Chapter 5
Duo of Digital and Material Artefacts Dedicated to the Learning of Geometry at Primary School

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Abstract Our research project questions the bonus brought by technology and the complementarities of material and digital frameworks in situations which link together digital tools and material tools. We will present here some features to define a duo of digital and material artefacts. We will illustrate our point with a situation to stimulate the use of a pair of compasses as a tool to construct a triangle with given lengths of the sides. We will show that digital technology can bring a didactic bonus, an extra value to a material tool for learning. Digital technology can raise functionalities which refer to a material tool, and vice versa, the material tool can enrich the digital tool. We will show how a duo of artefacts, both material and digital, used in a situation, brings processes of assimilation and adaptation of utilization schemes from one instrument to the other which assist in their instrumental geneses and lead to conceptualisation.

Keywords Digital and material artefacts · Didactic bonus · Duo of artefacts Utilization schemes · Instrumental geneses

5.1 Introduction

The aim of this research work is both the design, and the evaluation, of software allowing mathematics learning at primary school. This software consists of an experimental approach based on playing with representations of mathematical objects on the computer interface, and linking the digital objects with the use of real tools. For this kind of technological framework linked together with the use of concrete material tools we question the effectiveness in stimulating the pupil to uncover mathematical concepts: can the technology bring some extra value to the material tools, helping thus to overcome some difficulties or some epistemological obstacles?

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5.2 Digital and Material Artefacts

5.2.1 Literature Review

Moyer-Packenham et al. (2013) studied the effects of physical and virtual manipulatives in mathematics on student achievement during fraction instruction. They randomly assigned participants, from third- and fourth-grade classes, to one of two treatment groups. One treatment group used texts and physical manipulatives in a regular classrooms; the other treatment group used virtual manipulatives in a computer lab. Their results demonstrate that using either physical or virtual manipulatives produces similar student achievement for third- and fourth-grade students learning the fraction concept.

In mathematics classrooms, digital technology is often either seen as a replacement or an adjunct of a text or physical manipulative activity. The examination of the role of the conjunction between digital and physical manipulatives is not often studied. Maschietto and Soury-Lavergne created two situations calling up a duo of artefacts in order to make the pupils learn the number system (2013) and the geometry (2015). In both cases the duo consists in linking a given material artefact with a Cabri Elem[1] e-book which refers to the material artefact. They show that what happens in terms of learning when using the real artefact is different from what happens when using the digital artefact in the e-book.

We take the same basic idea, calling up a duo of material and digital artefacts in order to design and analyze another didactic situation (Brousseau, 1997). Our main idea is not only to substitute a computer framework to the use of a material tool but link a digital artefact to a given material artefact. We claim that on top of the achievement of a task, tools will also become a contribution of mathematics activities of the pupils, yielding the emergence of mathematical concepts. We believe that it is possible to use the potentialities and constraints of a digital technology in order to create an artefact linked to a material artefact in such a way that this linking is an added value for the conceptualization. This program implies to answer the following questions: What is the advantage of such a digital and material duo for the learning process? How can we design the linking together between the physical and digital tools in a complementary way, in order to favour the learning? Is it possible to overpass the technical aspect when calling up a duo? And finally, can the duo contribute to the pupil's individual elaboration of mathematics knowledge?

5.2.2 Instruments Which Benefit from One Another

The use of artefacts, whether they are material or digital, forces the user to build and develop cognitive structures, called schemes (Vergnaud, 2009) to properly use the

[1]The Cabri Elem software developed by the Cabrilog company is used in this project in the framework of the scientific collaboration between Cabrilog and the Frensh Institute for Education.

artefact when accomplishing a task. An artefact's potential uses and restrictions influence the actions and strategies used to solve a given problem. The artefact becomes an instrument through a process of instrumental genesis (Rabardel, 1995) when the user has appropriated it and integrated it in his or her activity. Depending on the type of the given task, while using the artefact, the user creates utilization schemes to properly use it to accomplish the given task. The utilization schemes organise the actions taken, they are developed in situation and bound to one use. An instrument is a dual entity, mixing the artefact as well as the utilization schemes created by the user to accomplish a given task. "*The instrumental genesis is a complex process, needing time, and linked to the artefact characteristics (its potentialities and its constraints) and to the subject's activity, his/her knowledge and former method of working*" (Trouche, 2004).

In a situation which uses a duo of material and digital artefacts, the two artefacts interact with each other as they are used, and thus the two instrumental geneses are mixed. According to Rabardel (ibid), the utilization schemes are bound to their artefacts as well as the objects with which the artefacts interact. "*However, utilization schemes cannot be applied directly. They must be adapted to the specificity of each situation. They are implemented in the form of a procedure relevant to the particularities of the situation*" (ibid, p. 85). When the assimilation of a situation does not allow accomplishment, the scheme is progressively adapted to become a new scheme. "*The implementation of utilization schemes in new but similar situations (assimilation process) leads to the generalization of schemes by extension of the classes of situations, of artefacts and objects they are relevant to. It also leads to their differentiation since most often they have to change to adapt to new and different aspects specific to situations*" (ibid). In a situation which uses a duo of artefacts, both material and digital, we question the scheme's process of assimilation and adaptation from one instrument to the other. We formulated and tested the following research hypothesis: a duo of artefacts, both material and digital, used in a situation, brings processes of assimilation and adaptation of utilization schemes from one instrument to the other which assist in its instrumental geneses and lead to conceptualisation.

5.3 A Duo of Artefacts, Both Material and Digital, to Introduce the Compasses in the Geometric Construction of a Triangle

5.3.1 A Material Artefact: The Material Compasses

We suggest a situation which consists in teaching the geometric construction of a triangle using a ruler and a pair of compasses, knowing the lengths of its three sides. There are two objectives: firstly, to give meaning to the use of the compasses in the construction of a triangle of which we know the lengths of its three sides.

The second objective is to make the pupil's knowledge of triangles grow. Often, the use of the compasses in a geometric construction is linked to a technical ability. The goal of using the compasses is never mentioned. "*There is a confusion between the ability to trace precise lines for a circle with the compasses and the knowledge of reasons why the tool is adequate*"[2] (Artigue & Robinet, 1982). To understand the procedure of constructing a triangle with compasses is one thing, but to understand why the compasses are adequate to accomplish this construction is another. Much like Artigue & Robinet (ibid), we believe that depending on the given task, the conceptions used in the use of compasses are different.

Multiple kinds of difficulties can be shown in the process of using a pair of compasses to construct a triangle of which we know the lengths of its three sides. We identify two of them: the difficulty relative to the dimensional deconstruction of the triangle (Duval, 2005) and the difficulty of the instrumental genesis of the compasses. Indeed, in this construction, the compasses are not the tool which allows the tracing of the outline of the triangle—it's the tool which allows the determination of the triangle's third vertex a 0 dimensional object, hardly apprehended by the pupils.

We discern two types of tools: those which produce the sought-out object, and those who produce an intermediary object which doesn't belong to the sought-out object. The ruler, used to trace a segment, produces a segment, and the compasses, used to trace a circle, produce a circle. But the compasses, used in the construction of a triangle, do not produce one of the triangle's sides. To move from the "compasses to trace a circle", instrument which produces the circle, to the "compasses to determine the triangle's third vertex", instrument which produces circles (circular arcs), an intermediary object, is at the heart of the design of our duo.

We will present later on our methodology regarding the design of a duo and the choices made in a digital environment:

- first of all, to help the pupil to create the compasses instrument to draw a triangle;
- second of all, to put into place the dimensional deconstruction of the triangle without necessarily, first, going to the 0D objects.

5.4 Methodology on the Design of the Duo

"*Tools only having meaning when relating to situations in which they are used*," (see Footnote 2) (Bruillard & Vivet, 1994). A duo of artefacts, material and digital, is attached to a didactical situation (Brousseau, 1997) and makes itself useful in this given situation. Our situation combines both digital and pen-and-paper environments, and allows the articulation between the manipulation of digital tools and the use of a

[2]Translated by the author from the original article in French.

material tool. After having done an epistemological and cognitive analysis of the material artefact, at the same time, we develop a digital artefact articulating with the material one as well as a situation in which the duo is used. In such a situation, our digital and material tools rely on one another, and the technology brings an additional element on a conceptual scale. A duo of digital and material artefacts is not necessarily characterised by a digital artefact which simulates precisely the material artefact. Nevertheless, it is necessary that the articulation between the two artefacts shows the links between the two and presents elements which follow a logical continuity in terms of learning. But some discontinuities are also necessary to promote the evolution of pupil knowledge.

5.4.1 A Situation Which Involves Material Compasses and E-Books

The technology used to design digital environments was the Cabri Elem software. The Cabri Elem technology allows us to create all of the elements with which the pupil will interact: the objects to be manipulated, the possibilities of actions that can be done on these objects as well as the environment's feed-backs. Such a digital environment is organised within an e-book. The user can go through the pages freely, and can do the different suggested tasks (Mackrell, Maschietto, & Soury-Lavergne, 2013). An e-book offers several tasks which bring the user to use appropriate strategies to accomplish each of them. Thus, Cabri Elem technology allows us to design tools and tasks in a digital environment, articulating with material tools. It allows the thought of articulation between the digital tool and the material tool.

Our situation includes two e-books and two pen-and-paper activities. Its orchestration (Trouche, 2004) was put in place with the intention of alternating activities set in a digital environment and in a pen-and-paper environment. Accomplishing the situation consists in successively handling an e-book and a pen-and-paper activity then a second e-book and a second pen-and-paper activity (Fig. 5.1). In the first e-book, the primary objective is to form triangles by manipulating digital segments. It provokes the elaboration of a rotation-dragging instrument to rotate the segments. This rotation-dragging brings the use of compasses in the pen-and-paper environment. The first pen-and-paper activity's objective is to include the material compasses in the triangle's construction. The connection between the first e-book and the first pen-and-paper activity is meant to implement the dimensional deconstruction of the triangle without having, at first, to get down to geometrical points. The second e-book is meant to bring in the circle as a tool in the geometric construction of the triangle. Playing with the tools available through the e-book's pages makes the circle tool a necessary strategy. Finally the objective of the second pen-and-paper activity is to construct triangles with the compasses and ruler. In the design of the duo, a range of didactic variables and

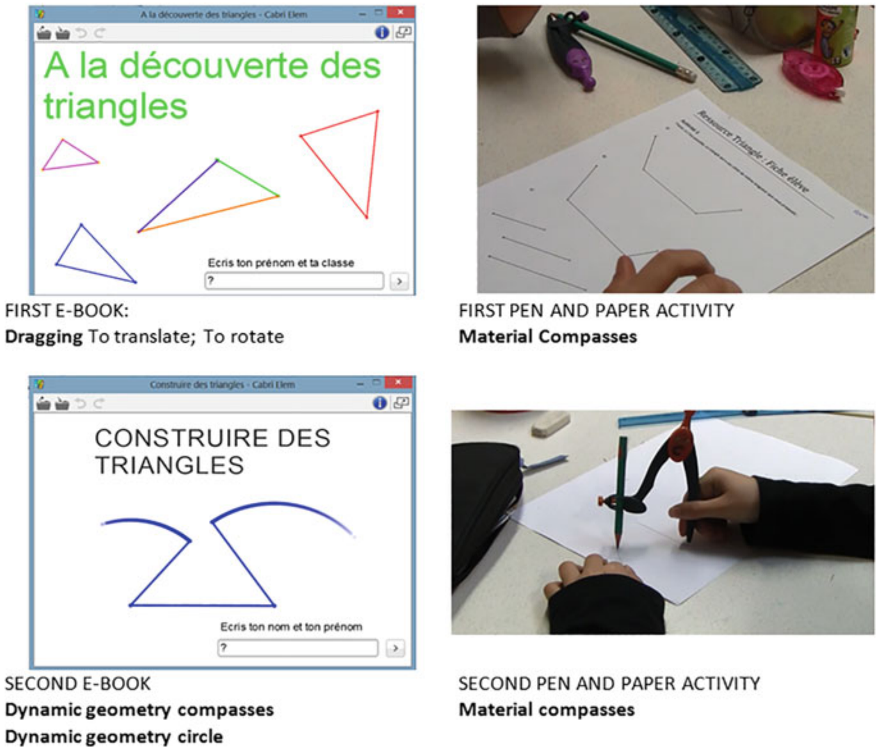


Fig. 5.1 A situation with the material compasses and e-books

mobilized artefacts induce the elaboration and evolution of strategies during the situation and of the connection between the digital artefacts and the material compasses.

5.4.2 A Choice from a Range of Didactic Variables

The first didactic variable is the length of the segments corresponding to the triangle's sides (a, b, c). For the three lengths $0 < a < b < c$, three values are associated to this variable. ($a, b, c = a + b$) The triangle is flat. (a, b, c with $c < a + b$) The lengths verify the triangular inequality and the triangle exists. (a, b, c with $c > a + b$) The triangle does not exist. The three values for this variable intervene in each part of the situation. The segment lengths suggested in the e-books and in the pen-and-paper activities allow the discovery of cases where the triangle does and does not exist. The flat triangle can be seen in the first e-book.

The second didactic variable concerns the possible dragging for the segments of the triangle's sides. Five values are associated to this variable: translation only;

rotation only; simultaneous translation and rotation; dissociated translation and rotation; no movement. The chosen values of this dragging variable at the different steps of the situation influence the implementation of strategies which, as we will later see, are the vehicles of learning.

The third didactic variable concerns the tools of geometry. We choose, in the design of the e-books, to provide the pupils with a toolbox available even when these tools are not useful to resolve the given tasks. In the first e-book all of the dynamic geometry tools are available even if none of them is necessary to resolve the given tasks. In the second e-book however these tools must be used to resolve the given tasks and the chosen values of the tool didactic variable influence the strategies that must be implemented. In the pen-and-paper environment all tools (pencil, ruler, setsquare and compasses) are always available. It is up to the pupil to choose which ones he must use to resolve the problem.

Our intention in designing a situation which makes use of a duo of artefacts, digital and material, in learning about the construction of the triangle with a ruler and the compasses is to characterise a milieu (Brousseau, 1997) that encourages the dimensional deconstruction of the triangle and the instrumental genesis of the compasses through this problem. We want the learning process to be included in the strategies implemented by the subject to resolve the set problems. In the next paragraph we detail the a priori analysis of the situation.

5.5 A Duo: Rotation Dragging and Material Compasses

5.5.1 Asymmetrically Dynamic Segments

The first duo mobilised in the situation consists of the material compasses and a digital artefact included in the first Cabri Elem e-book. The Cabri Elem software suggests the virtual compasses but both of the artefacts instrumented by the subject in this duo are the material compasses and the rotation-dragging of a point in a digital environment. In the e-book, the pupil is lead through the treatment of two tasks: forming triangles by the direct manipulation of segments with fixed and defined lengths, and determining whether three segments can be the three sides of a triangle (Fig. 5.2). The second task, on whether the triangle exists or not, is a mathematical question which problematizes the investigation of a triangle, thus resorting to drag the segments. The displayed segments on this page are represented as asymmetrical on screen and when they are moved.

Two types of movement are possible for a segment: translating the segment by holding the segment or its round end, and rotating the segment around its fixed round end by moving its cruciform end. The graphical distinction between the extremities, round or cruciform, lets the user anticipate the movement before doing it. The asymmetry of the digital segments relative to their movements is an essential part of the milieu constituted by this e-book. In fact, the digital

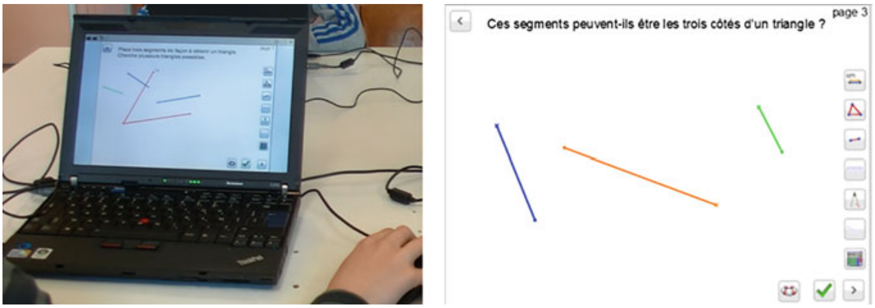


Fig. 5.2 Illustration of manipulations of digital segments in the e-book

environment places limits on the double dragging of the digital segments and forces a dissociation of the rotation and translation motions. When manipulating material objects, these two movements (rotation and translation) are realised simultaneously. Thus the digital environment highlights the rotation necessary to form a triangle from digital segments. Furthermore, the asymmetrical dragging of the segments induces an efficient winning strategy in building a triangle. The fact that both of the segment's ends do not rotate makes difficult an adjustment strategy. The easy adjustments to make are those by rotation, which lets the "broken line strategy" take shape. An efficient winning strategy for building a triangle from the asymmetrical segments given in the digital environment consists in forming, with three segments, a broken line whose extremities are cruciform. The triangle is then formed by rotating the two end-most segments of the broken line (Fig. 5.3). Thus the digital environment creates a milieu which highlights rotation, essential to form a triangle from digital segments, and leads to the implementation of a winning strategy that promotes learning. The broken line is one of the first steps in the dimensional deconstruction of the triangle. The activity in which we form a triangle in a digital environment with three segments by going through the broken line rests on a reconstruction of the two dimensional triangle starting from the one dimensional broken line. This strategy makes apparent that a triangle is a closed broken line.

In this first e-book, two dragging instruments emerge: translation-dragging and rotation-dragging. The rotation-dragging instrument is made to rotate the segments, particularly those segments at the ends of the broken line. It is essential to form a triangle in a digital environment. This instrument is what will spur the use of the compasses in the pen-and-paper environment. Firstly the segment, asymmetrical in its representation on the screen and in its movements, brings to mind the material

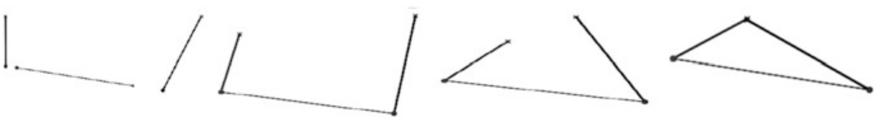


Fig. 5.3 The broken line strategy

Fig. 5.4 Juxtaposed strategies

compasses: one arm remains fixed while another arm turns. Secondly, in the digital environment, the rotation-dragging instrumentation produces utilization schemes which can be adapted to the use of the material compasses.

So as to permit the setup of anticipation strategies, the value of the dragging didactic variable is modified on one of the e-book's pages. Only the translation-dragging is possible on this page. Rotating the segments is no longer possible, and thus forming the triangle is no longer possible. Another strategy must be set up to predict whether the three segments can be the three sides of a triangle. Since the dynamic geometry compasses tool is available on the page, it can be used to set up a strategy. Strategies mobilising the dragging to translate the segments can also be elaborated. The two smaller segments can be juxtaposed over the longest one like a broken line, or in the style of triangular inequality (Fig. 5.4).

5.5.2 A New Function for the Compasses

The e-book is connected to the use of the material compasses in the pen-and-paper environment. This first pen-and-paper activity consists in constructing triangles whose sides are presented as segments drawn on the paper. The given segments are either in broken line formation or parallel to each other (Fig. 5.5). To complete these tasks, the pupil is given geometry tools: a pencil, a ruler, a setsquare and a pair of compasses.

The purpose of the first pen-and-paper activity is to mobilise the compasses in the construction of the triangle. Some of the milieu's elements, generated by the e-book, also appear in the pen-and-paper environment. The segments corresponding to the triangle's sides are already present on the paper, as they were in the digital environment. Furthermore, the broken line has completely integrated into the milieu. The broken line is part of the continuity of the digital and material duo of artefacts. This line whose extremities the pupils must rotate permits the switch from

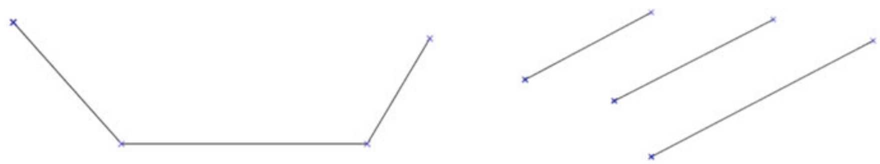


Fig. 5.5 First pen-and-paper activity

the rotation-dragging in the digital environment to the material compasses in the pen-and-paper environment. The material compasses are the artefact that replaces the rotation-dragging. The material compasses allow the subject to rotate a segment. The segment is stuck between the compasses' two arms. A new compasses instrument is created: the compasses to rotate a segment. In the first e-book, during the manipulation of digital segments, the pupil constructs schemes relative to the rotation-dragging to rotate the digital segment. These schemes can be associated to utilization schemes for the material compasses. A utilization schemes for rotating a digital segment can be described as: determine the two ends of the segment then grab the cruciform end and drag to rotate the end point. A utilization scheme for rotating a segment with the help of the compasses can be described as: distinguishing each arm of the pair of compasses, placing the needle on the steady end of the segment, spreading the arms and placing the pen lead on the end to be moved, and at least rotating the compasses whilst keeping the same spread and create a visible trace. Assimilations and adaptations between utilization schemes can be identified from one instrument to another. Whether in a digital or a pen-and-paper environment, in each utilization scheme for rotating a segment, these must be distinguished: the segment's extremities; the compasses arms. In each scheme the segment or the compasses must be rotated. When using the material compasses, there are some necessary adaptations: the compasses produce a visible line, marking the extremity of the rotating segment; since the original does not rotate, a stand-in for the rotated segment must be drawn.

5.6 The Material Compasses and the Dynamic Geometry Circle

5.6.1 The Dynamic Geometry Circle

The goal of the second e-book is to bring the circle as a tool used in the construction of the triangle. It is by playing with the allowed tools that the circle becomes the tool adapted to the situation (Fig. 5.6). At the beginning, the circle is used to verify if a broken line can be the outline of a triangle or not. The circle thus becomes the image of the trajectory of the extremity of a rotating segment.

This second e-book is connected to the use of the material compasses in the first pen-and-paper activity. Some elements of the milieu made in the pen-and-paper environment are kept in the second e-book so as to maintain continuity in the articulation of digital and material artefacts: the material compasses and the dynamic geometry circle. The broken line is still a key element of the milieu in the first pages of the e-book. It is still the element of continuity between the duo and the given situation. The milieu in the e-book is enriched by the dynamic geometry tools, more precisely, by the forbidden dynamic geometry tools.

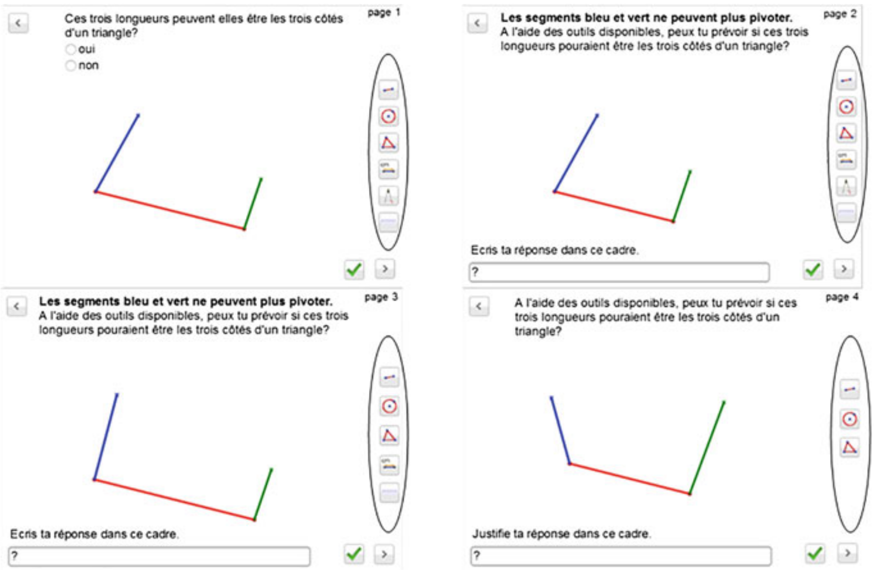


Fig. 5.6 Use of the dynamic geometry tools to involve the circle into the construction of a triangle

A tool initially used to verify then becomes a tool used to create. Afterwards, one must use the circle tool to determine the third vertex of the triangle (Fig. 5.7). The new instrument which is created in this e-book is the circle instrument used to identify a distance.

5.6.2 *The Material Compasses to Construct a Triangle*

The situation ends with a second pen-and-paper activity using the material compasses to construct a triangle. This second pen-and-paper activity consists, if possible, in constructing triangles of which the lengths of sides are given through numbers. This activity marks the end of the situation and allows the making of a summary of elements taught through the use of digital artefacts used with the material compasses.

5.7 Experiments and Results

Experiments of this situation were done over the course of three consecutive years in French primary school level CM2 classes (ten years' old pupils). These experiments were all done in the same elementary school. Every year, two classes and

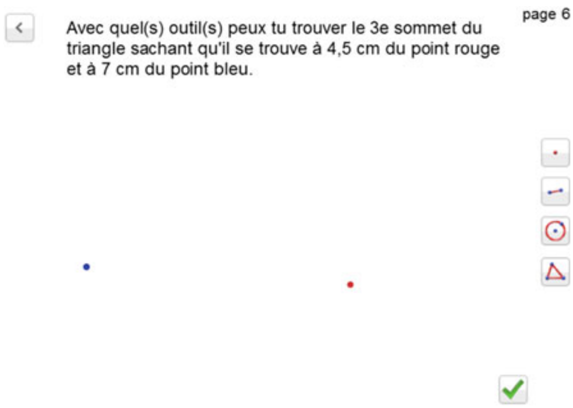


Fig. 5.7 The circle used to determine the third vertex of the triangle

their teachers participated in it. Over 130 pupils tested this situation (38 in 2014, 50 in 2015 and 44 in 2016) over three years. Footage recording the work of each pupil in the e-books were filmed. The pen-and-paper work of three out of four pupils was filmed on video cameras. The reactions of the pupils, as well as the remarks made by the teachers have allowed us to bring modifications to the situation each year. Additionally, these three years of experiments have allowed us to put our research hypotheses to the test.

5.7.1 A Rotation-Dragging Instrument Used to Rotate a Segment

The traces of the work of each pupil in the first e-book allow us to identify two levels of action in the e-book. Firstly, all the pupils interact with the given objects on the first page. We can observe interactions with digital segments in an attempt to move them and interactions with the given dynamic geometry tools. Right from the first page the pupils delve into the manipulation of segments. They grab and drag the segments. We can observe that pupils would like to rotate the segment by one of its ends: the rounded end is clicked upon and the cursor is moved in circular fashion. The two movements, by translation and by rotation around a given point are necessary for pupils. Secondly, strategies have been developed by the pupils to accomplish the given tasks. The double movement of segments is mastered when the pupil understands that only one action on the cruciform end allows the rotating of the segment. If the segment is grabbed by the rounded end or by a point of the segment then it is displaced. If the segment is grabbed by the cruciform end then the segment rotates around the other end which remains still. Finally, throughout the pages of the e-book, the double movement of digital segments is mastered by over 90% of the pupils. Over the course of the manipulation of asymmetrical digital segments in their movement, the pupils have created a utilization scheme to rotate a digital segment: distinguishing each end of the segment, then grabbing the cruciform end and dragging to rotate that end.

After every pupil completes the e-book, the teacher makes a synthesis of what has been done in the e-book with all the pupils. The videos show us that pupils caught three points: they have to form triangle; the rotation-dragging is essential; they can't always form a triangle with three given length.

5.7.2 The Material Compasses Used to Rotate a Segment

In the pen-and-paper environment, when a broken line into three segments is given to the pupils, more than 80% of them use a pair of material compasses (Fig. 5.8). These pupils say that the compasses are used here to replace the rotation-dragging used in the digital environment to rotate a segment. Several assimilation and adaptation processes of the utilization schemes used to rotate a digital segment are translated to a utilization scheme of material compasses to rotate a segment in the pen-and-paper environment. In the video we can see the pupils' organisation: distinguishing each arm of the pair of compasses, placing the needle on the steady end of the segment, spreading the arms and placing the pen lead on the end to be moved, and at least rotating the compasses whilst keeping the same spread and create a visible trace. The continuity between the rotation-dragging to move the end of the segment and the movement of the compasses which rotates upon it, give meaning to the compasses used to rotate segments of a broken line in the construction of the triangle. In the pen-and-paper environment, the initial segment does not rotate as it does in the digital environment. The compasses produce the trace of the rotating end—creating thus a circle, or a circular arc. Another trace representing the rotating segment must be drawn. 90% of the pupils who used the compasses to rotate a segment traced the new representations of the rotated segments (Fig. 5.8). These pupils thus drew the triangle obtained by closing the broken line.

To use the pair of compasses to rotate segments of the broken line also allows these pupils to recognize that a broken line may not allow the creation of a triangle (Fig. 5.9).

5.7.3 The Circle: A Tool in the Construction of the Triangle

In the second e-book, we observe in the videos that the presence of a broken line on the page leads the pupils to try to rotate its extremities. Even if it is explained that the broken line's segments can no longer rotate (Fig. 5.6), the pupils take one of the extremities and move the cursor in a circular motion. The segments' lack of movement forces them to set up strategies that mobilize the available dynamic geometry tools. When the compasses tool is available on the page, the pupils use it to rotate segments as with the material compasses in the first pen-and-paper activity.

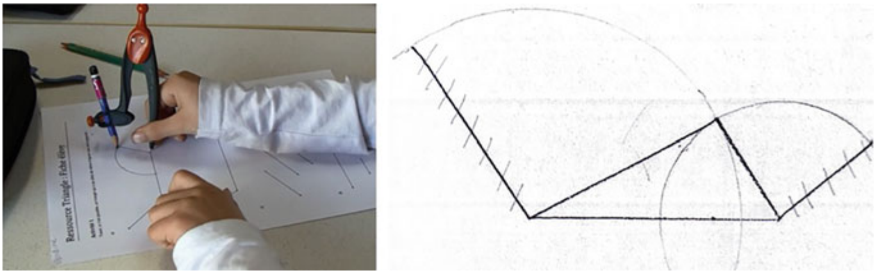
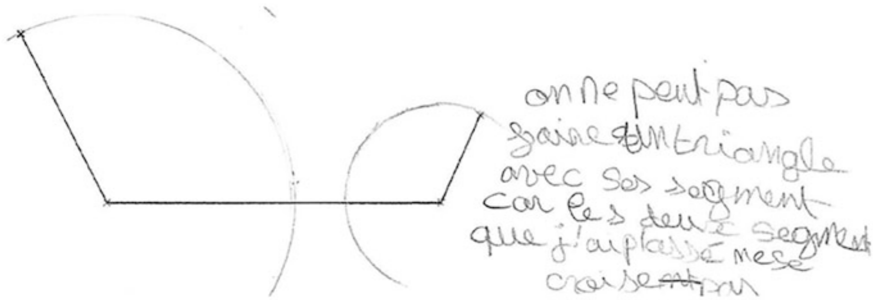


Fig. 5.8 The compasses used to rotate a segment



"One cannot make a triangle with these segments because the two segments that I've moved do not cross each other."

Fig. 5.9 Illustration of a broken line which may not allow the creation of a triangle

The circle tool is mobilized by 70% of pupils to replace the compasses tool. Only 60% of them (only 42% of all pupils) mobilize it correctly. The others activate the circle tool but cannot manage to use it to solve the problem. These pupils' difficulty lies in not knowing where to place the centre of the circle. Even if the utilization schemes for the compasses and dynamic geometry circle tool seem similar, as shown in Table 5.1, we can identify some obstacles in the utilization schemes for the circle tool. On the one hand, the centre of the circle is a 0 dimensional object hardly apprehended by primary school pupils as we already said before. On the other hand, the compasses tool allows a level of visual control that brings to mind the rotating digital segment (Fig. 5.10), which simplifies its use in the task.

So in this situation, the material compasses and the dynamic geometry circle are not a duo of artefacts. There is too wide a gap between the utilization schemes of each artefact to solve the task. Assimilation and adaptation processes of utilization schemes from one instrument to the other cannot be identified.

On the last two pages of the e-book the circle tool must be mobilized to identify a distance, the length of the triangle's side (Fig. 5.7). It is the only available tool to determine the desired distances. Only 15% of pupils manage to use the circle wisely. The perception of a circle as a set of points equidistant from a centre is not yet mastered by pupils at the end of elementary school.

Table 5.1 Utilization schemes for the dynamic geometry

Utilization schemes for the dynamic geometry compasses tool	Utilization schemes for the dynamic geometry circle tool
Click on the compasses tool Click to fix the needle Spread the arms Click to place the lead Pivot the lead-end to draw a trace	Click on the circle tool Click to define the circle's centre Spread the circle Click to define the circle's size and fix its image on the screen

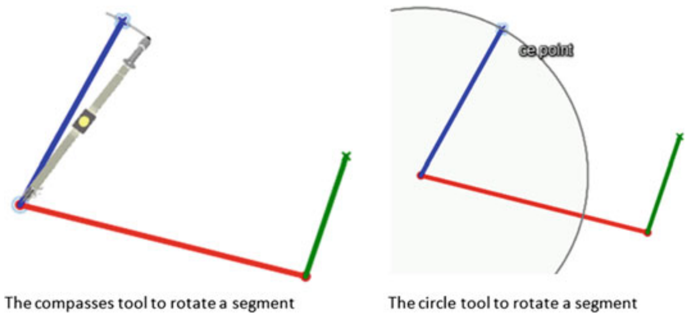


Fig. 5.10 Compasses tool versus circle tool to rotate a segment

5.7.4 A Triangle Is a Broken Line for More Than 75% of Pupils

The second pen-and-paper activity reveals what the pupils have learned in the situation. In the first e-book, the "broken line strategy" comes in as an efficient winning strategy to form a triangle in the digital environment (Fig. 5.3). The videos show us the implementation of a broken line strategy in the pen-and-paper environment. It is transposed into the pen-and-paper environment as a step in the triangle's construction (Fig. 5.8) by more than 75% of pupils. Pupils draw a broken line with the three segments, and then use the compasses to rotate the segments at both ends of the broken line, and finally they draw the triangle if it exists. Video capture allows us to point out certain pupils' remarks. One pupil says: "The broken line helps because before, we didn't know we had to use the compasses to construct a triangle."[3] Only 15% of pupils construct the triangles they are asked for with ruler and compasses without using the broken line. Some of them identify the two solutions for the third vertex. They draw the two triangles. The other 10% of pupils draw the triangle by using only the ruler by successive approximations.

[3]Translation of the original sentence «La ligne brisée ça nous aide parce qu'avant on savait pas qu'il fallait utiliser le compas pour tracer un triangle».

A triangle being a closed broken line is a new conception (Balacheff, 2013) of the triangle resulting from the design of the duo of digital and material artefacts. This conception allows a 1D understanding of the triangle: a triangle is a polygon with three sides. The broken line is a dimensional deconstruction of the 1D triangle, halfway between the 2D three-sided triangle and the triangle determined by its 0D vertices. Furthermore, this conception includes a control structure on the existence or not of the triangle. If the end segments of the broken line meet then the triangle exists. If the end segments of the broken line do not meet then the triangle does not exist.

The rotation-dragging and material compasses' duo for rotating the end segments of a broken line also create a new conception of the circle. A circle is the trajectory of the extremity of a rotating segment. Controls here do not only focus on the compasses' production. A new control is present: pivoting compasses corresponds to pivoting a segment between its arms. This conception highlights the notion of constant distance associated to the circle.

5.8 Conclusion

Our research project questions the added effect brought by digital technology in situations operating on the basis of linking together digital tools and material tools. With an example in geometry, we have illustrated the characteristic elements of a digital and material duo of artefacts, and its incidence on the learning process. The main idea behind a duo is that each artefact improves the other so that the duo encourages the pupils' construction of individual knowledge. Continuity and discontinuity in the two artefacts connection are essential and relay knowledge. In our example we have highlighted how the connection between manipulating in a digital environment and using the material compasses allows the pupils to elaborate a new instrument: the compasses to rotate a segment. The duo of artefacts in situation promotes the assimilation and adaptation of utilization schemes from one instrument to the other. Thus the situation mobilizing a duo of digital and material artefacts participates in the instrumental genesis of the compasses through the triangle-construction task and gives meaning to its usage in this task. Furthermore the duo brings about the elaboration of teaching strategies. The broken line strategy allows the setup of the triangle's dimensional deconstruction 2D/1D. Indeed the duo produces the circumstances that rely on the broken line to implement the triangle's dimensional deconstruction without necessarily going to the point a 0 dimensional object. This broken line also participates in the elaboration of a new conception of the triangle: a triangle is a closed broken line. Thus the digital artefact, or more precisely the duo of artefacts, brings additional value to the material tool which helps in breaking through certain difficulties or epistemological obstacles. The duo and the connection between digital and material artefacts in the situation give meaning to the material artefact's use in solving a task, enrich the system [subject/milieu] by relating problems and reiterating experiences while varying constraints, and are thus favourable to the elaboration of the subject's individual knowledge.

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