Chapter 10
Minding the Gap: A Comparison Between Pre-service and Practicing High School Teachers' Geometry Teaching Knowledge

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Abstract This study compares the Geometry Teaching Knowledge of pre-service teachers with that of current high school geometry teachers. Data was collected using items from the Mathematical Knowledge for Teaching Geometry (MKT-G) assessment described by Herbst and Kosko (Mathematical knowledge for teaching and its specificity to high school geometry instruction. Research trends in mathematics teacher education. Springer, New York, pp. 23–45, 2014), and a post-assessment survey. The study focuses on the differences found in responses to items belonging to four domains: Common Content Knowledge-Geometry (CCK-G), Specialized Content Knowledge-Geometry (SCK-G), Knowledge of Content and Students-Geometry (KCS-G), and Knowledge of Content and Teaching-Geometry (KCT-G). Data was analyzed using t-tests for independent groups. Practicing high school geometry teachers outperformed the pre-service teachers on the MKT-G assessment in all four domains. Awareness of geometry instructional techniques and methods used in the current high school geometry classrooms was investigated as well. Practicing high school geometry teachers reported using and learning different instructional techniques and methods in their classrooms and professional development when compared to pre-service teachers' techniques and methods used or learned in their education and mathematics courses.

Keywords Future teachers · Geometry teaching knowledge · Geometry teaching methods · Geometry teaching techniques · High school geometry Mathematical knowledge for teaching-geometry (MKT-G) · Practicing geometry teachers · Pre-service teachers · Professional development · Teacher education Teacher knowledge

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10.1 Introduction

Geometry is a field in mathematics that every student in the United States is required to study in order to fulfill high school graduation requirements. According to the Center for Public Education (2013), all states require that students have two or more mathematics credits of Algebra 1 or higher to graduate. Geometry is listed as the course to immediately follow Algebra 1. The Common Core State Standards Initiative (2010) stresses that geometry is a vital course when preparing students to enter a science, technology, mathematics, or engineering field. According to the National Center for Education Statistics (2012), American students' performance is consistently behind other countries involved in the PISA assessment organized by the Organization for Economic Cooperation and Development (OECD) in two content areas in mathematics: Geometry and Measurement. In 2007, U.S. 8th grade students average score in geometry on the Trends in International Mathematics Science Study (TIMSS) was 20 points lower than the TIMSS scale average, while these students scored at or above the TIMSS scale average on all other content domains (Aud et al., 2010). The literature shows that three possible reasons for poor performance in geometry and measurement are: not enough exposure and emphasis in K-12 curriculum implemented by the teacher, challenges associated with the teaching of geometry and measurement in the classroom, and limited knowledge of the teachers (Steele, 2013).

Teachers that have completed a bachelor's degree in mathematics and a traditional teacher preparation program are considered qualified teaching candidates. According to No Child Left Behind (2002), a highly qualified teacher holds a bachelor's degree in mathematics and has passed a state academic subject test. Teachers with a secondary teaching degree are expected to be able to successfully teach all courses of mathematics study taught in high school, including geometry. According to the topics addressed in teacher certification exams, a pre-service teacher should be prepared to teach geometry when entering the secondary classroom; however, Mitchell and Barth (1999) point out that individuals can pass state certification tests without having to pass all the domains assessed on the test. If a pre-service teacher does not pass the Geometry and Measurement section of the exam, they could still pass the exam, but that pre-service teacher might not have enough content knowledge in Geometry to be a successful Geometry teacher. There is a need to make sure all teachers teaching in secondary schools have enough knowledge of Geometry. Even though teachers follow a traditional teacher preparation program, they may not be prepared to teach the mathematics required of them when they leave the university and enter the secondary classroom.

10.1.1 Geometry Teaching Knowledge: Background

Deborah Ball and her colleagues developed the concept of Mathematical Knowledge for Teaching, also known as MKT. Using Shulman's major categories of teacher knowledge, they developed a theoretical framework for content knowledge for teaching mathematics. Throughout their research, they began to see that "pedagogical content knowledge begins to look as though it includes almost everything a teacher might know in teaching a particular topic" (Ball, Thames, & Phelps, 2008, p. 394). Ball began to focus on how, throughout history, the prevailing assumption that the mathematical knowledge a teacher requires consists of the mathematics that will be covered in the course they are teaching along with some additional study of mathematics at the college level. Deborah Ball and her colleagues decided to develop Shulman's model in the field of mathematics. The primary data used for the analysis was a National Science Foundation funded longitudinal study that documented an entire year of mathematics teaching in a third-grade public school classroom. Many studies have investigated the MKT domains.

The Teacher Education Development Study in Mathematics (TEDS-M) identifies two components to teachers' mathematical knowledge: mathematical content knowledge (MCK) and mathematical pedagogical content knowledge (MPCK) (Tatto et al., 2012). This study developed a framework to measure pre-service teachers' MCK and MPCK in different domains. The domains for MCK included number, geometry, algebra, and data, and in tasks that required knowing, applying, and reasoning. The domains for MPCK included mathematics curricular knowledge, knowledge of planning, and knowledge of enacting mathematics (Tatto et al., 2012). This study found that future teachers in America showed strength in number items but weakness in geometry and algebra items.

The German project COACTIV conducted a study of the connections between content knowledge and pedagogical content knowledge in secondary mathematics among secondary teachers (Krauss et al., 2008). They found that content knowledge and pedagogical content knowledge were distinct factors and highly correlated in the entire sample of teachers; however, teachers considered mathematical experts held knowledge that combined the content knowledge and the pedagogical content knowledge, while those that were not experts kept the factors separate. They concluded that pedagogical content knowledge may be supported by higher levels of content knowledge in ways that lower levels of content knowledge may not (Krauss et al., 2008).

Deborah Ball's model has been cited over 1800 times since it was published. Many studies have been conducted to try to solidify this model, and other studies have focused on specific domains of mathematical knowledge for teaching. For example, Hill, Ball, and Schilling (2008) focused on the domain called knowledge of content and students. They point out that there has been little research in conceptualizing, developing, and measuring teachers' knowledge in each of the domains (Ball et al., 2008). Even though there have been many studies referring to

Deborah Ball's MKT model, there is very little research on teachers MKT at the secondary level. Primarily, research has been conducted on teachers MKT of elementary algebra and number sense topics, but very few studies in elementary geometry. Another study of teachers' knowledge of Algebra points out that [while] "the University of Michigan's work marks considerable progress in defining and assessing teachers' mathematical knowledge for elementary and, more recently, middle-grades teaching, there is little systematic evidence about whether, or how different types of mathematical knowledge matter for effective teaching of algebra in grades 6–12" (McCrory, Floden, Ferrini-Mundy, Reckase, & Senk, 2012, p. 584).

In describing an MKT test designed to measure the knowledge needed to teach high school geometry, Herbst and Kosko (2014) pointed out that there had been little research into Ball's MKT model for high school specific subjects. At the time of the study reported here, there had not been any quantitative research on MKT-G of pre-service teachers, let alone a comparison between pre-service teachers and in-service teachers MKT of geometry. The literature calls for more research in pre-service and in-service teachers' MKT-G along with an investigation as to where these teachers gain this knowledge. Herbst and Kosko (2014) point out that there is more work to be done to refine the domains of Ball's MKT model with respect to Geometry and by doing so this "could inform the development of coursework in mathematics or mathematics education for future teachers" (Herbst & Kosko, 2014, p. 33).

10.2 Theoretical Framework

The theoretical framework used in this study follows the Domains of Mathematical Knowledge for Teaching-Geometry used by Herbst and Kosko (2014) to develop the MKT-G assessment. This assessment was founded on the framework by Deborah Ball and associates (2008). The original framework consisted of Common Content Knowledge, Specialized Content Knowledge, Knowledge of Content and Students, Knowledge of Content and Teaching, Knowledge of Content and Curriculum, and Horizon Content Knowledge. Herbst and Kosko's Mathematical Knowledge for Teaching-Geometry (MKT-G) assessment focuses on four of the six domains: Common Content Knowledge, Specialized Content Knowledge, Knowledge of Content and Students, and Knowledge of Content and Teaching.

Common Content Knowledge-Geometry (CCK-G) is defined as the geometry knowledge and skill also used in settings other than teaching. In particular, CCK-G is the mathematical knowledge needed to simply calculate the solution or correctly solve geometric problems such as those that students do. Specialized Content Knowledge-Geometry (SCK-G) is geometry knowledge and skill unique to teaching, not necessarily used in any other field. For example, the knowledge needed to see what a student's mistake was when solving a geometry problem incorrectly. Knowledge of Content and Students-Geometry (KCS-G) is knowledge that combines knowledge about students and knowing about geometry. KCS-G is the knowledge teachers need to predict how students may react to a new geometry topic, or what misconceptions and confusion students may have going into a geometry lesson. Knowledge of Content and Teaching-Geometry (KCT-G) is a domain that combines knowing about teaching and knowing about geometry. KCT-G primarily focuses on the planning of the teacher, the sequencing of geometry topics so that students are successful, or what geometry examples the teacher decides to show the students.

10.2.1 Purpose of Study

The purpose of this study was to compare what I call the *Geometry Teaching Knowledge* (GTK) of pre-service and practicing high school teachers; GTK includes MKT-G and awareness of geometric techniques and methods used in the geometry classroom. This study examined the differences in knowledge among different groups of teachers and where this knowledge is developed.

This study focused on the knowledge of high school pre-service teachers at a four-year university in the State of Texas (in the United States) and that of practicing high school geometry teachers from multiple school districts in north and central Texas.

10.3 Research Questions and Design

The research questions for this study are:

1. What do high school pre-service teachers and high school geometry teachers know about *Geometry Teaching Knowledge*? *Geometry Teaching Knowledge* consists of the following: Mathematical Knowledge for Teaching-Geometry (MKT-G) and awareness of geometry techniques and methods used in the high school geometry classroom.
2. How do pre-service and current high school teachers' *Geometry Teaching Knowledge* compare?
3. Where is awareness of geometry techniques and methods used in the classroom developed?

10.3.1 Sample

The study was conducted at a central Texas university and at school districts throughout the state of Texas. The sample was composed of 53 pre-service high school mathematics teachers at the university and 36 practicing high school geometry teachers in multiple school districts in north and central Texas. The pre-service teachers were chosen based off their completion of their coursework in the program. The pre-service teachers were in their Junior or Senior years of their degree program and had completed the required geometry content course. The geometry content course taught at this central Texas university is called *Modern Geometry*. This course focuses on Euclidian Geometry and historical aspects of Geometry. This course is a mathematics content course that is required of the secondary pre-service teachers, but there is little pedagogical content covered. Pre-service teachers at this point in their degree plan have at least taken two education courses: *Curriculum and Technology* and *Adolescent Growth and Development*. By choosing pre-service teachers at this point in their degree, there is a guarantee that the pre-service teachers have completed the majority of their required coursework for their specific graduation plan, and are about to enter their student teaching experiences.

The high school geometry teachers were current teachers in multiple school districts in central Texas. Their degrees were obtained from a variety of different universities, and their teaching experience ranged from one to twenty years of experience teaching geometry. Only high school teachers who were currently teaching or had taught geometry within the previous two years were selected to participate in the study.

The pre-service teachers were a convenience sample; however, this sample arguably represents the knowledge base of pre-service teachers about to enter their student teaching experiences. The university uses The Mathematics Education for Teachers II Report (2010), which gives requirements and suggestions for teacher preparation programs in the United States. These requirements are based off the Common Core Standards (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010).

10.3.2 Instrumentation

To investigate pre-service and practicing high school teachers' *Geometry Teaching Knowledge*, data was gathered by means of an online Mathematical Knowledge for Teaching-Geometry (MKT-G) assessment developed by Herbst and Kosko (2014) and a post-assessment survey. The MKT-G assessment consists of multiple choice questions administered through the online platform Lesson *Sketch*. The post-assessment survey consists of demographic questions and questions regarding the experiences of the pre-service and high school teachers with different methods of instruction. The following is a sample item from the post-assessment survey asked to both pre-service and high school teachers (Fig. 10.1).

a. Investigations (Example: Discovery lessons) ________

b. The use of a compass and protractor to construct figures ________

c. Computer Software (Geometer's Sketchpad, GeoGebra, etc.) ________

d. Manipulatives/Models ________

e. Other: (please describe) ____

Total: _______

Fig. 10.1 Example methods/technique problem

Read the following techniques and consider which ones you would use in your own Geometry Classroom. You are given a total of 10 points to distribute among 5 techniques however you would like based on what you would think would be best for your students (assign a value between 0 and 10 to all items), with the number of points assigned to the topic reflecting the importance of these techniques in your classroom. You must use all 10 points. Please make sure the points add up to 10 by including a total count at the end.

Pre-service teachers and practicing high school teachers were asked different questions regarding their awareness of instructional techniques and methods. Pre-service teachers were asked: what types of instructional techniques or methods have they seen in their geometry courses, what types of instructional techniques or methods have they seen in their education courses, and what types of instructional techniques or methods would they use in their ideal classroom. An ideal classroom was described as one for which they would have an unlimited budget and unlimited resources. Due to the selection of pre-service teachers, most of the participants had not been in a current high school geometry classroom as an observer or an instructor, which is why the first two questions addressed what they had seen as students in their geometry course and education courses. Practicing high school teachers were asked what types of instructional techniques or methods do they use in their current geometry classes, what types of instructional techniques or methods have they seen in their professional development, and what types of instructional techniques or methods would they use in their ideal classroom. All participants took the online Mathematical Knowledge for Teaching-Geometry assessment and all but one high school teacher completed the post-assessment survey.

10.4 Data Analysis

10.4.1 MKT-G Assessment Results

The MKT-G assessment was given to pre-service teachers and practicing high school Geometry teachers to assess their Mathematical Knowledge for Teaching

Table 10.1 Descriptive statistics of percentage correct by MKT-G domain and total score

Domain	Mean (%)	Standard deviation	N
CCK-G	64.80	21.94	87
SCK-G	60.00	14.49	87
KCS-G	39.24	19.87	87
KCT-G	36.95	23.52	87
Total	53.67	13.58	87

Geometry. The assessment includes items that address four of the domains of mathematical knowledge for teaching; Common Content Knowledge-Geometry (CCK-G), Specialized Content Knowledge-Geometry (SCK-G), Knowledge of Content and Students-Geometry (KCS-G), and Knowledge of Content and Teaching-Geometry (KCT-G). Because I was interested in comparing scores for each domain, I scored the responses by looking at how many items of each domain participants responded correctly.[1] Because there were different numbers of questions addressing each domain, I calculated the proportion of correct responses for each domain. All 87 participants were combined to form the following descriptive statistics of the proportion correct over each of the domains and the total score. A lower score indicates lower knowledge of a domain and the higher score indicates higher knowledge of a domain. The results are presented in Table 10.1. When comparing the means of each of the domains, all the participants preformed the best in the Common Content Knowledge-Geometry domain, and performed the worst in the Knowledge of Content and Teaching-Geometry.

In order to better understand the differences between pre-service teachers and high school geometry teachers, a comparison using the raw test scores in each domain was performed. The box plots in Fig. 10.2 show the difference between the two groups in each of the four domains and the total raw scores.

A *t*-test for independent groups was performed in each of the domains as well as with the total scores. The descriptive statistics for each domain and Cohen's *d* are presented in Table 10.2. A *t*-test for independent groups was performed in each of the domains as well as with the total scores. Pre-Service teachers had lower CCK-G scores on the MKT-G assessment than current high school Geometry teachers, $t(76.61) = -3.642$, $p < .001$, $d = -.832$. Cohen's effect size ($d = -.832$) suggests a moderate practical significance. Pre-service teachers had lower SCK-G scores on the MKT-G assessment than current high school Geometry teachers, $t(71.899) = -5.882$, $p < .001$, $d = -1.3873$, which suggests a large practical significance. Pre-Service teachers had lower KCS-G scores on the MKT-G assessment than did those that were current high school Geometry teachers, $t(72.16) = -3.285$, $p = .002$, $d = -.773$. Cohen's effect size ($d = -.773$) suggests a moderate to large practical significance. Pre-service teachers had lower KCT-G scores on the MKT-G assessment than current high school Geometry teachers, $t(80.76) = -6.516$,

[1]Because the samples were small, the scores could not be scaled using the Rasch model; hence this analysis does not consider the difficulty level of each of the questions.

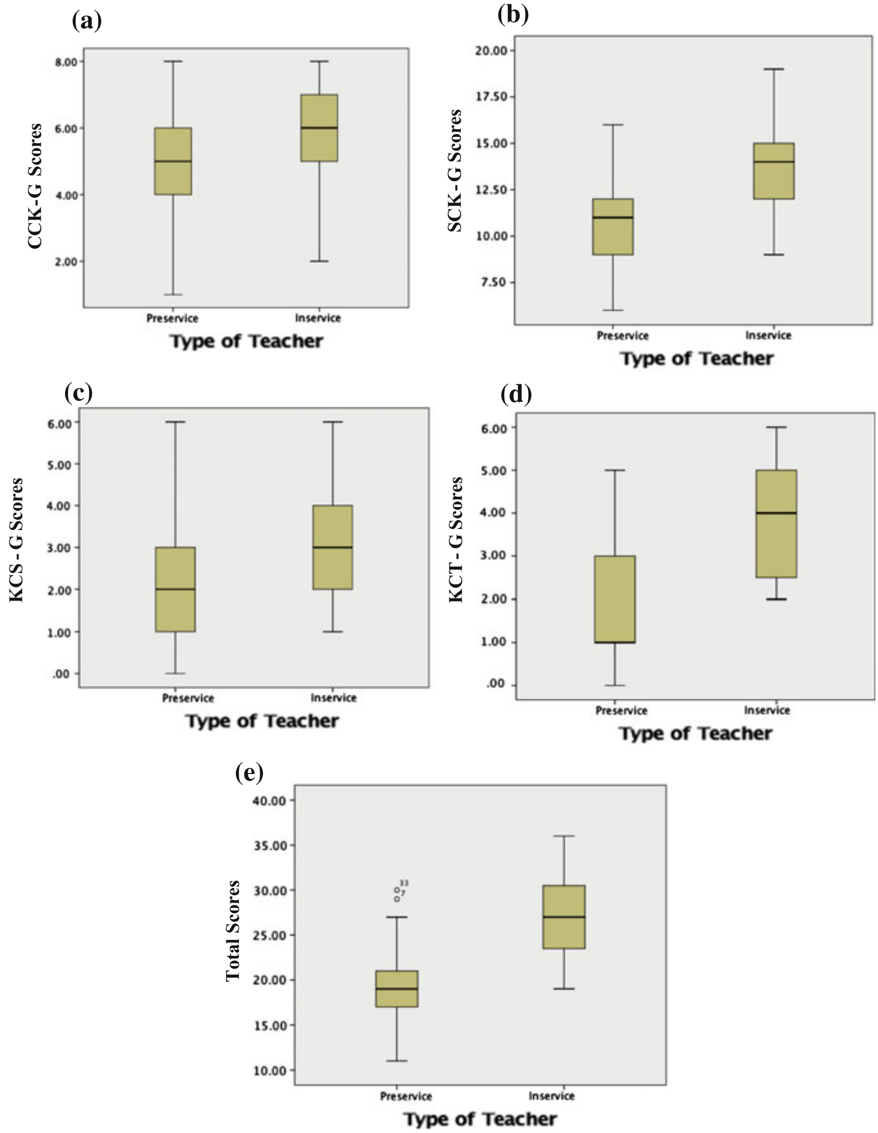

Fig. 10.2 **a** Boxplot comparing CCK-G scores of pre-service and in-service teachers. **b** Boxplot comparing SCK-G scores of pre-service and in-service teachers. **c** Boxplot comparing KCS-G scores of pre-service and in-service teachers. **d** Boxplot comparing KCT-G scores of pre-service and in-service teachers. **e** Boxplot comparing total scores of pre-service and in-service teachers

Table 10.2 Means, standard deviation, and Cohen's *d* by MKT-G domain and total score of pre-service and high school teachers

Domain	Pre-service		High school teachers		Cohen's *d*
	Mean (%)	Standard deviation	Mean (%)	Standard deviation	
CCK-G	58.09	20.74	78.30	20.25	−.832
SCK-G	53.43	11.85	69.31	12.77	−1.387
KCS-G	33.61	18.26	47.22	19.56	−.773
KCT-G	25.77	20.41	52.78	18.01	−1.45
Total	46.40	9.99	63.96	11.16	−1.80

Table 10.3 Correlations between MKT-G domains

	CCK-G	SCK-G	KCS-G	KCT-G
CCK-G	–			
SCK-G	.343**	–		
KCS-G	.391**	.389**	–	
KCT-G	.361**	.456**	.304**	–

$^{**}p < .01$

$p < .001$. Cohen's effect size ($d = -1.45$) suggests a large practical significance. Pre-service teachers had lower total scores on the MKT-G assessment than current high school Geometry teachers, $t(70.13) = -7.542$, $p < .001$. Cohen's effect size ($d = -1.80$) suggests a large practical significance.

Based on the *t*-tests performed, pre-service teachers had lower scores in all domains and in total scores. There is also large practical significance to all the comparisons.

Correlations between the domain scores are presented in Table 10.3, and suggest a moderate relationship between the different variables. These correlations were examined to make sure the results from this study were similar to the correlations reported by Herbst and Kosko (2014). These results show similar trends, which suggests that the four domains are interrelated, to a degree.

I calculated the correlations between each of the domains, total score, and the participants' years of teaching mathematics and years of teaching Geometry. The correlation between the number of years teaching mathematics and Common Content Knowledge-Geometry (CCK-G) and Knowledge of Content and Students-Geometry (KCS-G) were statistically significant, but weak. The correlation between Specialized Content Knowledge-Geometry (SCK-G), Knowledge of Content and Teaching-Geometry (KCT-G), and total score were statistically significant and moderate. The correlation between the number of years teaching Geometry and KCS-G was statistically significant, but weak. The correlation between CCK-G, SCK-G, KCT-G, and total score were statistically significant and moderate (Table 10.4).

Table 10.4 Correlations between Years experience and scores

	Years teaching math	Years teaching geometry
CCK-G	.239**	.323**
SCK-G	.361**	.352**
KCS-G	.265*	.286**
KCT-G	.448**	.397**
Total	.465**	.471**

*p < .05, **p < .01

10.4.2 Post-assessment Survey Results

As part of the Post-assessment Survey, participants were asked questions regarding their experiences with different Instructional Techniques and Methods that are frequently used in the geometry classroom. Pre-service teachers and current high school teachers were asked different questions regarding their knowledge. Pre-service teachers were asked what types of instructional techniques or methods have they seen in their geometry courses, what types of instructional techniques or methods have they seen in their education courses, and what types of instructional techniques or methods would they use in their ideal classroom. An ideal classroom was described as a situation in which they would have an unlimited budget and unlimited resources. High school teachers were asked what types of instructional techniques or methods they used in their current geometry classes, what types of instructional techniques or methods they had seen in their professional development, and what types of instructional techniques or methods they would use in their ideal classroom. Figure 10.3 shows the pre-service teacher survey results, specifically the distribution of experience with what types of instructional techniques or methods they had seen in their geometry courses, what types of instructional techniques or methods they had seen in their education courses, and what types of instructional techniques or methods they would use in their ideal classroom.

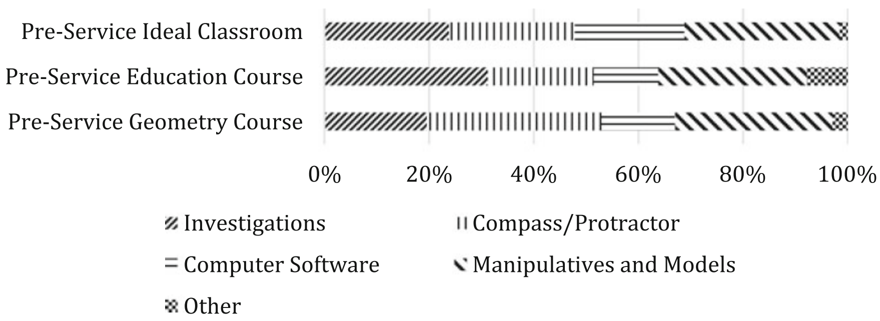

Fig. 10.3 Pre-service teacher survey results

For pre-service teachers' geometry courses, participants reported experiencing compass and protractor activities (33.3%) and manipulatives and models (30.1%) the most, and computer software (14.1%) the least. In their education courses, pre-service teachers reported seeing investigations (31.2%) the most and computer software (12.3%) the least. Pre-service teachers would use manipulatives and models (29.7%) the most and computer software (21%) the least in their ideal classrooms.

Figure 10.4 shows the practicing high school teachers' survey results, specifically what types of instructional techniques or methods they used in their current geometry classes, what types of instructional techniques or methods they had seen in their professional development, and what types of instructional techniques or methods they would use in their ideal classroom.

Practicing high school geometry teachers reported the use of *other* (35%) as most common in their classrooms. Other was defined as Lecture by 80% of the participants. They reported that computer software (11.6%) was used the least in their current geometry classes. High school teachers reported seeing investigations (27.3%) the most and compass and protractor activities (4.7%) the least in their professional development. When teachers were asked about their ideal classroom, high school teachers would use investigations (31.3%) the most and compass and protractor activities (15.7%) the least.

Pre-service teachers were asked which instructional techniques and methods they had used or seen in their geometry and education courses and practicing teachers were asked which instructional techniques and methods they had used or seen in their professional development. Attention was given to this comparison to investigate the methods taught at the university for pre-service teachers and the methods taught in the professional development opportunities given to practicing teachers. A chi-square test for independence was performed to examine the association between pre-service teachers' experience in their geometry and education courses to the practicing high school teachers' professional development. This test was found to be significant, $\chi^2(4, N = 86) = 123.84, p < .01$. This suggests that the pre-service teachers' distribution of what they see in their geometry and education

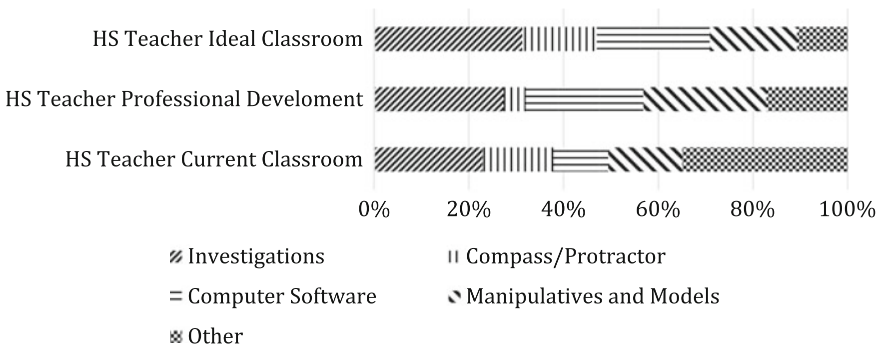

Fig. 10.4 Practicing high school teacher survey results

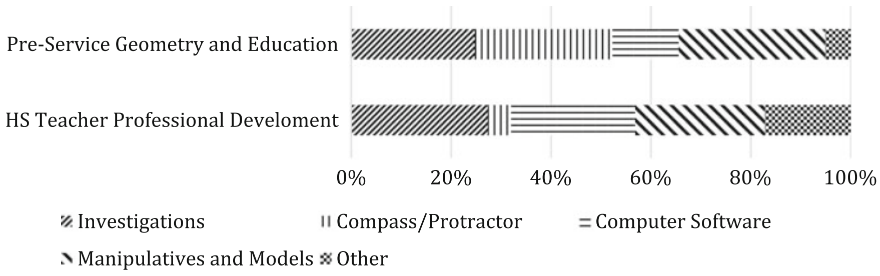

Fig. 10.5 Pre-service courses versus high school professional development

courses and what high school teachers have seen in their professional development are not independent. In Fig. 10.5, the strip diagrams show the distribution among the instructional techniques and methods of the pre-service teacher's Geometry Courses and what they would use in their ideal classroom.

Pre-Service teachers have seen more compass and protractor activities (27.5% of the time), and more manipulatives and models (29.3%) in their geometry and education courses when compared to high school teacher's professional development (4.7 and 25.9% respectively). High school teachers reported more investigations (27.3%), computer software (24.8%), and other (17.2%) in their professional development than pre-service teachers have seen in their geometry and education courses (24.7, 13.3, and 5.1% respectively). The responses for *other* in professional development included teaching strategies, classroom management, project based instruction, and direct teach/lecture, and the responses for *other* in their geometry and education courses included lesson plans, PowerPoints, projects, and lecture.

Pre-service teachers were asked which instructional techniques and methods they had used or seen used in their geometry and education courses, and practicing teachers were asked which instructional techniques and methods they used in their current classroom. This comparison was chosen because pre-service teachers would expect to see the instructional techniques and methods used in current high school classrooms during their courses at the university. A chi-square test of independence was performed to examine the relation between pre-service teachers' experience in their geometry and education courses to the current high school teachers' geometry classes. This test was found to be significant, $\chi^2(4, N = 86) = 196.19, p < .01$. This suggests that what pre-service teachers see in their geometry and education courses, and what high school teachers are using in their current geometry classes are not independent. In Fig. 10.6, the strip diagrams show the distribution among the instructional techniques and methods of the pre-service teacher's current geometry courses and what they would use in their ideal classroom.

Pre-service teachers reported more experience with compass and protractor activities (27.5%), and manipulatives and models (29.3%) than high school teachers reported using in their current classrooms (14.9 and 15.6% respectively). High school teachers reported more time spent on other (35%) than pre-service teachers

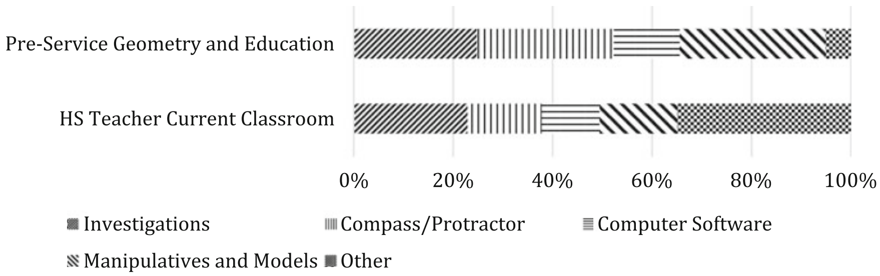

Fig. 10.6 Pre-service courses versus high school current class

claim in their geometry and education courses (5.1%). Lecture and Direct instruction is what 49% of the high school teachers described as *other*. Pre-service and high school teachers distributed points similarly to the investigations (24.7 and 22.9% respectively) and computer software (13.3 and 11.6% respectively).

Both groups were asked how they would spend time if they had an ideal classroom. An ideal classroom would consist of having unlimited resources and time. A chi-square test of independence was performed to examine the relation between Pre-Service teachers' ideal classroom and current high school teachers' ideal classroom. This test was found to be significant, $\chi^2(4, N = 86) = 59.93, p < .01$. This shows that what high school teachers think would be best for their ideal classroom and what the pre-service teachers think would be best for their ideal classroom are not independent. In Fig. 10.7, the strip diagrams show the distribution among the instructional techniques and methods of the pre-service and high school teachers' ideal classrooms.

Pre-service teachers thought that more compass and protractor activities (24% of the time) and manipulatives and models (29.7% of the time) were important to their ideal classes when compared to the high school teachers (15.7 and 18.4% respectively). The high school teachers thought more investigations (31.2%) and computer software (23.9%) would be important to their ideal classrooms, as well as a larger portion dedicated to other (10.7%) when compared to pre-service teachers' distribution of classroom time (23.8, 20.9, and 1.6% respectively). Lecture and Direct teach is what 49% of the high school teachers described as other.

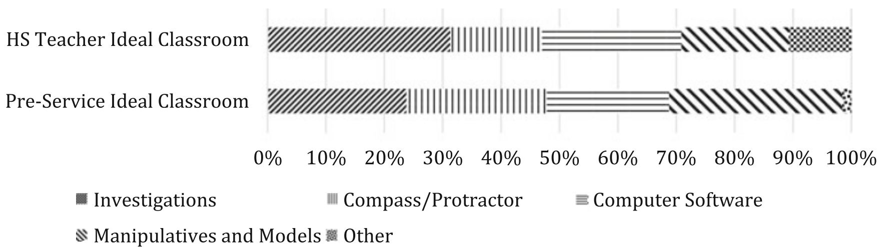

Fig. 10.7 High shool versus pre-service teachers' ideal classroom

10.5 Discussion

It could have been expected that the pre-service teachers would not do as well on the MKT-G as the practicing high school teachers because the high school teachers have been actively working with students and refining their geometry knowledge through practice, but this study sheds light on how the groups of teachers compare with one another. The primary domains where pre-service and high school teachers had the largest difference were Specialized Content Knowledge-Geometry (SCK-G) and Knowledge of Content and Teaching-Geometry (KCT-G). Specialized Content Knowledge-Geometry is "mathematical knowledge and skill unique to teaching" (Ball et al., 2008, p. 400). SCK-G is the knowledge of mathematics that is not necessarily used in any other field. Knowledge of Content and Teaching-Geometry is the category that "combines knowing about teaching and knowing about mathematics" (Ball et al., 2008, p. 401). KCT-G primarily focuses on the planning of the teacher, the sequencing of topics so that students are the most successful, or what examples the teacher decides to show the students. These results are not surprising when SCK-G is knowledge of geometry that would not be used in any other activity besides teaching high school geometry and KCT-G would require the pre-service teachers to have some idea of how to present material to students. The pre-service teachers were stronger in Common Content Knowledge-Geometry and Knowledge of Content and Students-Geometry, though they still score lower than the practicing teacher. Common Content Knowledge-Geometry is what they would get from their geometry courses at the university and the Knowledge of Content and Students-Geometry could come from them interacting with students through tutoring or remembering being a student themselves.

There were statistical differences between pre-service teachers and high school teachers in the knowledge of the different instructional techniques. This was unexpected, but this is a problem that needs to be addressed. One can understand teachers not being able to teach their ideal geometry class because of budgetary restrictions and time, and it seems that professional development would introduce current teachers to other instructional techniques that they might not be using in their current classroom, but the techniques presented in professional development would seem to transfer over to the teacher's ideal geometry class. It seems strange that pre-service teachers are being taught geometry and are in education courses, but their methods of teaching their ideal geometry class do not relate. Where are these pre-service teachers getting these ideas? It seems that there would be differences between the pre-service ideal classroom and the high school teachers' classroom because the pre-service teachers do not have as much classroom experience, and current high school teachers are drawing from their experiences being a geometry teacher. This also could relate to the MKT-G results showing that pre-service teachers have a lower score on the Knowledge of Content and Teaching-Geometry. One surprising result from these comparisons is the difference between the pre-service geometry and education courses and the professional development opportunities for high school teachers. It would seem that both of these types of teacher education would correspond in some way, but statistically they are different. The comparison between the pre-service teachers' geometry and education courses and the current high school geometry classroom is also interesting. If pre-service teachers are not being introduced to what the current high school teachers do in the geometry classroom, is this setting them up for failure?

10.5.1 Significance of the Study

This study sheds light on the *Geometry Teaching Knowledge* that high school pre-service and high school geometry in-service teachers. This study helps fill in the gap in research regarding Mathematical Knowledge for Teaching Geometry and awareness of geometric techniques and methods used in the geometry classroom that pre-service and high school geometry teachers possess and use. The instruments used to address these questions could be used in other pre-service mathematics teacher training programs and in professional development of high school teachers to address any gaps that may exist in their knowledge of geometry and of teaching geometry. This may impact future student performance in Geometry and Measurement since the three main reasons for a lag in performance are weak attention in K-12 curriculum, challenges associated with implementation of geometry and measurement in the classroom, and limited knowledge of the teacher (Steele, 2013).

10.5.2 Limitations of the Study

This study focused on a group of pre-service teachers from a single university in central Texas. The structure of this university's pre-service teacher training program could be different than other universities in Texas and in other states or countries. This study also focuses on currently practicing high school mathematics teachers in Texas. The knowledge level of geometry may be different depending on the state in which the teachers work. The professional development opportunities given to high school teachers varies depending on the district. In general, teachers are given a couple of days of professional development one week prior to the start of the school year and a day of professional development after the Christmas break. While some of the results may be extended beyond the scope of this university and state, any generalizing must be done cautiously.

The MKT-G assessment results were analyzed using the number correct in each of the domains and the total. Difficulty of each individual question was not considered because the sample was too small to estimate item difficulty parameters.

I developed the survey given to all the participants. The intention for the survey was to gather information about the knowledge of instructional methods and strategies of the participants. There is no guarantee that the survey accurately gathered all the knowledge of the participants.

10.5.3 Future Research

This study brought up issues of the differences in *Geometry Teaching Knowledge* between pre-service and currently practicing high school teachers. Pre-service teachers were weaker in all domains, but primarily in Specialized Content Knowledge-Geometry (SCK-G) and Knowledge of Content and Teaching-Geometry (KCT-G). There is a need for future research that focuses on these domains, specifically to target what can be done to increase scores in these domains for pre-service and high school teachers.

This study has shown there are differences in pre-service and high school teachers' experiences with instructional techniques and methods. Further research is needed to investigate the different instructional techniques and methods used in pre-service courses and professional development courses. These two forms of teacher education courses would correspond, and that knowledge would be transferred to the teachers' ideal geometry class. There is also a need for more research into ways they can implement what they learn in their teacher education courses into their current or future classroom.

Further research is needed to elaborate on the origin of *Geometry Teaching Knowledge* in pre-service and practicing high school teachers. If we can pinpoint where the majority of this knowledge is obtained, then we can make sure pre-service teachers have those experiences in their training programs to better prepare them for entering the high school classroom.

While this study is focused on *Geometry Teaching Knowledge*, there is a need to extend this type of research into other secondary mathematics courses (e.g., Algebra 2, Pre-Calculus, and Calculus), and even into post-secondary education. These results provide some insight into how this could be extended to other subjects, but specialized assessments will need to be developed.

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