Rhonda Douglas Brown

Neuroscience of Mathematical Cognitive Development

From Infancy Through Emerging Adulthood

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To Russ, Ainsley, and Paige Brown
and Theresa Douglas
In loving memory of Ronald Douglas
and Virgil and Martha Brown

Preface

This book examines the neuroscience of mathematical cognitive development from infancy through emerging adulthood, addressing both biological and environmental influences on brain development and plasticity. It was written as a resource for professors, researchers, clinicians, educators, and graduate students in the fields of developmental, cognitive, child clinical, educational, and school psychology; neuroscience and neuropsychology; and mathematics education and intervention. It was also designed to serve as a text for advanced undergraduate and graduate courses covering mathematical cognition. It includes background information on developmental psychology theory, brain development, and cognitive neuroscience research methods to make the volume more accessible to graduate students and professionals from other fields and to facilitate understanding and application of research results.

The book begins by presenting major theories for interpreting neuroscience studies of mathematical cognitive development and achievement, including evolutionary developmental psychology, a developmental systems approach, and the triple-code model of numerical processing. These theories provide a cohesive framework that is revisited throughout the text. A general paradigm for conducting studies using multiple levels of analysis to examine interactions between neural activity, behavior, and environmental contexts and experiences is discussed. Then, the book describes brain development and cutting-edge neuroscience research methods, including functional Magnetic Resonance Imaging (fMRI), Diffusion Tensor Imaging (DTI), Event-Related Potentials (ERP), and Transcranial Magnetic Stimulation (TMS), at a level that is comprehensible to those who may be unfamiliar with these neuroimaging techniques. The book includes chapters that discuss existing studies and new research findings from my work with colleagues at the University of Cincinnati using neuroscience research methods to examine quantity representation, calculation, and visuospatial cognition. Furthermore, it also presents neuroscience models and research for understanding mathematical difficulties and a diversity of exceptionalities, such as autism spectrum disorder and Turner's syndrome. A review of mathematics intervention programs is included that relates them to neuroscience theory and research to provide information to researchers, practitioners, and educators seeking strategies for improving developmental trajectories, individual outcomes, and educational practices for students experiencing mathematical difficulties. The book ends with a summary of conclusions that can currently be drawn from neuroscience studies of mathematical cognitive development and recommendations for future research.

This book is a product of the work of many scholars and researchers dedicated to understanding mathematical cognitive development so that we may, ultimately, improve children's learning and their success in life. Their names are too numerous to list here, but the references to each chapter provide some acknowledgement. I am especially inspired by the work of David F. Bjorklund, David C. Geary, Stanislas Dehaene, Daniel B. Berch, Robert Siegler, Douglas H. Clements, Ted Hasselbring, Lynn Fuchs, and Sharon Griffin. I am grateful to my colleagues Scott Holland, Peter Chiu, and Tzipi Horowitz-Kraus at the University of Cincinnati and Cincinnati Children's Hospital Medical Center for engaging me in neuroimaging research. I feel especially privileged to have Vincent J. Schmithorst as a long-time collaborator on the research presented in this book. I offer my sincere gratitude to David F. Bjorklund for reviewing and providing feedback on portions of the text; Bethany Reeb-Sutherland for providing a photograph from her lab; Ted Hasselbring for assisting with permissions for the figures from the FASTT Math program; Sharon Griffin for providing figures from the Number Worlds program; and Vicki Plano-Clark for her publication advice. I would especially like to thank Cheri Williams for mentoring and motivating me through the process of writing this book and for her friendship. I am also grateful for the camaraderie and support of my colleagues at the University of Cincinnati, especially Vicki Carr, Tina Stanton-Chapman, Heidi Kloos, and Cathy Maltbie; Jonathan Thomas at the University of Kentucky; and the graduate students who participated in my courses over the years, including Mathematics Cognition, especially Lori Kroeger, Nicole Hammons, Gail Headley, Laura Kelley, Lori Foote, Mindy Victor, Kate Doyle, Sammie Marita, Lindsay Owens, Rachel Lindberg, Jonathan Buening, Leslie Kochanowski, Sue Schlembach, Ann Rossmiller, and Ashley Vaughn. At Springer, I am grateful to Garth Haller, Judy Jones, and Michelle Tam for the opportunity to write this book and for their enthusiasm and assistance during this process. Finally, to my husband Russ, my daughters Ainsley and Paige, and my mother Theresa, I offer my deepest gratitude for their ever-present love, encouragement, and support.

Cincinnati, OH, USA Rhonda Douglas Brown

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Rhonda Douglas Brown, Ph.D. is currently an Associate Professor of Early Childhood Education and Human Development and Cognitive Developmental Psychology at the University of Cincinnati, where she teaches graduate courses in Mathematics Cognition; Cognitive Development; Brain Development, Cognition, & Learning; and Cognition & Culture, as well as undergraduate Educational Psychology. She co-founded and serves as the Director of the Developmental & Learning Sciences Research Center, which strives to generate research, educational experiences, and applications using an emergent, interdisciplinary approach from the fields of developmental psychology, cognitive science, neuroscience, and education. Dr. Brown conducts research examining neural correlates and cognitive mechanisms of mathematical and memory development using fMRI and behavioral measures with the aim of improving educational interventions for children with learning difficulties. Dr. Brown also serves as Co-Editor of the peer-reviewed journal *Child, Youth and Environments*. She has a profound interest in how children's environments affect their brain development, cognition, and learning. Her current research, sponsored by the National Science Foundation, examines how intentionally designed PlayScapes serve as environments for promoting children's science, technology, engineering, and mathematics (STEM) learning and executive function.

Chapter 1
Theories for Understanding the Neuroscience of Mathematical Cognitive Development

Rhonda Douglas Brown

Abstract Throughout history, humans have invented and used mathematics to solve meaningful problems critical to survival and prosperity. To advance our understanding of mathematical cognitive development and achievement, it is important to place research within theoretical frameworks that allow us to interpret and apply results. In this chapter, I discuss evolutionary developmental psychology as a metatheory for considering important questions relevant to understanding neuroscience research on mathematical cognitive development. Then, I use a developmental systems approach to describe how genetics, neural activity, and experiences in environmental niches dynamically interact in the development of evolved probabilistic cognitive mechanisms. As an example, I describe biologically primary mathematical abilities that may have been selected for in evolution to solve recurrent problems, passed on via genetics, and instantiated in human brain development. The process of their development into biologically secondary mathematical abilities, which are cultural inventions that build upon biologically primary abilities, is then described. I present Dehaene and colleagues' triple-code model of numerical processing as the predominant neuroscience-based theory of mathematical cognition. I conclude by arguing that there is a place for neuroscience in the field of cognitive development and advocating for the integration of scientific findings across levels of analysis.

Mathematical skills are important for human survival. Consider this interaction (originally in Portuguese) between a researcher posing as a customer and a 12-year-old boy with little formal education working as a street vendor in Recife, Brazil.

> Customer: I'm going to take four coconuts. How much is that?
> Child: Three will be 105, plus 30, that's 135…one coconut is 35…that is…140!

This excerpt from seminal work by Carraher and colleagues demonstrates that children construct mathematics in everyday cultural experiences that are important

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for survival (Carraher, Carraher, & Schliemann, 1985, p. 26). In Brazil and other places, it is fairly common for children of street vendors to help with the family business. From about the age of 8, children may carry out transactions while their parents are busy with other tasks. In doing so, they solve a number of mathematical problems involving addition, subtraction, multiplication, and sometimes division, without pencil and paper.

The Importance of Mathematics in Human Development

Throughout history, humans have invented and used mathematics to solve meaningful problems critical to survival and prosperity. To better understand the importance of mathematics in human development, let's consider a few remains that provide a glimpse into how humans have used mathematics to solve problems and how they have passed conceptual and procedural knowledge on to future generations through instructional texts. The summary that follows was mostly gleaned from the work of Merzbach and Boyer (2011) who provide a comprehensive history of mathematics.

The oldest mathematical artifact, estimated at 35,000 years old, is the Lebombo bone, a small piece of baboon fibula with 29 well-defined notches found in the Lebombo Mountains between South Africa and Swaziland. Although there are several hypotheses regarding the functional significance of the Lebombo bone, some scholars believe it was used as a lunar phase counter and binary calendar, indicating the quantification of time. Similarly, the Ishango bone pictured in Fig. 1.1 was found in Northeastern Congo and is estimated at 20,000 years old. It has tally marks organized into groups and is thought to represent a counting system or lunar phase calendar (e.g., Setati & Bangura, 2012).

Ancient mathematical texts of the Babylonians, in the form of clay tablets dating back to 2000 years BC, document the use of basic arithmetic, including multiplication and division, geometry, fractions, algebra, quadratic and cubic equations, and the Pythagorean theorem. For example, in his description of Mesopotamian mathematical cuneiform texts from the Norwegian Schøyen Collection, Friberg (2008) translated one tablet (MS 2830) that contains computations involving four different commodities and market rates:

> the question is how much a person can buy for 1 shekel of silver if *equal amounts* are bought of all four commodities (p. 1079).

Friberg also found evidence for what appears to have been ancient Geometry homework.

> It is likely that… each student was supposed to go home with his hand tablet and spend part of the evening writing down a detailed version of the solution procedure, to be brought back to school the next day (p. 1079).

Remnants of Babylonian mathematics exist today in that it used a base-60 numeral system that corresponds with our modern quantification of time in seconds

Fig. 1.1 The Ishango bone found in Northeastern Congo approximately 20,000 years ago has tally marks organized into groups that may represent a counting system or lunar phase calendar. Photograph from the Collections of the Royal Belgian Institute of Natural Sciences by Ben2 (Own work) [GFDL (http://www.gnu.org/copyleft/fdl.html), CC-BY-SA-3.0 (http://creativecommons.org/licenses/by-sa/3.0/), or CC BY-SA 2.5-2.0-1.0 (http://creativecommons.org/licenses/by-sa/2.5-2.0-1.0)], via Wikimedia Commons

and minutes. Similarly, early Egyptian texts, such as the Rhind papyrus (circa 1650 BC), served as an arithmetic and geometry instruction manual for students and the Moscow papyrus (circa 1980 BC) included story problems. In Greece, geometry was used to solve problems such as calculating the height of pyramids and the distance of ships from the shore. In 300 BC, Euclid wrote *Elements*, perhaps the greatest mathematical text to date, which formalized the use of axioms, theorems, and proofs that are taught in modern day geometry courses. In China, although older texts have been found, *The Nine Chapters on the Mathematical Art* dated at 179 AD contains word problems involving agriculture, architecture, engineering, and business. Europe was introduced to Indian and Islamic mathematics through the writings of Muḥammad ibn Mūsā al-Khwārizmī, dated around 825 AD, who described the Hindu-Arabic numeral system used throughout the world today, and the reduction and balancing of algebraic equations.

These artifacts demonstrate that humans have been using mathematics to solve real-world problems for more than 35,000 years, and likely during prehistoric periods. They also demonstrate that humans have been teaching and advancing mathematics and its applications throughout this history. As the Common Era progressed, Fibonacci applied the positional notation of Hindu-Arabic numerals to improve trade transactions; navigational demands led to the development of trigonometry (Grattan-Guinness, 2009); and the works of Galileo, Kepler, Descartes, Newton, and Leibniz during the scientific revolution of the seventeenth century led to major developments in astronomy, analytic geometry, physics, and calculus. In the twentieth century, Einstein used differential geometry to demonstrate general relativity, Turing used computability theory to develop computer science, and Mandelbrot used fractal geometry to describe nature. Currently, the application of mathematics to information technology, such as in bioinformatics, allows researchers addressing scientific questions to rapidly analyze volumes of data.

With these advances, one could argue that the scientific revolution continues. Currently, in the United States (U.S.) and other countries, since at least the 1990s, education policy has promoted STEM. *STEM* stands for the academic disciplines of Science, Technology, Engineering, and Mathematics (variations include STEMM, which incorporates Medicine, and STEAM, which incorporates Art). According to the economist Vilorio (2014), "STEM workers use their knowledge of science, technology, engineering or math to try to understand how the world works and to solve problems. Their work often involves the use of computers and other tools" (p. 3). In 2007, the America COMPETES Act (P.L. 110-69) increased funding for science and engineering research and STEM education from kindergarten through postdoctoral training. Projections from the U.S. Bureau of Labor Statistics estimate employment in STEM occupations to grow to more than nine million between 2012 and 2022 (as cited in Vilorio, 2014). Careers in STEM fields are expected to contribute to individual prosperity through higher wages and to the growth of economies in the U.S. and worldwide. Yet some scholars and educators believe that the current generation is not prepared for these careers (Vilorio, 2014; but see Charette, 2013). In the U.S., the National Assessment of Educational Progress (NAEP) online report for 2015 indicates that only 25% of 12th grade students performed at or above the *Proficient* level in mathematics, which has not changed significantly since 2005 (U.S. Department of Education, 2015).

Theoretical Frameworks

Clearly, mathematics achievement is at least as important today as it was to the Babylonians. To advance our understanding of mathematical cognitive development and achievement, it is important to place research within theoretical frameworks that allow us to interpret and apply results. From my perspective, and others', it makes sense to situate research on the neuroscience of mathematical cognitive development within the theoretical frameworks of Evolutionary Developmental Psychology (see Bjorklund & Pellegrini, 2002; Geary & Berch, 2016; Geary, Berch, & Koepke, 2015) and a Developmental Systems Approach (see Baltes, Reuter-Lorenz, & Rösler, 2006; Bronfenbrenner & Morris, 2006; Ford & Lerner, 1992; Gottlieb, Wahlsten, & Lickliter, 2006; Sameroff, 2009). In the following sections, I discuss evolutionary developmental psychology as a meta-theory for considering important questions relevant to understanding neuroscience research on mathematical cognitive development. Then, I use a developmental systems approach to describe how genetics, neural activity, and experiences in environmental niches dynamically interact in the development of evolved probabilistic cognitive mechanisms. As an example, I describe biologically primary mathematical abilities that may have been selected for in evolution to solve recurrent problems, passed on via genetics, and instantiated in human brain development. The process of their development into biologically secondary mathematical abilities, which are cultural inventions that build upon biologically primary abilities, is then described (Geary, 1995, 2007). I present Dehaene and colleagues' triple-code model of numerical processing as the predominant neuroscience-based theory of mathematical cognition (Dehaene, 1992, 2011; Dehaene & Cohen, 1995, 1997; Dehaene, Piazza, Pinel, & Cohen, 2003). I conclude by arguing that there is a place for neuroscience in the field of cognitive development and advocating for the integration of scientific findings across levels of analysis.

Evolutionary Developmental Psychology

Twenty years ago David Bjorklund and I proposed that research from the field of developmental cognitive neuroscience could be incorporated into the perspectives of evolutionary psychology and a developmental systems approach (Bjorklund, 1997a; Brown & Bjorklund, 1998; Byrnes & Fox, 1998a). This proposal reflects the emergence of the field of *evolutionary developmental psychology* over the last several decades. Although a comprehensive presentation of this field is beyond the scope of this book (see Bjorklund & Ellis, 2014; Bjorklund & Pellegrini, 2002; Geary, 2005; Geary & Berch, 2016; Geary & Bjorklund, 2000), this section describes premises that are most relevant to framing and interpreting neuroscience research on mathematical cognitive development.

Evolutionary developmental psychology postulates that the human mind consists of a set of information-processing mechanisms that are instantiated in the brain. These psychological mechanisms and the processes governing their development are adaptations that evolved gradually through the process of natural selection in response to pressures confronted by ancestral humans in environments during the period of evolutionary adaptation, such as hunting and foraging, the need to cooperate and compete with other members of social groups, and climate change (Bjorklund & Pellegrini, 2002; Geary, 2007). They are domain-specific and functionally specialized to produce behaviors that solve recurrent, real-world problems by extracting and processing specific aspects of physical and social environments. Natural selection may have also acted upon the evolution of domain-general mechanisms, such as components of executive function, including cognitive flexibility, working memory, and inhibition (e.g., Bjorklund & Kipp, 1996; Geary, 2007). Furthermore, some characteristics that influence survival and reproductive success may not have been selected for the function that they currently serve or may be by-products of adaptations co-opted to serve other functions, called *exaptations* (Gould & Vrba, 1982). That is, existing systems generate solutions to new problems.

The emphasis placed on phylogeny in evolutionary developmental psychology leads to functional analyses that consider the *distal* and *proximal* causes of adaptive behavior. Pertinent questions that apply functional analysis to cognitive neuroscience research include:

1. What neuroarchitecture supported cognition during the period of evolutionary adaptation?
2. What selection pressures were present during the period of evolutionary adaptation that may have led to the evolution of information-processing mechanisms underlying the cognition under consideration?
3. What adaptive problems needed to be solved?
4. What *distal* functions during the period of adaptation did the psychological mechanisms under investigation evolve to serve? and
5. How do the identified distal functions influence *proximal* causes of cognition and behavior? (Brown & Bjorklund, 1998, pp. 358–359).

Evolutionary developmental psychology recognizes that current human neuroarchitecture and its functions evolved gradually over millions of years. By asking these questions about distal explanations of behavior and how neural structures and their functioning support them, we enhance our understanding of cognitive development. By integrating distal functions and proximal causes of cognition and behavior, such as neural activity, we relate species-typical development to how individuals adapt to their particular life circumstances (Bjorklund, 2017; Bjorklund & Ellis, 2014; for a different perspective, see Witherington & Lickliter, 2017). One of the most promising aspects of neuroscience research involves identifying brain structures and networks that carry out specific information-processing functions and describing how these work together to produce complex cognition. When we consider distal and proximal functions, adaptive specializations and exaptations together, we can better understand the coexistence of domain-specific and domain-general abilities. For example, evolutionary adaptations in phylogenetically older and newer neural systems may coexist or result from the modification of existing systems. Interpreting neuroscience findings using a evolutionary developmental psychology lens may provide additional explanatory power and reconcile seemingly contradictory findings. Thus, it is fruitful to consider evolutionary accounts of cognitive processes in relation to ontogenetic developmental processes. A developmental systems approach provides a means for doing so.

Developmental Systems Approach

I believe that the value of neuroscience theories and research is more fully realized when placed within a *Developmental Systems Approach* (Brown & Bjorklund, 1998; Brown & Chiu, 2006; Kroeger, Brown, & O'Brien, 2012). Although there are conceptual and representational differences in developmental systems approaches (see Baltes et al., 2006; Bronfenbrenner & Morris, 2006; Ford & Lerner, 1992;

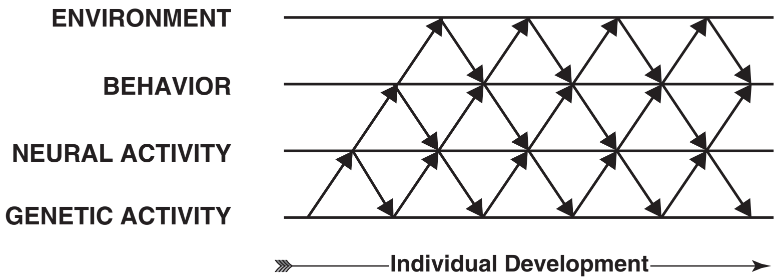

Fig. 1.2 Depiction of a developmental systems approach that characterizes human development throughout the lifespan as occurring within a hierarchically organized, integrated system of bidirectional interactions among genetic activity, neural activity, behavior, and environment. Source: Gottlieb, G. (1991). Experiential canalization of behavioral development: Theory. *Developmental Psychology, 27*(1), 4–13. https://doi.org/10.1037/0012-1649.27.1.4

Gottlieb et al., 2006; Sameroff, 2009), one basic model, shown in Fig. 1.2, depicts ontogenetic development throughout the lifespan as occurring within a hierarchically organized, integrated system of bidirectional interactions (bottom-up and top-down) among multiple factors, including genetic activity, neural activity, behavior, and environment (Gottlieb, 1991; Gottlieb et al., 2006). Within this approach, *probabilistic epigenesis* is the primary mechanism for ontogenetic change, rather than genetic determinism. New structural and functional properties may emerge through coactions (vertical and horizontal) with feedback between dynamic components of the developmental system. Integrating evolutionary developmental psychology and the developmental systems approach, Bjorklund, Ellis, and Rosenberg (2007) proposed the concept of *evolved probabilistic cognitive mechanisms*, which are

> information-processing mechanisms that have evolved to solve recurrent problems faced by ancestral populations; however, they are expressed in a probabilistic fashion in each individual in a generation, based on the continuous and bidirectional interaction over time at all levels of organization, from the genetic through the cultural. These mechanisms are universal, in that they will develop in a species-typical manner when an individual experiences a species-typical environment over the course of ontogeny (p. 22).

Interpreting cognitive developmental neuroscience within this framework involves describing bidirectional relationships between genes, the maturation of structures, the functions of structures, developmental processes, and experiences in environments. These relationships result in *plasticity*, but also produce various types of *constraints* on learning related to the fact that humans inherit a species-typical genome *and* species-typical environments similar to those of their ancestors (Bjorklund & Ellis, 2014; Gelman & Williams, 1998; Spelke & Kinzler, 2007). Specific neurons, brain structures, regions, and networks process certain types of information reflecting *architectural constraints*. Furthermore, infants may possess inherited perceptual and information-processing biases that orient their attention and help them make sense of certain stimuli in core domains, including language, number, physics, and theory of mind, which reflect *representational constraints* (Bjorklund & Ellis, 2014; Spelke & Kinzler, 2007). Although these constraints on learning suggest that human infants are born prepared to process and learn certain types of information, as Bjorklund (2003, 2017) points out, prepared is not preformed. Nevertheless, evolved cognitive mechanisms may form the basis for cognitive development through childhood and adolescence (Bjorklund & Pellegrini, 2002). Furthermore, adaptive functions may operate during a limited period of the lifespan, such as during infancy, by enhancing fit within a specific environment (e.g., Bjorklund, 1997b; Tooby & Cosmides, 1992). *Chronotopic constraints* place limitations on developmental timing such that some brain areas might be most receptive to certain types of experiences during sensitive periods of development; furthermore, early-developing brain areas may process different types of information than later-developing areas.

This description of constraints on learning illustrates the importance of understanding cognitive development using *multiple levels of analysis* and attempting to integrate findings into cohesive descriptions and explanations. As described in Brown and Bjorklund (1998), applying a developmental systems approach promotes the understanding that, as depicted in Fig. 1.2, neural activity influences cognition and behavior, which, in turn, influence neural organization and function. The mechanism of probabilistic epigenesis can be used to characterize relationships between genetic activity, structural maturation, and function, which includes internal experiences (Bjorklund & Pellegrini, 2002). For example, individual differences in brain morphology result from differences during the proliferation phases of brain maturation and pruning occurs as a result of the number and connectivity of neurons firing in response to environmental experience (Byrnes & Fox, 1998b; Gottlieb, 1991; for a brief description of these processes, see Chap. 2).

Biologically Primary and Secondary Mathematical Abilities

According to Geary (1995, 2007), *biologically primary mathematical abilities* represent a "core" modular system that responds to and processes certain types of information. These abilities are believed to have been important in the everyday lives of hunter-forager societies and hence were selected for during evolution. Biologically primary mathematical abilities include numerosity, ordinality, counting, simple arithmetic, estimation, and geometry, and tend to recruit brain regions in the *horizontal intraparietal sulcus* (for detailed descriptions of neuroscience research on these topics, see Chaps. 3 and 4). Geary (2007) stated "The emergence of primary abilities through an interaction between experience and inherent constraints ensures that all people develop the same core systems of abilities and, at the same time, allows these systems to be fine tuned to the nuances of the local social group, and the biological and physical ecologies in which they are situated" (p. 475). He proposes that implicit, "folk" knowledge or skeletal competencies are fleshed out during human development through species-typical parent-child and peer interactions, as well as children's own intrinsically motivated play and exploration of environments, and that they can be linked in novel ways (e.g., Bjorklund, 2006;

Bouchard, Lykken, Tellegen, & McGue, 1996; Greenough, Black, & Wallace, 1987; Scarr, 1993), which reflects a *neonativist* perspective. Geary also proposes that biologically primary abilities can be developed through more flexible top-down, conscious cognitive processing that engages executive function, including working memory, reasoning, and problem solving, and recruits the *prefrontal* and *anterior cingulate* cortices. Humans can innovate by creating novel solutions and they can also organize these solutions into a knowledge base that is culturally transferred across generations through texts, such as those described in the first section of this chapter, instructional practices, and other means. The need to acquire this corpus of knowledge accumulated from previous generations necessitates extended learning experiences throughout humans' long childhood. Geary defines *biologically secondary mathematical abilities* as the culture-specific skills that build upon biologically primary abilities. They are typically taught within the context of formal schooling and require extrinsic motivation and extensive practice. Examples of biologically secondary mathematical abilities include algebra, advanced geometry, and calculus. These skills are important to success in highly industrialized and technological societies; for example, in pursuing STEM careers. From the perspective of *evolutionary educational psychology*, Geary (Geary, 1995, 2007, 2010; Geary & Berch, 2016) suggests that explicit, teacher-directed instruction may not be best for the acquisition of early number skills related to biologically primary abilities, but may be the most effective instructional method for the acquisition of mathematical skills requiring biologically secondary abilities.

Let's return to the excerpt from Carraher et al. (1985) at the outset of this chapter depicting an interaction between a "customer" and a 12-year-old Brazilian street vendor. It provides a basic illustration of how children can use mental calculation to correctly solve everyday problems. Based on qualitative analyses, the researchers concluded that a frequent strategy for multiplication problems involved successively chaining addition. In the example, the boy decomposed a quantity into tens and units; that is, to add 35 to 105, he added 30 and later incorporated 5 into the result. Now consider the same child's response to a formal test given 1 week later by the same "customer" involving the same problem put into a more abstract context based on school mathematics, using paper and pencil.

> Child resolves the item 35 X 4 explaining out loud: 4 times 5 is 20, carry the 2; 2 plus 3 is 5, times 4 is 20. Answer written: 200 (Carraher et al., 1985, p. 26).

The researchers posed the question "How is it possible that children capable of solving a computational problem in the natural situation will fail to solve the same problem when it is taken out of its context?" (p. 25). Results from their qualitative analyses suggest that different problem solving procedures may have been used across the two contexts. Specifically, the child street vendors seemed to understand quantity, but were not proficient at formal, abstract, symbolic mathematics using school-based procedures. In this example, the boy confused addition procedures with multiplication procedures, which is a common error (e.g., Jordan & Hanich, 2000; Raghubar et al., 2009; Rourke, 1993), perhaps due to working memory and inhibition difficulties (e.g., Geary, 2011; but, see Cohen, Dehaene, Cohochon, Lehericy, & Naccache, 2000). Furthermore, the researchers noted that the children did not appear to engage in error detection procedures to make sure that their result made sense given the nature of the problem. They concluded that their results support sociocultural theorists' thesis that everyday problem solving may be performed using procedures different from those taught in schools (e.g., Cole, 1990; Donaldson, 1978; Lave, Murtaugh, & de la Rocha, 1984; Luria, 1976; Saxe, 1988). The researchers note that there are, of course, limitations to mental calculation, in that the strategies used by Brazilian street vendors become difficult to execute with greater quantities, which can be surmounted through written computation. Ideally, school-taught mathematics provides symbol systems and procedural tools for solving complex problems more efficiently across multiple contexts. As noted by Devlin (2005), "they underpin all our science, technology, and modern medicine, and practically every other aspect of modern life. Their development marks one of the crowning achievements of the human race. But that doesn't make them easy to learn or to apply. The problem is that humans operate on meanings. In fact, the human brain evolved as a meaning-seeking device. We see, and seek, meaning anywhere and everywhere." Indeed, Carraher et al. (1985) suggested that "educators should question the practice of treating mathematical systems as formal subjects from the outset and should instead seek ways of introducing these systems in contexts which allow them to be sustained by human daily sense" (p. 28). Although these results are interpreted in different ways by researchers of varying theoretical perspectives, I believe they are consistent with neuroscience theories of mathematical cognition.

Triple-Code Model of Number Processing

The *triple-code model of number processing* (e.g., Dehaene, 1992, 2011; Dehaene et al., 2003; Dehaene & Cohen, 1995, 1997) is the predominant neuroscience model of mathematical cognition. The model proposes that three representational systems may be recruited for mathematical cognition: Quantity, Verbal, and Visual (Dehaene et al., 2003). These systems are thought to be instantiated by three neural circuits that coexist in the parietal lobe and operate in an interactive fashion during specific tasks. Figure 1.3 shows the representational systems for numerical processing and associated brain areas proposed in the triple-code model.

The *Quantity system*, often referred to as *number sense* (see Dehaene, 2011), is proposed to use nonverbal semantic representations (i.e., meaning or cardinal value) of size and distance relations between numbers during mathematical cognition. Like the biologically primary abilities discussed in the previous section, this system engages a *horizontal* segment of the *intraparietal sulcus* (HIPS) brain regions, which are activated during tasks involving quantitative representation of numbers, including number comparison, subtraction, approximate addition, and numerosity estimation (for a review, see Dehaene et al., 2003). For example, knowing that 7 is close to 8 on a mental number line reflects functioning of the quantity system.

The *Verbal system* is proposed to represent and manipulate sequences of number words when naming numbers, counting, retrieving well-learned addition and multi-

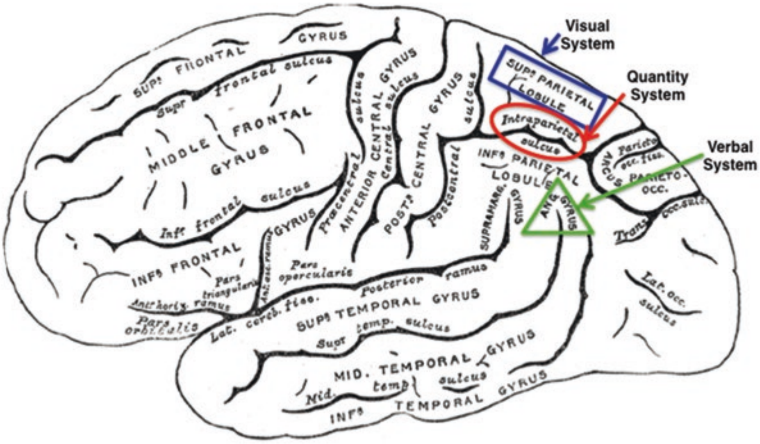

Fig. 1.3 The left hemisphere (lateral view) indicating representational systems and corresponding brain regions proposed in the triple-code model of numerical processing. Source: Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive Neuropsychology, 20,* 487–506. https://doi.org/10.1080/02643290244000239

plication facts from long-term memory, and performing exact calculation. For example, when you hear "two times three," the word "six" may pop into your mind. Use of the verbal system recruits general-purpose language modules, including the left perisylvian network, and a region of the *left angular gyrus* (for a review, see Dehaene et al., 2003).

The *Visual system* is proposed to represent and spatially manipulate numbers in their visual, symbolic format, which is commonly Arabic numerals (Dehaene & Cohen, 1995). For example, to solve $54 - 31 = ?$, you must first recognize the symbols for numbers and then you might subtract 1 from 4 to determine that the second digit of the solution is 3 and then subtract 3 from 5 to determine that the first digit of the solution is 2, and then combine these results to obtain 23 as the final solution. Use of the visual system recruits the *posterior superior parietal lobe* on tasks that involve spatial attention orienting and working memory, including number comparison, counting, approximation, subtraction of two digits, and carrying out two operations (for a review, see Dehaene et al., 2003).

Dehaene et al. (2003) speculate that the quantity system may be a good candidate for a domain-specific "core" system. They also believe that the involvement of the verbal and visual systems in number processing reflects the recruitment of more domain-general representations and processes that are not restricted to numerical functions, similar to the top-down processes discussed in the previous section. Dehaene and Cohen (1995, 1997) proposed two major transcoding routes to describe how the systems interact with one another in mathematical cognition. An *indirect semantic route* specialized for quantitative processing is hypothesized to perform several functions, including manipulating analog magnitude quantity representations to compare operands; using back-up strategies by manipulating visual Arabic representations when rote knowledge is not available in verbal memory, such as decomposing complex problems into new problems for which facts can be retrieved (e.g., 13 + 5 = 10 + 5 + 3 = 15 + 3 = 18) (LeFevre et al., 1996); and monitoring the plausibility of the direct asemantic route using estimation (Ashcraft & Stazyk, 1981; Dehaene & Cohen, 1991). A *direct asemantic route* is hypothesized to transcode numerical symbols (i.e., Arabic numerals) to auditory verbal representations in the left perisylvian language areas to guide retrieval of memorized arithmetic facts without semantic mediation. Furthermore, *prefrontal* areas and the *anterior cingulate* are hypothesized to coordinate the sequencing of processing through the systems, maintaining intermediate results in working memory, and detecting errors (Dehaene, 1992; Dehaene et al., 1996; Dehaene & Naccache, 2001; Kopera-Frye, Dehaene, & Streissguth, 1996; Shallice & Evans, 1978).

Overall, neuropsychological cases and studies using functional Magnetic Resonance Imaging (fMRI) and Diffusion Tensor Imaging (DTI) (for explanations of these neuroimaging techniques, see Chap. 2) provide support for the three parietal circuits for number processing shown in Fig. 1.3 in adults (for reviews, see Arsalidou & Taylor, 2011; Dehaene et al., 2003; Moeller, Willmes, & Klein, 2015; also, see Grabner et al., 2009; Klein et al., 2016; Schmithorst & Brown, 2004; van Eimeren et al., 2010) and children (e.g., Ansari & Dhital, 2006; Ansari, Garcia, Lucas, Hamon, & Dhital, 2005) and a distinction between the semantic, language-independent system used for *approximate* math and the asemantic language and culture-dependent system used for *exact* math (Ansari & Karmiloff-Smith, 2002). However, the triple-code model was initially developed using neurological case studies of adult patients with lesions (see Chap. 2 on neuroscience research methods). Therefore, developmental processes for the three parietal circuits and their associated functions need further investigation. Chapters 3 and 4 describe neuroscience research relevant to the triple-code model.

A Place for Neuroscience in Cognitive Development

Humans are fascinated with their own brains and minds. For the past 25 years or so, innovations in neuroimaging technologies (see Chap. 2), such as fMRI, have allowed us to visualize the working brain and, as a result, neuroscience research has proliferated at an astounding rate. Nevertheless, cognitive developmental and educational psychologists have been appropriately cautious regarding the value and interpretations of neuroscience research (e.g., Brown & Bjorklund, 1998; Crone, Poldrack, & Durston, 2010; Poldrack & Wagner, 2004; Spelke, 2002; Turner, 2014). For example, in 1997, Bruer published a now often-cited paper called *Education and the Brain: A Bridge Too Far* in which he stated, “Educational applications of brain science may come eventually, but as of now neuroscience has little to offer teachers in terms of informing classroom practice. There is, however, a science of mind, cognitive science, that can serve as a basic science for the development of an applied science of learning and instruction” (p. 4). Bruer (1997) and others (e.g., Alferink & Farmer-Dougan, 2010; Bowers, 2016; Geake, 2008; Goswami, 2006; Howard-Jones, 2014; Lindell & Kidd, 2011) cited counterproductive misapplications of brain science, such as notions that people are right-brained or left-brained and learning styles. Others, including myself, felt cautiously enthusiastic regarding the future of neuroscience in the field of cognitive development, recognizing neural activity as a valid component of the developmental system (Brown & Bjorklund, 1998; Byrnes & Fox, 1998a). Yet, we doubted that "education 20 years from now will be radically different than it is today because of a paradigm shift to neuroscience thinking by prominent educational researchers and theorists. However, we do believe that developmental neuroscience theorizing and research will provide a clearer, more accurate picture of children's developing cognitions and ways in which education can be tailored to the special characteristics of children's changing intellects, and thus to improved educational practices" (Brown & Bjorklund, 1998, p. 356).

As described in a previous section, I believe that the promise of neuroscience theories and research is more fully realized when placed within a developmental systems approach, which has become increasingly feasible due to technological and methodological innovations (Brown & Bjorklund, 1998; Brown & Chiu, 2006; Kroeger et al., 2012). For example, the recording of Event-Related Potentials (ERP) has emerged as a portable means for gathering brain data across the life span, including newborns (e.g., Molfese, Molfese, & Pratt, 2007), and protocols for familiarizing children to fMRI procedures have greatly improved success rates and the quality of data for children aged 5 and older (e.g., Byars et al., 2002; Rajagopal, Byars, Schapiro, Lee, & Holland, 2014). Although these advances make the use of neuroimaging techniques more practical for studying development, what can studies measuring brain activity tell us about cognitive development that we cannot gather from the behavioral level of analysis? Beyond mapping brain structures and functions, longitudinal neuroimaging studies can contribute to our understanding of developmental processes by revealing how earlier developing brain systems change to support later-developing, more sophisticated abilities (Rajagopal et al., 2014). Furthermore, neuroimaging studies go beyond localization by describing the temporal dynamics of neural processing and interregional interactions in the brain. Using a developmental systems approach, I believe that understanding how the neural level of analysis interacts with other levels of analysis can go a long way towards *explaining* cognitive development.

Whenever possible, it makes sense to examine cognitive development using multiple levels of analysis within the same study (Brown & Chiu, 2006). Of course it would be very difficult to design studies examining all of the factors that interact to produce cognitive development within a given domain, but some are now using neuroimaging, behavioral, and experiential measures to advance our understanding of cognitive and developmental processes by revealing bidirectional relations between hierarchical levels of analysis. For example, Chapter 6 describes how some neurodevelopmental disorders lead to differences in brain development that influence mathematical cognition, and Chap. 7 describes interventions that may influence behavior, cognition, and neural activity, potentially leading to gains in achievement.

Twenty years later, I believe that neuroscience research is mainstream in the field of cognitive development and is becoming integrated into our theories. At the time of this writing, a search of literature published through the end of 1998 in the

PsycINFO database using the terms “neuroscience” and “cognitive development” yielded 365 results. By the end of 2017, the same search yielded 4396 results. I also now believe that knowing more about neurological processes related to cognitive development will facilitate the design of better curricula, instructional practices, and interventions (see Kroeger et al., 2012). Scholars have been engaging in interdisciplinary discussions and translational research that meaningfully bridges the fields of neuroscience, cognitive development, and education (e.g., Coch, Michlovitz, Ansari, & Baird, 2009; Goswami, 2006; Howard-Jones et al., 2016; Mareschal, Butterworth, & Tolmie, 2013; Varma, McCandliss, & Schwartz, 2008). Interdisciplinary, collaborative research teams work on projects using multiple levels of analysis, including fMRI, behavior, and educational interventions (Brown & Chiu, 2006; Coch et al., 2009; Cohen et al., 2000; Hille, 2011; Johnson, Halit, Grice, & Karmiloff-Smith, 2002; Magill-Evans, Hodge, & Darrah, 2002; Supekar, Iuculano, Chen, & Menon, 2015). Professional societies and journals provide venues for disseminating this type of work (e.g., the International Mind, Brain, and Education Society). Professional development programs for researchers and educators integrating Mind, Brain, and Education are offered at several universities (e.g., http://education.jhu.edu/Academics/certificates/mindbrain/).

The most important reason for integrating these fields is to help children who are experiencing difficulties learning mathematics (Berch & Mazzocco, 2007). Neuroscientists, cognitive developmental psychologists, and math educators have made important discoveries. As researchers, our best approach for developing effective intervention practices is to triangulate our knowledge across these fields that have traditionally operated separately. By bringing these literatures together, we create opportunities for identifying conceptual knowledge and procedural skills that should be targeted in interventions and developing appropriate practices that affect student achievement. Although it is sometimes difficult for researchers from different fields to work together, we have to make the effort to learn each other’s languages and integrate our results to build a cohesive understanding of mathematical cognitive development. Chapter 8 discusses where we stand with respect to these efforts and directions for moving forward.

Next, in Chap. 2, I provide a basic description of brain development and neuroscience research methods as background information to the research presented in the remainder of the book.

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Chapter 2
Brain Development and Cognitive Neuroscience Research Methods

Rhonda Douglas Brown

Abstract In this chapter, I provide an overview of brain development, structure, and function as background for interpreting neuroscience research on mathematical cognitive development. The formation of the brain throughout prenatal development is described and the location and functions of the four major lobes of the brain and the major sulci and gyri are identified. I also explain the structure and functioning of neurons. Brain growth and regionally specific developmental changes in gray and white matter are detailed. Then, I describe cognitive neuroscience research methods including lesion studies, which measure changes in cognitive function related to brain injury, and Transcranial Magnetic Stimulation (TMS), which induces temporary lesions. Cutting-edge neuroimaging techniques that have provided opportunities for studying the living and working brain are explained, including functional Magnetic Resonance Imaging (fMRI), which measures changes in blood flow, Diffusion Tensor Imaging (DTI), which measures white matter connectivity patterns, Event-Related Potentials (ERP), which measures electrical activity, and functional Near-Infrared Spectroscopy (fNIRS), which uses light to measure changes in blood flow. I conclude by discussing advantages and limitations of using these cognitive neuroscience research methods. Despite their limitations, these methods provide us with tools for discovering how knowledge and thought are embodied in our brains.

Humans are fascinated with brains. Consider this quotation from *Nature* by Ralph Waldo Emerson, originally published in 1844.

> Man carries the world in his head, the whole astronomy and chemistry suspended in a thought. Because the history of nature is characterized in his brain, therefore he is the prophet and discoverer of her secrets. Every known fact in natural science was divined by the presentiment of somebody, before it was actually verified (Emerson in Ferguson & Carr, 1984, pp. 106–107).

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Our knowledge and thought are embodied in our brains, yet, we don't fully understand how. Throughout written history, humans have shown a curiosity and motivation to understand ourselves, our children, other people, and the world that surrounds us. We each have ideas about how the brain, mind, and cultural practices contribute to the development of our thinking, including our mathematical cognition. These theories and hypotheses are formally tested by researchers in the disciplines of neuropsychology, cognitive developmental psychology, developmental cognitive neuroscience, and educational psychology using a variety of research methods. In this chapter, I focus on the methods that are most relevant to the neuroscientific study of mathematical cognitive development. But first, I provide a brief and basic overview of brain development, structure, and function as background for interpreting neuroscience research. (For more comprehensive reviews of brain development and functional neuroanatomy, see Lagercrantz, 2016 or Stiles, 2008, and Afifi & Bergman, 2005, respectively).

Brain Development, Structure, and Function

The human brain is often described as a mass of clay, with genes and experience as its sculptors. Genes provide the instructions for the formation of the brain's basic structures and functions through *experience-expectant processes* within species-typical environments. Experience leads to individual differences in structures and functions through *experience-dependent processes* that allow our brains to be shaped by specific environments during their extended period of postnatal growth (Greenough, Black, & Wallace, 1987). For example, most humans acquire language, but whether the language we acquire is English, or Spanish, or Mandarin Chinese depends on our cultural experiences. As I discussed in Chap. 1, biologically primary abilities, such as language, are species-typical, whereas, biologically secondary abilities, such as reading and mathematics, may or may not develop depending on cultural practices and individuals' activities, including formal schooling. Thus, as described by a developmental systems approach, genetics influence neural activity, which influences, and is influenced by, experiences in environments.

The development of the most complex organ in the human body begins about 19 days after fertilization, occurs at a rapid pace during the first several years of life, and extends into early adulthood. The neurulation process begins with the formation of the *neural plate*, which is an oval-shaped disk of cells that will become the brain and spinal cord. By 20 days, the neural plate has folded inward, forming the *neural groove*, which then becomes fused from the middle outwards to form the *neural tube*, a hollow cord of cells, by 26 days. The top of the neural tube is the emerging brain and the rest becomes the spinal cord. If the neural tube does not close properly at the top or bottom, the nervous system is affected, resulting in conditions such as anencephaly or spina bifida, respectively (see Chap. 6 for more information on mathematical difficulties related to spina bifida). By 1 month after fertilization, the neural tube and ventricles have developed into the forebrain, midbrain, and hindbrain.

Around 5 weeks, the forebrain enlarges and divides into two vesicles in the front, called the telencephalon and the diencephalon. The *telencephalon* develops into the cerebral cortex, basal ganglia, and limbic structures and begins dividing along its midline to form the left and right *hemispheres*, or halves, of the brain. The *diencephalon* develops into the thalamus and hypothalamus. The hindbrain develops into the *metencephalon*, which forms the cerebellum and pons, and the *myelencephalon*, which forms the medulla oblongata. Each of the ventricles contains proliferative zones where *neurons*, or nerve cells, and supporting *glial cells* are generated from stem cells during a process called *neurogenesis*, or *proliferation*, which begins at 3 weeks, peaks at 7 weeks, and is mostly complete by 18 weeks. This rapid cell division produces 100 billion neurons at the rate of over 500,000 per minute and leads to the initial formation of distinct brain regions (Eliot, 1999). New neurons go through a *migration* process, following chemical cues to their permanent locations in the brain by travelling away from the ventricle along radial glial cells to form layers that make up the *cerebral cortex*, the seat of human cognition. The outer surface, called the *neocortex* because it is the newest area of the brain to evolve, varies from about 2 to 4 mm thick and is organized into six horizontal layers of cells. The phylogenetically older *allocortex* is comprised of structures and regions deep inside the brain with three or four layers.

By 7 months, all of the major structures of the brain are in place and the once smooth surface of the neocortex is now convoluted with *sulci*, which are fissures that allow the growing brain to fold in on itself, creating more surface area within the constraints of the skull, and *gyri*, which are elevated ridges. Sulci and gyri are used as reference points for locating brain regions and structures. Figure 2.1 illustrates the four major lobes of the brain and the major sulci and gyri from the left lateral (side) view of the exterior surface of the neocortex and the right sagittal (middle) view of the interior regions, including some of the allocortex. The *frontal lobe* is located anterior to (in front of) the *central sulcus*, or fissure, and superior to (above) the *lateral* or *Sylvian fissure*. The *temporal lobe* is located inferior to (below) the lateral fissure and anterior to the angular gyrus. The *parietal lobe* is posterior to (behind) the central sulcus and anterior to the parieto-occipital sulcus. The *occipital lobe* is located at the back of the brain, posterior to the parieto-occipital sulcus. The *limbic system* involves areas of the frontal, temporal, and parietal lobes and includes the amygdala, uncus, parahippocampal gyrus, cingulate gyrus, and paraolfactory area, as well as other internal structures.

Table 2.1 lists the major structures and functions associated with each lobe and the limbic system. Some major functions are *lateralized*, relying more on one hemisphere than another. For example, generally, language functions are left lateralized and visuospatial functions are right lateralized. It is important to keep in mind that the hemispheres control the *contralateral* (or opposite) sides of the body, so the left hemisphere controls the right side of the body and vice versa; thus, handedness also relates to lateralization.

Although the major brain structures are in place by 7 months of gestation, neurons are not fully functional. They have a basic structure, but have not yet set up communication networks that allow the brain to transfer, process, and integrate

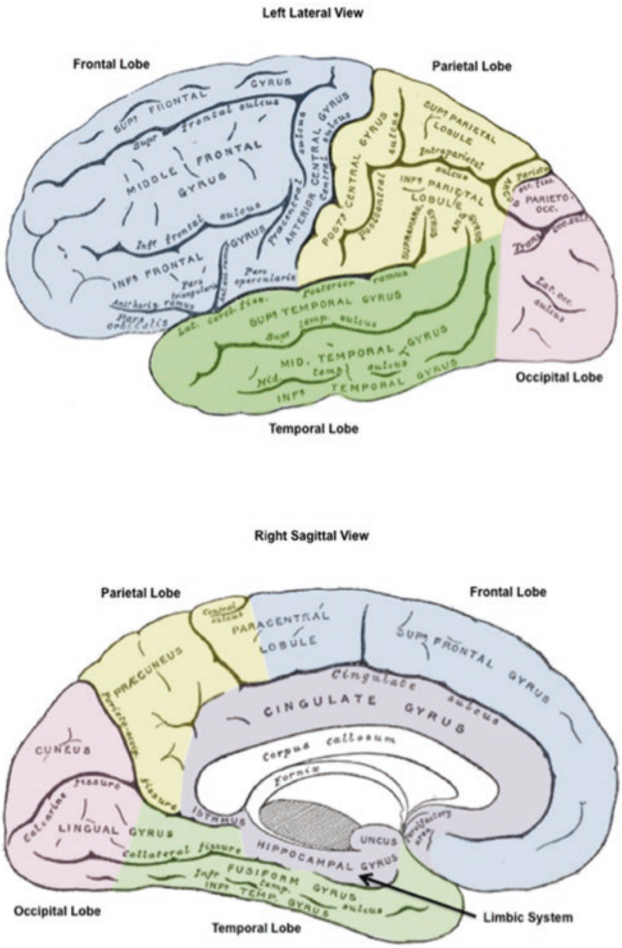

Fig. 2.1 The four major lobes of the brain and the major sulci and gyri from the left lateral (side) view of the exterior surface of the neocortex and the right sagittal (middle) view of the interior regions, including the limbic system

information from the external world through our senses and engage in higher-order thinking. Figure 2.2 provides an illustration of communication between fully developed neurons. Each neuron has a cell body with a nucleus that contains deoxyribonucleic acid (DNA). When a neuron is at rest, ions on the inside of the cell membrane are more positively charged than those on the outside. A neuron transfers information at the rate of 200 times per second by firing electrical impulses called *action potentials* down a long fiber called an *axon* that extends from its cell body to other neurons. Multiple axon terminals branch off from the end of an axon that contains vesicles of a specialized chemical called a *neurotransmitter*. Positive calcium ions trigger the vesicles to drift to the cell membrane, fuse with it, and release the neurotransmitter.

Table 2.1 Major structures and functions associated with each lobe and the limbic system of the brain

Region	Major functions
Frontal lobe	• Body movements • Speech (productive language) • Executive functions, including attention, inhibition, planning, reasoning, problem solving, and abstract thought • Working memory • Personality • Emotions
Temporal lobe	• Hearing • Recognizing faces • Understanding language • Memory • Sequencing and organization • Emotion
Parietal lobe	• Taste • Perceiving touch, pressure, pain, and temperature • Body awareness • Visuospatial perception and processing • Interpreting language and words
Occipital lobe	• Vision • Visuospatial processing
Limbic system	• Olfaction (smell) • Fight or flight response • Long-term memory • Emotion and motivation

When neurotransmitter molecules are released into gaps between neurons, called *synapses*, they may bind to specialized receptors on the *dendrites*, or roots, of other neurons. Receptors and neurotransmitters operate like locks and keys. When a specific neurotransmitter molecule enters a receptor site, positive sodium ions flow into the cell, which may lead to a positive charge that moves down the axon to the axon terminals. There are up to 100 different neurotransmitters, with some serving excitatory functions that pass a signal on (e.g., glutamate) and others serving inhibitory functions that stop a signal from passing on (e.g., gamma-aminobutyric acid or GABA).

To become functional, once in its final location, a neuron goes through processes of *differentiation* and *synaptogenesis* during which it grows and extends its dendrites and axonal terminals to form synapses with other neurons. These processes occur rapidly during the first several years of life when the brain is becoming organized into functional networks (e.g., Huttenlocher & Dabholkar, 1997). At the peak of synaptogenesis, 15,000 synapses are produced on every cortical neuron, which corresponds to a rate of 1.8 million new synapses per second between 2 months of gestation and 2 years after birth.

Indeed, infants have many more neurons and synapses than adults because they are *over*produced early in development. Thus, critical, yet perhaps counterintuitive, processes during brain development include *apoptosis*, or cell death, and *synaptic*

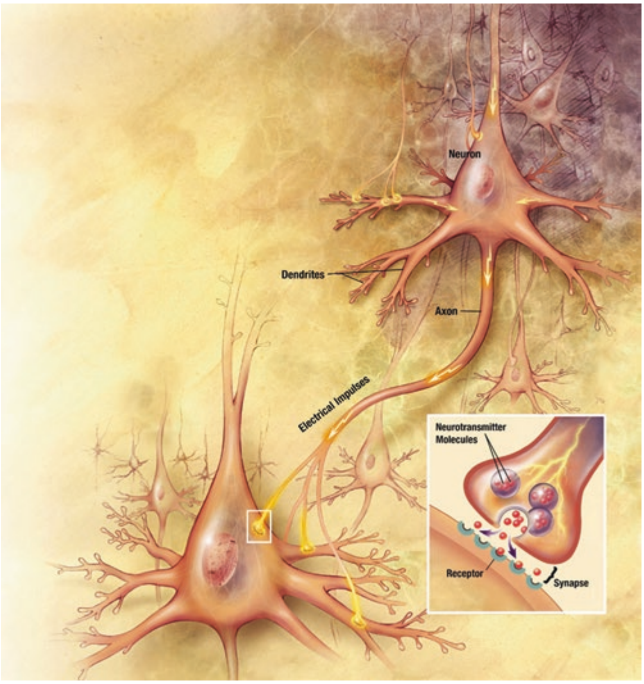

Fig. 2.2 Neurons communicate through electrical impulses that travel along axons to synapses at dendrites where neurotransmitter molecules are released at receptor sites

pruning based on experience-dependent neural activity (Fox, Levitt, & Nelson, 2010; Stiles, 2009). Neurons that connect with other neurons, and are used to perform specific sensory, motor, and cognitive functions, survive and form hundreds of synapses with other cells, but those that are not used die off (Bertenthal & Campos, 1987; Changeux & Dehaene, 1989; Edelman, 1987; Greenough et al., 1987). It is estimated that about 50% of neurons produced during gestation do not survive (Tau & Peterson, 2010). This reciprocal relationship between brain activity and structural organization allows human brains to adapt to environments and experiences and become more specialized. For example, Kuhl et al. (2006) provided evidence that infants' ability to perceive phonemes from nonnative languages declines during the first year of life, while their abilities to perceive contrasts in their native language increases, which the authors attributed to neural commitment. According to Casey, Giedd, and Thomas (2000), "neurons that fire together wire together" (p. 246). More specifically, cognitive development coincides with the suppression of competing, less frequent behaviors through the loss of synapses and the strengthening of remaining connections through repeated exposure. Thus, the primary mechanism for learning and memory is experience-dependent modification of synapses (Shepherd, 2004). Ultimately, each neuron may be connected to thousands of other neurons.

Another major development related to brain function and microstructure is that axons become insulated in a coating of a fatty substance called *myelin* that is produced by glial cells. You may have heard the terms *gray matter*, which refers to the cell bodies and dendrites that form major brain regions, and *white matter*, which refers to the myelinated tracts of axons that lie beneath the cortex connecting brain areas into networks. The left and right hemispheres are connected by five commissures of white matter that span the longitudinal fissures, the largest of which is the *corpus callosum*, indicated in the right sagittal (middle) view of Fig. 2.1, which is comprised of approximately 200 million myelinated axons (Giedd et al., 2015). Myelination facilitates the conduction of action potentials, thereby increasing the speed, frequency, and synchrony of neural firing patterns, and reduces interference from nearby signals (Giedd et al., 2015).

The human brain grows very rapidly during infancy and early childhood. Approximately 80% of total brain volume is reached by 1.5 years (Groeschel, Vollmer, King, & Connelly, 2010), 95% is reached by 6 years (Lenroot et al., 2007), and peak size is reached at age 10.5 in girls and 14.5 years in boys, presumably due to dendritic growth (80% occurs after birth) and synaptogenesis (e.g., Giedd et al., 2015; Huttenlocher, 1994). Then, brain size declines slightly into early adulthood, presumably due to pruning that occurs during the teens and twenties (e.g., Giedd et al., 2015; Huttenlocher, 1994; Whitford et al., 2007). Thus, total brain volume reflects a dynamic interaction between concurrent progressive and regressive processes, with different tissue types, regions, and structures following different time courses (Durston et al., 2001). Cortical gray matter volume doubles to triples during the first year of life and increases by another 15–20% during the second year of life (Gilmore et al., 2012; Knickmeyer et al., 2008). Total volume of subcortical structures also doubles during the first year of life, except for the hippocampus, which shows a slower growth rate (Gilmore et al., 2012). In their longitudinal study examining developmental trajectories of brain matter volume between the ages of 5 and 25 years, Giedd et al. (2015) reported that gray matter volumes generally follow inverted U-shaped curves with peaks occurring in the primary sensorimotor areas first between the ages of 2 and 4 (Gogtay et al., 2004), in the parietal lobes at 7.5 years for girls and 9 years for boys, in the temporal lobes at 10 years for girls and 11 years for boys, and in the frontal lobes at 9.5 years for girls and 10.5 years for boys. Cerebellum size peaks at 11.3 years for girls and 15.6 years in boys (Tiemeier et al., 2010). On average, male brains are approximately 10% larger across the lifespan (Giedd et al., 2015). Throughout the lifespan, the human brain shows *plasticity*, continuing to respond to experience through synaptogenesis. Furthermore, although most neurons are formed before birth, evidence for the generation of new neurons after birth exists for the olfactory bulb, involved in smell, and the dentate gyrus of the hippocampus, involved in storing new memories (Eriksson et al., 1998; Nelson, de Haan, & Thomas, 2006).

Unlike gray matter, white matter increases with development, with most myelination occurring between the prenatal period and age 2 for the sensory and motor areas, but continuing to increase about 1–2% per year into late adolescence and early adulthood for the parietal and frontal areas (Giedd, Blumenthal, Jeffries, Castellanos, et al., 1999; Giedd et al., 2015; Miller et al., 2012; Sowell et al., 1999; Yakovlev & Lecours, 1967). The number of white matter tracts is relatively stable by age 4, but fiber density within tracts decreases with age (Dennis et al., 2014; Lim, Han, Uhlhaas, & Kaiser, 2015; Richmond, Johnson, Seal, Allen, & Whittle, 2016). The posterior corpus callosum reaches maturity during adolescence (Durston et al., 2001; Giedd, Blumenthal, Jeffries, Rajapakse, et al., 1999; Thompson et al., 2000). With development and experience, white matter tracts increase their connectivity and organization, becoming more streamlined, thereby increasing speed and efficiency of information processing (Richmond et al., 2016). Through these developmental processes, functional networks are established. In the next section, I describe how researchers use case studies and medical imaging techniques to discover relationships between brain structures, cognition, and behavior.

Cognitive Neuroscience Research Methods

Lesion Studies

Historically, *lesion studies* have been used to discover relationships between brain structures, cognition, and behavior. This method involves conducting case studies of patients with brain injury, disease, or neurodevelopmental disorders to assess loss or impairment of specific cognitive functions. The comparison of patients in lesion studies provides neuropsychologists with opportunities for discovering *double dissociations* that reveal brain regions or structures supporting distinct cognitive functional systems. When patients who have damage to circumscribed brain areas lose specific conceptual or procedural knowledge, we can draw inferences regarding where and how cognitive functions are instantiated in the brain. For example, Lemer, Dehaene, Spelke, and Cohen (2003) provided evidence for distinct *quantity* and *verbal* systems for numerical processing (see triple-code model in Chap. 1) by studying two patients with different types of lesions and *acalculia*, a broad term used to describe mathematical difficulties. Patient LEC had a focal lesion in the left parietal lobe and experienced difficulties performing approximation and subtraction tasks that require understanding of the meaning of numbers, which indicates dysfunction of the proposed quantity system, but LEC could accurately retrieve multiplication facts. Conversely, patient BRI had hypometabolism in the left temporal lobe and experienced difficulties with multiplication fact retrieval and exact addition on large problems, which indicates dysfunction of the verbal system, but BRI had proficient approximation and subtraction abilities. This type of study provides some evidence that the proposed quantity system is localized in the left parietal lobe and the proposed verbal system is localized in the left temporal lobe (also see Cappelletti, Butterworth, & Kopelman, 2001; Cipolotti & Butterworth, 1995; Cohen, Dehaene, Chochon, Lehéricy, & Naccache, 2000; Dagenbach & McCloskey, 1992; Dehaene & Cohen, 1997; Delazer & Benke, 1997; Grafman, Kampen, Rosenberg, Salazar, & Boller, 1989; Lampl, Eshel, Gilad, & Sarova-Pinhas, 1994;

Pesenti, Seron, & Van Der Linden, 1994; Pesenti, Thioux, Samson, Bruyer, & Seron, 2000; van Harskamp & Cipolotti, 2001; van Harskamp, Rudge, & Cipolotti, 2002; Whalen, McCloskey, Lesser, & Gordon, 1997).

However, lesion studies must be interpreted with some degree of caution. Although some brain regions may be highly specialized, permitting relatively straightforward inferences about relationships between structure and function, complex cognitive functions likely involve networks of areas distributed across the brain (Dehaene, 2011). Other, nonmathematical processes required by tasks, such as attention and working memory, may be disrupted (e.g., Whalen et al., 1997) or compensatory mechanisms may lead to the recruitment of other regions that could be affected because they are connected to the damaged regions (Nelson et al., 2006; Nelson & Bloom, 1997). Furthermore, case studies of patients with lesions are highly limited in their generalizability, especially regarding cognitive development, since they are typically conducted on adults with developed brains and acquired knowledge and skills, or on patients with neurodevelopmental disorders, which may have altered fundamental brain organization and affected multiple cognitive systems (Kolb & Fantie, 2009). Interestingly, these limitations can be addressed by using transcranial magnetic stimulation to induce lesions in individuals with typical development.

Transcranial Magnetic Stimulation (TMS)

Transcranial magnetic stimulation (TMS) takes advantage of Faraday's principles of electromagnetic induction. For this technique, researchers send a pulse of current through a coil placed over a participant's head to generate a magnetic field so that it passes through the participant's scalp and skull, inducing a current in his brain (for a review, see Pascual-Leone, Walsh, & Rothwell, 2000). This magnetic stimulation temporarily disrupts ongoing cortical activity in the targeted brain region, creating a transient lesion, allowing us to observe how behavior and performance are related to brain structures and distributed networks, functions, and the timing of processing. For example, Chap. 5 includes a description of a study (Sack et al., 2007) that demonstrates the importance of the right parietal lobe to visuospatial cognition by disrupting it using TMS and examining the frontoparietal network using functional Magnetic Resonance Imaging, which is described in the next section.

Over the past three decades, developmental cognitive neuroscience research has increased exponentially, which has led to important discoveries. Many of these discoveries have been made possible by technological innovations in *neuroimaging techniques* that have provided opportunities for studying the living and working brain (for reviews, see Amso & Casey, 2006; Lenroot & Giedd, 2007; Twardosz, 2007). Using these noninvasive technologies, researchers can localize cognitive functions in the brain and identify networks of neuronal activity in real time as information is transferred from region to region within the brain. Furthermore, we can better understand the development of typical and atypical cognitive processing, thereby contributing to the science of learning and, ultimately, design of effective interventions. In the following sections, I provide broad overviews of the major technologies used in much of the developmental cognitive neuroscience research presented in this book, focusing on functional Magnetic Resonance Imaging.

Functional Magnetic Resonance Imaging (fMRI)

Functional Magnetic Resonance Imaging (fMRI) has rapidly become a popular method for noninvasively identifying brain regions and structures that are active while participants perform specific cognitive tasks inside a scanner. You may have experienced clinical MRI on your knee, heart, brain, or other part of your body during which you laid down on a table, were moved into the bore of a magnet, and heard some loud banging noises as a series of digital photographs were taken. Although there are similarities in the experiences of a patient undergoing clinical MRI for the purposes of diagnosis and a participant in a research study using fMRI, there are some critical differences that permit the study of relationships between brain structures and cognitive functioning. Below, I provide a basic description of fMRI and an example of Institutional Review Board approved procedures used in studies I have conducted with colleagues at the Imaging Research Center at Cincinnati Children's Hospital Medical Center. Many researchers at other hospitals and universities use similar procedures.

Basically, fMRI measures changes in blood flow in the brain. More specifically, fMRI measures the time course of neuronal activity in specific brain structures or regions during cognitive tasks. When neurons are active, their local blood supply and oxygen content changes, allowing us to identify the brain structures that are active during a cognitive task. Most studies use *Blood Oxygenation Level Dependent (BOLD)* contrast (Ogawa, Lee, Kay, & Tank, 1990). This method generates high spatial resolution brain images that reflect neuronal activity by measuring changes in the magnetic properties of hemoglobin correlated with the oxygenation of blood in the cerebral vessels. Since deoxygenated hemoglobin is paramagnetic and oxygenated hemoglobin is diamagnetic, changes in magnetic susceptibility result in changes to the overall magnetization of the hemoglobin in brain regions experiencing increased blood flow due to activity. The brain images that are shown in the results of fMRI studies display this change in magnetization due to localized neuronal activation using colored pixels. It is important to note that exactly what is represented by these colored pixels varies from study to study. For example, they could represent regions of brain activation or *de*activation, or areas in which activation or *de*activation is correlated with task performance or other measures. When reading studies using fMRI, you must carefully examine figures and captions and results sections to accurately interpret and draw conclusions from brain images.

Most fMRI studies are conducted at hospitals and universities because they have the necessary infrastructure and personnel. Typical research grade scanners have magnetic field strengths of 3 Teslas (T) or above. Thus, standardized MRI safety restrictions and specific exclusion criteria for participation in a study are applied. Below, I describe what you would experience if you were participating in a typical fMRI study (also see Byars et al., 2002).

Upon arrival at the hospital, you are met by a researcher and escorted to the imaging center. The researcher explains the study and its procedures and she addresses any questions or concerns that you may have. Next, you provide your consent for participation in the study. Then, you complete a MRI safety and screening form that gathers your surgical history and information regarding the presence of any metal in your body that would lead to exclusion from the study since the strong magnetic field could tug at it, such as metal orthopedic pins or plates above the waist, orthodontic braces or a permanent retainer, or any medical devices, such as a pacemaker or nerve stimulator, as well as any injury or work involving metal. Next, you complete a demographic questionnaire that includes questions regarding your age, handedness, previously diagnosed neurological impairments (e.g., Autism), psychological disorders (e.g., Attention Deficit/Hyperactivity Disorder) or learning disabilities, and participation in intervention programs. Then, a registered radiological technologist, who runs the scanning session, reviews the form with you, describes the procedures used in MRI scanning, and addresses any questions that arise. Next, you remove all jewelry, pens, pencils, cellular phone, belt, and any other metallic objects from your clothing since these objects could be projectiles in the strong magnetic field of the scanner. Then, the researcher administers any neuropsychological measures that are included in the study, such as standardized intelligence tests, achievement tests, and so forth. Next, you participate in a brief training session on the specific experimental and comparison tasks that will be used in the study. On a desktop computer, you solve the same types of problems for each of the tasks that will be used in the fMRI scanner. On the monitor, you see a task name and directions, which are narrated by the researcher. As part of the training session, the researcher shows you how to use hand-held button boxes for making response selections during the tasks. You are told to hold *very* still and not to talk while making your responses in the scanner because motion artifacts can make the images uninterpretable. The researcher addresses any of your questions regarding the tasks at this time. Then, you are escorted into the scanner room, shown in Fig. 2.3, and asked to lie on your back on the bed of the scanner. The radiological technologist explains that you can stop the scanning process at any time by signaling the control room through the intercom system, closed circuit video camera, or by squeezing a "panic bulb" clipped to your clothing. A radiofrequency (RF) coil, a cylindrical device open at both ends that serves as a receiving antenna, is placed over your head along with headphones to reduce scanner noise and allow you to hear auditory stimuli and instructions. The radiological technologist also helps you put on goggles that display visual stimuli and places the button box for making response selections in your hand.

At the beginning and end of the scanning session, you watch a movie you have selected from the center's video library via the MR-compatible audiovisual system while locator and anatomical scans and fine-tuning processes are conducted, which also may help you relax. Prior to each scan, the researcher uses the intercom system

Fig. 2.3 fMRI scanner room at Cincinnati Children's Hospital Medical Center. Photograph courtesy of the University of Cincinnati

to tell you that the scan is beginning, provide directions for the task, check that you are comfortable, and remind you to remain as still as possible despite the loud noises you hear during scanning.

Immediately preceding each math task, you view specific directions projected onto the goggles and the researcher reads them out loud using the intercom system. When the task begins, you see a math problem in the center of the visual field, with answer choices shown on the left and right sides of the visual field. You are instructed to press the left button on the hand-held box to select the answer choice on the left and the right button to select the answer choice on the right. Your button presses (left or right) and response time for each problem are stored along with the functional neuroimaging data for each trial. To control for neural activation related to movements (i.e., button presses), visual processing, and cognitive functions associated with generating responses during the experimental trials, control trials are included during scanning sessions. For example, control trials for some tasks we have used consist of three identical numbers. Using the button box, you are asked to indicate which number from the left or right visual field is the same color as the number presented in the center of the visual field. Experimental and control trials can be presented in random order within blocks or interleaved in an event-related design. After scanning is completed, the researcher escorts you to a nearby testing room where you complete paper-and-pencil assessments. The total time required for participation in a typical study is approximately 2.5 h, with scanner time limited to less than 1.5 h. At a later time, a neuroradiologist reviews all scans and reports any clinically significant findings to the researcher, who then provides the results to you or your parent or guardian following hospital procedures.

What types of information can be gleaned from fMRI studies of mathematical cognition? A variety of questions can be answered using fMRI depending on the type of experimental design, tasks, and measures used. Fundamentally, fMRI has been used to answer questions about which brain regions and structures perform particular cognitive functions. Changes in neural activity in a particular structure or brain region can be associated with differences in the underlying cognitive functions recruited by different tasks (Brown & Chiu, 2006). Some studies are designed to examine group differences in brain activation during the performance of specific cognitive tasks. For example, participants may be classified into groups, those with Mathematical Difficulties (MD, or even subtypes of MD) and those with Typical Achievement (TA) based on standardized test results (e.g., percentile scores), to investigate relationships between pre-existing group membership and brain activation during the performance of specific math tasks (i.e., quasi-experimental, between-subjects designs). Accuracy and reaction time on the math tasks can be recorded and correlated with neural activation in particular Regions of Interest (ROIs). This method can be used with a variety of measures, including standardized paper-and-pencil tests, researcher-created assessments or tasks, demographic information, and pertinent learning history. These measures collected outside of the scanner can be correlated with brain activation during tasks completed inside the scanner. For example, Kroeger (2012) created an assessment to examine the types of errors made when calculating multi-digit arithmetic problems, which can be correlated with brain activation during in-scanner cognitive tasks. Furthermore, longitudinal studies can provide information about how functional networks change with age and experience.

A variety of data analysis protocols and software is available, such as SPM (Wellcome Dept. of Cognitive Neurology, London, UK). Typically, motion correction procedures are performed prior to statistical analysis of the functional data (Szaflarski et al., 2006; Thevenaz, Ruttimann, & Unser, 1998). In our studies, for each participant, activation T-maps are computed using the General Linear Model (GLM) assuming the Hemodynamic Response Function (HRF). Using SPM routines, T-maps are converted into 3D stereotaxic coordinate space (e.g., Talairach & Tournoux, 1988 or Montreal Neurologic Institute [MNI] template) to localize anatomical regions and allow for comparisons between participants and groups.

Diffusion Tensor Imaging (DTI)

fMRI procedures often also include *Diffusion Tensor Imaging (DTI)*, which can be used to measure patterns and organization of connections from one brain area to another, axonal density and size, and myelination (Basser, 1997; Le Bihan, 1991; Le Bihan et al., 2001; Paus, 2010). DTI is a noninvasive neuroimaging technique that measures the diffusion process of water within brain tissue (Le Bihan & Breton, 1985). The anisotropic diffusion of water along axons is less restricted than diffusion perpendicular to the axonal direction, which permits the generation of 3D images of white matter tracts (Schmithorst, Wilke, Dardzinski, & Holland, 2005).

According to Qiu, Mori, and Miller (2015), quantitative measures of diffusion include *fractional anisotropy (FA)*, which indicates “the degree to which water diffusion is restricted in one direction relative to other directions” (p. 855), *mean diffusivity (MD)*, which “corresponds to the directionally averaged magnitude of diffusion” (p. 856) and reflects tissue density, and *axial diffusivity (AD)*, and *radial diffusivity (RD)*, which “reflect the rate of microscopic water motion parallel and perpendicular, respectively, to the direction of axonal fibers in a regional tissue” (p. 856). These measures have been used in studies characterizing typical development of white matter, including age-related changes showing increases in white matter volume and regionally specific myelination (e.g., Giedd et al., 2015; Paus et al., 2001) and sex differences (e.g., Clayden et al., 2012; Lenroot & Giedd, 2010; Schmithorst, 2009; Schmithorst, Holland, & Dardzinski, 2008). Studies have consistently shown linear increases in FA due to myelination and decreases in MD during early life, childhood, and adolescence (Dubois et al., 2008; Hüppi & Dubois, 2006; Schmithorst, Wilke, Dardzinski, & Holland, 2002; Wang et al., 2012). As noted by Giedd et al. (2015), several studies suggest a positive relationship between developmental changes in white matter fiber organization and cognitive function, including measures of language and memory (Nagy, Westerberg, & Klingberg, 2004), reading (Deutsch et al., 2005), response inhibition (Liston et al., 2006), and intelligence (Clayden et al., 2012; Mabbott, Noseworthy, Bouffet, Laughlin, & Rockel, 2006; Mabbott, Rovet, Noseworthy, Smith, & Rockel, 2009; Paus et al., 2001; Schmithorst et al., 2002, 2005; Tamnes et al., 2010). Dysfunction or disorganization of white matter tracts is associated with disorders (Le Bihan et al., 2001). For example, Kucian et al. (2014) found that children with arithmetic learning difficulties, called *developmental dyscalculia*, show reduced FA and fiber impairments in the superior longitudinal fasciculus adjacent to the intraparietal sulcus (IPS), which is a key region specified in the triple-code model of numerical processing (see Chap. 6 for information on mathematical difficulties).

Event-Related Potentials (ERP)

When we engage in cognitive tasks, such as deciding whether or not the arithmetic problem 7 + 16 = 25 is correct, we perform mental operations that activate multiple brain areas. One way to study this activation involves measuring time courses of electrical activity across the whole brain. *Electroencephalography (EEG)* indexes synaptic activity by recording electrical signals of synchronized populations of neurons through electrodes on the scalp. If you were participating in an ERP study of mathematical cognition, you would be fitted with an elastic cap that has between 20 and 256 sensors with standard positions, like the one shown in Fig. 2.4. While you are wearing the cap that records electrical activity in your brain, you would watch a large number of trials of math problems, such as 7 + 16 = 25, presented on a monitor and click one button on a mouse if the solution is correct or a different button on the mouse if it is incorrect. EEG raw data is collected during stimulus presentation. The

Fig. 2.4 Infant participating in a study measuring event-related potentials. Photograph courtesy of Bethany Reeb-Sutherland, Ph.D.

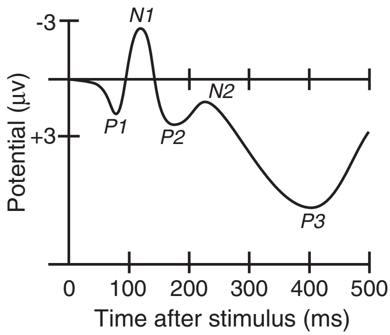

Fig. 2.5 Plot of an ERP waveform. Source: Original: Choms Vector: Mononomic (Own work based on: Constudevent.gif) [GFDL (http://www.gnu.org/copyleft/fdl.html) or CC-BY-SA-3.0 (http://creativecommons.org/licenses/by-sa/3.0/)], via Wikimedia Commons

electrical activity at each electrode site is averaged and the *event-related potential (ERP)* can be identified, which provides information regarding the timing and sequencing of particular neural events across the whole brain on the scale of tens to hundreds of milliseconds (Nelson et al., 2006; Steinhauer, 2014; for a review, see Luck, 2012). The EEG system displays the amplified EEG signals onto a separate monitor and stores data. Specifically, the system marks stimulus onset, allowing the sections of the EEG signal that reflect the processing of target stimuli to be identified. Once the study is complete, data is preprocessed to remove artifacts and correct for eye movements. Then, the EEG signals for the clean trials are averaged, time-locked to the onset of the target stimulus.

Cognitive developmental psychologists study ERP components. An *ERP component* is "a voltage deflection that is produced when a specific neural process occurs in a specific brain region" (Luck, 2012, p. 526). For example, in our hypothetical experiment, when you viewed the math problem 7 + 16 = 25 and decided whether the solution is correct or incorrect, many components were elicited that comprise an observed ERP waveform, such as the one shown in Fig. 2.5. Data analysis techniques are used to isolate ERP components of interest from other brain activity (Kappenman & Luck, 2012; Luck, 2005; Steinhauer, 2014). Similar to fMRI, ERP studies use a subtraction method between experimental condition trials and control condition trials to extract ERP profiles for specific cognitive processes. Components in waveforms are labeled according to the polarity of the peaks (P = Positive or N = Negative) and the timing by order (e.g., P1, P2, P3) or in milliseconds (e.g., P400). ERP epochs are typically 1 s long, beginning with stimulus presentation. During the first 200 milliseconds (ms), ERP components primarily reflect perception of the physical characteristics of the stimulus. For example, a negative deflection around 100 ms (N1) indicates processing in the visual cortex, which is followed by a positive deflection around 200 ms (P2) that reflects more complex pattern recognition (P200). The ERP components that occur later in the time-series reflect higher-level cognitive processing (e.g., P3 in Fig. 2.5). Due to variations in plotting conventions, when reading the results of an ERP study it is important to pay attention to which direction waveforms are plotted (i.e., Fig. 2.5 is plotted with positive deflections downward). Often, voltage maps are used to display the scalp distribution of these profiles during a specific time frame.

ERP methods are best suited for answering questions relevant to the timing of cognitive processes, particularly those that unfold over a short time frame, around approximately 2 s or less. Furthermore, ERP allows researchers to study cognition in the absence of behavioral responses, which is particularly useful for studying infants for whom fMRI often requires sedation (Dehaene et al., 1998; Luck, Vogel, & Shapiro, 1996). However, it does not have the spatial resolution of fMRI, so locations are limited to general regions, rather than specific structures. Thus, ERP is often combined with fMRI to provide more detailed information regarding locations of neural activity.

Functional Near-Infrared Spectroscopy (fNIRS)

Functional Near-Infrared Spectroscopy (fNIRS), sometimes referred to as *optical brain imaging*, detects brain activity by noninvasively measuring changes in near-infrared light to monitor the concentration and oxygenation of hemoglobin (Chance, Zhuang, UnAh, Alter, & Lipton, 1993; Hoshi & Tamura, 1993; Kato, Kamei, Takashima, & Ozaki, 1993; Maki et al., 1995; Villringer, Planck, Hock, Schleinkofer, & Dirnagl, 1993). Thus, similar to fMRI, fNIRS measures brain activity by examining changes in hemodynamic responses (i.e., BOLD response). Similar to ERP procedures, an infant participating in an fNIRS study would wear a cap with fiber optic cables attached for transmitting and receiving light. A computer records data as the infant engages in cognitive tasks. fNIRS has some advantages over fMRI in that it has greater temporal resolution, it can be used for some experimental paradigms that are not amenable to fMRI (i.e., is not as sensitive to motion), it's portable, and the costs are lower. Although it has higher spatial resolution than ERP, fNIR cannot be used to measure cortical activity beyond 5–10 mm beneath the skull. However, it can be simultaneously combined with fMRI to gain more precise localization information.

Despite their limitations, these cognitive neuroscience research methods provide us with tools for discovering how knowledge and thought are embodied in our brains. Chapters 3–6 present research that combines neuroimaging techniques and behavioral measures to help us better understand how the brain, mind, and cultural practices contribute to mathematical cognitive development.

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Chapter 3
Quantity Representation

Rhonda Douglas Brown and Vincent J. Schmithorst

Abstract In this chapter, we draw on evolutionary developmental psychology theory and Dehaene and colleagues' triple-code model to describe quantity representation, which is the basis for a set of numerical abilities selected during evolution, including numerosity, which involves quickly determining the quantity of a set without counting, and ordinality, which involves recognizing that one set contains more than another without counting. We present research using innovative behavioral and cognitive neuroscience methods indicating that sensitivity to magnitude is present at birth and increases in precision into adulthood, including work investigating two quantity representation systems: the Parallel Individuation (PI) system that allows humans to precisely track a small number of individual objects through space and time; and the Approximate Number system, or number sense, that allows humans to approximate the numerosities of sets of items without using symbols. Research establishing a relationship between quantity representation and mathematics achievement during childhood and adolescence is also described. We present results from a functional Magnetic Resonance Imaging (fMRI) study demonstrating that brain activation in the inferior occipital gyrus, lingual gyrus, and bilateral intraparietal sulcus (IPS) during magnitude comparison is positively related to adolescents' mathematics achievement, whereas deactivation of the Default Mode Network (DMN) during magnitude comparison is negatively related to adolescents' mathematics achievement, indicating that abstract quantity representation may be foundational for the development of calculation skills.

Take a look at Fig. 3.1 and, without counting, answer *How many blocks are on the right side*?

Your ability to quickly determine that there are *2* blocks in the array on the right side of the figure is called *numerosity*, which involves apprehending the number of objects, without counting. Now, take a look at Fig. 3.1 again and answer *Which array has more, the one on the left or the one on the right*? On this *magnitude comparison* task, your ability to quickly determine that the array on the *left* side has

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Fig. 3.1 Example of a trial from a magnitude comparison task

more blocks demonstrates sensitivity to the *ordinality* of numerical relationships, which involves recognizing that one set contains more than another, without counting.

Would it surprise you to hear that infants possess these abilities? During the first days of life, newborns can discriminate numerosities of small sets. How do we know? Infants can't talk and have little motor control to indicate their understanding. Researchers studying infants' early numerical processing carefully design studies using innovative methodologies that capitalize on what infants can do versus what they cannot (for reviews, see Cantrell & Smith, 2013; Mou & vanMarle, 2014). For example, in their seminal study using a *habituation/dishabituation paradigm*, Antell and Keating (1983) presented newborns with cards showing arrays containing the same number of dots until their visual fixation decreased to a criterion level (*habituation*) and then presented them with a new card containing a different number of dots and measured whether infants' looking time increased (*dishabituation*), which indicates that they recognize a difference between the new card and the familiar ones. They found that infants could discriminate small sets of dots (2 vs. 3), but not larger sets (4 vs. 6). Thus, although infants cannot tell us that there are 2 blocks on the right side of Fig. 3.1, they are sensitive to quantities in their environments and can discriminate arrays of up to 3 or 4 items (e.g., Starkey & Cooper, 1980; Starkey, Spelke, & Gelman, 1983; van Loosbroek & Smitsman, 1990). Rather than arrays, Wynn and colleagues showed that 5- to 6-month-old infants can discriminate between 2 and 3 jumps performed by puppets (Sharon & Wynn, 1998; Wynn, 1996; Wynn, Bloom, & Chiang, 2002). Studies have also provided some evidence for cross-modal matching between 2 or 3 items (e.g., Féron, Gentaz, & Streri, 2006; Kobayashi, Hiraki, & Hasegawa, 2005; Starkey et al., 1983; Starkey, Spelke, & Gelman, 1990; for different findings and interpretations, see Mix, Levine, & Huttenlocher, 1997 and Moore, Benenson, Reznick, Peterson, & Kagan, 1987). In research using *food choice* and *manual search paradigms*, infants watch as an experimenter hides food or other types of items by placing them in opaque buckets (Feigenson & Carey, 2003, 2005; Feigenson, Carey, & Hauser, 2002; vanMarle, 2013). Such studies have found that 10- to 12-month-old infants are more likely to retrieve food from a bucket containing more items from sets of 1 to 3 items, but not when there are more than 3 items.

Using sets with larger numbers of objects, other research has demonstrated that infants can tell the difference between quantities larger than 4, but the ratio of the larger to smaller quantities must be large. Thus, discrimination of ordinality is a function of the number of items in the sets being compared, conforming to Weber's law, which characterizes the perceptual discriminability of many sensory stimuli. For example, Izard, Sann, Spelke, and Streri (2009) familiarized newborns to a continuous stream of auditory sequences of syllables that repeated a fixed number of times and measured looking times for visual arrays that either had the *same* number of objects as syllables in the auditory sequence or a *different* number of objects as the auditory sequence. They found that newborns looked longer at the visual arrays with the matching number of objects when the numbers differed by a ratio of 3:1 (4 vs. 12 and 6 vs. 18), indicating that they recognize *numerical equivalence* between auditory and visual information, but not when they differed by a 2:1 ratio (4 vs. 8).

Although sensitivity to magnitude appears to be present at birth, its precision increases with age into adulthood. Six-month-old infants can discriminate larger sets of visual and auditory quantities and event sequences, but only if the larger set is double the smaller set (i.e., 2:1 ratio); for example, a puppet jumping up and down 4 times compared to 8 times (Lipton & Spelke, 2003; Wood & Spelke, 2005; Xu, 2003; Xu & Spelke, 2000; Xu, Spelke, & Goddard, 2005). Ten-month-old infants can discriminate sets differing at a 3:2 ratio (vanMarle, 2013; vanMarle & Wynn, 2011; Xu & Arriaga, 2007). Three-year-olds can discriminate numerosities at a ratio of 4:3, 6-year-olds at a ratio of 6:5, and some adults can discriminate at a 11:10 ratio (Barth, Kanwisher, & Spelke, 2003; Halberda & Feigenson, 2008; Piazza et al., 2010; Siegler & Lortie-Forgues, 2014). Thus, there are developmental increases in the precision of nonsymbolically representing quantity that may be due to brain development and experience (Izard et al., 2009), as well as individual differences in acuity (for a review, see Halberda & Odic, 2015; Libertus & Brannon, 2010).

Cognitive Systems for Quantity Representation

In Chap. 1, I described the importance of mathematics in human phylogenetic and ontogenetic development. Early quantity or magnitude representation is conceptualized by Geary (1995, 2005) as a biologically primary ability that was selected during evolution to solve recurrent problems faced by our ancestors and is universally acquired by humans to guide everyday activities that are important to survival, such as foraging. The results described in the previous section indicate that humans (for a review of other species, see Starr & Brannon, 2015) can represent quantity at birth in an abstract way—before they learn language, number names, counting routines, or numeric symbols (e.g., Starkey et al., 1990). But, how can we explain differences between studies showing that the upper limits of infants' discrimination of quantity is 3 or 4 items and those showing that infants can discriminate larger sets of quantities according to ratios increasing in precision with age and experience? Do these differences reflect distinct cognitive mechanisms and underlying neuroarchitectures? Are they due to infants' difficulty with enumerating small and larger numerosities, other cognitive constraints, differential levels of experience, or imprecision in experimental methods (Hyde & Mou, 2016)? Some theorists propose that humans use two evolutionarily older cognitive systems for abstract quantity representation: one for distinguishing between small quantities of 1–3 individual items in parallel, called the *parallel individuation (PI) system*, and one for approximating the numerosities of sets of items, called the *approximate number system (ANS)*, which corresponds to the *quantity system* of the triple-code model, or *number sense* (e.g., Dehaene, 2011; Feigenson, Dehaene, & Spelke, 2004; Gallistel & Gelman, 2000; Hyde, 2011; Hyde & Spelke, 2011; for a review, see Hyde & Mou, 2016). These dissociable systems are proposed to support nonsymbolic numerical cognition through different *types* of mental representations, each with distinct functions and constraints.

The PI, which has also been referred to as the *object-tracking system* or the *object file system*, allows human infants and some other species to precisely track a small number of individual objects through space and time (for a review, see Hyde & Mou, 2016). This system may not have evolved for numerical processing, but may be engaged when infants precisely discriminate between sets of 1–3 objects by representing individual items separately, rather than summarizing a set's numerosity using a symbol (Hyde, 2011). Because each item is represented individually, one-to-one correspondence procedures can be used for some comparison and simple arithmetic tasks by matching symbols for individual items back-and-forth across sets (Carey, 2009; Feigenson et al., 2004), but these procedures are labor intensive, requiring attentional and working memory resources. Thus, the system is constrained to three sets of 3–4 individual items (for a review, see Cowan, 2001).

In contrast to the PI system, the ANS is a general, imprecise, ratio-based mechanism, often described as "noisy," that allows humans to nonsymbolically represent approximate quantities (e.g., Lemer, Dehaene, Spelke, & Cohen, 2003; Gallistel & Gelman, 2000, 2005; for a review, see Starr & Brannon, 2015). According to Hyde and Mou (2016), "The output of this system, more broadly, is one of many types of estimates the brain makes to rapidly summarize the complex and uncertain visual environment…in the absence of exact verbal counting, the uncertainty or imprecision in ANS representations increases logarithmically as numerosity increases" (p. 54). This phenomenon produces several signatures of the ANS described by Starr and Brannon (2015), among others. As previously noted, it adheres to Weber's law, which specifies that numerosity discrimination depends on ratio instead of numerical distance (e.g., Dehaene, 2011; Feigenson et al., 2004; Revkin, Piazza, Izard, Cohen, & Dehaene, 2008; Starr & Brannon, 2015). That is, infants can discriminate a set of 4 items from a set of 8 items more easily than they can discriminate a set of 8 items from a set of 12 items, even though the difference is 4 items in both cases because mental representations of larger numerosities are more imprecise. *Distance effects* occur such that discriminating between set sizes that are more discrepant, such as 2 from 8, is easier than those that are closer in number, such as 2 from 3. *Magnitude effects* occur in that discrimination between set sizes is easier for fewer objects. Thus, it is easier to discriminate 3 from 4 than 8 from 9. There are upper limits on the ANS in that it cannot discriminate small differences between larger numbers, such as 27 from 28.

To understand the neural basis of these cognitive systems for nonsymbolic quantity representations, researchers have combined techniques for imaging brain structure and function with paradigms for measuring behavioral performance on tasks (i.e., accuracy and reaction time). Furthermore, they have considered whether these types of representations and their associated neural activity are related to standardized measures of mathematics concepts and achievement.

Neural Basis of Quantity Representation

The neural basis of quantity representation has been studied during different periods of development by combining a number adaptation paradigm with the use of neuroimaging techniques (for a review, see Hyde & Mou, 2016; see Chap. 2 for descriptions of neuroimaging techniques). The *number adaptation paradigm* is a type of *habituation/dishabituation paradigm*, discussed at the outset of this chapter, that has been especially efficacious for understanding functional specialization of the brain for quantity representation during infancy, before verbal and visual symbols for number are acquired. For example, Izard, Dehaene-Lambertz, and Dehaene (2008) measured Event-Related Potentials (ERPs) using a number adaptation paradigm to investigate neural responses to changes in numerosity in 3-month-old infants. In their study, electrical activity evoked by the brain was recorded on the surface of the scalp as 3-month-old infants watched a continuous stream of images of sets of objects. Within a given block of trials, most images contained the same type and number of objects, but some test images were interleaved into the stream that contained a different type and/or number of objects. *Neural adaptation*, reflecting *habituation*, occurs when repeated presentation of the same stimulus leads to diminished neural responses in regions specialized for processing a particular type of representation, such as quantity, despite changes in other features, such as size, density, and position. When a stimulus is presented that is recognized as different, the neural response is reactivated, reflecting *dishabituation*. Using cortical source modeling, Izard et al. (2008) established a *double dissociation* between object and number processing in early infancy. They found that when the *type* of object changed, a *ventral pathway* in the left temporal cortex was activated in infants' brains with an antagonistic response in the right temporal cortex, but when the *number* of objects changed, a *dorsal pathway* in the right inferior parietal and frontal cortex was activated. Furthermore, for number change, there was decreased response in left anterior temporal regions and increased response in right anterior temporal cortex. Izard et al. (2008) concluded that brain specialization for object and number processing is already present in early infancy and that there may be an antagonistic relation between the ventral network for object identity and the dorsal network for number and space (see Chap. 5 for further discussion of these networks). Contrary to behavioral studies, no differences in neural responses were found between small (2 vs. 3) and large (4 vs. 8 and 4 vs. 12) numerosities, leading the authors to suggest that human infants and nonhuman primates have an analog representation of numerosities for small and large numbers that shows developmental continuity by increasing in precision and guiding the acquisition of arithmetic and mathematical concepts.

Similarly, using a number adaptation paradigm and event-related functional Near-Infrared Spectroscopy (fNIRS), Hyde, Boas, Blair, and Carey (2010) found that 6-month-old infants' neural responses to *number* were right lateralized in the parietal lobe and were dissociated from their neural responses to *shape* in the occipital lobe, which indicates that infants use the right intraparietal sulcus (IPS) to process number even before they learn symbolic number systems (Hyde et al., 2010). In a different study using fNIRS and a number alternation paradigm with 6.5-month-old infants, Edwards, Wagner, Simon, and Hyde (2016) found that only 1 right parietal channel out of 24 posterior channels responded to numerosity. Furthermore, they included two conditions to rule out alternative hypotheses that responses to number change are due to aspects of stimuli that tend to be confounded with number (i.e., item size, total area, spacing, luminance) or increased attention related to visual interest. They found a *double dissociation* between infants' right parietal brain response to number change and their more general bilateral occipital and middle parietal visual attention response to interesting, colorful, audio-visual animations with no number change.

Other researchers using fNIRS with 5- to 7-month-old infants have found that anterior areas of the temporal lobes are involved in individuating two objects (Wilcox, Haslup, & Boas, 2010; Wilcox, Stubbs, Hirshkowitz, & Boas, 2012), providing support for the hypothesis that small and large quantities are processed by different neural systems. In contrast to Izard et al.'s (2008) results, Hyde and Spelke (2011) found differences in 6- to 7.5-month-old infants' neural responses to small and large quantities. They used ERPs during a number alternation paradigm that involved viewing alternating images of dot arrays containing either small (1, 2, and 3) or large (8, 16, and 32) sets of objects across 3 blocks in which there were no changes in numerosities, small changes (1:2 ratio), and large changes (1:3 or 1:4 ratio). For small numerosities, but not large, an early positive component (P400) was greater over left and right occipital and temporal scalp sites for larger cardinal values. Conversely, for large numerosities, but not small, a mid-latency positivity (P500) was greater over posterior parietal scalp sites as ratio decreased, but was not sensitive to changes in cardinal value. These results indicate a *double dissociation* between small and large number processing that is present early in development, corresponding to the PI system and the ANS described in the previous section.

Moving beyond infancy, the use of other neuroimaging techniques with higher spatial resolution, particularly fMRI, becomes more feasible (Byars et al., 2002). Cantlon, Brannon, Carter, and Pelphrey (2006) used event-related fMRI with a number adaptation paradigm to study the ANS (Cantlon et al., 2006). They found that both 4-year-old children and adults engage bilateral IPS for processing the numerosity of object arrays, which was dissociated from shape processing. Children's IPS activation was more right lateralized than adults who showed greater and more bilateral activation in the IPS.

Relationships Between Neural Correlates of Quantity Representation and Mathematics Achievement

Although there are important questions surrounding the nature of the PI system and the ANS, it seems clear that infants and young children can represent quantity non-symbolically; that is, they have number sense. Further questions arise concerning whether this number sense forms the basis for biologically secondary mathematical abilities learned in school. Does the IPS that underlies quantity representation during infancy and early childhood also support formal mathematics achievement during childhood, adolescence, and emerging adulthood?

As noted by Hyde and Mou (2016), some studies have provided evidence for a relationship between the ANS and mathematics achievement scores (e.g., Bonny & Lourenco, 2013; Chen & Li, 2014; Fazio, Bailey, Thompson, & Siegler, 2014; Gilmore, McCarthy, & Spelke, 2010; Halberda & Feigenson, 2008; Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Halberda, Mazzocco, & Feigenson, 2008). For example, Starr, Libertus, and Brannon (2013) found that individual differences in the discrimination of nonsymbolic magnitude at age 6 months are related to achievement on symbolic mathematics tasks at age 3, after controlling for IQ. Likewise, preschoolers who perform better when deciding which array has more dots have higher mathematics achievement 1–2 years later (Chu, vanMarle, & Geary, 2015; Libertus, Feigenson, & Halberda, 2011; Mazzocco, Feigenson, & Halberda, 2011) and individual differences at age 3 are related to scores on standardized mathematics achievement tests concurrently and at age 5 (Mazzocco et al., 2011).

Whether this relationship is unidirectional, bidirectional, or mediational remains unclear (Hyde & Mou, 2016). Some studies have not found a relationship between the ANS and mathematics achievement or have found that domain-general abilities mediate the relationship (e.g., Fuhs & McNeil, 2013; Gilmore et al., 2013; Lyons, Ansari, & Beilock, 2012; Lyons, Price, Vaessen, Blomert, & Ansari, 2014; Sasanguie, De Smedt, Defever, & Reynvoet, 2012; for a review, see De Smedt, Noël, Gilmore, & Ansari, 2013). For example, vanMarle, Chu, Li, and Geary (2014) found that the relationship between the ANS and mathematics achievement in preschoolers is mediated by the learning of *symbolic* quantity knowledge. Other research indicates that as children begin to map quantities with symbols, important developmental changes occur in the neural communication between the right parietal region and other brain regions. For example, Park, Li, and Brannon (2014) presented 4- to 6-year-olds with magnitude comparison tasks using lines, dot arrays, and Arabic numerals while undergoing fMRI. They found a right parietal region that had a higher BOLD signal to numerical stimuli (dot arrays and Arabic numerals) than nonnumerical stimuli (lines). Furthermore, they found significant effective connectivity from the right parietal region to the left supramarginal gyrus and the right precentral gyrus, which was related to performance on a standardized mathematics test. They concluded that effective neural connectivity underlying symbolic number processing might be critical to associations between quantities and symbols and predict mathematics achievement.

In a fascinating study using fMRI, Emerson and Cantlon (2012a, 2012b) had children aged 4- to 11-years-old match arrays of dots to Arabic numerals and found that functional connectivity of white matter between frontal and parietal regions measured during this task was predictive of scores on the Test of Early Mathematical Abilities (TEMA-3; Ginsburg & Baroody, 2003), in contrast to a control network involving matching faces, words, or shapes that showed little correlation with TEMA scores. They found that connectivity between frontal and parietal regions

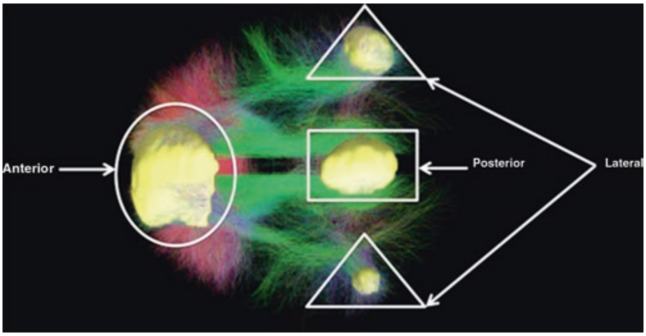

Fig. 3.2 Depiction of connectivity between the anterior, posterior, and lateral brain regions of the Default Mode Network from a superior view. Source: Andreashorn (Own work) [CC BY-SA 4.0 (http://creativecommons.org/licenses/by-sa/4.0)], via Wikimedia Commons

correlated with performance on the in-scanner task and with TEMA scores. The researchers concluded that this frontal-parietal network is mathematics-specific, independent of children's verbal IQ scores.

These studies provide evidence for a relationship between quantity representation and mathematics achievement during childhood, but does this relationship persist into adolescence and emerging adulthood? During the transitional period of adolescence, the brain continues to mature and streamline its connections. One brain circuit that undergoes critical development from ages 7 to 15, including increases in anterior-posterior connectivity, is the Default Mode Network (Sato et al., 2014). The *Default Mode Network* (DMN), depicted in Fig. 3.2, is a circuit of brain regions that includes *anterior* structures (medial prefrontal and orbitofrontal cortex), *posterior* structures (the cingulate and precuneus), and *lateral* structures (angular gyri).

DMN *activation* is associated with mind-wandering; therefore, DMN *deactivation* is important for suppressing distractions during cognitively demanding tasks (Mason et al., 2007; McKiernan, D'Angelo, Kaufman, & Binder, 2006). Studies using fMRI have revealed that school-aged children (9- and 12-year-olds) show reduced or absent DMN deactivation compared to adults for magnitude, approximate, and exact calculation tasks (Davis et al., 2009; Kucian, von Aster, Loenneker, Dietrich, & Martin, 2008). However, research on DMN deactivation and mathematical cognition during adolescence is sparse. To study relationships between mathematical cognition and achievement during adolescence, we examined brain *activation* and *deactivation* of the DMN for a variety of tasks. Here, we present results for a magnitude comparison task. Chapter 4 presents results for exact and approximate calculation and error detection.

When you looked at Fig. 3.1 and determined, without counting, which side has more blocks, it was probably easy for you as an adult; and, according to the theory and research presented at the beginning of this chapter, it should be easy for children and adolescents as well. We used this magnitude comparison task in an fMRI study in which a small sample of adolescents (16 7th and 8th graders; 13.5-years-olds on

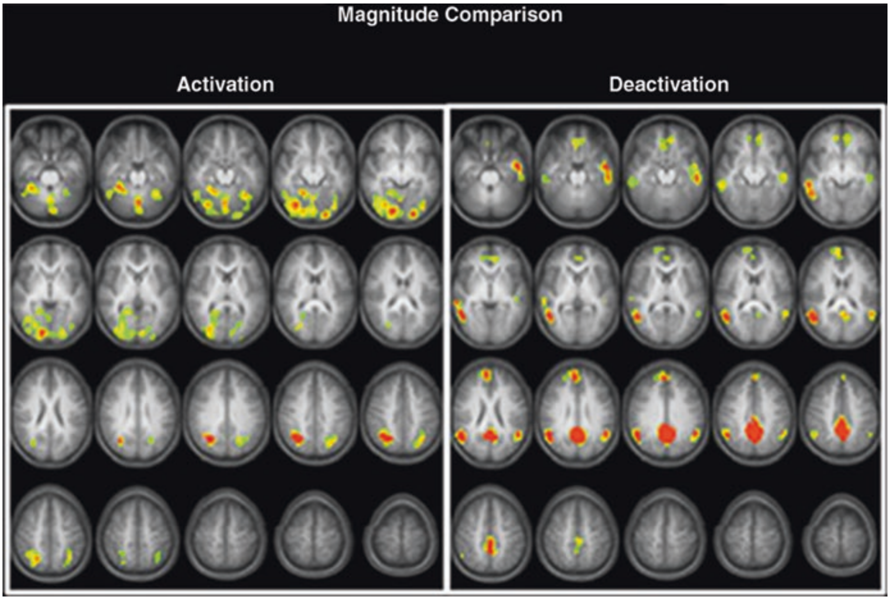

Fig. 3.3 Regions with significant ($p < 0.01$ FWE corrected) correlations between functional activation (positive) and deactivation (negative) and magnitude comparison task performance. Images in radiologic orientation. Slice locations: Z −25 mm to Z = +70 mm

average) were shown arrays of blocks and asked to determine, without counting, the side of the screen that displayed either more (50% of trials) or less (50% of trials) blocks. Trials were presented in random order on a MR-compatible video projector and correct responses were presented on the left (50% of trials) and the right (50% of trials). Participants held a button box in each hand and were instructed to press the button in the left hand to choose the answer on the left side of the screen or the button in the right hand to choose the answer on the right side of the screen. The number of blocks on each side ranged from 1 to 9, with a difference of at least 2, unless the largest number of blocks was 2 or 3. To subtract the effects of general visual processing, participants also completed a control task in which they were asked to select whether a large block appeared on the left or right side of the visual field.

As shown in Fig. 3.3, we found that *activation* in the inferior occipital gyrus (top 2 rows of images) and bilateral IPS (bottom 2 rows of images) positively correlated with performance on this magnitude comparison task. *Deactivation* in all (anterior, posterior, and lateral) DMN regions *negatively* correlated with performance on the magnitude comparison task. Furthermore, we found robust and significant negative correlations between average fMRI activation in IPS regions during magnitude comparison and posterior and lateral overall DMN deactivation. It is important to note that deactivation has a negative correlation value, indicating an increasing amount of deactivation, which corresponds to making more neural resources available for cognitive processing. These results support the conclusion that low levels of competence in quantity processing (which is associated with low levels of activation of the IPS) increases the difficulty of the task, resulting in a need for greater neural resources, and, therefore, greater deactivation of the DMN. Thus, IPS activation and DMN deactivation may act in coordination, rather than as independent processes.

Our primary question concerned whether neural activity during magnitude comparison contributes to mathematics achievement as measured by standardized tests. We related *activation* of the inferior occipital cortex, lingual gyrus, bilateral IPS, and left prefrontal cortex and *deactivation* of the anterior, posterior, and lateral DMN during the task to math fluency and calculation as measured by percentile scores on the Woodcock-Johnson Tests of Achievement (WJ III; Woodcock, McGrew, & Mather, 2001) and overall math achievement (composite of math fluency and calculation subtests). The math fluency subtest is designed to measure efficient fact retrieval for 2-operand problems. Participants solved as many simple addition, subtraction, and multiplication problems as they could during a 3-min time limit. The calculation subtest is designed to measure accurate calculation for more complex addition, subtraction, multiplication, division, and multiple operation problems, as well as geometric, trigonometric, logarithmic, and calculus operations, according to capabilities.

As shown in Fig. 3.4 we found that activation in the inferior occipital gyrus, the lingual gyrus (top 2 rows of images), and bilateral IPS (bottom 2 rows of images) during the magnitude comparison task positively correlated with math achievement and calculation subtest scores. Activation in these same areas as well as the right prefrontal cortex was positively correlated with math fluency subtest scores. (Images are presented in radiological orientation; therefore, left activation appears on the right side of the image and right activation appears on the left side of the image.) A significant negative interaction between magnitude comparison performance and calculation scores was found in the lingual gyrus. These findings are consistent with our hypothesis that quantity representation is related to math achievement as measured by skill at solving 2-operand problems efficiently (math fluency) and solving paper-and-pencil multi-digit mathematical problems (calculation), similar to the types of problems found in school contexts. This suggests that abstract quantity representation may be foundational for the development of calculation skills. The positive correlations between inferior occipital regions and math achievement, math fluency, and calculation may have been related to the translation of abstract symbols (blocks) into quantity representations, which is also likely to be an important skill for the development of higher-order mathematical cognition. Haist, Wazny, Toomarian, and Adamo (2015) found similar results for the relationship between numerosity precision and mathematics achievement for children and adults, but not adolescents.

Concerning the DMN, as shown in Fig. 3.5, *deactivation* in all (anterior, posterior, and lateral) regions during the magnitude comparison task *negatively* correlated with math achievement, math fluency, and calculation. Thus, adolescents who were less proficient at mathematics deactivated the DMN to make more neural resources *available* for the task (i.e., potentially for increased attention), while it

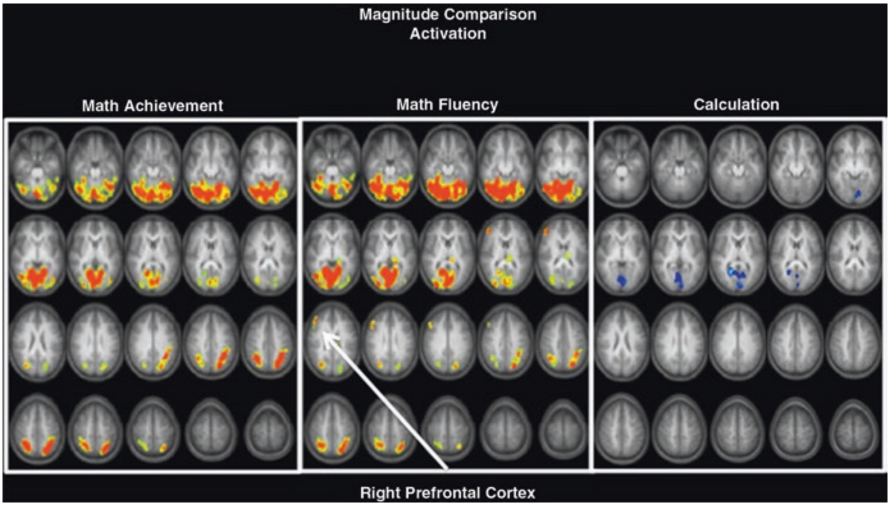

Fig. 3.4 Regions with significant ($p < 0.01$ FWE corrected) positive correlations between functional activation during magnitude comparison and Woodcock-Johnson III percentile scores for math achievement and math fluency. Regions with significant ($p < 0.01$ FWE corrected), negative magnitude task performance by calculation percentile score interaction for functional activation. Images in radiologic orientation. Slice locations: Z −25 mm to Z = +70 mm

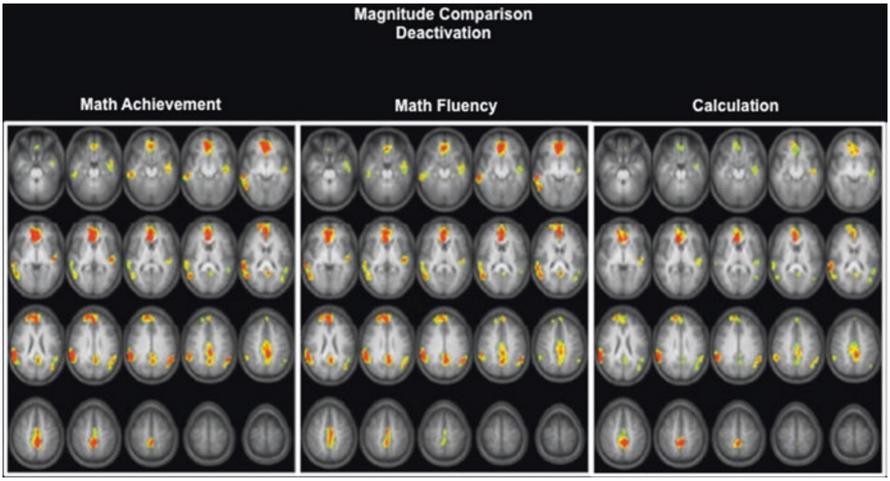

Fig. 3.5 Regions with significant ($p < 0.01$ FWE corrected) negative correlations between functional deactivation during magnitude comparison and Woodcock-Johnson III percentile scores for math achievement, math fluency, and calculation. Images in radiologic orientation. Slice locations: Z −25 mm to Z = +70 mm

was likely not necessary for adolescents who were more proficient at mathematics to do so. In general, adolescents found the magnitude comparison task quite easy (mean accuracy = 95%). Indeed, DMN deactivation was not detected at the group level.

Interestingly, we did not find significant correlations between in-scanner performance on the magnitude comparison task and math achievement, math fluency, or calculation scores. Thus, at the behavioral level, there was no significant relationship between magnitude comparison and standardized measures of mathematics achievement, yet a relationship was revealed when behavior was related to brain functioning. This study can be interpreted using a Developmental Systems Approach given that measures at multiple levels of analysis were used. It provides evidence that activation of critical brain regions (inferior occipital gyrus, lingual gyrus, bilateral IPS) while using quantity representations, as predicted by Dehaene and colleagues' triple-code model (e.g., Dehaene, 1992, 2011; Dehaene & Cohen, 1995, 1997; Dehaene, Piazza, Pinel, & Cohen, 2003), is related to math achievement and that deactivation of the DMN to employ domain-general operations is negatively related to Mathematical achievement. In Chap. 4, we examine the roles of activation and deactivation of the DMN during tasks that are more similar to the types of problems children encounter in school contexts.

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Chapter 4
Calculation

Rhonda Douglas Brown, Vincent J. Schmithorst, and Lori Kroeger

Abstract In this chapter, we present neuroscience research that addresses the development of the more complex skill of calculation from childhood into adulthood. Cognitive processes related to mathematics achievement are described including the quantity, verbal, and visual systems of Dehaene and colleagues' triple-code model and domain-specific and domain-general processes. We present results from our research using functional Magnetic Resonance Imaging (fMRI) to examine relationships between neural correlates of calculation and mathematics achievement. Activation in critical brain regions and deactivation of the Default Mode Network (DMN) for a variety of tasks, including exact and approximate calculation and error detection, are illustrated. We also discuss our research using exploratory group Independent Component Analysis (ICA) to reveal separate components of functional activation in bilateral inferior parietal, left perisylvian, and ventral occipitotemporal areas during the mental addition and subtraction of fractions. Taken together, our work provides support for the triple-code model for a variety of tasks. Furthermore, it indicates that domain-specific neuroarchitecture for quantity processing and domain-general processes related to the DMN may act in coordination to perform calculation.

The basic numerosity and ordinality competencies reviewed in Chap. 3 emerge with little, if any, instruction, and may be online at birth, or shortly thereafter. These biologically primary abilities exist across cultures and in other primate species whether or not they evolved to serve a quantitative function (Beran & Beran, 2004; Boysen & Berntson, 1989; Cantlon, Merritt, & Brannon, 2016; Hauser, 2000). Beyond infancy, these skills are transformed through cultural interactions within families, schools, and in other settings into biologically secondary mathematical skills that are developed through prolonged learning experiences (Geary, 1995, 2007). In this chapter, we present neuroscience research that addresses the development of the more complex skill of calculation from childhood into adulthood.

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Cognitive Systems and Neural Basis Related for Mathematics Achievement

In Chap. 1, Dehaene and colleagues' (Dehaene, 1992, 2011; Dehaene & Cohen, 1995, 1997; Dehaene, Piazza, Pinel, & Cohen, 2003) triple-code model of numerical processing was presented to describe how mathematical cognition is instantiated in the brain (see Fig. 1.3). To briefly summarize, the triple-code model predicts that three distinct representational systems associated with specific neural substrates may be used in mathematical cognition, depending on the task. According to the model, the *quantity system*, often referred to as *number sense*, uses nonverbal, meaning-based representations of size and distance relations between numbers on a mental number line and engages the bilateral horizontal intraparietal system (hIPS) of the brain to mediate performance on magnitude comparison and approximate calculation tasks. The *verbal system* represents numbers in a linguistic format and engages a region of the left angular gyrus and other language networks to retrieve well-learned arithmetic facts. The *visual system* represents and spatially manipulates numbers in Arabic format and engages regions in the posterior superior parietal lobe to perform arithmetic when well learned facts cannot be retrieved.

As described in Chap. 1, neuropsychological cases, functional Magnetic Resonance Imaging (fMRI), and Event-Related Potential (ERP) studies have provided support for the triple-code model for tasks involving number processing and calculation in adults (for a review, see Dehaene et al., 2003; Grabner et al., 2009; van Eimeren et al., 2010). Children engage a similar network of brain regions for simple number processing, with activation of hIPS increasing with age (Ansari & Dhital, 2006; Ansari, Garcia, Lucas, Hamon, & Dhital, 2005). However, children also recruit the inferior frontal cortex to a greater extent than adults (Cantlon et al., 2009), implicating a potential frontal to parietal shift with development and learning. Davis et al. (2009) found that children with Math Difficulties (MD) and those with Typical Achievement (TA) activated the same network of brain regions for exact and approximate calculation tasks, but children in the MD group had significantly greater activation in the parietal, frontal, and cingulate cortices. They concluded that differences in these brain areas associated with domain-general cognitive resources, such as executive functioning and working memory, imply the use of more developmentally immature and less efficient strategies. The authors also found decreased task-related deactivation in children with MD in the anterior and posterior cingulate, which is part of the Default Mode Network (DMN; Raichle & Snyder, 2007).

Convergent evidence from longitudinal studies indicates that both domain-specific and domain-general cognitive processes contribute to mathematics achievement. Regarding domain-specific processes, studies have found that basic numerical competencies, such as number naming, recognition, and comparison (Geary, 2011; Landerl, Bevan, & Butterworth, 2004; Locuniak & Jordan, 2008), and early competence with simple arithmetic, including counting and decomposition procedures and arithmetic fact retrieval (Geary, 2011; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Mazzocco & Thompson, 2005), contribute to mathematics achievement.

Regarding domain-general abilities, speed of processing and working memory, particularly the central executive component, contribute to mathematics achievement (Bull, Espy, & Wiebe, 2008; Geary, 2011; Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Mazzocco & Kover, 2007; Passolunghi, Vercelloni, & Schadee, 2007; Swanson, Jerman, & Zheng, 2008). For example, Geary (2011) analyzed the initial state and growth of mathematics achievement from the beginning of 1st grade through 5th grade and found that specific quantitative skills, such as early fluency in apprehending quantity (see Chap. 3), combining small sets of items and Arabic numerals, accurate counting procedures for solving addition problems, number line knowledge, and the functioning of the visuospatial sketchpad (see Chap. 5) were uniquely predictive of mathematics achievement while controlling for domain-general abilities. He also found that domain-general abilities, such processing speed and the central executive component of working memory, predicted math achievement and growth, controlling for intelligence. Regarding the central executive component of working memory, Geary (2011) found increases in its contribution to mathematics achievement as children progressed through grades in school. He explained these results by noting that the easier items on the mathematics achievement measure for the earlier grades minimally engage the central executive; however, the items become more difficult with successive grades, which requires more engagement of the central executive component of working memory. As complex tasks become more familiar and performance becomes more automatic and long-term memory-based, the role of the central executive lessens. Indeed, Geary (2011) also found that use of memory-based processes to solve addition problems, such as arithmetic fact retrieval and decomposition, also predicted mathematics achievement, with the benefits of basic fact knowledge in 1st grade increasing with each successive grade. Furthermore, studies of Math Disabilities/Difficulties (MD; see Chap. 6) have distinguished between difficulties in mathematical processing arising from a domain-specific core deficit in numerosity versus domain-general deficits (Butterworth & Reigosa, 2007; Reigosa-Crespo et al., 2012; Rubinsten & Henik, 2009). The longitudinal studies described above focus on elementary and early middle-school aged children. Few studies have followed children into adolescence.

Relationships Between Neural Correlates of Calculation and Mathematics Achievement

A key neural network that undergoes development from ages 7 to 15 is the Default Mode Network (DMN), depicted in Fig. 3.2 (Mason et al., 2007; McKiernan, D'Angelo, Kaufman, & Binder, 2006; Sato et al., 2014). You may recall from Chap. 3 that the DMN is a circuit of brain regions that includes *anterior* structures (medial prefrontal and orbitofrontal cortex), *posterior* structures (the cingulate and precuneus), and *lateral* structures (angular gyri). The DMN is important for domain-general functions related to attention, suppressing distractions during cognitively demanding tasks, and engaging in effortful processing. Previous research has demonstrated that adults *de*activate the DMN during magnitude and exact and approximate calculation tasks; whereas school-aged children (9- and 12-year-olds) show reduced or absent DMN deactivation (Davis et al., 2009; Kucian, von Aster, Loenneker, Dietrich, & Martin, 2008). However, research on DMN deactivation and mathematical cognition during adolescence is sparse.

To study relationships between mathematical cognition and achievement during adolescence, we used functional Magnetic resonance imaging (fMRI) to examine brain *activation* in the inferior occipital cortex, lingual gyrus, bilateral IPS, and the left prefrontal cortex and *deactivation* of the anterior, posterior, and lateral DMN for a variety of tasks. Here, we present results for exact and approximate calculation and error detection. (Results for magnitude comparison were presented in Chap. 3). For all tasks, mathematical problems were presented in random order on a MR-compatible video projector in horizontal and vertical orientations. Correct responses were presented on the left and the right, and, when applicable, the choices varied as to whether the correct response was the higher or lower number (i.e., as possible, features of stimuli were evenly counterbalanced across trials). Participants held a button box in each hand and were instructed to press the button in the left hand to choose the answer on the left side of the screen or the button in the right hand to choose the answer on the right side of the screen.

Exact Calculation

Take a look at Fig. 4.1 and choose the correct answer to each problem.

How did you solve the addition problem on the left? As an adult, you probably instantly recognized 11 as the correct response to 6 + 5 because you have associated the problem and the answer by repeatedly calculating it or by memorizing it through the drill-and-practice methods of schooling. A child in elementary school may solve

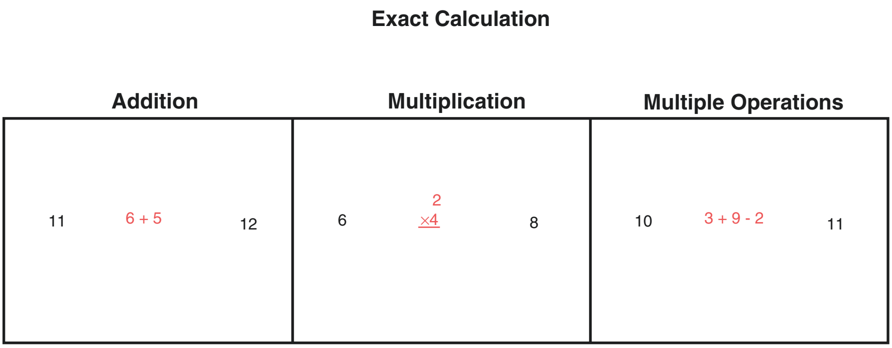

Fig. 4.1 Examples of problems used for three types of exact calculation tasks: addition, multiplication, and multiple operations

it by reasoning that 6 + 6 = 12, so 6 + 5 must be 1 less. How did you come up with the answer to the multiplication problem in the center? Again, as an adult you probably retrieved the answer to this math fact from memory, without the need for calculation. Elementary school children may solve it through repeated addition: 2 + 2 + 2 + 2 = 8 or they may know that 2 × 3 is 6 and then add 2 more to get 8. How did you solve the problem on the right? Chances are that the correct answer didn't just pop into your mind. You most likely had to retrieve the answer to 3 + 9, hold the answer 12 in working memory, and then determine that 12 − 2 = 10. In our study, we examined adolescents' brain activation and deactivation of the DMN while they were deciding the exact answer for addition, multiplication, and multiple operations problems like those shown in Fig. 4.1 and related the results to their mathematics achievement.

Addition

While undergoing fMRI, adolescents (14 7th and 8th graders; 13.5-years-old on average) were shown 1-digit, 2-operand addition problems like the one on the left side of Fig. 4.1 and had 3.5 s to select the side of the screen displaying the correct answer. The other response choice differed by either 1 or 2 from the correct answer so that participants had to determine the exact answer. To control for visual activation and response selection, we compared brain activation during mathematical problems to brain activation during a color matching task that gave adolescents 2.5 s to choose which of two identical numbers matched the color of the same number shown in the middle of the screen.

As shown in Fig. 4.2, we did not find significant correlations between activation in any brain regions and performance on these problems. However, *de*activation in anterior DMN regions was negatively correlated with performance. Deactivation is quantified using negative numbers; thus, the interpretation of this result is that greater deactivation of anterior DMN regions during exact addition was associated with lower performance.

Our primary question concerned whether neural activity during exact addition contributes to mathematics achievement as measured by standardized tests. We related *activation* of the inferior occipital cortex, lingual gyrus, bilateral IPS, and left prefrontal cortex and *deactivation* of the anterior, posterior, and lateral DMN during the task to math fluency and calculation as measured by percentile scores on the *Woodcock-Johnson Tests of Achievement* (*WJ III*; Woodcock, McGrew, & Mather, 2001) and overall mathematics achievement (composite of math fluency and calculation subtests). The math fluency subtest is designed to measure efficient fact retrieval for 2-operand problems. Participants solved as many simple addition, subtraction, and multiplication problems as they could during a 3-min time limit. The calculation subtest is designed to measure accurate calculation for more complex addition, subtraction, multiplication, division, and multiple operations problems, as well as geometric, trigonometric, logarithmic, and calculus operations, according to capabilities.

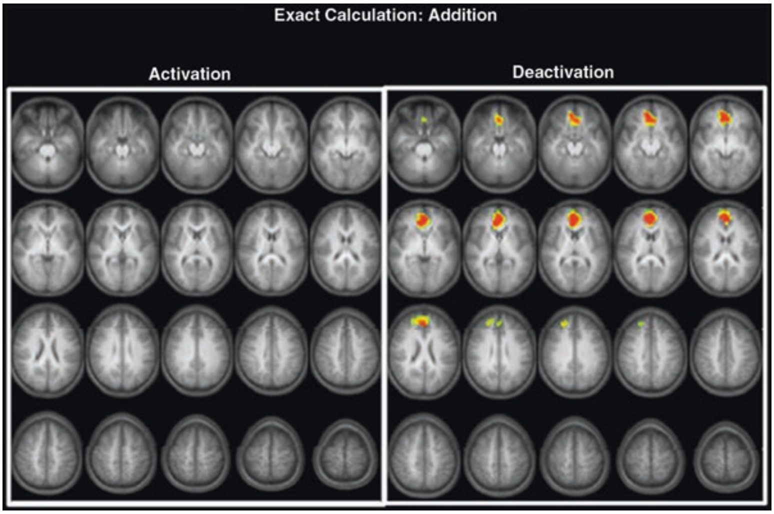

Fig. 4.2 Regions with significant ($p < 0.01$ FWE corrected) correlations between functional activation (none) and deactivation (negative) and exact addition task performance. Images in radiologic orientation. Slice locations: Z −25 mm to Z = +70 mm

Interestingly, activation during exact addition did not correlate with any of these measures. However, as shown in Fig. 4.3, deactivation in anterior, posterior, and right lateral DMN regions during addition negatively correlated with overall math achievement and math fluency. (Images are presented in radiological orientation; therefore, left activation appears on the right side of the image and right activation appears on the left side of the image.) In anterior and right lateral DMN regions a significant interaction indicating a smaller negative correlation between DMN deactivation and task performance for participants with higher calculation subtest scores was found.

Our interpretation of these results is that individuals who are less proficient with their math facts may need to rely more on counting and other strategies, so they deactivate the DMN to a greater extent to make resources available for deliberative, slower processes (i.e., attention, working memory, strategy use). However, higher achieving participants who can readily retrieve math facts from memory rely less on cognitive processes that require deactivation of the DMN. By 7th and 8th grade, many adolescents who are proficient at math can quickly retrieve addition facts. Indeed, the exact addition task was relatively easy for the participants in this study (M = 83.3% correct).

Multiplication

While undergoing fMRI, adolescents (N = 16) were shown 2-operand multiplication problems like the one in the center panel of Fig. 4.1 and had 3.5 s to select the side of the screen displaying the correct answer. The other response choice differed by

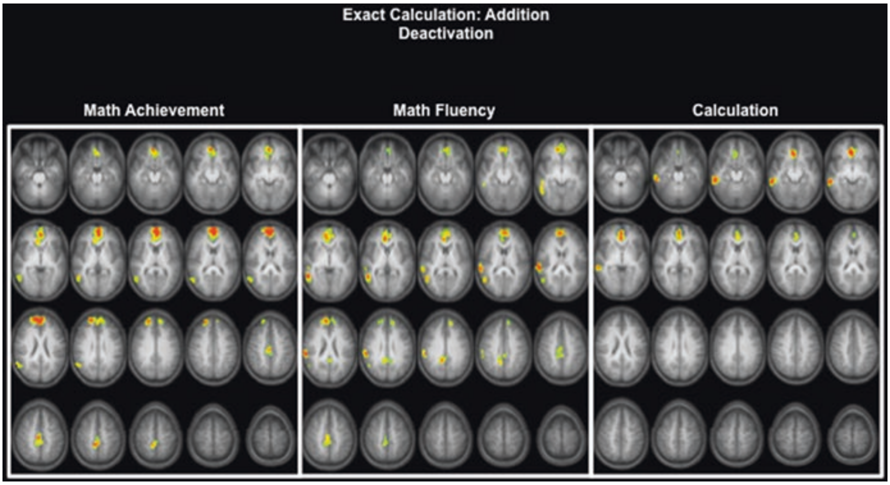

Fig. 4.3 Regions with significant ($p < 0.01$ FWE corrected) negative correlations between functional deactivation during exact addition and Woodcock-Johnson III percentile scores for math achievement and math fluency. Regions with significant ($p < 0.01$ FWE corrected) exact addition task performance by calculation percentile score interaction for functional deactivation. Images in radiologic orientation. Slice locations: Z −25 mm to Z = +70 mm

either 1 or 2 from the correct answer so that participants had to determine the exact answer. To control for visual activation and response selection, we compared brain activation during mathematical problems to brain activation during a color matching task that gave adolescents 2.5 s to choose which of two identical numbers matched the color of the same number shown in the middle of the screen.

Similar to the results for addition problems, as shown in Fig. 4.4, we did not find any significant correlations between brain activation during multiplication and task performance, math achievement, math fluency, or calculation. However, deactivation in posterior and lateral DMN regions positively correlated with exact multiplication task performance.

Furthermore, as shown in Fig. 4.5, deactivation in lateral DMN regions positively correlated with math achievement, deactivation in anterior DMN regions positively correlated with math fluency, and deactivation in right lateral DMN regions positively correlated with calculation.

Thus, deactivation of the DMN during exact multiplication had an opposite relationship with standardized measures of mathematics achievement in comparison to exact addition; that is positive, rather than negative correlations. Thus, individuals with higher mathematics achievement deactivated the DMN to a greater extent than those with lower mathematics achievement, making more neural resources available for determining the exact answer.

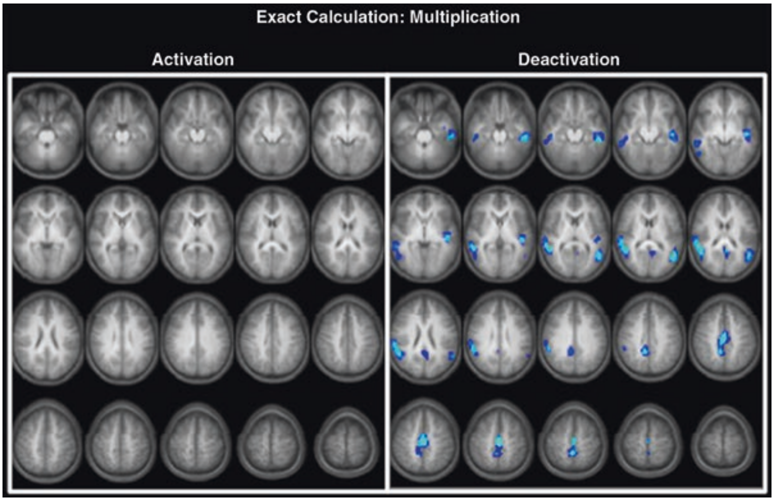

Fig. 4.4 Regions with significant ($p < 0.01$ FWE corrected) correlations between functional activation (none) and deactivation (positive) and exact multiplication task performance. Images in radiologic orientation. Slice locations: Z −25 mm to Z = +70 mm

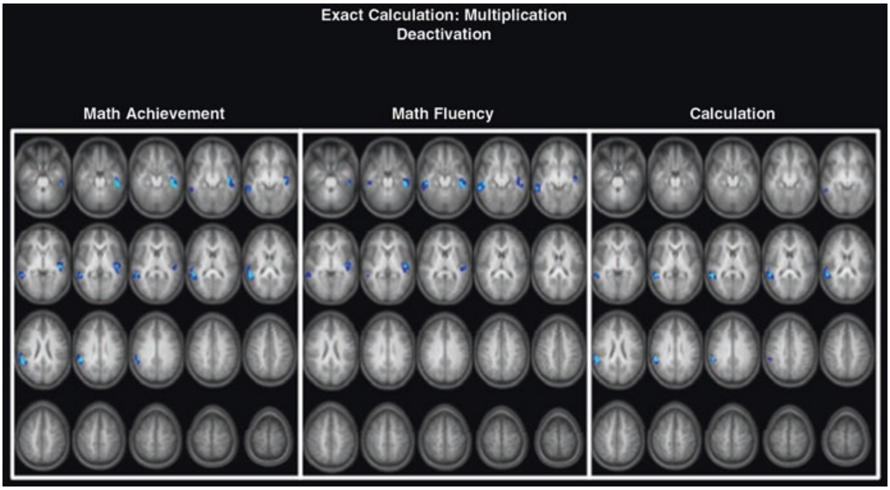

Fig. 4.5 Regions with significant ($p < 0.01$ FWE corrected) positive correlations between functional deactivation during exact multiplication and Woodcock-Johnson III percentile scores for math achievement, math fluency, and calculation. Images in radiologic orientation. Slice locations: Z −25 mm to Z = +70 mm

Multiple Operations

While undergoing fMRI, adolescents ($N = 10$) were presented with 3-operand problems involving addition and subtraction like the one on the right side of Fig. 4.1 (addition first on half of the trials) and had 5 s to select the side displaying the correct answer. The other response choice differed by either 1 or 2 from the correct answer so that participants had to determine the exact answer. To control for visual activation and response selection, we compared brain activation during mathematical problems to brain activation during a color matching task that gave adolescents 2.5 s to choose which of two identical numbers matched the color of the same number shown in the middle of the screen.

As shown in Fig. 4.6, in contrast to our results for 2-operand addition and multiplication, for these more complex 3-operand problems, activation in the left prefrontal cortex (PFC) negatively correlated with performance. Deactivation in the right lateral DMN region positively correlated with task performance.

As shown in Fig. 4.7, activation in the left PFC and right IPS during the multiple operations problems negatively correlated with math achievement and activation in the left PFC and left IPS negatively correlated with math fluency. We did not find significant correlations with calculation scores. The PFC activation may be due to maintaining the solution to the first operation in verbal working memory while completing the second operation (De Pisapia, Slomski, & Braver, 2007), which may be more effortful for individuals who are less proficient in mathematics.

As shown in Fig. 4.8, deactivation in posterior and right lateral DMN regions positively correlated with math achievement and deactivation in anterior DMN regions positively correlated with math fluency and calculation.

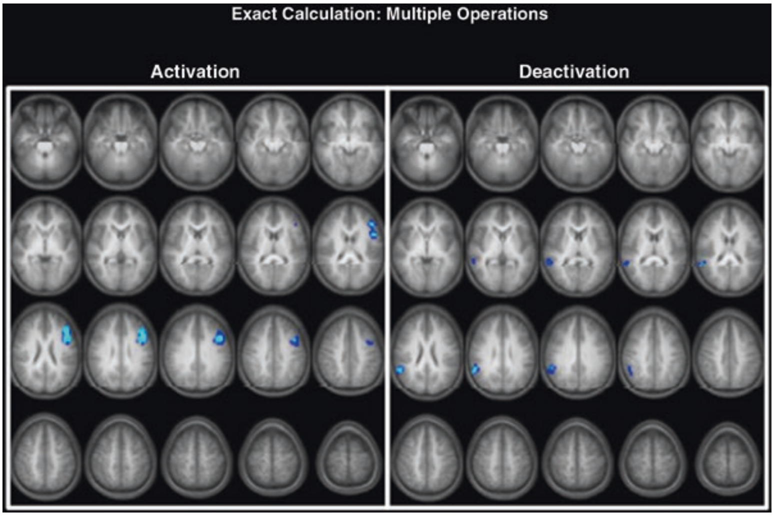

Fig. 4.6 Regions with significant correlations ($p < 0.01$ FWE corrected) between functional activation (negative) and deactivation (positive) and task performance on exact multiple operations problems. Images in radiologic orientation. Slice locations: Z −25 mm to Z = +70 mm

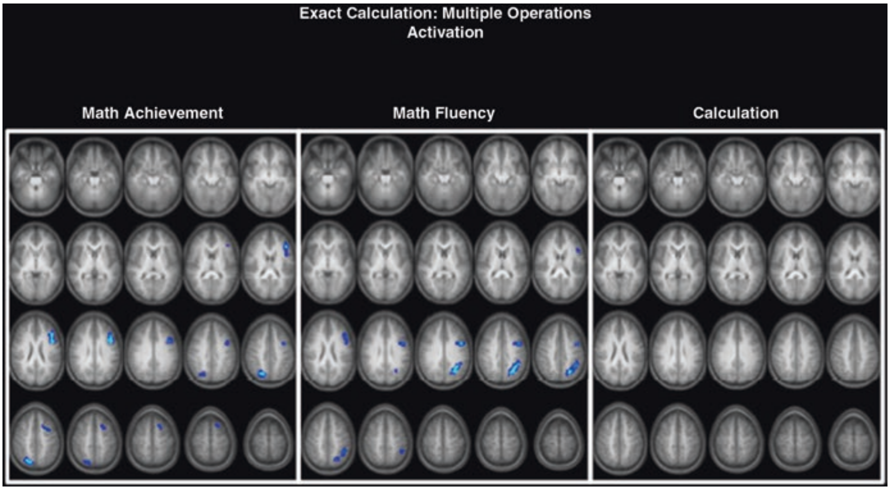

Fig. 4.7 Regions with significant ($p < 0.01$ FWE corrected) correlations between functional activation during exact multiple operations problems and Woodcock-Johnson III percentile scores for math achievement (negative), math fluency (negative), and calculation (none). Images in radiologic orientation. Slice locations: Z −25 mm to Z = +70 mm

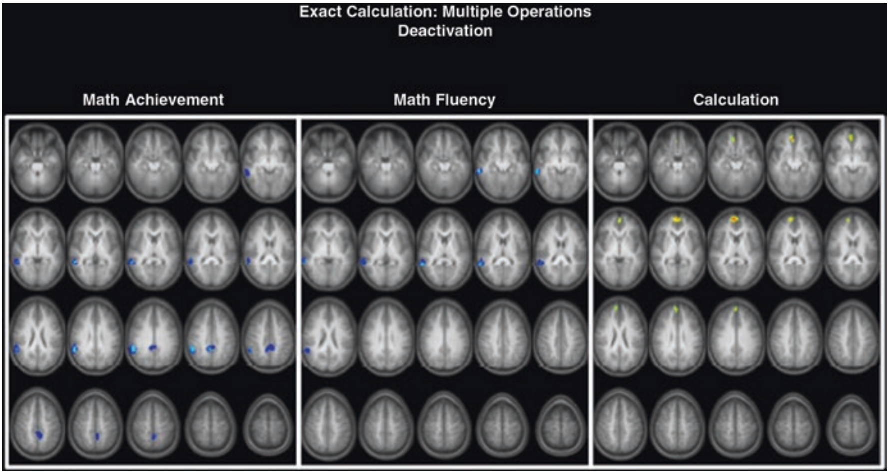

Fig. 4.8 Regions with significant ($p < 0.01$ FWE corrected) positive correlations between functional deactivation during exact multiple operations problems and Woodcock-Johnson III percentile scores for math achievement, math fluency, and calculation. Images in radiologic orientation. Slice locations: Z −25 mm to Z = +70 mm

For this task, participants with higher mathematics achievement may have deactivated the DMN to a greater extent to make neural resources available during this more difficult task that likely involves working memory.

Approximate Calculation

Now, take a look at Fig. 4.9 and choose the *closest* answer to each problem.

How did you choose your answers to these problems? Did you select your answers rather quickly, without performing any calculations? You may have recognized the large differences between the two choices, which may have made it easy to select your answer. For the addition problem on left side of Fig. 4.9, you may have recognized that the first addend is close to 1000, so the answer would have to be 1375, the choice on the right, because the choice on the left is below 1000. For the subtraction problem in the center panel, you may have estimated that 800 – 400 is closer to 400 than 45, and for the multiplication problem on the right side, you may have quickly recognized the appropriate number of zeros in the closest answer choice (i.e., 300 vs. 3000). Although these problems use different operations and may engage different types of strategies, they fundamentally assess approximate, rather than exact calculation.

While undergoing fMRI, adolescents ($N = 16$) were presented with these types of problems and had 5 s to select the side of the screen displaying the closest answer. The problems involved 3-digit addition and subtraction, and 1-, 2-, or 3-digit multiplication with two possible answers, a reasonable estimate and one that was off by approximately 1 order of magnitude. To control for visual activation and response selection, we compared brain activation during mathematical problems to brain activation during a color matching task that gave adolescents 2.5 s to choose which of the two

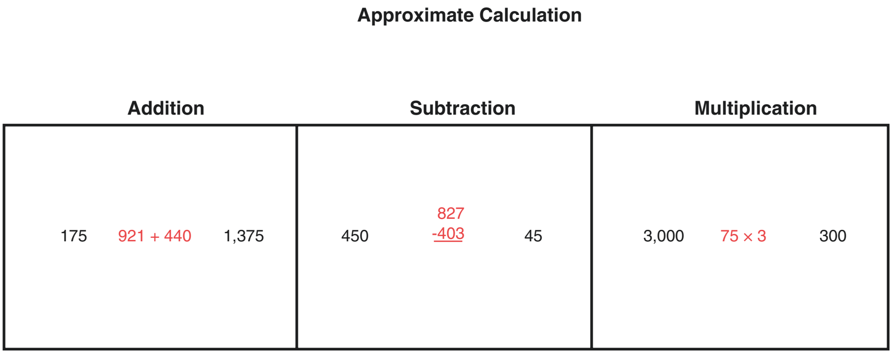

Fig. 4.9 Examples of problems used for three types of approximate calculation tasks: addition, subtraction, and multiplication

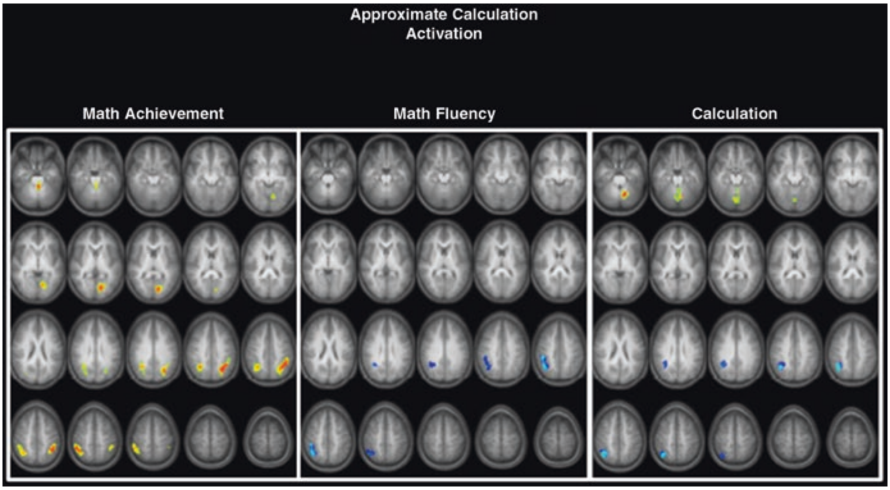

Fig. 4.10 Regions with significant ($p < 0.01$ FWE corrected) positive correlations between functional activation during approximate calculation and Woodcock-Johnson III percentile scores for math achievement and calculation. Regions with significant ($p < 0.01$ FWE corrected) negative approximate calculation task performance by math fluency and calculation percentile score interactions for functional activation. Images in radiologic orientation. Slice locations: Z −25 mm to $Z = +70$ mm

identical 2- or 3-digit numbers matched the color of the same number shown in the middle of the screen.

As shown in Fig. 4.10, activation in bilateral IPS and the left lingual gyrus during approximate calculation positively correlated with mathematics achievement. Activation in these areas plus the right inferior occipital gyrus positively correlated with calculation subtest scores. Significant negative approximate calculation task performance by subtest interactions for activation in right IPS were also found for math fluency and calculation.

As shown in Fig. 4.11, for math achievement, a significant interaction was found in anterior and left lateral DMN regions, indicating a smaller correlation between DMN deactivation and task performance for participants with higher scores, but a larger correlation in the right lateral DMN region. For math fluency, a significant interaction was found in anterior, posterior, and right lateral DMN regions, indicating a smaller correlation between DMN deactivation and task performance for participants with higher scores, while the opposite relationship was found in the left lateral DMN region. For the calculation subtest, a significant interaction indicating a smaller correlation between DMN deactivation and task performance for participants with higher scores was found in anterior, posterior, and lateral DMN regions.

Similar to the results for the magnitude comparison task presented in Chap. 3, for approximate calculation, activation in the IPS and lingual gyrus was correlated with standardized measures of math achievement. These two tasks were designed to measure approximate quantity representation and the results are consistent with predictions

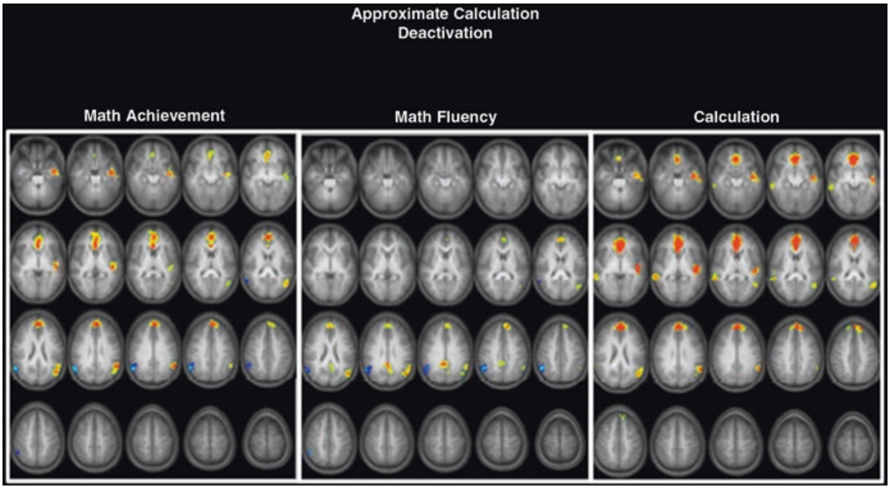

Fig. 4.11 Regions with significant ($p < 0.01$ FWE corrected) approximate calculation task performance by percentile score interactions (math achievement, math fluency and calculation) for functional deactivation. Images in radiologic orientation. Slice locations: Z −25 mm to Z = +70 mm

regarding the *quantity* and *verbal* systems of Dehaene and colleague's triple-code model of numerical processing. Thus, abstract quantity representation, or *number sense*, may be foundational to mathematics achievement (e.g., Gallistel & Gelman, 1992; Geary, 2010; Geary, Hoard, Nugent, & Bailey, 2012).

For deactivation, the significant interactions involving task performance and standardized math achievement may be related to differences in strategy use. Adolescents with higher math achievement scores are likely using more efficient strategies involving approximation, making the task easier (i.e., negative correlations), while adolescents with lower math achievement may be using more inefficient strategies, such as calculating the solution, which involves working memory and executive function, making the task more difficult (i.e., positive correlations; see Torbeyns, De Smedt, Peters, Ghesquière, & Verschaffel, 2011).

Error Detection

Have you ever looked at an answer to a math problem and recognized that something's wrong? Take a look at Fig. 4.12 and answer *Yes* or *No* if you think the solution to the problem is correct.

Were you able to quickly determine that the answer is *No*? What is the nature of this error? The problem in Fig. 4.12 represents a *wrong operation* error. That is, addition was performed, rather than multiplication. Other common errors include *string intrusions*, in which the answer generated is the next higher or lower number from

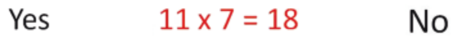

Fig. 4.12 Example of a trial from an error detection task

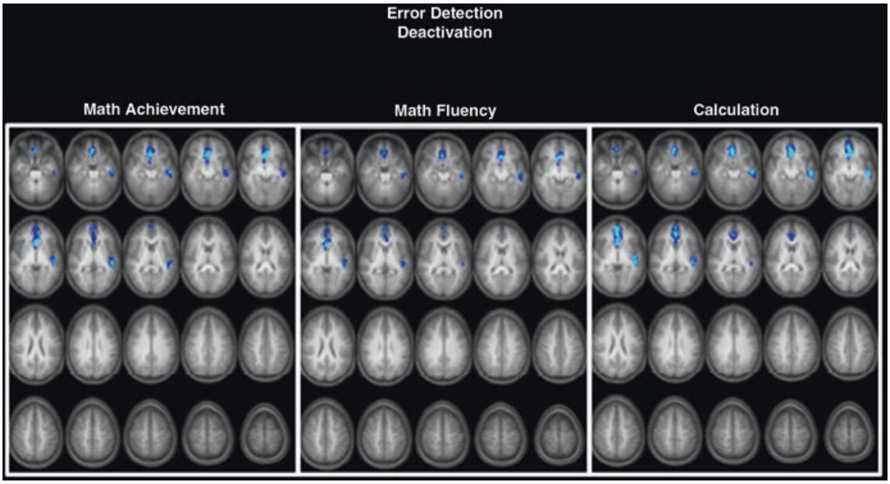

Fig. 4.13 Regions with significant ($p < 0.01$ FWE corrected) error detection task performance by percentile score interactions (math achievement, math fluency, and calculation) for functional deactivation. Images in radiologic orientation. Slice locations: Z −25 mm to Z = +70 mm

one of the operands (e.g., 4 × 5 = 6), *associated fact* errors, in which the answer generated is 1 or 2 off from the correct solution (e.g., 5 + 8 = 12), and *global* errors, in which the answer combines the numerals of the two operands (e.g., 2 × 8 = 28).

To see what is happening in the brain when adolescents look at a solution to a problem and decide whether it is correct or not, we showed them 1- and 2-digit addition, subtraction, and multiplication problems like the one in Fig. 4.12 and gave them 3.5 s to indicate *Yes* if the answer is correct or *No* if it is not correct. We carefully designed the incorrect problems so that they involved the common errors that were described above made by children with mathematical difficulties (Raghubar et al., 2009). To control for visual processing, we compared brain activation during this type of error detection to a task in which we gave adolescents 2.5 s to indicate *Yes* if two identical numbers were the same color or *No* if they were not.

For this error detection task, we did not find any significant correlations between activation and performance, math achievement, math fluency, or calculation. As shown in Fig. 4.13, significant interactions were found in anterior and left lateral DMN regions, indicating a larger correlation between DMN deactivation and Error Detection performance for participants with higher math achievement, math fluency, and calculation scores (for more detailed results, see Kroeger, 2012).

In contrast to our results for approximate calculation, adolescents with higher standardized achievement scores showed greater correlations between error detection task performance and DMN deactivation. We attribute this result to the higher achieving adolescents more efficiently suppressing fast, automatic responses, which requires more neural resources for executive function and response comparison.

Across all of the tasks described in the previous sections, we conclude that deactivation of the DMN plays an important role in mathematical cognition in adolescents. There were no significant correlations between IPS activation and task performance or standardized math achievement scores for the more difficult tasks of exact multiplication, exact calculation involving multiple operations, or error detection; whereas many significant relationships were found related to deactivation of the DMN. These findings suggest that interventions for more complex calculation procedures could target skills related to the DMN, including developing efficiency in strategy use and working memory, in addition to more domain-specific quantitative skills.

Fractions

In an earlier study, we examined adults' use of the representational systems proposed in Dehaene and colleagues' triple-code model of numerical processing for an even more complex task: the mental addition and subtraction of fractions (Schmithorst & Brown, 2004). While undergoing fMRI using a block-periodic design (i.e., rather than event-related), adults ($N = 15$) were presented with three fraction problems and had 10 s to mentally calculate the solution. The problems involved addition and subtraction of fractions with single-digit numerators and denominators (e.g., 2/3 – 1/4), which sometimes resulted in negative solutions. Improper fractions were used, but denominators were 5 or less. All participants completed a practice session before scanning to ensure that they could perform the task. A visual reminder that $a/b + c/d = (ad + bc)/bd$ was written on a board in the scanning area. To control for visual activation, participants were presented with 3 sets of 4 numbers without divisor, addition, or subtraction symbols in the same positions on the screen as the fractions every 10 s.

We used exploratory group *Independent Component Analysis* (*ICA*; for details, see McKeown et al., 1998) and found separate components of functional activation in bilateral inferior parietal, left perisylvian, and ventral occipitotemporal areas. An additional component of activation was found in the medial-superior occipital gyrus. We suggested that the bilateral inferior parietal component corresponds with the *quantity system* of the triple-code model, which may have been used during this task to process the relative positions of fractions on a mental number line as well as relations between the sizes of fractions involved in proportional reasoning (Dehaene, 1989; Dehaene, Dupoux, & Mehler, 1990; Ischebeck, Schocke, & Delazer, 2009; Restle, 1970). The left perisylvian component, which included Broca's and Wernicke's areas involved in language functions and basal ganglia, may have reflected use of the *verbal system* of the triple-code model for fact retrieval involved in determining common denominators (e.g., transforming 2/3 into 8/12 and 1/4 into

3/12) and adding and subtracting fractions (e.g., 8 − 3 = 5; González & Kolers, 1982). We suggested that the bilateral inferior ventral occipitotemporal component of the ventral visual pathway, including the inferior temporal gyrus and fusiform gyrus, corresponds to the *visual system* of the triple-code model, which may have been used to recognize numerators in comparison to denominators and spatially manipulate the fractions in Arabic format to calculate solutions that could not be retrieved from memory (Ashcraft & Stazyk, 1981; Cohen & Dehaene, 1991; Dahmen, Hartje, Büssing, & Sturm, 1982; Dehaene & Cohen, 1991; Weddell & Davidoff, 1991). For the fourth component of medial-superior occipital gyrus activation, we suggested that neural correlates of the *visual system* might extend beyond the ventral visual pathway into other secondary visual areas (See Chap. 5 for more research on this pathway). Thus, the addition and subtraction of fractions employed all three systems proposed in the triple-code model. Our results were consistent with previous neuroimaging studies examining other types of calculation problems (Chochon, Cohen, van de Moortele, & Dehaene, 1999; Dehaene, Spelke, Pinel, Stanescu, & Tsivkin, 1999; Gruber, Indefrey, Steinmetz, & Kleinschmidt, 2001; Naccache & Dehaene, 2001; Pinel, Dehaene, Rivière, & Le Bihan, 2001; Simon, Mangin, Cohen, Le Bihan, & Dehaene, 2002; Stanescu-Cosson et al., 2000).

Taken together, our work provides support for the triple-code model for a variety of tasks. Furthermore, it indicates that domain-specific neuroarchitecture for quantity processing and domain-general processes related to the DMN may act in coordination to perform calculation.

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Chapter 5
Visuospatial Cognition

Jonathan Buening and Rhonda Douglas Brown

Abstract In this chapter, we present theory and research on early- and later-developing visuospatial cognition into adulthood and its importance to mathematical cognitive development. We describe the development of dorsal and ventral visual pathways associated with the visuospatial functions of spatial awareness and pattern processing. Research using cognitive neuroscience techniques, including functional Magnetic Resonance Imaging (fMRI), Electroencephalography (EEG), and Transcranial Magnetic Stimulation (TMS), is presented on the following topics relevant to visuospatial cognition and its development: visual attention and search, visual perception and judgment, geometry, visual imagery and mental rotation, and visuospatial working memory. We conclude that the parietal lobe plays an important role in general visuospatial cognition and that the right hemisphere is dominant for certain visuospatial skills. Other brain areas related to visuospatial cognition include the superior frontal gyrus/sulcus, anterior insular cortex, temporal-occipital cortex, dorsolateral prefrontal cortex, precentral gyrus, and left hemisphere dorsal anterior cingulate cortex.

Transport yourself back to school, perhaps back to a geometry class. Imagine posters of different shapes that may have been up on the walls. Which ones draw your attention? Does your attention shift from one shape to another? Perhaps you detect patterns within a more complex geometric design. Are there any 3-dimensional (3D) representations of shapes within the room? If so, pick one, close your eyes, and imagine you're holding it in your hand. Now turn it over and look at all sides of the shape. Put that shape down, and with your eyes still closed, think about what other images you remember viewing just a moment ago.

If you performed any of the tasks described just now, you made use of an array of visuospatial skills. *Visuospatial cognition* encompasses a variety of skills, including searching for and locating objects, shifting spatial attention, holding items in your visual memory, performing mental rotations, and detecting patterns, among others.

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Neural Basis of Visuospatial Cognition

Research conducted with adults has described two major cortical streams related to visuospatial cognition: the dorsal visual pathway and the ventral visual pathway (e.g., Goodale & Milner, 1992; Mishkin & Ungerleider, 1982). Anatomically, both pathways begin at the retina where visual information is first received, and proceed to the primary visual cortex, referred to as V1, in the occipital lobe. From there, however, the pathways diverge, with the *dorsal visual pathway* projecting forward to the temporal lobe and then to the inferior parietal lobe, and the *ventral visual pathway* proceeding downward to the inferior temporal lobe. Each pathway has been associated with specific visuospatial functions. The dorsal stream, commonly referred to as the *where* or *how* stream, involves tasks primarily related to spatial awareness and action planning (Chinello, Cattani, Bonfiglioli, Dehaene, & Piazza, 2013; Stiles, Paul, & Ark, 2008). The ventral visual pathway, known as the *what* stream, is used in tasks involving part-whole or global-local visual pattern processing (Stiles et al., 2008). Figure 5.1 depicts each pathway from the left hemisphere lateral view.

Stiles et al. (2008) reviewed the literature on the development of brain networks and functions related to visuospatial processing. While many visuospatial skills are established early in life, many of those skills show protracted development. For example, they describe studies showing that children as young as 5 can perform mental rotations; however, they also noted that their speed and accuracy continues

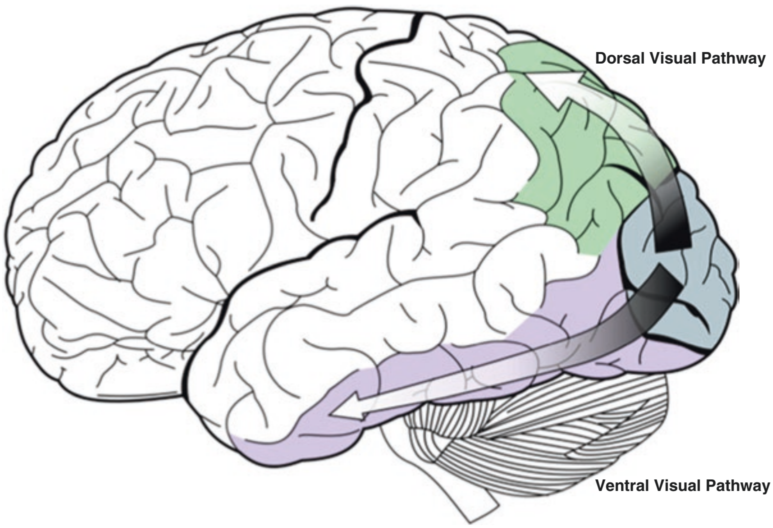

Fig. 5.1 The left hemisphere (lateral view) indicating the dorsal and ventral visual pathways. Photograph by Selket (From File: Gray728.svg) [GFDL (http://www.gnu.org/copyleft/fdl.html), CC-BY-SA-3.0 (http://creativecommons.org/licenses/by-sa/3.0/) or CC BY-SA 2.5-2.0-1.0 (http://creativecommons.org/licenses/by-sa/2.5-2.0-1.0)], via Wikimedia Commons

to develop and improve through adolescence. They drew similar conclusions from their review of ventral stream skills, such as pattern processing. Also common to their findings is evidence of connectivity between regions of the brain. For example, with respect to spatial location, a network of neural systems emerges as early as the first year of life, involving areas in both the frontal and parietal lobes.

Some studies have sought to specifically explore the nature of developmental changes within the two major visuospatial streams. As described previously, each stream is associated with functions that answer a general question: the *where* versus the *what*. Therefore, you might expect that there would be certain correlations between those associated functions. By that same logic, you may also expect there to be little correlations among functions between the two streams. Chinello et al. (2013) investigated these issues by administering a series of tasks to both kindergarten-aged children and adults to measure six specific abilities: numerical acuity, finger gnosis (i.e., finger recognition or localization), visuospatial memory, grasping precision, face recognition, and object recognition. As hypothesized, Chinello and colleagues found that, for children, there are significant correlations among tasks falling within a particular functional domain (dorsal or ventral), but little correlation on tasks between domains. However, adults showed lower correlations overall, whether within or between domains, suggesting that certain visuospatial skills that are more interrelated during early childhood may become more specialized during adulthood. Overall, Chinello and colleagues concluded that functions between the two major visuospatial streams are unrelated and follow their own specific developmental trajectories (for a different perspective, see Pisella et al., 2013). More specifically, they found that the development of finger gnosis, spatial abilities, and nonsymbolic numerical abilities were correlated independently of chronological age. Chinello and colleagues attributed the source of this correlation to finger counting, speculating that finger gnosis would improve with finger counting by increasing individual finger representations and their relative position in space and that finger counting would improve spatial abilities by tracking multiple items in parallel with the corresponding mental representation of numerical quantities. Indeed, in children attending kindergarten, strategies for solving addition problems, including finger counting, were significantly correlated with spatial, but not verbal subtests of the Wechsler Preschool and Primary Scale of Intelligence (WPPSI; Geary & Burlingham-Dubree, 1989).

Thus, while the various structures and networks of the brain related to visuospatial cognition are well identified, the developmental pathways of these structures, along with connectivity between them, are not clearly understood. Therefore, in this chapter, we review some of the major functions that fall within the realm of visuospatial cognition, identify the major areas of the brain associated with those functions, and describe what is known about how those functions change with development.

Before proceeding, however, it is important to place the role of visuospatial cognition within the context of mathematical cognitive development, the focus of this text in general. Intuitively, it perhaps seems obvious that visuospatial skills would play a major role in mathematical cognitive development. In Chap. 4, results on the mental addition and subtraction of fractions were presented from Schmithorst and Brown (2004) that indicated a bilateral inferior ventral occipitotemporal component of the ventral visual pathway, including the inferior temporal gyrus and fusiform gyrus. These areas comprise the *visual system* of the triple-code model (see Chap. 1), which may have been used to recognize numerators in comparison to denominators and spatially manipulate the fractions in Arabic format to calculate solutions that could not be retrieved from memory. We also found a component of medial-superior occipital gyrus activation and suggested that neural correlates of the *visual system* might extend beyond the ventral visual pathway into other secondary visual areas.

Wai, Lubinski, and Benbow (2009) cite longitudinal studies showing correlations between spatial ability and later Science Technology Engineering and Mathematics (STEM) related degrees and careers. Thus, spatial ability could have important implications for guiding coursework, career pursuits, and appropriate interventions. Wai et al. (2009) conducted a study involving a sample of 400,000 high school students, based on available data on spatial ability, along with degree and career data from an 11-year follow-up and found that spatial ability was indeed a reliable predictor of advanced STEM degrees, as well as occupational outcomes. Figure 5.2 shows example items from their measures of spatial ability. Importantly, their sample was not solely comprised of academically high-achieving students, suggesting that these implications are applicable to the general population. Given findings such as these, the importance of visuospatial ability to mathematical cognitive development cannot be underestimated. Keeping this in mind, the following sections will review specific functions within the scope of visuospatial cognition.

Visual Attention and Search

Let's return to our imaginary geometry class. As you look around, are there shapes that seem to draw your attention involuntarily? In contrast, what if we asked you to find and look at a specific one—a circle, a parallelogram, or a cone? What if we asked you to look around and find examples of specific shapes hidden within objects or patterns in the room? Or what if we asked you to note all of the spherical objects you could see—where are they located in relation to each other? These are examples of activities that involve visual attention and visual search.

One important concept to remember about these functions is that they are not uniform in nature. There is no single definition of attention, but several types. For example, consider if you found yourself in a room where there was a fluorescent light flickering above. In that case, your attention would shift involuntarily to the light as a result of a stimulus. However, if we ask you to search for something specific, as we did above when mentioning a circle, parallelogram, or cone, your attention became goal-oriented in nature. Similarly, searching for one particular shape in a room means you have to match criteria in a specific, localized fashion; but asking you to locate all of the representations of that shape requires a more globalized view.

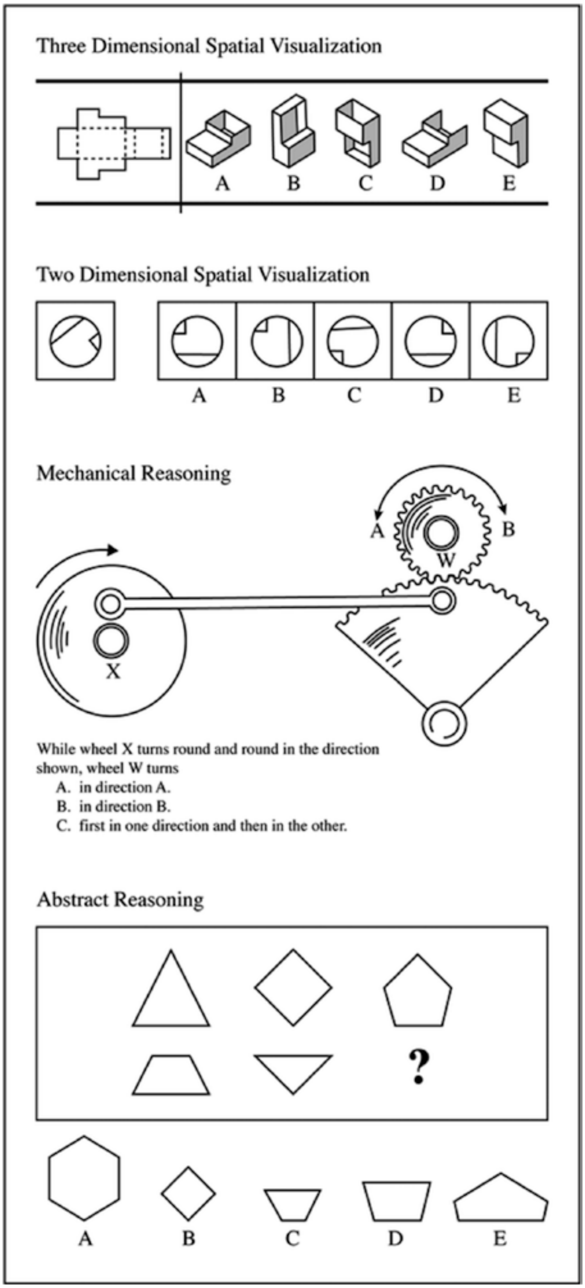

Fig. 5.2 Example items from four tasks measuring spatial ability: 3D spatial visualization, 2D spatial visualization, mechanical reasoning, and abstract reasoning. Source: Wai, J., Lubinski, D., & Benbow, C. P. (2009). Spatial ability for STEM domains: Aligning over 50 years of cumulative psychological knowledge solidifies its importance. *Journal of Educational Psychology*, *101*(4), 817-835. https://doi.org/10.1037/a0016127

The dorsal stream is used in tasks involving spatial attention and orientation, spatial localization, and mental rotation. Briefly, research on *spatial attention*, and more specifically, *spatial orientation*, has shown that a posterior parietal network is involved in disengaging attention from one location, shifting it to a different location, and inhibiting a return of attention to the original location, which is thought to be an evolutionarily important phenomenon (e.g., Posner, 1980; Posner & Petersen, 1990). Like numerosity and ordinality, these abilities appear to be present very early in life, but show improvement between 2 and 4 months, and are mostly functional by 6 months (e.g., Butcher, Kalverboer, & Geuze, 1999; Clohessy, Posner, Rothbart, & Vecera, 1991; Hood, 1993; Simion, Valenza, Umiltá, & Dalla Barba, 1995; Valenza, Simion, & Umiltà, 1994). With age and experience, visual attention skills become more right lateralized. For example, Smith and Chatterjee (2008) reported that 12- to 14-year-olds with slower responses in both local and global attention tasks displayed more bilateral activation than better performing peers, who displayed a greater tendency towards right hemisphere activation. In adults, the right hemisphere, particularly the superior parietal cortex, is the dominant locale for visuospatial skills. In their study, Everts et al. (2009) administered visual search tasks while participants were in an MRI scanner. Participants also completed assessments measuring their visuospatial ability. Results showed that visual search was associated with bilateral frontal, superior temporal, and occipital regions, indicating interconnectivity of visuospatial networks. Additionally, Everts and colleagues found that lateralization of activity within the right hemisphere, particularly in the frontal and parietal regions, increased not only with age, but also with increased performance on visuospatial assessments. These results are consistent with Chinello et al.'s (2013) findings suggesting that age and greater efficiency contribute to increased specialization in the brain related to visuospatial function.

Visual Perception and Judgment

Visual perception is a broad term that encompasses several possible functions. Back in our geometry class, imagine if we were to show you a series of geometric shapes on note cards. In addition to different shapes being present, imagine too that there are differences in shading, the thickness of lines, the orientation of the shapes in relation to each other, the number of certain shapes appearing in a row, and so on. Whenever you notice those different aspects, you are activating a particular function within the realm of visual perception.

In terms of the areas of the brain involved in various visuospatial perception tasks, we see many commonalities with those already described in the dorsal and ventral streams, as well as in the areas related to visual attention and search. Ebisch et al. (2012) conducted a study investigating the nature of fluid intelligence by having participants perform four tasks while undergoing fMRI. Tasks specific to visuospatial perception included *induction*, for which participants must determine common characteristics among stimuli; *visualization*, for which participants must manipulate visual images; and *spatial relationship*, for which participants are required to identity spatial patterns or orientations among objects. Brain areas that showed common activation across different tasks included the bilateral superior frontal gyrus/sulcus, inferior parietal lobe, intraparietal sulcus (IPS), posterior parietal cortex, superior parietal cortex, anterior insular cortex, temporal-occipital cortex, dorsolateral prefrontal cortex, precentral gyrus, and left hemisphere dorsal anterior cingulate cortex (ACC). These results indicate that visual perception involves a distributed frontoparietal network. For the visualization task, there was significant activation of the bilateral inferior parietal lobe, as well as the right hemisphere dorsal ACC. These results are consistent with other findings stressing not only the importance of the parietal lobe in general visuospatial cognition, but also the dominance of the right hemisphere with respect to certain visuospatial functions. Additionally, Ebisch and colleagues found that participants who scored higher on fluid intelligence assessments showed greater functional connectivity between these different brain areas, once again suggesting that specialization of the brain occurs not only with age, but also with increased skill and efficiency.

Other studies have shown similar results, and have sought to expand on them by including developmental data as well. Eslinger et al. (2009) conducted an fMRI study in which participants aged 8–19 completed *relational reasoning* tasks. Specifically, participants had to identify a correct response in order to complete a series of images based on dimensions of color and shape. They found a network of related structures involved in these types of tasks including the superior parietal cortex, the dorsolateral prefrontal cortex, the superior premotor/supplementary motor region, and the occipital-temporal cortex, with the greatest level of activation in the right and left superior parietal cortices. In this primary area, Eslinger and colleagues found increased activation with age. They noted a contrast in the bilateral nature of activation in their study compared to lateral specialization in other studies, although this could be accounted for by the broad nature of the visuospatial tasks used. Interestingly, they found that certain areas showed decreased activation with age, notably the prefrontal-frontal cortex and the cingulate. As previously discussed, with age, children likely become more efficient in performing visuospatial tasks, which translates into a decreased need for executive function, attention, working memory, and so on, functions associated with the decreased brain activation areas mentioned above (i.e., the Default Mode Network [DMN]).

Regarding lateralization, Fink et al. (2000) conducted an fMRI study in which adult male participants had to judge if lines were correctly bisected. Scan results showed distinct and significant activation of the right hemisphere superior parietal cortex as well as the right inferior parietal cortex. Other studies have attempted to investigate this apparent hemispheric specialization and its place within the functional connectivity network that seems to exist in visuospatial skills. Sack et al. (2007) used transcranial magnetic stimulation (TMS; see Chap. 2 for a description) as a way to disrupt neural activity within the parietal lobe, and then measure effects on behavioral performance, as well as neural activity in both stimulated and unstimulated areas of the brain through fMRI scans. The adult participants in the study were asked to identify either angle-based or color-based targets on a series of clock images, both in a control trial and in a trial where TMS was applied. Results demonstrated that, consistent with past research, the parietal and frontal lobes showed activation during both visual tasks. As predicted, right parietal TMS resulted in decreased behavioral performance as well as decreased activity in the stimulated area. However, decreased activation was also seen in the right postcentral gyrus and the middle frontal gyrus. These extended decreased activation areas were not observed when left parietal TMS was applied. These results further support the importance of the right parietal lobe in visuospatial tasks, and also demonstrate the functionally connective nature of the frontoparietal network with respect to visuospatial cognition.

Building on this study, de Graaf, Roebroeck, Goebel, and Sack (2010) conducted research to further investigate the specific patterns of this functional connectivity. They had adult participants perform the same tasks described above used by Sack et al. (2007), and then used fMRI to measure brain activation. Furthermore, de Graaf and colleagues employed a connectivity analysis technique known as *Granger Causality Mapping (GCM)* to investigate more closely the interactions within the established visuospatial network. Their results indicated a strong connectivity between the posterior parietal cortex and the middle frontal gyrus, which is consistent with past findings. They also identified directional aspects of this network, displaying a flow of information from the frontal to the parietal lobe. While the flow was mainly from the middle frontal gyrus to the posterior parietal cortex, other frontal regions were indicated as well, such as the insula. Other areas possibly involved in this network include the superior frontal sulcus, postcentral gyrus, and occipital cortex.

Many of the studies of visual perception described to this point indicate a certain degree of specialization that occurs in the brain with age, experience, or efficiency. Another possible mediator of specialization that is of particular interest in this text is task-specific—namely, tasks related to mathematical function. As previously noted, associations have been established between visuospatial cognition and mathematical skills. Some mathematical tasks relate directly to visuospatial cognition, such as those involving geometry (see Fig. 5.3). Izard, Pica, Dehaene, Hinchey, and Spelke (2011) sought to understand what sort of inherent ability we might have for geometric tasks as they relate to spatial perception. They conducted a study in which participants had to identify figures in a series of images that were different in some way; those differences could be related to one of a few geometric features, such as angle, size, or sense. By comparing results between children, adults, and an Amazonian tribe with no formal geometry education, Izard and colleagues found that certain geometric intuitions do indeed appear to be universal. However, developmental differences were noted with respect to certain tasks. In task trials involving stimuli related to length and angle, children performed well. However, in tasks requiring spatial sense, children were outperformed by adults, indicating a protracted development for certain types of visuospatial perception.

Other studies have added to the research on specific math-related visuospatial skills by investigating activity within specific brain circuits. Mangina et al. (2009) note that task stimuli involving size and dimension correlate more with mathematical

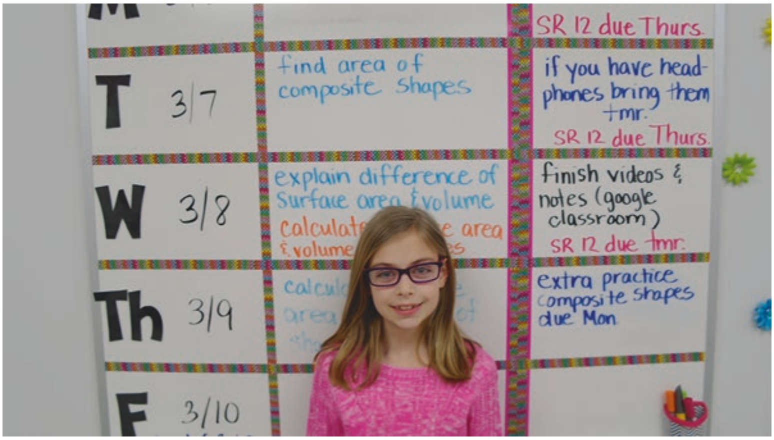

Fig. 5.3 Some mathematical tasks relate directly to visuospatial cognition, such as those involving geometry, and are incorporated into mathematics curricula across grade levels

skills, and tasks involving orientation relate more to reading skills. In an effort to see if these differences are reflected in brain activity, they administered different perceptual tasks to participants that related specifically to direction, spatial orientation, size, or dimension. The adult participants were required to identify similar stimuli according to one of the factors listed above while fMRI scans were conducted to measure brain activity. As with previous visuospatial studies, Mangina and colleagues identified overall activity within a distributed network that included the prefrontal cortex as well as the occipitotemporal and parietal regions. However, tasks related more to mathematical skills (i.e., those involving size and dimension) revealed activation in the bilateral posterior parietal, premotor, and prefrontal regions. These findings contrast with reading-related skills, which showed activation in the occipitotemporal and sensorimotor cortices.

Overall then, while studies of visual perception cover a broad array of topics, they establish clear evidence of a frontoparietal network and a high level of connectivity between these brain regions, indicating that disruption to activity in one area can affect activity in another. The available data suggest that both lateral and focal specialization within this network is mediated by contributing factors such as age, experience, and task specificity.

Visual Imagery and Mental Rotation

So far we've discussed attention to and perception of actual stimuli in our environment. But, in addition to what is directly perceivable, visuospatial cognition also encompasses our ability to picture objects and spatial relationships in our minds.

For instance, imagine a map from your home to place of work. Perhaps consider a map that includes not only routes and direction, but also one that includes actual images as well, as if it was a picture from a satellite. How many landmarks, buildings, natural objects, and so on can you picture from that route and with how much detail? What color are the buildings? How big are the trees? Where are structures located in relation to one another? As you drive along your route, how does your perspective of a building change as you drive towards it, past it, and then away from it? What you pictured above involves not only visual imagery, but manipulation of that imagery as well.

It is first important to realize the different types of visual imagery. Let us return for a moment to the beginning of this chapter when we discussed two primary cortical streams involved in visuospatial cognition. Remember how we distinguished between the dorsal stream, the *where* pathway concerned with spatial awareness, and the ventral stream, the *what* pathway concerned with object recognition? The available research on the basics of visual imagery indicates two basic types of imagery that seem to fit within these two cortical streams.

One type is known as *spatial imagery*, and refers to a person considering the spatial relationship between objects and how they might exist or move in relation to each other. So, when we asked you to imagine the placement of structures along your route, you were making use of spatial imagery. The other type is known as *object imagery*, and as its name implies, it refers to a person actually picturing an object in his or her mind (Kozhevnikov & Blazhenkova, 2013). When we asked you to imagine the specific buildings and landmarks, you were making use of object imagery. Research supports both functional and anatomical differences between these two types of imagery, with processes related to spatial imagery being contained more within the occipitoparietal dorsal pathway and object imagery being contained more within the occipitotemporal ventral pathway (Kozhevnikov & Blazhenkova, 2013).

Much of the cognitive neuroscience research on visual imagery has investigated spatial imagery. Kozhevnikov and Blazhenkova (2013) refer to past research that indicates that structures within the dorsal pathway seem to show less neural activity in people that perform better on spatial reasoning tasks. This inverse relationship between high performance on tasks and lower activity in those task-related areas of the brain is known as *neural efficiency* (Kozhevnikov & Blazhenkova, 2013), the idea that a person who is skilled in these sorts of tasks is able to make better and more efficient use of brain resources during task performance. For example, Kozhevnikov and Blazhenkova describe fMRI studies indicating that participants who are more skilled in object imagery tasks typically show less activation in the occipital complex when asked to study and visualize line drawings of common objects than participants who are more skilled in spatial imagery tasks.

There are other differences between the two imagery types. With regards to development, spatial ability seems to increase and peak during adolescence, followed by a decline into adulthood (Kozhevnikov & Blazhenkova, 2013). Object ability, however, seems to increase continually with age. These fundamental differences between the two imagery types may have educational implications as well, especially considering the fact that the increased use of spatial reasoning typically leads to increased success in mathematical problem solving, as noted in Kozhevnikov and Blazhenkova (2013), as well in other studies mentioned previously in this chapter. There seem to be sex differences involved in visual imagery as well. Perhaps the area of visual imagery for which this is most evident is *mental rotation*, which involves a person's ability to imagine how an object would look in a different orientation. If you think back to when we asked you how a building would look depending on your angle to it while driving, you may have employed mental rotation during that exercise. As it relates to the two distinct types of imagery, mental rotation falls under the umbrella of spatial imagery. Going back to sex differences, Kozhevnikov and Blazhenkova (2013) note that available research points to females being more skilled in tasks of object imagery, and males being more skilled in tasks of spatial imagery, such as mental rotation.

Roberts and Bell (2000) used electroencephalography (EEG) to investigate differences in brain activity with regard to both sex and age during mental rotation tasks. They noted that while previous research had indicated that men tend to display greater right parietal activation than women during mental rotation tasks, little research had been done to include the factor of development. To that end, they conducted a study with a group of 8-year-old children and a group of college students, both male and female. Participants were shown a figure of a gingerbread man. Two versions of that figure were then presented, each rotated at a different angle from the original. Participants were asked to identify which of the two choices matched the original, which would involve mental rotation of the figure to its original position. In terms of developmental differences, adult participants displayed more activation in all brain areas measured than children, with the exception of the lateral frontal area. Meanwhile, the comparison of men and women showed that men displayed more activation in the posterior temporal, parietal, and occipital regions, consistent with previous research. Interestingly, these activation differences were not seen between male and female children, suggesting a developmental factor in mental rotation with regards to sex differences. One result that was somewhat inconsistent with past research was that men displayed greater parietal activation in the left hemisphere as opposed to the right. However, the researchers suggested that this result may be in line with a male advantage for mental rotation tasks, as previous research has indicated that simple rotation tasks typically elicit greater left hemisphere activation than complex tasks. In this case, if the task was indeed easier for the male participants, then greater left hemisphere activation seems plausible.

Other studies have further investigated these apparent sex differences. Neubauer, Bergner, and Schatz (2010) noted some of the inconsistencies in the stability of mental rotation performance, citing studies that show performance can be enhanced through practice and training. Additionally, the advantage that males have on these tasks seems to disappear when the task is presented in 3D, rather than 2-dimensionally (2D). They conducted an EEG study in which they presented a mental rotation task involving 3D cubes to adolescents. Different trials were conducted involving both 2D presentation (on a screen) and 3D presentation (using 3D glasses). Pre- and post-tests were also completed after a training specifically designed for mental rotation tasks. Neubauer and colleagues found that sex differences in terms of task performance only held for the 2D version of the task. Additionally, females were able to increase their performance to a greater degree following the training. Consistent with previous research, activity was generally right lateralized in parietal regions for both sexes. One interesting note was that increased neural efficiency after training was seen in males overall, but in females neural efficiency only increased for the 3D task, rather than the 2D task. These findings suggest that mental rotation task performance is malleable, indicating that activation and performance can depend on factors such as experience and the nature of the stimuli.

Development seems to be an important factor in tasks of mental rotation. A number of studies have investigated how children perform on these tasks, and what patterns of brain activation they display. Heil and Jansen-Osmann (2007) cite previous research indicating that although adults show increased right parietal activity during mental rotation, children display greater left hemisphere activation. To validate this finding, they conducted an EEG study with a group of 7- and 8-year-old children involving mental rotation of alphabetical letters. Consistent with the pattern seen in the research on spatial reasoning in general, the most pronounced activation was seen in the parietal lobe. Additionally, results showed increased activity in the left hemisphere. The researchers speculated that this hemispheric difference between children and adults stems from how the two groups approach the task. Children may engage in more complex part-representations of mental rotation, whereas adults may engage in a simpler whole-representation of the entire figure. These approaches could correspond to differences in areas of activation. To build upon this study, Lange, Heil, and Jansen (2010) conducted an additional EEG study with the goal of determining whether this hemispheric difference in children may be stimulus dependent. Instead of alphabetical letters, children were required to mentally rotate line drawings of animals. While activity was again displayed in the parietal region, there was no lateralization effect. This finding seems to support the results obtained by Neubauer et al. (2010), which point to a variability in mental rotation tasks based on task stimuli.

Visuospatial Working Memory

Throughout the chapter, we've presented you with a variety of imaginary tasks. Many of these tasks required you to picture shapes you are familiar with. Others required you to navigate through an imaginary space. What can you remember about those tasks right now? Are you still able to picture the shapes you found hidden in objects in your geometry class? What did they look like? Can you picture them with the same detail you did some moments ago? What about when we asked you to drive past a building—can you still remember the exact route you took in your mind?

Just now, the questions we asked caused you to make use of your visuospatial memory. Related to general visuospatial memory is *visuospatial working memory*

(VSWM). Working memory in general is not a concept specific to visuospatial cognition. In fact, working memory involves a few different functions such as memory, attention, and perception, and it is what allows you to hold pieces of information in your mind while you operate on that information and use it towards a goal (Scherf, Sweeney, & Luna, 2006). Therefore, VSWM involves such abilities as being able to picture a novel object you just observed, or being able to remember a spatial pattern of movement. Anyone who has ever played the game Simon is probably familiar with latter, in which you must remember and repeat a pattern of sequential lighted buttons.

Just as with other concepts within visuospatial cognition, there are certain areas of the brain that have been implicated for VSWM tasks. However, some background is required. Two important components of general working memory have repeatedly been identified in the literature: the *phonological loop*, relating to verbal information, and the *visuospatial sketchpad* , relating to visual information (Baddeley & Hitch, 1974). As Vecchi, Phillips, and Cornoldi (2001) note, there is evidence that these two components operate somewhat independently, such that increased load on one component will lead to increased significant interference on tasks requiring that same component, but not on tasks requiring the opposite component. Other theoretical views indicate that working memory performance may vary according to the type of stimuli being processed, as well as the degree of necessary active processing (Vecchi et al., 2001).

In terms of the type of information being processed, the subdivisions follow the same properties of the primary dorsal and ventral networks mentioned previously in this chapter. Remembering information related to the *where* of an object (e.g., the route that car takes to get home) activates the dorsal network. Remembering information related to the *what* of an object (e.g., the size and color of a car) activates the ventral network. However, there are also differences involving the degree of active processing. Passive storage of information requires only remembering previously learned information as it was originally presented (Vecchi et al., 2001). However, active processing involves not only remembering that information, but also being able to manipulate it. Our previous discussion of mental rotation is an example of this active processing.

Research has shown that there are observable developmental differences with respect to VSWM and its associated subdivisions. One basic observed outcome is that VSWM skills increase with age during childhood, although this increase may relate more to improvements in working memory in general (Vecchi et al., 2001). However, when looking specifically at passive versus active processing, a distinction does seem to arise. Given effective training strategies, children perform as well as adults on tasks involving passive storage, but not on tasks involving active processing. On the other end of the spectrum, older adults show a decline in their active processing ability, while their passive storage remains stable (Vecchi et al., 2001). Since these results are mostly based on VSWM test outcomes, one possible explanation for this is the novelty of VSWM tasks versus verbal working memory tasks. Pure VSWM information is free of context, which can lead to an increased requirement of executive processing in the frontal lobe. It could be that the decline in

VSWM skills in older adults has more to do with these brain areas than functions of VSWM specifically.

Other researchers have investigated the aforementioned increase in VSWM skills with age and attempted to explain them using the different components of general working memory. Pickering (2001) cites previous research indicating that when remembering visual information, young children tend to encode the information visually; as children age, they tend to translate that same information phonologically. For example, older children may use verbal labels more than actual visual imagery. Additionally, increasing VSWM skills may reflect an overall maturation in children's executive functioning, leading to more efficient use of both verbal- and visual-related processing. Pickering also notes that although verbal translation of information may account for some of this increase, it cannot explain it completely. Other factors may contribute, such as increased attentional capacity, processing speed, visuospatial knowledge, and memory strategies.

A number of cognitive neuroscience studies have been conducted to further investigate some of these developmental differences. Klingberg, Forssberg, and Westerberg (2002) conducted an fMRI study with participants ranging in age from 9 to 18. Participants were presented with a VSWM task in which they had to remember positions of a number of red circles within a grid. Results showed that certain areas of the brain were activated during the task for all age groups, including the prefrontal, cingulate, parietal, and occipital cortices. Additionally, a positive correlation was noted between age and activation in the superior frontal sulcus, the intraparietal and superior parietal cortices, and the left occipital cortex. These results are somewhat inconsistent with other studies of visuospatial cognition in which age usually correlated with lower frontal activity. However, VSWM tasks are somewhat unique in that they are not likely to improve with training and experience, leading to continued frontal activation. These results are consistent with other studies, however, in providing evidence of a frontoparietal network for visuospatial cognition. Klingberg and colleagues suggest that concurrent neurological developments, such as myelination within the parietal cortex and synaptic pruning, may contribute to the increased activation seen in this network with age.

Scherf et al. (2006) conducted an fMRI study with a group of children, adolescent, and adult participants aged 8–47. In this study, participants had to move their eye gaze to a spatial target based on previous presentation of a visual stimulus in that target area. As predicted, certain regions displayed activation for all three age groups, including the right dorsolateral prefrontal cortex, the right ACC, the bilateral interior insula, the right superior temporal gyrus, the right interoccipital sulcus, and the right basal ganglia. Interestingly, the parietal cortex was not included in these results. Developmentally, children displayed greater activation in the thalamus and cerebellum in comparison to adolescents. Additionally, children displayed less activation in the dorsolateral prefrontal, parietal, premotor, and cingulate cortices than adolescents or adults. Overall, these results are consistent with other findings indicating that the dorsal pathway has a more protracted developmental trajectory than the ventral pathway. While both adolescents and adults showed activation in the aforementioned areas, Scherf and colleagues noted that adult activation was more left lateralized in regions consistent with use of the phonological loop. As mentioned previously, this may reflect a strategy of verbally recoding visual information for increased memory effectiveness.

Because of these apparent developmental differences in VSWM, other researchers have attempted to investigate the associated educational implications by exploring associations between VSWM and certain academic abilities, including mathematical performance. Dumontheil and Klingberg (2012) cited meta-analyses showing the IPS to be vital in numerical processing. As this area of the brain has been noted to be important to VSWM as well, they conducted a longitudinal study investigating the relationship between working memory ability, arithmetic performance, and brain activation in children aged 6–16 years. Participants' VSWM, visuospatial reasoning, and arithmetic abilities were assessed using standardized assessments. A subset of participants also took part in an fMRI scan while performing a VSWM task. Their results showed that VSWM and visuospatial reasoning correlated positively with arithmetic performance, but there was no relationship with age. However, there were developmental differences in brain activation, with increased activation in the left parietal sulcus relative to the rest of the brain predicting *poorer* arithmetic performance 2 years later, but greater activation in the whole brain VSWM network predicting better arithmetical performance.

Whereas Dumontheil and Klingberg's (2012) study investigated VSWM and associated brain activation trends with respect to mathematics, others have sought to explore the relationship between mathematical performance and the type of working memory employed as children grow older. Soltanlou, Pixner, and Nuerk (2015) conducted a study in which children in grades 3 and 4 were assessed on both verbal and visual working memory tasks and their multiplication skills. As observed in other studies, results showed that, overall, children's working memory skills increased with age. However, they also found that while verbal working memory was a predictor of multiplication problem solving ability in grade 3, it was not predictive of performance in grade 4. The opposite was true for VSWM, being a predictor of performance in grade 4, but not in grade 3. According to Soltanlou and colleagues, these results are consistent with other studies showing a weak connection between the phonological loop and mathematical ability in adults versus children. The implication is that the verbal component of working memory is important when younger children are learning math skills, but that as they develop, their understanding of mathematics becomes more abstract and visually based. Results such as these show the general developmental trends seen in VSWM have a complex interaction with children's mathematical abilities and cognition.

Conclusions

Overall, research indicates the important role of the parietal lobe in general visuospatial cognition, but also the dominance of the right hemisphere with respect to certain visuospatial skills. Other brain areas related to visuospatial cognition include the superior frontal gyrus/sulcus, anterior insular cortex, temporal-occipital cortex, dorsolateral prefrontal cortex, precentral gyrus, and left hemisphere dorsal anterior cingulate cortex.

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Chapter 6
Mathematical Difficulties and Exceptionalities

Rachel Lindberg and Rhonda Douglas Brown

Abstract In this chapter, we review research on mathematical difficulties and exceptionalities. Mathematical difficulties are distinguished from general learning difficulties, and include developmental dyscalculia and mathematical learning disabilities. We discuss research on cognitive processing associated with mathematical difficulties, including the approximate number system, or number sense, fact retrieval, delayed procedural development, fractions and proportional reasoning, visuospatial reasoning, working memory, and time estimation. We also present neuroscience research indicating specific effects related to mathematics for children with a diversity of neurodevelopmental disorders, syndromes, and conditions, including Autism spectrum disorder, Fragile X syndrome, Turner syndrome, 22q11.2 deletion syndrome, Williams syndrome, Spina Bifida, prenatal alcohol exposure, premature birth, developmental coordination disorder, attention deficit hyperactivity disorder, epilepsy, traumatic brain injury, schizophrenia, and depression. Neuroscience research related to individual differences in language and reading and giftedness, including synesthesia, is also discussed. We conclude by raising considerations and limitations for interpreting neuroscience research on mathematical difficulties and exceptionalities, including small sample sizes, group assignment, inferences from lesion and neuroimaging studies, and the disease model.

Mathematical Difficulties

We encounter numbers on a daily basis and in a number of formats, including Arabic (2), number words (two), Roman numerals (II), time (2:00 p.m.), finger signs, and words with numeric meaning (duo). Individuals with *dyscalculia* have difficulty mastering these numerical understandings and show deficits in counting skills, magnitude processing, arithmetic, using digits and quantities, and spatial representations as well as difficulties in domain-general skills such as working memory and

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attention (Kucian & von Aster, 2015). Kaufmann et al. (2013) suggested that dyscalculia subtypes arise from both domain-general and domain-specific processing deficits. From a developmental perspective, von Aster and Shalev (2007) proposed that any of the four steps of number acquisition may be negatively affected in individuals with dyscalculia, including (1) innate number representation, (2) associations between nonverbal characteristics and linguistic symbolization, (3) Arabic notation and value, and (4) exact number line with ordinal number positions. Research on neural correlates of dyscalculia report reduced brain activation in core areas of number processing and increased activation in frontal regions (Ashkenazi, Rosenberg-Lee, Tenison, & Menon, 2012; Kucian & von Aster, 2015; Kuhn, 2015; Stark, Eve, & Murphy, 2016). In other words, individuals with dyscalculia do not activate the same brain regions as their typically developing (TD) peers. This finding of a different developmental trajectory suggests compensatory networks for number processing (Kucian & von Aster, 2015; Stark et al., 2016).

Mathematical difficulties (MD) are typically characterized by impairments in arithmetic problem solving (Karagiannakis, Baccaglini-Frank, & Papadatos, 2014). That is, individuals with mathematical difficulties have impairments in cognitive processes that are specifically related to mathematical understanding. To distinguish mathematical difficulties from general learning difficulties and low mathematics achievement, researchers search for indicators of numerical cognition, arithmetic reading, working memory, numerical processing speed and accuracy, and visuospatial processing (Tolar, Fuchs, Fletcher, Fuchs, & Hamlett, 2016). Although more general cognitive processes such as working memory and intelligence (IQ) are associated with mathematical difficulties, the literature uses indicators of number-specific cognitive processes more often than comparisons between IQ and achievement to identify mathematical difficulties (Bartelet, Ansari, Vaessen, & Blomert, 2014; Tolar et al., 2016). For example, Tolar et al. (2016) evaluated mathematical problem solving among 813 third-grade students and used percentiles among the sample to classify students into learning disabled (LD), low achievement (LA), and no learning difficulties groups. They found significant differences between the groups in achievement, cognitive processing, and attention. Furthermore, Tolar et al. (2016) found that when groups were defined by mathematical problem solving achievement, they differed in basic arithmetic skills, but when groups were defined by IQ discrepancy, they differed in word problem solving. These findings indicate a need to specify types of learning difficulties in order to meet the needs of individual students. In the literature, four subtypes of mathematical learning difficulties have been identified: number sense, procedural, semantic, and spatial (Bartelet et al., 2014; Geary, 2010; Jordan, Hanich, & Kaplan, 2003; Karagiannakis et al., 2014; Mazzocco & Myers, 2003; Raghubar et al., 2009). For example, Geary (2010) described a *number sense subtype* that involves difficulties with exact, small quantity and approximate representational systems; a *procedural subtype* that involves working memory difficulties during counting; a *semantic subtype* that involves fact retrieval deficits; and a *visuospatial subtype* that involves difficulties in aligning numerals in multi-digit calculations.

Mathematical learning disabilities (MLD) represent a discrepancy between mathematical performance and expected performance based on age, intelligence, and educational level (Geary, 2011). Individuals with MLD are delayed in the development of procedural skills and have difficulty storing and retrieving arithmetic facts while those classified as having low mathematics achievement demonstrate average intelligence and subtle deficits in attention (Geary, 2011, 2013). Therefore, tests of intelligence may help identify whether individuals have MLD or low mathematics achievement. To document growth trajectories of children with MLD and low mathematics achievement, Geary, Hoard, Nugent, and Bailey (2012) tracked and compared number processing, fact retrieval, and arithmetic competency among children from kindergarten through fifth grade. Results are shown in Fig. 6.1. Although no significant achievement score differences were found between children with MLD and low achievement, children with MLD demonstrated a developmental delay of 1 year for simple problems and 1–2 years for more complex problems and experienced more difficulties retrieving facts from long-term memory. Difficulty in fact retrieval may be due to the inability of children with MLD to inhibit irrelevant information in working memory during retrieval or lower executive attention (Geary, 2011; Geary, Hoard, & Bailey, 2012; Kuhn, 2015).

To identify neurological underpinnings of cognitive deficits, many studies investigating MLD use fMRI to study brain activation during various mathematical tasks. Such research indicates that children with MLD demonstrate significantly greater activation in domain-general brain regions, which may be due to greater cognitive demands and reliance on developmentally immature methods of problem solving (Davis et al., 2009; Geary, 2010). According to Davis et al. (2009), abnormal brain activation may lead to reallocation of cognitive resources to other areas of the brain in order to maintain performance. In the following section, we discuss various cognitive processing difficulties associated with mathematical difficulties in greater detail.

Cognitive Processing Difficulties

Approximate Number System or Number Sense On number line estimation tasks, children with MLD use less mature strategies and make more estimation errors, suggesting deficits in mental representation of numerical magnitude (van't Noordende, van Hoogmoed, Schot, & Kroesbergen, 2016), or the *quantity system* of the triple-code model (see Chap. 1). However, research also indicates that individuals with MLD have difficulties with accessing magnitude representations on symbolic number comparison tasks (Andersson & Östergren, 2012; de Smedt & Gilmore, 2011; Defever, de Smedt, & Reynvoet, 2013). For example, children with MLD have greater difficulty comparing two numbers that are presented in two different formats (the Arabic numeral 2, five dots), but do not have difficulty when comparing two numbers presented in the same format (three dots, six dots). Research on brain activation related to symbolic number processing demonstrates that children with dyscalculia show differential brain activation in the intraparietal sulcus (IPS), bilaterally, during number comparison than children without dyscalculia (Mussolin et al., 2010).

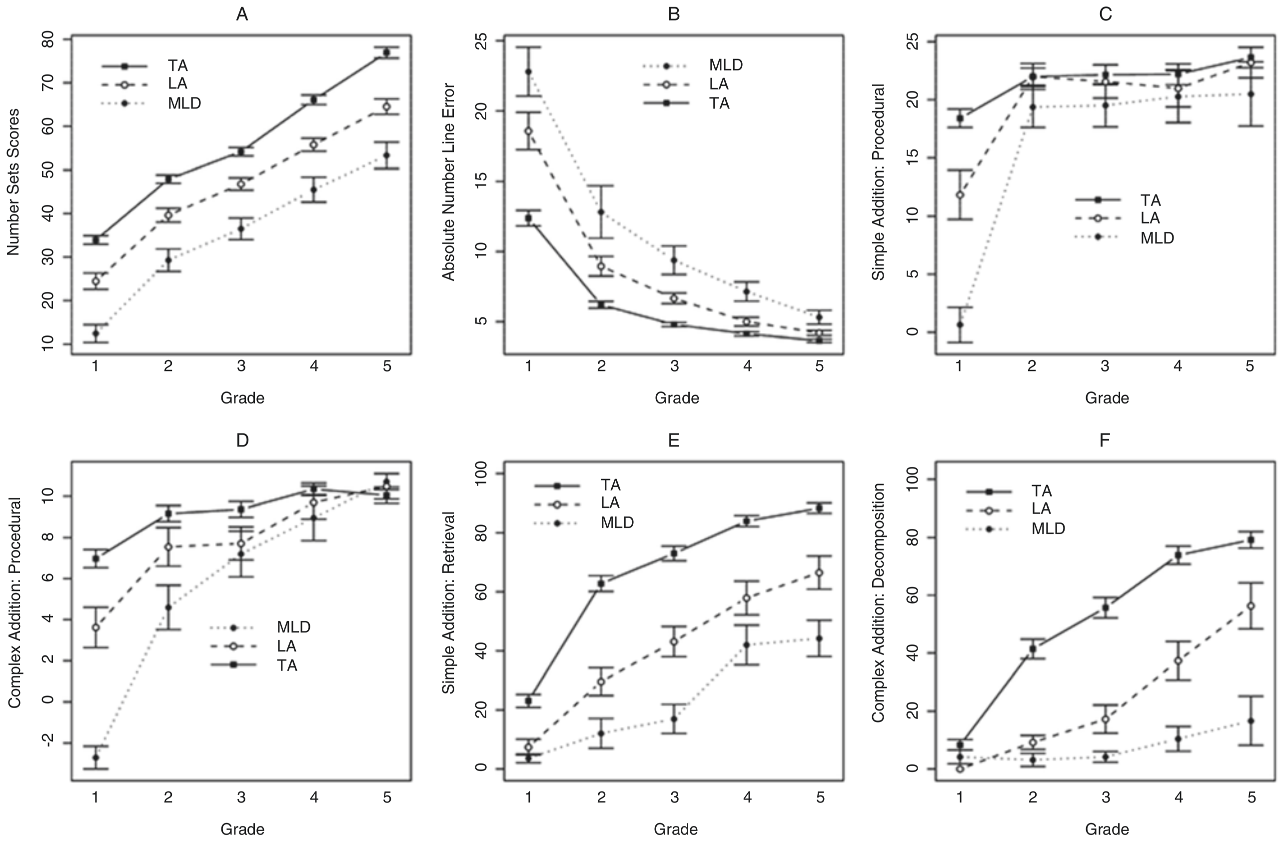

Fig. 6.1 Number sets scores (Panel **a**), absolute errors for number line (Panel **b**), procedural competence scores for simple (Panel **c**) and complex (Panel **d**) addition, percentage of simple addition problems correctly solved using retrieval or decomposition (Panel **e**), and percentage of complex addition problems correctly solved using decomposition (Panel **f**). *MLD* mathematical learning disability, *LA* low achieving, *TA* typically achieving. Source: Geary, D. C., Hoard, M. K., Nugent, L., & Bailey, D. H. (2012). Mathematical cognition deficits in children with learning disabilities and persistent low achievement: A five-year prospective study. *Journal of Educational Psychology, 104*(1), 206–223. https://doi.org/10.1037/a0025398

Fact Retrieval Mastery of arithmetic facts and speed of fact retrieval, corresponding to the *verbal system* of the triple-code model, are impaired among those with MLD. Poor fact mastery and retrieval are associated with developmental delays, little developmental growth over time, and immature calculation strategies (Baroody, Bajwa, & Eiland, 2009; Jordan et al., 2003). Abnormal activation in the prefrontal cortex among children with MLD may indicate differential neural pathways for information retrieval. In their study of children with MLD and low mathematics achievement, Geary, Hoard, Nugent, and Bailey (2012) found that children with MLD had the lowest verbal IQ and achievement scores, relied most heavily on finger and verbal counting to solve mathematical problems, had pervasive working memory deficits, and took longer to respond to problems. Individual differences in each area of brain activation may differentiate individual subtypes of MLD and lead to different developmental and information retrieval patterns.

Delayed Procedural Development Procedural errors during problem solving may represent developmental delays in mathematical processing. Errors in mathematical facts (i.e., *verbal system* of the triple-code model) and visual monitoring (*visual system* of the triple-code model) are most characteristic of children with MLD (Raghubar et al., 2009). To study procedural development in adolescents, Rosenzweig, Krawec, and Montague (2011) used a think-aloud method where students verbally reported their strategies for solving mathematical problems of varying complexity. They found those with MLD used more task-unrelated verbalizations on the most complex problems, suggesting potential difficulty in analyzing complex procedures or translating mental procedures into verbalizations.

Fractions and Proportional Reasoning Hecht and Vagi (2010) found that individuals with MLD showed significantly fewer gains in fraction computation and word problem solving over time, indicating potential difficulties in both conceptual knowledge and attention. These findings support the literature on slower growth trajectories and attentional deficits in MLD.

Visuospatial Representation Solve the following word problem:

> Carol is wrapping gifts and tying them with ribbon. Each gift requires 3 feet of ribbon and she can only wrap 1 gift at a time. If Carol has 28 feet of ribbon, how many presents can she tie with ribbon?

When solving the problem, did you picture the problem in your mind? If so, did you visualize pictorial representations of the objects, or did you visualize a schematic or diagram of the problem? Different types of visualizations can be used to help solve mathematical problems, although varying definitions of imagery can contribute to different findings on the relationship between visuospatial representation and mathematics achievement. Van Garderen and Montague (2003) examined this relationship among students who were either classified as gifted, TD, or having MLD. They found that gifted students used more visualizations overall and primarily used schematic representations, TD students used both schematic and pictorial representations, and students with MLD used the fewest visualizations and favored pictorial representations. Furthermore, van Garderen and Montague (2003) noted that schematic representations were more often associated with correct responses, while pictorial representations were typically associated with incorrect responses. Difficulties with visuospatial information and representation among individuals with MLD may relate to their inability to inhibit irrelevant information from working memory by means of including irrelevant information in their visualizations.

Working Memory Research suggests that working memory includes the *visuospatial sketchpad* for number line representation, the *phonological loop* for maintaining arithmetic results, and the *central executive* for sequencing complex arithmetical procedures (Ashkenazi, Rosenberg-Lee, Metcalfe, Swigart, & Menon, 2013; Menon, 2016; Meyer, Salimpoor, Wu, Geary, & Menon, 2010). To examine each of these components and their relation to brain activation, Ashkenazi et al. (2013) studied the performance of children on a block recall task. They found the central executive and phonological working memory functions were similar between TD individuals and individuals with MLD, but visuospatial working memory was significantly lower and brain activations demonstrated differential use in those with MLD. These findings are consistent with other research on visuospatial working memory in which reduced brain activity was observed in children with MLD (Menon, 2016; Rotzer et al., 2009).

Time Estimation Accurate time perception allows us to predict, anticipate, and respond to daily situations, such as estimating how long we need to get ready in the morning or guessing the current time without looking at a clock. Time perception consists of three components: clock or regular interval pulses, memory for duration and number of pulses, and comparison between current and previously remembered duration (Hurks & van Loosbroek, 2014). To compare time perception between children with TD and those with MLD, Hurks and van Loosbroek (2014) measured verbal time estimation, time production, and time reproduction. They found that children with MLD overestimated duration of a time interval and underproduced a reproduction of a time sample. Thus, difficulties in time perception may be due to the translation from experienced duration to a verbal statement.

Multiple Difficulties So far, we have discussed how individuals with MLD are alike. However, even within this group, individual differences in strategies, working memory, visuospatial representation, and other cognitive processes contribute to differing developmental trajectories. Research suggests that individual differences in fact retrieval strategies may be related to delayed development (Geary, Hoard, Byrd-Craven, & DeSoto, 2004) and impaired verbal retrieval (Berteletti, Prado, & Booth, 2014) may prevent the shift from counting to more complex retrieval strategies. Kaufmann, Wood, Rubinsten, and Henik (2011) conducted a meta-analysis of 19 studies to examine individual differences between symbolic and nonsymbolic representation, number magnitude processing, and neural correlates of calculation in children. They found that children activate a broad network of brain regions when performing calculation and symbolic and nonsymbolic representation activated different parietal regions. Additionally, children with dyscalculia had less robust number-activations in the intraparietal sulcus (IPS), indicating that they used compensatory neural pathways or activated inefficient frontal brain regions during calculation tasks. Therefore, any combination of cognitive processing deficits may contribute to MLD and be represented by differential brain activation.

Limited Opportunities Apart from cognitive abilities, environmental opportunities for learning and social experiences may contribute to learning success and difficulty. Take a moment to think about the environmental influences on your learning during childhood. Did you have many same-age peers or siblings? What kinds of toys and activities did you have to engage you physically and mentally? Did you attend a preschool or daycare center? These and other factors can contribute to differences in early mathematical learning. Jordan and Levine (2009) reported that poor early learning experiences and instruction led children entering school to rely on counting strategies longer, have poor calculation fluency, and difficulties in reading and language skill acquisition. Children with low SES show delays in number competence and little growth during early school years if they begin at a low level (Jordan & Levine, 2009). With regard to such environmental factors, positive early learning experiences prior to formal education are essential for successful and developmentally appropriate learning.

Exceptionalities

Neurodevelopmental Disorders, Syndromes, and Conditions

Mathematical Learning Difficulties may be related to neurodevelopmental disorders, syndromes, and conditions. Neurodevelopmental disorders have a basis in biology and are thought to directly influence brain structure and function, which in turn affects cognition and mathematical learning and performance. Children with 22q11.2 gene deletion syndrome demonstrate difficulties in procedural calculation, mathematical word problem solving, and understanding numerical quantities (Dennis, Berch, & Mazzocco, 2009). Similarly, girls with Turner syndrome are slower at number comparison, processing, and numerical estimation, while girls with fragile X syndrome show weaker numerical understanding. In other developmental disorders, profiles of difficulties are less clear, as greater individual differences exist. According to Dennis et al. (2009), trajectories of MLD include early mathematical impairments that predict later difficulties, which may span a lifetime. Regarding results from neuroimaging studies, some findings appear to be common in those with MLD, including reduced white matter and parietal lobe activation (Dennis et al., 2009). Again, it is important to note that mathematical abilities are associated with a complex developmental system of biological, genetic, and environmental factors. In this section, we will discuss some of the research on neurodevelopmental disorders, syndromes, and conditions, and their relation to MLD.

Autism Spectrum Disorder *Autism spectrum disorder (ASD)* is an early onset neurodevelopmental disorder that primarily affects social communication and interaction (for a review, see Pelphrey, Yang, & McPartland, 2014). Some people believe that individuals with ASD are more likely to show mathematical giftedness. However, conclusions from Chiang and Lin's (2007) meta-analysis suggest that the full range of mathematical ability is represented in the ASD population, with a subset of those with ASD experiencing significant mathematical difficulties. These difficulties may involve early-developing skills that require perceiving and representing multiple objects simultaneously, such as numerosity and subitizing discussed in Chap. 3. For example, O'Hearn et al. (2016) found that, while engaging in an enumeration task, adults with autism showed greater activation, used more regions with smaller set sizes, and showed a lack of deactivation for competing processes compared to TD adults. Oswald et al. (2016) found that adolescents with ASD were five times more likely to have mathematical difficulties than to be mathematically gifted, with perceptual reasoning accounting for much of the differences in mathematical performance between individuals with ASD and their TD peers. Regarding neural functioning, Damarla et al. (2010) found that, although their behavioral performance on the Embedded Figures Task was similar to controls, a sample of adolescents and young adults with ASD showed less activation in dorsolateral frontal areas and more activation in bilateral parietal and right occipital areas, possibly indicating reliance on visuospatial strategies. Furthermore, they showed lower functional connectivity between frontal areas involved in working memory and executive function and posterior (parietal and occipital) areas involved in visuospatial processing, which was positively correlated with size of the corpus callosum. Yamada et al. (2012) found similar results using the Raven's Standard Progressive Matrices test. However, Iuculano et al. (2014) found that school-age children with ASD used more sophisticated decomposition strategies and solved numerical problems better than TD peers. Interestingly, they found that activation in ventral temporal-occipital areas typically associated with face processing predicted problem solving abilities in children with ASD, but not their TD peers. They concluded that individuals with ASD use cortical regions involved in perceptual expertise in novel ways.

Fragile X and Turner Syndromes *Fragile X* is caused by a disruption in the production of fragile X mental retardation protein (FMRP), leading to less than optimal neural development as well as deficits in visual perception skills, working memory, and executive function (Murphy, 2009). More is known about MLD in girls with fragile X syndrome, as this population is more often studied in the literature. According to Rivera, Menon, White, Glaser, and Reiss (2002), females with fragile X have acalculia, or inability to complete simple arithmetic problems, as well as patterns of weakness in arithmetic reasoning and digit span. Individuals with fragile X show no differences in brain activation in the bilateral prefrontal cortex, motor, or parietal cortices between easy and difficult tasks, which may indicate deficiencies in number sense and rational number knowledge. *Turner syndrome* results from partial or complete loss of an X chromosome and has cognitive phenotypes similar to fragile X, including weaknesses in visuospatial perception, mathematics, sustained attention, and executive function (Mazzocco & Hanich, 2010).

22q11.2 Genetic Deletion and Williams Syndrome Chromosome *22q11.2 deletion*, or velo-cardio-facial syndrome, and also called DiGeorge syndrome, is caused by deletion on chromosome 22. Individuals with 22q11.2 deletion syndrome often have specific facial features, such as a long face; flat cheeks; wide-set eyes with hooded eyelids; small, low-set ears with squared upper ears; a narrow nose; a small, downward mouth; cleft lip or palate; and an underdeveloped chin. 22q11.2 deletion syndrome is associated with abnormalities in posterior and frontoparietal neural networks (Brankaer, Ghesquière, De Wel, Swillen, & De Smedt, 2016). Regarding cognitive phenotype, children with 22q11.2 deletion syndrome exhibit deficits in visuospatial working memory and executive control as well as domain-specific impairments in numerical representations and slower execution of calculations and word problem solving (de Smedt et al., 2007; De Smedt, Swillen, Verschaffel, & Ghesquière, 2009). de Smedt et al. (2009) found that individuals with 22q11.2 deletion syndrome were significantly slower than their TD peers on number comparison, strategy use, and problem solving with larger quantities. Deficits in visuospatial processes are also found in individuals with *Williams syndrome*, which is associated with atypical brain activation patterns in the parietal lobe and decreased grey matter (O'Hearn & Luna, 2009). Despite impairments to visuospatial processes, visual abilities such as perception of motion and face and object recognition remain intact among those with Williams syndrome. O'Hearn and Luna (2009) also found impairments in number magnitude representation and approximate numbers, but strong memory for mathematical facts and exact number representation.

Spina Bifida *Spina bifida myelomeningocele* is a neurological disorder that affects development of the spine and brain and is accompanied by difficulties with arithmetic, estimation, and word problem solving (English, Barnes, Taylor, & Landry, 2009). In a longitudinal study, Barnes et al. (2014) found that children with spina bifida performed worse than TD peers on mathematical calculations, mathematical and reading fluency, and passage comprehension. Despite demonstrating strong verbal and word reading skills, individuals with spina bifida show impairments specific to mathematics learning.

Prenatal Alcohol Exposure and Premature Birth Individuals with *prenatal alcohol exposure* present with neurocognitive deficits in visuospatial processing, attention, and working memory as well as mathematical processing, particularly among physically affected (dysmorphic) individuals (Santhanam, Li, Hu, Lynch, & Coles, 2009). Research on *premature birth* (less than 32 weeks) focuses on brain structure and function such that reduced grey matter in premature brains negatively correlates with reaction times of numerical processing (Starke et al., 2013). Children born prematurely tend to show reduced brain size and function, which relates to difficulties in classifying numbers. Similar results in reduced brain morphology were found by Klein et al. (2014). Children born prematurely demonstrated greater activation of the inferior frontal gyrus during a number comparison task, which was linked to lower estimated IQ. Taken together, research suggests that premature birth is associated with abnormal brain morphology and function, which, in turn, affects numerical processing and intelligence.

Developmental Coordination Disorder *Developmental coordination disorder (DCD)* is characterized by impairment in motor coordination and has been connected to mathematical learning difficulties (Pieters, Roeyers, Rosseel, Van Waelvelde, & Desoete, 2015). One characteristic of mathematical difficulties that appears in individuals with DCD is developmental delay, in which children demonstrate skills equal to younger peers based on age and educational level. For example, Pieters, Desoete, Van Waelvelde, Vanderswalmen, and Roeyers (2012) examined differences in fact retrieval and procedural calculation between children with mild and severe DCD. Overall, difficulties in both fact retrieval and procedural calculation were reported for individuals with DCD and those with mild DCD had academic profiles similar to children 1 year younger, while those with severe DCD had academic profiles similar to children 2 years younger.

Attention Deficit/Hyperactivity Disorder While reading a long journal article or book chapter such as this, it is normal for attention to wander or to have small, regular breaks, so go stretch for a minute before coming back to your reading. Were you able to jump back into the reading fairly easily? Can you remember the information you previously read? As you are reading, do you experience significant difficulties in avoiding physical or mental distractions? Individuals with *attention deficit/hyperactivity disorder (ADHD)* have difficulty sustaining attention and retaining information in working memory and would likely be unable to sustain attention to this reading for long. Previous research on the neural correlates of ADHD consistently demonstrates weaker activation in attention-related areas, which may represent lack of arousal or attention during simple tasks (Lenartowicz et al., 2014; van Ewijk et al., 2015). To capture attention and incentivize individuals with ADHD, research advocates the use of greater external rewards and feedback. Hammer et al. (2015) measured the effects of rewards and feedback on working memory and brain activation in individuals with ADHD, observing greatest performance when individuals with ADHD received high rewards and feedback, during which brain activation equaled that of TD peers. Therefore, offering greater external rewards and feedback may be an avenue to help children with ADHD sustain attention and inhibit distracting behaviors and thoughts.

Epilepsy and Traumatic Brain Injury *Epilepsy* is a chronic disease characterized by seizures caused by abnormal neuronal discharges (Lv et al., 2014). Seizures may lead to cognitive impairments in memory, attention, language, and executive function, depending on where the neuronal discharges are located. For individuals with epilepsy, brain plasticity may allow secondary, compensatory pathways to form in order to maintain cognitive functions. Lv et al. (2014) examined cognitive impairments due to epilepsy and found that those with right temporal lobe epilepsy demonstrated lower functional connectivity in regions of the right prefrontal lobe, but greater functional connectivity in adjacent regions, indicating compensatory pathways to allow continual functioning of visuospatial working memory. The result of physical injury rather than seizures, *traumatic brain injury (TBI)* may lead to similar deficits in information processing speed, visuospatial processing, memory, and executive function. Specific to visuospatial processing, research suggests damage to the left hemisphere results in local or detail-oriented errors, while damage to the right hemisphere results in global or big-picture errors; thus, as discussed in Chap. 5, visuospatial processing is lateralized (Schatz, Ballantyne, & Trauner, 2000). Overall, mathematical-related difficulties for individuals with TBI compared to those with TD include lower performance on math fluency, calculation, applied problems, and verbal working memory (Raghubar, Barnes, Prasad, Johnson, & Ewing-Cobbs, 2013).

Schizophrenia and Depression *Schizophrenia* involves breakdown of thought, emotion, and behavior, leading to faulty perceptions and mental fragmentation. *Depression* is characterized by persistent depressed mood, withdrawal from relationships, and loss of interest in activities. While using fMRI to measure brain activation during mental arithmetic among individuals with schizophrenia and depression, Hugdahl et al. (2004) detected significantly lower response accuracy and slower reaction times. Furthermore, individuals with schizophrenia activated parietal lobe regions and showed deficits in the left hemisphere, while individuals with depression activated frontal lobe regions and showed deficits in the right hemisphere. Although dysthymic disorder is characterized by sad and irritable mood and is only associated with depressive symptoms, individuals with dysthymic disorder demonstrate patterns of neurological activation and mathematical learning difficulties similar to those with depression. Indeed, children with dysthymic disorder displayed abnormal brain activity in regions of the left prefrontal cortex, which are associated with working memory and are impaired in individuals with depression (Vilgis, Chen, Silk, Cunnington, & Vance, 2014).

Individual Differences in Language and Reading

Deaf or Hard of Hearing Earlier in this chapter, we discussed how acquisition of linguistic skills is important for numerical processing and how innate abilities for processing small quantities provide the foundation for secondary mathematical abilities, which rely on language skills and formal schooling. Consider your own learning of numbers—did you speak numbers as you learned them or repeat what you heard about quantities and arithmetic formulas? Children who are deaf or hard of hearing are not afforded this luxury and research indicates they struggle to learn the fundamentals required to achieve good mathematics performance and have difficulties in symbolic number processing (Rodríguez-Santos, Calleja, García-Orza, Iza, & Damas, 2014). In their investigation of nonsymbolic and symbolic number representation task performance among children, Rodríguez-Santos et al. (2014) observed equal task performance between TD and deaf or hard of hearing children, but the later were significantly slower at accessing quantity information from symbolic representations. Further research may help determine whether slower reactions and symbolic processing speed are more related to differential neural pathways or developmental delay.

Multilingualism Multilingualism has obvious linguistic benefits, but it is unclear whether cognitive costs exist with multilingual learning, particularly when the language used during instruction differs from the language used during knowledge retrieval (Saalbach, Eckstein, Andri, Hobi, & Grabner, 2013). *Cognitive costs* refer to the disadvantages or abnormal brain functions that occur in information storage and retrieval due to the extra cognitive resources dedicated to becoming proficient in multiple languages. Research on cognitive costs of multilingualism indicate that gained knowledge is tied to the language of learning while switching between languages lowers accuracy and speed in addition to utilizing differential neural networks for solving problems (Grabner, Saalbach, & Eckstein, 2012; Saalbach et al., 2013). Mondt et al. (2011) used fMRI to examine whether language of instruction influenced calculation skills and found that children who did not complete a mathematical task in the same language they used to learn the information activated more diffuse areas, suggesting that multilingual children who use different languages for information learning and retrieval use secondary, and potentially weaker, neural pathways for information recall.

Reading Disabilities and Dyslexia *Dyslexia* is primarily a language-based disorder in which individuals exhibit difficulties with phonological awareness and processing and demonstrate deficits in translating symbolic representations and information recall (Olulade, Gilger, Talavage, Hynd, & McAteer, 2012). Individuals with dyslexia may have comorbid difficulties with mathematical learning or *dyscalculia*. Research on such comorbidity reveals significant impairments on fact retrieval and exact mental calculation and arithmetical fact retrieval, although significantly slower reading speed accounts for some variance between individuals with TD and those with dyslexia and dyscalculia (Mammarella et al., 2013). Functional differences in nonverbal areas of the brain during information processing were examined by Olulade et al. (2012) who found that individuals with reading difficulties demonstrated less activity in the left frontal and occipital-temporal areas during a reading task. Thus, individuals with dyslexia may exhibit deficits in reading speed and word processing as demonstrated by abnormal brain activation during reading, and, when comorbid with MLD, demonstrate impairments in poor mental calculation and arithmetic fact retrieval.

Giftedness

Up to this point, we have explored conditions that lead to mathematical difficulties and low achievement. Now, we turn to the opposite end of the performance spectrum to consider neural correlates of *mathematical giftedness*, which is characterized by exceptional mathematical abilities that emerge early in life without formal training. Prescott, Gavrilescu, Cunnington, O'Boyle, and Egan (2010) investigated whether mathematical giftedness is associated with enhanced functional connectivity during complex spatial tasks. They found that mathematically gifted individuals showed greater *intra*hemispheric frontoparietal connectivity, which is involved in complex spatial analysis, and greater *inter*hemispheric dorsolateral prefrontal and premotor connectivity, which indicates better integration of inputs from more areas of the brain without additional processing costs. Consistent with these findings, Fehr, Weber, Willmes, and Herrmann (2010) reported the case of mathematical prodigy CP who engaged in daily training in mental calculations and exhibited faster processing speed and a larger network that included greater activation in frontal and posterior brain regions as well as typical patterns of activation in the middle frontal and inferior parietal regions.

When you see a string of numbers or are given certain mathematical problems or formulas, do you automatically associate them with particular sights, shapes, sounds, or smells? For some individuals, every number has its own color, emotion, and personality. For example, when DT thinks of the number 1, it is a flash of white light and when he thinks of the number 6, he sees it as a black hole, a place to climb into, and a retreat from the world. This condition, in which stimulation in one sensory or cognitive stream involuntarily leads to experiences in other sensory or cognitive systems, is called *synesthesia*. Baron-Cohen and colleagues (2007; Bor, Billington, & Baron-Cohen, 2007) have published single case studies of savant DT, who not only has a form of synesthesia that creates complex 3D mental landscapes when he is stimulated by a stream of numbers, but also has Asperger syndrome, which is considered an ASD. As noted by Baron-Cohen and colleagues, this combination seems to have endowed DT with exceptional numerical memory and calculation abilities. For example, DT knows the value of Pi up to 22,514 decimal places and he can rapidly perform mental calculations. To better understand DT's abilities, Bor et al. (2007) used fMRI to compare his neural activity to TD controls without synesthesia or ASD. During a digit span task, DT showed hyperactivity in bilateral prefrontal cortex during encoding in comparison to controls, indicating that his processing of number sequences is different. The authors speculated that TD's synesthesia generates highly chunked representations of numbers that enhance encoding of digits and facilitate recall and calculation, which is supported by their findings that DT's neural activity and performance did not differ for structured and unstructured digit sequences, whereas controls showed higher bilateral prefrontal activation and a performance advantage for structured sequences. DT's more conceptual form of synesthesia in combination with his focus on local details associated with Asperger syndrome may explain his exceptional abilities. Brogaard, Vanni, and Silvanto (2013) provided a different case study of JP who has an exceptional ability to draw complex geometric images by hand and reports perceiving mathematical formulas and objects as geometric images. They found that JP demonstrated greater activation in the left temporal, parietal, and frontal lobes for formulas that induced geometric images in comparison to those that did not, suggesting that his synesthetic experiences are based on particular concepts, rather than general perceptions.

Table 6.1 displays each of the specific mathematical learning exceptionalities discussed in the chapter, along with accompanying cognitive processing phenotypes and neurological correlates.

Table 6.1 Neurodevelopmental conditions, associated cognitive processing difficulties, and neural correlates

Neurodevelopmental condition	Associated cognitive processing difficulties	Neural correlates
Autism spectrum disorder	• Low mathematics achievement • Slow growth rates	• Less activation in left frontal areas • Greater activation in IPS, visuospatial areas
Fragile X	• Visual perception, working memory, executive function • Arithmetic reasoning, digit span, acalculia (number sense)	• Greater activation in bilateral prefrontal cortex, motor, parietal cortices
Turner syndrome	• Sustained attention, visuospatial perception, executive function • Symbolic numerical magnitude processing	
22q11.2 deletion (velo-cardio-facial syndrome)	• Visuospatial perception, executive function • Numerical representation, slower calculation, and word problem solving	• Abnormal activation in posterior and fronto-parietal networks
Williams syndrome	• Visuospatial representation, numerical magnitude processing, approximate number systems	• Abnormal parietal lobe activation, decreased grey matter
Spina Bifida	• Arithmetic estimation, calculation, word problem solving	
Prenatal alcohol exposure	• Visuospatial representation, attention, working memory • Math processing	
Premature birth	• Numerical processing and classification, low IQ	• Reduced brain size, reduced grey matter • Greater activation of inferior frontal gyrus
Developmental coordination disorder	• Developmental delay, procedural calculation, math fact retrieval	
Attention deficit/ hyperactivity disorder (ADHD)	• Sustained attention, working memory, information retention	• Low activation in attention-related areas
Epilepsy	• Visuospatial working memory, attention, language, executive function	• Less functional connectivity in right prefrontal lobe • Greater functional connectivity in adjacent and other areas
Traumatic brain injury	• Individual variation (e.g., information processing speed, memory, executive function)	• Depends on location of injury and cognitive processes affected
Schizophrenia	• Low math performance, slow reaction times	• Parietal lobe activation
Depression	• Low math performance, slow reaction times, working memory	• Frontal lobe activation • Abnormal activation in left PFC

(continued)

Table 6.1 (continued)

Neurodevelopmental condition	Associated cognitive processing difficulties	Neural correlates
Deaf or hard of hearing	• Symbolic number processing • Developmental delay	
Multilingualism	• Low accuracy and speed (language switching)	• Abnormal activation of diffuse areas
Reading disabilities and dyslexia	• Exact mental calculation, math fact retrieval, slow reading speed, slow word processing	• Low activation in left frontal and occipital-temporal areas during reading tasks
Giftedness	• Individual variation (e.g., exceptional math reasoning skills, mental calculation) • Individual variation in cognitive costs	• Greater intrahemispheric frontoparietal activation • Greater interhemispheric dorsolateral prefrontal and premotor connectivity, • Larger general network activation
Synesthesia	• Individual variation (e.g., exceptional number processing, numerical memory, math calculation)	• Depends on cognitive processes affected

Note. The table is not exhaustive in listing neurodevelopmental conditions, associated cognitive processing, or neural correlates, only summarizing findings reported in this chapter

Considerations and Limitations

The human brain is highly plastic. Neural pathways have the ability to change in response to different experiences. For example, an individual with TBI to the right prefrontal lobe may activate adjacent prefrontal regions and other areas in order to maintain cognitive functions. Delazer, Benke, Trieb, Schocke, and Ischebeck (2006) examined the case of HR, whose reduction of cortical perfusion in the superior and posterior parietal lobe and posterior temporal lobe resulted in unaffected verbal intelligence, but at the cost of visuospatial processing, numerical approximation and estimation skills, dot counting, and positioning on a number scale. HR displayed normal language area activation during mathematical tasks, but activated inferior temporal structures during a multiplication task, which may be indicative of the brain requiring additional effort or using compensatory pathways to complete the calculation task (Delazer et al., 2006). Previous research on compensatory pathways reveals how brain plasticity plays a role in creating secondary neural networks to sustain cognitive function among individuals with dyscalculia (Kaufmann et al., 2011; Kucian & von Aster, 2015), MLD (Davis et al., 2009; Geary, 2010), ADHD (Hammer et al., 2015), and TBI (Lv et al., 2014).

Whether research is conducted with case studies of cognitive factors or neuroimaging studies using fMRI, a major limitation across research on MLDs is the use of small samples (Desoete, Praet, Titeca, & Ceulemans, 2013; Kucian et al., 2014; Raghubar et al., 2013; Rosenzweig et al., 2011; Vilgis et al., 2014), which typically limits power and effect sizes of significant findings, thereby limiting generalizability and reliability of the research. Samples themselves may be limited due to age or group assignment. Research with homogenous age groups assumes that profiles of strengths and weaknesses of numerical and mathematical learning difficulties are stable across development, such that adult data can be used to understand children with similar learning difficulties, and vice versa (Ansari, 2010; Ansari & Karmiloff-Smith, 2002). Regardless of age, participants are often divided into TD and MLD groups based on performance during measures of intelligence or mathematical ability. However, groups can vary depending on what measures of performance and cutoff scores are used (Kaufmann et al., 2011; Mazzocco & Myers, 2003; Mondt et al., 2011; Passolunghi & Mammarella, 2010), which further limits generalizability and reliability as individuals near cutoff scores are essentially interchangeable between groups, and individuals with exceptional mathematical abilities (e.g., gifted) are placed into TD groups. Related to group assignment, not all research uses disorder subtypes to further separate groups by individual differences, such as dysthymic disorder primarily sad or irritable type (Vilgis et al., 2014), lifelong versus learned bilingualism (Saalbach et al., 2013), and ADHD primarily attentive or hyperactive/impulsive type (Hammer et al., 2015). Together, these limitations suggest individual differences within groups are not adequately addressed in the research.

Additional limitations relating to generalizability are evident in lesion and neuroimaging studies. Brain activation among groups of participants with the same learning and mathematical deficits may be reliable if samples are homogenous. However, samples of individuals with neurodevelopmental disorders are not always checked to account for medication use (van Ewijk et al., 2015) or intervention (Raghubar et al., 2013), which can drastically affect brain activation and presence of learning deficits depending on the type and duration of medication or intervention. Diagnosis and treatment of individuals with learning difficulties can rely on research that does not directly study individuals with learning deficits. For example, research used to diagnose deficits in developmental dyscalculia is derived from studies of individuals with brain lesions, which assumes the profile of deficits are the same between dyscalculia and brain lesion patients (Ansari & Karmiloff-Smith, 2002). In line with research on individual differences among those with learning difficulties, a limitation in neuroimaging research is the uncertainty of associations between brain activation and cognitive deficits. Researchers are unsure whether numeracy deficits characteristic of genetic disorders are more related to domain-general or domain-specific processes (Ansari & Karmiloff-Smith, 2002; Barnes et al., 2014). Further research on neuroimaging and brain lesion studies is needed to identify mathematical deficits and their accompanying patterns of brain activation.

Research on MLDs is limited due to the adoption of the “disease model” of abnormal learning by early medical researchers, which focuses on medical aspects of conditions and considers individuals with conditions such as reading difficulties to be “diseased” (Gilger & Hynd, 2008). The continued use of a disease model suggests negative genetic underpinnings of all conditions outside of the normal range. Therefore, less is known or discussed about the high end of the spectrum, such as mathematical giftedness. As research on mathematical difficulties often divides participant samples between TD and developmentally delayed or challenged, mathematical exceptionalities and gifted individuals are not often identified and these individuals become mixed with TD individuals.

Gilger and Hynd (2008) state that most research on learning difficulties that does not follow the disease model is performed by professionals typically focusing on educational issues, rather than underlying causes. What little is known about causes is often limited to neurological foundations of particular neurodevelopmental disorders or else replaced in favor of correlational studies whose associations with environmental factors and cognitive deficits do not imply causality. Research suggests the brain is the basis of behavior, while individual differences are the result of complex, integrated effects of genes and environment (Gilger & Hynd, 2008). Genetics play a role in risk for difficulties and giftedness, but are not isolated causes as they interact with environmental and cognitive factors at the individual and group level to create a highly complex profile of the individual. For example, two individuals with synesthesia can demonstrate exceptional skills in numerical processing, but have different experiences. As we saw with case studies of synesthesia, both JP and DT were diagnosed with synesthesia, yet JP's experiences created pictorial representations which he would draw by hand when presented with image-inducing formulas (Brogaard et al., 2013) while DT's experiences created detailed mental landscapes when presented with a stream of numbers (Bor et al., 2007). According to Gilger and Hynd (2008), giftedness and reading difficulties, as well as general learning difficulties, are seen as opposite ends of a learning continuum. This view of learning as a natural spectrum to which internal and external factors may influence reinforces the need for further research on learning differences and individual factors that affect them.

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Chapter 7
Enhancing Mathematical Cognitive Development Through Educational Interventions

Lori Kroeger and Rhonda Douglas Brown

Abstract In this chapter, we presents theory and research on mathematical interventions, which are programs used to provide supplementary opportunities to children who are struggling with learning mathematical concepts, facts, and procedures either in small groups or individually (Tiers 2 and 3 of the Response to Intervention framework). Fifteen mathematical interventions and curricula for students in Pre-Kindergarten through Post-Secondary levels are listed that meet the Institute of Education Sciences' What Works Clearinghouse's standards, with scientific evidence of potentially positive effects on mathematical outcomes. Three additional programs with digital components are highlighted: The Number Race; Fluency and Automaticity through Systematic Teaching with Technology (FASTT Math); and SRA Number Worlds® with Building Blocks®. For each of these programs, we provide an overview, describe the user's experience, and summarize theoretical frameworks and efficacy studies. Furthermore, we describe how components of the programs are related to neuroscience theory and research on mathematical cognition and development, particularly Dehaene and colleagues' triple-code model of numerical processing.

Mathematical interventions are broadly defined as programs that provide some level of supplemental support to children during the learning process. These programs are most frequently used to provide additional support to children who are struggling to learn mathematical concepts and procedures. Using the *Response to Intervention (RTI)* framework, a tiered model of instruction with continuous assessment and evaluation of student progress over an extended period of time, these interventions are most likely to involve a teacher working with targeted small groups of children (Tier 2) or one-on-one with a student (Tier 3) (for a review, see Lyon, Fletcher, Fuchs, & Chhabra, 2006). However, some mathematical interventions are used to provide additional opportunities to children who are advanced in learning math.

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Table 7.1 Mathematical intervention programs with evidence of positive effects listed by the What Works Clearinghouse

Intervention program or curriculum	Grades
Accelerated Math	2–8
Building Blocks® for Math (SRA Real Math)	Pre-Kindergarten
Cognitive Tutor® Algebra I	8–Post-Secondary
Core-Plus Mathematics	9–10
DreamBox Learning	Kindergarten–1
Everyday Mathematics®	3–5
I CAN Learn® Pre-Algebra and Algebra	8
Lindamood Phoneme Sequencing® (LiPS®)	1–4
Odyssey® Math	4–8
Pre-K Mathematics	Pre-Kindergarten
Saxon Math	1–8
Teach for America (TFA)	Kindergarten–12
The Expert Mathematician	8
University of Chicago School Mathematics Project (UCSMP) Algebra	8
University of Chicago School Mathematics Project (UCSMP) Multiple Courses	7–10

Note. Adapted from https://ies.ed.gov/ncee/wwc/FWW/Results?filters=,Math

As noted by Swanson (2008), RTI and neuroscience research on mathematical cognitive development are complementary approaches that can help explain and predict individual differences that emerge in children at risk for learning disabilities after exposure to validated instructional procedures.

How do teachers, intervention specialists, school psychologists, administrators, and parents know which intervention programs are effective, result in lasting changes in learning, and meet the specific needs of children in their learning environments? To inform decision making, the *Institute of Education Sciences (IES)* within the United States Department of Education established the *What Works Clearinghouse (WWC)* over 15 years ago to review research, determine studies with research methodologies that meet rigorous standards, and summarize scientific evidence on the effectiveness of educational programs, products, practices, and policies. The WWC's *Find What Works* online resource provides evidence snapshots, reports, including thorough descriptions, and a comparison tool for over 140 mathematical interventions. Currently, this resource lists 15 mathematics interventions and curricula for students in Pre-Kindergarten through Post-Secondary levels with scientific evidence of potentially *positive* effects on outcomes that meets the WWC's standards, presented in Table 7.1. In addition to these programs, there are many others with varying levels of scientific evidence (for a review, see Kroeger, Brown, & O'Brien, 2012). In the following section, several intervention programs that are consistent with neuroscience research on mathematical cognition are described in detail.

Mathematical Intervention Programs Related to Neuroscience Research

A comprehensive review of mathematical intervention programs is beyond the scope of this chapter. Instead, we provide descriptions of three mathematical intervention programs with digital components, including The Number Race, FASTT Math, and SRA Number Worlds® with Building Blocks®, and relate them to neuroscience research on mathematical cognition, particularly the triple-code model (Dehaene, 1992, 2011; Dehaene & Cohen, 1995, 1997; Dehaene, Piazza, Pinel, & Cohen, 2003) described in Chap. 1. For each program, we provide an overview, describe the user's experience, and summarize theoretical frameworks and efficacy studies.

The Number Race

Overview With a focus on number sense development, The Number Race game was designed by Wilson and Dehaene in direct alignment with the triple-code model to help strengthen the user's brain circuits for each of the three representational systems detailed by the model (see http://www.thenumberrace.com/nr/home.php). By representing number in each of the three codes, quantity, verbal, and visual, the user is engaging each of the specific regions of the brain associated with the codes while playing the game. The Number Race seeks to accomplish six goals: strengthen the brain mechanisms of number processing, establish the mental number line, teach and practice counting, teach and practice early addition and subtraction, encourage fluency, and help children with dyscalculia. While the game does support representations of number in all three codes of the model, it is limited in its overall scope and should be used in conjunction with a more comprehensive mathematics program.

User Experience When playing the game for the first time, the user builds a profile, where he/she self-selects the playing level as beginner, intermediate, or advanced. After building the profile or selecting it, the user chooses which world he/she will play in: the sea or the jungle. The game begins with a voice instructing the user to "Choose which side you want." On a split screen two quantity representations are displayed. On the left is the Arabic digit and dots representing a quantity; the right side has a similar display of a different quantity. In addition to the Arabic digits and dots, the game reads the number name aloud. At this point, the user drags the character to the quantity he/she wants to select. For instance, if the representations are 3 and 1, the user will hear "three" while three tokens are stacking up in the middle of the screen, followed by "one," while that token aligns next to the others. This visual representation with tokens supports development of the number line concept as the user can visually compare the quantities side-by-side. Once the user selects the quantity, a voice tells the user which quantity he/she has chosen and who has more. For instance, if the user does select the larger quantity, the game says, "You have more. I have the least." The user then wins the corresponding number of tokens that he/she drags onto a number line at the bottom of the screen. While the user is placing tokens on the number line, he/she hears a voice counting the spaces. After the tokens are placed on the number line, the user hears the addition problem that corresponds to the move. For example, if the user won three tokens, he/she would hear "3 plus 1 equals 4" and see the addition problem before the pawn is moved. The Number Race user is initially instructed to compare sets of objects or numbers 1 through 10. In more advanced levels, the objects are removed, using only Arabic digits and auditory number names. Additionally, the game board has spaces that, if landed on, will result in the player moving backward the number of spaces corresponding to the quantity chosen. In that case, the game provides visual and auditory cues for the subtraction problem. The goal of the game is to reach the end of the board first.

To support the development of fluency, The Number Race maintains a 75% success rate for the user by relying on an algorithm that automatically adjusts the numerical distance between the quantities and the length of the allowed response time. This approach supports the development of knowledge and skills at a level that is both appropriate, yet challenging to individual users, providing positive experiences with both numbers and math procedures.

Theoretical Frameworks The Number Race was designed and developed in direct alignment with the triple-code model. To support development of all three representational systems, the game presents quantity arrays and number lines, engaging the quantity system; digits, engaging the visual system; and spoken words, engaging the verbal system. The Number Race game "is one of only a few games that were specifically designed to teach and practice the various representations of numbers and the transformations between them, with a special focus on the quantity representation" (Wilson & Dehaene, 2004). This opportunity to practice comparing numbers across the three systems "encourages processing quantity and transforming the numbers from their symbolic representation to the quantity representation" (Wilson & Dehaene, 2004).

Efficacy Studies Two studies have shown support for The Number Race game. The first (Wilson, Revkin, Cohen, Cohen, & Dehaene, 2006), conducted over 4 months, included nine participants, aged 7–9 years old. Pre-testing indicated that all participants were experiencing mathematical learning difficulties. Over a period of 5 weeks, participants played the game for 30 min per day. At the end of the intervention period, participants were tested again on a number of mathematical concepts, including enumeration, nonsymbolic and symbolic numerical comparison, addition, subtraction, counting, transcoding, and understanding of the base-10 system. Post-test results indicated "progress in several core areas of numerical cognition: number comparison, subitizing, and subtraction" (Wilson et al., 2006, p. 14).

In a second study (Räsänen, Salminen, Wilson, Aunio, & Dehaene, 2009), children identified with low numeracy played The Number Race, or Graphogame-Math, or did not participate in either intervention, but completed pre- and post-tests. Both treatment groups showed improvement in number comparison over the control group, but gains were short-lived. It is important to note that both of these studies involved small sample sizes and were conducted by the developers of the program.

Fluency and Automaticity Through Systematic Teaching with Technology (FASTT Math)

Overview FASTT Math is a program that places emphasis on increasing accurate and fluent basic fact retrieval (see http://www.hmhco.com/products/fastt-math/index.htm). This program, designed for both struggling and accelerated students in 2nd grade and up, uses a baseline assessment to create an individualized learning program that can be used to provide core or supplemental instruction in math fact learning. Through the use of adaptive technology, the user is scaffolded through 18 independent games that foster growth on targeted math facts.

User Experience When the user begins the intervention, he/she completes a Placement Assessment designed to determine typing speed, which facts are already known, and which are ready to be learned. For the typing test, the user is shown numbers, one at a time, and instructed to type the number as fast as he/she can. In the second phase of the Placement Test, the user is given math fact problems and asked to type the answer as quickly as he/she can. For example, if shown 3 × 8, the user would type 24 as quickly as he/she is able. When the Placement Assessment is complete, the user can see her/his Fact Grid. The Fact Grid provides a graphic representation of *Fast Facts*—those already learned facts that can be retrieved from memory in less than 1 s. In Fig. 7.1, these are the 82 highlighted facts. The *Study Facts* are those that have been identified from the Placement Assessment as to be learned because they were not retrieved in less than 1 s. These are the 16 facts in the lower right corner of Fig. 7.1. The *Focus Facts* are 4 × 9 and 9 × 4, which have been selected by the software program for focused practice during the user's current session.

Once the ready to be learned facts have been identified through the Placement Assessment, the user begins the Adaptive Instruction component of the intervention. FASTT Math is driven by an algorithm that accounts for the speed of typing and known facts and delivers instruction on to-be-learned facts only. The instruction within the intervention is built on the principle of expanding recall, a system in which the *Focus Facts* are interspersed with *Fast Facts* (see Fig. 7.2). As the user gains fluency with the Focus Facts, the algorithm increases the amount of time between presentations of the target facts, requiring the retrieval of facts over a longer and longer period of time. Throughout this game-like component of the FASTT Math intervention the user is scaffolded through games targeting number facts that have not yet been learned while also reinforcing well-known facts.

Fig. 7.1 FASTT Math Fact Grid from http://www.hmhco.com/products/fastt-math/index.htm with permission from Houghton Mifflin Harcourt

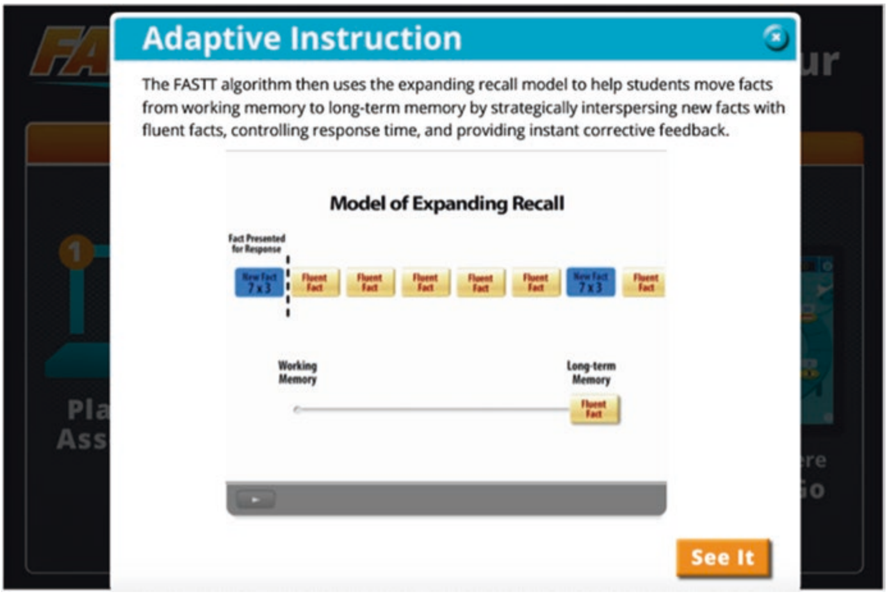

Fig. 7.2 FASTT Math Model of Expanding Recall from http://www.hmhco.com/products/fastt-math/index.htm with permission from Houghton Mifflin Harcourt

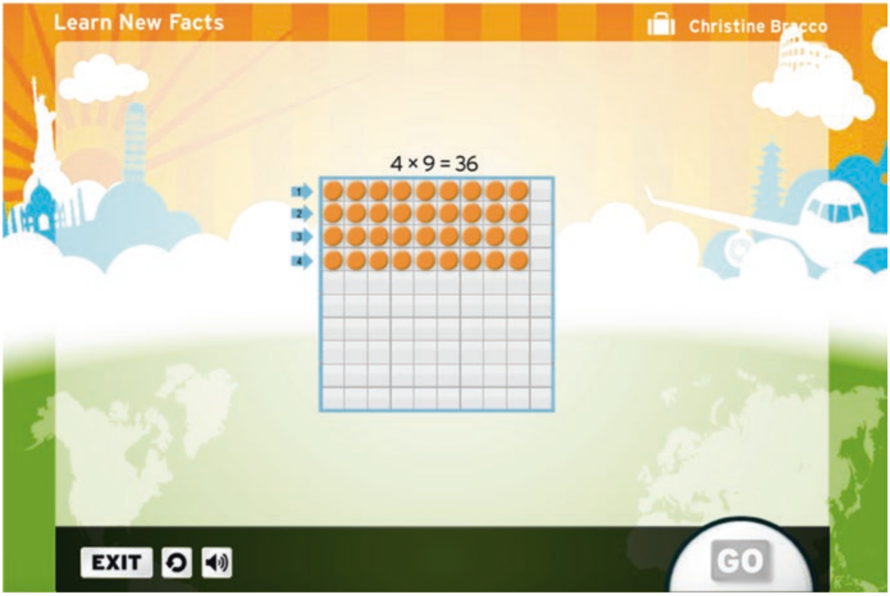

Fig. 7.3 FASTT Math See It Grid from http://www.hmhco.com/products/fastt-math/index.htm with permission from Houghton Mifflin Harcourt

Within this component, the user completes a round of 60 practice items, in which the two *Focus Facts* are interspersed with *Fast Facts*. The goal is to type the answers as quickly as possible, pressing the spacebar to move to the next problem. Providing both audio and visual instruction on the *Focus Facts*, this 10 min practice session starts by displaying a focus fact; for example, 4 × 9 = 36. While this is displayed on the screen, the user hears, "four times nine is thirty-six." The process is repeated for 9 × 4. The users is instructed to rehearse those facts until he/she feels ready to practice them. If the user needs or wants additional support, he/she can click a *See It* button, which then provides a visual representation of the fact on a 100-grid showing 4 rows of 9 dots, as shown in Fig. 7.3. This leads to a thorough conceptual explanation of the fact and its reciprocal.

After completing the Adaptive Instruction, the user is taken to her/his Dashboard. The Student Dashboard supports a growth-mindset by providing a platform for actively monitoring one's learning progress and success. This is where the user tracks her/his progress, sees which facts are mastered and those still being learned, checks on her/his personal best times and scores, and customizes her/his experience by changing the screen interface, as shown in Fig. 7.4. This Dashboard may promote motivated learning and executive function in that the user can engage in self-regulation and monitoring of her/his own learning.

To continue to build her/his fact retrieval skills, the user can play Fluency Games for Practice and STRETCH-To-Go games. The Fluency Games allow the user to

Fig. 7.4 FASTT Math Student Dashboard from http://www.hmhco.com/products/fastt-math/index.htm with permission from Houghton Mifflin Harcourt

practice newly learned facts. Through an engaging, personalized interface, the user solves fact problems. The user can see her/his progress on the screen. In the Race Car gaming interface, the FASTT Math racecar moves into first place as the student answers the problems correctly and quickly. *Study Facts* answered correctly in less than 1 s become *Fast Facts* in the next challenge.

To extend growth to users who have mastered basic facts, the STRETCH-To-Go component allows them to apply basic facts to more advanced objectives, providing an appropriate level of challenge. One objective in the STRETCH-To-Go platform is to identify equivalent equations. In the Equal and Out game, the user must identify the pair of numbers that correctly completes an equation, as shown in Fig. 7.5. For this game, the user drags numbers onto the scale to balance the equation. The algorithm responds to the user's answers, both correct and incorrect, to maintain an appropriate level of challenge.

Theoretical Frameworks The FASTT Math Program is designed around four theoretical frameworks: the developmental trajectory of math fact fluency, building declarative knowledge through repeated exposure to math facts, purposeful coupling of number and language to optimize memory, and employing technology to increase learner motivation. The developers of FASTT Math draw on the work of developmental and cognitive psychologists suggesting that young children must first learn the general properties of numbers and the conceptual processes of addition prior to learning multiplication, and that this learning is supported by the use of

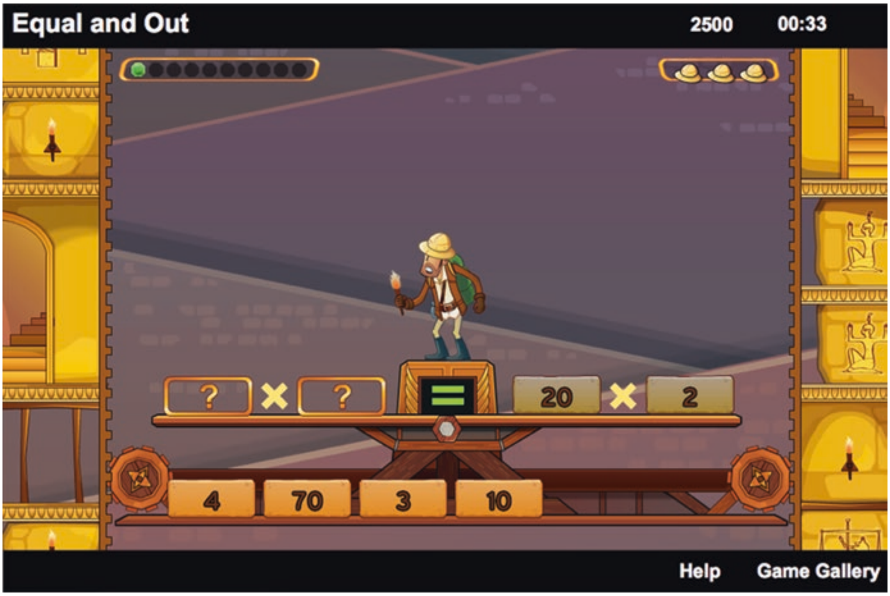

Fig. 7.5 FASTT Math Equal and Out Game in the STRETCH-To-Go Component from http://www.hmhco.com/products/fastt-math/index.htm with permission from Houghton Mifflin Harcourt

increasingly sophisticated strategies (Fosnot & Dolk, 2001; Van de Walle, Karp, & Bay-Williams, 2010). Early-developing strategies are both cumbersome and prone to error (Ashcraft, 1992; Fuson, 1982, 1988, 1992; Kilpatrick, Swafford, & Findell, 2001; Siegler, 1988). Children with mathematical learning difficulties are often challenged by number sense, the learning of basic properties of number and the relationships between numbers and objects, often relying on early-developing strategies (Fleischner, Garnett, & Shepard, 1982; Geary et al., 2009; Hasselbring, Goin, & Bransford, 1988; Torbeyns, Verschaffel, & Ghesquière, 2004; Vaidya, 2004). Reliance on these immature strategies leads to a negative impact on future mathematical learning. The FASTT Math program "helps students abandon the use of inefficient strategies for determining the results of basic number combinations and promotes student automaticity with basic math facts" (*FASTT Math Next Generation Research Foundation paper*, 2012, p. 4).

The second research base upon which FASTT Math is developed suggests that declarative knowledge can be built and sustained through instruction and practice with a small set of targeted math facts (c.f., Hasselbring et al., 1988). This instruction and practice must be purposeful; if not, the child may create inaccurate number associations, resulting in mislearned facts that, when called upon, may be retrieved in place of the correct response (Dehaene, 2011). When the association between numbers has been well learned, the processing of those math facts moves from the quantity system in the parietal regions to one that engages in automatic retrieval from the verbal system in the temporal regions of the brain associated with language (Dehaene, 2011). FASTT Math developers optimize this coupling between number and language by using "controlled response times to reinforce the memory connection and inhibit the use of counting or other non-automatic strategies, thereby 'moving' the fact into the student's declarative knowledge network" (*FASTT Math Next Generation Research Foundation paper*, 2012, p. 13), which relies on the verbal code in triple-code model for fast retrieval.

Accurate fact retrieval leads to increased reliance on long-term memory, freeing working memory resources for more complex operations (Baroody, Bajwa, & Eiland, 2009), the third theoretical framework of the FASTT Math intervention. Once these basic facts have been well learned, the user can begin to practice more advanced concepts in the STRETCH-To-Go component, allowing them to rely on the retrieval strategy while solving complex problems. "As students play in this STRETCH-To-Go component, they gain opportunities to understand inverse relationships, recognize unknowns, and apply mathematical properties. Specifically, this aspect of the software links students' fluent facts to related computations with multiples of 10. For example, if *3 + 8* is a fluent fact, then the STRETCH-To-Go games could include computations such as *30 + 80* as well as *80 + 30*, relating meaning for the commutative property with a fluent fact" (*FASTT Math Next Generation Research Foundation paper*, 2012, p. 5).

The final theoretical framework upon which FASTT Math relies is the premise that technological environments can positively affect learning of mathematical content and motivation to learn. The gaming environment provides a level of intrinsic motivation and allows for individualized learning (Clements, 2002; Kamii, 2000). The computerized format also permits instant corrective feedback by providing the correct problem and answer (Van de Walle et al., 2010).

Efficacy Studies The sole empirical study validating the FASTT Math program examined the efficacy of the program with a sample of 160 children (ages 7–14). Participants were assigned to either a treatment group (those with mathematical learning difficulties) or a contrast group (those with no identified mathematical learning disabilities). The treatment group received daily instruction using the FASTT Math system. A comparison of the pre- and post-test assessments revealed that the experimental group increased the number of facts that were retrieved from memory by 45–73% over the duration of the study. Furthermore, participants in the experimental group developed fluent facts at a rate twice that of the control group. However, it is important to note that the study was conducted by the developer of the program and inferential statistics were not reported.

SRA Number Worlds® with Building Blocks®

Overview As a Common Core State Standards-based intervention, SRA Number Worlds® with Building Blocks® is a math curriculum designed to develop proficiency in mathematical literacy and fluency for children in Pre-Kindergarten through

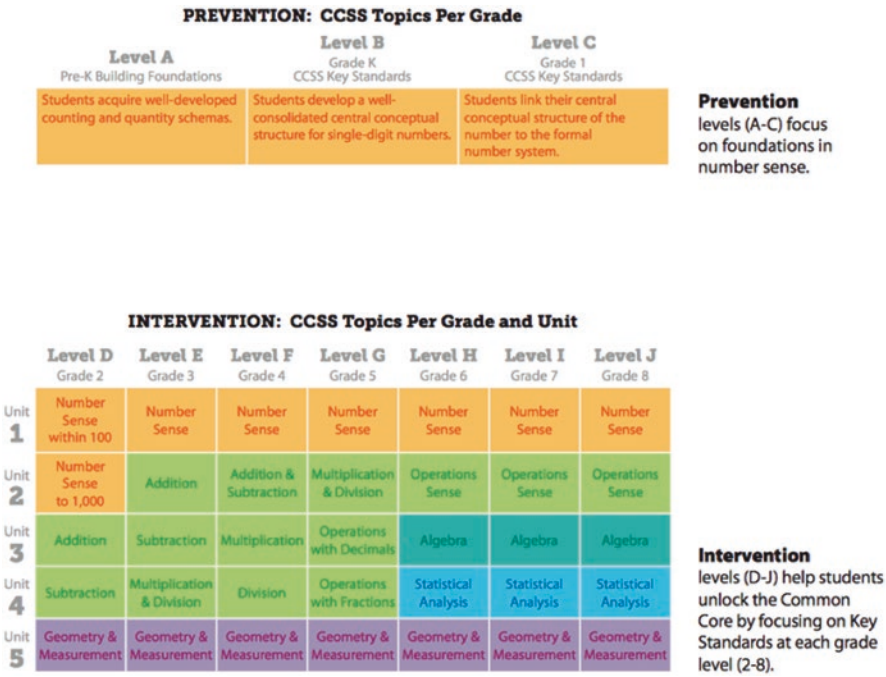

Fig. 7.6 SRA Number Worlds Topics by Grade from Griffin (2015) with permission from Sharon Griffin

8th grade, bringing struggling learners' performance on par with their peers (see http://www.mheducation.com/prek-12/program/microsites/MKTSP-TIG05M0.html). It is the only mathematics intervention with an embedded prevention program for Pre-Kindergarten through 1st grade (*SRA Number Worlds: Research and efficacy*, 2015). Designed from a project-based learning approach, Number Worlds focuses on solving ill-formed problems with many possible solutions; developing functional knowledge with cognitive flexibility; engaging in self-directed, active, and engaged learning; collaborating; and building habits of reflection and self-appraisal in all learning experiences (Barrows & Kelson, 1993). As shown in Fig. 7.6, Number Worlds focuses on developing number sense, procedural skills, fluency, operations sense, and spatial understanding.

User Experience At the beginning of the program, the user completes a Number Knowledge Test. This individually administered, oral assessment indicates whether the user is performing below, above, or at grade level proficiency. From this assessment, the teacher can also identify which concepts have been mastered and which are yet to be learned. The Number Knowledge Test provides the teacher with direction about which Level Placement Test to administer and can be used to track the user's progress over a particular instructional period or the entire academic year.

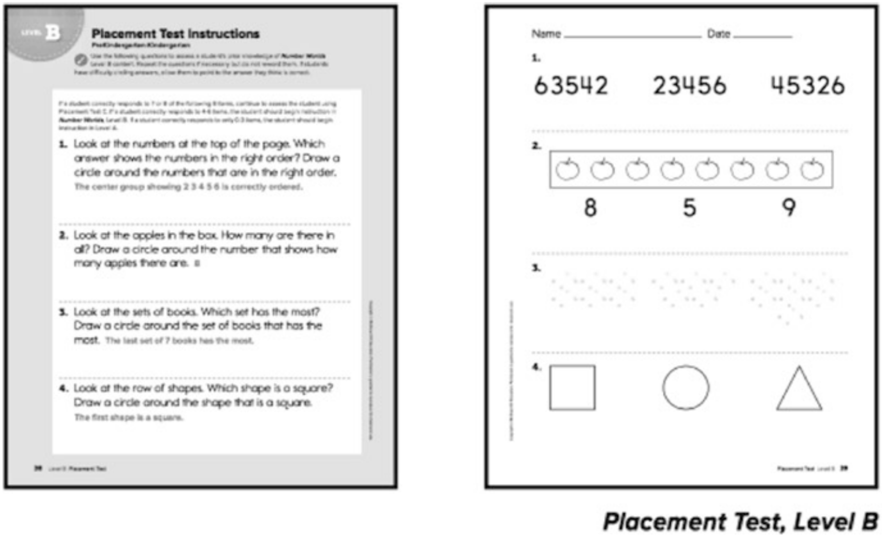

Fig. 7.7 Example of a SRA Number Worlds Placement Test from Griffin (2015) with permission from Sharon Griffin

The Level Placement Test (see Fig. 7.7 for an example) is used to determine the appropriate instructional level by assessing pre-existing knowledge. The user completes a range of items on the test, which approximates her/his abilities. If a student makes several consecutive correct responses, it is assumed that the student would also respond correctly to the preceding items, which are easier (*Using the Number Worlds Placement Tests*, n.d.).

Once a level has been determined and assigned, the user gains access to her/his Dashboard. This interface provides a to-do-list of activities and assessments. The assignments are hyperlinked, making it easy for the user to access the assignment he/she wants to work on with the goal of completing it by the due date. From the Dashboard, the user can also access Building Blocks software, a series of approximately 200 online learning activities for grades Pre-Kindergarten through 8 that adapt to the user's responses to ensure an appropriate level of challenge. In these engaging, game-like modules, the user practices the skills that have been identified on the Level Test as to be learned. Extra practice items are also available for developing mastery. For example, in one Building Blocks activity, the user is instructed to match the product to its correct quantity representation by dragging it to the visual representation. If the user answers the problem correctly, he/she is advanced to the next activity in her/his learning trajectory. If the user responds incorrectly, he/she is prompted to think about her/his response and try again. If he/she commits an error a second time, the user is provided with a hint or a short tutorial before being prompted to try again. If a pattern of errors occurs, the algorithm recognizes that the user needs additional support and provides a detailed tutorial that breaks the skill into smaller steps or activities designed to develop the associated foundational skills.

Fig. 7.8 SRA Number Worlds Dragon Quest Game from Griffin (2015) with permission from Sharon Griffin

From the Dashboard, the user can also access Math Tools and Games. By choosing from a variety of interactive games, the user receives additional practice with the conceptual knowledge and procedural skills that he/she is working to build. In the Dragon Quest game shown in Fig. 7.8, multiple users can play with each other while building their number sense knowledge and skills. Each user clicks a spinner and moves her/his pawn the corresponding number of spaces on the game board.

The Number Worlds system also provides multiple Interactive Tools that can be used by teachers and users as they work through the activities in the system, including a white board, timer/stopwatch, protractor, ruler, and a customizable number line. Another beneficial feature within the system is the option to print and export work.

Theoretical Frameworks The Number Worlds intervention is built upon five guiding principles that support children's learning of conceptual and procedural mathematical skills. Guiding Principle 1, *Building upon children's current knowledge*, draws upon constructivism, acknowledging that children use "their existing knowledge to construct new knowledge that is within reach— that is one step beyond where they are now" (*SRA Number Worlds: Research and efficacy*, 2015, p. 7). The second Guiding Principle, *Follow the natural developmental progression when selecting new knowledge to be taught*, is based on the work of Case (1992), which suggested that "because there are limits in development on the complexity of information children can handle at any particular age/stage, it makes no sense to attempt to speed up the developmental process by accelerating children through the curriculum" (*SRA Number Worlds: Research and efficacy*, 2015, p. 7). Recognizing working memory limitations, the developers of Number Worlds designed the program so that the user works on a restricted number of concepts and skills at one time (for an in-depth theoretical explanation, see Griffin, 2009). The Number Worlds curriculum and activities adhere to Guiding Principle 3, *Teach computational fluency as well as conceptual understanding*. Guiding Principle 4 is to *Provide plenty of opportunity for hands-on exploration*, *problem solving*, and *communication*. Guiding Principle 5, *Expose children to the major ways number is represented and talked about in developed societies*, is the basis for the exposing users to multiple ways of representing numbers that are built into the intervention curriculum. This final Guiding Principle is aligned with the triple-code model by recognizing that quantity and numerical information are processed using different representational systems operating in distinct brain regions. Number Worlds provides access to and practice with multiple representation of numbers, including pictorial representations of quantity and digits, aligning with the quantity and visual systems of the triple-code model, respectively.

The development of Building Blocks technology-enhanced materials was funded by the National Science Foundation (NSF) and is based on the work of Simon (1995) and Clements and Sarama who created research-based mathematical learning trajectories, which they define as descriptions of children's thinking and learning in a specific mathematical domain and a route through a set of instructional tasks designed to facilitate mental processes and actions that will advance them through a developmental progression (see Clements, 2007; Clements & Sarama, 2004, 2011, 2014; Sarama & Clements, 2009). That is, they defined key mathematical cognitive developments and sequenced them along research-based learning trajectories. Building Blocks activities were developed to support defined skills at points along the trajectories.

Efficacy Studies Griffin (2004) reported that several evaluation studies demonstrated that children from low-income communities who participated in the Number Worlds program showed significant gains in conceptual knowledge of number and number sense in comparison to a matched control group who received different school readiness training. Furthermore, she noted

> These gains enabled them to start their formal schooling in grade one on an equal footing with their more advantaged peers, to perform as well as groups of children from China and Japan on a computation test administered at the end of grade one, and to keep pace with their more advantaged peers (and even outperform them on some measures) as they progressed through the first few years of formal schooling (Griffin, 2004, p. 178; Griffin & Case, 1997).

Several major studies evaluating the efficacy of Building Blocks have been published in peer-reviewed journals. Clements and Sarama (2007) described an initial summative evaluation of Building Blocks in classrooms with children at risk for later school failure, including state funded and Head Start programs. They reported that the classrooms with children who learned mathematics using Building Blocks increased their scores on tests of early mathematics and geometry significantly more than the comparison group, with achievement gains nearing those found for individual tutoring. In a another study, Clements and Sarama (2008) used randomized trials and found that, after 26 weeks of instruction, children in classrooms using the Building Blocks curriculum had significantly greater increases in scores from pre-test to post-test than the comparison group and the control group. Children in Building Blocks classrooms showed the greatest gains in verbal counting, recognition of number and subitizing, comparison of shape, and shape composition. Furthermore, they were more accurate and increased their use of more sophisticated strategies. Another study using a randomized block design involving 1375 preschools in 106 classrooms provided evidence that children in the Building Blocks group learned more mathematics than children in the control group (Clements, Sarama, Spitler, Lange, & Wolfe, 2011; for longitudinal results, see Clements, Sarama, Wolfe, & Spitler, 2013). It is important to note that the implementation of Building Blocks in these studies was likely more comprehensive than the online learning activities that are embedded within Number Worlds.

Other Intervention Programs

There are other well-designed and researched mathematical intervention programs and protocols that are consistent with neuroscience research. For example, Math Recovery® (MR); program (http://www.mathrecovery.org/) is primarily a professional development program that provides training in identifying, teaching, and assessing children's understanding of mathematics from kindergarten through 5th grade (e.g., Wright, Ellemor-Collins, & Tabor, 2012). Other types of mathematical intervention focus on the explicit instruction and deliberate practice of specific skills. For example, Hot Math (http://www.intensiveintervention.org/chart/academic-intervention-chart/13683) targets problem solving skills (for efficacy studies see, Fuchs, Fuchs, & Hollenbeck, 2007; Fuchs et al., 2008) and Pirate Math (http://www.intensiveintervention.org/chart/academic-intervention-chart/13648) targets counting procedures for solving simple addition and subtraction problems (for efficacy studies, see Fuchs et al., 2008, 2009, 2010).

Conclusions and Future Directions for Research

Efficacy studies provide scientific evidence of interventions that promote positive mathematical learning outcomes for students from early childhood through emerging adulthood. This chapter provided a few illustrations of mathematical intervention programs that are based on, or consistent with, theory and research from the fields of neuroscience and cognitive development. Future research could more firmly establish the effects of mathematical interventions by conducting pre- and post-intervention studies using fMRI and other neuroimaging techniques to show how interventions actually change brain functioning.

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Chapter 8
Conclusions and Future Directions for the Neuroscience of Mathematical Cognitive Development

Rhonda Douglas Brown

Abstract In this chapter, I call attention to progress that has been made over the past 20 years in children's mathematics achievement and in using neuroscience to understand mathematical cognitive development. Evidence supporting the triple-code model of numerical processing is mounting and we are beginning to understand how domain-specific and domain-general cognitive processes related to mathematics are instantiated in the brain and how they change with age and experience. I note that there is a great deal of work ahead in studying and applying the results of neuroscience research on mathematical cognitive development. Future studies should focus on longitudinal changes for the various components of numerical processing and how they interact in children with and without mathematical difficulties and on the effects of intervention and instructional approaches using a pre-/post-design.

In Chap. 1, I noted that the *National Assessment of Educational Progress (NAEP)* online report for 2015 indicates that only 25% of 12th grade students performed at or above the *Proficient* level in mathematics, which has not changed significantly since 2005 (U.S. Department of Education, 2015). Yet, results from the *Trends in International Mathematics and Science Study (TIMSS)* 2015 indicate that over the course of 20 years, mathematics achievement has risen to its highest levels, with approximately 40% of 4th graders and 30% of 8th graders reaching the highest benchmarks (see Fig. 8.1; Mullis, Martin, Foy, & Hooper, 2016). Of the 17 countries with 20-year trend data (1995–2015) for 4th grade, 14 had higher average mathematics achievement in 2015 than 1995, just 2 had lower achievement, and 1 was unchanged. There were 16 countries with 20-year trend data for 8th grade, and in both mathematics and science there were 9 countries with higher achievement in 2015, 3 countries with lower achievement, and 4 countries where average achievement was unchanged (Mullis, Martin, & Loveless, 2016). Mathematics education is advancing students' achievement.

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https://doi.org/10.1007/978-3-319-76409-2_8

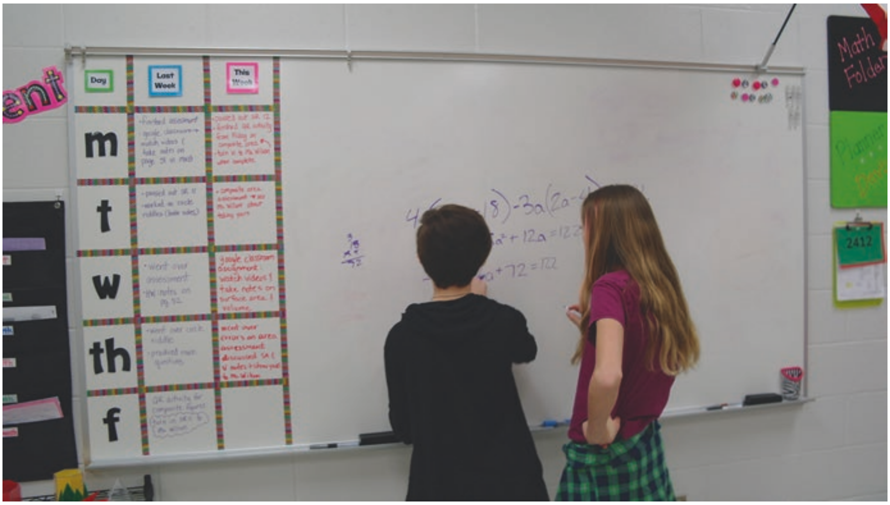

Fig. 8.1 Approximately 30% of 8th graders reached the highest benchmarks according to the results from the 2015 Trends in International Mathematics and Science Study (TIMSS)

Like the performance of 7th and 8th graders on the TIMSS 2015, a great deal of progress has been made over the past 20 years in using neuroscience to understand mathematical cognitive development. Evidence supporting the triple-code model of numerical processing (Dehaene, 1992, 2011; Dehaene & Cohen, 1995, 1997; Dehaene, Piazza, Pinel, & Cohen, 2003) is mounting and we are beginning to understand how domain-specific and domain-general cognitive processes related to mathematics are instantiated in the brain and how they change with age and experience. In my view, one of the most important conclusions from the research thus far is that, fundamentally, mathematics involves understanding quantities and the relationships between them, but it also involves verbal skills, visuospatial skills, *and* executive function skills. I placed emphasis on *and* because I believe some recent philosophies and practices in education put all or most of the weight on developing children's understanding of quantity, but not necessarily on the other components. Consider this excerpt published in the journal *Mind, Brain, and Education*:

> …a local district was considering adoption of a new elementary mathematics curriculum and was struggling to choose between one instructional program focused on teaching "procedures" and another that claimed to enhance children's understanding of "mathematical concepts." The debate over which of these two methods was "best" became quite heated and was accompanied by the formation of factions within the educational community. Frustrated with this situation, the superintendent asked one of us (Ansari) to talk with the teachers about the differences between procedural and conceptual aspects of math learning from a research perspective. In an afternoon presentation and discussion with all the math teachers from the district present, behavioral and neuroscientific data on math learning were reviewed and the teachers ' questions were addressed from a research-based viewpoint. Later, the superintendent reported that this event had lowered tensions and helped teachers in the opposing camps to recognize that a false dichotomy had been established; that rather than either/or, both instructional approaches had some value (Coch, Michlovitz, Ansari, & Baird, 2009, p. 31).

Fig. 8.2 Research indicates that fast retrieval of math facts is important for mathematics achievement

Some teachers are especially concerned with focusing too much on memorization of addition, subtraction, multiplication, and division facts. Yet, the bulk of research from cognitive developmental psychology and the neuroscience of mathematical cognitive development indicates that fast fact retrieval makes an important contribution to mathematics achievement, as the school in Fig. 8.2 understands, and it doesn't have to be at the expense of conceptual understanding. Many of us may have not so fond memories of timed math fact worksheets during our school days. But, as discussed in Chap. 7, computerized games and apps, such as FASTT Math, make learning math facts much more fun and engaging for children than it used to be.

Indeed, by understanding double dissociations (see Chap. 2), we know that fact retrieval involves the left lingual gyrus, whereas quantity representation involves the intraparietal sulcus (IPS). Thus, focusing solely on conceptual aspects of mathematics will not necessarily lead to math fact fluency. Performing complex mathematics involves an orchestration of many brain regions and networks. And from evolutionary developmental psychology, we understand that current human neuroarchitecture and its functions evolved gradually over millions of years—evolution works with what it's got. Thus, we see heterogeneity in *mathematical learning disabilities (MLD)*, including the number sense, procedural, semantic, and visuospatial subtypes discussed in Chap. 7. Taking research on the neuroscience of mathematical cognitive development into account, balanced curricula and targeted interventions make sense. As noted by Katzir and Paré-Blagoev (2006),

> Neuroscience has provided fascinating glimpses into the brain's development and function; advances in our knowledge of the brain hold promise for improving the education of young children. When applied correctly, brain science may serve as a vehicle for advancing the application of our understanding of learning and development. Reciprocally, education may serve as an important vehicle in formulating important research questions for neuroscientists and in providing more precise guidelines for behavioral measurements used in neuroscience. Brain research can challenge common-sense views about teaching and learning by suggesting additional systems that are involved in particular tasks and activities (p. 70).

As an example of education informing research, a seasoned mathematics teacher noticed that even college students struggled with reading mathematical text. As a Ph.D. student, she conceptualized *Symbolic Mathematics Language Literacy (SMaLL)*, which she defined as the ability to read and write symbolic mathematics using the conventions of the writing system for the language of mathematics. For example, in reading mathematical text, $f(x)$ makes more sense mathematically than $(f)x$. Using a task similar to the error detection task presented in Chap. 4, her research found that SMaLL is related to measures of mathematics achievement (Headley, 2016). This research could also be easily conducted as a neuroimaging study to understand the neural and cognitive systems involved in reading mathematical text. Furthermore, an intervention study could be conducted with students who struggle with reading mathematical text using a pre-intervention/post-intervention design to understand how instructional practices change neural activity related to mathematical cognition.

Similar work on interventions for dyslexia has revealed different patterns of neural changes for different individuals, with some experiencing normalizing effects, while others show compensatory effects (for a review, see Katzir & Paré-Blagoev, 2006). Furthermore, as noted by Swanson (2008), neuroscience research can help explain and predict individual differences in response to intervention and document changes in functional brain anatomy that may result from intervention. Future studies should also focus on longitudinal changes for the various components of numerical processing and how they interact in children with and without mathematical difficulties. Although not all types of mathematical cognition are conductive to studies using neuroimaging techniques, such as solving especially long paper-and-pencil problems, there is a great deal of work ahead in studying and applying the results of neuroscience research on mathematical cognitive development.

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