Chapter 8
An Overview of Traffic Control Schemes for Freeway Systems

8.1 Freeway Traffic Management and Control

The need for the development of surveillance and control strategies for freeway traffic networks has increased in the past decades because of the persistent growth of traffic congestion and the resulting negative effects on people and on the ecosystem. Freeway networks, although designed to meet the mobility needs of high traffic flows, have suffered in recent years the increasing demand which can be rarely solved with proper infrastructure interventions (see Chap. 1 for a more detailed discussion on these aspects). Consequently, the adoption of specific control measures represents, in many cases, the only possible answer to improve the performance of freeway traffic systems.

Moreover, in recent years, the development of information systems supporting the drivers when travelling along freeways has strongly increased thanks to the progress in detection, transmission and data processing technologies. In fact, an important aspect in efficiently managing a freeway network is the implementation of a reliable *traffic monitoring system* or, analogously, a *traffic surveillance system*, able to elaborate the information coming from sensors located throughout the network, to detect possible critical situations and to provide, both to controllers and to road users, useful information about the current state of the system and, even in some cases, a prediction of its evolution in the short–medium term.

Besides monitoring the traffic state, a further advancement in the management of a freeway traffic system consists in controlling and regulating traffic flows in order to improve the performance of the system itself. *Freeway traffic control systems* have been developed and are still under investigation by scientists, in order to act on the system in real time, depending on the present system state and, in some cases, also on its predicted evolution. One of the main objectives of a freeway traffic control tool is the reduction of congestion, i.e. the reduction of the travel times for drivers (see Fig. 8.1). Clearly, reducing congestion and delays for travellers often entails the reduction of other negative effects of traffic, more related to sustainability and quality of life of citizens. Nevertheless, in some recent freeway traffic control systems, these

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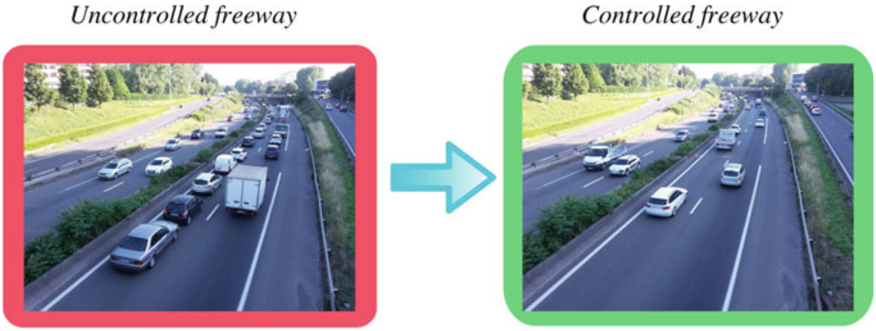

Fig. 8.1 Congestion reduction as a main objective of traffic control

related objectives are explicitly taken into account, i.e. some traffic controllers are specifically devised to reduce traffic emissions, noise, accidents and so on.

8.1.1 Traffic Control Strategies

Freeway traffic can be controlled in different ways. The most traditional way is the implementation of *road-based traffic control*, which is realised by regulating traffic at a macroscopic level. In particular, referring to road-based traffic control, it is possible to regulate the access of traffic flows to the freeway, by implementing *ramp management* policies, or to control the movement of vehicles inside the freeway, via *mainstream control*, or to route vehicles on specific paths, implementing suitable *route guidance* strategies. Of course, these control strategies can be properly combined via *integrated control*, in order to achieve better performance for the freeway system.

An example of a freeway regulated via road-based traffic control strategies is depicted in Fig. 8.2, showing two controlled on-ramps, two installations of mainstream control and one junction in which route guidance indications are provided. Note that road-based traffic control strategies act at a system level, e.g. the traffic lights at the on-ramps regulate the access to the freeway of the whole flow of incoming vehicles, as well as variable speed limits or routing indications displayed on Variable Message Signs (VMSs) are the same for all the drivers passing in front of them.

The technological development of electronic devices present on board of vehicles is allowing and will allow in the near future the wide diffusion of control policies specifically devised for each driver, according to a *vehicle-based traffic control* logic. Vehicle-based traffic control is a new concept, surely promising for the next years, but on which very few research results have been developed so far. Analysing the literature on this topic, it is worth mentioning works referring to control schemes in which the control actions are determined considering the whole traffic system but are

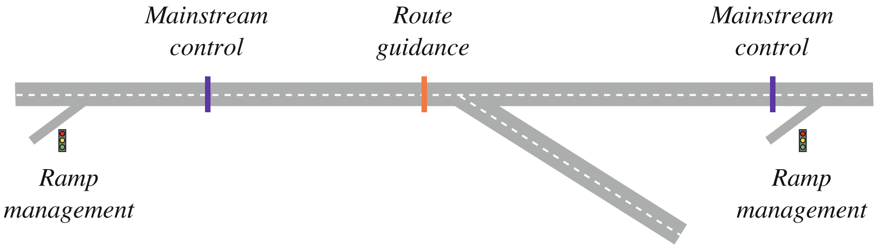

Fig. 8.2 Road-based traffic control strategies

transferred to vehicles via Vehicle-to-Infrastructure (V2I) communication systems (see e.g. [1–6]), and other works addressing the issue of coordination mechanisms for Connected and Automated Vehicles (CAVs) (see e.g. [7, 8] and the references therein).

In this chapter and in the following ones, we will mainly focus on road-based traffic control strategies, which researchers have studied for some decades and which still are the most interesting traffic control options for real implementations.

Ramp Management Ramp management strategies are applied in order to control the flow of vehicles entering the freeway mainstream. The most widespread strategy belonging to this category is surely *ramp metering*, which regulates the access of traffic flows to the mainstream through traffic signals installed at the on-ramps. The idea of controlling traffic flows by metering the on-ramps has been exploited since the 60s in the U.S., with very simple control strategies based on historical data [9, 10]. The very first implementation of ramp metering occurred in 1963 on the Eisenhower Expressway in Chicago, U.S., where a police officer was in charge of regulating traffic at the on-ramp in order to allow a safer and smoother merging into the freeway.

Most of the ramp metering strategies are devoted to reduce the *onset of congestion* phenomena, by managing the amount of traffic flows entering the freeway and by facilitating the merge of the on-ramp flows with the mainstream. Some traffic phenomena associated with merging areas are discussed in [11], based on real observations in the U.S., in which the effects of ramp metering strategies are analysed in detail.

Besides the prevention of traffic breakdowns, further benefits of ramp metering applications are widely documented in the literature (see e.g. [12, 13], which also refer to field implementations). For instance, an important phenomenon associated with congestion is the *blockage of off-ramps*, i.e. the fact that vehicles which would like to exit the freeway are delayed because they are stuck in the traffic jam, hence further increasing congestion. If a ramp metering strategy acts effectively in reducing the congestion, this can also reduce or eliminate the off-ramp blockage phenomenon, and such amelioration is more relevant in case the congestion would involve more than one off-ramp (see Fig. 8.3).

Fig. 8.3 Off-ramps in A1 freeway, close to Florence, Italy (courtesy of Autostrade per l'Italia SpA, photo from Archivio Videofotografico Autostrade per l'Italia)

In addition, the use of traffic lights at the on-ramps may increase *safety* during the merge phases. This is due to the fact that ramp metering prevents the entry of long platoons of vehicles, and, also, vehicles are induced to enter the freeway with lower speeds, reducing the risk of collisions. A better merging behaviour by vehicles can be also translated in a reduction of *pollutant emissions* in the environment.

Despite the clear positive effects achieved on the efficiency of the freeway infrastructure when implementing ramp metering policies, some critical issues may arise. One of the main drawbacks due to the application of ramp metering is the creation of long queues at the entering on-ramps. In strongly congested scenarios, the presence of *limited storage space* (which is very common, especially in urban freeways) may induce a queue spillback that can compromise the functionality of the adjacent infrastructure. In addition, the formation of long queues may generate dissatisfaction in the road users.

Ramp metering has been applied successfully for some decades and still is very widespread worldwide. Many real applications may be found in the United States, in Europe (especially in the Netherlands) and in Australia.

Mainstream Control Mainstream control is used to regulate traffic flows of vehicles travelling in the mainstream, generally showing proper indications to drivers through VMSs or with mainstream traffic lights. It is indeed proven that the operability and safety of freeway traffic may be potentially improved through control actions on the mainline (see for instance Fig. 8.4, representing a forming congestion in a freeway controlled with variable speed limits).

Fig. 8.4 Variable speed limits in A20 freeway, close to Rotterdam, the Netherlands (courtesy of Rijkswaterstaat, Photo: Essencia Communication/Rob de Voogd)

At a general level, these mainstream control actions have the aim of homogenising the traffic conditions, preventing the formation of recurrent congestions and reducing the probability of collisions among vehicles. An additional objective is to face the formation of phenomena of non-recurrent congestion, by increasing the efficiency of the system under conditions of limited capacity.

One of the most widespread mainstream control measures in freeway networks is represented by *variable speed limits*, widely applied in Northern Europe. This methodology aims to improve *mobility* and *safety* conditions in freeways by suggesting or imposing appropriate speed limits, displayed by means of VMSs. The development of V2I technologies, enabling the communication of specific messages, including also speed limits, on board of vehicles, could increase the effectiveness of this methodology in the next future.

The basic underlying idea is to reduce the speed of vehicles travelling upstream the congested area in order to *homogenise* the overall traffic conditions. Note that homogenisation means reduction of the speed differences among the vehicles composing the traffic flow, thereby limiting the onset phenomena of stop-and-go waves, that often cause accidents and traffic breakdowns (see e.g. [14, 15] for a detailed analysis on the main effects of variable speed limits). The presence of variable speed limits can have an impact also on the distribution of vehicles among the different lanes. This aspect is investigated in [16], referring to a real setup in the Netherlands, where the change in lane distribution due to variable speed limits is analysed, with reference to the merging process due to traffic flows coming from on-ramps.

Another relevant implementation of mainstream control is *mainline metering*, which involves the use of traffic lights along the mainstream. This control action is often actuated before bottlenecks in order to avoid their activations, and the associated negative consequences of system performance degradation. Mainline metering was experienced for the first time in the U.S. in the late 50s, to increase the throughput of the tunnels under the Hudson River, connecting New York City with New Jersey. The tunnel was controlled through an inflow traffic control system using real-time traffic measurements from the bottleneck location [17]. Another relevant example of mainline metering is the entrance control system with traffic lights, that had been implemented at the San Francisco–Oakland Bay Bridge for more than 35 years [18].

Another way to control the traffic flow in the mainstream is via *lane control*, in order to warn the drivers about the presence of possible queues (that may be caused by adverse weather conditions, accidents, work zones and so on) or to redirect the vehicle flows to different lanes. In Northern European countries, in particular in the Netherlands and in Germany, a widespread lane control measure is the temporary use of the *shoulder lane*. During peak hours, in order to increase the vehicle throughput, shoulder lanes are utilised as extra-lanes, and their opening or closure is communicated via VMSs. Another form of lane control consists in the exclusive use of the shoulder lane for specific classes of vehicles, such as heavy vehicles or public transport means. Further policies frequently adopted to control drivers in the mainstream are the ‘keep your lane’ strategy, forcing drivers to maintain their lane, or the ‘early merge’ strategy, encouraging drivers to merge into the open lane before the lane closure.

Finally, among the mainstream control strategies, it is possible to include also *section control*, often called also *average speed enforcement* or *point-to-point speed enforcement*. It is a speed control system, which measures the travel time of vehicles between different positions (normally with cameras) to verify the speed limit compliance. The effects of section control on freeway traffic are of several types, the most relevant ones being related to more homogenised traffic flow, increased traffic capacity, and, above all, a consequent reduction of accidents [19]. The effectiveness of section control is verified, for instance in [20], on the basis of floating car data.

Route Guidance In freeway traffic networks, drivers have often to face routing decisions, in case there are different *alternative paths* to reach their destinations (see Fig. 8.5). Among these alternatives, drivers would like to choose the most convenient path, which can correspond to the shortest, fastest or cheapest choice, depending on their preferences. Since traffic conditions vary over time, the most effective route guidance systems are the *dynamic* ones, i.e. those which are based on real-time measurements coming from the freeway network.

Route Guidance and Information Systems (RGISs) are devised in order to provide the users with information about the current state of the system (such as the presence of congestion, traffic incidents, working zones and so on) in the alternative routes or, in some cases, to give specific routing indications to the drivers. Such information can be communicated to drivers by displaying messages on VMSs or by providing them with specific (and even personalised) information by using special in-car communication devices. Even though in the future this latter option will probably become

Fig. 8.5 Alternative paths in A20 freeway, close to Rotterdam, the Netherlands (courtesy of Rijkswaterstaat, Photo: Essencia Communication/Rob de Voogd)

the most frequent, actual RGISs basically rely on the use of VMSs to communicate routing indications to the drivers. The main scientific approaches analysed in this chapter will refer to this communication option.

Different route guidance systems have been developed all over the world, simply indicating estimated travel times for alternative paths or directly suggesting paths to drivers (see e.g. [21, 22] for a survey and classification of route guidance systems).

Integrated Control Strategies Phenomena of recurrent and non-recurrent congestion in freeway systems can be relieved more efficiently if different control strategies are integrated and combined towards a common objective. It is quite evident, indeed, that the best achievements in controlling traffic in a freeway network are obtained if traffic is regulated exploiting all the possible control actions. Applying, for instance, ramp metering can provide effective results in reducing congestion phenomena but it is undeniable that, for some specific traffic scenarios, acting on the system only by regulating the access of vehicles from the on-ramps can be a limitation, while controlling also the mainstream flow or routing vehicles through alternative paths can make the overall control action more effective.

On the other hand, it is apparent that a control scheme which combines different control strategies is more challenging from the design point of view and good performance results can be obtained only if the different control strategies are properly integrated in order to achieve the same objective for the controlled system.

8.1.2 Freeway Traffic Control Schemes

The road-based traffic control strategies described in Sect. 8.1.1 are of different types and act on the freeway by intervening on different parts of the system, i.e. on the on-ramps, on the mainstream or on the diverging junctions. Regardless of their different natures, all these control strategies should act according to suitably devised traffic control algorithms to be applied online, on the basis of real-time measurements coming from the freeway network. In this sense, we are dealing with *feedback* (*closed-loop*) control schemes, since the values of the control inputs depend on the measurements of the system state. A recent work dealing with feedback control laws applied to general acyclic traffic networks can be found in [23], where robust global exponential stabilisation is proven (the robustness is referred to any uncertainty related to the Fundamental Diagram, as well as the uncertain nature of the traffic model in the congested case).

Few and very old-fashioned control schemes represent an exception to feedback control strategies, i.e. they are not based on real-time measurements but, instead, are derived off-line on the basis of historical data. Examples of this type of controllers are fixed-time ramp metering strategies (see e.g. [9, 10]), dating back to the 60s, which rely on simple static models and on past traffic data. In this book, we only consider feedback control schemes, since the control strategies that are computed off-line and are applied to the system independently from the real system state are no more of interest for real applications.

Figure 8.6 reports a very general scheme of a feedback loop for a controlled freeway traffic system. The dynamics of the freeway traffic system is affected by two different types of inputs:

- the *control inputs* are computed by a traffic controller and transferred to the real system through proper *actuators*. For instance, in case of ramp metering, the control inputs are the flows that should enter the mainstream from the on-ramp and the actuators are the traffic lights (see e.g. [24] for a discussion about how ramp metering control inputs can be translated into specific traffic light settings

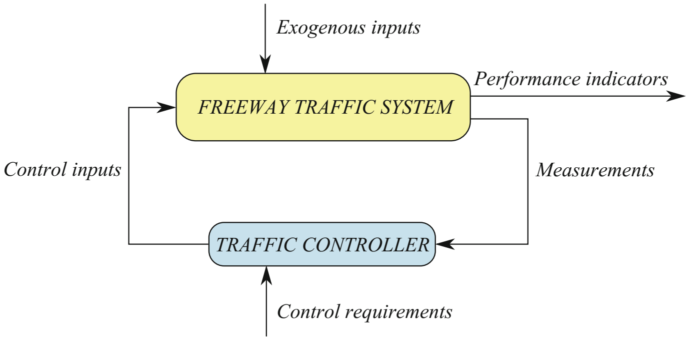

Fig. 8.6 A controlled freeway traffic system

according to the applied metering policy). In case of variable speed limits, the control inputs are the speeds of vehicles and the actuators are normally VMSs; these actuators are used also in case of route guidance control, in which the control variables are generally the splitting rates of vehicles at junctions;

- the *exogenous inputs* represent external conditions which influence the traffic system state. Typical examples of exogenous inputs affecting a freeway traffic system are the external demand (vehicles that require to enter the considered freeway), weather conditions, accidents and turning rates: some of these exogenous inputs are measurable, detectable or predictable, but they cannot be manipulated nor controlled.

The control inputs are computed by a *traffic controller*, which includes a control algorithm, that can vary from a very simple control law to highly sophisticated control frameworks. In any case, traffic controllers base the computation of the control inputs on real-time *measurements* (e.g. measurements of flows, densities, mean speeds, queue lengths), which are collected through proper *sensors*. A discussion about the possibility of measuring traffic variables in freeway systems or to estimate such variables all over the freeway network can be found in Chap. 7.

The effectiveness of the traffic controller is defined according to pre-defined *control requirements*, indicating specific functions or characteristics of the controller, as well as suitable behaviours desirable for the controlled system. Control requirements may regard, for instance, the computational time necessary to determine the control law, the use of specific types of measurements/estimates, the use (or not) of prediction models, as well as the definition of ad hoc control objectives. Strongly correlated with the control requirements, suitable *performance indicators* are defined for the freeway traffic system under investigation, e.g. the total time spent by the drivers in the system, the total delay in queues, the overall emissions, which can be referred to the entire freeway or to specific road portions. Performance indicators can be used to assess the behaviour of the system in real time, but also such indicators can be employed, via simulation, to verify the effectiveness of a given control approach, normally compared with the uncontrolled case or with other control schemes.

8.1.3 Classification of Freeway Traffic Control Schemes

The first freeway traffic control systems were developed and implemented in the U.S. in the 60s [25]. Since then, a very wide literature on freeway traffic control has been developed (see e.g. the survey papers [26, 27]). In recent years, the technological developments, especially in sensors, communication devices and processors, have allowed the actual transfer of many research results from a theoretical to a practical level. Also, the technological innovation in the context of traffic management, surveillance and control has put into evidence in some cases that it is necessary to revise conventional algorithms and control schemes in order to fully exploit the potential of new technologies.

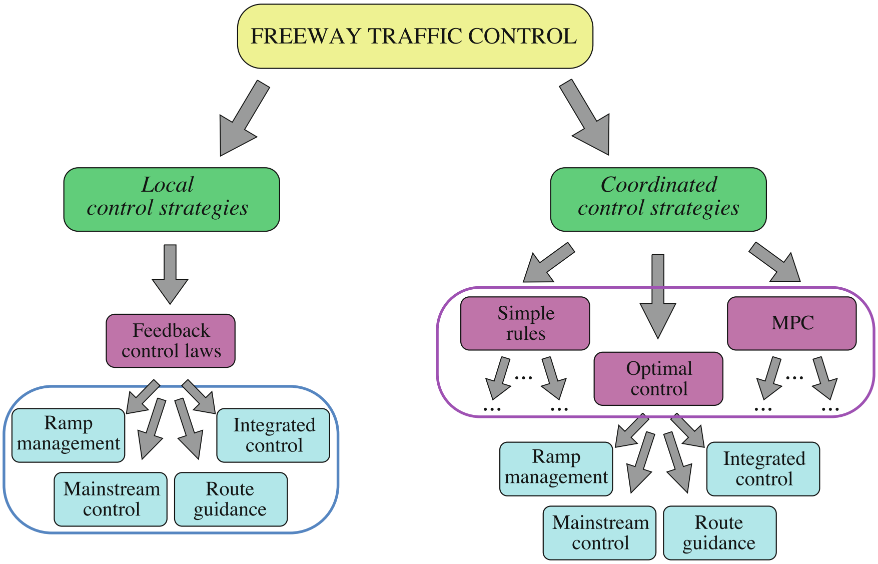

Fig. 8.7 Classification of conventional freeway traffic control approaches

This chapter is devoted to report and classify the main *conventional approaches* for freeway traffic control, according to the scheme reported in Fig. 8.7. In particular, these conventional approaches are first divided in two main categories:

- *local traffic control strategies*: they are the simplest feedback strategies, in which the control action of each controller depends on local measurements of the system state, normally coming from sensors placed in the vicinity of the corresponding actuators;
- *coordinated traffic control strategies*: the control actions actuated in different portions of the freeway are not independent and are computed taking into account measurements of the whole system state. Coordinated strategies are, in general, more effective than local ones to regulate traffic flows in a freeway network but more difficult to be designed and managed.

Both local and coordinated traffic control strategies can be further subdivided according to different criteria. The two most relevant criteria for this classification are

- the considered control methodology;
- the adopted control action.

As shown in Fig. 8.7, *local control strategies* do not differ too much in terms of control methodology, since they are mainly based on feedback control laws or on more sophisticated schemes (e.g. hierarchical), anyhow relying on feedback control concepts. Local control strategies are instead strongly differentiated on the basis of the adopted control action, i.e. ramp management, mainstream control, route guidance and integrated control. Hence, in this book, and in particular in Sect. 8.3, local control strategies are classified according to the type of control action.

Coordinated control strategies, also, can be classified depending on the type of control method and the adopted control action, as shown in Fig. 8.7. Differently from local strategies, the most meaningful classification for coordinated traffic control schemes seems related to the control methodology, and this is the criterion used in this book for their categorisation. Specifically, in Sect. 8.4, coordinated traffic control schemes are divided in schemes resulting from the coordination of simple feedback strategies, control schemes relying on optimal control approaches and Model Predictive Control (MPC) frameworks.

Chapter 9 will investigate some new concepts of traffic management, related to the *implementability* of freeway traffic control systems, i.e. the computational efficiency of the control algorithms so as to make them suitable for real-time use in possibly large freeway networks. Hence, Chap. 9 will include an overview of innovative approaches in this direction, also including event-triggered control frameworks, as well as decentralised and distributed control schemes.

Chapter 10, instead, will be focused on a new vision for freeway management and control, related to the system *sustainability*, i.e. the improvement of the quality of life of citizens as well as the efficient use of the natural resources. According to this vision, freeway traffic needs to be controlled not only for guaranteeing a significant efficiency in using the road network capacity and the improvement of global mobility, but also to limit emissions and reduce fuel consumptions. Moreover, it is particularly relevant in this context to distinguish different typologies of vehicles, leading to multi-class traffic control schemes. Hence, Chap. 10 will include an overview of the most innovative approaches including sustainability-related factors in the design of the control schemes.

8.2 Objectives of Traffic Controllers

The objectives of traffic controllers are strictly related to the improvement of the traffic conditions in the freeways, i.e. to the reduction of congestions and to the mitigation of the associated negative effects. The main goal pursued by traffic controllers is surely the reduction of the travelling times, since this is the most direct impact for travellers. As aforementioned, more recently, other control objectives have been introduced by researchers, in order to take into account environmental issues, safety aspects and, more in general, factors related to the quality of life of citizens.

In order to achieve the aforementioned benefits for freeway traffic systems, traffic controllers must be properly designed and implemented, also taking into account the requisites coming from the real application context. Among the wide variety of control approaches for freeway traffic present in the scientific literature, it is not straightforward to properly categorise all the objectives of the different freeway traffic control schemes. In the following, these objectives are classified into four main groups, respectively, corresponding to

- tracking of set-point values;
- improvement of the system performance in terms of congestion reduction;
- improvement of the system performance in terms of emission reduction;
- balancing of some system variables.

8.2.1 Tracking of Set-Point Values

A large number of traffic controllers for regulating freeway traffic have been devised in order to track some specified set-point values for the traffic variables. As it is widely applied in control theory, a *set-point* is the desired or target value for a variable of the system.

In freeway traffic, the most common choice is to fix reference values for the *traffic densities* and to design the traffic control schemes in order to track these set-points. Let us refer to a macroscopic discrete-time traffic flow model for a freeway stretch (see Sects. 3.3.1 and 4.2.1, respectively, for the CTM and for METANET), in which the stretch is composed of N road sections and the time horizon is discretised into K time intervals, where $\rho_i(k)$ is the traffic density in section i at time kT. Let us denote with $\rho_i^*(k)$ the set-point value for the traffic density in section i at time kT, $i = 1, \ldots, N$, $k = 0, \ldots, K$.

Set-point values for the traffic density which are different in each road section and are time-varying surely represent the most general case. Indeed, in sophisticated control schemes, the set-points can be defined according to the present traffic conditions; this is particularly suitable in hierarchical control schemes in which a supervisor computes the set-points in real time. On the opposite side, the simplest choice for these density target values is to maintain them as fixed values. In many cases, the desired density is set equal to the critical density, i.e. $\rho_i^*(k) = \rho_i^{\mathrm{cr}}$, $i = 1, \ldots, N$, $k = 0, \ldots K$. Designing a traffic controller in order to track the critical density is equivalent to maximise the flow, i.e. to exploit the road capacity as much as possible.

In case a set-point is fixed, it is very useful to define the *error signal*, given by the difference between the set-point and the dynamic variable and generally denoted as $e_i(k)$, referred to section i at time kT. Such error is computed as $e_i(k) = \rho_i^*(k) - \rho_i(k)$ in case the reference value is associated with traffic density. The basic idea is that, as in a standard tracking control problem, the tracking error should go to zero, hence implying stability concepts that are investigated in some research papers, as discussed later on. Note that similar considerations can be made also in case the set-points are defined for other traffic variables, such as the mean speed or the traffic flow.

A concept similar to the tracking of set-point values is related with the definition of proper *thresholds* for the traffic variables. This choice can be motivated by the fact that the real traffic control problem is not related to track a given value but to avoid specific critical situations. For instance, considering again the traffic density, the essential goal of a traffic controller is, when the density is high, to reduce it to the critical density (or another value defined according to the traffic conditions). In the opposite case in which the density is lower than the critical value and the traffic is flowing freely, there is no interest (and, often, no chance) to increase the density to the critical value. Note that the case of lower densities often corresponds to a situation in which the system does not need even to be controlled. Similar arguments can be used if a threshold is defined for the mean speed but, in that case, the traffic controller acts in the opposite way, i.e. it aims to avoid that the mean speed becomes lower than the threshold. In case thresholds are considered, it is no more relevant to define an error, but it is more useful to compute and to penalise the cases in which the threshold is overcome.

8.2.2 Improvement of the System Performance: Congestion Reduction

Instead of considering set-point or threshold values for the traffic variables, another possibility is to design the traffic controller in order to explicitly improve the performance of the freeway system, by defining suitable performance indices. The most relevant and common performance indicators are associated with *congestion reduction*. In this context, let us introduce the three most widespread indices, very often used in freeway traffic control schemes, that are the Total Time Spent, the Total Travel Distance and the Mean Speed [28].

The *Total Time Spent* (TTS) represents the time spent in the freeway by all the vehicles [veh h] in the considered time horizon. It is computed as the sum of two terms, that are the *Total Travel Time* (TTT), i.e. the total time spent by all vehicles [veh h] in the mainstream, and the *Total Waiting Time* (TWT), i.e. the total time spent by all vehicles [veh h] waiting at the on-ramps. Reducing the TTS is equivalent to reduce congestion and, equivalently, to increase the throughput exiting from the network [12]. This is due to the fact that reducing the delays suffered by vehicles implies that they will reach their destination in shorter times, i.e. improving the level of service of the infrastructure.

In addition, the *Total Travel Distance* (TTD) is the total distance [veh km] covered by all the vehicles in the considered time horizon. On the basis of the TTS and the TTD, it is possible to compute the Mean Speed (MS) [km/h] of the vehicles travelling in the considered system in the whole time horizon.

Let us refer to the METANET model for a freeway stretch with on-ramps described in Sect. 4.2.2. In this model, the freeway stretch is composed of N road sections, each one with length L_i, $i = 1, \ldots, N$, the time horizon is discretised into K time intervals with sample time T, $\rho_i(k)$ is the traffic density in section i at time kT, $q_i(k)$ is the traffic flow leaving section i during time interval $[kT, (k+1)T)$, $l_i(k)$ is the queue length of vehicles waiting in the on-ramp of section i at time kT. According to this model, the cited indices are computed as

$$TTS = TTT + TWT = T\sum_{k=0}^{K}\sum_{i=1}^{N}\rho_i(k)L_i + T\sum_{k=0}^{K}\sum_{i=1}^{N}l_i(k) \tag{8.1}$$

$$TTD = \sum_{k=0}^{K}\sum_{i=1}^{N}L_i q_i(k)T \tag{8.2}$$

$$MS = \frac{TTD}{TTS} \tag{8.3}$$

Let us report also how these indices are computed in case a multi-class traffic model is adopted, referring specifically to the multi-class METANET model for freeway stretches described in Sect. 4.3.1. In this model, again, the freeway stretch is divided into N road sections, with length L_i, $i = 1, \dots, N$, the time horizon is discretised in K time intervals with sample time T, and, in addition, C classes of vehicles are explicitly modelled. To account for different vehicle classes, the parameter η^c, $c = 1, \dots, C$, is used, being a conversion factor of vehicles of class c into cars. Moreover, $\rho_i^c(k)$ is the traffic density of class c in section i at time kT, $q_i^c(k)$ is the traffic flow of class c leaving section i during time interval $[kT, (k+1)T)$, $l_i^c(k)$ is the queue length of vehicles of class c waiting in the on-ramp of section i at time kT. In the multi-class case, the previous indices are computed as follows:

$$TTS = TTT + TWT = T\sum_{k=0}^{K}\sum_{i=1}^{N}\sum_{c=1}^{C}\eta^c\rho_i^c(k)L_i + T\sum_{k=0}^{K}\sum_{i=1}^{N}\sum_{c=1}^{C}\eta^c l_i^c(k) \tag{8.4}$$

$$TTD = \sum_{k=0}^{K}\sum_{i=1}^{N}\sum_{c=1}^{C}L_i\eta^c q_i^c(k)T \tag{8.5}$$

while the MS is still given by (8.3). Note that in the multi-class case, the TTS is expressed in [PCE h] and the TTD in [PCE km].

The computation of the same indices in case the CTM is used or macroscopic traffic models for freeway networks are adopted is very similar to the presented one, with only slight differences in the notation.

8.2.3 *Improvement of the System Performance: Emission Reduction*

The reduction of congestions is not the unique performance index to be considered in a freeway traffic system to be controlled. Many other aspects can be taken into account, such as the reduction of noise, pollution, as well the increase of safety. The performance indices associated with emission reductions are of particular interest, especially for the purposes of the present book, and will be detailed below, referring to the two emission models, COPERT and VERSIT+, described in Chap. 6.

As motivated in Chap. 6, when adopting emission models, it is useful to consider multi-class traffic flow models, allowing to explicitly consider the different emission factors of the multiple vehicle classes. In particular, let us refer to the multi-class METANET model for a freeway stretch described in Sect. 4.3.1, in which the emissions are computed, on the basis of COPERT and VERSIT+ models, as described in Sects. 6.3.2 and 6.4.2, respectively. In both cases, $E_i^M(k)$ represents the mainstream emissions in section i at time step k, and $E_i^R(k)$ indicates the on-ramp emissions in section i at time step k. Note that these emissions are given in [g/km] if COPERT model is applied, while they are expressed in [kg/s] for VERSIT+.

Analogously to the TTS previously described, a performance index associated with the *Total Emissions* (TE) in the freeway system in the whole time horizon can be defined. The TE are given by the sum of the *Mainstream Emissions* (ME) and the *Ramp Emissions* (RE), and are computed as follows:

$$TE = ME + RE = \sum_{k=0}^{K} \sum_{i=1}^{N} E_i^M(k) + \sum_{k=0}^{K} \sum_{i=1}^{N} E_i^R(k). \qquad (8.6)$$

Note that this performance index can be applied also in case emission models different from COPERT and VERSIT+ are considered, provided that $E_i^M(k)$ and $E_i^R(k)$ are properly computed.

8.2.4 *Balancing of the System Variables*

Another possible objective of a traffic controller is to homogenise and balance traffic variables. This balancing approach can be applied following different concepts.

A first option is to design the traffic controller in order to balance the state variables along the freeway. This *space-balancing* is normally applied to the *traffic densities*, in order to obtain a homogenisation of the traffic conditions along the freeway, or to the *on-ramp queues*, in order to make the ramp metering actions more fair for drivers entering from different on-ramps. Another type of space-balancing is in some cases associated with the *control variables*: when applying variable speed limits, for instance, it is desirable that successive VMSs encountered by drivers display speed limits that are not too oscillating.

A concept of balancing can be applied also in time, to equalise the values of variables in consecutive time steps. The *time-balancing* is often applied to the *control variables*, in order to reduce oscillations in time, which could reduce the performance of the control actions. Consider for instance the control variables of ramp metering controllers, i.e. the entering flows from on-ramps: if these values have high variations from one time step to another, it is hard and often ineffective to actuate them through red and green phases of the traffic lights present at the on-ramps.

8.3 Local Control Strategies

Referring to the general scheme depicted in Fig. 8.7, let us start by describing local control strategies, that are the simplest feedback strategies, in which the control action of each controller depends on *local measurements*, i.e. measurements in the vicinity of the corresponding actuators. In particular, considering a generic freeway traffic system represented in a discrete-time framework with k indicating the time step, let us denote with $u(k)$ the generic control action computed by a given traffic controller at time step k. According to a local control strategy, the control action $u(k)$ depends on local measurements of the system state, denoted in general as $x^{\text{loc}}(k)$, which can include only one variable or more than one, depending on the specific case.

Let us refer to the example of controlled freeway depicted in Fig. 8.2, with two controlled on-ramps, two installations of mainstream control and one junction in which route guidance indications are provided. Figure 8.8 shows a scheme of local control strategies applied to that freeway stretch, in which four traffic controllers are present, denoted with A, B, C and D. In particular, traffic controller A applies a ramp management action, controller B implements mainstream control, controller C realises a route guidance strategy, and controller D implements an integrated control action, by combining ramp management and mainstream control.

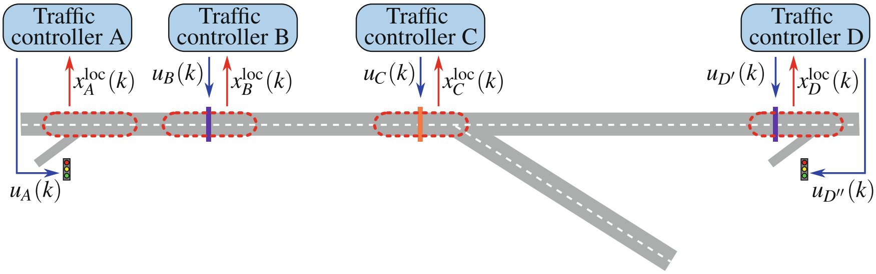

Fig. 8.8 Local control strategies

Different local strategies for freeway traffic control have been developed by researchers during the years and can be found in the literature. Among these control strategies, a very relevant class of controllers often exploited for freeway traffic is given by regulators of proportional and integral type, properly designed in order to track a specified *reference value*. Let us denote this reference value of the local system state as $x^{\text{loc},*}(k)$, referred to time step k. The error at time step k, denoted as $e^{\text{loc}}(k)$, can be then computed as $e^{\text{loc}}(k) = x^{\text{loc},*}(k) - x^{\text{loc}}(k)$.

In a *proportional* controller (often called regulator of P-type), the control action is proportional to the error, i.e.

$$u(k) = K_P e^{\text{loc}}(k) \tag{8.7}$$

where K_P is the proportional gain.

An *integral* controller (often called regulator of I-type) is characterised by a control action computed as

$$u(k) = u(k-1) + K_I e^{\text{loc}}(k) \tag{8.8}$$

where K_I is the integral gain.

A *proportional-integral* controller (often called regulator of PI-type) is characterised by a control action which includes the proportional and the integral actions, i.e.

$$u(k) = u(k-1) + K_P \left[e^{\text{loc}}(k) - e^{\text{loc}}(k-1) \right] + K_I e^{\text{loc}}(k) \tag{8.9}$$

Note that, in most cases, the control variables are bounded for physical reasons and must belong to a range $[u^{\min}, u^{\max}]$. Hence, the value of $u(k)$ resulting from the control laws (8.7)–(8.9) should be truncated if it is out of the requested range.

The control strategies of local type can be categorised according to different criteria. In this book, we have classified them according to the type of control action adopted, since the proposals made by researchers strongly differ according to this aspect. Hence, local control strategies are divided in:

- local ramp management strategies;
- local mainstream control strategies;
- local route guidance strategies;
- local integrated control strategies.

8.3.1 Local Ramp Management Strategies

Different feedback ramp metering control strategies of local type have been proposed in the literature and have been applied in real cases. The main objective of local ramp metering is to properly regulate the inflow from one on-ramp in order to reduce congestion in the mainstream downstream the on-ramp, which corresponds to reduce undesirable phenomena such as capacity drop or blockage of off-ramps.

The local ramp metering control strategies developed by researchers in the last decades differentiate both for the type of control law and for the type and number of local measurements needed to the traffic controller. Specifically, a common distinction is between local measurements taken *upstream* the on-ramp and measurements taken *downstream*.

Let us report in the following some of the most widespread ramp metering local strategies, by referring to the METANET model for a freeway stretch with on-ramps described in Sect. 4.2.2, in which the generic ramp metering control variable is $r_i^{\mathrm{C}}(k) \in [r_i^{\min}, r_i^{\max}]$, representing the flow that should enter section i from the on-ramp during time interval $[kT, (k+1)T)$.

One of the earliest feedback ramp metering strategies is the *demand-capacity* strategy [29], which is an open-loop disturbance-rejection policy in which the local control action depends on the flow measured upstream the on-ramp and the occupancy measurement downstream the on-ramp. Specifically, the flow that should enter section i from the on-ramp during time interval $[kT, (k+1)T)$ is given by

$$r_i^{\mathrm{C}}(k) = \begin{cases} q_i^{\max} - q_i^{\mathrm{up}}(k-1) & \text{if } o_i^{\mathrm{down}}(k) \leq o_i^{\mathrm{cr}} \\ r_i^{\min} & \text{otherwise} \end{cases} \tag{8.10}$$

where the two required measurements are $q_i^{\mathrm{up}}(k)$, i.e. the flow measured upstream the on-ramp, and $o_i^{\mathrm{down}}(k)$, i.e. the occupancy measured downstream the on-ramp. Moreover, in (8.10), $q_i^{\max}$ is the mainstream capacity downstream the on-ramp, and o_i^{cr} is the critical occupancy (at which the flow reaches its maximum value). The basic philosophy of this strategy is, in case of under-critical traffic conditions, to allow to enter in the mainstream an on-ramp flow such that the downstream freeway capacity is reached; if instead the mainstream situation is congested, only a minimum on-ramp flow is allowed to enter.

Another simple and very common local ramp metering strategy is the so-called *percent-occupancy* strategy, which is one of the most widespread ramp metering schemes in the U.S., due to its simplicity of implementation and observed effectiveness [13]. The percent-occupancy strategy provides a proportional control of the occupancy measurement and depends on the occupancy measurement taken upstream of the on-ramp. Specifically, according to the percent-occupancy strategy, the on-ramp flow of section i for time interval $[kT, (k+1)T)$ is computed as

$$r_i^{\mathrm{C}}(k) = r^{\mathrm{H}} - \frac{r^{\mathrm{H}} - r^{\mathrm{L}}}{o^{\mathrm{H}} - o^{\mathrm{L}}} \left[o_i^{\mathrm{up}}(k) - o^{\mathrm{L}} \right] \tag{8.11}$$

where the only measurement is $o_i^{\mathrm{up}}(k)$, i.e. the the occupancy measured upstream the on-ramp, while r^{H} and r^{L} are parameters (corresponding to a high and a low threshold for the on-ramp flow) and, analogously, o^{H} and o^{L} are other parameters (corresponding to a high and a low threshold for the occupancy). According to (8.11), the on-ramp flow is a decreasing linear function of the mainline occupancy, with $r_i^{\mathrm{C}}(k) = r^{\mathrm{H}}$ when $o_i^{\mathrm{up}}(k) = o^{\mathrm{L}}$, and, vice versa, $r_i^{\mathrm{C}}(k) = r^{\mathrm{L}}$ when $o_i^{\mathrm{up}}(k) = o^{\mathrm{H}}$.

One of the most well-known local ramp metering strategies is *ALINEA* [30], that is a control law of I-type, in which the flow entering from the on-ramp is computed according to an error signal expressed in terms of difference between a set-point value and the occupancy measured downstream the on-ramp. In particular, the flow that should enter section i from the on-ramp during time interval $[kT, (k+1)T)$ is computed according to the following control law

$$r_i^{\mathrm{C}}(k) = r_i^{\mathrm{C}}(k-1) + K_R \left[o_i^* - o_i^{\mathrm{down}}(k) \right] \tag{8.12}$$

where $o_i^{\mathrm{down}}(k)$ is the occupancy measured downstream the on-ramp, o_i^* is a set-point value for the downstream occupancy, and K_R is the integral gain. Note that, in case the main objective of the traffic controller is to reduce congestion and to maximise the throughput, a good choice for the set-point is $o_i^* = o_i^{\mathrm{cr}}$. In some papers, the control law of ALINEA is expressed similarly to (8.12) but in terms of density instead of occupancy. The ALINEA controller is able to react to differences of $o_i^* - o_i^{\mathrm{down}}(k)$ in a less abrupt way compared with the demand-capacity strategy, as discussed in [12]. Note that the set-point could be time-varying and, in a hierarchical control scheme, it could be communicated to the controller by a supervisor. ALINEA has been applied in real cases for some decades, especially in Europe, to maximise the freeway throughput [28, 31].

A specific case of ramp metering installation is addressed in [32], where *dual-branch on-ramps* controlled with ALINEA are considered. In on-ramps of this type, it is very important to take account of balancing concepts, both in terms of queue lengths and in terms of waiting times experienced in the two branches. Different balancing policies are analysed in [32], referring to the real ramp metering system of the Monash Freeway, in Melbourne, Australia.

Different versions of ALINEA have been proposed in the literature. In [33], three versions are analysed, which are the flow-based strategy called *FL-ALINEA*, the version relying on the upstream occupancy called *UP-ALINEA*, and the one based on the upstream flow named *UF-ALINEA*. In [34], the adaptive *AD-ALINEA* strategy is proposed, being suitable for cases in which the critical occupancy cannot be a priori estimated. Indeed, AD-ALINEA includes an estimation algorithm based on the Kalman filter, which uses real-time measurements to estimate the critical occupancy that guarantees throughput maximisation according to the present traffic conditions. Again in [34], an upstream-measurement based version of the AD-ALINEA is investigated, called *AU-ALINEA*. AU-ALINEA may be useful in real cases in which no measurement devices are present downstream the on-ramp.

Another relevant extension of ALINEA is the so-called *PI-ALINEA*, in which a proportional term is added to the integrative term, resulting in a PI regulator. The PI-ALINEA control law assumes the following form:

$$r_i^{\mathrm{C}}(k) = r_i^{\mathrm{C}}(k-1) - K_P \left[o_i^{\mathrm{down}}(k) - o_i^{\mathrm{down}}(k-1) \right] + K_R \left[o_i^* - o_i^{\mathrm{down}}(k) \right] \tag{8.13}$$

where K_P is another regulator parameter.

A comparison between ALINEA and PI-ALINEA was carried out in [35], based on a theoretical stability analysis. In particular, [35] addresses the case of *distant downstream bottlenecks*, i.e. bottlenecks with smaller capacity than the merging area which are present further downstream the on-ramp. For that case, it is argued in the paper that it is advisable to use measurements from these downstream bottlenecks rather than from the merging area. According to the stability analysis of the closed-loop ramp metering system reported in [35], it can be stated that PI-ALINEA is able to guarantee a better control performance than ALINEA. A similar case of distant downstream bottlenecks is analysed in [36], where ALINEA and PI-ALINEA are compared via a simulation analysis, considering three different types of bottlenecks, i.e. an uphill, a lane drop and an uncontrolled on-ramp, showing again a better performance of PI-ALINEA compared with ALINEA.

Another interesting and realistic case is associated with many *bottlenecks with random location*, that can form downstream the metered on-ramp. This aspect has been addressed for instance in [37], considering incidents or lane changes in merge areas as possible causes of random-location bottlenecks and referring to a real case in Melbourne, Australia. In that work, the authors propose a generalisation of PI-ALINEA: a PI-ALINEA controller is defined for each possible bottleneck and a properly defined decision policy selects the controller corresponding to the most critical situation, in order to be actuated by the traffic light at the on-ramp.

A different methodological approach has been proposed in [38, 39] for ramp metering control. This approach is based on *iterative learning control*, which is a simple and robust feed-forward control method particularly suitable for addressing modelling uncertainties and non-linear dynamics (very common in the traffic case) and which exploits the repetitiveness of traffic phenomena to learn and improve the performance of the traffic controller. Indeed, traffic patterns are in general repeated similarly every day, and it is possible to find also similarities on monthly and yearly bases. According to the authors of [38, 39], this learning mechanism can allow the controller to improve its performance over time, differently from standard feedback regulators. On the other hand, compared with more sophisticated approaches as neural networks or fuzzy logic, iterative learning control has some advantages, as discussed in [39]. In particular, the control scheme proposed in [38] has the objective of driving the traffic density to converge to a desired density value, by combining the iterative learning control law with a generic feedback control law. The specific case in which the iterative learning control law is combined with ALINEA is addressed in [39].

Note that most of the ramp metering controllers should act in connection with *queue control* strategies, since the on-ramps are normally characterised by a limited space, that in some real cases can be quite large but, in others, can be very restrictive. In addition, it is important to point out that a long queue formed at an on-ramp may cause traffic problems to the adjacent streets, also affecting the possibly close urban traffic network. This means that, in case of maximum queue limits, the ramp metering controller should take into account the queue upper bound and regulate the on-ramp flow accordingly. This aspect can strongly limit the performance of ramp metering actions that has been proven, instead, to be very high in ideal conditions, i.e. without on-ramp storage space limitations. Queue control strategies can be implemented in different ways; for instance, the authors of [33] propose a proportional regulator which could work in conjunction with feedback ramp metering policies, such as ALINEA or PI-ALINEA.

8.3.2 Local Mainstream Control Strategies

Local mainstream control strategies regulate the mainstream flow of vehicles according to local measurements of the system state. Mainstream control can be actuated in different ways, and the most general concept of this type of control action has been proposed in [40, 41], where *mainstream traffic flow control* is defined as a general tool to regulate traffic in the mainstream, adopting different types of actuators, such as mainstream traffic lights, variable speed limits, or more advanced systems according to which suitable indications are provided directly to drivers on board of vehicles.

The objective of mainstream control strategies can be of different types. Safety increase has been one of the first goals of mainstream control, but, over the years, the most common objective has become to homogenise traffic conditions along the mainstream and to avoid the activation of bottlenecks, in order to mitigate all the negative effects of such phenomena, such as capacity drop, off-ramp blockage, and stop-and-go waves. Mainstream control can be realised in different ways, by imposing or suggesting variable speed limits to the drivers (normally by means of VMSs), by applying mainline metering actions or with lane control policies.

One of the first feedback local strategies for variable *speed limits* can be found in [42], where an ALINEA-like mainstream regulator is proposed, along with a switched activation-deactivation mechanism for the controller.

An interesting dynamic speed limit control algorithm, named *SPECIALIST*, was proposed in [43]. It is a feed-forward control scheme based on the shock wave theory aiming at maximising the discharge rate of vehicles. The main goal of this algorithm is to eliminate moving shock waves, i.e. short moving jams that propagate upstream causing an increase of travel times and unsafe situations for drivers, as well as a rise in noise and pollution for the environment.

Starting from the SPECIALIST algorithm, a more sophisticated variable speed limit control scheme was presented in [44], with the twofold objective of maximising the discharge rate at the bottleneck and to reduce speed variations upstream. To reach this result, a variable speed limit control is applied upstream the bottleneck in order to dissipate the possible forming queue. In addition, another variable speed limit control is applied further upstream to solve the queue generated by the first variable speed limit and to better regulate the inflow to the bottleneck.

A local mainstream traffic flow feedback controller, enabled via variable speed limits, is discussed in [45]. It is a controller of I-type, designed in order to prevent the congestion formation at an active bottleneck, hence eliminating the negative effects of capacity drop and blocking of off-ramps. Let us refer to the METANET model for a freeway network described in Sect. 4.2.3, where the control variable is given by

$b_m(k) \in [b^{\min}, 1]$, representing the variable speed limit rate to be displayed in each section of link m during time interval $[kT, (k+1)T)$. In [45], the variable speed limit rate $b_m(k)$ is computed according to the following control law

$$b_m(k) = b_m(k-1) + K_I \left[o_m^* - o_m^{\text{down}}(k) \right] \tag{8.14}$$

where K_I is the integral gain, $o_m^{\text{down}}(k)$ is the occupancy measured at the bottleneck downstream, and o_m^* is the occupancy set-point value, typically set equal to the critical occupancy, to maximise the flow. In [45], this variable speed limit controller is tested via micro-simulation for the case of an on-ramp merge bottleneck, showing very high performance results.

Controllers of I-type and PI-type are investigated also in [46] for application of variable speed limits. In particular, in [46], continuous-time traffic flow models are adopted and the effectiveness of the controllers is shown both through analytical results and with numerical evaluations.

A more sophisticated control scheme is discussed in [47], where a *cascade control* framework is devised, with two nested control loops. According to the cascade control scheme proposed in [47], the variable speed limit rate $b_m(k)$ is computed with the *secondary loop* controller of I-type, as

$$b_m(k) = b_m(k-1) + K_I \left[q_m^{\text{C},*}(k) - q_m^{\text{C}}(k) \right] \tag{8.15}$$

where $q_m^{\text{C}}(k)$ is the controlled mainstream flow measured downstream the VMS location, and $q_m^{\text{C},*}(k)$ is the reference value. Such value is computed according to the control law of PI-type of the *primary loop* given by

$$q_m^{\text{C},*}(k) = q_m^{\text{C},*}(k-1) + K'_p \left[\rho_m^{\text{down}}(k-1) - \rho_m^{\text{down}}(k) \right] + K'_I \left[\rho_m^* - \rho_m^{\text{down}}(k) \right] \tag{8.16}$$

where K'_p and K'_I are gains, while $\rho_m^{\text{down}}(k)$ is the measured downstream density and ρ_m^* is the set-point value for this density.

In [47], an interesting analysis of practical application aspects regarding the implementation of variable speed limits is reported. Indeed, in practice, there are several constraints and practical limitations for speed limits to be displayed on VMSs, that controllers computing variable speed limits must take into account. First of all, variable speed limits can assume discrete values belonging to a pre-defined set. Also, the speed limits to be shown on a given VMS cannot change too fast in time and, analogously, there is a limited space variation of speed limits displayed on consecutive VMSs. The interested reader can find more details on these practical issues in [47].

The cascade controller described in [47] was extended in [48] to account for the case of *multiple bottleneck locations*. In particular, in [48], the variable speed limit control is used to deal with multiple bottlenecks in different downstream locations. At each of these locations, a suitable sensor is placed to measure the density and a controller of PI-type similar to (8.16) is applied for each location. A decision logic is used to identify the most critical downstream bottleneck, in order to compute the reference value $q_m^{\mathrm{C},*}(k)$ for the secondary loop controller given by (8.15).

An integral regulator is used in [49] for *mainline metering*, with reference to the case of a generic merge area, i.e. a freeway infrastructure characterised by a high number of lanes merging into a lower number of lanes. This is the case for instance of the merging of two highways, toll plazas, or working zones which reduce the number of available lanes. In case the incoming flow exceeds the capacity, the capacity-drop phenomenon occurs, with consequent delays for the drivers. As outlined in [49], there are different real configurations of merging areas, for instance lane changing may be allowed or not, the merging can be asymmetric or symmetric, i.e. some lanes may have priorities over others or not, and so on. Each of these cases should be addressed in a specific way, with proper policies. The control law adopted in [49] is analogous to ALINEA, but the control variable is the flow entering the merge area. Once this flow is computed, it is necessary to define how to distribute such a flow among the controllable lanes. Depending on the type of merging, different distribution policies are suggested in [49].

A similar case of mainline metering for merging zones is analysed in [50], specifically addressing the case of work zones. A regulator of PI-type is applied and particular attention is paid to the location of the traffic lights. In [50], it is shown that choosing the right location for the traffic lights, at a given distance from the merge area, has a relevant impact on the performance of the traffic controller.

8.3.3 Local Route Guidance Strategies

Route guidance systems aim at routing vehicles along alternative paths either by providing the drivers with specific information about these paths (e.g. expected travel time, presence of work zones, accidents, and so on) or by directly suggesting the path to follow. Since in this chapter we are considering freeway traffic control schemes, the latter option is taken into consideration, i.e. the case in which a traffic controller computes the *splitting rates* at a given junction, corresponding to the portions of vehicles which should choose each alternative path. Then, these control inputs are transferred to the system with proper actuators, that are normally VMSs before the junction displaying appropriate messages to the drivers. Alternatively, it is possible to adopt proper interfaces on board of vehicles in which specific indications are provided to the drivers. In the following, we will refer to the case of VMSs, being at present the most conventional solution, but most of the reported results could also be generalised to different actuator devices.

For route guidance, local control strategies are those in which the splitting rates computed by the traffic controller associated with a given road junction and displayed on a VMS placed before that junction are based on local information and are not correlated with other splitting rates referred to other junctions. It is worth noting that, in the context of route guidance, the concept of *local state* should be distinguished from the cases just analysed and related to ramp management and mainstream control. If for these latter control actions the local state is generally a measurement of occupancy, flow or density close to the actuator, for route guidance the corresponding ‘local measurement’ is mostly referred to *travel times* along the alternative paths to reach a common destination. This is, in fact, the most relevant information on the basis of which the control action is derived.

It is important to make a relevant distinction between

- instantaneous travel times;
- predictive travel times.

The easiest way to account for travel times along alternative paths is to measure *instantaneous travel times* (also called reactive travel times). The instantaneous travel time can be defined as the travel time of a virtual vehicle travelling along a given path facing the current traffic conditions. This variable can be measured in real time, assuming that the freeway stretches are equipped with sensors providing the mean speed. Of course, instantaneous travel times can be misleading or inappropriate for a traffic controller in case of traffic conditions changing fast in the considered freeway stretch, because the driver that will follow a given path will experience traffic conditions that are different from those that are present on those links when he leaves the junction.

For these reasons, in many cases, it can be more appropriate to consider *predictive travel times* (often called also experienced travel times). Experienced travel times can be known only after completion of the corresponding trip, hence it is necessary to predict them in real time in order to properly feed a route guidance traffic controller. This is normally done by running online a traffic model in order to predict future traffic conditions in the alternative paths.

Another very relevant distinction when discussing route guidance strategies is related to the main principles adopted by the traffic controller in terms of route choice. Two main alternative approaches are available, coming from the theory of traffic assignment for transport networks, that are the Wardrop’s principles of user equilibrium and system optimum [51]. In particular,

- the *user equilibrium* corresponds to a purely selfish behaviour of drivers who want to minimise their travel times; in the resulting equilibrium condition, the alternative routes that are actually used are characterised by the same travel times, which are lower than those of the unused routes;
- the *system optimum* concept, instead, is related to a social behaviour of users that allows to minimise the total travel times, according to a system perspective.

These two principles correspond to two different points of view, respectively, the one of road users, aiming at minimising their travel times, and the one of authorities, trying to improve the global performance of the system [52]. These two different views, the individual and the system perspectives, should be accurately analysed and studied for real implementations, taking into account also the rationality and selfishness levels of the drivers [53].

Analogously to ramp management and mainstream control, researchers have exploited the possibility of applying *feedback control strategies* for route guidance as

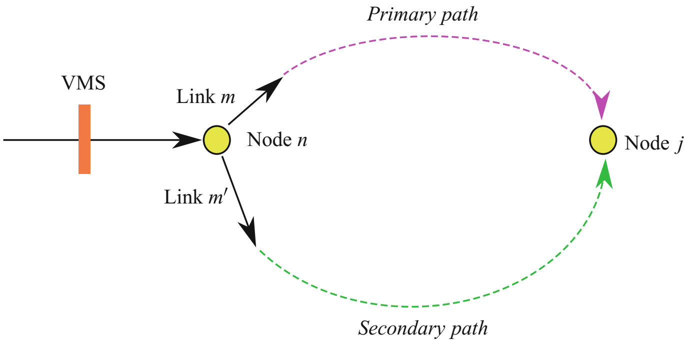

Fig. 8.9 Routing choices at a junction

well. In this case, the control law is computed taking into account the instantaneous travel times along the alternative paths, originated from the considered junction.

Let us refer to the METANET model for a freeway network described in Sect. 4.2.3, in which the control variable is the splitting rate at a given node. Considering the simple case of only two alternative paths originating from node n, let us denote with m and m' the two links exiting node n, corresponding respectively to the primary and secondary path (see Fig. 8.9). The primary path is the one characterised by the shortest travel time, in case of regular traffic conditions. In particular, the control variable is the splitting rate $\beta^{\mathrm{C}}_{m,n,j}(k) \in [0, 1]$, representing the portion of flow present in node n at time instant kT which should choose link m to reach destination j. The other control variable is $\beta^{\mathrm{C}}_{m',n,j}(k)$, referred to link m', but it is easily computed from $\beta^{\mathrm{C}}_{m,n,j}(k)$, since $\beta^{\mathrm{C}}_{m',n,j}(k) = 1 - \beta^{\mathrm{C}}_{m,n,j}(k)$.

Feedback regulators of P-type or PI-type have been proposed for route guidance systems [54, 55]. According to a *proportional control law*, the portion of flow present in node n at time instant kT which should choose link m to reach destination j is computed as

$$\beta^{\mathrm{C}}_{m,n,j}(k) = \beta^{\mathrm{N}}_{m,n,j}(k) + K_P \Delta\tau_{n,j}(k) \tag{8.17}$$

where $\beta^{\mathrm{N}}_{m,n,j}(k)$ is the nominal splitting rate, K_P is a gain, $\Delta\tau_{n,j}(k)$ is the instantaneous travel time difference between the secondary and primary direction from n to j. Note that $\beta^{\mathrm{C}}_{m,n,j}(k)$ is bounded and should be truncated in the interval [0, 1]. According to (8.17), the splitting rate for the primary path is decreased in case the instantaneous travel time difference becomes negative, i.e. in case the secondary path is characterised by a lower travel time. In this way, the traffic controller aims at equalising the travel times along the two alternative paths, in accordance with the user equilibrium principle.

In proportional-integral regulators, the splitting rate is instead computed as

$$\beta^{\mathrm{C}}_{m,n,j}(k) = \beta^{\mathrm{C}}_{m,n,j}(k-1) + K_P \left[\Delta\tau_{n,j}(k) - \Delta\tau_{n,j}(k-1)\right] + K_I \Delta\tau_{n,j}(k) \tag{8.18}$$

where K_P and K_I are other controller gains. Feedback strategies of P-type and PI-type for route guidance are compared via simulation in [56].

An alternative to the feedback control approach for route guidance is given by *iterative strategies*, in which the control action is computed by iteratively running different simulations in real time with different route guidance, in order to achieve conditions of either user equilibrium or system optimum (see e.g. [57, 58]). Although iterative strategies are very efficient in establishing these ideal conditions (more than feedback strategies that only approximate such conditions), they require a very high computational effort.

For this reason, the authors of [59] proposed a *predictive feedback* approach which incorporates the advantages of feedback and iterative strategies. In particular, in [59], the METANET model is run in real time, at each predicting time step, in order to forecast the travel times of vehicles which leave node n until they reach destination j. Note that the simulation model is initialised with the traffic state measured when the simulation is run and, for the whole prediction horizon, the splitting rates are assumed to be constant and equal to those implemented in the real system when the simulation starts. The predicted travel times computed by the simulation model for the alternative paths are used to calculate the time difference $\Delta\tau_{n,j}(k)$, which is applied as input for feedback regulators of I-type or PI-type, as those given by (8.17) and (8.18). A similar feedback approach, relying on predictive capabilities, was described in [60], referring to a real application in the Scottish freeway network.

8.3.4 Local Integrated Control Strategies

Local integrated control strategies are based on the principle that different control actions can be combined to obtain higher performance for the freeway system.

The integration of *ramp metering* with *variable speed limits* is discussed in [61], where the algorithm SPECIALIST proposed in [43] is extended to include also on-ramps controlled via ramp metering strategies, resulting in the so-called *SPECIALIST-RM*. According to the same logic of SPECIALIST, the proposed integrated control algorithm relies on shock wave theory and primarily aims at mitigating moving jams in freeways.

Another work dealing with ramp metering integrated with variable speed limits is proposed in [62]. As argued in [62], this integration is applied in order to overcome the limit of ramp metering strategies, which is related to the possible restrictions on queue lengths at the on-ramps. The basic idea developed in that paper is to exploit ramp metering as much as possible until the on-ramp is completely full of vehicles or the minimum metering lower bound is reached, and, when these limits are met, to activate variable speed limit control integrated with ramp metering. This integration is achieved by extending the *cascade control* law proposed in [47], through the application of a split-range-like scheme. A relevant practical issue is also addressed in [62], regarding the possible different control periods for the implementation of ramp metering and variable speed limits. If these periods are equal, the implementation is straightforward, whereas it is more elaborate in case of different periods. For this latter case, often occurring in real implementations, the authors propose an appropriate methodology and provide some insights.

The integration of ramp metering control and variable speed limits is addressed also in [63], where iterative learning control techniques are exploited. As already explained above for local ramp metering strategies, iterative learning control can be effective for freeway traffic, since it requires a low modelling knowledge and can be applied also in case of model uncertainties and disturbances to the system difficult to estimate, thanks to the capability of the traffic controller to learn from previous executions.

The integration of *variable speed limits* with *lane change control* is investigated in [64], with specific reference to a bottleneck or an accident causing a reduction of the number of lanes. The authors claim that, in a situation of that type, the only application of variable speed limits can provide very limited results, while the integration with lane change recommendations can highly improve the performance of the system, avoiding the capacity drop. The variable speed limit controller proposed in [64] is designed according to an analytical method based on the CTM, by exploiting feedback linearisation techniques. Stability properties are also proven in that paper under the assumption of speed limits varying continuously. Since instead in real cases the speed limits displayed on VMSs must comply with practical constraints of discretisation, space and time variation limitations, the authors of [64] also discuss these aspects and propose an integrated control strategy respecting these constraints.

8.4 Coordinated Control Strategies

In contrast with local control strategies described in Sect. 8.3, coordinated control strategies compute the control law taking into account *measurements* of the system state that are *not local*, but are related to a wider area (see Fig. 8.7 for the classification scheme adopted in this chapter). Also, the control actions applied on different actuators placed in different locations are not independent, as in local control strategies, but are in some way related and synchronised. Thanks to these coordination mechanisms and to information on the traffic state of an entire region of the network, a coordinated strategy is in general more effective than the combination of multiple independent local strategies. It goes without saying that designing coordinated control strategies requires more complex control schemes, in which a large number of information should be dealt with and more complicated control algorithms are needed.

Referring again to the freeway depicted in Fig. 8.2 with two controlled on-ramps, two installations of mainstream control and one junction with route guidance indications, a scheme of coordinated control strategies is reported in Fig. 8.10, to be compared with local control strategies shown in Fig. 8.8. According to the example reported in Fig. 8.10, the different control laws are computed by a centralised traffic controller on the basis of the knowledge of the whole system state $\underline{x}(k)$.

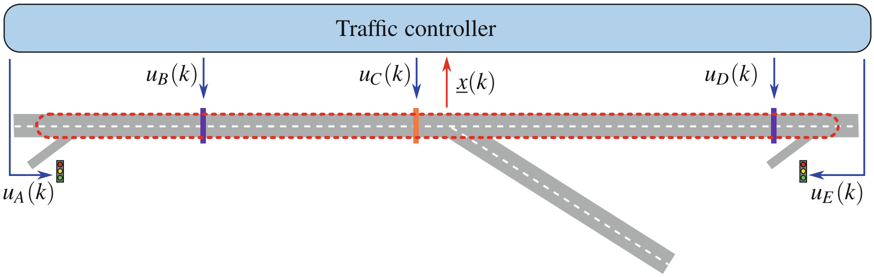

Fig. 8.10 Coordinated control strategies

The scheme shown in Fig. 8.10 represents the most general case of coordinated control strategies in which different control actions are combined, i.e. it is an example of *coordinated and integrated control strategies*. There are also many cases in which control strategies are not integrated but are coordinated. For instance, in coordinated ramp metering, the control actions implemented in different on-ramps are properly synchronised and based on traffic measurements coming from the whole region of the traffic network these on-ramps belong to. Analogously, it is possible to define coordinated mainstream control and coordinated route guidance strategies.

Traffic engineers have studied different approaches for designing coordinated control strategies, which are still under development in order to find the correct compromise between effectiveness and ease of implementation. Differently from local control strategies, which have been classified according to the type of control action (see Sect. 8.3), coordinated control strategies are categorised according to the control method adopted, since this represents the main differentiation feature. In particular, coordinated control strategies are classified, according to the coordination method, in:

- coordination of simple feedback control strategies;
- coordination via optimal control;
- coordination via Model Predictive Control.

8.4.1 Coordination of Simple Feedback Control Strategies

Different approaches to coordinate traffic control actions can be found in the literature. Most of them are based on optimisation or optimal control techniques (see Sects. 8.4.2 and 8.4.3), which can lead to efficient but very complicated control schemes. A less sophisticated but more practice-oriented way to coordinate different traffic control actions is to adopt simple regulators or heuristic rules, the main of which are summarised in this section.

Coordinated strategies have been developed especially for *ramp metering* controllers. One of the first attempts to design coordinated ramp metering strategies was the generalisation and extension of ALINEA in a multivariable regulator strategy called *METALINE* [65]. According to this feedback strategy, the controlled flow of one on-ramp depends not only on occupancy measurements immediately downstream the on-ramp but on more measurements referred to a larger area.

After these early approaches, most of the literature developed later for simple coordinated ramp metering control strategies included heuristic and rule-based algorithms. For instance, heuristic coordinated ramp metering algorithms, such as the *Zone* and the *Stratified Zone* algorithms, the *Bottleneck* algorithm, and the *Helper* algorithm, were developed and applied in some freeways in the U.S. [66, 67].

More recently, the rule-based coordinated ramp metering strategy *HERO* was developed and described in [68]. It is based on a control scheme adopting ALINEA at the local level and some coordination mechanisms for the control actions of different on-ramps, particularly useful in case of limited on-ramp space. In [68], it is shown that the effectiveness of local ALINEA controllers is low in case of limited queues, and it is exactly in this case that a coordination mechanism is useful to improve the performance of the freeway traffic system. In order to preserve easy applicability to real freeway networks, HERO is a simple reactive and rule-based strategy which decides in real time whether to activate or not the coordination mechanism among the on-ramps depending on real-time measurements.

Another research work dealing with coordinated ramp metering is [69], in which the main question addressed by the authors is when to start the control of on-ramps, having in mind that the final goals of coordinated ramp metering are to increase the throughputs and, at the same time, to mitigate traffic instabilities which characterise high flow traffic states, often improving the probability of crashes. To this aim, a production stability indicator is defined on the basis of a Macroscopic Fundamental Diagram (MFD) characterising the system at a network level and relying on a risk assessment technique, since traffic breakdown is seen as a risk. This approach is validated with empirical traffic data of the city of Shanghai, China.

A comparison between local and coordinated ramp metering strategies is reported in [70], with reference to field results based on traffic data of the A6W freeway in Paris, France. In particular, the considered local strategy is ALINEA, whereas the coordinated strategy is *CORDIN*, a new strategy proposed in that paper, which is based on a heuristic approach adopting ALINEA control laws that are properly corrected in order to achieve a better coordination. The comparative tests are devoted to evaluate the system performance, not only in terms of total travel times, but also in terms of travel time reliability. In that paper, it is shown that CORDIN performs better than ALINEA in terms of travel time reductions, but the two strategies are rather similar from a reliability point of view.

A quite general framework for coordinating different actuators is proposed in [71], referring in particular to *integrated traffic control*, combining ramp metering and variable speed limits. By exploiting some ideas developed by the same authors in previous papers (see e.g. [37, 48]), the feedback integrated control scheme proposed in [71] aims at controlling a freeway stretch in which multiple bottlenecks are present, in order to maximise the throughput. A set of controllers of PI-type is used, each associated with a measurement from a possible bottleneck (all the bottlenecks are downstream the actuators); a decision algorithm computes the smallest smoothed flow to be implemented, after distributing the flows among the available actuators in order to balance the delays upstream them.

8.4.2 Coordination via Optimal Control

The most advanced coordinated control strategies are based on the solution of an optimisation problem. In some cases, this problem is assumed to be solved off line, leading to optimal control approaches as the ones treated in this section, while in other cases it needs to be solved online, by applying MPC schemes, which are dealt with in Sect. 8.4.3.

Compared with the local control strategies or the simple heuristic coordination schemes described above, optimisation-based control strategies allow to obtain results that are in general more efficient. Indeed, the control actions are computed considering the overall freeway system and its dynamic evolution over a given time horizon, leading to optimal (or suboptimal) solutions to be implemented in the network.

Generally speaking, *optimal control* theory is concerned with the control of a dynamic system in the optimal way (a very detailed review of optimal control techniques can be found in the books [72–74]). The dynamic process is described by differential or difference equations, and the objective is to optimise an objective function (normally a cost function to be minimised) related to state and control variables. The problem constraints represent the dynamics of the system and bounds on the control variables.

Most of the research works on traffic control use discrete-time macroscopic models to describe the dynamic evolution of freeway systems. Two of the most widespread traffic flow models are the CTM of first-order type (see Sect. 3.3) and the more complex second-order model METANET (see Sect. 4.2), both represented in a discrete-time framework with k indicating the time step. These models are non-linear and can be in general written in the following form:

$$\underline{x}(k+1) = \underline{f}\left[\underline{x}(k), \underline{u}(k), \underline{d}(k)\right] \tag{8.19}$$

where $\underline{x}(k)$ is the vector gathering all the system state variables, $\underline{u}(k)$ is the vector of control inputs, and $\underline{d}(k)$ is the vector including exogenous inputs.

The *system state* is represented only by traffic densities when the CTM is used, and by both traffic densities and mean speeds in case METANET is instead applied. In both cases, conservation equations for the on-ramp queue lengths can be added to the model, so that the state vector may also include queue lengths.

The *control variables* used in the optimal control formulation depend on the choice of the control strategy to be applied to the system, as already discussed in the previous sections. In case a ramp metering control scheme is defined, the control variables are associated with the vehicles entering the freeway from the on-ramps, in terms of traffic flows or control rates. If instead variable speed limits are taken into account, the control variables are the values of speed suggested to the drivers via VMSs or equivalent control rates. In case route guidance is chosen, the control variables are normally the splitting rates, i.e. the ratios of vehicles arriving in a node and choosing a given path or direction. A combination of these variables is considered in case of integrated control strategies.

The *exogenous inputs* of the traffic model include all the external uncontrollable variables affecting the system. These exogenous inputs can be referred to external input signals (e.g. traffic demands or turning rates), as well as to modelling uncertainties and measurement noises.

In order to properly state a freeway traffic optimal control problem, a suitable *objective function* must be defined. This function is related to the final goal of the designed traffic controller, as described in Sect. 8.2. In particular,

- if the goal of the traffic controller is to track specific set-points (see Sect. 8.2.1), the cost function in the optimal control problem normally takes into account the *quadratic deviations* of the considered variables from their *reference values*, i.e. the quadratic errors (the choice of the quadratic form is not compulsory but it is the most common in optimal control problems); in case the goal is to avoid that given variables exceed threshold values, suitable *penalty terms* are added in the cost function;
- if the goal is associated with the improvement of the system performance, e.g. congestion reduction or emission reduction (see Sects. 8.2.2 and 8.2.3), the relative *performance indices*, i.e. TTS and TE, are minimised in the optimal control problem;
- if the goal is to balance system or control variables over space, *quadratic deviations* of these variables between consecutive road sections are minimised in the cost function and, analogously, quadratic deviations between consecutive time steps are minimised in case of time-balancing (see Sect. 8.2.4).

Note that the cost function can be also a combination of more terms, in order to take into account different goals. This is often dealt with by minimising a cost function given by a *weighted sum* of the different terms. Such terms can be conflicting or not; in the former case, the optimal control problem has a multi-objective nature and different Pareto-optimal solutions can be analysed in order to better understand which traffic control strategies are more suitable to be implemented in practice.

The general formulation of the optimal control problem over a finite horizon of K time steps is the following.

Problem 8.1 Given the system initial conditions $\underline{x}(0) = \underline{x}_0$ and the estimated sequence of exogenous inputs $\underline{d}(k)$, $k = 0, \ldots, K-1$, find the optimal control sequence $\underline{u}(k)$, $k = 0, \ldots, K-1$, that minimises

$$J = \vartheta[\underline{x}(K)] + \sum_{k=0}^{K-1} \varphi\left[\underline{x}(k), \underline{u}(k), \underline{d}(k)\right] \tag{8.20}$$

subject to the system dynamics expressed by (8.19), with $k = 0, \ldots, K-1$, and

$$\underline{u}^{\min} \le \underline{u}(k) \le \underline{u}^{\max} \qquad k = 0, \ldots, K-1 \tag{8.21}$$

□

In (8.20) $\vartheta[\cdot]$ represents the final cost, $\varphi[\cdot]$ is the stage cost, while vectors $\underline{u}^{\min}$, $\underline{u}^{\max}$ include, respectively, lower and upper bounds for the control variables. Note that the formulation of Problem 8.1 is suitable for discrete-time models. An analogous problem statement can be provided for continuous traffic models, but it is not reported here, since they are less common in freeway traffic control approaches (see e.g. [75, 76]).

It is worth noting that the structure of the problem to be solved depends both on the type of traffic model chosen for (8.19) and on the type of objective function. In most cases, the optimal control problem has a *non-linear* nature, since the most common traffic models are non-linear, e.g. CTM and METANET. In these cases, the numerical solution is often hard to find, because of the problem dimensions and complexity, and, also, there are generally no guarantees about the global optimality of the obtained solution. For this reason, some of the efforts made by researchers are still devoted to find efficient solution algorithms for non-linear optimal control problems for traffic networks. In other cases, the adopted traffic models are simplified or rewritten in suitable forms (see e.g. the Switching Mode Model (SMM) described in Sect. 3.3.6) or properly linearised, in order to obtain more tractable formulations.

The scientific literature on traffic control in freeway networks via the solution of an optimal control problem in the form of Problem 8.1 is very wide. This literature may be classified on the basis of the type of problem to be solved, on the adopted solution method or on how specific aspects of the problem (e.g. exogenous inputs) are treated.

In some works, first-order models are adopted to represent the dynamic evolution of traffic. For instance, in [77, 78], a discretised version of the LWR is considered for coordinated ramp metering and the resulting non-linear optimal control problem is solved with *gradient-based* techniques. The same first-order model is also applied in [79], in which the proposed non-linear feedback control law is obtained via neural networks.

Another type of first-order models is considered in [80], where a modified version of the CTM, i.e. the Asymmetric Cell Transmission Model (ACTM) (see Sect. 3.3.6), is used for a coordinated ramp metering case. The resulting non-linear optimal control problem is proven to be equivalent to a linear formulation, under specific conditions. Similarly, in [81], the adoption of another version of the CTM, i.e. the LN-CTM (see Sect. 3.3.6), leads to a non-linear problem to be solved. The authors of [81] show that, under some conditions, the solution of a linear problem is equivalent to the one of the original problems for a freeway in which variable speed limits and ramp metering are applied.

A very efficient numerical solution algorithm has been adopted in the optimal freeway traffic control tool *AMOC* [82, 83], based on the second-order METANET to represent the traffic system dynamics. AMOC is a general framework conceived to control traffic networks with different topology and with different types of control measures, such as ramp metering, mainstream control and route guidance [40, 84]. AMOC includes different solution methods, all gradient-based, which are compared in [83]. From this comparison, it results that the most promising choice is given by the *feasible direction algorithm* applying the derivative backpropagation method *RPROP* [85] (the original form of the same algorithm was proposed in [86]). This algorithm has been used also in recent works for more complicated optimal control problems, for instance for multi-class traffic regulation [87] or applications to urban networks [88].

Other solution methods to Problem 8.1 have been proposed in the literature. For instance, in [89], the non-linear optimal control problem is solved in a receding-horizon scheme in which a parameterised control law is found by using multi-layer feed-forward neural networks. In other works, the discrete adjoint method is used for solving the arising non-linear optimal control problem, e.g. in a coordinated ramp metering traffic case [90] and in a dynamic traffic assignment problem [91]. The adjoint method is also employed in [92], where a traffic control problem is solved for cybersecurity applications, i.e. in order to evaluate the potential for an adversary having access to control the freeway infrastructure.

An alternative to gradient-based algorithms for solving the non-linear optimal control problems arising in traffic systems is represented by *derivative-free* algorithms, that can be useful for complex cases, in which the gradient computation is very time consuming or even impossible (e.g. if the objective function is not differentiable). A comparative analysis between gradient-based and derivative-free algorithms is reported in [93] for a specific traffic control problem dealing with a coordinated ramp metering strategy to reduce congestion and emissions in a two-class flow environment.

While in these cited works the exogenous inputs are assumed to be known over the considered time horizon, there are other works in which such inputs are seen as *disturbances* or *uncertainties*, explicitly modelled and taken into account in the determination of the control law. For instance, stochastic disturbances, acting on the system and on the measurement channel, are considered and modelled as noises in [94]. Moreover, in [95], the traffic control problem is formulated as an H_∞ control problem which accounts for uncertainties of different natures associated with the macroscopic traffic model, while a robust control approach is proposed in [96], taking into account both model uncertainties and disturbances in the freeway network. In [97], the control law is determined by exploiting the Pontryagin maximum principle and the robustness of the controlled system with respect to uncertainties in the input parameters is analysed.

8.4.3 Coordination via Model Predictive Control

A further possibility to coordinate different control actions for traffic regulation is via *Model Predictive Control* (the interested reader can find theoretical and practical issues on MPC in the books [98–100]). Analogously to optimal control methods described in Sect. 8.4.2, MPC aims at controlling a dynamic system in an optimal way, by taking into account its dynamic evolution and by considering a suitable objective function to be optimised. The difference is that MPC is an *online* control scheme, i.e. it is applied to the system in real time by iteratively solving a *Finite-Horizon Optimal Control Problem* (FHOCP) that is updated on the basis of real system measurements.

Considering a generic MPC scheme in a discrete-time framework, a FHOCP is solved at each time step $k, k = 0, \ldots, K$. This optimisation problem is characterised by an objective function and some constraints. Among them, the system model equations are included, in the form of (8.19), thus allowing the prediction of the system evolution over a given *prediction horizon* of K_p time steps. It is worth noting that the prediction model is initialised with the system state $\underline{x}(k)$ measured at time step k.

In particular, the FHOCP to be solved at time step k can be stated as follows.

Problem 8.2 Given the initial conditions on the system state $\underline{x}(k)$ and the estimated sequence of exogenous inputs $\underline{d}(h), h = k, \ldots, k + K_p - 1$, find the optimal control sequence $\underline{u}(h), h = k, \ldots, k + K_p - 1$, that minimises

$$J(k) = \vartheta[\underline{x}(k + K_p)] + \sum_{h=k}^{k+K_p-1} \varphi\left[\underline{x}(h), \underline{u}(h), \underline{d}(h)\right] \tag{8.22}$$

subject to

$$\underline{x}(h + 1) = \underline{f}\left[\underline{x}(h), \underline{u}(h), \underline{d}(h)\right] \qquad h = k, \ldots, k + K_p - 1 \tag{8.23}$$

$$\underline{u}^{\min} \leq \underline{u}(h) \leq \underline{u}^{\max} \qquad h = k, \ldots, k + K_p - 1 \tag{8.24}$$

and further possible additional constraints on the state variables $\underline{x}(h)$, $h = k + 1, \ldots, k + K_p$ and control variables $\underline{u}(h), h = k, \ldots, k + K_p - 1$. □

By solving the FHOCP at time step k, the optimal state trajectory $\underline{x}^\circ(h|k), h = k + 1, \ldots, k + K_p$, and the optimal control sequence $\underline{u}^\circ(h|k)$, $h = k, \ldots, k + K_p - 1$, are found over the whole prediction horizon. Note that the notation $f^\circ(h|k)$ means that this is the optimal value of the generic variable f, referred to time step h and obtained by solving the FHOCP at time step k.

MPC relies on a *receding-horizon* framework. This means that, after solving the FHOCP at time step k, only the first element of the whole optimal control sequence is implemented in the system, i.e. $\underline{u}^\circ(k|k)$, and at the subsequent time step $k + 1$

the optimisation procedure is repeated again, updated with the system state values $\underline{x}(k+1)$, over a prediction horizon that is shifted one time step ahead.

The objective function (8.22) can be used to pursue different goals of the traffic controller, e.g. tracking set-points, improving the system performance, or balancing the system variables, as discussed in Sect. 8.4.2. Moreover, according to the adopted traffic model, the type of constraints and objective function, Problem 8.2 can have a different structure, e.g. linear, quadratic or non-linear, convex or nonconvex (more detailed considerations on these aspects can be found in [101]).

Problem 8.2 provides the basic statement of a generic FHOCP, but some modifications can often be found. In some cases, in order to reduce the number of variables, the prediction horizon is distinguished from the *control horizon* of K_c time steps (with $K_c < K_p$). The idea is that, after the control horizon has passed, the control actions are forced to be constant. The parameters K_c and K_p should be carefully chosen, to account for the trade-off between accuracy of the controller and computational complexity.

Another possible variation in the formulation of the FHOCP is given by the distinction between the simulation time step index k referred to the model sample time T and the controller time step index k_c referred to the *controller sample time* T_c (with T_c integer multiple of T), this latter representing the rate at which the control actions are updated. This difference is normally related to implementation issues, since the model sample time is chosen small enough to correctly capture the traffic dynamic evolution, while the controller sample time is dictated by the type of actuator.

As widely recognised, MPC has many *advantages*. First of all, it has prediction capabilities, so that the control action computed at time step k is optimal not only for the present situation but also for the future evolution of the system. Second, MPC allows to explicitly handle constraints on the system, and, finally, it has a closed-loop nature, since the control action is determined on the basis of real-time measurements. The main *drawbacks* of MPC are related with computational issues: since MPC is applied in real time, the FHOCP must be solved very fast. This represents a big challenge for real-case traffic networks, because the FHOCP often has a non-linear form and a large number of variables is involved in the optimisation. Some implementation-oriented strategies to deal with MPC schemes for real applications are discussed in Chap. 9.

The first works regarding MPC frameworks for freeway traffic control are [102–104], respectively for variable speed limits, ramp metering, and their coordinated action. In those works, the adopted prediction model is METANET and the resulting optimisation problem is, then, non-linear. A similar framework is considered in [105], where a traffic simulator is used to represent the real freeway, causing a model mismatch between the simulation and the prediction model.

Different research works have dealt with MPC for freeway traffic in the last decade, also considering specific and peculiar cases. For instance, a *mixed traffic network* with two urban regions and a freeway is addressed in [106], where the MFD and the ACTM are used, respectively, to model urban and freeway traffic. The control measures are represented by perimeter controllers and ramp metering. The case of *discrete variable speed limits* is addressed in [107], resulting in discrete control variables and, consequently, in a non-linear mixed-integer problem to solve. Since it is very computationally demanding to solve this type of problem in real time, some methods are proposed to find a reasonable solution in acceptable computational times. Freeway control via *reversible lanes* is studied in [108], where METANET is modified to take into account this type of control action. A search-tree method is used to solve the resulting mixed-integer non-linear problem.

A more advanced control scheme including MPC is reported in [109] for coordinated ramp metering. This is a *hierarchical control* scheme composed of three layers: the upper estimation/prediction level provides estimates and predictions of the future system disturbances; the intermediate layer includes the optimal control tool AMOC [83] used in a receding-horizon framework; the lower level adopts local feedback ALINEA-based controllers, in which the optimal state trajectory found at the upper level is used to fix the set-point values.

A two-level hybrid control scheme, represented with the formalism of discrete-time discrete-event automata, is described in [110] for regulating traffic conditions in freeway systems. In that paper, both ramp metering and variable speed limits can be applied but the MPC regulator to be used at the lower level is chosen by the higher level of the control scheme, on the basis of the present operating conditions. A similar concept applied to a multi-class traffic case is developed in [111].

In order to deal with more tractable problem formulations, some model simplifications or relaxations have been studied in some papers. For instance, heuristic restrictions and relaxations are applied to solve the nonconvex optimisation problem in [112], while a simple first-order model but extended to account for the capacity drop is adopted in [113], enabling a good prediction accuracy and, at the same time, a fast computation of the optimal solution.

In other research works on MPC for freeway traffic, the exogenous inputs are explicitly modelled as *disturbances* or *uncertainties*. In [114], the exogenous inputs affecting the traffic demands are explicitly considered as additive and bounded quantities and the input-to-state practical stability of the system controlled via MPC is proven. In a framework similar to the one considered in [114], a new concept of 'natural robustness' is introduced for a traffic system subject to these types of disturbances and controlled with MPC [115, 116]. A robust control approach is proposed in [117] to handle uncertainties in freeway traffic networks via a min–max scheme. In particular, to reduce the computational complexity, a scenario-based receding horizon parametrised approach is adopted.

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