# Chapter 13
# Comparison and Modelling of Pension Systems

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**Abstract** The purpose of this work is a comparison of pension systems of the selected countries—the pension systems and reforms of Austria, the Czech Republic, Slovakia, Sweden, Poland, and Chile will be our subjects of interest. Firstly, we focus on a short historical overview of the development and classification of pension systems in general. Consequently, the main part of this chapter deals with different scenarios, which should show whether the systems would be stable in the future. For these purposes, we developed utility in Mathematica. We tested normality of salary samples from Slovakia by robust tests for normality and computed pensions in several scenarios.

**Keywords** Pension systems and reforms · Modelling of pension systems
Lorenz curves · Gini coefficients · Robust testing for normality · Interest rates

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## 13.1 Introduction

Pension systems are a rather new invention, in the history of humanity. In former tribal societies and high cultures (as ancient Egypt, ancient Rome, and ancient China), there was also no real need for such systems. First, the populations were rather young, not many old people in comparison with the whole population and secondly one can doubt that such systems could be established and maintained by such ancient cultures. The first care system for elderly people besides the own family was the so-called Knappschaften, which was kind of social miners insurances. The next big step toward a widespread social security and pension system was initiated by the Bismarck's social reform. As a starting point, one can refer to the so-called Kaiserliche Botschaft (of November the 17th 1881). In the following years, the German Reichstag enacted several different social laws. The law for the pension insurance, the so-called Invaliditts- und Alterssicherung 1889 (Gesetzliche Rentenversicherung = GRV), became effective on January 1, 1891.

Generally, there are three so-called pillars or tiers, which define pension systems. These pillars or tiers are used by OECD. For more you can see classification of the pension system of the analyzed countries (see Table 13.1). One must say that these two terms sometimes are used synonymously. The first tier is mandatory and redistributive. The goal of this tier is to prevent people from old-age poverty. This first tier is divided into three main types. Basic schemes pay kind of flat-rate benefits where an additional retirement income does not change the entitlement.

The aim of this chapter is to simulate the stability of pension system. For this purpose, we focus on three pension systems—Austrian, Slovakian, and Swedish pension systems. We illustrate this problem at a pay-as-you-go pillar studied by [5] and [6]. Therein, the probability of oversizing the limiting value of pillar is studied under normality (see [6] p. 241) and for the light-tailed claims (therein pp. 241–243). Reference [6] considered the Cramer–Lundberg model in the case of a homogeneous portfolio with the attention focused on ruin probability for it under both light or heavy tails. They illustrated such situation in the setup of oversizing of the limiting value

**Table 13.1** Classification of the pension systems of the analyzed countries

| Taxonomy of selected pension systems | | | | | |
|---|---|---|---|---|---|
| | First tier | | | Second tier | |
| | Public | | | Public | Private |
| | Resource-tested | Basic | Minimum | Type | Type |
| Austria | | | | DB | |
| Czech Republic | | Yes | Yes | DB | |
| Chile | Yes | | Yes | | DC |
| Poland | | | Yes | NDC | DC |
| Slovak Republic | | | Yes | Points | DC |
| Sweden | | | Yes | NDC | DC |

of the fund for the pay-as-yo-go pillar in Slovakia (see also [5]). Reference [4] gives further consequences for insurance. We continue in research of sequence of papers [1, 7].

## 13.2 Comparison of Pension Systems in Analyzed Countries

### *13.2.1 Austrian Pension System*

The Austrian pension system consists of three different pillars. The main pillar is a public pay-as-you-go system. The former "Abfertigung," the Austrian form of the severance pay, was expanded to the "Abfertigung Neu." It is entirely funded by the employers. In a major pension reform in the early 2000s, a voluntary pension tier was established. The so-called Geförderte Staatliche Zukunftsvorsorge systematically is a state-aided funding principle.

According to the OECD taxonomy, the Austrian pay-as-you-go system is a defined-benefit public schema. There is also a so-called income-tested top-up for low-income pensions (Ausgleichszulage). The normal pension age is 65 for men and 60 for women. However, one must say that Austria has one of the lowest real retirement age of all OECD countries. On average, the Austrians retire at 58.1 (old age and disability pension combined). A factor that will stress the Austrian pension system for many years to come is the long transition period of the harmonization of women's pension age. Not until 2033, the retirement age of women will reach equality. The conditions to receive pension payments are the following. One has to pay 180 months of contributions within the last 30 years, or 300 months during the complete working career. There is an exception to this rule. Since 2005, it is possible to receive pension payments with only 7 years of contribution, if the remaining insurance period of 8 years can be reached, by child-raising periods.

### *13.2.2 Chilean Pension System*

In 1980, Chile replaced its pay-as-you-go public pension system with a system of individual accounts, the Chilean model. It is based on three tiers. The first tier is a poverty prevention tier, the second is an individual accounts tier, and the third is a voluntary saving tier. Next, I will describe the tiers in more detail.

**First Tier**

For all people who are older than 65 years and pass means test and lived in Chile for at least 20 years, and who did not contribute to individual accounts the state pays a basic pension of 75000 pesos, about 154$ per month (wage indexed started from

2008). There is also a second form of state-paid pension, the Pension Solidarity Complement (PSC). It is paid for people who contributed to individual accounts and pass means test. The amount and the calculation of the PSC depend on the height of the pension of the individual accounts tier.

**Second Tier**

The second tier is the major pension tier in Chile. It is a mandatory individual accounts system. Each employee contributes 10% of his/her wage or salary earnings into individual accounts. The contribution into the system is capped up to 67.4 UF (unidades de formento, a Chilean term which is used in many purposes, including pension contributions. In 2012, one UF equals about $22.46). The employers directly forward the contribution to a so-called AFP (Administradora de Fondos de Pensiones, these are private managed pension funds). Each employee chooses his/her desired AFP. An important point in the Chilean Pension System is that the employers generally do not contribute to the individual accounts, they only have to contribute to a survivor and disability insurance for their employees, and therefore they have to pay about 1.49% of the employees wage. Concerning the AFPs, one can say that during the time the employee can switch them at any time. But have to pay a certain fee for that. There are 5 AFPs, Funds A to E, which have different levels of risk and potential return. All AFPs must adhere to the rules drawn up by the government. The pension is calculated based on the accumulated assets. Age and gender are taken into account. There is the possibility of early retirement, if the pension equals at least 80% of the Pension Solidarity Complement. The assets accumulated can be withdrawn in four different ways. Also for funeral expenses, 15 UFs are reserved from the account balance.

**Third Tier**

This is a voluntary system. Workers can contribute to saving products which are authorized by the Chilean government, such as voluntary savings accounts managed by AFPs, mutual funds, and other savings products. Contributors may pay up to 50 UF per month to this pension tier. There is also the possibility to transfer savings accounts to the individual accounts, to increase future monthly pension annuity. Contributors receive certain tax preferences for this kind of payments. The government also tried to encourage employers with tax incentives, to contribute to voluntary savings accounts for their employees.

### *13.2.3 Slovakian Pension System*

Due to the fact that Slovakia was not an independent state until January 1, 1993, apart from a short period between 1939 and 1945, a historical summary makes no real sense. Since the pension reform of 2005, the Slovakian pension system consists of a reformed PAYG (Pay-as-you-go) state pension system and a funded pension system, which is divided into a mandatory personal pension tier and a voluntary pension tier.

Since the pension reform of 2005, people entering the labor market and all self-employed have to participate in the new reformed system. It consists of a social insurance public pension pillar and a funded pension pillar. People younger than 52 who already paid into the former pension system can choose if they also want to enter the mandatory private saving tier. The legal retirement age is 62 years for men and as of 2015 also for women. People gain eligibility after at least 10 years of contribution. The height of the pension benefits is calculated according to a point formula. Each contributor earns annual pension points (ratio of individual earnings to economy-wide average earnings). The sum of the pension points over the career multiplied by the pension point value is the pension entitlement. A point is worth 8.9955 Euro (2009) and indexed to average earnings. Pension payments are indexed to the arithmetic average of earnings growth and inflation.

There is no minimum pension, however a minimum pension base that is equal to the minimum wage of 295.5 Euro. In the new system, an incentive mechanism is established. The pension payments are increased by 0.5 percent for each 30-day period worked beyond retirement age. On the other hand, the pension is reduced the same percentage for each 30-day period worked less the retirement age. Nevertheless, there are three conditions necessary for receiving early retirement payments. Not before the age of 60, the fifteen-year contribution and the minimum pension have to be higher than 223.2 Euro.

### *13.2.4 Swedish Pension System*

Sweden had one of the most generous pension systems in the world. Due to financial difficulties during the eighties, Sweden decided to overcome the former pension system, which was a combination of a flat-rate basic pension and an earnings-related, contribution-financed, defined-benefit pension system. Within only a few years, Sweden changed its pension system considerable. It is now a multi-pillar system, which in its present design is considered as one of the most stable and reliable in the world. In the following, we will describe this new system. The Swedish pension system can be divided into three different pillars. The most important part is the national pension system. It accounts for about 3/4 of the pension payments and consists of three tiers. Further, there exists an occupational pillar, which accounts for about a fifth of the payments. Finally, there is also a voluntary fund-based pillar and it accounts only for 5% of the payments.

The Swedish national pension system is based on three tiers. In the following classification, labeling starts with 0. The tier zero is a guaranteed pension. The first tier is the so-called income pension, a pension system meanly based on a pay-as-you-go scheme. The last tier of the national pension system is a fund-based premium pension (bonus-pension). For people who were born before 1938, there applies the old ATP system. For persons born between 1938 and 1953, there applies a mixture of the old and the new reformed system.

**Table 13.2** Income values converted in EUR, 2000–2010

| Year | Austria | Sweden | Slovakia | Czech Republic |
|---|---|---|---|---|
| 2000 | 1.987,42 | 2.076,96 | 379,41 | 480,73 |
| 2001 | 2.002,92 | 2.177,46 | 410,44 | 522,87 |
| 2002 | 2.034,92 | 2.266,79 | 448,48 | 564,55 |
| 2003 | 2.064,33 | 2.344,95 | 476,83 | 597,50 |
| 2004 | 2.091,67 | 2.411,95 | 525,29 | 635,17 |
| 2005 | 2.142,00 | 2.467,78 | 573,39 | 667,10 |
| 2006 | 2.208,33 | 2.523,62 | 622,75 | 710,82 |
| 2007 | 2.288,17 | 2.557,12 | 668,72 | 762,13 |
| 2008 | 2.354,58 | 2.713,45 | 723,03 | 821,59 |
| 2009 | 2.378,08 | 2.802,78 | 744,50 | 848,93 |
| 2010 | 2.392,92 | 2.847,44 | 769,00 | 867,84 |
| $N$ | 11 | 11 | 11 | 11 |
| $\mu$ | 2.176,85 | 2.471,85 | 576,53 | 679,93 |
| $\sigma$ | 154,78 | 249,65 | 138,84 | 133,52 |

## 13.3 Dissimilarity of Income Levels

Due to the fact that incomes and wages are the basis of future pensions, we give a short overview of the different income levels of the above countries and check them according to their statistical similarity. In the following table, the income values are converted in EUR (Table 13.2).

## 13.4 Modeling of Pension Systems

Here, we continue in research based on [5]. This approach originally deals with the Slovakian pension system. Their fear is based on assumption that the $1^{st}$ pension pillar, so-called a pay-as-you-go system, is not sufficient to cover the liabilities of the future pensioners, because the number of contributors in relation to the pensioners worsens, so this fear is comprehensible. Therein is considered a closed group of Slovakian people, all aged 50 in the year 1998, and interest is in the estimation of the total claim amount for this group in the year 2010 when the members are supposed to retire. For this purpose, they also assume a linear relationship between the salary $S_t$ and pension $P_t$ at time $t$.

Therefore, [5] is interested in estimation of the probabilities $P\left(\sum_{k=1}^{N} X_k > C\right)$, where $X_i$ are individual monthly claims of the members of the above-mentioned group and $C$ is a critical (limiting) value of the fund representing the amount the fund has gathered from the contributions of the active members or from other sources. It is possible to consider $N$ as a constant or a random variable as it was treated in [5, 8]. In [6], the case that $N$ is a random variable was considered. Then following [5], it is quite natural to choose a binomial model for $N$, namely $N \sim bi(n, p)$ with $n = 130000$ and $p$ representing the probability of surviving a 50-year person from the group to the age 62 years. Note that such probabilities are regularly published by Slovak Statistical Office (see [9]). Then, one is looking for the largest $C$ such that $P(\sum_{k=1}^{N} X_k > C) = p$ with $p$ given in advance, e.g., 0.1 or 0.05.

Typically, it is possible to model salaries as normal variables in short terms and lognormal at long terms. In [5] is used the normal distribution which led to the following upper bound

$$\bar{p} = 1 - \Phi\left(\frac{C/(kN_t) - \mu}{\sigma}\right). \tag{13.1}$$

Here, $\Phi$ is cdf of standardized normal distribution, $C$ is a critical level as given above, $\mu$ and $\sigma^2$ are parameters of normal distribution of salaries, $k = \frac{P_t}{S_t}$, and $N_t$ is the number of claims.

Consequently, we will simulate the example given in the mentioned paper [5] and we will also show other settings based on estimated Austrian and Swedish numbers. For the implementation of the model, in the following designated as simply tool, we used the mathematical programming language Mathematica Version 8.0.

At the beginning, we will reconstruct the example of the paper [5]. Therefore, we need the average maximum Slovakian salaries from 1998 to 2002, which are shown in the following table (note that this data was used for testing for normality in the paper [10]):

As it is mentioned above, typically it is possible to model salaries as normal variables in short terms and lognormal at long terms. In [5] is used the normal distribution which led to the upper bound in Eq. (13.1). In the case of Table 13.3, we have $\hat{\mu} = 29396.4$ and $\hat{\sigma} = 3903.35$. Therefore, the first screenshot of the tool is shown in Figs. 13.1 and 13.2, which show the development of (13.1), given $N_t = 130000$ and $k \in (0.5, 0.67)$ and $C \in (15 * 10^6, 29 * 10^8)$.

**Table 13.3** Slovakian Salaries (Slovakian Koruna), 1998–2002

| Year | 1998 | 1999 | 2000 | 2001 | 2002 |
|---|---|---|---|---|---|
| Salary | 24233 | 26862 | 30021 | 31825 | 34041 |

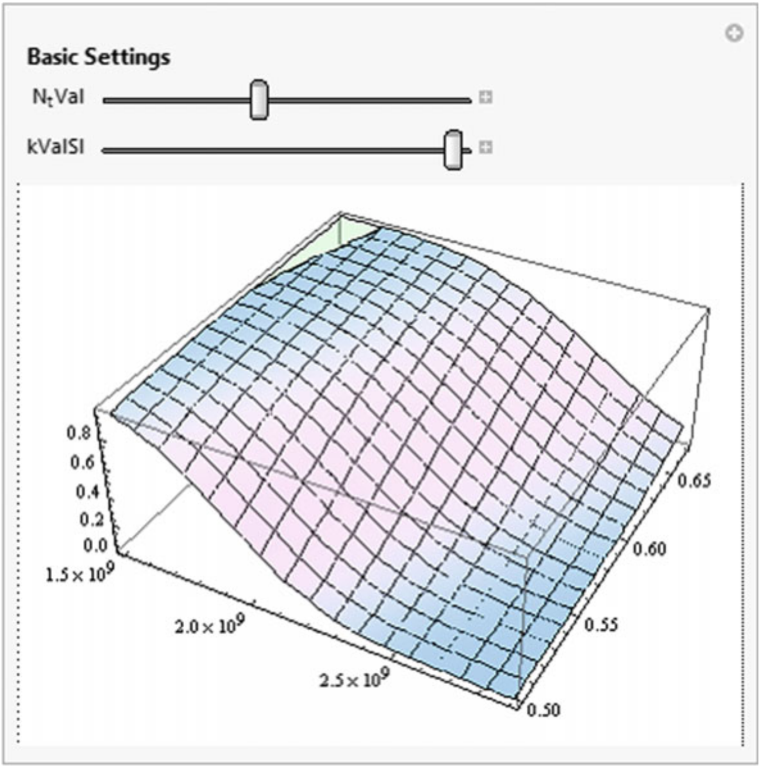



**Fig. 13.1** Slovakian wages

We also implement the same model for the minimum wages (for reasons of simplification we assume $N_t = 130000$). Next, Table 13.4 presents average Slovakian minimum and maximum wages and Figs. 13.3 and 13.4 show the different development of the ruin probability when we use average Slovakian minimum and maximum wages.

From Fig. 13.4, we can see that if all Slovaks only would receive minimum average wage, there would not be any problems with future pension payments at all. The ruin probability is practically zero. On the other hand, if ever Slovak would earn the maximum average wage the ruin probabilities are very high (pink surface) many different settings.

Alternatively, we want to show the situation of Austrian incomes. In addition, the Austrian incomes we got from Statistik Austria source—see Table 13.5. As we can see from this table, the Austrian wages moved slower than the Slovakian ones. Therefore,

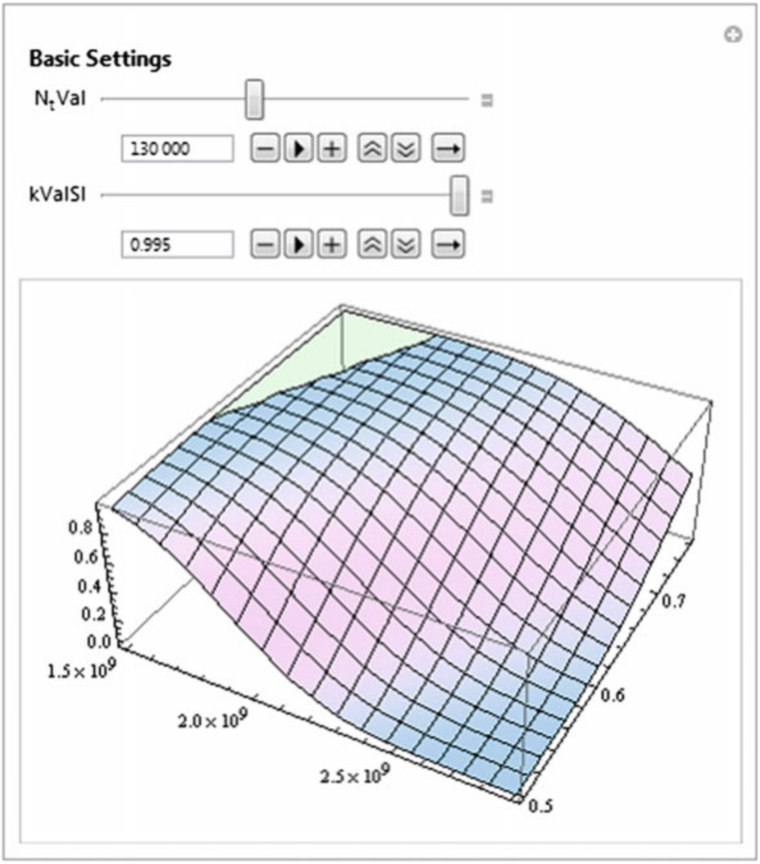



**Fig. 13.2** Slovakian wages with active sliders

Fig. 13.5 shows the basic model with the Austrian situation, given the wages from Table 13.5 and the following starting settings: $N_t = 75500$ and $k \in (0.5, 0.67)$ and $C \in (10 * 10^8, 20 * 10^8)$.

Finally, we also show the development of $p$ (ruin probability) for the Swedish incomes—data are also presented in Table 13.5. Consequently, Fig. 13.6 shows the basic model with the Swedish situation, given the wages from Table 13.5 and the following starting settings: $N_t = 124000$ and $k \in (0.5, 0.67)$ and $C \in (6 * 10^9, 3 * 10^{10})$.

**Table 13.4** Monthly mean brutto salary in Slovakia (EUR), 1992–2013 [9]

| Year | Max. salary group | Min. salary group | Year | Max. salary group | Min. salary group |
|---|---|---|---|---|---|
| 1992 | 255.6 | 138.3 | 2003 | 1148.1 | 294.7 |
| 1993 | 329.4 | 152.6 | 2004 | 1315.6 | 314.8 |
| 1994 | 386.4 | 172.6 | 2005 | 1418.1 | 339.9 |
| 1995 | 451.2 | 199.2 | 2006 | 1511.6 | 364.9 |
| 1996 | 645.9 | 210.8 | 2007 | 1609.4 | 398.2 |
| 1997 | 827.4 | 187.9 | 2008 | 1705.1 | 431.5 |
| 1998 | 807.7 | 206.9 | 2009 | 1728.9 | 474.9 |
| 1999 | 895.4 | 214 | 2010 | 1790 | 491 |
| 2000 | 1000.7 | 226.1 | 2011 | 1835 | 503 |
| 2001 | 1060.8 | 242.1 | 2012 | 1923 | 521 |
| 2002 | 1134.7 | 284.4 | 2013 | 1934 | 532 |

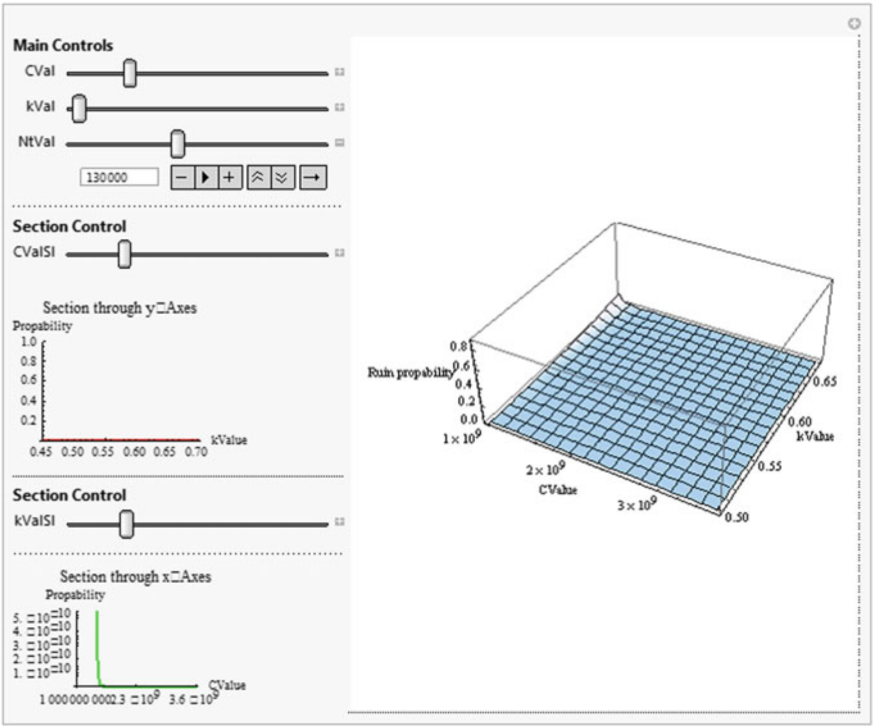



**Fig. 13.3** Slovakian wages—minimum average wages 1998–2007

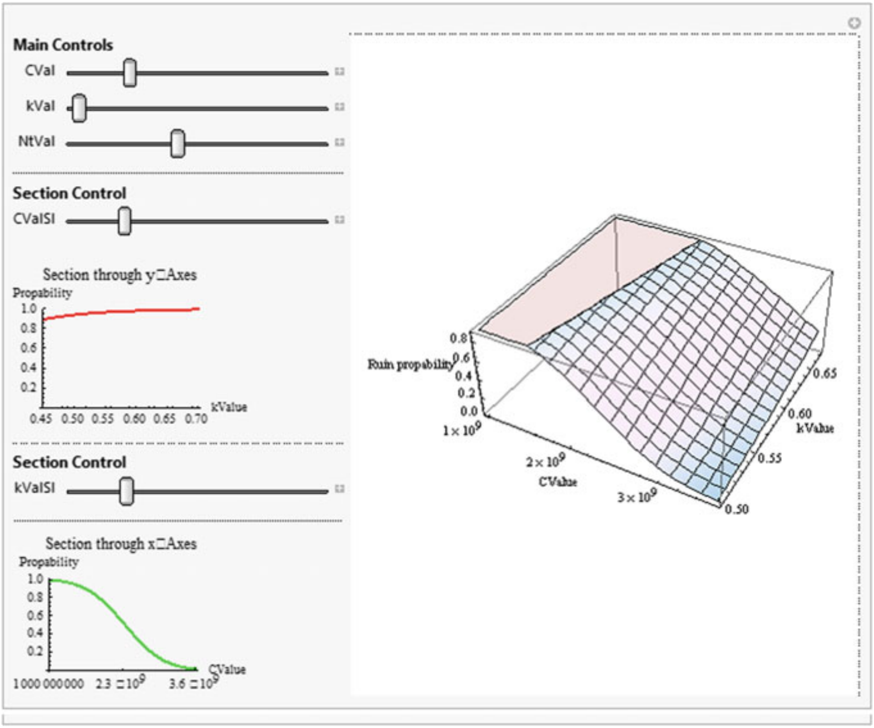



Fig. 13.4 Slovakian wages – maximum average wages 1997–2007

**Table 13.5** Austrian average salaries (Euro) and Swedish average salaries (1000 Swedish krona), 1998–2007

| Year | Austria | Sweden | Year | Austria | Sweden |
|---|---|---|---|---|---|
| 1998 | 22857 | 182.0 | 2003 | 24772 | 222.7 |
| 1999 | 23311 | 190.7 | 2004 | 25100 | 227.9 |
| 2000 | 23849 | 200.9 | 2005 | 26500 | 234.7 |
| 2001 | 24035 | 210.5 | 2006 | 27458 | 242.0 |
| 2002 | 24419 | 217.4 | 2007 | 28262 | 251.9 |

## 13.5 Calculated Gini Coefficients and Lorenz Curves

This chapter concentrates on the wage distribution. Therefore, the earnings distributions of the different countries are shown. Concrete the deciles, out of this a measure for evenly distribution is calculated, namely the Gini coefficient. The Lorenz curve is linked with the Gini coefficient. These measures are calculated with the R-Package `ineq`.

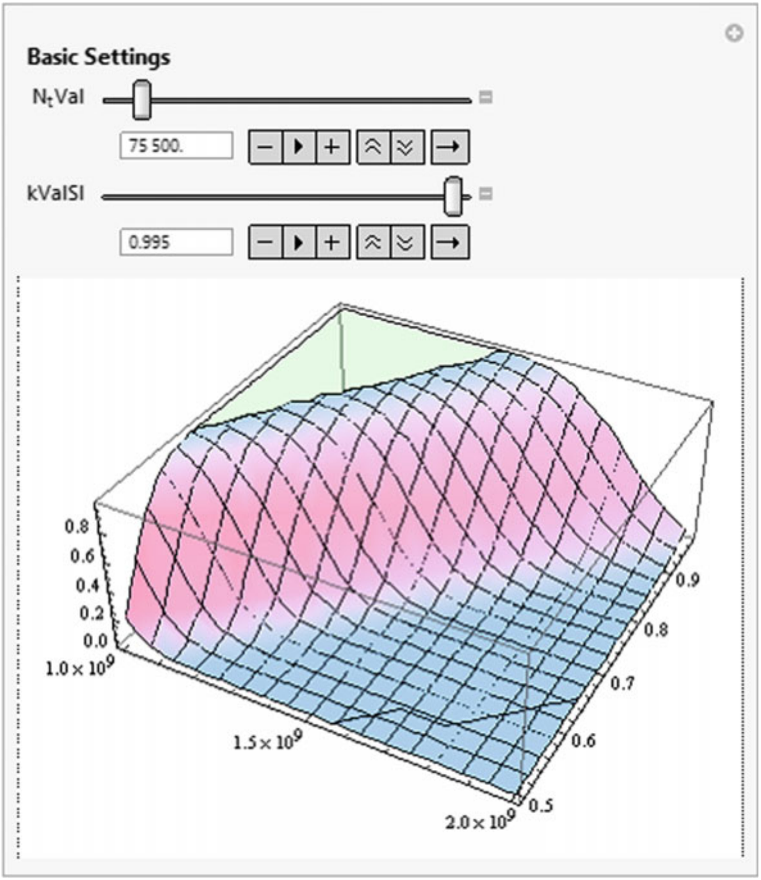



**Fig. 13.5** Austrian wages

### *13.5.1 Calculation of Gini Coefficient*

The below-mentioned table consists of the available income distribution. The data are mainly from ILO, the International Labor Organization. This leads to a more or less unified data set. Only for Sweden, there is no suitable data, so the Gini coefficient is calculated based on data from Eurostat. We considered another income distribution for Austria and form data of Statistik Austria. The reason for that is that the first decile of the Austrian income data of ILO looks unreasonably low. Also, the ratio of last and first decile (83.3) is unreasonably high. This is typically a characteristic of low-income and developing countries. In contrast to the ILO data, the data provided by Statistics Austria are based on annual wages (Table 13.6).

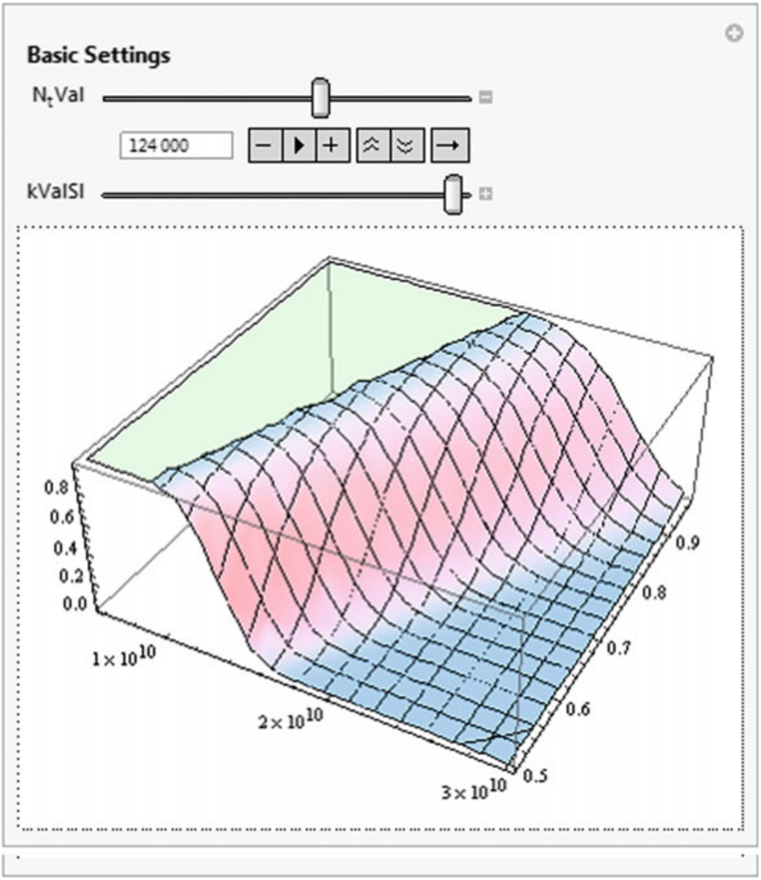



**Fig. 13.6** Sweden wages

### *13.5.2 Lorenz Curves*

In this section, Lorenz curves for all countries are shown. Therefore, Fig. 13.7 shows the Lorenz curves of all considered countries, i.e., for Austria, Chile, the Czech Republic, Poland, Slovakia, and Sweden.

## 13.6 Impact of Interest Rates on Pensions

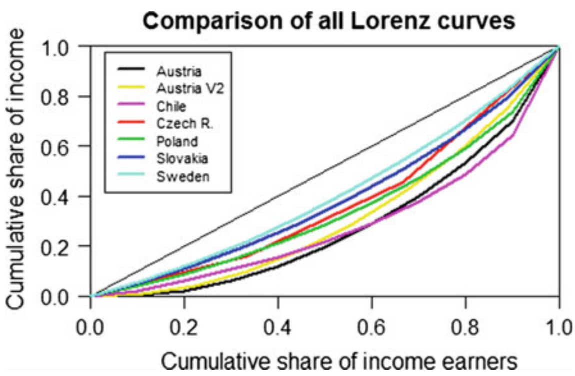
Here, we study the influence of interest rates on pensions (see [13]). Generally, there are three main points that lead to low fund returns.

- High fund management fees.

**Table 13.6** Countries and their mean monthly earnings of employees by decile (local currency/most recently of data) and the Gini coefficient

| Decile | Austria | Austria | Chile | Czech Rep. | Poland | Slovakia | Sweden |
|---|---|---|---|---|---|---|---|
| | EUR/2011 | EUR/2012 | CLP/2011 | CZK/2013 | PLN/2010 | EUR/2013 | SEK/2013 |
| Decile1 | 87 | 2400 | 89805 | 11972 | 1350 | 386 | 122709 |
| Decile2 | 407 | 7309 | 167754 | | 1660.5 | 469 | 155295 |
| Decile3 | 920 | 13232 | 191513 | | 2010.9 | 554 | 182202 |
| Decile4 | 1412 | 19031 | 217659 | | 2360.7 | 637 | 206667 |
| Decile5 | 1854 | 24540 | 256606 | 22557 | 2719.6 | 718 | 229912 |
| Decile6 | 2269 | 29700 | 305286 | | 3104.2 | 815 | 253738 |
| Decile7 | 2688 | 35327 | 375511 | | 3564.2 | 934 | 280526 |
| Decile8 | 3218 | 43215 | 475303 | | 4144.4 | 1106 | 316268 |
| Decile9 | 4079 | 57736 | 657366 | 41600 | 5073.2 | 1452 | 373978 |
| Decile10 | 7247 | | 1534495 | | 9440.2 | | |
| Gini coefficient | 0.438 | 0.367 | 0.43 | 0.259 | 0.314 | 0.224 | 0.181 |

**Fig. 13.7** Comparison of Lorenz curves




- Fund mismanagement.
- Lower than expected interest rates.

In [13] the following scenario based on the Slovakian data from Table 13.4 was considered. In this table the monthly mean gross salary is shown for both rich and poor males who were 45 years old in 1993 (max. and min. salary) and who invested in a fund with a certain interest rate until 2009. The authors assumed a contribution rate $u = 0.09$, and fix interest rates of $r \in \{0.005, 0.01, 0.02\}$, and additionally a non-fix interest function $r(t) = 0.05exp(-t/10)$, whereas $t$ is the calendar year 1993.

With this parameters and the following equation of equivalence, they calculated the pension height and the replacement rate:

**Table 13.7** Estimation of pension at age 62 for 45 year old male, $r(t) = 0.05exp(-t/10)$

| Distribution | r | v | Pmax | Pmin |
|---|---|---|---|---|
| Weibull | 0.005 | 0.27154 | 3056.08 | 842.3 |
| $\hat{\gamma} = 0.033$ | 0.01 | 0.30135 | 3298.08 | 913.12 |
| $\hat{\beta} = 1.548$ | 0.02 | 0.36914 | 3827.27 | 1069.53 |
| | r(t) | 0.24407 | 2292.01 | 620.108 |
| Gamma | 0.005 | 0.21724 | 3056.07 | 842.3 |
| $\hat{\gamma} = 0.084$ | 0.01 | 0.2437 | 3298.09 | 913.12 |
| $\hat{\beta} = 2.297$ | 0.02 | 0.3039 | 3827.27 | 1069.54 |
| | r(t) | 0.19283 | 2179.2 | 589.59 |
| Logistic | 0.005 | 0.41675 | 3056.08 | 842.3 |
| $\hat{\mu} = 27.35$ | 0.01 | 0.46088 | 3298.08 | 913.13 |
| $\hat{\sigma} = 9.948$ | 0.02 | 0.56116 | 3827.27 | 1069.53 |
| | r(t) | 0.37605 | 2331 | 630.657 |
| Makeham | 0.005 | 0.158 | 3855.45 | 1021.24 |
| | 0.01 | 0.1344 | 3310.29 | 872.22 |
| | 0.02 | 0.1193 | 3062.15 | 804.8 |
| | r(t) | 0.1085 | 2400.89 | 649.566 |

$$\sum_{t=1}^{17} 0.95 \times 0.09 \times X_{t+1992} \times (1 + r_t)^{17-t} {}_t p_{45} =$$

$$= \sum_{t=0}^{\infty} 0.95 \times v \times P \times (1 + r_t)^{-t} {}_t p_{62}.$$

Here, the left-hand side (LHS) and right-hand side (RHS) mean the following. LHS: Left side sums up contributions during working life (45–62). Parameter of the LHS: 0.95 is a constant which discounts the contribution by 5% (normally the fund fee). The constant $0.09 = u$ is the contribution rate, $X_{t+1992}$ is the salary in year $t + 1992$ (see table), $r_t$ is the interest rate in year $t$, and ${}_t p_{45}$ stands for the probability a person aged $x = 45$ survive the next $t$ years. These probabilities were modeled with different distributions that generally fit mortality rates quite well.

RHS: Right side is life annuity of the surviving pensioner, $v$ stands for replacement rate, and $P$ is the pension.

The equation was then solved and resulted in the following results for different mortality distributions and interest rates (Table 13.7).

Out of this concrete numbers of the replacement rate and the pension height, the chapter also concluded the following. The more realistic non-fix interest rates lead to the lowest pension and replacement rate. And maybe the most important conclusion is that for the poorer males the expected pensions are too low in comparison with

**Table 13.8** Expected value of pension at age 62 for 45 years old male with fixed $v$ for two groups salary

| | $\sigma$ | $\mathbb{E}[P_{max}]$ | Std. err. | $\mathbb{E}[P_{min}]$ | Std. err. |
|---|---|---|---|---|---|
| $f(t) = 0.05 \exp\left(-\frac{t}{10}\right)$ | 0.001 | 3704.093 | 1.303 | 998.555 | 0.353 |
| | 0.01 | 3786.651 | 13.267 | 1016.706 | 3.657 |
| | 0.05 | 6982.14 | 119.479 | 1966.152 | 35.305 |

**Table 13.9** Expected value of pension at age 62 for 45 years old male with fixed $v$ for two groups salary

| | $\sigma$ | $\mathbb{E}[P_{max}]$ | Std. err. | $\mathbb{E}[P_{min}]$ | Std. err. |
|---|---|---|---|---|---|
| $f(t) = 0.02 + 0.05 \exp\left(-\frac{t}{10}\right) \cos t$ | 0.001 | 3287.102 | 1.100 | 851.710 | 0.284 |
| | 0.01 | 3357.465 | 11.183 | 871.762 | 2.929 |
| | 0.05 | 6087.512 | 106.88 | 1570.891 | 27.392 |

minimal pension of 250 Euros guaranteed in the first pension pillar. So consequently they should rather stay in the 1st pension pillar (see [13]).

### 13.6.1 *Computation of Pension Under Stochastic Interest Rates: An Example*

Now consider that paths of interest rate are given by the process

$$r_t = f(t) + \sigma W_t,$$

where $f \in C[t_0, \infty)$ (Figs. 13.8 and 13.9).

Obviously, $r_t$ is normally distributed with $\mathbb{E}[r_t] = f(t)$ and $\text{Var}[r_t] = \sigma^2 t$. The number of replications was $10^4$ (Tables 13.8 and 13.9).

## 13.7 Testing for Normality – RT Class Tests

The general RT class is based on robustification of the classical Jarque-Bera test. The general RT class test statistic is defined by [11] for the purpose of robust testing for normality against Pareto tails and more analyzed in [12].

For the example purposes, we consider some classical non-robust tests of normality with higher power against the broad scale of alternative distributions—the

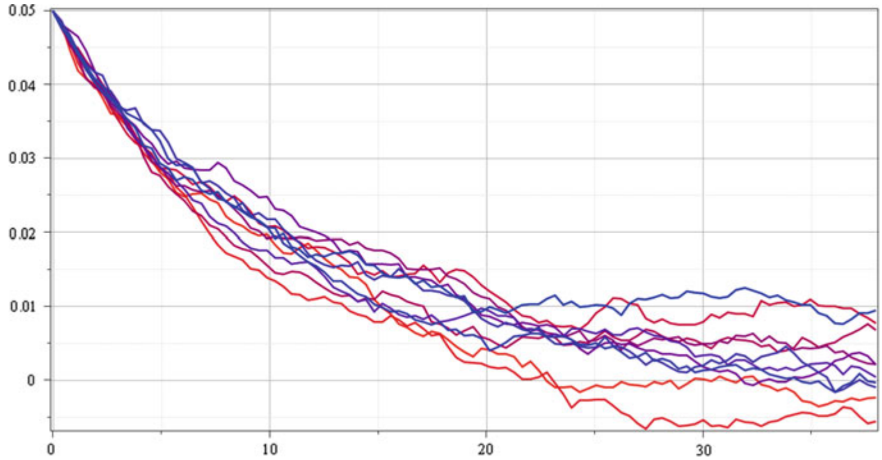



**Fig. 13.8** Ten paths of $r_t$ with monotonic expectation

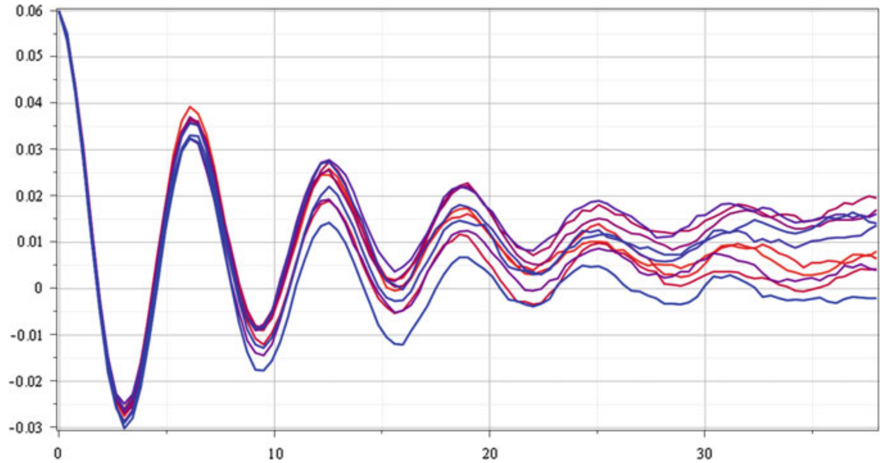



**Fig. 13.9** Ten paths of $r_t$ with oscillatory expectation

Shapiro–Wilk test (SW) as the most popular omnibus test of normality for a general use, the Jarque-Bera test (JB) as the most widely adopted omnibus test of normality in finance and related fields, and the Anderson-Darling test (AD) and the Lilliefors test (LT) as the most famous tests of normality based on the empirical distribution function – accompanied with several new tests for normality based on robust characteristics, in particular, the medcouple test (MC-LR) introduced by [2], the robust Jarque-Bera test (RJB) introduced by [3], and the selected robust tests from the RT class, namely MMRT1, MMRT2, TTRT1 and TTRT2 – for more details of these RT class tests see [12]. We also suppose the data set of max. and min. salary group for Slovakia presented in Table 13.4.

**Table 13.10** Test statistics and p values of analyzed normality tests for Slovakian data of max. and min. salary group

| | Max. salary | | Min. salary | |
|---|---|---|---|---|
| | Statistic | p-value | Statistic | p-value |
| $AD$ | 0.333 | 0.487 | 0.708 | 0.055 |
| $JB$ | 1.248 | 0.303 | 1.955 | 0.135 |
| $LT$ | 0.119 | 0.630 | 0.160 | 0.191 |
| $RJB$ | 1.024 | 0.354 | 1.320 | 0.256 |
| $SW$ | 0.945 | 0.305 | 0.897 | 0.038 |
| $MC_{LR}$ | 1.198 | 0.591 | 5.525 | 0.039 |
| $MMRT1$ | 1.118 | 0.417 | 2.642 | 0.119 |
| $MMRT2$ | 1.137 | 0.421 | 2.836 | 0.092 |
| $TTRT1$ | 2.367 | 0.427 | 4.874 | 0.200 |
| $TTRT2$ | 1.013 | 0.504 | 2.014 | 0.258 |

Based on results presented in Table 13.10, we can conclude that the hypothesis of normality of analyzed data sets is not rejected by the majority of tests for normality, at 5% significance level. Only Shapiro–Wilk test rejects the hypothesis of normality in the case of minimum salary data, at 5% significance level. We can also see higher robustness of the TTRT1, TTRT2, MC-LR, and RJB tests in comparison with the classical normality tests such as the classical Jarque-Bera test, Shapiro-Wilk test, etc.

## 13.8 Summary

As conclusion, we can say that there are a few basic pension system concepts, which are combined in different ways. Consequently, each pension system of the three countries is unique. From our simulation study, we can see that in the future each country would face difficulties financing their pension system, because of the rising of the old dependency rate.

For proper pension system management, one should at least use two different approaches. Namely, ruin probability and a kind of income distribution measure, and their suitable data representation (curves, indices) ROC-shaped curves have similarities to probability distribution of the ruin, so we can use the indices of the ROC curve on it. For the income distribution point of view, we used the Lorenz curves and Gini coefficients.

In a society, a certain level of financial balancing is wishful. Therefore and because of the 80:20 rule (∼20% of fund owners, hold ∼80% of the fund assets), the wealthy should invest in private pension fund. The so generated taxes could be redistributed to the poorer in the pension system.

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## References

1. Adams, J.H.: A nonlinear regression model of incurred but not reported losses. In: CASE Forum Summer, pp. 1–14 (2007)
2. Brys, G., Hubert, M., Struyf, A.: Goodness-of-fit tests based on a robust measure of skewness. Comput. Stat. **23**, 429–442 (2008)
3. Gel, Y.R., Gastwirth, J.L.: A robust modification of the Jarque Bera test of normality. Econ. Lett. **99**, 30–32 (2008)
4. Potocký, R.: Modelling of the claims for non-life insurance. Forum Statisticum Slovacum **3**(4), 122–126 (2007)
5. Potocký, R., Stehlík, M.: Analysis of pensions in the 1st pilar under the expected demographic development. In: Proceedings of 10th Slovak Conference on Demography, Smolenice (2005)
6. Potocký, R., Stehlík, M.: Stochastic models in insurance and finance with respect to basel II. JAMSI, **3**(2), 237–245 (2007)
7. Potocký, R., Stehlík, M.: Improved nonlinear regression modeling of parameters of IBNR reserves. Commun. Dependability Qual. Manag. **11**(4), 54–60 (2008)
8. Potocký, R., Waldl, H., Stehlík, M.: On sums of claims and their applications in analysis of pension funds and insurance products. Prague Econ. Pap. **3**, 349–370 (2014)
9. Statistical database of indicators of economic and social-economic development in the Slovak Republic, Statistical Yearbook of Slovakia, 1998–2013
10. Stehlík, M., Střelec, L.: On normality assumptions for claims in insurance. Acta Universitatis Agriculturae et Silviculturae Mendelianae Brunensis **57**(3), 141–146 (2009)
11. Stehlík, M., Fabián, Z., Střelec, L.: Small sample robust testing for normality against Pareto tails. Commun. Stat. - Simul. Comput. **41**(7), 1167–1194 (2012)
12. Stehlík, M., Střelec, L., Thulin, M.: On robust testing for normality in chemometrics. Chemom. Intell. Lab. Syst. **130**, 98–108 (2014)
13. Stehlík, M., Potocký, R., Kiselak, J., Jordanova, P.: On generalized interest rate dynamics. Appl. Math. Inf. Sci. **9**(2L), 325–338 (2015)