Chapter 2
A Review of Simulation Usage in the New Zealand Electricity Market

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Abstract In this chapter, we outline and review the application of simulation on the generation offer and consumption bids for the New Zealand electricity market (NZEM). We start by describing the operation of the NZEM with a particular focus on how electricity prices are calculated for each time period. The complexity of this mechanism, in conjunction with uncertainty surrounding factors such as consumption levels, motivates the use of simulation. We will then discuss simulation–optimization methods for optimal offer strategies of a generator, for a particular time period, in the NZEM. We conclude by extending our ideas and techniques to consumption bids and interruptible load reserve offers for major consumers of electricity including large manufacturers such as the steel mill.

Keywords Electricity markets · Price simulation · Demand response

2.1 Introduction to Wholesale Electricity Markets

Electricity markets have become prevalent around the world in the past two to three decades. The first example of privatization of an electric power system took place in Chile in the early 1980s. The idea behind the Chilean model was to bring rationality and transparency to the operations of the power system that would ultimately be reflected in power prices. Other rationales for the eventuation of electricity markets include better reliability and signalling appropriate levels of investment in infrastructure in the energy sector through proper pricing of this commodity. England–Wales, New Zealand, Australia, the Nord Pool, Spain and the

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Pennsylvania–Jersey–Maryland (PJM) markets are amongst the oldest electricity markets with an abundance of available data.

2.1.1 Pricing of Electricity

Arguably, proper pricing of electricity is the corner stone of the electricity market paradigm. This is a key for signalling scarcity, and it is the market signal that would drive investment decisions. While in most commodity markets the price of a good is determined through supply and demand, in the case of electricity, the physical constraints governing an electricity system also impact prices. Electricity is not a storable commodity. It is injected into a transmission grid at certain nodes of that transmission grid often referred to as grid injection points (GIPs) and flows through the grid complying with physical constraints. Electricity is withdrawn at grid exit points (GXPs) and delivered to consumers. Due to the physical constraints on the flow of electricity, in all electricity markets, the dispatch of the generation of electricity is left to an independent system operator (ISO). In most electricity markets, an additional function of the ISO is to determine the price of electricity at different nodes of the transmission network.

Typically in a wholesale electricity market, for each period of the day, each generator offers in generation quantities for each of its plants (possibly located at different GIPs), at certain prices. In its most general form, the generation offers are supply functions (also known as offer curves) denoted $p = S(q)$, where $S(q)$ is the marginal price of producing quantity q. In all electricity markets, $S(q)$ is required to be a monotone increasing function. It is important to note that these supply functions are offered by a deadline well ahead of the pertaining (market) time period; therefore, participants do not know other generator offers or a complete picture for demand.

These supply offers are collected by the ISO. The ISO estimates the demand (in the case of inflexible demands), over that period. The ISO then solves a side constrained network optimization problem where the objective is to minimize the total cost of production of electricity. The constraints of this optimization problem reflect that demand must be met at every node of the network and that physical flow constraints such as transmission line capacities and Kirchhoff's laws must be complied with. Often reactive power modelling is left out of the ISO's dispatch problem, and the problem is in fact a direct current equivalent load flow model [17, 24]. When flexible demand is offered into the market, in the form of a demand-side bid, the objective of the ISO's optimization problem becomes welfare maximization, producing system optimal amounts of generation and consumption for a time period.

A general model for the ISO's economic dispatch problem (EDP) in its simple cost-minimizing form is formulated below.

$$
\begin{array}{llll}
\text{EDP: } \textit{minimize} & \sum_i \sum_{m \in \mathscr{O}(i)} \int_0^{q_m} C_m(x)dx & & \\
\quad\quad \textit{s.t.} & g_i(y) + \sum_{m \in \mathscr{O}(i)} q_m & = D_i,\ i \in \mathscr{N},\ [\pi_i] & \\
 & q_m \in Q_m, \quad m \in \mathscr{O}(i), & \quad\quad i \in \mathscr{N}, & \\
 & y \in Y. & &
\end{array}
\tag{2.1}
$$

We use i as the index for the nodes in the transmission grid. We use m as the index for the generators, and $\mathscr{O}(i)$ indicates the set of all generators located at node i. Generator m can supply quantity q_m, and the demand at node i is denoted by D_i. Q_m indicates the capacity of generator m. Here the components of vector x measure the dispatch of each generator, and the components of the vector y measure the flow of power in each transmission line. We denote the flow in the directed line from i to k by y_{ik}, where by convention we assume $i < k$. (A negative value of y_{ik} denotes flow in the direction from k to i.) It is required that this vector lie in the set Y, which means that each component satisfies the thermal limits on each line and satisfies loop flow constraints that are required by Kirchhoff's Law. The function $g_i(y)$ defines the amount of power arriving at node i for a given choice of y. This notation enables different loss functions to be modelled. For example, if there are no line losses, then we obtain

$$
g_i(y) = \sum_{k<i} y_{ki} - \sum_{k>i} y_{ik}.
$$

With quadratic losses, we obtain

$$
g_i(y) = \sum_{k<i} y_{ki} - \sum_{k>i} y_{ik} - \sum_{k<i} \frac{1}{2} r_{ki} y_{ki}^2 - \sum_{k>i} \frac{1}{2} r_{ik} y_{ik}^2.
$$

The price of electricity is determined as the shadow price π_i of the node balance constraints above that indicate demand must be met at all nodes. This price is the system cost of meeting one more unit of demand at node i. This method of determining the electricity price is sometimes referred to as locational marginal pricing (LMP). New Zealand and the PJM market in the USA are examples of electricity markets with LMP. It is worth noting that some wholesale electricity markets operate by assuming that demand and supply are located at the same node, and trading takes place in that one node. This means that a single price of electricity is arrived at. Nevertheless, in order to ensure that the demand is met at all nodes and that the flow complies with physical constraints, a balancing market would follow in real time where the residuals of the single node market are traded. The UK wholesale electricity market is an example of a single node market.

2.2 The New Zealand Electricity Market

Following a transition from a centralized system, to a deregulated electricity market, an immediate natural question for a generator is what supply offer function will optimize their returns. In a strictly monitored market such as the PJM, there is not much room for a generator to exercise market power. In such a market, the marginal cost of generation of electricity is relatively well known. Much of the supply is procured from thermal plants (e.g. gas and coal) with known cost of fuel or nuclear plants with a minimal marginal cost of generation. Here, it is relatively simple for a market monitor to observe the supply offers and question any offers that are significantly above the marginal cost of production.

Not all electricity markets are strictly monitored however. Markets such as Nord Pool and the New Zealand market are dominated by hydroelectric generation. While one can argue that inflows into hydro-lakes are free, there is an opportunity cost attached to using the water now or saving it for a future period. This is particularly important as the inflows are uncertain and dry periods can have disastrous consequences for the electricity system. This opportunity cost is referred to as the *value of water*. When all market participants are risk neutral, this value can be found by solving a large-scale stochastic program that minimizes the expected cost of production of electricity, using various generation sources in a coordinated fashion, over a long time horizon (e.g. a year that is divided into 52 weeks; see e.g. [18, 20]). In a real market, however, generators face various risks and it is not possible to ascertain their level of risk. Even if this information were available, it would not always be possible to solve an equivalent centralized problem to obtain the value of water [19, 23]. Hence, the New Zealand market was designed not to be a strictly regulated market. The question therefore remains, how can a generator offer supplies into this market so as to maximize their profits. The answer to this question, and a very similar question for the demand side, utilizes simulation intensely and is the topic of the remainder of the chapter.

2.2.1 The Need for Simulation: Pricing in the NZEM

While it would be highly desirable to obtain a simple analytical answer to the question of optimizing generation offers, this is not possible due to the nature of price determination. As laid out in Sect. 2.1.1, the nodal price of electricity is the value of the optimal shadow prices for the demand constraints. There is no explicit analytical form for these prices, which are clearly affected endogenously, as the firm varies their supply offer. The best way to tackle the problem of offer optimization over an electricity market is to simulate the ISO's problem and obtain prices. It is fortunate that the Electricity Authority (EA), who exercise oversight over the NZEM, has made publicly available an accurate replica of the market clearing optimization problem that is solved in New Zealand in every half hour time period. This replica is referred to as the vectorized Scheduling, Pricing and Dispatch (vSPD) that is available from the EA's web site.[1]

[1] See http://www.emi.ea.govt.nz/.

The market clearing side constrained network optimization vSPD contains over 250 nodes(GIPs and GXPs) and over 450 arcs (that form the backbone transmission network for New Zealand). The database for vSPD contains historical offer and demand information dating back to 2000. The generator offers for New Zealand are in the form of five step, step functions, where each step is referred to as a tranche. Each historical tranche of each offer is available, indicating the quantity and price pair that comprise that tranche, for each generator. Furthermore, the database contains information on the thermal capacity of the transmission lines, availability of generation units, demand data and various other necessary information for replicating any historical period. This is a rich and ideal set-up for simulation.

Another feature of the NZEM is the co-optimization of energy and reserve. Electricity markets need to be robust to failure. To that end, reserve generation is procured for every electricity market. In New Zealand, the procurement of reserves takes place in conjunction with procurement of energy. There are a number of constraints relating energy and reserves. We mention this feature of the NZEM here for completeness; however, we will refrain from dwelling on this point for the sake of simplicity. We will return to this point in Sect. 2.4 when we discuss consumption and reserve offer strategies for a major consumer of electricity.

2.3 Optimal Offers for Generation

We start this section by formulating an analytical description of the generator optimization problem under uncertainty. We will lay out a simulation–optimization approach for this problem which has been in use by generators over the NZEM. Under a number of strong assumptions, the problem of generator offer optimization can be solved analytically. Our setting is a realistic electricity market where such strong assumptions are not justified. However to place the problem in context and gain some intuition, we start with this analytically tractable case.

2.3.1 A Simplified Problem

The problem of bid–offer optimization was first approached by Klemperer and Meyer [14] who were interested in modelling an oligopoly facing uncertain demand, where each firm bids a supply function as its strategy. This is in contrast to previous models in the economics literature where firms were restricted to strategize over their quantities only (Cournot models) or their prices only (Bertrand models) and allows a firm to adapt better to an uncertain environment. Green and Newbery address the same question but in the context of the British spot market [13].

To begin, let us assume that there are only two generators supplying the market (i.e. we are dealing with a duopoly) and suppose that the offer curve of the competitor is given by $q = S(p)$. Let us also assume that the demand curve is given by $q = D(p)$, that is the market will absorb quantity q if the price is p. For their analysis, Klemperer and Meyer use the concept of the residual demand curve faced by the generator. Consider the curve given by $q = D(p) - S(p)$. This determines what quantity must be offered into the market if we desire the price to be p based on the demand curve and the competitor's offer strategy. Note that this approach makes the simplified assumption that *all transactions occur at a single node*. The inverse of this curve describes how the price is influenced by the quantity we offer and is referred to as the residual demand curve. With this information at hand, it is now easy to optimize the profits of the generator in question (see Fig. 2.1).

Recall that Klemperer and Meyer point out that supply functions allow a firm to adapt better to an uncertain environment. If there are multiple possible residual demand curves that a generator may face, the supply function response may allow selecting a point on each of these residual demand curves that would optimize the generator's profit given that that residual demand curve has realized. This is referred to as a strong supply function response (see Fig. 2.2). A number of papers construct the residual demand curve by simulating the (single node) market and explicitly building the supply function response; see e.g. [9, 10]. In [9], the residual demand curve takes on a step function form and the authors develop a nonlinear integer programming model of the generator's revenue optimization problem. They develop a combined coordinate search, branch and bound method to solve this problem. Torre et al. exploit the nature of the previous problem to develop a more efficient solution method in [10].

In a sequence of papers, Anderson and Philpott have also addressed the profit maximization problem of a price-maker generator under various assumptions. In [3], they assume that a price-maker generator knows its competitors' offer curves, but is faced with uncertain demand. They first establish the existence of a strong supply function response, for such a generator, that would be optimal for any realization of the uncertain demand. This strong supply function response is guaranteed to exist when the generation costs of the generator in question are increasing and convex,

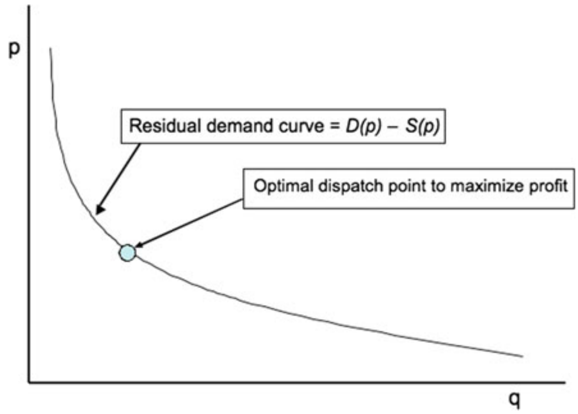

Fig. 2.1 Optimal point for a generator to get dispatched along a residual demand curve

and the competitor offers are log-concave. They discuss a procedure where the true aggregate offer stack of the competitors is approximated by a log-concave function. Note that this (aggregate) offer stack would be a step function in almost all real-world electricity markets. They construct a strong supply function response S_g, for the generator in question. Subsequently, they approximate S_g in order to comply with market rules. Finally, they provide bounds on the performance of such an offer strategy.

In [2], Anderson and Philpott generalize their model by allowing uncertainty not only in the demand but also allow the competitor offers to be unknown. They introduce the concept of a market distribution function $\psi(q, p)$ pertaining to a specific generator at a specific transmission node. They define $\psi(q, p)$ to be the probability of not being fully dispatched if the generator submits a quantity q at price p. Let $R(q, p)$ denote the or profit that the generator makes if it is dispatched q at a clearing price of p. They demonstrate that if the generator submits the curve s, and the pertinent market distribution function ψ is continuous then the expected profit of the company is given by

$$V(s) = \int_s R(q, p) d\psi(q, p).$$

They proceed to provide conditions that guarantee (local) optimality of an offer stack s that would maximize $V(s)$. To address the question of estimating the market distribution function see [4, 22].

2.3.2 *Using Simulation for the General Problem*

The work described thus far only deals with generators that are located at a single node of the market or alternatively assumes that the wholesale market is a single node market. As noted in Sect. 2.1 however, most wholesale electricity markets use locational marginal pricing where the price of electricity is different from node to

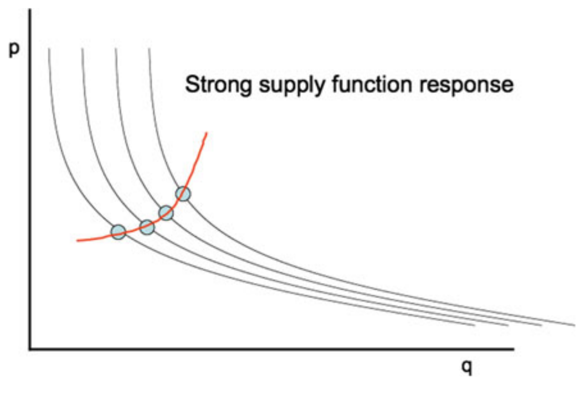

Fig. 2.2 Building a strong supply function response from a distribution of residual demand curves

node. To capture the effects of the transmission network, a generator must look at the variations in the prices from the dispatch problem EDP as a function of how it offers into the market. The revenue optimization problem is now posed as a bilevel program, or a mathematical program with equilibrium constraints (MPEC) and becomes a non-convex optimization problem.

$$
\begin{array}{llll}
\textit{maximize} & R(x,\pi) & & \\
\textit{s.t.} & (x,\pi) \in \ \textit{arg min} & \sum_i \sum_{m\in\mathcal{O}(i)} \int_0^{q_m} C_m(x)dx & \\
& \textit{s.t.} & g_i(y) + \sum_{m\in\mathcal{O}(i)} q_m \qquad = D_i, & i \in \mathcal{N}, \\
& & q_m \in Q_m, \quad m \in \mathcal{O}(i), & i \in \mathcal{N}, \\
& & y \in Y. &
\end{array}
$$

Here x denotes the vector of quantities dispatched at each node if the generator offered at that node (or is 0 if the generator in question does not own generation at a particular node), and π is the vector of electricity prices. Note that the inner optimization problem, namely the economic dispatch problem EDP can be replaced with its necessary and sufficient conditions for optimality as it is a convex problem (see e.g. Chap. 4 of [5]). In this case, the reformulation is referred to as an MPEC [16]. Furthermore, as described in Sect. 2.1, the offers submitted to the market are for a (near) future period. In particular, over the NZEM, generator offers are "locked in" two hours ahead of each time period. Therefore, generators have at best probabilistic knowledge of demand and competitor offers. The amount of randomness depends on what the generator (plant owner) is assumed to know before submitting the offer curve. Competing generators' offer curves may be modelled stochastically (if unknown) or deterministically (if known). A realistic problem is likely to contain some of each: the availability of another power plant owned by the same firm is probably known, while the availability of a wind farm is likely to be unknown. submitted very close to the time of production. We will assume that the sizes of loads require a stochastic model. The model may also include stochastic transmission line outages.

Pritchard considers this stochastic version of the above MPEC in [21]. An algorithm is developed where first the market is simulated under varying (quantity, price) offers of the generator in question. The market clearing prices faced by this generator are recorded, and a global optimization is performed that determines the best supply function offer resulting in optimal expected profits for our generator. We will proceed with detailing the steps.

We begin by subdividing the $q - p$ plane containing the offer stack, with a finite rectangular grid by considering a range of price and quantities, each subdivided into intervals. For examples, a price range may be from a \$1.00 to \$1000.00 with finer step sizes for likely prices (tens to few hundred dollars) and coarser steps further out in the range. This will restrict the class of admissible supply functions to those which follow the edges of this grid. Then there are only finitely many admissible offer stacks, each consisting of a finite sequence of horizontal or vertical line segments that are grid edges. Note that for any grid edge e, being dispatched on edge e is independent of which other edges have been included in the offer stack. Therefore,

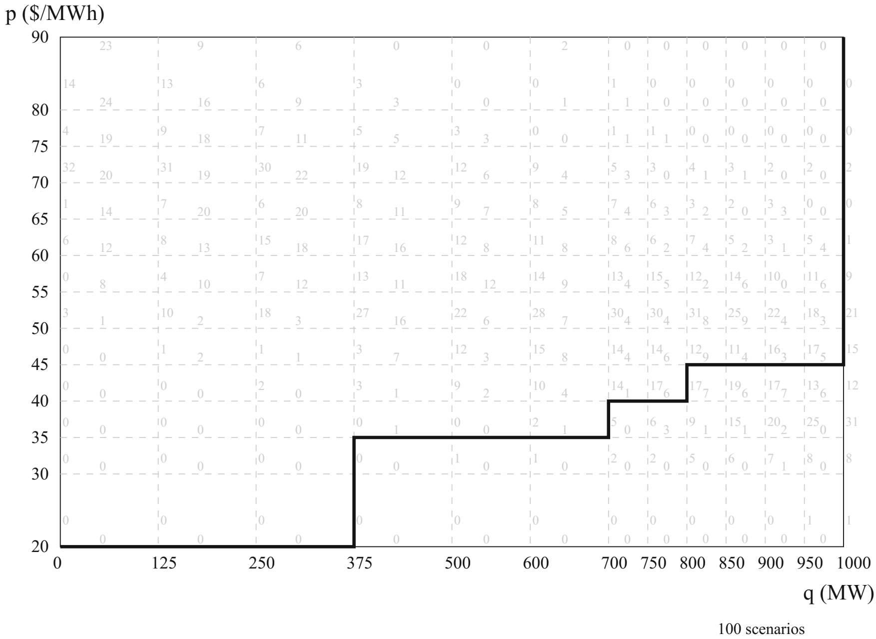

Fig. 2.3 A grid for building an optimal offer stack based on edge values using simulation

the expected payoff from any offer stack can be computed by adding the expected payoffs from the edges comprising this offer stack.

Let $V(e)$ denote the value of including edge e in the offer stack. To estimate $V(e)$ by simulation, we start with n randomly (and independently) chosen scenarios $\{\omega_1, \ldots, \omega_n\}$, where each "scenario" is a realization of the random elements of the problem (e.g. competitors' offer curves, loads, outages, etc.). Such scenarios may be extrapolated from historical information or may be based on richer ensemble forecast information. For each scenario ω_i and each edge e, we can compute the payoff $V_i(e)$ if ω_i results in a point of dispatch along e (i.e. when the offer curve includes e), or 0 if no such dispatch occurs. We can now approximate $V(e)$ by $\hat{V}(e) = \frac{1}{n}\sum_{i=1}^{n} V_i(e)$. Note that $\hat{V}(e)$ is a consistent unbiased estimator of $V(e)$. Figure 2.3 illustrates a (q, p) grid with along with frequency of dispatch attached to each edge.

To build the optimal offer stack resulting in the optimal expected profits for the generator, we can utilize dynamic programming. Due to the monotonicity constraint on any admissible offer stack, once at a vertex k of the grid, we must choose to continue right or up from that point. Therefore, the maximum expected payoff attached to a vertex k is given by

$$W(k) = \max(W_r(k), W_u(k)).$$

where

$$W_r(k) = \begin{cases} -\infty & \textit{if } k \textit{ is on } q = q_{max} \\ V(e_r(k)) + W(\nu_r(k)) & \textit{otherwise.} \end{cases}$$

In the above equation, $\nu_r(k)$ is the vertex adjacent to k on the right and $e_r(k)$ is the edge joining these vertices. $W_u(k)$ has an analogous description involving the edge and the neighbour above k. Given that $V((q_{max}, p_{max})) = 0$, we can start at the upper right-hand corner of the (q, p) grid and utilize a Bellman recursion to determine the optimal generator stack.

2.4 Bid Optimization for Large Consumers of Electricity

Similar to generators, large consumers in an electricity market, who are exposed to spot market prices, are often able to influence the clearing price through their decisions. These users, often large industrial sites or potentially aggregated blocks of residential or commercial users who wish to actively participate in the electricity market, can carefully choose their consumption level to influence price. There is a large amount of uncertainty associated with this problem, especially for participants who bid in the co-optimized ancillary service markets. For the same reasons as outlined above in the generation case, the problem of choosing an optimal consumption level, with an associated optimal reserve offer, is too broad to undertake analytically. As an alternative, numerical simulations can be used to approach this problem. A methodology to tackle this problem numerically was presented by Cleland et al. in [8]. This methodology is similar to what has already been presented for the generation case; however, it has nuances stemming from the co-optimization of energy and reserve. We present a concise detailed version here.

2.4.1 Reserve Co-optimization

Modern markets often incorporate the provision of ancillary services (AS) into the market dispatch problem. These ancillary services such as primary, secondary and tertiary contingency reserve or regulating reserve [11, 12] are often procured differently throughout the world. New Zealand has fully co-optimized primary and secondary contingency reserve via separate markets [1]. In Spain, for example, secondary reserve is procured for both contingency and regulation purposes, but primary reserve is a non-remunerated mandatory service [15]. In New Zealand, consumers are capable of participating in the AS markets through the provision of IL, for which they are paid the spot market reserve price for the FIR (primary) and SIR (secondary) markets. This benefits the consumer (industrial site) directly through additional revenue. But also indirectly, as the provision of IL capable reserve may release spinning reserve plant back to the energy market which may alleviate constraints.

The NZEM operates under $N-1$ reserve requirements, and sufficient reserve is procured to secure against the largest risk setter in each of New Zealand's two islands. In theory, this prevents under scheduling of reserve, in practice it can have a notable effect upon energy prices. When a risk setting asset (generator or transmission) is the marginal energy unit, the cost of securing the output from this unit (the reserve price) is incorporated into the energy price. For a marginal generator, the final energy price π is thus linked to the marginal energy, p_e, and marginal reserve, p_r, offer prices. We illustrate this through a very simple example. Let x_1 and x_2 and x_r represent the system dispatches from firms 1, 2 and reserve, respectively. Similarly, let p_1, p_2 and p_r denote the offered prices of energy and reserve by the firms, and q_1, q_2 and q_r the quantities available at the respective price. The small dispatch problem, meeting demand d in a single node network, is formulated as

$$\begin{aligned} \min \quad & p_1x_1 + p_2x_2 + p_rx_r \\ \text{s/t} \quad & x_1 + x_2 = d \qquad [\pi] \\ & x_r \geq x_1 \qquad [\lambda_{r1}] \\ & x_r \geq x_2 \qquad [\lambda_{r2}] \\ & x_i \leq q_i \quad i \in \{1, 2, r\} \\ & x_i \geq 0 \quad i. \in \{1, 2, r\} \end{aligned} \tag{2.2}$$

When $c_1 + r < c_2$, and $d < q_1$, meeting a marginal unit of demand will require procurement of an extra unit of energy. Hence, $\pi = p_1 + p_r$; this is easily verified by writing the KKT conditions.

If the marginal generator is transmitted from a neighbouring reserve zone (in New Zealand, these are differentiated by the two major island land masses), then the nodal energy prices become linked via the marginal reserve price in Eq. 2.3.

$$\pi_2 = \pi_1 + p_{r,2} \tag{2.3}$$

where π_1 and π_2 denote the locational marginal prices in nodes 1 and 2, respectively.

The above two examples are very simple illustrations of the interaction of energy and reserve prices. In reality, not only does reserve have to be covered for each of the North and South islands of New Zealand, energy and reserve are also restricted through constraints that express physical limitations such as ramp rate of a turbine in the event of an emergency shortage where reserves are called upon. The joint optimization of consumption and reserve offers is therefore a significant challenge theoretically. However, it may be approached numerically through simulations. Large consumers are an inviting target for this approach due to the convergence of means (manned control rooms, real-time prices, advanced metering) and motive (profit maximization), which is often missing from smaller consumers. These consumers thus satisfy many of the conditions which are a requirement for demand elasticity [6].

2.4.2 Optimization of Consumption and Reserves

We start by determining the consumption levels for our major consumer. These are naturally derived from the plant operation modes. In order to compute consumer profits, a utility figure for electricity consumption in a designated period may be necessary; note that this figure is inputted by the user, and they are free to experiment with a range of utilities. As the operational decisions for the plant are made ahead of time, the energy offers and other consumption quantities for the period in question are uncertain. Therefore, we need to consider a distribution. To address this, the user will input a base scenario. This can be a scenario derived from historical offers, e.g. the equivalent period on the previous day or a period closely matched to hydrology or demand conditions. We develop a "rest of New Zealand" set of scenarios that are generated from randomly scaled versions of demand (in nodes other than the one in question) for the base scenario. In particular, we can use a log-normal distribution for each island and the number of these scenarios can be chosen by the user.

For each demand level, corresponding to a plant operational mode, a distribution of energy prices at the consumer (site) node is determined. The site can then use this information to determine, under uncertainty, their optimal operating level. This can be done in expectation, or with any risk measure, as the distribution of prices attached to each consumption level is provided. Prices are used as they represent the only source of permitted variability in the site profitability calculation. A graph of the price distributions, found using simulation, is presented in Fig. 2.4.

As observed in Sect. 2.4.1, there may be a significant interaction between the market clearing price of electricity and the offered reserve prices. To take full account of this and determine a combined optimal consumption and reserve offer, we require a grid containing all admissible reserve supply stacks for the site. In other words, the quantity, price plane of possible offers, is subdivided into a finite grid consisting of rectangular cells, identical to what was presented for the generator offer case. This simplifies our problem as admissible offer stacks are those which follow the edges of the cells. we now output energy (and reserve) price distributions attached to each level of consumption. However this time, the price distribution attached to each consumption level is derived from the optimal reserve offer for the corresponding consumption level, for the period. For each consumption level (drawn from plant operation mode), we simulate different market scenarios using vSPD, as before. Each simulation will record the point of dispatch on any admissible reserve offer stack confined to our reserve grid. This is effectively done by tracing out the intersections of the "reserve residual demand" on the grid (as outlined in Sect. 2.3). We are now in a position to find an optimal reserve offer stack, for this consumption level using dynamic programming. The states of this DP are the vertices of the reserve grid. It is clear that the value to go attached to the top right corner of the reserve grid is zero (no reserves above our max quantity and max price will be procured). We solve the DP using backward recursion. The actions for this DP amount to amending a vertical (moving up) or a horizontal (moving right) segment to the reserve offer stack constructed thus far. Our choices are limited to up and right moves as the stack must be increasing.

The overall approach separates the co-optimization problem into three sequential steps. The influence of each consumption level on energy and reserve prices under uncertainty is determined in phase one. In the second phase, the optimal reserve offer stack attached to each consumption level is determined using the dynamic programming, very similar to the case for generator offers. Lastly, the optimal consumption level with its associated reserve offer stack is determined through a repetition of phase one, with the optimal reserve offer level in place. Cleland et al. have reported on the effectiveness of this methodology under various performance measures on experiments that span 13 months of data. The results are outlined in [7].

2.5 Conclusions

Pricing of electricity is a complex process that relies on solving a large-side constrained network optimization problem for every time period of every market. Many decisions, such as offer strategies for generators and consumption bids for major users of electricity, ought to be made based on a good understanding of electricity

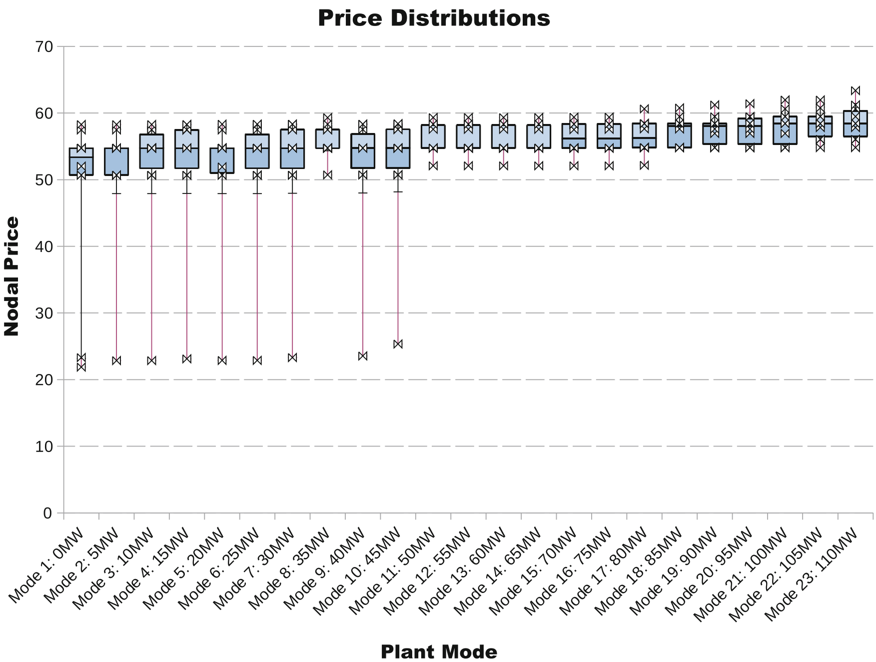

Fig. 2.4 Distribution of market clearing prices found through simulation

prices. We laid out in this chapter, two major applications of simulation–optimization over a deregulated electricity market. These applications have been developed and are in use in the NZEM.

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