

A Two-Factor State Theory



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Abstract When studying longitudinal phenomena, the notions of traits and states can be a useful classification. Specifically, traits represent basic human characteristics that have a permanency or enduring property, while on the other hand, states are environmental or ephemeral that are more time specific. Admittedly, research often focuses on traits and the relationships of these traits to other important variables. Moving in a different direction, this contribution focuses on the more ephemeral aspects of longitudinal variables, that is, states. A very practical justification for this direction is model fit indices. A probably more important rationale for expanding the state model is to obtain a more accurate reflection of the situation under study. To establish a common foundation, a longitudinal factor analytic model and a latent curve model are presented. Next, a statistical model of the ephemeral effects or state, which is analogous to Spearman's Two-Factor Theory is given. Lastly, a substantive illustration demonstrates the worthwhileness of this Two-Factor State Theory.

Keywords Traits • States • Longitudinal factor analysis • Latent curve analysis

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1 A Two-Factor State Theory

1.1 Introduction

When studying longitudinal phenomena, the notions of traits and states (Lord and Novick 1968, pp. 27–28) can be a useful classification. Specifically, traits represent basic human characteristics that have a permanency or enduring property, while on the other hand, states are environmental or ephemeral that are more time specific (Tisak and Tisak 2000). Admittedly, research often focuses on traits and the relationships of these traits to other important variables. In addition, one might be interested in the decomposition of observed measure variance into trait, state, and error variances for any psychological variable (Alessandri et al. 2012).

Moving in a different direction, this contribution focuses on the more ephemeral aspects of longitudinal variables, that is, states. A very practical justification for this direction is model fit indices. Concretely, the modeling of states may improve the acceptability of one's statistical model or more precisely one's Structural Equations Model (SEM) without the inclusion of "nuisance" parameters. A probably more important rationale for expanding the state model is to obtain a more accurate reflection of the situation under study. Parenthetically, this circumstance is analogous to the dichotomy between common and specific factors in classical factor analysis (Thurstone 1947). To improve model fit, one could include additional common factors, however, this approach might lead to theoretical unimportant factors, which could reflect undesirable or nuisance features of the items, such as sentence length.

To establish a common foundation, a longitudinal factor analytic model and a latent curve model are presented. Since these models are well established, the exposition will be terse. However, to facilitate an understanding, the common notation used in LISREL (Jöreskog and Sörbom 1996) is used. Next, a statistical model of the ephemeral effects or state, which is both conceptually and structurally analogous to Spearman's Two-Factor Theory (Spearman 1904) is given. Concretely, in Spearman's Two-Factor Theory, there is a general or g -factor that is common to all the items, and there are specific factors, which are unique to each item. Analogously, in the proposed Two-Factor State Theory, there is a temporal/general or t -factor that is present at each time point and that impacts each of the factors or saliences. Additionally, there are temporal-specific effects, which are unique to each factor or salience at each time point. Lastly, a substantive illustration demonstrates the worthwhileness of this Two-Factor State Theory.

1.2 A Basic Longitudinal Factor Analytic Model (FAM)

In this and the following two sections, three related longitudinal models are presented. The first is a longitudinal factor model (FAM), which is a standard

measurement model with measured variates, latent variables or factors, and measurement errors. The second or latent curve model (LCA) restricts each of the longitudinal factors to have a specific structure and includes temporal effects at this second-level. Finally, the third latent curve model with state structure (LCA-S) permits the usually uncorrelated state effects to be correlated.

Initially, consider a basic longitudinal factor analytic model (Tisak and Meredith 1989):

$$\mathbf{y}^{(k)} = \boldsymbol{\tau}_y + \Lambda_y \boldsymbol{\eta}^{(k)} + \boldsymbol{\epsilon}^{(k)}, \tag{1}$$

where $k = 1, 2, \dots, g$ indicates the populations or groups. $\mathbf{y}^{(k)}$ is an observed random vector of size mp (m is the number of measurement periods and p is the number of variables). The unobserved random vectors, $\boldsymbol{\eta}^{(k)}$ and $\boldsymbol{\epsilon}^{(k)}$, are of size mr and mp , respectively. Here r indicates the number of factors at each time point. The intercepts, $\boldsymbol{\tau}_y$, and slopes, Λ_y , have dimensions $mp \times 1$ and $mp \times mr$, respectively. Notice that both the intercepts and slopes exhibit the property of stationarity (invariance across time) and invariance across populations.

Concretely,

$$\boldsymbol{\tau}_y = [1_m \otimes \boldsymbol{\tau}] \text{ and } \Lambda_y = [I_m \otimes \boldsymbol{\lambda}],$$

where 1_m and I_m are a unit vector and identity matrix of size m ; \otimes is the kronecker product.

For this first-order model, the Means and Covariance Structure (MACS) are

$$\boldsymbol{\mu}_y^{(k)} = \boldsymbol{\tau}_y + \Lambda_y \boldsymbol{\mu}_\eta^{(k)} \text{ and } \Sigma_y^{(k)} = \Lambda_y \Sigma_\eta^{(k)} \Lambda_y' + \Theta_\epsilon^{(k)}, \tag{2}$$

where $\boldsymbol{\mu}_\eta^{(k)}$ is a $mr \times 1$ mean vector and $\Sigma_\eta^{(k)}$ is a $mr \times mr$ covariance matrix of the first-order factors. Lastly, $\Theta_\epsilon^{(k)}$ is a $mp \times mp$ covariance matrix of the unique factors. More specifically, in this longitudinal situation it has the following form.

$$\Theta_\epsilon^{(k)} = \begin{bmatrix} \Theta_{11}^{(k)} & \dots & \Theta_{1m}^{(k)} \\ \vdots & \ddots & \vdots \\ \Theta_{m1}^{(k)} & \dots & \Theta_{mm}^{(k)} \end{bmatrix},$$

where $\Theta_{tt}^{(k)}$ is a diagonal matrix of uniqueness of size p with $t = 1, 2, \dots, m$.

1.3 A Basic Latent Curve Model (LCM)

Next, consider a basic latent curve model (Meredith and Tisak 1990) that contains both traits and states:

$$\boldsymbol{\eta}^{(k)} = \boldsymbol{\alpha} + \Gamma \boldsymbol{\xi}^{(k)} + \boldsymbol{\zeta}^{(k)}, \tag{3}$$

where $\boldsymbol{\alpha}$ is a vector of temporal effects that impacts everyone in the same fashion; $\boldsymbol{\zeta}^{(k)}$ are individual temporal or state influences; and $\boldsymbol{\xi}^{(k)}$ is a set of individual saliences that determines how individuals change across time (these are the trait aspect of the model). Parenthetically in the parlance of latent curve analysis, salience is the weighting or individual change; it is analogous to a common factor in factor analysis.

The set of basis curves, Γ , describe general change across time. In general, they have the following form:

$$\Gamma = \begin{bmatrix} \gamma_{11} & & \gamma_{1r} \\ \gamma_{21} & \cdots & \gamma_{2r} \\ \gamma_{31} & \ddots & \gamma_{3r} \\ \vdots & & \vdots \\ \gamma_{m1} & \cdots & \gamma_{mr} \end{bmatrix}.$$

Notice that the elements, γ_{ij} , ($t = 1, 2, \dots, m; j = 1, 2, \dots, r$) can be fixed or parameters to be estimated, and if they are to be estimated, then identification constraints will be needed.

For this second-order model, the Means and Covariance Structure (MACS) are

$$\boldsymbol{\mu}_{\boldsymbol{\eta}}^{(k)} = \boldsymbol{\alpha} + \Gamma \boldsymbol{\kappa}^{(k)} \text{ and } \Sigma_{\boldsymbol{\eta}}^{(k)} = \Gamma \Phi^{(k)} \Gamma' + \Psi^{(k)}, \tag{4}$$

where $\boldsymbol{\kappa}^{(k)}$ are the means for latent factors or salience weights and the covariance matrices of trait and state factors are given by $\Phi^{(k)}$ and $\Psi^{(k)}$, respectively. Further, $\Phi^{(k)}$ is usually a symmetrical matrix and $\Psi^{(k)}$ is usually assumed to be a diagonal matrix, that is, temporal or state variables are unrelated.

1.4 A Latent Curve Model with State Structure (LCM-S)

Clearly from (4) an additional structure could be imposed on either the trait, $\Gamma \Phi^{(k)} \Gamma'$, or on the state, $\Psi^{(k)}$, aspects of the model (Tisak et al. 2017), however, for this project the diagonal covariances of the state or temporal factors, $\boldsymbol{\zeta}^{(k)}$, will be generalized to include correlated state factors. Concretely, the state factors associated with each trait factor will be allowed to correlate across time:

$$\Psi_{\boldsymbol{\zeta}}^{(k)} = \begin{bmatrix} \Psi_{11}^{(k)} & \cdots & \Psi_{1m}^{(k)} \\ \vdots & \ddots & \vdots \\ \Psi_{m1}^{(k)} & \cdots & \Psi_{mm}^{(k)} \end{bmatrix}, \tag{5}$$

where $\Psi_{tt}^{(k)}$ is a diagonal matrix size r ($t = 1, 2, \dots, m$). Note that this pattern is analogous to Spearman's Two-Factor Theory (Spearman 1904). Concretely, for each time of measurement, there will be a general state factor, which influences all the factors in the second-order model, and specific state factors, which are uncorrelated.

2 A Substantive Illustration of the Impact of State Variables in the Development of Positive Orientation

2.1 Introduction

The positive psychology movement (Seligman and Csikszentmihalyi 2000) has generated interest in the positive features of individual functioning. These findings have lead Caprara and colleagues (Caprara et al. 2010) to address what is common to self-esteem, life satisfaction, and optimism. In particular, they identified a common latent factor named positive orientation (POS). Additionally, in a longitudinal study (Alessandri et al. 2012), it was reported how POS relates to three additional constructs: (1) the quality of affective experiences (Watson et al. 1988); (2) the quality of social interactions (Hartup 1993); and (3) psychological resilience (Block and Kremen 1996). Given this longitudinal study of positive orientation, positive and negative affects, quality of social experiences, and psychological resilience across three time periods, this contribution generalizes the latent curve model with uncorrelated temporal effects to one that includes the suggested two-factor model on the state or temporal effects.

2.2 Method

2.2.1 Participants

As part of a longitudinal study the participants were male ($N = 45$) and female ($N = 81$) adolescents, who had complete data, from Genzano, Italy, a residential community near Rome. Notice that the original sample at Time 1 had 228 observations and that the attrition was mainly due to relocation from the area. For additional information on the attrition, see Alessandri et al. (2012). The first assessment (T1) was in 2000 at the age of 16; the second (T2) was in 2002 at the age of 18; and the third (T3) was in 2004 at the age of 20.

2.2.2 Measures

1. *Self-esteem*. Assessed by the 10 items of the Self-Esteem Scale (RSGE) of Rosenberg (1965). Coefficient alpha's at T1, T2, and T3 were respectively, 0.80, 0.81, and 0.83.
2. *Life satisfaction*. Assessed by the five items of the Satisfaction with Life Scale (Diener et al. 1985). Coefficient alpha's at T1, T2, and T3 were respectively, 0.90, 0.91, and 0.93.
3. *Optimism*. Assessed by the 10 items of the Life Orientation Test (SWLS) of Scheier et al. (1994). Coefficient alpha's at T1, T2, and T3 were respectively, 0.79, 0.83, and 0.81.
4. *Positive affectivity*. The Positive and Negative Affect Schedule (PANAS-P) of Watson et al. (1988). For positive affectivity, there were 10 items. Coefficient alpha's at T1, T2, and T3 were respectively, 0.81, 0.78, and 0.83.
5. *Negative affectivity*. The Positive and Negative Affect Schedule (PANAS-N) of Watson et al. (1988). For negative affectivity, there were 10 items. Coefficient alpha's at T1, T2, and T3 were respectively, 0.87, 0.80, and 0.81.
6. *Perceived quality of interpersonal relationships*. Assessed by the nine items of the Quality of Friendships Questionnaire (QDA) of Capaldi and Patterson (1989). Coefficient alpha's at T1, T2, and T3 were respectively, 0.81, 0.79, and 0.73.
7. *Psychological resilience*. Assessed by the 14 items of the Ego Resiliency Scale (ER89) of Block and Kremen (1996). Coefficient alpha's at T1, T2, and T3 were respectively, 0.73, 0.74, and 0.73.

Note that because of the small samples, the measures were aggregated into scales and the first three scales formed the construct of positive orientation. As described in the next section, these aggregations will lead to a "measurement model", which contains both measured variable (without errors) and latent variables with measurement errors.

2.3 Statistical Analyses

2.3.1 Program and Model Fit

For estimating the hypothesized model, we utilized LISREL 8.80 (Jöreskog and Sörbom 1996). To evaluate the fit of the models, chi-square and restricted chi-square tests were used. Additionally, the root mean square of approximate (RMSEA) of Steiger and Lind (1980) was used. Browne and Cudeck's (1993) guidelines are that $RMSEA < 0.05$ is a close fit, and $RMSEA < 0.08$ is a reasonable or near fit, but $RMSEA > 0.10$ is a poor fit.

2.3.2 Structural Equation Models (SEM)

Two major Structural Equation Models (SEM) were evaluated:

1. A Longitudinal Factor Analytic Model (FAM) with the corresponding modeling equation (1) and MACS (2). Specifically, on this first-order model both invariance and stationarity conditions were imposed on the intercepts and slopes. There were seven measured variables (scales) that were assessed at the three time points; hence, there were 21 variables. The first-order construct of positive orientation was obtained from self-esteem, life satisfaction, and optimism. The remaining four variables were treated without measurement error, that is unstructured except for the invariant and stationary intercepts and slopes. The covariance matrix of the unique factors was zero, except for self-esteem, life satisfaction, and optimism. Each of these variables were allowed to covary with themselves across time, and they had positive variances. This model reduced the 21 measured variables to 15 latent variables.
2. A Latent Curve Model (LCM) with the corresponding modeling equation (3) and MACS (4). For each of the five longitudinal variables (positive orientation, positive affectivity, negative affectivity, quality of relationships, ego resiliency), the temporal effects, α , were set to zero, and a single latent curve was used. Concretely, the 15×5 matrix of basis curves (Γ) is

$$\Gamma = \begin{bmatrix} 1 & 0 & 0 & & \\ \gamma_{21} & 0 & 0 & & \\ \gamma_{31} & 0 & 0 & & \\ 0 & 1 & \dots & 0 & \\ 0 & \gamma_{22} & & 0 & \\ 0 & \gamma_{32} & & 0 & \\ & \vdots & \ddots & \vdots & \\ 0 & 0 & & 1 & \\ 0 & 0 & \dots & \gamma_{25} & \\ 0 & 0 & & \gamma_{35} & \end{bmatrix}.$$

Further, the covariance matrix of the temporal (state) variables, $\Psi^{(k)}$, was as usual constrained to be a diagonal matrix. Lastly, one of the major interests in this study was the covariance matrix of the latent factor, $\Phi^{(k)}$, because it gives the relationships among the individual saliences.

Based on the findings of the two previous models, a modified third model was explored. This model with correlated temporal or state variables demonstrates the point of the manuscript.

Table 1 A summary of the fit indices for the different models assessed

Model	Df	Chi-Square	RMSEA	p-value
FAM	194	255.521	0.0358	0.00201
LCM-S	344	462.302	0.0459	0.00002
LCM	404	659.149	0.0790	0.00000

2.4 Results

The simple modeling fitting results for the two models (FAM and LCM) are given in the first and third rows of Table 1. Notice that FAM has a very acceptable RMSEA of 0.0358, while LCM only has a marginally acceptable RMSEA of 0.0790. Further using a restricted chi-square test ($\chi_R^2(210) = 403.628$, $p = 0.00000$), the reduced LCM was significantly different from the general FAM.

Given these results, how should one proceed? One could report the Latent Curve Model, or one could try to generalize it by modifying the number of basis curves (the trait aspect) or by modifying the covariance matrix of the temporal or state factors. Concretely, since $\Sigma_{\eta}^{(k)} = \Gamma\Phi^{(k)}\Gamma' + \Psi^{(k)}$, $\Gamma\Phi^{(k)}\Gamma'$ and $\Psi^{(k)}$ represents the trait and the state aspect of the model.

If one changes the trait aspect, that is, the number of basis curves, there are numerous combinations, which could lead to adding “nuisance parameters” to the model. Hence, one avenue to explore is to add structure to the previously diagonal matrix, $\Psi^{(k)}$. Using the Two-Factor State Theory, $\Psi^{(k)}$, has the form depicted in (5). The fit indices of this model, which includes correlated temporal effects, are given in the second row of Table 1 (LCM-S Model). Notice that LCM-S has a very respectable RMSEA of 0.0459. Further, when one compares the more general LCM-S to the more specific LCM, the restricted chi-square (LCM versus LCM-S) equals $\chi_R^2(60) = 196.847$, $p = 0.00000$. Thus, correlated temporal factors should not be ignored. In conclusion, there is a model (LCA-S) between the general (FAM) and the reduced (LCM) models, which is an improvement in terms of the fit indices over the (LCM).

Notice that the degrees for freedom for these models represent the difference between the observed data means (21) and the data covariances (231) for the two genders for a total of 504 and the number of parameters estimated in each model. To illustrate, the FAM has 310 parameters that are estimated; so the degrees of freedom for this model is $504 - 310$ or 194.

3 Discussion

Earlier it was pointed out that the notions of traits and states can be an important and useful classification in longitudinal studies. These two entities are expressed in (3) and the corresponding means and covariance structure in (4). Concretely, traits

may be expressed by the formulation: $\Gamma\xi^{(k)}$, and states or temporal effects may be represented in the random vector, $\zeta^{(k)}$.

Psychological constructs, like positive orientation, are not directly observable. Instead, constructs are latent entities introduced to explain the recurrent organization of an individual's internal states, such as feelings and emotions, as well to be used as causes of human behaviors (Borsboom et al. 2003). Researchers often studies those constructs at different timescales, depending on whether they are interested, for example, in the longitudinal development of individual's traits or aptitudes, or in the online tracking of individuals daily functioning. Whatever the timing of the study, psychological constructs usually reveal both trait and state variance, that researchers need to isolate and separately investigate. Whereas constructs characterized by trait variance only are rare, pure state-like constructs are often the exception. This contribution moves in a different direction, that is, states. Fit indices are a very practical justification for enhancing the structure on states. However, a more important justification for a more developed state model is an increased accuracy of the situation under study.

Additionally, whereas temporal consistency is one of the more distinctive characteristics of traits, states may often reveal a significant degree of continuity. Carry-over effects, denoting the tendency for a previous state to spill over time into the following state are often observed and often expected on a theoretical basis. For example, emotional states often display a high temporal continuity, denoting a tendency of the emotional dynamics to slow down until a state called emotional inertia (Kuppens et al. 2010).

To account for a significant continuity of states, researchers need tools able to allow the modeling of temporal variances in psychological attributes, such as the Latent Curve Model with state structure. Introduced as an expansion of the Latent Curves-Latent State-Trait modeling framework, the LCA-S is sensible to the continuity of states, allowing their inclusion in the model as covariances among subsequent states. Results presented in this paper point to this model as an interesting alternative to the general (FAM) and the reduced (LCM) models, which ensure a gain in terms of fit indices over the (LCM).

A longitudinal study on the development of position orientation illustrates the importance of states in one's statistical model. In Table 1, the Latent Curve Model (LCM) has a questionable mode fit index (RMSEA) of 0.0790. Further note that this LCM has a minimum formulation on the temporal effects, that is, these effects exist, but they do not correlate. On the other hand, if one generalizes the LCM to include structured states (LCM-S), the index of RMSEA is greatly improved. Lastly, when one compares the more general LCM-S to the more specific LCM, the restricted chi-square (LCM versus LCM-S) is significant. Thus, correlated temporal factors are statistically significant.

We surmise that in many situations, the LCA-S model may represent a more realistic alternative to simple LCM models. For example, we expect the LCA-S model to be of great value in allowing the modeling of intensive short-term studies (such as daily studies, weekly studies) whereas a significant continuity in trait

variance can be expected. Of course, the results presented in this paper are preliminary, and more work is recommended to examine the stability of the LCA-S model, under different empirical data conditions, and different variance/covariance structures.

Moreover, it is likely that the benefit introduced by the use of the LCA-S model are directly correlated with the length of the temporal lag, being probably higher for lags introducing more temporal variance in psychological constructs. In conclusion, we recommend to routinely consider the LCA-S as an alternative to simple LCM models, most of all, in all those conditions where including more common factors lacks theoretical justification and thus risks overfitting the model without any practical contribution to the understanding of the phenomenon under study.

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