^{'}}\dot{\theta}(t) + \frac{2mg(z_G - z_B)}{\rho L^5 I_2^{'}}\theta(t) + \frac{M_d(t)}{0.5\rho L^5 I_2^{'}} + M_\eta(w,q) \end{pmatrix} \tag{11.44}$$

while for matrices M and B_o it holds

$$M = \begin{pmatrix} 1 & -Z_{\dot{Q}}L/m_3^{'} \\ -M_{\ddot{w}}(LI_2^{'-1}) & 1 \end{pmatrix} \; B_o = \begin{pmatrix} \frac{Z_{\delta B}^{'}U^2}{m_3^{'}L} & \frac{Z_{\delta S}^{'}U^2}{m_3^{'}L} \\ \frac{M^{'}\delta BU^2}{L^2 I_2^{'}} & \frac{M^{'}\delta SU^2}{L^2 I_2^{'}} \end{pmatrix} \tag{11.45}$$

It holds that the depth of the vessel measured in the inertial reference frame and the velocity of the submarine along the z-axis of the body-fixed frame are related as follows:

$$\begin{aligned} \dot{h} &= wcos(\theta) - U_o sin(\theta) \Rightarrow \\ \ddot{h} &= \dot{w}cos(\theta) - wsin(\theta)\dot{\theta} - U_o cos(\theta)\dot{\theta} \Rightarrow \\ \ddot{h} &= \dot{w}cos(\theta) - wQsin(\theta) - U_o Qcos(\theta) \end{aligned} \tag{11.46}$$

Moreover, solving with respect to w. from the first row of Eq. (11.46) one obtains

$$w = (cos(\theta)^{-1})(\dot{h} + U_o sin(\theta)) \tag{11.47}$$

Additionally, from Eq. (11.43) one gets

$$\begin{aligned} \dot{w} &= f_w(w,\theta,Q,t) + B_{o_{11}}u_1 + B_{o_{12}}u_2 \\ \dot{Q} &= f_\theta(w,\theta,Q,t) + B_{o_{21}}u_1 + B_{o_{22}}u_2 \end{aligned} \tag{11.48}$$

Substituting Eq. (11.47) and the first row of Eq. (11.48) into the third row of Eq. (11.46) gives

$$\ddot{h} = [f_W(w,\theta,Q,t) + B_{o_{11}}u_1 + B_{o_{12}}u_2]cos(\theta) - \frac{(\dot{h} + U_0 sin(\theta))}{cos(\theta)}Qsin(\theta) - U_0 Qcos(\theta) \tag{11.49}$$

Next by denoting

$$\begin{aligned} f_w(w,\theta,Q,t) &= g_h(h,\dot{h},\theta,\dot{\theta},t) \\ f_\theta(w,\theta,Q,t) &= g_\theta(h,\dot{h},\theta,\dot{\theta},t) \end{aligned} \tag{11.50}$$

from Eq. (11.49) and the second row of Eq. (11.48) one obtains

$$\ddot{h} = g_h(h, \dot{h}, \theta, \dot{\theta}, t)cos(\theta) - \frac{(\dot{h}+U_0 sin(\theta))}{cos(\theta)}\dot{\theta}sin(\theta) - U_0\dot{\theta}cos(\theta) + \\ +B_{0_{11}}cos(\theta)u_1 + B_{0_{12}}cos(\theta)u_2 \tag{11.51}$$

$$\ddot{\theta} = g_\theta(h, \dot{h}, \theta, \dot{\theta}, t) + B_{0_{21}}u_1 + B_{0_{22}}u_2 \tag{11.52}$$

Thus, from Eqs. (11.51) and (11.52) and by defining the state vector

$$x = [h, \dot{h}, \theta, \dot{\theta}]^T \tag{11.53}$$

the dynamic model of the submarine is written as

$$\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_3 \end{pmatrix} = \begin{pmatrix} g_b(x,t)cos(x_3) - \frac{x_4+U_0 sin(x_3)}{cos(x_3)}x_4 sin(x_3) - U_0 x_4 cos(x_3) \\ g_\theta(x,t) \end{pmatrix} + \\ + \begin{pmatrix} B_{0_{11}} & B_{0_{12}} \\ B_{0_{21}} & B_{0_{22}} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \tag{11.54}$$

or equivalently in the MIMO form

$$\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_3 \end{pmatrix} = \begin{pmatrix} f_1(x,t) \\ f_2(x,t) \end{pmatrix} + \begin{pmatrix} g_{11}(x,t) & g_{12}(x,t) \\ g_{21}(x,t) & g_{22}(x,t) \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \tag{11.55}$$

11.3.3 Estimation of the Submarine's Unknown Dynamics

11.3.3.1 Differential Flatness of the Submarine's Model

It can be proven that the submarine's MIMO nonlinear model given in Eq. (11.55) is a differentially flat one, having as flat output the vector

$$y = [x_1, x_3]^T = [h, \theta]^T \tag{11.56}$$

As explained above it holds that $x_2 = \dot{x}_1$ and $x_4 = \dot{x}_3$, which also means

$$x_2 = [1\ 0]\dot{y} \\ x_4 = [0\ 1]\dot{y} \tag{11.57}$$

Moreover, by solving Eq. (11.55) with respect to the control input u one obtains

$$\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} g_{11}(x) & g_{12}(x) \\ g_{21}(x) & g_{22}(x) \end{pmatrix}^{-1} (\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} - \begin{pmatrix} f_1(x) \\ f_2(x) \end{pmatrix}) \tag{11.58}$$

and since the elements of the state vector x are functions of the flat output, one has $u_1 = f_a(y, \dot{y}, \ddot{y})$ and $u_2 = f_b(y, \dot{y}, \ddot{y})$. Therefore, one finally has that all elements of the submarine's state vector and the control inputs can be written as functions of the flat output and its derivatives [57, 145, 254, 267, 322, 472, 476, 519]. Consequently, the system is a differentially flat one.

By exploiting the differentially flat description of the system. the submarine's model can be written in the linear canonical (Brunovsky) form. To this end the following transformed control inputs are defined

$$
\begin{aligned}
v_1 &= f_1(x,t) + g_{11}u_1 + g_{12}u_2 \\
v_2 &= f_2(x,t) + g_{21}u_1 + g_{22}u_2
\end{aligned} \tag{11.59}
$$

Therefore, one gets

$$
\begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \dot{y}_3 \\ \dot{y}_4 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \tag{11.60}
$$

while it is considered that the complete state vector of the submarine $y = [h, \dot{h}, \theta, \dot{\theta}]$ is available through measurements.

11.3.3.2 Approximation of the Submarine's Unknown Dynamics

The control signal of the MIMO nonlinear system which has been transformed into the Brunovsky form as described by Eq. (11.60) contains the unknown nonlinear functions $f(x)$ and $g(x)$ which can be approximated by

$$
\begin{aligned}
\hat{f}(x|\theta_f) &= \Phi_f(x)\theta_f \\
\hat{g}(x|\theta_g) &= \Phi_g(x)\theta_g
\end{aligned} \tag{11.61}
$$

where

$$
\Phi_f(x) = (\xi_f^1(x), \xi_f^2(x), \cdots \xi_f^n(x))^T \tag{11.62}
$$

with $\xi_f^i(x), \imath = 1, \cdots, n$ being the vector of kernel functions (e.g. normalized fuzzy Gaussian membership functions), where

$$
\xi_f^i(x) = (\phi_f^{i,1}(x), \phi_f^{i,2}(x), \cdots, \phi_f^{i,N}(x)) \tag{11.63}
$$

thus giving

$$\Phi_f(x) = \begin{pmatrix} \phi_f^{1,1}(x) & \phi_f^{1,2}(x) & \cdots & \phi_f^{1,N}(x) \\ \phi_f^{2,1}(x) & \phi_f^{2,2}(x) & \cdots & \phi_f^{2,N}(x) \\ \cdots & \cdots & \cdots & \cdots \\ \phi_f^{n,1}(x) & \phi_f^{n,2}(x) & \cdots & \phi_f^{n,N}(x) \end{pmatrix} \tag{11.64}$$

while the weights vector is defined as

$$\theta_f{}^T = \left(\theta_f^1, \theta_f^2, \cdots \theta_f^N\right) \tag{11.65}$$

$j = 1, \cdots, N$ is the number of basis functions that is used to approximate the components of function f which are denoted as $i = 1, \cdots, n$. Thus, one obtains the relation of Eq. (11.61), i.e. $\hat{f}(x|\theta_f) = \Phi_f(x)\theta_f$.

In a similar manner, for the approximation of function g one has

$$\Phi_g(x) = \left(\xi_g^1(x), \xi_g^2(x), \cdots \xi_g^N(x)\right)^T \tag{11.66}$$

with $\xi_g^i(x), \imath = 1, \cdots, N$ being the vector of kernel functions (e.g. normalized fuzzy Gaussian membership functions), where

$$\xi_g^i(x) = \left(\phi_g^{i,1}(x), \phi_g^{i,2}(x), \cdots, \phi_g^{i,N}(x)\right) \tag{11.67}$$

thus giving

$$\Phi_g(x) = \begin{pmatrix} \phi_g^{1,1}(x) & \phi_g^{1,2}(x) & \cdots & \phi_g^{1,N}(x) \\ \phi_g^{2,1}(x) & \phi_g^{2,2}(x) & \cdots & \phi_g^{2,N}(x) \\ \cdots & \cdots & \cdots & \cdots \\ \phi_g^{n,1}(x) & \phi_g^{n,2}(x) & \cdots & \phi_g^{n,N}(x) \end{pmatrix} \tag{11.68}$$

while the weights vector is defined as

$$\theta_g = \left(\theta_g^1, \theta_g^2, \cdots, \theta_g^p\right)^T \tag{11.69}$$

where the components of matrix θ_g are defined as

$$\theta_g^j = \left(\theta_{g_1}^j, \theta_{g_2}^j, \cdots \theta_{g_N}^j\right) \tag{11.70}$$

$j = 1, \cdots, N$ is the number of basis functions that is used to approximate the components of function g which are denoted as $i = 1, \cdots, n$. Thus one obtains about matrix $\theta_g \in R^{N \times p}$

$$\theta_g = \begin{pmatrix} \theta_{g_1}^1 & \theta_{g_1}^2 & \cdots & \theta_{g_1}^p \\ \theta_{g_2}^1 & \theta_{g_2}^2 & \cdots & \theta_{g_2}^p \\ \cdots & \cdots & \cdots & \cdots \\ \theta_{g_N}^1 & \theta_{g_N}^2 & \cdots & \theta_{g_N}^p \end{pmatrix} \tag{11.71}$$

It holds that

$$g = \begin{pmatrix} g_1 \\ g_2 \\ \cdots \\ g_n \end{pmatrix} = \begin{pmatrix} g_1^1 & g_1^2 & \cdots & g_1^p \\ g_2^1 & g_2^2 & \cdots & g_2^p \\ \cdots & \cdots & \cdots & \cdots \\ g_n^1 & g_n^2 & \cdots & g_n^p \end{pmatrix} \tag{11.72}$$

Using the above, one finally has the relation of Eq. (11.61), i.e. $\hat{g}(x|\theta_g) = \Phi_g(x)\theta_g$. If the state variables of the system are available for measurement then a state-feedback control law can be formulated as

$$u = \hat{g}^{-1}(x|\theta_g)[-\hat{f}(x|\theta_f) + y_m^{(r)} - K^T e + u_c] \tag{11.73}$$

where $\hat{f}(x|\theta_f)$ and $\hat{g}(x|\theta_g)$ are fuzzy models to approximate $f(x)$ and $g(x)$, respectively. u_c is a supervisory control term, e.g. H_∞ control term that is used to compensate for the effects of modelling inaccuracies and external disturbances. Using the submarine's state-space description of Eq. (11.60) the control term u_c is defined as

$$u_c = -\tfrac{1}{r} B^T P e \tag{11.74}$$

Moreover, K^T is the feedback gain matrix that assures that the characteristic polynomial of the resulting closed-loop dynamics will be a Hurwitz one.

11.3.4 Flatness-Based Adaptive Fuzzy Control of the Submarine Dynamics

Next, taking into account also the effects of additive disturbances to the submarine the dynamic model of Eq. (11.55) becomes

$$\begin{aligned} \ddot{x}_1 &= f_1(x,t) + g_1(x,t)u + \tilde{d}_1 \\ \ddot{x}_3 &= f_2(x,t) + g_2(x,t)u + \tilde{d}_2 \end{aligned} \tag{11.75}$$

or, in matrix form

$$\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_3 \end{pmatrix} = \begin{pmatrix} f_1(x,t) \\ f_2(x,t) \end{pmatrix} + \begin{pmatrix} g_1(x,t) \\ g_2(x,t) \end{pmatrix} u + \begin{pmatrix} \tilde{d}_1 \\ \tilde{d}_2 \end{pmatrix} \tag{11.76}$$

The following control input is defined

$$u = \begin{pmatrix} \hat{g}_1(x,t) \\ \hat{g}_2(x,t) \end{pmatrix}^{-1} \cdot \{ \begin{pmatrix} \ddot{x}_1^d \\ \ddot{x}_3^d \end{pmatrix} - \begin{pmatrix} \hat{f}_1(x,t) \\ \hat{f}_2(x,t) \end{pmatrix} - \begin{pmatrix} K_1^T \\ K_2^T \end{pmatrix} e + \begin{pmatrix} u_{c_1} \\ u_{c_2} \end{pmatrix} \} \tag{11.77}$$

where $[u_{c_1} \ u_{c_2}]^T$ is a robust control term that is used for the compensation of the model's uncertainties as well as of the external disturbances and the vector of the feedback gain is $K_i^T = [k_1^i, k_2^i, \cdots, k_{n-1}^i, k_n^i]$.

Substituting Eqs. (11.77) into (11.76) the closed-loop tracking error dynamics of the submarine is obtained

$$\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_3 \end{pmatrix} = \begin{pmatrix} f_1(x,t) \\ f_2(x,t) \end{pmatrix} + \begin{pmatrix} g_1(x,t) \\ g_2(x,t) \end{pmatrix} \begin{pmatrix} \hat{g}_1(x,t) \\ \hat{g}_2(x,t) \end{pmatrix}^{-1} \cdot$$
$$\{ \begin{pmatrix} \ddot{x}_1^d \\ \ddot{x}_3^d \end{pmatrix} - \begin{pmatrix} \hat{f}_1(x,t) \\ \hat{f}_2(x,t) \end{pmatrix} - \begin{pmatrix} K_1^T \\ K_2^T \end{pmatrix} e + \begin{pmatrix} u_{c_1} \\ u_{c_2} \end{pmatrix} \} + \begin{pmatrix} \tilde{d}_1 \\ \tilde{d}_2 \end{pmatrix} \tag{11.78}$$

Equation (11.78) can now be written as

$$\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_3 \end{pmatrix} = \begin{pmatrix} f_1(x,t) \\ f_2(x,t) \end{pmatrix} +$$
$$+ \{ \begin{pmatrix} g_1(x,t) - \hat{g}_1(x,t) \\ g_2(x,t) - \hat{g}_2(x,t) \end{pmatrix} + \begin{pmatrix} \hat{g}_1(x,t) \\ \hat{g}_2(x,t) \end{pmatrix} \} \begin{pmatrix} \hat{g}_1(x,t) \\ \hat{g}_2(x,t) \end{pmatrix}^{-1} \cdot \tag{11.79}$$
$$\cdot \{ \begin{pmatrix} \ddot{x}_1^d \\ \ddot{x}_3^d \end{pmatrix} - \begin{pmatrix} \hat{f}_1(x,t) \\ \hat{f}_2(x,t) \end{pmatrix} - \begin{pmatrix} K_1^T \\ K_2^T \end{pmatrix} e + \begin{pmatrix} u_{c_1} \\ u_{c_2} \end{pmatrix} \} + \begin{pmatrix} \tilde{d}_1 \\ \tilde{d}_2 \end{pmatrix}$$

and using Eq. (11.77) this results into

$$\begin{pmatrix} \ddot{e}_1 \\ \ddot{e}_3 \end{pmatrix} = \begin{pmatrix} f_1(x,t) - \hat{f}_1(x,t) \\ f_2(x,t) - \hat{f}_2(x,t) \end{pmatrix} + \begin{pmatrix} g_1(x,t) - \hat{g}_1(x,t) \\ g_2(x,t) - \hat{g}_2(x,t) \end{pmatrix} u -$$
$$- \begin{pmatrix} K_1^T \\ K_2^T \end{pmatrix} e + \begin{pmatrix} u_{c_1} \\ u_{c_2} \end{pmatrix} + \begin{pmatrix} \tilde{d}_1 \\ \tilde{d}_2 \end{pmatrix} \tag{11.80}$$

The following description for the approximation error is defined

$$w = \begin{pmatrix} f_1(x,t) - \hat{f}_1(x,t) \\ f_2(x,t) - \hat{f}_2(x,t) \end{pmatrix} + \begin{pmatrix} g_1(x,t) - \hat{g}_1(x,t) \\ g_2(x,t) - \hat{g}_2(x,t) \end{pmatrix} u \tag{11.81}$$

Moreover, the following matrices are defined

$$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \tag{11.82}$$

$$K^T = \begin{pmatrix} K_1^1 & K_2^1 & K_3^1 & K_4^1 \\ K_1^2 & K_2^2 & K_3^2 & K_4^2 \end{pmatrix}$$

Using matrices A, B, K^T, Eq. (11.80) is written in the following form

$$\dot{e} = (A - BK^T)e + Bu_c + B\{\begin{pmatrix} f_1(x,t) - \hat{f}_1(x,t) \\ f_2(x,t) - \hat{f}_2(x,t) \end{pmatrix} + \\ + \begin{pmatrix} g_1(x,t) - \hat{g}_1(x,t) \\ g_2(x,t) - \hat{g}_2(x,t) \end{pmatrix} u + \tilde{d}\} \tag{11.83}$$

Next, the following approximators of the unknown system dynamics are defined

$$\hat{f}(x) = \begin{pmatrix} \hat{f}_1(x|\theta_f) \; x \in R^{4\times 1} \; \hat{f}_1(x|\theta_f) \; \in \; R^{1\times 1} \\ \hat{f}_2(x|\theta_f) \; x \in R^{4\times 1} \; \hat{f}_2(x|\theta_f) \; \in \; R^{1\times 1} \end{pmatrix} \tag{11.84}$$

with kernel functions

$$\phi_f^{i,j}(x) = \frac{\prod_{j=1}^n \mu_{A_j}^i(x_j)}{\sum_{i=1}^N \prod_{j=1}^n \mu_{A_j}^i(x_j)} \tag{11.85}$$

where $l = 1, 2$ and $\mu_{A_j^i}(x)$ is the i-th membership function of the antecedent (IF) part of the l-th fuzzy rule. Similarly, the following approximators of the unknown system dynamics are defined

$$\hat{g}(x) = \begin{pmatrix} \hat{g}_1(x|\theta_g) \; x \in R^{4\times 1} \; \hat{g}_1(x|\theta_g) \; \in \; R^{1\times 2} \\ \hat{g}_2(x|\theta_g) \; x \in R^{4\times 1} \; \hat{g}_2(x|\theta_g) \; \in \; R^{1\times 2} \end{pmatrix} \tag{11.86}$$

The values of the weights that result in optimal approximation are

$$\begin{matrix} \theta_f^* = arg\, min_{\theta_f \in M_{\theta_f}} [sup_{x \in U_x}(f(x) - \hat{f}(x|\theta_f))] \\ \theta_g^* = arg\, min_{\theta_g \in M_{\theta_g}} [sup_{x \in U_x}(g(x) - \hat{g}(x|\theta_g))] \end{matrix} \tag{11.87}$$

where the variation ranges for the weights are defined as

$$\begin{matrix} M_{\theta_f} = \{\theta_f \in R^h : \; ||\theta_f|| \leq m_{\theta_f}\} \\ M_{\theta_g} = \{\theta_g \in R^h : \; ||\theta_g|| \leq m_{\theta_g}\} \end{matrix} \tag{11.88}$$

For the value of the approximation error defined in Eq. (11.81) that corresponds to the optimal values of the weights vectors θ_f^* and θ_g^* one has

$$w=\left(f(x,t)-\hat{f}(x|\theta_f^*)\right)+\left(g(x,t)-\hat{g}(x|\theta_g^*)\right)u \tag{11.89}$$

which is next written as

$$\begin{array}{c} w=\left(f(x,t)-\hat{f}(x|\theta_f)+\hat{f}(x|\theta_f)-\hat{f}(x|\theta_f^*)\right)+ \\ +\left(g(x,t)-\hat{g}(x|\theta_g)+\hat{g}(x|\theta_g)-\hat{g}(x|\theta_g^*)\right)u \end{array} \tag{11.90}$$

which can be also written in the following form

$$w=\left(w_a+w_b\right) \tag{11.91}$$

where

$$w_a=\{[\hat{f}(x,\theta_f)-\hat{f}(x|\theta_f^*)]+[\hat{g}(x,\theta_g)-\hat{g}(x|\theta_g^*)]\}\cdot u \tag{11.92}$$

$$w_b=\{[f(x,t)-\hat{f}(x|\theta_f)]+[g(x,t)-\hat{g}(x|\theta_g)]\}u \tag{11.93}$$

Moreover, the following weights error vectors are defined

$$\begin{array}{c} \tilde{\theta}_f=\theta_f-\theta_f^* \\ \tilde{\theta}_g=\theta_g-\theta_g^* \end{array} \tag{11.94}$$

Following the previous analysis it is pointed out that the use of differential flatness theory enables to solve the problem of control of the nonlinear dynamics of the autonomous submarine in a conclusive manner: (i) by showing that a dynamical system is differentially flat it is possible to express its dynamics through specific primary variables which are the so-called flat outputs. All state variables of the system can be written as differential functions of the flat outputs, (ii) by showing that a dynamical system is differentially flat it can be assured that its transformation to the linear canonical (Brunovsky) form can be achieved, (iii) by expressing a differentially flat system into its equivalent linearized form the design of a state feedback controller for it can be completed in a few stages.

11.3.5 Lyapunov Stability Analysis

The following quadratic Lyapunov function is defined for the control loop of the autonomous submarine

$$V=\frac{1}{2}e^TPe+\frac{1}{2\gamma_1}\tilde{\theta}_f^T\tilde{\theta}_f+\frac{1}{2\gamma_2}tr[\tilde{\theta}_g^T\tilde{\theta}_g] \tag{11.95}$$

Parameter γ_1 is the learning rate used in the adaptation of the weights of the neurofuzzy approximator for $f(x)$, while parameter γ_2 is the learning rate used in the adaptation of the weights of the neurofuzzy approximation for $g(x)$. It holds that

$$\dot{V} = \frac{1}{2}\dot{e}^T Pe + \frac{1}{2}e^T P\dot{e} + \frac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T\tilde{\theta}_f + \frac{1}{\gamma_2}tr[\dot{\tilde{\theta}}_g^T\tilde{\theta}_g] \tag{11.96}$$

The tracking error dynamics is described by

$$\begin{aligned}\dot{e} = (A - BK^T)e + Bu_c + B\{\begin{pmatrix} f_1(x,t) - \hat{f}_1(x,t) \\ f_2(x,t) - \hat{f}_2(x,t)\end{pmatrix}\}+ \\ + \begin{pmatrix} g_1(x,t) - \hat{g}_1(x,t) \\ g_2(x,t) - \hat{g}_2(x,t)\end{pmatrix} u + \tilde{d}\}\end{aligned} \tag{11.97}$$

and defining the approximation error

$$w = \begin{pmatrix} f_1(x,t) - \hat{f}_1(x,t) \\ f_2(x,t) - \hat{f}_2(x,t)\end{pmatrix} + \begin{pmatrix} g_1(x,t) - \hat{g}_1(x,t) \\ g_2(x,t) - \hat{g}_2(x,t)\end{pmatrix} u \tag{11.98}$$

the previous relation can be also written as

$$\dot{e} = (A - BK^T)e + Bu_c + B(w + \tilde{d}) \tag{11.99}$$

From Eq. (11.96) one obtains

$$\begin{aligned}\dot{V} = \frac{1}{2}\{e^T(A - BK^T)^T + u_c^T B^T + \\ +(w + \tilde{d})^T B^T\}Pe + \frac{1}{2}e^T P\{(A - BK^T)e+ \\ +Bu_c + B(w + \tilde{d})\} + \frac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T\tilde{\theta}_f + \frac{1}{\gamma_2}tr[\dot{\tilde{\theta}}_g^T\tilde{\theta}_g]\end{aligned} \tag{11.100}$$

which in turn gives

$$\begin{aligned}\dot{V} = \frac{1}{2}e^T\{(A - BK^T)^T P + P(A - BK^T)\}e+ \\ \frac{1}{2}2e^T PBu_c + \frac{1}{2}2B^T Pe(w + \tilde{d})+ \\ +\frac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T\tilde{\theta}_f + \frac{1}{\gamma_2}tr[\dot{\tilde{\theta}}_g^T\tilde{\theta}_g]\end{aligned} \tag{11.101}$$

Assumption 1: For given positive definite matrix Q there exists a positive definite matrix P, which is the solution of the following matrix equation

$$\begin{aligned}(A - BK^T)^T P + P(A - BK^T)- \\ -PB(\frac{2}{r} - \frac{1}{\rho^2})B^T P + Q = 0\end{aligned} \tag{11.102}$$

Substituting Eqs. (11.102) and (11.74) into $\dot{V}$ yields after some operations

$$\begin{array}{c}\dot{V}=\frac{1}{2}e^T\{-Q+PB(\frac{2}{r}-\frac{1}{\rho^2})B^TP\}e+\\ e^TPB\{-\frac{1}{r}B^TPe\}+B^TP(w+\tilde{d})+\frac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T\tilde{\theta}_f+\frac{1}{\gamma_2}tr[\dot{\tilde{\theta}}_g^T\tilde{\theta}_g]\end{array} \tag{11.103}$$

Therefore it holds

$$\begin{array}{c}\dot{V}=-\frac{1}{2}e^TQe-\frac{1}{2\rho^2}e^TPBB^TPe+e^TPB(w+\tilde{d})+\\ \frac{1}{\gamma_1}\dot{\tilde{\theta}}_f^T\tilde{\theta}_f+\frac{1}{\gamma_2}tr[\dot{\tilde{\theta}}_g^T\tilde{\theta}_g]\end{array} \tag{11.104}$$

It also holds that

$$\begin{array}{c}\dot{\tilde{\theta}}_f=\dot{\theta}_f-\dot{\theta}_f^*=\dot{\theta}_f\\ \dot{\tilde{\theta}}_g=\dot{\theta}_g-\dot{\theta}_g^*=\dot{\theta}_g\end{array} \tag{11.105}$$

The following weights adaptation law is used (Fig. 11.13)

$$\begin{array}{c}\dot{\theta}_f=-\gamma_1\Phi(x)^TB^TPe\\ \dot{\theta}_g=-\gamma_2\Phi(x)^TB^TPeu^T\end{array} \tag{11.106}$$

This is a gradient-type learning method for the weights of the neurofuzzy approximators [33, 431, 463]. Assuming N fuzzy rules and associated kernel functions the matrices dimensions are $\theta_f{\in}R^{N\times 1}$, $\theta_g{\in}R^{N\times 2}$, $\Phi(x){\in}R^{2\times N}$, $B{\in}R^{4\times 2}$, $P{\in}R^{4\times 4}$ and $e{\in}R^{4\times 1}$. Therefore it holds

$$\begin{array}{c}\dot{V}=-\frac{1}{2}e^TQe-\frac{1}{2\rho^2}e^TPBB^TPe+e^TPB(w+\tilde{d})+\\ +\frac{1}{\gamma_1}(-\gamma_1)e^TPB\Phi(x)(\theta_f-\theta_f^*)+\\ +\frac{1}{\gamma_2}(-\gamma_2)tr[ue^TPB\Phi(x)(\theta_g-\theta_g^*)]\end{array} \tag{11.107}$$

or

$$\begin{array}{c}\dot{V}=-\frac{1}{2}e^TQe-\frac{1}{2\rho^2}e^TPBB^TPe+e^TPB(w+\tilde{d})+\\ +\frac{1}{\gamma_1}(-\gamma_1)e^TPB\Phi(x)(\theta_f-\theta_f^*)+\\ +\frac{1}{\gamma_2}(-\gamma_2)tr[ue^TPB(\hat{g}(x|\theta_g)-\hat{g}(x|\theta_g^*)]\end{array} \tag{11.108}$$

Taking into account that $u\in R^{2\times 1}$ and $e^TPB(\hat{g}(x|\theta_g)-\hat{g}(x|\theta_g^*))\in R^{1\times 2}$ it holds

$$\begin{array}{c}\dot{V}=-\frac{1}{2}e^TQe-\frac{1}{2\rho^2}e^TPBB^TPe+e^TPB(w+\tilde{d})+\\ +\frac{1}{\gamma_1}(-\gamma_1)e^TPB\Phi(x)(\theta_f-\theta_f^*)+\\ +\frac{1}{\gamma_2}(-\gamma_2)tr[e^TPB(\hat{g}(x|\theta_g)-\hat{g}(x|\theta_g^*))u]\end{array} \tag{11.109}$$

Since $e^TPB(\hat{g}(x|\theta_g)-\hat{g}(x|\theta_g^*))u{\in}R^{1\times 1}$ it holds

$$\begin{array}{c}tr(e^TPB(\hat{g}(x|\theta_g)-\hat{g}(x|\theta_g^*)u)=\\ =e^TPB(\hat{g}(x|\theta_g)-\hat{g}(x|\theta_g^*))u\end{array} \tag{11.110}$$

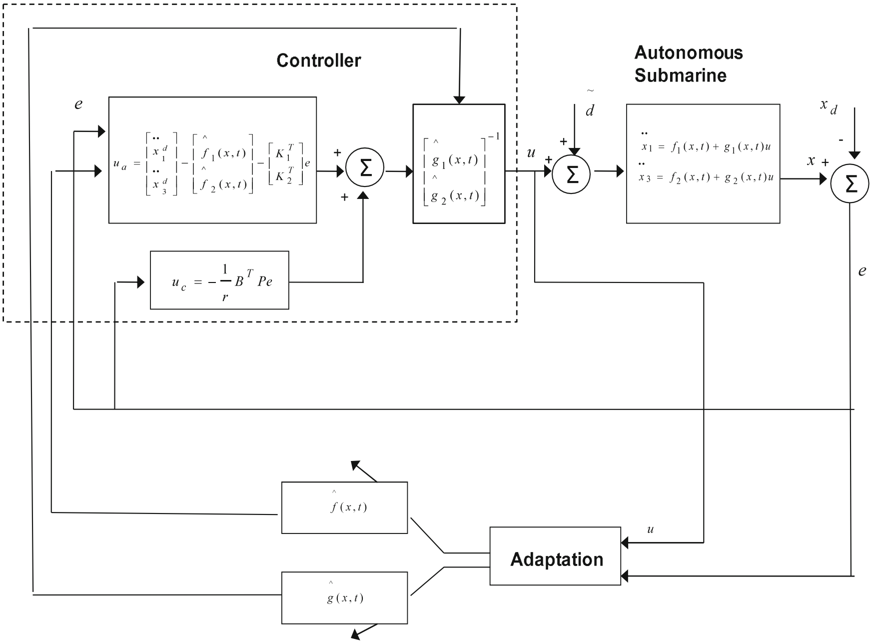

Fig. 11.13 Diagram of the flatness-based adaptive fuzzy controller for the autonomous submarine

Therefore, one finally obtains

$$\begin{aligned}\dot{V} = -\tfrac{1}{2}e^T Qe - \tfrac{1}{2\rho^2}e^T PBB^T Pe + e^T PB(w+\tilde{d})+ \\ +\tfrac{1}{\gamma_1}(-\gamma_1)e^T PB\Phi(x)(\theta_f - \theta_f^*)+ \\ +\tfrac{1}{\gamma_2}(-\gamma_2)e^T PB(\hat{g}(x|\theta_g) - \hat{g}(x|\theta_g^*))u\end{aligned} \tag{11.111}$$

Next the following approximation error is defined

$$w_\alpha = [\hat{f}(x|\theta_f) - \hat{f}(x|\theta_f^*)] + [\hat{g}(x|\theta_g) - \hat{g}(x|\theta_g^*)]u \tag{11.112}$$

Thus, one obtains

$$\begin{aligned}\dot{V} = -\tfrac{1}{2}e^T Qe - \tfrac{1}{2\rho^2}e^T PBB^T Pe+ \\ +e^T PB(w+\tilde{d}) + e^T PBw_\alpha\end{aligned} \tag{11.113}$$

Denoting the aggregate approximation error and disturbances vector as

$$w_1 = w + \tilde{d} + w_\alpha \tag{11.114}$$

the derivative of the Lyapunov function becomes

$$\dot{V} = -\tfrac{1}{2}e^T Qe - \tfrac{1}{2\rho^2}e^T PBB^T Pe + e^T PBw_1 \tag{11.115}$$

which in turn is written as

$$\begin{aligned}\dot{V} = -\tfrac{1}{2}e^T Qe - \tfrac{1}{2\rho^2}e^T PBB^T Pe+ \\ +\tfrac{1}{2}e^T PBw_1 + \tfrac{1}{2}w_1^T B^T Pe\end{aligned} \tag{11.116}$$

Next, the following Lemma is introduced:

Lemma: The inequality given below holds:

$$\tfrac{1}{2}e^T PBw_1 + \tfrac{1}{2}w_1^T B^T Pe - \tfrac{1}{2\rho^2}e^T PBB^T Pe \leq \tfrac{1}{2}\rho^2 w_1^T w_1 \tag{11.117}$$

Proof: The binomial $(\rho a - \frac{1}{\rho}b)^2 \geq 0$ is considered. Expanding the left part of the above inequality one gets

$$\begin{aligned}\rho^2 a^2 + \tfrac{1}{\rho^2}b^2 - 2ab \geq 0 \Rightarrow \tfrac{1}{2}\rho^2 a^2 + \tfrac{1}{2\rho^2}b^2 - ab \geq 0 \Rightarrow \\ ab - \tfrac{1}{2\rho^2}b^2 \leq \tfrac{1}{2}\rho^2 a^2 \Rightarrow \tfrac{1}{2}ab + \tfrac{1}{2}ab - \tfrac{1}{2\rho^2}b^2 \leq \tfrac{1}{2}\rho^2 a^2\end{aligned} \tag{11.118}$$

The following substitutions are carried out: $a = w_1$ and $b = e^T PB$ and the previous relation becomes

$$\tfrac{1}{2}w_1^T B^T Pe + \tfrac{1}{2}e^T PBw_1 - \tfrac{1}{2\rho^2}e^T PBB^T Pe \leq \tfrac{1}{2}\rho^2 w_1^T w_1 \tag{11.119}$$

The previous inequality is used in $\dot{V}$, and the right part of the associated inequality is enforced

$$\dot{V} \leq -\frac{1}{2}e^T Qe + \frac{1}{2}\rho^2 w_1^T w_1 \tag{11.120}$$

The attenuation coefficient ρ can be chosen such that the right part of Eq. (11.120) is always upper bounded by 0. For instance, it suffices at every iteration of the control algorithm to have

$$\begin{aligned}-\tfrac{1}{2}e^T Qe + \tfrac{1}{2}\rho^2 ||w_1||^2 \leq 0 \Rightarrow -\tfrac{1}{2}||e||_Q^2 + \tfrac{1}{2}\rho^2 ||w_1||^2 \leq 0 \Rightarrow \\ \tfrac{1}{2}\rho^2 ||w_1||^2 \leq \tfrac{1}{2}||e||_Q^2 + \Rightarrow \rho^2 \leq \tfrac{||e||_Q^2}{||w_1||^2}\end{aligned} \tag{11.121}$$

Again without knowledge of the uncertainties and disturbance term $||w_1||$ a sufficiently small value of ρ can assure that the above inequality holds and thus that the loop's stability is ascertained. Actually, ρ should be given the least value which permits to obtain a solution of the Riccati equation, given in Eq. (11.102).

Equation (11.120) can be used to show that the H_∞ performance criterion is satisfied. The integration of $\dot{V}$ from 0 to T gives

$$\begin{array}{c}\int_0^T \dot{V}(t)dt \leq -\frac{1}{2}\int_0^T ||e||^2 dt + \frac{1}{2}\rho^2 \int_0^T ||w_1||^2 dt \Rightarrow \\ 2V(T) + \int_0^T ||e||_Q^2 dt \leq 2V(0) + \rho^2 \int_0^T ||w_1||^2 dt\end{array} \tag{11.122}$$

Moreover, if there exists a positive constant $M_w > 0$ such that

$$\int_0^\infty ||w_1||^2 dt \leq M_w \tag{11.123}$$

then one gets

$$\int_0^\infty ||e||_Q^2 dt \leq 2V(0) + \rho^2 M_w \tag{11.124}$$

Thus, the integral $\int_0^\infty ||e||_Q^2 dt$ is bounded and according to Barbalat's Lemma one obtains $lim_{t\to\infty} e(t) = 0$.

It is of worth mentioning that in case that the complete state vector of the submarine is not completely measurable one can implement an observer-based adaptive fuzzy control scheme based on differential flatness theory. The observer-based adaptive fuzzy control, making use of differential flatness theory, extends the class of systems to which indirect adaptive fuzzy control can be applied. This control method enables control of MIMO nonlinear systems without the need to measure all state vector elements [454]. The only assumption needed for the design of the observer-based controller and for succeeding H-infinity tracking performance for the control loop is that there exists a solution for two Riccati equations associated with the linearized error dynamics of the differentially flat model. This assumption holds for several nonlinear systems, thus providing a systematic approach to the design of observer-based controllers.

11.3.6 Simulation Tests

The results about the stability and robustness features of the submarine's control loop were also confirmed through simulation experiments. In the simulation tests, the dynamic model of the submarine was considered to be completely unknown and was identified in real-time by the previously analyzed neurofuzzy approximators. The estimated unknown dynamics of the system was used in the computation of the control inputs which were finally exerted on the submarine's model. The sampling period was set to $T_s = 0.01 sec$. Apart from modelling uncertainty it was considered that the submarine's model was also affected by external perturbations. The numerical values of the gains which have been used in the solution of the Riccati equation have been defined as $r = 0.1$ and $\rho = 1.0$.

The state feedback gain was $K \in R^{2\times 4}$. The basis functions used in the estimation of $f_i(x,t),\ i = 1,2$ and $g_{ij}(x,t),\ i = 1,2,\ j = 1,2$ were $\mu_{A_j}(\hat{x}) = e^{(\frac{\hat{x}-c_j}{\sigma})^2},\ j = 1,\cdots,3$. Since there are four inputs x_1, x_2 and x_4, x_4 and the associated definition set

Table 11.2 Parameters of the fuzzy rule base

Rule	$c_1^{(l)}$	$c_2^{(l)}$	$c_3^{(l)}$	$c_4^{(l)}$	$v^{(l)}$
$R^{(1)}$	−1.0	−1.0	−1.0	−1.0	3
$R^{(2)}$	−1.0	−1.0	−1.0	0.0	3
$R^{(3)}$	−1.0	−1.0	−1.0	1.0	3
$R^{(4)}$	−1.0	−1.0	0.0	−1.0	3
$R^{(5)}$	−1.0	−1.0	0.0	0.0	3
$R^{(6)}$	−1.0	−1.0	0.0	1.0	3
...	...	...	...	...	...
...	...	...	...	...	...
$R^{(81)}$	1.0	1.0	1.0	0.5	3

(universe of discourse) consists of 3 fuzzy sets, for the approximation of functions $f_i(x,t)$ $i=1,2$, there will be 81 fuzzy rules of the form:

$$\begin{array}{l} R^l : IF \;\; x_1 \;\; is \;\; A_1^l \;\; AND \;\; x_2 \;\; is \;\; A_2^l \\ AND \;\; x_3 \;\; is \;\; A_3^l \;\; AND \;\; x_4 \;\; is \;\; A_4^l \;\; THEN \;\; \hat{f}_i^l \;\; is \;\; b^l \end{array} \tag{11.125}$$

and $\hat{f}_i(x,t) = \frac{\sum_{l=1}^{81} \hat{f}_i^l \prod_{i=1}^{4} \mu_{A_i}^l(x_i)}{\sum_{l=1}^{81} \prod_{i=1}^{4} \mu_{A_i}^l(x_i)}$. Indicative (dimensionless) values for the placement on a spatial grid of the centers $c_i^{(l)}$, $i=1,\cdots,4$ and the variances $v^{(l)}$ of each rule are as follows (Table 11.2).

As noted, in the considered fuzzy rule-base there are four input parameters in the antecedent parts of the fuzzy rules, i.e. $x_1 = h$, $x_2 = \dot{h}$, $x_3 = \theta$ and $x_4 = \dot{\theta}$. Each parameter is partitioned into 3 fuzzy sets. Therefore, by taking all possible combinations between the fuzzy sets one has $3^4 = 81$ fuzzy rules. The finer the partition of the input variables into fuzzy sets is, the more accurate the approximation of the nonlinear system dynamics by the neuro-fuzzy model is expected to be (although some of the rules of the fuzzy rule base may not be sufficiently activated due to little coverage of the associated region of the state-space by input data). However, considering a large number of fuzzy sets for each input variable induces the curse of dimensionality which means that there is an excessive and rather unnecessary increase in the number of the adaptable parameters that constitute the neuro-fuzzy model.

The associated results are presented in Figs. 11.14, 11.15 and 11.16. It can be observed that the adaptive fuzzy control scheme achieved fast and accurate tracking of the reference setpoints. After finding the solution of the algebraic Riccati equation given in Eq. (11.102) the computation of an H-infinity feedback control term was possible and this provided the submarine's control loop with additional robustness. Taking into account that in real operating conditions the control of a submarine cannot rely on the assumption about a precise mathematical model and about complete knowledge of external perturbations, the significance of the proposed adaptive fuzzy control scheme becomes obvious.

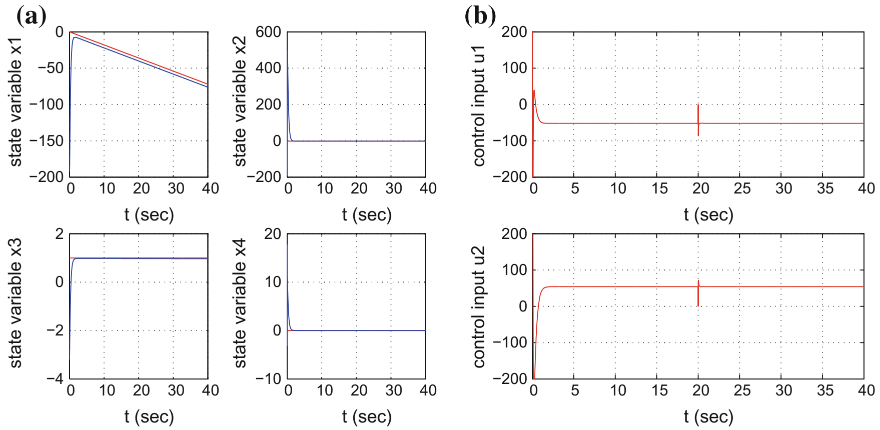

Fig. 11.14 Setpoint 1: **a** Convergence of the state variables x_i, $i = 1, \cdots, 4$ of the submarine to the desirable setpoints, **b** Variations of the control inputs (bow and stern hydroplane reflections)

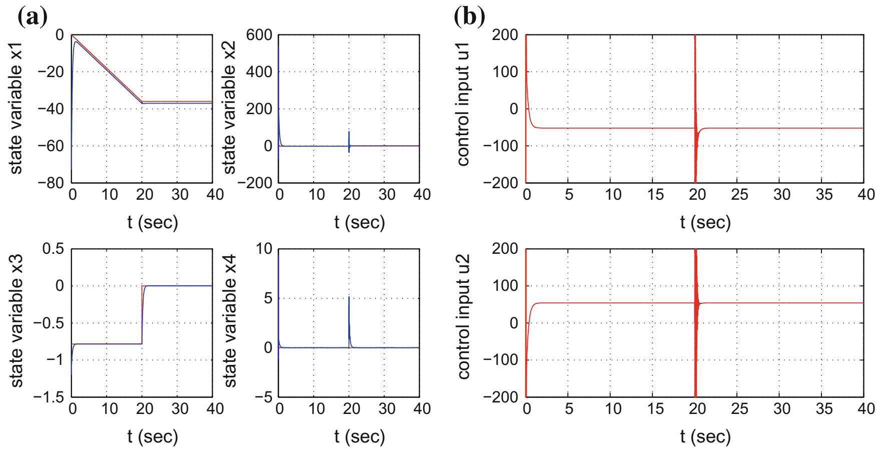

Fig. 11.15 Setpoint 2: **a** Convergence of the state variables x_i, $i = 1, \cdots, 4$ of the submarine to the desirable setpoints, **b** Variations of the control inputs (bow and stern hydroplane reflections)

There have been numerous examples of the use of model-based flatness-based control, given in [450, 457]. If the model of the control system is a precise one flatness-based control is anticipated to have an excellent performance. The control problem becomes more complicated in the case of absence of a precise mathematical model for the controlled system. It is even more difficult when there is no prior knowledge about the system's dynamics that can be used in the design of the flatness-based controller. The solution to the latter control problem is obtained with the use of the proposed flatness-based adaptive fuzzy control method. Although the dynamic model of the system is completely unknown, it is assured through Lyapunov stability

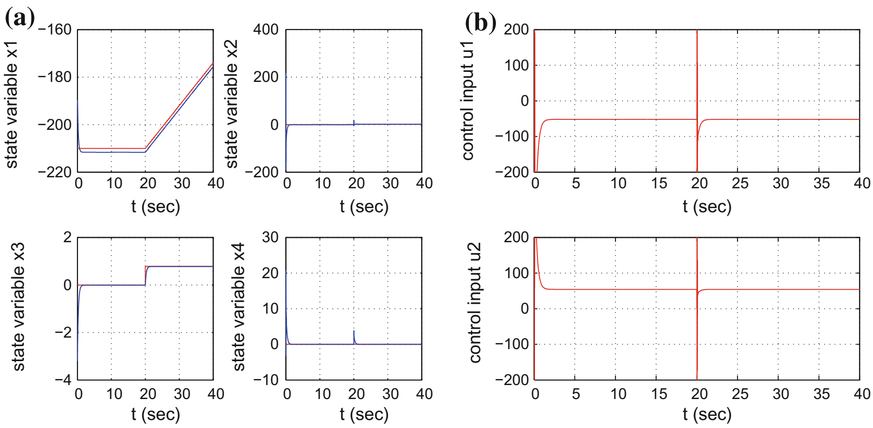

Fig. 11.16 Setpoint 3: **a** Convergence of the state variables x_i, $i = 1, \cdots, 4$ of the submarine to the desirable setpoints, **b** Variations of the control inputs (bow and stern hydroplane reflections)

analysis that this unknown system dynamics will be online identified by neurofuzzy approximators and that the state variables of the system will converge to the desirable setpoints. The robustness of the proposed adaptive fuzzy control method depends on the selection of parameters, such as the attenuation coefficient ρ which is used in the solution of the associated Riccati equation.

The reference trajectories can be generated using the differential flatness properties of the system. This means that all state variables of the system are expressed as differential functions of the flat outputs. Next, reference trajectories are defined for the flat outputs and these are also used for computing the reference setpoints for the rest of the state variables of the submarine's model.

11.4 Nonlinear Optimal Control of Autonomous Submarines

11.4.1 Outline

As previously noted, research on nonlinear control of autonomous underwater vessels has grown rapidly during the last years since there is need to develop robotic systems capable of functioning autonomously in an underwater environment [37, 251, 258, 602]. In this section, a nonlinear optimal (H-infinity) control method is developed aiming at solving the problem of depth and heading control of an autonomous submarine. It has been pointed out that navigation of autonomous underwater vessels (AUVs) and particularly of submarines exhibits several difficulties due to strong nonlinearities and the multivariable coupling characterizing the associated dynamic model [143, 386, 411, 416, 457, 462]. Moreover, submersible robotized vessels are subject to model uncertainty and parametric variations while they are also affected by external perturbations [253, 275, 420, 500, 635]. For these reasons the control problem of a submarine's depth and heading angle is a nontrivial one. Apart from the developments of the previous sections, other results for the solution of this problem with the use of optimal control theory can be found in [44, 50, 98, 287, 346, 423, 608]. The approach to be developed in this section is relies on approximate linearization of the submarine's dynamics and on application of optimal (H-infinity) control to the model that is obtained from the linearization procedure.

The dynamic model of the submarine, describing coupling between its depth and its heading angle, undergoes approximate linearization, around a temporary operating point (equilibrium) which is recomputed at each iteration of the control algorithm [461, 466]. The equilibrium is defined by the present value of the submarine's state vector and the last value of the control inputs vector that was exerted on it. The linearization takes place through Taylor series expansion and the computation of the associated Jacobian matrices [33, 463, 564]. The modelling error which is due to truncation of higher order terms from the Taylor series is considered to be a perturbation that is compensated by the robustness of the control algorithm.

For the approximately linearized model of the submarine, the optimal (H-infinity) control problem is solved [132, 305, 450, 457, 459]. Actually the designed H-infinity controller stands for a solution to a min-max differential game. In such a game the controller tries to minimize a quadratic cost functional based on the submarine's state vector error, while the model uncertainty and external perturbation terms try to maximize it. The computation of the feedback gain of the H-infinity controller requires the solution of an algebraic Riccati equation which also takes place at each step of the control method.

The stability of the control method is proven through Lyapunov analysis. First, it is demonstrated that the control loop satisfies the H-infinity tracking performance criterion. This provides the control scheme with elevated robustness against model uncertainty and external perturbations. Moreover, under moderate conditions it is shown that the control loop exhibits global asymptotic stability properties. Finally to implement state estimation-based control of the submarine without the need to measure its entire state vector, the H-infinity Kalman Filter is used [169, 511]. This stands for an optimal state estimator, when the monitored system's model is characterized by parametric uncertainty or is subject to external perturbations.

11.4.2 Approximate Linearization of the AUV's Model

Using the description of the state-space model of the submarine given in Eq. (11.54) one has about functions $g_h(x,t)$ and $g_\theta(x,t)$

$$\begin{pmatrix} g_h(x,t) \\ g_\theta(x,t) \end{pmatrix} = \begin{pmatrix} 1 & -Z_Q^{'}L/m_3^{'} \\ -M_{\ddot{w}}(LI_2^{'-1}) & 1 \end{pmatrix}^{-1} \cdot$$
$$\begin{pmatrix} \frac{Z_w^{'}U}{Lm_3^{'}}w(t) + \frac{1}{m_3^{'}}(\dot{Z}_{\dot{\theta}}^{'} + m^{'})U\dot{\theta}(t) + \frac{Z_d(t)}{0.5\rho L^3 m_3^{'}} + Z_\eta(w,Q) \\ \frac{M_v^{'}U}{L^2 I_2^{'}}w(t) + \frac{M_{\dot{\theta}}^{'}U}{LI_2^{'}}\dot{\theta}(t) + \frac{2mg(z_G - z_B)}{\rho L^5 I_2^{'}}\theta(t) + \frac{M_d(t)}{0.5\rho L^5 I_2^{'}} + M_\eta(w,Q) \end{pmatrix} \tag{11.126}$$

The effects of the wave and currents forces and the effects of hydrodynamic forces are considered as disturbances and thus are omitted from the model of the submarine's dynamics. By grouping coefficients the previous equation given in Eq. (11.126) can be written as

$$\begin{pmatrix} g_h(x,t) \\ g_\theta(x,t) \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} a_1 \frac{1}{cos(x_3)}[x_2 + U_0 sin(x_3)] + a_2 x_4 \\ b_1 \frac{1}{cos(x_3)}[x_2 + U_0 sin(x_3)] + b_2 x_4 \end{pmatrix} \tag{11.127}$$

or equivalently

$$\begin{pmatrix} g_h(x,t) \\ g_\theta(x,t) \end{pmatrix} = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} \begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix} \begin{pmatrix} \frac{1}{cos(x_3)}[x_2 + U_0 sin(x_3)] \\ x_4 \end{pmatrix} \tag{11.128}$$

and by performing additional operations between coefficients one has

$$\begin{pmatrix} g_h(x,t) \\ g_\theta(x,t) \end{pmatrix} = \begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} \begin{pmatrix} \frac{1}{cos(x_3)}[x_2 + U_0 sin(x_3)] \\ x_4 \end{pmatrix} \tag{11.129}$$

According to the above, the AUV's model is written in the generic form:

$$\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_3 \end{pmatrix} = \begin{pmatrix} F_1(x) \\ F_2(x) \end{pmatrix} + \begin{pmatrix} G_{11}(x) \\ G_{21}(x) \end{pmatrix} u_1 + \begin{pmatrix} G_{12}(x) \\ G_{22}(x) \end{pmatrix} u_2 \tag{11.130}$$

where one has that

$$F_1(x) = p_{11}\frac{1}{cos(x_3)}(x_2 + U_0 sin(x_3)) + p_{12}x_4 - \frac{x_2 + U_0 sin(x_3)}{cos(x_3)} x_4 sin(x_3) - U_0 x_4 sin(x_3) \tag{11.131}$$

$$F_2(x) = p_{21}\frac{1}{cos(x_3)}(x_2 + U_0 sin(x_3)) + p_{22}x_4 \tag{11.132}$$

while it also holds that

$$\begin{matrix} G_{11}(x) = B_{011}cos(x_3) & G_{12}(x) = B_{012}cos(x_3) \\ G_{21}(x) = B_{021} & G_{22}(x) = B_{022} \end{matrix} \tag{11.133}$$

Next, the Jacobian matrices of the submarine's dynamic model are computed. For the Jacobian matrix $\nabla_x F$ one has:

$$\nabla_x F = \begin{pmatrix} \frac{\partial F_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} & \frac{\partial F_1}{\partial x_3} & \frac{\partial F_1}{\partial x_4} \\ \frac{\partial F_2}{\partial x_1} & \frac{\partial F_2}{\partial x_2} & \frac{\partial F_2}{\partial x_3} & \frac{\partial F_2}{\partial x_4} \\ \frac{\partial F_3}{\partial x_1} & \frac{\partial F_3}{\partial x_2} & \frac{\partial F_3}{\partial x_3} & \frac{\partial F_3}{\partial x_4} \\ \frac{\partial F_4}{\partial x_1} & \frac{\partial F_4}{\partial x_2} & \frac{\partial F_4}{\partial x_3} & \frac{\partial F_4}{\partial x_4} \end{pmatrix} \tag{11.134}$$

About the first row of the Jacobian matrix $\nabla_x F$ one has: $\frac{\partial F_1}{\partial x_1} = 0$, $\frac{\partial F_1}{\partial x_2} = 1$, $\frac{\partial F_1}{\partial x_3} = 0$, $\frac{\partial F_1}{\partial x_4} = 0$

About the second row of the Jacobian matrix $\nabla_x F$ one has: $\nabla_x F$ one has: $\frac{\partial F_2}{\partial x_1} = 0$, $\frac{\partial F_2}{\partial x_2} = p_{11}\frac{1}{cos(x_3)} - \frac{x_4 sin(x_3)}{cos(x_3)}$, $\frac{\partial F_2}{\partial x_3} = \frac{p_{11}U_0}{cos(x_3)^2} - \frac{U_0 2sin(x_3)cos(x_3)^2 + U_0 sin(x_3)^2}{cos(x_3)^2} - U_0 x_4 cos(x_3)$, $\frac{\partial F_2}{\partial x_4} = p_{12} - \frac{x_2 + U_0 sin(x_3)}{cos(x_3)} sin(x_3) - U_0 sin(x_3)$

About the third row of the Jacobian matrix $\nabla_x F$ one has: $\frac{\partial F_3}{\partial x_1} = 0$, $\frac{\partial F_3}{\partial x_2} = 0$, $\frac{\partial F_3}{\partial x_3} = 0$, $\frac{\partial F_3}{\partial x_4} = 1$

About the fourth row of the Jacobian matrix $\nabla_x F$ one has: $\nabla_x F$ one has: $\frac{\partial F_4}{\partial x_1} = 0$, $\frac{\partial F_4}{\partial x_2} = p_{21}\frac{1}{cos(x_3)}$, $\frac{\partial F_4}{\partial x_3} = p_{21}\frac{U_0}{cos(x_3)^2}$, $\frac{\partial F_4}{\partial x_4} = p_{22}$

For the Jacobian matrix $\nabla_x G_1$ one has:

$$\nabla_x G_1 = \begin{pmatrix} \frac{\partial G_{11}}{\partial x_1} & \frac{\partial G_{11}}{\partial x_2} & \frac{\partial G_{11}}{\partial x_3} & \frac{\partial G_{11}}{\partial x_4} \\ \frac{\partial G_{21}}{\partial x_1} & \frac{\partial G_{21}}{\partial x_2} & \frac{\partial G_{21}}{\partial x_3} & \frac{\partial G_{21}}{\partial x_4} \\ \frac{\partial G_{31}}{\partial x_1} & \frac{\partial G_{31}}{\partial x_2} & \frac{\partial G_{31}}{\partial x_3} & \frac{\partial G_{31}}{\partial x_4} \\ \frac{\partial G_{41}}{\partial x_1} & \frac{\partial G_{41}}{\partial x_2} & \frac{\partial G_{41}}{\partial x_3} & \frac{\partial G_{41}}{\partial x_4} \end{pmatrix} \tag{11.135}$$

About the first row of the Jacobian matrix $\nabla_x G_1$ one has: $\frac{\partial G_{11}}{\partial x_1} = 0$, $\frac{\partial G_{11}}{\partial x_2} = 0$, $\frac{\partial G_{11}}{\partial x_3} = 0$, $\frac{\partial G_{11}}{\partial x_4} = 0$

About the second row of the Jacobian matrix $\nabla_x G_1$ one has: $\frac{\partial G_{21}}{\partial x_1} = 0$, $\frac{\partial G_{21}}{\partial x_2} = 0$, $\frac{\partial G_{21}}{\partial x_3} = -B_{011} sin(x_3)$, $\frac{\partial G_{21}}{\partial x_4} = 0$

About the third row of the Jacobian matrix $\nabla_x G_1$ one has: $\frac{\partial G_{31}}{\partial x_1} = 0$, $\frac{\partial G_{31}}{\partial x_2} = 0$, $\frac{\partial G_{31}}{\partial x_3} = 0$, $\frac{\partial G_{31}}{\partial x_4} = 0$

About the fourth row of the Jacobian matrix $\nabla_x G_1$ one has: $\frac{\partial G_{41}}{\partial x_1} = 0$, $\frac{\partial G_{41}}{\partial x_2} = 0$, $\frac{\partial G_{41}}{\partial x_3} = 0$, $\frac{\partial G_{41}}{\partial x_4} = 0$

For the Jacobian matrix $\nabla_x G_2$ one has:

$$\nabla_x G_1 = \begin{pmatrix} \frac{\partial G_{12}}{\partial x_1} & \frac{\partial G_{12}}{\partial x_2} & \frac{\partial G_{12}}{\partial x_3} & \frac{\partial G_{12}}{\partial x_4} \\ \frac{\partial G_{22}}{\partial x_1} & \frac{\partial G_{22}}{\partial x_2} & \frac{\partial G_{22}}{\partial x_3} & \frac{\partial G_{22}}{\partial x_4} \\ \frac{\partial G_{32}}{\partial x_1} & \frac{\partial G_{32}}{\partial x_2} & \frac{\partial G_{32}}{\partial x_3} & \frac{\partial G_{32}}{\partial x_4} \\ \frac{\partial G_{42}}{\partial x_1} & \frac{\partial G_{42}}{\partial x_2} & \frac{\partial G_{42}}{\partial x_3} & \frac{\partial G_{42}}{\partial x_4} \end{pmatrix} \tag{11.136}$$

About the first row of the Jacobian matrix $\nabla_x G_2$ one has: $\frac{\partial G_{12}}{\partial x_1} = 0$, $\frac{\partial G_{12}}{\partial x_2} = 0$, $\frac{\partial G_{12}}{\partial x_3} = 0$, $\frac{\partial G_{12}}{\partial x_4} = 0$

About the second row of the Jacobian matrix $\nabla_x G_2$ one has: $\frac{\partial G_{22}}{\partial x_1} = 0$, $\frac{\partial G_{22}}{\partial x_2} = 0$, $\frac{\partial G_{22}}{\partial x_3} = -B_{011} sin(x_3)$, $\frac{\partial G_{22}}{\partial x_4} = 0$

About the third row of the Jacobian matrix $\nabla_x G_2$ one has: $\frac{\partial G_{32}}{\partial x_1} = 0$, $\frac{\partial G_{32}}{\partial x_2} = 0$, $\frac{\partial G_{32}}{\partial x_3} = 0$, $\frac{\partial G_{32}}{\partial x_4} = 0$

About the fourth row of the Jacobian matrix $\nabla_x G_2$ one has: $\frac{\partial G_{42}}{\partial x_1} = 0$, $\frac{\partial G_{42}}{\partial x_2} = 0$, $\frac{\partial G_{42}}{\partial x_3} = 0$, $\frac{\partial G_{42}}{\partial x_4} = 0$

By considering the time varying equilibrium (linearization point) (x^*, u^*), where x^* is the present value of the submarine's state vector and u^* is the last value of the control inputs vector that was exerted on it, the linearized description of the AUV's model becomes

$$\dot{x} = Ax + Bu + \tilde{d} \tag{11.137}$$

where matrices A and B are given by

$$A = [\nabla_x F + \nabla_x G_1 u_1 + \nabla_x G_2 u_2] \mid_{(x^*, u^*)} \tag{11.138}$$

$$B = [\nabla_u F + \nabla_u G_1 u_1 + \nabla_u G_2 u_2] \mid_{(x^*, u^*)} = [G_1, G_2] \tag{11.139}$$

and $\tilde{d}$ is a term denoting modelling error and external perturbation effects.

11.4.3 Design of an H-Infinity Nonlinear Feedback Controller

11.4.3.1 Equivalent Linearized Dynamics of the Submarine

After linearization round its current operating point, the submarine's dynamic model is written as

$$\dot{x} = Ax + Bu + d_1 \quad (11.140)$$

Parameter d_1 stands for the linearization error in the submarine's dynamic model appearing in Eq. (11.140). The reference setpoints for the submarine's state vector are denoted by $\mathbf{x_d} = [x_1^d, \cdots, x_4^d]$. Tracking of this trajectory is succeeded after applying the control input u^*. At every time instant the control input u^* is assumed to differ from the control input u appearing in Eq. (11.140) by an amount equal to Δu, that is $u^* = u + \Delta u$

$$\dot{x}_d = Ax_d + Bu^* + d_2 \quad (11.141)$$

The dynamics of the controlled system described in Eq. (11.140) can be also written as

$$\dot{x} = Ax + Bu + Bu^* - Bu^* + d_1 \quad (11.142)$$

and by denoting $d_3 = -Bu^* + d_1$ as an aggregate disturbance term one obtains

$$\dot{x} = Ax + Bu + Bu^* + d_3 \quad (11.143)$$

By subtracting Eq. (11.141) from (11.143) one has

$$\dot{x} - \dot{x}_d = A(x - x_d) + Bu + d_3 - d_2 \quad (11.144)$$

By denoting the tracking error as $e = x - x_d$ and the aggregate disturbance term as $\tilde{d} = d_3 - d_2$, the tracking error dynamics becomes

$$\dot{e} = Ae + Bu + \tilde{d} \quad (11.145)$$

The above linearized form of the submarine's model can be efficiently controlled after applying an H-infinity feedback control scheme.

11.4.4 The Nonlinear H-Infinity Control for the Autonomous Submarine

The initial nonlinear model of the autonomous submarine is in the form

$$\dot{x} = \tilde{f}(x, u) \quad x \in R^n, \ u \in R^m \quad (11.146)$$

Linearization of the system (autonomous submarine) is performed at each iteration of the control algorithm round its present operating point $(x^*, u^*) = (x(t), u(t - T_s))$, where T_s is the sampling period. The linearized equivalent model of the system is described by

$$\dot{x} = Ax + Bu + L\tilde{d} \quad x{\in}R^n,\ u{\in}R^m,\ \tilde{d}{\in}R^q \tag{11.147}$$

where matrices A and B are obtained from the computation of the Jacobians of the submarine's state-space model and vector $\tilde{d}$ denotes disturbance terms due to linearization errors. The problem of disturbance rejection for the linearized model that is described by

$$\begin{aligned} \dot{x} &= Ax + Bu + L\tilde{d} \\ y &= Cx \end{aligned} \tag{11.148}$$

where $x{\in}R^n$, $u{\in}R^m$, $\tilde{d}{\in}R^q$ and $y{\in}R^p$, cannot be handled efficiently if the classical LQR control scheme is applied. This is because of the existence of the perturbation term $\tilde{d}$. The disturbance term $\tilde{d}$ apart from modeling (parametric) uncertainty and external perturbations can also represent noise terms of any distribution.

As already explained in previous examples on the H_∞ control approach, a feedback control scheme is designed for trajectory tracking by the submarine's state vector and simultaneous disturbance rejection, considering that the disturbance affects the system in the worst possible manner. The disturbances' effects are incorporated in the following quadratic cost function:

$$J(t) = \frac{1}{2}\int_0^T [y^T(t)y(t) + ru^T(t)u(t) - \rho^2\tilde{d}^T(t)\tilde{d}(t)]dt, \quad r, \rho > 0 \tag{11.149}$$

It has already been proven that the significance of the negative sign in the cost function's term that is associated with the perturbation variable $\tilde{d}(t)$ is that the disturbance tries to maximize the cost function $J(t)$ while the control signal $u(t)$ tries to minimize it. The physical meaning of the relation given above is that the control signal and the disturbances compete to each other within a min-max differential game. This problem of min-max optimization can be written as

$$min_u max_{\tilde{d}} J(u, \tilde{d}) \tag{11.150}$$

As already analyzed, the objective of the optimization procedure is to compute a control signal $u(t)$ which can compensate for the worst possible disturbance, that is externally imposed to the system. However, the solution to the min-max optimization problem is directly related to the value of the parameter ρ. This means that there is an upper bound in the disturbances magnitude that can be annihilated by the control signal.

11.4.4.1 Computation of the Feedback Control Gains

For the linearized system given by Eq. (11.148) the cost function of Eq. (11.149) is defined, where the coefficient r determines the penalization of the control input and the weight coefficient ρ determines the reward of the disturbances' effects.

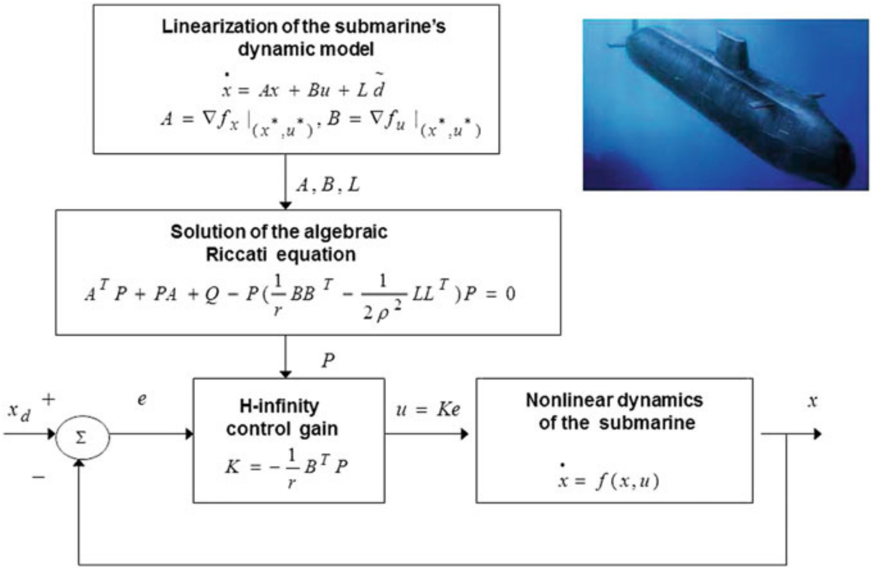

Fig. 11.17 Diagram of the control scheme for the autonomous submarine

Remaining at the assumptions made in previous applications of H-infinity control it is considered that (i) The energy that is transferred from the disturbances signal $\tilde{d}(t)$ is bounded, that is $\int_0^\infty \tilde{d}^T(t)\tilde{d}(t)dt < \infty$, (ii) matrices $[A, B]$ and $[A, L]$ are stabilizable, (iii) matrix $[A, C]$ is detectable. Then, the optimal feedback control law is given by

$$u(t) = -Kx(t) \tag{11.151}$$

with

$$K = \frac{1}{r} B^T P \tag{11.152}$$

where P is a positive semi-definite symmetric matrix which is obtained from the solution of the Riccati equation

$$A^T P + PA + Q - P(\tfrac{1}{r} BB^T - \tfrac{1}{2\rho^2} LL^T)P = 0 \tag{11.153}$$

where Q is also a positive definite symmetric matrix. The worst case disturbance is given by

$$\tilde{d}(t) = \frac{1}{\rho^2} L^T P x(t) \tag{11.154}$$

The diagram of the considered control loop is depicted in Fig. 11.17.

11.4.5 Lyapunov Stability Analysis

Through Lyapunov stability analysis it will be shown that the proposed nonlinear control scheme assures H_∞ tracking performance for the submarine, and that in case of bounded disturbance terms asymptotic convergence to the reference setpoints is achieved. The tracking error dynamics for the autonomous submarine is written in the form

$$\dot{e} = Ae + Bu + L\tilde{d} \tag{11.155}$$

where in the submarine's case $L = I \in R^4$ with I being the identity matrix. Variable $\tilde{d}$ denotes model uncertainties and external disturbances of the submarine's model. The following Lyapunov function is considered

$$V = \frac{1}{2}e^T Pe \tag{11.156}$$

where $e = x - x_d$ is the tracking error. By differentiating with respect to time one obtains

$$\begin{array}{c}\dot{V} = \frac{1}{2}\dot{e}^T Pe + \frac{1}{2}eP\dot{e} \Rightarrow \\ \dot{V} = \frac{1}{2}[Ae + Bu + L\tilde{d}]^T Pe + \frac{1}{2}e^T P[Ae + Bu + L\tilde{d}] \Rightarrow\end{array} \tag{11.157}$$

$$\begin{array}{c}\dot{V} = \frac{1}{2}[e^T A^T + u^T B^T + \tilde{d}^T L^T]Pe + \\ + \frac{1}{2}e^T P[Ae + Bu + L\tilde{d}] \Rightarrow\end{array} \tag{11.158}$$

$$\begin{array}{c}\dot{V} = \frac{1}{2}e^T A^T Pe + \frac{1}{2}u^T B^T Pe + \frac{1}{2}\tilde{d}^T L^T Pe + \\ \frac{1}{2}e^T PAe + \frac{1}{2}e^T PBu + \frac{1}{2}e^T PL\tilde{d}\end{array} \tag{11.159}$$

The previous equation is rewritten as

$$\begin{array}{c}\dot{V} = \frac{1}{2}e^T(A^T P + PA)e + (\frac{1}{2}u^T B^T Pe + \frac{1}{2}e^T PBu) + \\ + (\frac{1}{2}\tilde{d}^T L^T Pe + \frac{1}{2}e^T PL\tilde{d})\end{array} \tag{11.160}$$

Assumption: For given positive definite matrix Q and coefficients r and ρ there exists a positive definite matrix P, which is the solution of the following matrix equation

$$A^T P + PA = -Q + P(\frac{2}{r}BB^T - \frac{1}{\rho^2}LL^T)P \tag{11.161}$$

Moreover, the following feedback control law is applied to the system

$$u = -\frac{1}{r}B^T Pe \tag{11.162}$$

By substituting Eqs. (11.161) and (11.162) one obtains

$$\dot{V} = \frac{1}{2}e^T[-Q + P(\frac{2}{r}BB^T - \frac{1}{\rho^2}LL^T)P]e+ \\ +e^T PB(-\frac{1}{r}B^T Pe) + e^T PL\tilde{d} \Rightarrow \tag{11.163}$$

$$\dot{V} = -\frac{1}{2}e^T Qe + \frac{1}{r}e^T PBB^T Pe - \frac{1}{2\rho^2}e^T PLL^T Pe \\ -\frac{1}{r}e^T PBB^T Pe + e^T PL\tilde{d} \tag{11.164}$$

which after intermediate operations gives

$$\dot{V} = -\frac{1}{2}e^T Qe - \frac{1}{2\rho^2}e^T PLL^T Pe + e^T PL\tilde{d} \tag{11.165}$$

or, equivalently

$$\dot{V} = -\frac{1}{2}e^T Qe - \frac{1}{2\rho^2}e^T PLL^T Pe+ \\ +\frac{1}{2}e^T PL\tilde{d} + \frac{1}{2}\tilde{d}^T L^T Pe \tag{11.166}$$

Lemma: The following inequality holds

$$\frac{1}{2}e^T PL\tilde{d} + \frac{1}{2}\tilde{d}L^T Pe - \frac{1}{2\rho^2}e^T PLL^T Pe \leq \frac{1}{2}\rho^2\tilde{d}^T\tilde{d} \tag{11.167}$$

Proof: The binomial $(\rho\alpha - \frac{1}{\rho}b)^2$ is considered. Expanding the left part of the above inequality one gets

$$\rho^2 a^2 + \frac{1}{\rho^2}b^2 - 2ab \geq 0 \Rightarrow \frac{1}{2}\rho^2 a^2 + \frac{1}{2\rho^2}b^2 - ab \geq 0 \Rightarrow \\ ab - \frac{1}{2\rho^2}b^2 \leq \frac{1}{2}\rho^2 a^2 \Rightarrow \frac{1}{2}ab + \frac{1}{2}ab - \frac{1}{2\rho^2}b^2 \leq \frac{1}{2}\rho^2 a^2 \tag{11.168}$$

The following substitutions are carried out: $a = \tilde{d}$ and $b = e^T PL$ and the previous relation becomes

$$\frac{1}{2}\tilde{d}^T L^T Pe + \frac{1}{2}e^T PL\tilde{d} - \frac{1}{2\rho^2}e^T PLL^T Pe \leq \frac{1}{2}\rho^2\tilde{d}^T\tilde{d} \tag{11.169}$$

Equation (11.169) is substituted in Eq. (11.166) and the inequality is enforced, thus giving

$$\dot{V} \leq -\frac{1}{2}e^T Qe + \frac{1}{2}\rho^2\tilde{d}^T\tilde{d} \tag{11.170}$$

Equation (11.170) shows that the H_∞ tracking performance criterion is satisfied. The integration of $\dot{V}$ from 0 to T gives

$$\int_0^T \dot{V}(t)dt \leq -\frac{1}{2}\int_0^T ||e||_Q^2 dt + \frac{1}{2}\rho^2\int_0^T ||\tilde{d}||^2 dt \Rightarrow \\ 2V(T) + \int_0^T ||e||_Q^2 dt \leq 2V(0) + \rho^2\int_0^T ||\tilde{d}||^2 dt \tag{11.171}$$

Moreover, if there exists a positive constant $M_d > 0$ such that

$$\int_0^\infty ||\tilde{d}||^2 dt \leq M_d \tag{11.172}$$

then one gets

$$\int_0^{\infty}||e||_Q^2 dt \leq 2V(0) + \rho^2 M_d \quad (11.173)$$

Thus, the integral $\int_0^{\infty}||e||_Q^2 dt$ is bounded. Moreover, $V(T)$ is bounded and from the definition of the Lyapunov function V in Eq. (11.156) it becomes clear that $e(t)$ will be also bounded since $e(t) \in \Omega_e = \{e|e^T Pe \leq 2V(0) + \rho^2 M_d\}$. According to the above and with the use of Barbalat's Lemma one obtains $lim_{t\to\infty}e(t) = 0$.

Elaboratiing on the above, it can be noted that the proof of global asymptotic stability for the control loop of the autonomous submarine relies on Eq. (11.170) and on the application of Barbalat's Lemma. It uses the condition of Eq. (11.172) about the boundedness of the square of the aggregate disturbance and modelling error term $\tilde{d}$ that affects the model. However, the proof of global asymptotic stability is not restricted by this condition. By selecting the attenuation coefficient ρ to be sufficiently small and in particular to satisfy $\rho^2 < ||e||_Q^2/||\tilde{d}||^2$ one has that the first derivative of the Lyapunov function is upper bounded by 0. Therefore for the i-th time interval it is proven that the Lyapunov function defined in Eq. (11.156) is a decreasing one. This also ensures that the Lyapunov function of the system defined in Eq. (11.156) will always have a negative first-order derivative.

11.4.6 Robust State Estimation with the Use of the H-Infinity Kalman Filter

The control loop for the autonomous submarine can be implemented with the use of information provided by a small number of sensors and by processing only a small number of state variables. To reconstruct the missing information about the state vector of the autonomous submarine it is proposed to use a filtering scheme and based on it to apply state estimation-based control [169, 457, 511]. The recursion of the H_∞ Kalman Filter, for the model of the submarine, can be formulated in terms of a *measurement update* and a *time update* part

Measurement update:

$$\begin{aligned}
&D(k) = [I - \theta W(k)P^-(k) + C^T(k)R(k)^{-1}C(k)P^-(k)]^{-1} \\
&K(k) = P^-(k)D(k)C^T(k)R(k)^{-1} \\
&\hat{x}(k) = \hat{x}^-(k) + K(k)[y(k) - C\hat{x}^-(k)]
\end{aligned} \quad (11.174)$$

Time update:

$$\begin{aligned}
&\hat{x}^-(k+1) = A(k)x(k) + B(k)u(k) \\
&P^-(k+1) = A(k)P^-(k)D(k)A^T(k) + Q(k)
\end{aligned} \quad (11.175)$$

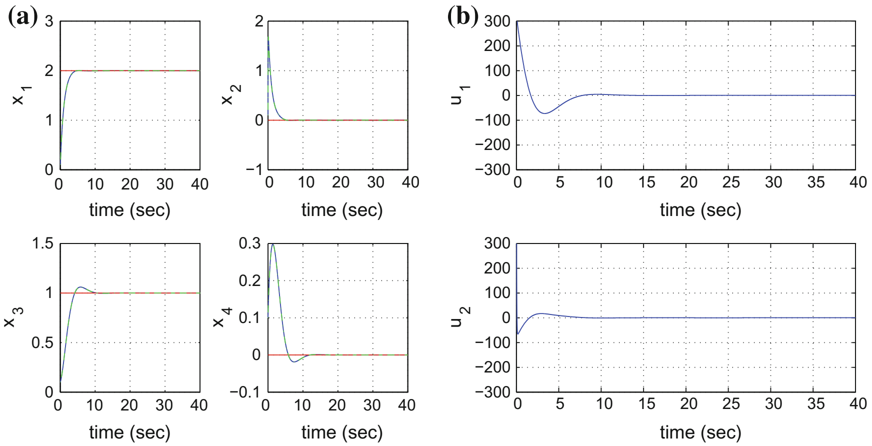

Fig. 11.18 Tracking of setpoint 1: **a** Convergence of the state variables of the submarine x_i $i = 1, \cdots, 4$ (blue lines) to the reference setpoints (red lines) and associated state estimates (green lines) **b** variation of the submarine's control inputs u_i,$i = 1, 2$

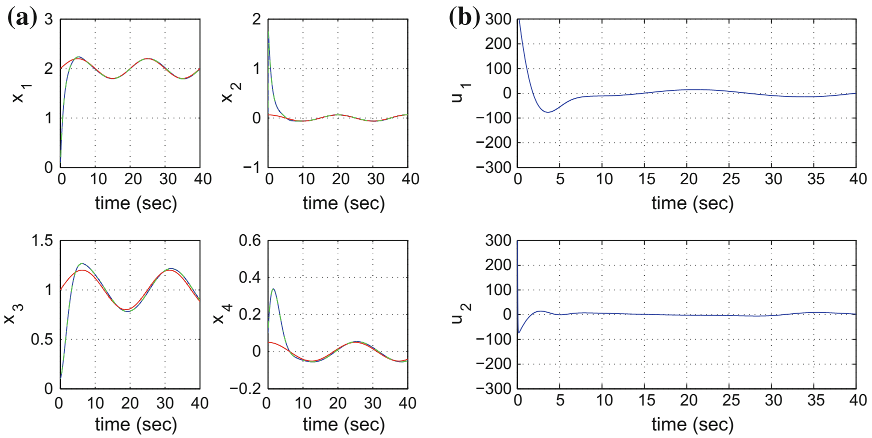

Fig. 11.19 Tracking of setpoint 2: **a** Convergence of the state variables of the submarine x_i $i = 1, \cdots, 4$ (blue lines) to the reference setpoints (red lines) and associated state estimates (green lines) **b** variation of the submarine's control inputs u_i,$i = 1, 2$

where it is assumed that parameter θ is sufficiently small to assure that the covariance matrix $P^{-}(k)^{-1} - \theta W(k) + C^T(k)R(k)^{-1}C(k)$ will be positive definite. When $\theta = 0$ the H_∞ Kalman Filter becomes equivalent to the standard Kalman Filter. One can measure only a part of the state vector of the submarine, and can estimate through filtering the rest of the state vector elements. Moreover, the proposed Kalman filtering method can be used for sensor fusion purposes.

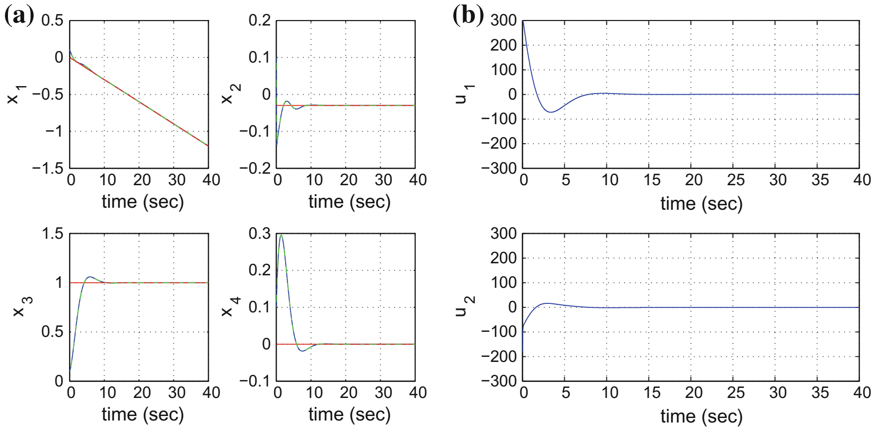

Fig. 11.20 Tracking of setpoint 3: **a** Convergence of the state variables of the submarine x_i $i = 1, \cdots, 4$ (blue lines) to the reference setpoints (red lines) and associated state estimates (green lines) **b** variation of the submarine's control inputs u_i,$i = 1, 2$

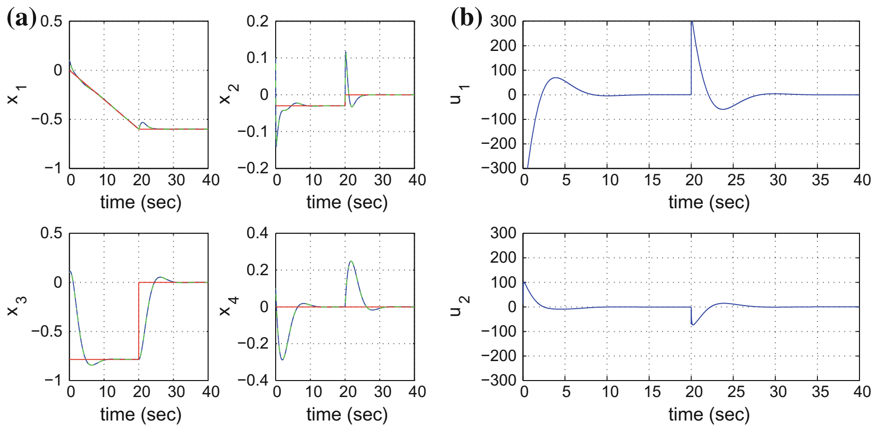

Fig. 11.21 Tracking of setpoint 4: **a** Convergence of the state variables of the submarine x_i $i = 1, \cdots, 4$ (blue lines) to the reference setpoints (red lines) and associated state estimates (green lines) **b** variation of the submarine's control inputs u_i,$i = 1, 2$

11.4.7 Simulation Tests

The performance of nonlinear H-infinity control for the autonomous submarine was tested through simulation experiments.After applying H-infinity control to the dynamic model of the submarine which has been obtained through Taylor series expansion it has become possible to make its state variables converge to the associated reference setpoints. The obtained results are depicted in Figs. 11.18, 11.19, 11.20, 11.21, 11.22 and 11.23. It can be noticed that fast and accurate tracking of

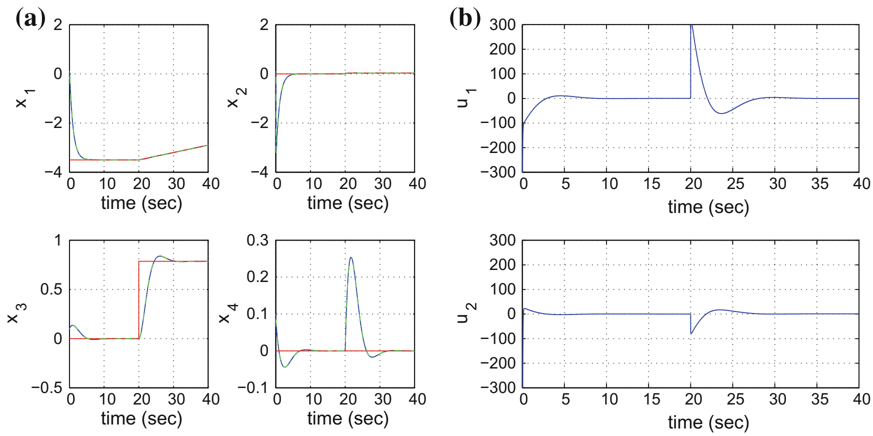

Fig. 11.22 Tracking of setpoint 5: **a** Convergence of the state variables of the submarine x_i $i = 1, \cdots, 4$ (blue lines) to the reference setpoints (red lines) and associated state estimates (green lines) **b** variation of the submarine's control inputs $u_i, i = 1, 2$

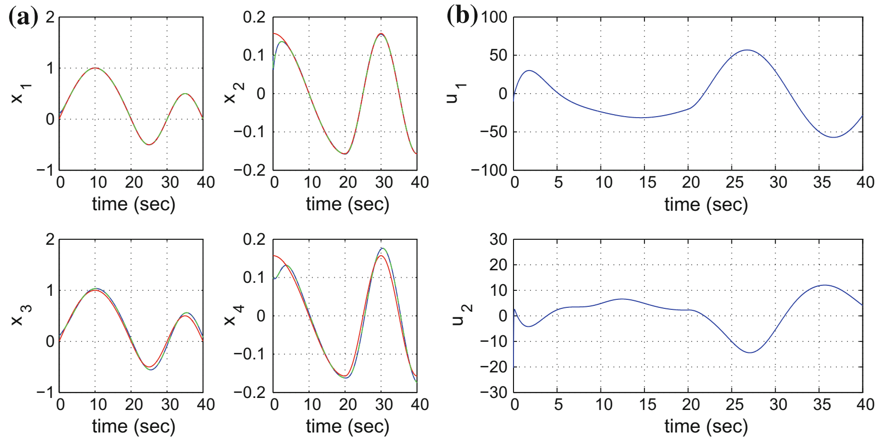

Fig. 11.23 Tracking of setpoint 6: **a** Convergence of the state variables of the submarine x_i $i = 1, \cdots, 4$ (blue lines) to the reference setpoints (red lines) and associated state estimates (green lines) **b** variation of the submarine's control inputs $u_i, i = 1, 2$

the reference setpoints was achieved while the variation of the submarine's control inputs remained smooth and within moderate ranges. For the computation of the feedback control gain the algebraic Riccati equation appearing in Eq. (11.161) had to be repetitively solved at each step of the control method.

In the presented simulation experiments state estimation-based control has been implemented. Out of the 4 state variables of the autonomous submarine only 2 where considered to be measurable. These were the submarine's depth h and its heading angle θ. The rest of the state variables, describing rate of change of the vessel's depth and rate of change of its heading angle were indirectly estimated with the use of the H-infinity Kalman Filter. The real value of each state variable has been plotted in blue, the estimated value has been plotted in green, while the associated reference setpoint has been plotted in red. It can be noticed that despite model uncertainty the H-infinity Kalman Filter achieved accurate estimation of the real values of the state vector elements. In this manner the robustness of the state estimation-based H-infinity control scheme was also improved.

Comparing to control methods for autonomous underwater vessels which are based on global linearization techniques, the main properties of the nonlinear H-infinity control scheme are outlined as follows: (i) it is applied directly on the nonlinear dynamical model of the submarine and does not require the computation of diffeomorphisms (change of variables) that will bring the system into an equivalent linearized form, (ii) the computation of the feedback control signal follows an optimal control concept and requires the solution of an algebraic Riccati equation at each iteration of the control algorithm, (iii) the control method retains the advantages of optimal control, that is fast and accurate tracking of reference setpoints under moderate variations of the control inputs.