Chapter 7
Makes Religion Happy or Makes Happiness Religious? An Analysis of a Three-Wave Panel Using and Comparing Discrete and Continuous-Time Techniques

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7.1 Introduction

Application of continuous-time methods in behavioural science is still rare. The analysis of longitudinal data almost always takes place in discrete time. In this chapter we explain in detail the serious problems connected with a discrete-time analysis and how to solve these problems by continuous-time modelling. We do this by means of an empirical example, the effect of religiosity on life satisfaction, which has been the subject of several discrete-time analyses in the past. In the next paragraphs, we discuss the theoretical background of the example, previous discrete-time studies and the discrete-time model adapted from Meulemann (2017). It is this model that will first be estimated in discrete time and next criticized and improved from a continuous-time perspective.

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As human beings die and are aware of being mortal, they must distinguish between This World and the World Beyond. Every human being has to get along with what this world is for and whether and what is beyond this world. Traditionally, religion promises to provide answers to these kinds of questions. It explains bad luck and injustice showing up in every human life within an overarching order. It provides a "sacred canopy" which "nomizes" life (Berger 1967). It is a resource to cope with life. The religiosity of a person is supposed to increase his or her life satisfaction. This is called the *nomization hypothesis*.

However, the more one is satisfied with life, the more one looks optimistically at it and takes it as it is. The more one is inclined to take the answers given by religion for granted, the more one will choose a religious belief that justifies and a religious community that reinforces one's satisfaction. In brief, the more one will become religious. This self-selection of satisfied people into religiosity is called the *optimism hypothesis*.

The nomization hypothesis has been examined longitudinally in four panel studies. First, in 16 yearly waves of the German Socio-Economic Panel (GSOEP), a fixed effects regression—that is, a regression of the change of the dependent variable on changes of the independent variables—shows a positive impact of the frequency of church attendance on the general life satisfaction (Headey et al. 2008, p. 18). Second, in a 1-year panel in the USA, regressions controlling for the former level of the dependent variables (yet not for the former levels of the independent variables) show a positive impact of the frequency of church attendance on general life satisfaction. However, a real effect is doubtful, because in the short time interval of 1 year, church attendance and life satisfaction change only slightly (Lim and Putnam 2010, p. 924). Third, in a further 1-year panel in the USA, not church membership but assessment of the importance of religion in life increases a specific life satisfaction—namely, with the family (Regnerus and Smith 2005, pp. 39–40). Fourth, in a 12-year panel study controlling for the former dependent variables, neither public nor private religious practices have an effect on general life satisfaction (Levin and Taylor 1998). We conclude that a positive impact of religiosity on life satisfaction has been confirmed strictly—over a longer time span and by the appropriate means of a fixed effects regression—only once: In the GSOEP study. As plausible as the nomization hypothesis seems to be, it is not yet strongly founded empirically. To our knowledge the optimism hypothesis as a causal hypothesis has never been examined empirically.

At first sight the nomization and optimism hypotheses seem to contradict each other. However, both could also be operating simultaneously in a reciprocal relationship across time. Whether the effect is in one of the two directions, in both directions or in none, and whether the sign of the effect is positive or negative can only be tested, if both are measured more than once—that is, longitudinally. In the following, how religiosity and life satisfaction measured at age 30 affect each other at age 43 and how religiosity and life satisfaction measured at age 43 affect each other at age 56 are examined. Thus, stabilities and cross-effects of life satisfaction and religiosity are analysed across time. At all three ages, 30, 43 and 56, religiosity is split up in its two main dimensions, practice and belief—measured as church

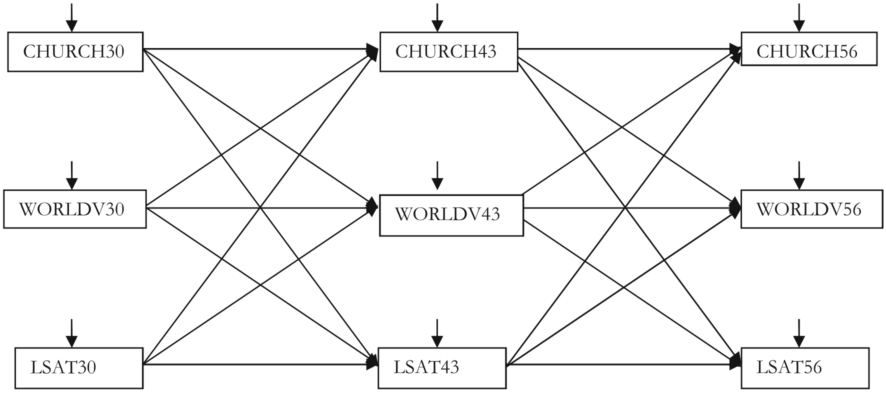

Fig. 7.1 Religiosity as church attendance (CHURCH) and Christian world view (WOLRDV) and life satisfaction (LSAT) at ages 30, 43 and 56

attendance and Christian worldviews. Thus, there are three variables measured three times, and the path diagram of the hypothesized causal system is given in Fig. 7.1.

In this so-called Markov chain model, causal hypotheses connect neighbour time points only. We suppose that at each time point, only experiences of the preceding period can have an impact. Impacts of earlier periods are supposed to be mediated and controlled by the immediately preceding ones. LSAT30, for example, can have no impact on LSAT56 that has not been already taken up by its impact on LSAT43 and the two stabilities in between. Stated differently, at each time point, the variables contain all information relevant to predict the future, and previous time points do not improve prediction.

As a structural equation model, the model has the rather simple recursive structure. The model structure would have been nonrecursive (interdependent), if in addition to lagged reciprocal effects (e.g. CHURCH30 to WORLDV43 and WORLDV30 to CHURCH43) also instantaneous reciprocal effects would have been specified (e.g. CHURCH43 to WORLDV43 and WORLDV43 to CHURCH43). In cross-sectional research, such a nonrecursive structure would be the only possibility to analyse reciprocal effects. In longitudinal research, often both lagged and instantaneous effects (instead of correlated residuals) are specified, in particular when the observation intervals between waves are big (e.g. Abele et al. 2011). We did not do this, because it leads to special problems that can be avoided by continuous-time analysis techniques.

The causal system in Fig. 7.1 has been analysed by Meulemann (2017). He used discrete-time (DT) structural equation modelling (Bollen and Brand 2010), controlled for unobserved heterogeneity by a random person factor, and used full information maximum likelihood (FIML) estimation, which takes care of arbitrary missing values based on the so-called missing-at-random (MAR) assumption

(Wothke 2009). In the following, we reanalyse the same data using continuous-time (CT) structural equation modelling (Oud and Delsing 2010; Oud and Jansen 2000) in order to show exemplarily the added value of CT over DT. In Meulemann (2017) the effects of some exogenous variables were also part of the model. To ease comparison, in this chapter, we skip both in DT and CT exogenous effects and limit the model to the relationships between the three endogenous variables. However, as in Meulemann's DT model, a single random person factor or "trait" for all three endogenous variables simultaneously was added to the CT model. A single trait not only saves degrees of freedom, but controlling for a general level in these variables with similar standardized score scale (see next section) was considered sufficient.

After presenting in the next section more specific information about the data and measurement, we report in Sect. 7.3 detailed results of the DT model. Section 7.4 explains the essence and advantages of CT over DT, while the CT results are given in Sect. 7.5. The DT analysis has been done by SAS-CALIS (SAS Institute Inc. 2013), the CT analysis by the R package CT-SEM (Driver et al. 2017).[1]

7.2 Data and Measurement

The sample is the Cologne High School Panel (*Kölner Gymnasiastenpanel, KGP*). Tenth grade high school students in the German federal state *Northrhine-Westfalia* have been first interviewed in written form in their classroom about their life plans in 1969 and reinterviewed orally three times in 1984, 1997 and 2010 about their life career between early and late midlife. The modal ages of the respondents in the reinterviews were 30, 43 and 56. Of the 3240 respondents in 1969, 1301 (40.1%) have been reinterviewed at ages 30, 43 and 56. However, time intervals between interviews were not for all respondents exactly equal to 13 years. The exact time intervals for each individual subject were known and will be used in the CT analysis. The sample is socially selective, because it has been drawn from students from the highest stratum of the German tripartite secondary school system.

The frequency of church attendance (CHURCH) has been measured by a single question with six options. The Christian worldview (WORLDV) has been ascertained by three statements of an inventory of Felling et al. (1987): "Life has meaning for me only because there is a God", "Life has a meaning because there is something after death" and "I believe that human existence has a clear meaning and follows a specific plan". For each statement, there were five response options; responses have been averaged. Life satisfaction (LSAT) has been measured on a scale from 0 to 10 as follows: "How satisfied are you nowadays altogether with your life" with a numbered response scale from 0 to 10, the extremes of which were labelled "totally unsatisfied" and "totally satisfied". Means were considerably

[1]The programming code of the analyses is available as supplementary material at the book website http://www.springer.com/us/book/9783319772189.

above the middle point and increased between the first and third observation time point slightly from 7.6 to 7.7; standard deviations decreased from 1.5 to 1.4. Detailed information about the data set is given in Weber (2017). For all three variables, values have been transformed into the standardized scores of the cumulative frequencies under the normal distribution (z-scores). This was done for the cumulative distribution of the three time points combined, so that the differences between them have been kept.

7.3 DT Model Results

As shown in Fig. 7.1, the model has 9 variables, that is, 54 nonidentical elements in the augmented moment matrix. Of our 1301 subjects, 1262 have complete information; 39 subjects have in total 107 missing values, which are taken care of by the missing value procedure of FIML. The model starts off at age 30 with the three initial means and six (co)variances, in total nine parameters, of three so-called predetermined variables. At age 43, the same variables are endogenous, and each depends on every other one at age 30. Thus, 9 regression parameters are needed; and, just as at age 30, 9 parameters for intercepts and residual (co)variances—resulting in altogether 18 parameters. At age 56, the model is exactly the same as at age 43 such that in a DT analysis, one normally would use again 18 parameters. It is customary in DT to test for equality of each of the corresponding parameters and set them equal if the test is passed. From a CT perspective, this is already questionable in the frequent case of unequal intervals, because then significant differences would show up in DT even if the underlying parameters are equal. In CT, however, time-varying parameters would not be handled stepwise at each of the discrete time points separately but by a function covering the whole time range of the model. In order to make the DT analysis comparable to the CT analysis, therefore, we set all parameters at age 56 equal to the respective ones at age 43. That is, there are no additional parameters: The model specified in both cases is time-invariant. Finally, one parameter for the trait variance is added which shows up in each of the six equations.

Altogether, the model has 28 parameters—leaving 26 degrees of freedom. The DT model has a $-2\times$Log-Likelihood (-2LL) of 26,563.86. The Chi-square value of 176.04 for testing the model against the saturated model is for 26 degrees of freedom significant.[2] This is expected, however, for such a big sample. The popular fit measure RMSEA (Browne and Cudeck 1993) with value 0.067 indicates that the model fits "reasonably". The results are presented in Table 7.1.

[2]The fact that the degrees of freedom left is a positive number, so making the Chi-square test possible, is only a necessary condition for identification of the model. A sufficiency proof of the identification of both the DT and the CT version of the model for as few as three observation time points is given in Appendix B of Angraini et al. (2014).

Table 7.1 Church attendance, Christian worldview and life satisfaction at ages 30, 43 and 56 in DT model

Initial parameters (age 30)				
	Means	(Co)variances		
		CHURCH	WORLDV	LSAT
CHURCH	0.0819***	0.4751***		
WORLDV	−0.0436	0.1903***	0.5024***	
LSAT	−0.0541	−0.1076***	−0.0437*	0.8847***
Dynamic parameters (ages 43, 56)				
	Intercepts	Regressions		
		CHURCH	WORLDV	LSAT
CHURCH	−0.0516**	0.4892***	−0.0279	−0.1188***
WORLDV	0.0235	−0.0244	0.4432***	−0.1117***
LSAT	0.0318	−0.2049***	−0.1410***	0.2591***
		Residual (co)variances		
		CHURCH	WORLDV	LSAT
	CHURCH	0.3074***		
	WORLDV	0.0468***	0.4679***	
	LSAT	−0.0589***	−0.0741***	0.6339***
	Trait variance	0.2051***		

−2LL = 26,563.86; Chi-square = 176.04 with df = 26; RMSEA = 0.067
*** $p<0.001$, ** $p<0.01$, * $p>0.05$

The results are similar to the ones in Meulemann (2017), although no exogenous variables are taken into consideration. First, all autoregressions are significantly positive, but church attendance and Christian worldviews turn out to be more persistent properties than life satisfaction. They can be seen as internalized early in life and held upright fairly well against shocks from outside. Also, there is substantial autoregression of life satisfaction, but this seems to be more susceptible to outside influences such as success and failure in life than religiosity.

Contrary to expectation, neither church attendance nor Christian worldviews have a positive effect on life satisfaction, nor life satisfaction on church attendance or Christian worldviews. Rather, all cross-regressions between religiosity and life satisfaction are significantly negative. On the one hand, religiosity seems to be rather a cost than a benefit in terms of life satisfaction. It costs time to go to church, and it may cost self-actualization to believe. On the other hand, neither is there a tendency of satisfied people to become religious. Moreover, the two cross-regressions between the two dimensions of religiosity are also negative but only slightly negative and not significantly. Practice and belief do not buttress each other.

The residual variances are found in conformity with the autoregressions to be smaller for the two religiosity dimensions than for life satisfaction. Finally, there is a strong trait variance representing unobserved heterogeneity. This may stem from private and occupational life success but also from personality factors and from socialization in family and school.

7.4 Discrete-Time Modelling Problems Solved by Continuous-Time Modelling

The first problem of a discrete-time model is that it ignores the processes taking place in continuous time between the measurement time points. The path diagram in Fig. 7.1 makes this clear. The arrows jump from one point in time to the next one, assuming that nothing happens between measurements. In fact, the discrete-time autoregression coefficients (horizontal arrows) and cross-regression coefficients (nonhorizontal arrows) in Fig. 7.1 are complicated mixtures of underlying continuous-time auto- and cross-effects in a constant interchange and dependent on the chosen observation interval. So, the true underlying auto- and cross-effects in continuous-time (CT) coefficients should be differentiated from the resulting auto- and cross-regressions in discrete-time (DT) coefficients. A variable with a high auto-effect, meaning that there is a strong tendency to sustain its value over time, tends also to retain the influence of other variables better and over a longer time interval than a variable with a low auto-effect. So, even a relatively small CT cross-effect can result in a relatively high DT cross-regression coefficient, if the variable influenced has a high auto-effect. But the converse can also be true: A relatively strong CT cross-effect having only small impact over a discrete-time interval (low DT cross-regression coefficient) because of a rather low auto-effect in the dependent variable.

Oud and Delsing (2010) show that, going in this way from DT to the underlying CT results, paradoxical changes can take place: Equal DT coefficients become different in CT, the strength order of coefficients reverses from DT to CT (e.g. if in DT the effect of CHURCH on WORLDV is larger than in the opposite direction, it becomes the other way around in CT), and nonzero coefficients in DT vanish or even change sign in CT. So, the first contribution of a CT analysis is to disentangle the true underlying CT auto- and cross-effects from the DT mixtures. One finds these mixtures in the autoregression matrix $\mathbf{A}_{\Delta t}$ (autoregression coefficients on the diagonal and cross-regression coefficients off-diagonally) in discrete-time equation (7.1), which further contains the DT intercepts $\mathbf{b}_{\Delta t}$ and prediction errors $\mathbf{e}_{t-\Delta t}$. The observation interval Δt as a subscript of $\mathbf{A}_{\Delta t}$ and $\mathbf{b}_{\Delta t}$ reminds that the discrete-time mixtures may differ for different observation intervals.

$$\mathbf{x}_t = \mathbf{A}_{\Delta t}\mathbf{x}_{t-\Delta t} + \mathbf{b}_{\Delta t} + \mathbf{e}_{t-\Delta t} \tag{7.1}$$

$$\frac{\mathrm{d}\mathbf{x}(t)}{\mathrm{d}t} = \mathbf{A}\mathbf{x}(t) + \mathbf{b} + \mathbf{e}(t) \tag{7.2}$$

The so-called drift matrix $\mathbf{A}$ in CT equation (7.2) analogously has the underlying CT auto-effects on its diagonal and the underlying CT cross-effects off-diagonally. Differential equation (7.2) explains the derivative $\mathrm{d}\mathbf{x}(t)/\mathrm{d}t$ or change in $\mathbf{x}$ at t for the interval Δt going to zero: $\Delta t \to 0$. Because of $\Delta t \to 0$, $\mathbf{A}$ and the CT intercepts $\mathbf{b}$ do not depend on the interval Δt any more. Oud and Delsing (2010) show how the DT mixtures $\mathbf{A}_{\Delta t}$ and $\mathbf{b}_{\Delta t}$ arise from the underlying CT $\mathbf{A}$ and $\mathbf{b}$ and explain in detail how a CT analysis gets the true underlying $\mathbf{A}$ and $\mathbf{b}$ back again from the mixtures. Autoregression matrix $\mathbf{A}_{\Delta t}$ in Eq. (7.1) and drift matrix $\mathbf{A}$ in Eq. (7.2) relate by the matrix exponential function as given in Eq. (7.3). It is by the highly nonlinear character of this relation that most of the paradoxical differences between DT and CT results can be explained. For the relation between DT and CT intercepts $\mathbf{b}_{\Delta t}$ and $\mathbf{b}$ as well as between errors $\mathbf{e}_{t-\Delta t}$ and $\mathbf{e}(t)$, we refer to Oud and Delsing (2010).

$$\mathbf{A}_{\Delta t} = \mathrm{e}^{\mathbf{A}\cdot\Delta t} \tag{7.3}$$

CT modelling can also be approached from the perspective of the lagged and instantaneous effects mentioned earlier. A big time lag is expected to result in relatively low values of the lagged coefficients (in any case in the autoregression coefficients but after some time interval also in the cross-regression coefficients), leading for many analysts to the correct feeling that lagged coefficients alone are not sufficient and to the decision to add the corresponding instantaneous effects to give a more appropriate picture of the underlying effects. However, Oud and Delsing (2010) describe a second problem of DT modelling and call it the lagged and instantaneous effects dilemma. It consists in the fact that the corresponding lagged and instantaneous coefficient values give quite different results and that in general the longer the lag, the higher the instantaneous coefficients become in comparison to the lagged ones. In the study by Vuchinich et al. (1992), for example, the dilemma was whether to choose for instantaneous or lagged effects between parental disciplinary behaviour and child antisocial behaviour. The authors found significant instantaneous effects but no significant lagged effects between these variables. Another example is in Becker et al. (2017), who related church attendance and education. When relating these variables instantaneously, they found a highly significant positive effect of education on church attendance. However, the lagged effects of education on church attendance were significantly negative. The dilemma is solved by Bergstrom (1966) in a nonrecursive model that imposes such restrictions on both types of coefficients—lagged and instantaneous—that one combined set of values results which approximates the true underlying CT coefficients (see details in Oud and Delsing 2010). In fact, Bergstrom presented his DT approximation of underlying differential equation (7.2) as a justification of the very use of nonrecursive models in economics. Instead of Bergstrom's approximate procedure, we use an exact procedure, but both solve the lagged and instantaneous effects dilemma of DT modelling.

The consequence of CT modelling to solve both problems described is that we should not causally interpret the DT autoregressions and cross-regressions in $\mathbf{A}_{\Delta t}$ and intercepts in $\mathbf{b}_{\Delta t}$ of Eq. (7.1) nor the instantaneous coefficients discussed in the previous paragraph but the CT auto-effects and cross-effects in drift matrix $\mathbf{A}$ and the intercepts in $\mathbf{b}$ of differential equation (7.2). This does not mean that the autoregression and cross-regression coefficients are useless. They tell a quite important but different story. Autoregressions and cross-regressions give the response over specific intervals for a unit impulse at the starting point. In particular, autoregression tells what after a specific interval Δt is left from a unit quantity in the variable at the starting time point. A cross-regression tells what after specific interval Δt the increase is in the dependent variable as a result of a unit increase in the independent variable at the starting point. However, a DT analysis gives these resulting quantities only for one specific interval Δt.

A third contribution of a CT analysis is that it provides the entire autoregression and cross-regression functions over the whole continuous time scale, that is, for all intervals, by modelwise interpolating between and predicting after the observation time points. An autoregressive function enables to answer, for example, after which interval only half of the unit value is left. A cross-regression function starts at zero for a zero interval (a causal effect needs some time to operate), then goes to a maximum at some point on the time scale and finally converges to zero again in a stable model. It enables to answer, for example, at what interval the maximal effect of the independent variable is reached and at what interval the effect becomes virtually zero. In addition to the autoregression and cross-regression functions, CT also provides the mean and covariance functions and so displays the means and variances/covariances not only for the discrete observation time points in the study but for all points in continuous time.

One main problem is the dependence of DT results on the chosen time interval. This leads to incomparability of results over different observation intervals within and between studies. If unaccounted for, it can easily lead to contradictory conclusions. In a multivariate model with three or more variables, one researcher could find a positive effect between two variables x and y, while another researcher, again in DT, finds no or a negative effect between the same variables, just because of a different observation interval length. This might well be the case, for example, in our study. After 13 years an originally strong effect of religiosity on life satisfaction might have faded away. Because results depend on the specific length of the chosen observation interval, even the use of equal intervals in DT studies does not solve the problem (Oud and Delsing 2010). Another interval might have given different results to both researchers as we discussed in the first problem of a discrete-time model.

A fourth contribution of CT analysis is therefore making the different and possibly contradictory effects in DT independent of the interval for equal as well as unequal intervals. So, by reporting the CT results instead of or in addition to the DT results, one enables other researchers with different or equal intervals to make a correct comparison with one's own results.

The last but not the least important contribution of CT analysis is in missing data handling (Oud and Voelkle 2014). In its attempt to limit the quantity of missing data, a DT analysis classifies the data in a restricted number of equidistant time groups: 1, 2, 3,.... The implication is that all data in one such group come from exactly the same time point. This is seldom the case. Measurements differ almost always in time, be it hours or even minutes. By putting data actually coming from different time points in the same group, the results of the analysis will become at least inaccurate and possibly unacceptable. Some missing value patterns can be handled in DT by so-called phantom variables, but this approach is limited to rather simple cases. For example, suppose one has a panel data set with four waves, 2 years between wave 1 and 2 and between wave 2 and 3 but only 1 year between wave 3 and 4. In DT one could choose time groups 1, 2, 3, 4, 5 and 6 and use phantom variables for totally missing groups 2 and 4. In CT the missing data problem is translated into an unequal measurement interval problem, and the missing data vanish. Each datum gets exactly the treatment it needs by combining it with its exact time interval. In this way, even a data set with all subjects having different measurement time points and different intervals is unproblematic. The use of different intervals is advocated by Voelkle and Oud (2013). While the previously mentioned advantages and solutions of CT do not lead to a different model fit, if no extra restrictions are imposed, giving data their exact time intervals in CT for each subject separately instead of the approximate equidistant ones in DT will change the data and therefore also lead to a different model fit in CT.

7.5 CT Model Results

The CT model results are reported in Table 7.2. The table contains also the model implied DT dynamic parameter values. The small differences of those values as well as of the initial parameters and −2LL with the ones in Table 7.1 are exclusively caused by the fact that CT inserts for all subjects individually the exact measurement intervals, while DT assumes equal intervals for all subjects.

When interpreting the values of the dynamic parameters in Table 7.2, it should be kept in mind that the scale range of autoregression from 1 (maximum autoregression in a stable model, no decay) to 0 (minimum autoregression, no predictability at all) translates to a range from 0 to $-\infty$ for the auto-effect in the CT drift matrix. So, the autoregression of 0.4892 for CHURCH in Table 7.1, which is highly significantly deviating from minimum 0 ($p < 0.001$), corresponds to the auto-effect −0.0647, also deviating significantly from 0 but which in this case is from the maximum value in a stable model. The story to be told for the autoregressions/auto-effects in general turns out similar in DT and CT. CHURCH is the most persistent and predictable variable, followed by WORLDV and LSAT, respectively. As in Table 7.1 also, all cross-effects are negative. Of course, the implied DT dynamic parameters in Table 7.2, which are easily calculated by the matrix exponential in Eq. (7.3) for $\Delta t = 13$, do not differ much from the ones in Table 7.1, because they only improve on the inexact measurement time points used in Table 7.1. If formally tested, the differences between the results in Table 7.1 and the implied DT results in Table 7.2 would in this case probably not be significant. Also the CT auto-effects and the sign of the CT cross-effects resemble those in Table 7.1. But, different from Table 7.1, the relatively low negative cross-effects between CHURCH and WORLDV turn out to be significant in CT ($p < 0.05$). Different also from Table 7.1 is that the strength order of reciprocal effects between CHURCH and WORLDV reverses in CT, the effect of WORLDV on CHURCH becoming more negative than in the opposite direction. Interesting is that all diffusion (co)variances

Table 7.2 Church attendance, Christian worldview and life satisfaction at ages 30, 43 and 56 with exact time intervals in CT model

Initial parameters				
	Means	(Co)variances		
		CHURCH	WORLDV	LSAT
CHURCH	0.0802***	0.4750***		
WORLDV	−0.0453	0.1903***	0.5024***	
LSAT	−0.0558	−0.1076***	−0.0437**	0.8447***
Dynamic parameters				
	Intercepts	Drift coefficients		
		CHURCH	WORLDV	LSAT
CHURCH	−0.0054*	−0.0647***	−0.0110*	−0.0313***
WORLDV	0.0029	−0.0128*	−0.0704***	−0.0316***
LSAT	0.0037	−0.0538***	−0.0403***	−0.1293***
		Diffusion (co)variances		
		CHURCH	WORLDV	LSAT
	CHURCH	0.0482***		
	WORLDV	0.0123***	0.0776***	
	LSAT	0.0179***	0.0174***	0.1630***
	Trait variance	0.2083***		
Implied DT dynamic parameters for $\Delta t = 13$ years				
	Intercepts	Regressions		
		CHURCH	WORLDV	LSAT
CHURCH	−0.0541	0.4813	−0.0276	−0.1189
WORLDV	0.0229	−0.0255	0.4363	−0.1136
LSAT	0.0315	−0.2068	−0.1425	0.2474
		Residual (co)variances		
		CHURCH	WORLDV	LSAT
	CHURCH	0.3089		
	WORLDV	0.0454	0.4761	
	LSAT	−0.0636	−0.0785	0.6377
	Trait variance	0.2083		

$-2LL = 26{,}579.13$
$^{***}p<0.001$, $^{**}p<0.01$, $^{*}p>0.05$

are positive, but because of the effects in the rest of the model, this results in negative values for the covariances of CHURCH and WORLDV with LSAT in DT (−0.0589 and −0.0741 in Table 7.1 and −0.0636 and −0.0785 in Table 7.2). Again, differences in interpretation between DT and CT in this case should not be exaggerated. Nevertheless, it is important to realize that, being independent of any specific interval, it is more reliable to interpret CT results than DT or implied DT results.

Beyond the more fundamental model specification and especially its independence of a specific DT time interval, CT has the advantage over DT of clearly depicting the process over the total period. Figures 7.2, 7.3, 7.4 and 7.5 display for

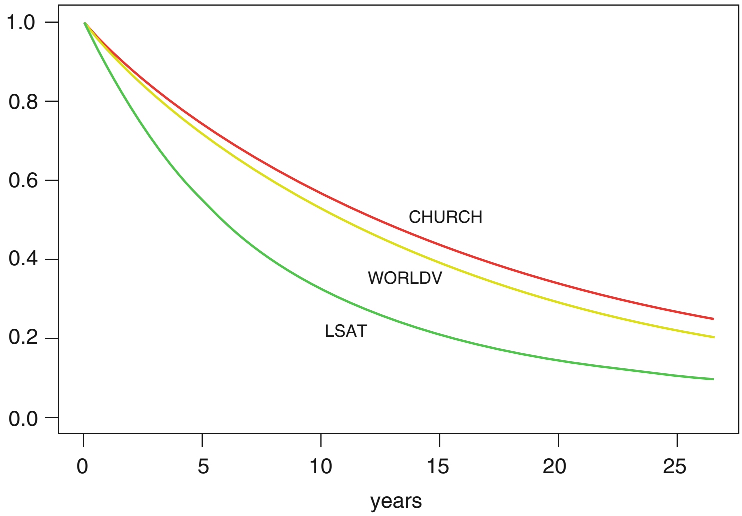

Fig. 7.2 Autoregression functions

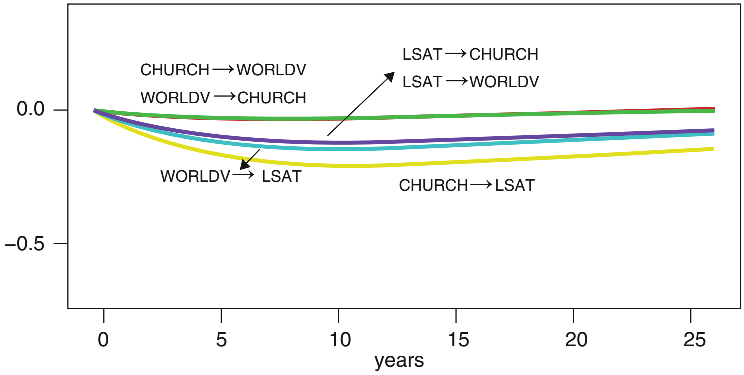

Fig. 7.3 Cross-regression functions

increasing intervals in continuous time over 26 years the estimated autoregressions and cross-regressions as well as the expected means and (co)variances in the studied group of subjects. Figure 7.2 shows that the autoregression is for CHURCH highest over the whole CT time scale and such that after 12 years still half of its value is left and after 26 years still more than 20%. Predictability of Christian worldview (WORLDV) on the basis of its previous value is at all intervals less, but the difference with CHURCH is small: after 11 years half of its value and about 20% after 26 years. Predictability of LSAT is considerably lower: half of its value only after 6 years and about 10% after 26 years.

All cross-regressions between the three variables in Fig. 7.3 turn out to be negative until the final interval of 26 years. However, not much is happening

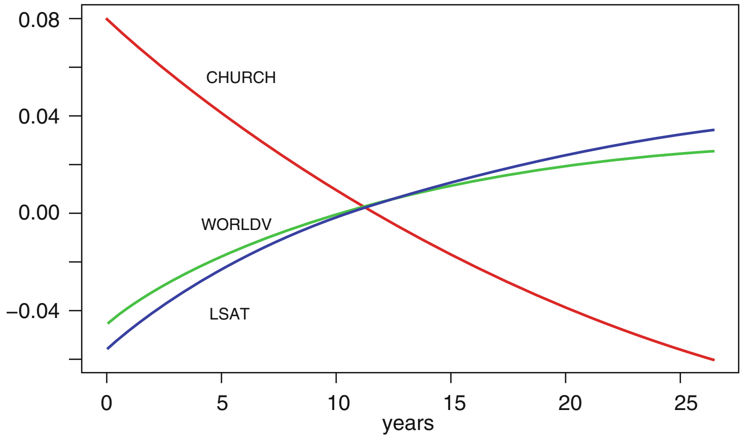

Fig. 7.4 Means across continuous time

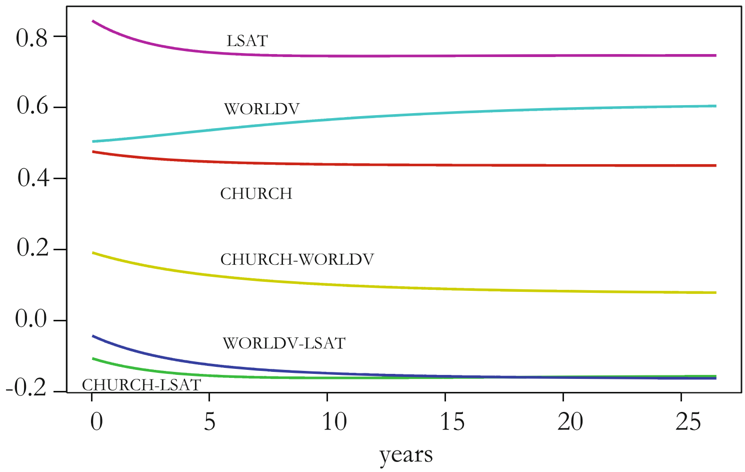

Fig. 7.5 Variances/covariances across continuous time

between CHURCH and WORLDV, neither in the short term nor in the long term. There are more substantial negative contributions from LSAT to these variables and from these variables to LSAT. Biggest is the negative contribution from CHURCH to LSAT, which reaches its maximum of −0.208 after 11 years, followed by the negative contribution from WORLDV to LSAT with maximum −0.146 after 10 years. The cross-regression functions in the opposite direction from LSAT to CHURCH and WORLDV, respectively, are almost identical and reach both their negative maxima of −0.120 and −0.116 after 11 years. It should be noted that these substantial contributions do not fade away rapidly, because after 26 years none of them is less negative than −0.075.

It will be no surprise that the CHURCH mean in Fig. 7.4 decreased over the 26 years period. However, WORLDV and LSAT showed a small increase over the same period. While in Fig. 7.5 the variances of CHURCH and LSAT kept being rather stable across time, the variance in WORLDV increased somewhat over the 26 years. There is a small positive correlation between CHURCH and WORLDV, which decreases somewhat across time. The other variables show small negative correlations across time.

7.6 Conclusion

We all lead our lives without really noticing how time passes by. Yet time and again we stop and look back. We register what has happened in the meantime—in our lives as well as in the lives of our fellow men. We notice the differences between some former and some current state; but we do not notice what has happened in between. We wonder how a difference could come up. If we are untroubled, we stick to contrasting former and current states. If we are pensive, we try to imagine a process that has led step by step from there to here. We live in continuous time, but we reflect upon our lives in discrete time.

Research on life histories, which is a reflection upon other people's life, cannot but do the same. It asks what people think and do at different times, it notices stabilities or differences and it tries to explain them. If it follows conventional wisdom, it takes the differences as given and looks for factors that may have determined them; time is split up into a sequence of discrete snapshots. If it is sophisticated, it assumes a process that has led from this to that value and constructs the values in between according to the rules of the process. In this chapter, we have compared two modelling approaches with the very same question and data. The question how religiosity and life satisfaction determine each other over a life span of 26 years of 1301 persons has been modelled by structural equations in discrete and in continuous time.

If we compare the results of the actual discrete-time analysis, supposing the data are collected at exactly the same discrete time points for all people, with the discrete-time results as implied by the parameters of continuous-time analysis, which accounts of the true individual measurement time points and intervals, the differences are small. Both analyses agree in three points. First, church attendance and Christian worldviews have stronger autoregressions than life satisfaction; the former seem to be habits of action and thought internalized early in life and the latter more easily subject to shocks from outside. Second, there are—contrary to expectation—negative rather than positive impacts of religiosity on life satisfaction and vice versa. In fact, both questions in the title should be answered negatively, on the basis of the discrete time as well as the continuous-time analysis. Religiosity, on the one hand, behaves rather as a cost than as a benefit in terms of life satisfaction. On the other hand, there is no self-selection of satisfied people into religiosity. Third, there is also a small negative reciprocal impact between the two dimensions of religiosity. Practice and belief interfere rather with one another than that they reinforce each other. Obviously, inconsistent patterns of both dimensions in cross-sectional population surveys which have often been interpreted as “believing without belonging” (Davie 2010) also show up in the longitudinal perspective on life histories.

Over and above these common results, the continuous-time analysis provides insights from the drift and the diffusion matrix that have no counterpart in discrete time; moreover, continuous-time analysis enables to depict the course of parameters, including means and variances/covariances, over time instead of only at a few specific discrete time points. The drift matrix in Table 7.2 reveals the underlying process and its realization across time in Figs. 7.2 and 7.3. The drift and diffusion matrix provided two main new insights into our specific data set. First, while the discrete-time dynamic parameters in Table 7.1 as well as in Table 7.2 showed that worldviews have a stronger negative impact on church attendance than vice versa, the corresponding drift parameters in Table 7.2 reversed this order. In addition, while both effects in Table 7.1 are nonsignificant, in Table 7.2 both are significant. So, practice seems to precede belief in a sense. Second, while the discrete-time residual covariances in Table 7.1 as well as in Table 7.2 between the two dimensions of religiosity and life satisfaction are negative, the respective diffusion covariances in Table 7.2 were positive. In both cases, the causal system as displayed in Fig. 7.1 is controlled for by unobserved heterogeneity—a person factor or trait. So, omitted effects seem to impact religiosity and life satisfaction in the same direction.

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