Chapter 5
Microscopic and Mesoscopic Traffic Models

5.1 Uses and Applications of Traffic Models

Chapters 3 and 4 of this book are focused on *macroscopic* traffic models, which represent the dynamics of traffic flow by means of aggregate variables. The main classifications of macroscopic traffic models distinguish them according to the number of variables whose dynamics is explicitly taken into account, corresponding to first-order, second-order or higher-order models. Macroscopic models, generally allow to represent large road networks with an acceptable computational load. This computational advantage characterising macroscopic models is counterbalanced by the drawback that these models cannot capture some specific traffic phenomena related to the behaviour of individual drivers.

On the opposite side, *microscopic* models describe the dynamic behaviour of each single vehicle in the traffic stream, trying to capture the interactions among vehicles and between vehicles and the road infrastructure. These models can be very detailed and accurate in representing specific features of traffic but, of course, are very demanding from a computational point of view. Another important drawback of microscopic models is that they are often characterised by a very high number of parameters which must be properly calibrated. In case of models including heterogeneity among drivers or vehicles and stochasticity, the number of parameters becomes higher. Section 5.2 is devoted to some of the microscopic models present in literature, but it does not aim to exhaustively cover the wide variety of microscopic traffic models. The interested reader can refer to [1–3] and the references therein for a comprehensive overview on the topic.

A very interesting use of microscopic models is their utilisation inside *traffic simulation tools* (see Sect. 5.2.4). Indeed, the complexity of the traffic stream behaviour and the difficulties in performing experiments with real-world cases make computer simulation an important analysis tool in the traffic engineering field. By making use of different traffic models, generally of microscopic type, one can simulate large-scale real-world situations in great detail [4, 5].

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An intermediate class of traffic models, which bridges the gap between the higher level of detail of microscopic models and the aggregate description of macroscopic models, is constituted by the so-called *mesoscopic* models. These models represent a link between microscopic and macroscopic modelling, where the characteristic aspects of both levels of description are combined. In mesoscopic models, the traffic flow dynamics is described in aggregate terms using probability distribution functions and the dynamics of these distributions is governed by individual drivers' behaviour. In fact, even if mesoscopic models do not distinguish individual vehicles (as it happens instead with microscopic models), they specify individual behaviours in probabilistic terms. In this sense, mesoscopic models provide an intermediate option with their ability to model large road networks with limited coding and calibration effort, while providing a better representation of the traffic dynamics and individual travel behaviour than their macroscopic counterparts. Some mesoscopic traffic models are presented in Sect. 5.3, which, again, does not represent an exhaustive survey of all the mesoscopic models appeared in the literature. The interested reader can refer to [1, 2] and the references therein for a more detailed discussion on mesoscopic models.

Taking into account all the traffic models present in the literature and partly described in this book, i.e. macroscopic, mesoscopic and microscopic models, it can be stated that the variety of modelling options is very wide. Of course, each model is characterised by its own strengths and weaknesses, thus making the choice of the most suitable model to be adopted strictly dependent on the objective of the study under concern and on the scale of the system to be investigated.

Microscopic models are surely more suitable for applications in small size road networks or, better, for specific road sections, especially in the urban context. Moreover, a very common use of microscopic models is for *simulation*, especially in case of offline decisions, such as for long-term planning or road design. In these cases, it is more relevant to have a highly detailed model, possibly stochastic, able to provide accurate predictions of the system dynamics, even if it requires a high computational load, rather than a fast but less accurate simulation. The use of macroscopic models is instead particularly relevant for *model-based estimation and control* purposes, especially when real-time applications are considered and large traffic networks are involved. In addition, if optimal control is applied, not only a small problem to be solved is preferable (i.e. with less variables, as macroscopic models can provide) but also the structure of the problem becomes relevant, and hence linear or linearisable traffic models are aimed for. These aspects will be further discussed in Chap. 7 and Chaps. 8–10, respectively, on traffic state estimation and traffic control, where all the reported approaches are based on macroscopic modelling. It is also worth noting that microscopic models can be used for real-time estimation and control, not as a basis for the method but for validation purposes. There are indeed many research works in which new estimation and control methods are developed and their effectiveness is tested and validated by means of traffic simulators.

It is certainly unquestionable that the new developments in technologies and computing devices will change the possible applications of traffic models. The development of faster computers will probably give a chance to the use of microscopic models for real-time applications, as well as the development of new data sources (e.g. probe vehicles) capturing more detailed aspects of the traffic flow and the individual behaviour of drivers will require the use of more specific traffic models, especially of mesoscopic and microscopic types. Surely, as suggested in [2], the development of multi-class models, as well as the improvement of hybrid models properly combining macroscopic, mesoscopic and microscopic features, seems the most promising future direction for traffic modelling.

5.2 Microscopic Traffic Models

Microscopic traffic models describe the behaviour of each single vehicle in the traffic stream and how it interacts with the other vehicles and with the road infrastructure. Specifically, in microscopic models, the vehicle–driver relation and vehicle–vehicle interactions are represented via differential equations in which the longitudinal (car-following) and/or the lateral (lane-changing) behaviour of individual vehicles can be taken into account. Since microscopic models allow to explicitly represent the dynamics of each single vehicle, it is straightforward to model different typologies of vehicles, e.g. cars and trucks, by properly setting the model parameters to represent the different behaviours of the different classes.

Several microscopic models, considering at different extents the different aspects of individual vehicle dynamics, are present in the literature. Among them, let us consider in this section of the book the following classes of models: car-following models, lane-changing models and cellular automata models.

Car-following models, also known as *follow-the-leader models*, were introduced in the 50s [6–8]. These models represent the position and speed dynamics of each vehicle through continuous-time differential equations, in which it is basically assumed that the speed dynamics of a single vehicle depends on its speed, as well as on the distance from the preceding vehicle and the speed of this latter. In more sophisticated models, the behaviour of a driver depends on a platoon of preceding vehicles instead of on one single leader. As discussed in [1, 9], these models have seen various developments after their first appearance. In a first version proposed by Pipes [7], the distance between the two vehicles (leader and follower) is determined as the safe distance computed on the basis of the vehicle length. Later, in [10], the concepts of perception time, decision time and braking time were introduced, allowing to identify the necessary safety distance to avoid collisions between two vehicles. In other models, stimulus–response concepts were introduced, including terms related to the acceleration [11] and sensitivity factors [12], calculated on the basis of the speed difference between the leader and the follower. Further models including the acceleration dynamics were presented in [13, 14]. Section 5.2.1 reports a brief overview of the main car-following models present in the literature.

Lane-changing models seek to describe the behaviour of drivers when a change of lane occurs, regardless of the reason yielding the lane changing (overtaking of a vehicle, merging to and from secondary roads or freeway on-ramps, need to avoid obstacles and so on). The representation of this phenomenon in a reliable manner is, however, one of the most complex problems that the traffic theoreticians have had to face. The lane-changing behaviour can be schematically subdivided into three steps: the decision on lane changing, the selection of the desired lane and the gap acceptance decision. Most of the modelling efforts focused on the last aspect, i.e. the representation of the gap acceptance. Several lane-changing models can be found in the literature, such as the lane-changing urban driving model described in [15] or the advanced model aiming to capture the merging behaviour in severe jammed traffic conditions proposed in [16]. Some more details on lane-changing models are reported in Sect. 5.2.2.

Another class of microscopic models is represented by *cellular automata models* (see, e.g. [17–19]), where the road topology is described by means of a grid of cells and a discrete-time dynamics is adopted. The dimension of a single cell is generally chosen in such a way that each cell can be occupied by only one vehicle (or it can remain empty), whereas the discretisation in time is carried out considering the reaction time of drivers. The traffic dynamics, given by the movement of vehicles, is represented in terms of the state (free or occupied) of the road cells. The speed is instead defined as the number of cells overtaken by a vehicle in a time step. The dynamic evolution of the speed is defined considering some factors that are the acceleration needed to reach a desired speed, the slowing down in order to decrease the speed according to the distance gap to the preceding vehicle, and a random term accounting for a deceleration which spontaneously decreases the vehicle speed according to a certain probability. Even though cellular automata models are less accurate than car-following ones, they allow to effectively replicate many traffic phenomena with a lower computational burden. An overview of cellular automata models can be found in Sect. 5.2.3.

Microscopic models are often adopted in *traffic simulation tools*, and a review of their application in this field is reported in [4, 5]. Section 5.2.4 reports a description of the most common traffic simulators.

5.2.1 Car-Following Models

Car-following models describe the *longitudinal* interactions of vehicles in a road, i.e. the behaviour according to which a driver follows the preceding vehicle in traffic. The first car-following models appeared in the 50s [7] and, since then, a great number of models of this type were proposed by researchers.

Car-following models differentiate for the considered assumptions, but they present a common notation which considers a pair of vehicles: the preceding vehicle (i.e. the *leader*) is denoted with $n-1$, whereas the vehicle following the leader (i.e. the *follower*) is denoted with n, as shown in Fig. 5.1. The following notation is used:

- L_{n-1}, L_n are the lengths of vehicles $n-1$, n, respectively [m];
- $x_{n-1}(t)$, $x_n(t)$ are the positions of vehicles $n-1$, n, respectively, at time t [m];

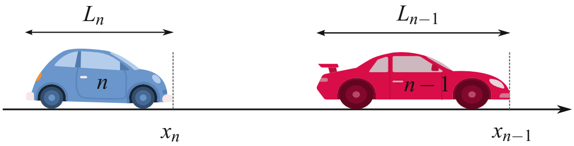

Fig. 5.1 The main notation of car-following models

- $v_{n-1}(t)$, $v_n(t)$ are the speeds of vehicles $n-1$, n, respectively, at time t [m/s];
- $a_{n-1}(t)$, $a_n(t)$ are the accelerations of vehicles $n-1$, n, respectively, at time t [m/s^2];
- $\Delta x(t) = x_{n-1}(t) - x_n(t)$ is the space headway between vehicle $n-1$ and n at time t [m];
- $\Delta v(t) = v_{n-1}(t) - v_n(t)$ is the speed difference between vehicle $n-1$ and n at time t [m/s];
- $s_n(t) = \Delta x_n(t) - L_{n-1}$ is the spacing from the front edge of vehicle n to the rear end of vehicle $n-1$ [m];
- T is the reaction time [s].

Several overview papers on car-following models have appeared in the literature, such as the historical survey proposed in [9], the one in [20] specifically focused on driver heterogeneity aspects, and the more recent review especially addressing how human factors are incorporated in car-following models [21]. Each of these papers suggests a different classification of car-following models. In this book, we propose a classification based on four categories: Gazis–Herman–Rothery or stimulus–response models, safety-distance or collision-avoidance models, reference-signal models, and models including human factors.

GHR or Stimulus–Response Models Gazis–Herman–Rothery (GHR) models are probably the most studied models of car-following type. The basic concept of GHR models [12] is the definition of the acceleration of vehicle n at time t as

$$a_n(t) = c\, v_n^m(t) \frac{\Delta v(t-T)}{\Delta x(t-T)^l} \tag{5.1}$$

where c, l and m are the model parameters to be determined.

GHR models are also known as *stimulus–response models*; the stimulus being defined by the speed difference between the preceding vehicle and the follower, and the response being the braking or acceleration of the follower delayed by the reaction time. If GHR models have the great advantage of being simple, they also received a few critiques since they are rather unrealistic to represent some traffic situations. Actually, in free-flow conditions, when the distance headway is very large, the model assumes that drivers keep reacting to speed differences. Moreover, the traffic is considered homogeneous, i.e. all the vehicles are assumed to react in the same way. This is clearly not true in real situations in which heavy vehicles typically behave differently from cars, for instance, slow trucks are not able to adapt their speed to one of the possible leading fast cars.

Different GHR models have been studied and developed during the last decades, also trying to overcome the limitations mentioned above. Among others, it is worth citing the *asymmetrical* version of GHR models in which different parameter values are used for acceleration and deceleration situations (see, e.g. [22]). There are also versions of the GHR model which use different parameter values for congested and non-congested situations (see, e.g. [23]). This allows to model the fact that drivers may have shorter reaction times in congested situations, since they are more alert. A significant amount of work has been devoted to find suitable calibration procedures for the GHR model parameters. The most reliable parameter values, according to [9], are those indicated in [11, 24–26].

An interesting extension of GHR models is based on the use of *fuzzy logic* [27]. In this framework, concepts like "too close" or "too fast" are described using fuzzy sets, and logical rules are introduced to model the corresponding behaviour of drivers. The fuzzy sets may overlap, so that probabilistic density functions must be used to deduce how the driver perceives the considered variable (for instance, given a certain speed of the leader vehicle, the fuzzy model describes whether it is regarded as low, moderate or high by the follower). The first fuzzy version of the GHR model was proposed in [28]. More recently, a fuzzy model has been presented in [29]. A discussion on calibration and validation of car-following models based on fuzzy logic is contained in [30].

Safety-Distance or Collision-Avoidance Models Safety-distance models are also known as collision-avoidance models, since their basic relationship indicates a safety distance between vehicles in order to avoid collisions. This is specified by the so-called *Pipes' rule*, stating that a good rule for following another vehicle at a safe distance is to maintain a distance that is at least the length of a car for every ten miles an hour (i.e. 16.1 km/h) of speed [7]. This rule can be mathematically expressed as follows:

$$D_n(t) = L_n \left[1 + \frac{v(t)}{16.1} \right] \tag{5.2}$$

where $D_n(t)$ is the *prescribed headway* between vehicle $n-1$ and vehicle n. In alternative, (5.2) can be expressed as

$$D_n(t) = L_n \left[1 + \frac{v(t-T)}{16.1} \right] \tag{5.3}$$

if the reaction time T is taken into account.

Safety distance models differ from GHR models since they assume that drivers react to spacing with respect to the preceding vehicle, rather than to the relative speed. This idea was elaborated in [31], where the proposed model assumes that each vehicle always tries to keep the minimum safety distance from the preceding vehicle, defined as

$$\Delta x(t-T) = \alpha v_{n-1}^2(t-T) + \beta v_n^2(t) + \gamma v_n(t) + d \tag{5.4}$$

where α, β and γ are model parameters, whereas d is the minimum allowed spacing between subsequent vehicles.

Models implementing the same philosophy are those presented in [10, 32, 33]. Yet, the most widely used safety-distance model is the *Gipps* model [34], which is the car-following model implemented in the well-known traffic simulation software Aimsun (see Sect. 5.2.4). The Gipps model assumes that any vehicle tends to travel at the speed which allows to avoid a rear crash if the vehicle performs an emergency braking. Despite presenting several advantages, the Gipps model has the limitation that the following vehicle can only travel exactly at the safe distance with respect to the preceding vehicle, which is clearly unrealistic. More realistic safety-distance car-following models overcome such limitation by better defining the safe distance, for instance, as a function of the relative speed between the leading vehicle and the follower, such as in [35].

Reference-Signal Models This category includes models in which a desired reference signal is explicitly introduced to describe the tendency of any individual driver to adjust his/her behaviour to track that signal. The nature of the reference signal differs from model to model. More specifically, the reference signal can be a prescribed space headway or a desired speed or an adequate time gap.

The first example of model of this class was introduced by *Helly* in [13] and is often known in the literature as the *linear model*. In this model, the acceleration of any vehicle linearly depends on the relative speed and on the difference between the relative distance and the prescribed space headway. The latter is defined by including a term accounting for the follower's acceleration, in contrast with (5.2). This can be expressed mathematically as follows:

$$a_n(t) = C_1 \Delta v(t-T) + C_2 \left[\Delta x(t-T) - D_n(t)\right] \tag{5.5}$$

where the prescribed headway is computed as

$$D_n(t) = \alpha + \beta v_n(t-T) + \gamma a_n(t-T) \tag{5.6}$$

where C_1, C_2, α, β and γ are parameters to be identified on the basis of real data. In particular, Helly observed that C_1 could be considered as dependent on the relative distance between vehicles, whereas C_2 could be made speed dependent. Several works were then devoted to calibrate the Helly model parameters (see, for instance, [36–39]).

Another example of reference-signal models is the so-called *intelligent driver model*, proposed in [40, 41]. In this model, there are two reference signals, the desired speed and the desired space headway, i.e.

$$a_n(t) = a_n^{\max} \left[1 - \left(\frac{v_n(t)}{\tilde{v}_n(t)}\right)^{\beta} - \left(\frac{\tilde{s}_n(t)}{s_n(t)}\right)^{2}\right] \tag{5.7}$$

where $a_n^{\max}$ is the maximum acceleration/deceleration of vehicle n, $\tilde{v}_n(t)$ is the speed reference signal, $\tilde{s}_n(t)$ is the spacing reference signal and β is a model parameter. It is worth noting that, when the spacing between two subsequent vehicles is high, the third term becomes negligible, so that the considered vehicle just follows the speed reference signal. In car-following situations, the spacing reference signal can depend on several factors, such as the speed of vehicle n, the relative speed between vehicles n and $n-1$, the maximum acceleration, the desired time gap and so on.

A further example of reference-signal models is the *optimal speed model*, introduced in [14]. In this model, the reference signal is a speed assumed to be optimal for the considered vehicle, taking into account the distance from the preceding vehicle. Hence, the acceleration of vehicle n can be determined according to the difference between the actual speed and the optimal speed v_n^*, i.e.

$$a_n(t) = \alpha \left[v_n^*(\Delta x_n(t)) - v_n(t) \right] \tag{5.8}$$

where α is a model parameter. Variations of the original optimal speed model were proposed in [42], also to counteract the tendency of the model to produce unrealistic accelerations or decelerations. Further extensions can be found, for instance, in [43–46].

Models Including Human Factors The car-following models described so far are mainly based on physical signals. Nevertheless, as highlighted in [47], the human driving behaviour is not only influenced by physical signals but also by psychological aspects. Moreover, many assumptions of standard car-following models are not always true in real cases, for instance, drivers often adopt strategies that are adequate for the current situation but not optimal, drivers do not continuously react to stimuli, each driver has a different driving style and so on. Based on these considerations, a wide literature has been developed in order to encompass psychophysical aspects, typical of perceptual psychology, into car-following models.

The most famous car-following models which include human factors are the so-called *psychophysical* or *action point* models. The basic idea is that *perception thresholds* characterise the human capability of perceiving spacing and speed differences (see [48, 49] for perception-based experiments to quantify the thresholds). In practice, drivers do not continuously react to speed differences and spacings but only when the current action significantly differs from the action which is regarded as appropriate for the given situation. In other terms, the existence of these perception thresholds makes the acceleration (or, more in general, behavioural changes) occur at asynchronous time instants, named *action points*. The thresholds and time intervals between two subsequent action points are stochastic quantities. Referring in particular to the vehicle acceleration, it is kept constant by the driver until it is significantly different from the acceleration required to maintain the proper spacing with respect to the preceding vehicle. This implies that, in case of large spacing, the following driver tends to act rather independently, i.e. such driver is not influenced by the relative speed, as if this were imperceptible. At small spacings, instead, the driver alertness is higher. The thresholds, and the regimes they define, are typically presented in a relative space–speed diagram for a vehicle pair.

One of the first psychophysical models was introduced by *Wiedermann* [50], a modified version of which has been used in the software tool Vissim (see Sect. 5.2.4). The car-following model implemented in the software tool Paramics (see Sect. 5.2.4) is based instead on the psychophysical model reported in [51]. Other psychophysical models were investigated and can be found in the literature (see, e.g. [51–54]).

Another class of models including human factors are those modelling the driving behaviour related to the *visual angle* subtended by the preceding vehicle. The first car-following model of this type was introduced in [55], where the basic assumption is that drivers approaching a vehicle react to the changes in the apparent size of this vehicle. Then, compared to classical car-following models, the relative spacing and speed are replaced by the visual angle and the angular speed. Different versions of car-following models based on visual angles were developed (see, for instance, [56, 57]).

More sophisticated car-following models were defined by researchers in order to represent aspects related to *risk* and *driving errors*. For instance, driving in risky situations was modelled in [58] as a human decision-making problem, relying on prospect theory [59], and properly defining the subjective probability of being involved in a collision with the preceding vehicle. This model was then extended in [60] to consider response and behaviour of drivers in different surrounding traffic conditions. Further efforts were devoted to include, in car-following models, driving errors and distraction situations, which are the main cause of crashes in real traffic circumstances. For instance, in [61], the Helly model is extended to consider that the time headway is influenced by different aspects, such as visual conditions and driver state, in [62] the intelligent driver model is modified to consider the reactions of the driver to the surrounding traffic environment, and in [63] the Gipps model is extended by considering human perception limitations in processing information and adjusting speed accordingly.

5.2.2 Lane-Changing Models

While car-following models have the main objective of representing the longitudinal interactions among vehicles inside the traffic flow, lane-changing models are instead devoted to describe *lateral* interactions on the road. These two primary modelling tasks have often been treated separately, even if they are two fundamental components of the microscopic traffic flow modelling theory. Although car-following models have been widely studied for many years, lane-changing aspects have received some attention only in recent years [64–66]. This recent interest in lane-changing behaviours has been mainly due to the increasing evidence of their negative impact on traffic safety and traffic congestion.

The impact of lane-changing movements on traffic safety was investigated in some works, such as in [67, 68]. Many studies show that the stress of drivers significantly increases during lane-changing manoeuvres, thus making them more error-prone and dangerous. Moreover, the lane-changing process plays a role in capacity drop phenomena related to bottleneck discharge rate reduction at the onset of congestion [69] and also in the formation and propagation of stop-and-go oscillations [70, 71]. More recently, in [72] it has been shown that lane changing is a primary trigger of oscillations and is responsible for transforming minor and localised oscillations into substantial disturbances.

Research efforts to represent the lane-changing aspects have rapidly increased over the last decade. The main lane-changing models in the literature can be distinguished into two groups: models related to the lane-changing decision-making process (i.e. how a driver reaches the lane-changing decision), and models devoted to quantify the impact of lane-changing behaviours on surrounding vehicles. It can be noted that a comprehensive lane-changing model should take into account both these aspects together with car-following behaviours in order to fully represent the dynamics of vehicles, but a widely recognised modelling tool covering all these aspects is not yet available [66].

The different models developed in the literature differentiate for the way in which they represent the lane-changing decision-making process, but, in any case, they must take into consideration the interactions of the vehicle aiming to change lane with the other vehicles in the surroundings. In particular, as shown in the scheme presented in Fig. 5.2, let us consider the lane changer vehicle, denoted as LC, which is travelling in the lane called *initial lane* and would like to move to the so-called *target lane*. Vehicle LC has to interact, in some way, with the preceding vehicle (i.e. the leader) and the following vehicle (i.e. the follower) in the initial lane, denoted as L^I and F^I, respectively, and with the preceding and following vehicle in the target lane, denoted as L^T and F^T, respectively.

The lane-changing decision-making process is based on several factors, one of which is the so-called *gap acceptance process* which precedes an overtaking manoeuvre. In this process, a driver who wants to overtake a vehicle preceding him estimates both the space he needs and the available space. On the basis of the comparison between required and available space, the driver decides whether to start the lane-changing manoeuvre or not. Several gap acceptance models are present in the liter-

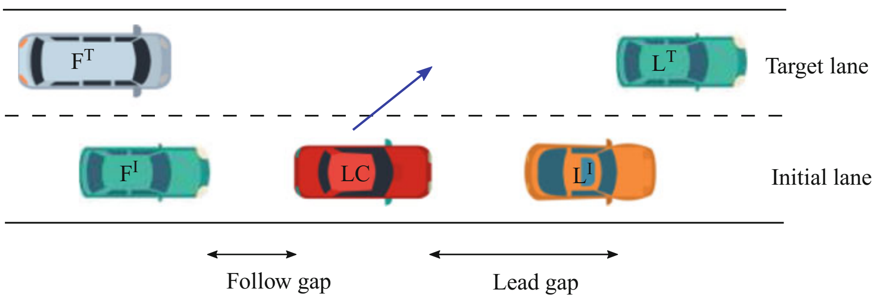

Fig. 5.2 Generic lane-changing process

ature, not only for freeway systems but for any kind of road (and also for pedestrian flows). These models are stochastic and they are based on the definition of a *gap acceptance function* defining the probability that an arbitrary driver accepts an available gap, thus starting the overtaking manoeuvre. A description of gap acceptance models can be found in [16].

A basic model describing a more general lane-changing decision-making process is due to *Gipps* [15], in which various driving situations in an urban street context are considered. In the Gipps model, the driver's behaviour is governed by two basic considerations typically arising in an urban network: the willingness to maintain a desired speed and the desire to be in the correct lane for an intended turning manoeuvre. The drivers' behaviour is considered as deterministic and, thus, a set of deterministic rules to be sequentially evaluated is defined.

One of the first works related to the lane-changing decision process in the freeway context is [73], in which lane-changing movements are classified as either mandatory (when the lane change is necessary due to a lane drop, an accident or the use of an exit junction) or discretionary (when a driver evaluates that in the target lane better driving conditions can be experienced compared to those found in the current lane) and a lane-changing probability is introduced to make the model more realistic. Several variations and extensions were proposed, as, for instance, in [74], in which a novel logic for simplifying and modelling lane-changing decisions is defined in terms of single-lane accelerations.

In [16], the utility theory is applied to model the decision process of lane changing, whereas in [75] Markov processes are used to model mandatory lane-changing actions. Furthermore, several lane-changing decision models based on fuzzy logic have been developed in the last decades [29, 76].

The models discussed so far largely ignore the impact of lane changing on surrounding vehicles. Several studies, such as [69, 77], address the influence between lane-changing and critical traffic phenomena, such as breakdowns, capacity drop and traffic oscillations. Some models for representing the impact of lane changing on surrounding vehicles are reported in [66], where it is also discussed how this aspect still needs to be investigated to define accurate lane-changing models.

5.2.3 Cellular Automata Models

Cellular Automata (CA) models, sometimes also called *Particle Hopping* models, were first proposed in 1948 [78] and then revitalised in the 80s with the work reported in [79]. CA models are basically characterised by four components, i.e. the physical environment, the states of cells, the neighbourhoods of cells and the local transition rules. The physical environment in which CA models are applied for modelling traffic flow is a road segment, which is discretised into cells of the same length, typically equal to the vehicle length, so that any cell can be exactly occupied by a single vehicle. CA models are discrete-time models in which time is discretised and the sample time is generally set equal to 1 s. The speed of a vehicle is then computed as the number of cells that a vehicle hops in one time step (implying that speed is discretised as well). The state of each cell can be either equal to 0 (if the cell is empty) or equal to 1 (if it is occupied).

One of the most famous CA models is the one developed by *Nagel* and *Schreckenberg* [17], which has a stochastic nature. According to this model, the road is discretised into cells (approximately 7.5 m long) and a maximum speed $v^{\max}$ is considered. At each time step, the model evolves according to the following predefined rules:

- *acceleration*: if the speed v of a vehicle is lower than $v^{\max}$ and if the distance to the vehicle in front is larger than $v + 1$, then the speed is increased by one;
- *deceleration*: if a vehicle in cell i finds the next vehicle in cell $i + j$, with $j \leq v$, then the vehicle decelerates to $j - 1$;
- *randomisation*: the nonzero speed of each vehicle is decreased by one, with probability p;
- *vehicle motion*: each vehicle is advanced by v cells.

The update of the states of cells can be done in different ways, i.e. in the direction of travel, in the opposite direction or even in a random order, without affecting the model behaviour. CA models are very simple and computationally low demanding, and hence large size road networks with a high number of vehicles can be analysed (and simulated) in short computational times, and this is surely a relevant advantage of such models, especially for real-time applications.

Moreover, different traffic Fundamental Diagrams can be established by varying the model parameters, specifically by varying $v^{\max}$ and p. Also, CA models describe the spontaneous formation of traffic congestion and stop-and-go waves. As observed in the various survey papers about CA models (see, e.g. the review papers [80–82]), a large number of variations and extensions to the basic CA model have been defined and studied. Let us report in the following a CA model including lane-changing phenomena for a two-class traffic case.

A Two-Class CA Model with Lane Changing The considered model was defined in [83], being based on the model reported in [84]. This model refers to the case in which two classes of vehicles, i.e. cars and trucks, are present in a multi-lane freeway stretch. Specifically, two-lane freeway stretches are taken into account, in which cars can overtake other vehicles by occupying the left lane, with lane-changing rules inspired from [85], while trucks are forced not to overtake other vehicles.

As it is common in CA models, the space is discretised, specifically each lane is subdivided into cells with length equal to 1.5 m. It is assumed that cars have an occupancy of 3 cells, whereas trucks occupy 8 cells. The speed is expressed as the number of cells that one vehicle can go over in one time step, being 1 s the sample time.

The model introduced in [84] presents some important features that makes it more accurate than the original simple model reported in [17]. With reference to Fig. 5.3, considering three vehicles, denoted as n, m and l, the main notation of the model is the following:

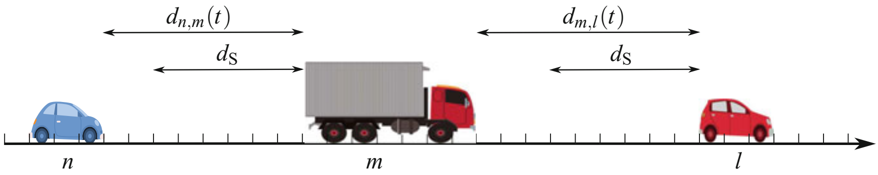

Fig. 5.3 The main notation in the two-class CA model

- d_S is a fixed safety distance between vehicles [number of cells];
- $d_{n,m}(t)$ and $d_{m,l}(t)$ are the number of free cells between vehicles n and m, and between m and l, respectively, at time t [number of cells];
- $v_n(t)$ is the speed of vehicle n at time t, i.e. the number of cells that vehicle n can go over in one time step (analogously $v_m(t)$ and $v_l(t)$ for vehicles m and l) [number of cells];
- $b_n(t) \in \{\text{on}, \text{off}\}$ is the state of the brake light of vehicle n at time t (analogously $b_m(t)$ and $b_l(t)$ for vehicles m and l);
- $l_n(t) \in \{\text{straight}, \text{right}, \text{left}\}$ is the position that vehicle n would like to occupy at time t, which can be obtained by going straight, moving to right or moving to left (analogously $l_m(t)$ and $l_l(t)$ for vehicles m and l);
- $\psi_n \in \{\text{car}, \text{truck}\}$ is the typology of vehicle n (analogously ψ_m and ψ_l for vehicles m and l).

The main rules adopted in the model presented in [84] are the following:

- *anticipation*: a generic vehicle n does not only consider the distance from the preceding vehicle m but it also estimates how far this vehicle will move during the time step; this is done by introducing and computing $d^{\text{eff}}_{n,m}(t)$ as

$$d^{\text{eff}}_{n,m}(t) = d_{n,m}(t) + \max\left\{v^{\min}_m(t) - d_S, 0\right\} \tag{5.9}$$

where $v^{\min}_m(t)$ is given by

$$v^{\min}_m(t) = \min\left\{d_{m,l}(t), v_m(t)\right\} - 1 \tag{5.10}$$

- *brake lights*: again considering a generic vehicle n, a time interval $\tau^S_n(t)$ is referred to the interaction with the brake light of the vehicle in front; specifically, vehicle n reacts to the state $b_m(t)$ of the brake light if $\tau^H_{n,m}(t) < \tau^S_n(t)$, where quantities $\tau^H_{n,m}(t)$ and $\tau^S_n(t)$ are defined follows:

$$\tau^H_{n,m}(t) = \frac{d_{n,m}(t)}{v_m(t)} \tag{5.11}$$

$$\tau^S_n(t) = \min\left\{v_n(t), \nu\right\} \tag{5.12}$$

where ν is a model parameter;

- *slow-to-start*: vehicle n brakes according to a probability which depends on $v_n(t)$, $b_m(t)$, $\tau^{\mathrm{H}}_{n,m}(t)$ and $\tau^{\mathrm{S}}_n(t)$.

In the two-class CA model with lane changing proposed in [83], the algorithm updating the position and the speed of every vehicle for every time step is composed of four phases: definition of entrances from the on-ramps, check for possible lane changes, application of vehicle motion and definition of exits through the off-ramps. Each of these four phases is detailed in the following.

1. *Entrances from on-ramps*. The presence of vehicles at the on-ramps is modelled by means of queues, where vehicles wait to access the mainstream. The queue can contain up to $q^{\max}$ vehicles, and the number of vehicles accessing the queue is generated at each time step according to a given probability p_{in} depending on the vehicle class. Moreover, the number of vehicles which enter the mainstream depends on the space available in the mainstream (this number is reduced if the freeway is congested) and on a maximum value of κ vehicles (where κ is a given parameter related to the on-ramp capacity).
2. *Lane change*. As in [84], two different rules are adopted to define the lane-changing process, from the right lane to the left one and vice versa. Moreover, it is imposed that trucks cannot move to the left lane; hence, these two-lane change rules are applied only to cars. Let us consider these two different rules separately.

    - Rule for moving *from right to left*: let us consider vehicle n in the right lane and let us identify the preceding vehicle m in the same lane, the preceding vehicle s in the left lane and the vehicle r before vehicle s in the left lane (see Fig. 5.4); the variable $l_n(t)$ is first set as follows:

    $$l_n(t) = \text{straight} \tag{5.13}$$

    Then, it is checked if the lane change is possible for vehicle n, i.e.

    $$\begin{aligned}&\text{If}\quad (b_n(t) = \text{off}) \ \wedge\ \big(d_{n,m}(t) < v_n(t)\big) \ \wedge\ \Big(d^{\text{eff}}_{n,s}(t) \geq v_n(t)\Big) \ \wedge\ \big(d_{r,n}(t) \geq v_r(t)\big)\\ &\text{then}\quad l_n(t) = \text{left}\end{aligned} \tag{5.14}$$

    If, by applying (5.14), it results $l_n(t) = \text{left}$, then vehicle n moves to the left lane.

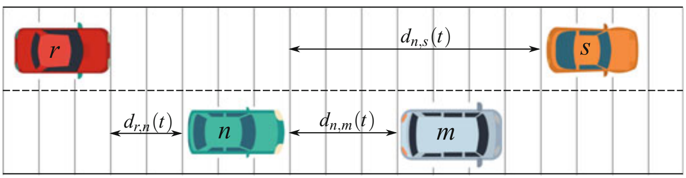

Fig. 5.4 Lane change from right to left in the two-class CA model

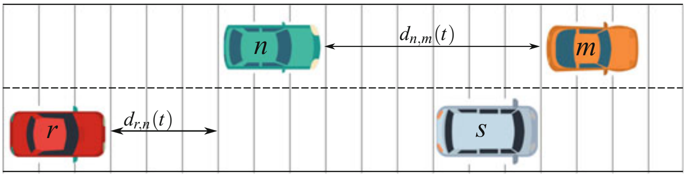

Fig. 5.5 Lane change from left to right in the two-class CA model

- Rule for moving *from left to right*: let us consider vehicle n in the left lane and let us identify the preceding vehicle m in the left lane, the preceding vehicle s in the right lane and vehicle r before vehicle s in the right lane (see Fig. 5.5); the variable $l_n(t)$ is initially fixed as

$$l_n(t) = \text{straight} \tag{5.15}$$

Then, the possibility of lane change is checked for vehicle n, i.e.

$$\begin{aligned} &\text{If} \quad (b_n(t) = \text{off}) \ \wedge \ \left(\tau^{\mathrm{H}}_{n,s}(t) > \xi\right) \ \wedge \ \left(\tau^{\mathrm{H}}_{n,m} > \upsilon \vee v_n(t) > d_{n,m}(t)\right) \\ &\qquad \wedge \ \left(d_{r,n}(t) > v_r(t)\right) \\ &\text{then} \quad l_n(t) = \text{right} \end{aligned} \tag{5.16}$$

where ξ and υ are other parameters. Once (5.16) has been applied, if $l_n(t) =$ right, then vehicle n moves to the right lane.

Summarising, for the two classes of vehicles, the lane changing rule is the following:

$$\begin{aligned} &\text{If} \quad \psi_n = \text{truck} \\ &\text{then} \quad l_n(t) = \text{straight} \qquad \text{(lane change not allowed)} \\ &\text{else} \quad \text{conditions (5.13)–(5.16) hold} \qquad \text{(lane change allowed)} \end{aligned} \tag{5.17}$$

3. *Vehicle motion*. The vehicle motion phase is the core of the algorithm and is executed for every vehicle in each lane at each time step. Vehicle motion is based on a set of rules in order to obtain the speed $v_n(t+1)$ of vehicle n through some consecutive steps, in which the intermediate values $v_n(t+1/3)$ and $v_n(t+2/3)$ are computed. More specifically, let us consider vehicle n and the next vehicle in front m, and let us set $b_n(t+1) = \text{off}$. According to the *acceleration* phase, $v_n(t+1/3)$ is computed as

$$v_n(t+1/3) = \begin{cases} v_n(t) & \text{if } (b_n(t) = \text{on}) \ \vee \ \left(b_m(t) = \text{on} \right. \\ & \qquad \left. \wedge \ \tau^{\mathrm{H}}_{n,m}(t) < \tau^{S}_n(t)\right) \\ \min\{v_n(t)+1, v_{\max}\} & \text{otherwise} \end{cases} \tag{5.18}$$

where $v_{\max}$ is another parameter representing the maximum speed. The *braking* phase allows to compute $v_n(t + 2/3)$ as follows:

$$v_n(t + 2/3) = \min\left\{v_n\,(t + 1/3)\,,\; d^{\text{eff}}_{n,m}(t)\right\} \tag{5.19}$$

and the following rule is applied:

$$\begin{aligned} &\text{If} \quad v_n(t + 2/3) < v_n(t) \\ &\text{then} \quad b_n(t + 1) = \text{on} \end{aligned} \tag{5.20}$$

According to the *randomisation* phase, the value of $v_n(t + 1)$ is obtained as

$$v_n(t + 1) = \begin{cases} \max\{v_n(t + 2/3) - 1, 0\} & \text{with probability } p \\ v_n(t + 2/3) & \text{otherwise} \end{cases} \tag{5.21}$$

and the following rule is applied:

$$\begin{aligned} &\text{If} \quad p = p_0 \;\wedge\; v_n(t + 1) < v_n\,(t + 2/3) \\ &\text{then} \quad b_n(t + 1) = \text{on} \end{aligned} \tag{5.22}$$

where p_0 is a parameter. Finally, according to the *move* rule, the position of each vehicle is updated according to the speed just determined, i.e.

$$x_n(t + 1) = x_n(t) + v_n(t + 1) \tag{5.23}$$

4. *Exits from off-ramps*. The number of vehicles exiting a freeway stretch is defined by means of a probability p_{out} depending on the vehicle class. Note that more advanced approaches should consider the assignment of the final destination to every vehicle. Moreover, it would be possible to model the off-ramp as a finite-capacity buffer, so that, when the buffer is full, a queue grows backwards in the freeway stretch creating a spillback phenomenon.

5.2.4 Traffic Simulation Tools

Traffic simulation tools are software systems with a large variety of applications, both in the urban and in the freeway context. These tools generally implement different types of traffic models, such as microscopic and mesoscopic models, and provide a visual framework useful for experimental studies, also in case of large-scale traffic systems.

In the following, an analysis of the characteristics of the main traffic simulators available on the market is reported, without presuming to provide in this book a complete list of all the traffic simulation tools present worldwide. The interested

Table 5.1 Traffic simulation tools

Name	Developer	Type of license
Paramics	Quadstone	Commercial
Aimsun	Aimsun	Commercial
PTV Vissim	PTV	Commercial
TSIS-CORSIM	McTrans	Commercial
MATSim	Open Community	Open Source
MITSIMLab	Massachusetts Institute of Technology	Open Source
SUMO	DLR	Open Source

reader can find more details in the books [4, 5], which are specifically dedicated to the topic.

Some of the traffic simulators that are presently used by traffic experts and research centres working on modelling, planning and control of road traffic systems are listed in Table 5.1. Among the commercial traffic simulators, it is worth citing Paramics, Aimsun, PTV Vissim and TSIS-CORSIM. *Paramics*, developed by Quadstone, is a microscopic traffic simulation software used by researchers, engineers and planners worldwide, and it provides solutions for both freeway and urban networks, including public transport, pedestrian modelling and ITS applications. *Aimsun* is an integrated transport modelling software which has grown from being a micro-simulator to becoming a fully integrated application with features of travel demand modelling, macroscopic functionalities and mesoscopic–microscopic hybrid simulation allowing to represent the traffic behaviour in a very detailed way, while preserving computational efficiency. *PTV Vissim* is a microscopic multimodal traffic flow simulation software package developed by PTV. It is conceived for motorised private transport, goods transport, rail and road public transport, pedestrians and cyclists, and allows to make a detailed analysis and planning of urban and extra-urban road infrastructure. *TSIS-CORSIM* is a microscopic traffic simulation software package for urban signalised traffic systems, freeway traffic systems or combined urban-freeway systems. It is based on microscopic traffic models to represent the movements of individual vehicles, including the influences of geometric conditions, drivers' behaviours, presence of traffic control implementations and so on.

Besides these commercial software tools, many open-source traffic simulators have been developed by open communities or research groups worldwide. Among them, it is worth mentioning MATSim, MITSIMLab and SUMO. *MATSim* is an agent-based micro-simulator, in which every part of the traffic system is represented as an agent specified by a dynamic behaviour, and the evolution of the entire system is given by interactions among the various agents. The intermodal simulation is supported as well and advanced users can extend the source code, written in Java, to create customised releases adapted to their own purposes. *MITSIMLab* is an open-source application, written in C++, developed at the MIT Intelligent Transportation Systems Program. This platform includes MITSIM, i.e. the traffic simulation tool, implementing microscopic traffic models, and TMS, i.e. the traffic management simulator, which models the implementation of traffic control strategies, such as ramp metering, mainline control, route guidance and so on. *SUMO* is a free and open traffic simulation suite developed in C++, basically devoted to urban mobility, including intermodal traffic composed of road vehicles, public transport and pedestrians.

Each traffic simulation tool has its own characteristics and it is sometimes difficult to find the best tool to be used for the simulation of a given traffic case. Some works in the literature deal with the comparison among the characteristics and the performance of different software tools for traffic simulations. These comparisons, and the conclusions drawn in these works, are of course dependant on the considered test case and on the software version that has been adopted. For instance, [86] reports a comparison among three traffic simulation software programs, that are CORSIM, Vissim and Paramics, referring to a test case of an intersection between the U.S. Highway 50 and the Missouri Flat Road interchange near Placerville, California, U.S. In this study, the application to the test case showed for instance that Paramics and Vissim are characterised by a larger number of parameters compared to CORSIM, allowing to more accurate simulations but making the set-up phase more difficult.

Another comparison among traffic simulation tools was done and reported in [87], where Aimsun, Paramics and Vissim are analysed with specific attention to the effectiveness of car-following models. This comparative analysis was carried out considering a real car-following experiment, set in Germany, in which instrumented vehicles were used to record the speeds and relative distances on a one-lane road. The same setting was implemented with the three traffic simulators and the simulated results were compared with field data. The results show that the lowest errors are obtained with the Gipps-based models implemented in AIMSUN, while higher errors are obtained with the psychophysical models used in Paramics and Vissim.

Another comparison related with the car-following rules was discussed in [88], where the simulators Aimsun, Paramics, Vissim and MITSIM were compared considering the same test case. According to this study, the number of parameters present in Vissim and Paramics is very high, whereas MITSIM and Aimsun are characterised by fewer parameters, and, also, in Aimsun the parameters have a more intuitive meaning. In this study, some specific microscopic aspects are analysed in detail and the way how they can be represented with the four traffic simulators is described. For instance, referring to the reaction time of drivers, in [88] it is observed that AIMSUN uses a driver reaction time equal to the simulation time step, which is equal for all drivers, MITSIM assigns possibly different individual reaction times to every vehicle, while Vissim and Paramics do not model reaction times explicitly.

The results obtained from the described comparisons highlight how each simulator has strengths and weaknesses; the choice is subject to specific user needs and a trade-off between different features and performance. Despite the commercial simulators offer the most comprehensive options with programming frameworks that are carefully designed and optimised, guaranteeing support to the users, the open-source simulators have the strength that the user can use the source code and properly modify it. This aspect is relevant for two main reasons: the former is the possibility for the users to create an ad-hoc version of the software that meets their precise needs and the latter lies in the contribution that individual users can give to the developers' community.

5.3 Mesoscopic Traffic Models

The class of mesoscopic traffic models represents an intermediate approach between macroscopic traffic models, relying on the dynamics of aggregate variables, and microscopic traffic models, representing instead the dynamics of each vehicle in the traffic flow. Mesoscopic models describe the traffic flow dynamics in an aggregate way but represent the individual behaviour of drivers using probability distribution functions. In the literature, different mesoscopic modelling approaches are present [1]. Among them, three main classes can be identified related to *headway distribution models*, *cluster models* and *gas-kinetic models*. Sections 5.3.1–5.3.3 describe, respectively, these three types of mesoscopic models.

5.3.1 Headway Distribution Models

In headway distribution models, attention is posed on the statistical properties of *time headways*. Starting from an empirical observation of the distribution of time headways (or, alternatively, of vehicle spacings) and assuming that they are independent and identical distributed random variables, headway distribution models are based on the definition of suitable probability density functions for such distributions.

In a first set of works dealing with headway distribution models (see, for example, [89–91]), *stationary* distribution models were addressed. These models have shown to effectively fit empirical data in free-flow traffic conditions but they are not completely adequate in congested situations. *Mixed* headway distribution models tackle this drawback by distinguishing between free-driving vehicles and following vehicles, with the headways of the two categories characterised by different probability density distributions (see, e.g. [92]).

The characteristic of some stationary distribution models of being mainly suitable for free-flow conditions is often motivated by the fact that they support an incomplete representation of the interactions among vehicles, which are typically weak and negligible in free-flow conditions and consistent, instead, in congested traffic cases. More recently, *dynamic* headway distribution models have been developed to improve the way in which the dynamic role of traffic is considered. To this end, in [93] different vehicle types in the different phases of traffic are explicitly modelled, whereas random matrix theory is used in [94] to predict headway distributions in a model in which traffic is represented as a set of strongly linked particles under fluctuations. A further work on the topic is, for instance, [95], in which a variance-driven adaptation mechanism is defined, according to which drivers increase their safety time gaps when the local traffic dynamics is unstable or largely varying.

5.3.2 Cluster Models

Cluster models represent the dynamics of traffic flow by describing the formation of *clusters of vehicles*, i.e. groups of vehicles which share a specific property. Clusters usually emerge because of restricted lane-changing possibilities or due to prevailing weather or ambient conditions. Different aspects of clusters can be considered, such as their size (the number of vehicles in a cluster) and their speed. Generally, the size of a cluster is dynamic, i.e. clusters can grow and decay. Clusters are typically considered as homogeneous, in a sense that the conditions of vehicles inside a cluster, e.g. their headways or the speed differences, are not explicitly taken into account (see, for example, [96, 97]).

In particular, cluster models deal with the rules of cluster formation, the conditions under which clusters can appear and their characteristics. The basic idea is to find a physically motivated assumption for the transition rates of the attachment and detachment of individual vehicles to a cluster consistent with the empirical observations in real traffic.

Cluster models are first referred to the simplified case in which only one cluster is present in the traffic system [97], and then extended to a multi-cluster case [98]. In the case in which a *single cluster* is considered, the cluster is specified by its size n, which is the number of aggregated vehicles. Its internal parameters, namely the headway distance and, consequently, the speed of vehicles in the cluster, are treated as fixed values independent of the cluster size n. As depicted in Fig. 5.6, a cluster grows when free vehicles join it at its upstream boundary, and it becomes instead shorter when vehicles located near its downstream boundary accelerate to leave it.

The processes yielding changes in the cluster size are described as random processes in which the probability function $P(n, t)$ for the cluster to have size n at time t is defined. This function evolves, thanks to the so-called *one-step master equation* expressed as follows:

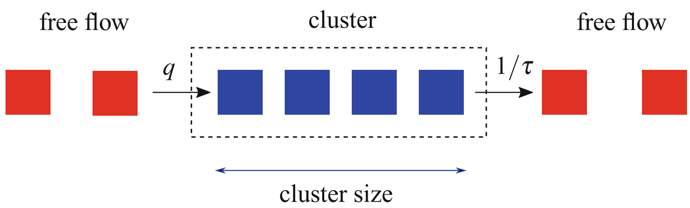

Fig. 5.6 Sketch of a single cluster

$$\frac{\partial P(n,t)}{\partial t} = w_+(n-1)P(n-1,t) + w_-(n+1)P(n+1,t) - [w_+(n)P(n,t) + w_-(n)P(n,t)] \tag{5.24}$$

where $w_+(n)$ and $w_-(n)$ are the attachment rate and the rate of vehicles leaving the cluster when it has size n, respectively. These rates can be considered as constant values and expressed as $w_+ = q$, $w_- = 1/\tau$, where q is the traffic flow before the cluster and τ is the characteristic time needed to the first vehicle in the cluster to leave it and to go out from its downstream boundary at a distance approximately equal to the headway distance in the free-flow state.

On the basis of the balance equation (5.24) and the Fokker–Planck approximation to calculate mean first passage times or escape rates, it is possible to determine the dynamics of the traffic pattern formation and, specifically, the time in which the traffic conditions vary from free-flow to congested, including the influence of the parameters affecting the discharge and adhesion rates.

The single-cluster case can then be extended to consider the presence of *several clusters* of different sizes [98]. It is in this case necessary to model the dynamics of all the sizes of the clusters and to make the transition rates of the attachment and detachment of individual vehicles to a cluster consistent with the empirical observations in real traffic. To this end, the analogy with first-order phase transitions and nucleation phenomena in physical systems (like supersaturated vapour) is exploited.

In order to make a comparison with real measurements, the results are represented with a Fundamental Diagram of traffic flow (i.e. the steady-state flow–density relation), and, then, compared with empirical data. It is also possible to analyse different traffic conditions (free-flow, congested mode and heavy viscous traffic) and to include on-ramp effects.

5.3.3 Gas-Kinetic Models

Among mesoscopic approaches, the most known models are gas-kinetic models, in which an analogy between the dynamics of gases and the dynamics of traffic flows is exploited. In these models, some concepts of statistical physics are introduced, such as the *reduced phase-space density*, which is related to the expected number of vehicles present in an infinitesimal region, travelling with a speed defined on the basis of a probability distribution function. Such a concept can be seen as the mesoscopic version of the macroscopic traffic density. Moreover, the distribution function of the speed is affected by three processes: the process of convection, the process of acceleration towards the desired speed and the process of deceleration due to the interaction among vehicles.

An initial proposal of these models was presented by *Prigogine* and *Herman* in [99, 100]. These works introduce the concept of reduced phase-space density $\tilde{\rho}(x, v, t)$. Specifically, the reduced phase-space density $\tilde{\rho}(x, v, t)$ can be used to compute the expected number of vehicles present at time t in the infinitesimal region between position x and $x + \mathrm{d}x$, with $\mathrm{d}x \to 0$, moving with a speed between v and $v + \mathrm{d}v$, with $\mathrm{d}v \to 0$. This expected number of vehicles can be obtained as $\tilde{\rho}(x, v, t)\mathrm{d}x\, \mathrm{d}v$.

The first relation encountered in gas-kinetic traffic flow models is the following partial differential equation:

$$\frac{\partial \tilde{\rho}(x, v, t)}{\partial t} + v \frac{\partial \tilde{\rho}(x, v, t)}{\partial x} = \left(\frac{\partial \tilde{\rho}(x, v, t)}{\partial t} \right)_{\mathrm{acc}} + \left(\frac{\partial \tilde{\rho}(x, v, t)}{\partial t} \right)_{\mathrm{int}} \tag{5.25}$$

where

- the second term of the left-hand side is the so-called *convection* term describing the propagation of the phase-space density with speed v;
- the first term of the right-hand side is the *acceleration/relaxation* term modelling the fact that vehicles tend to reach an equilibrium or desired speed;
- the second term of the right-hand side represents the *interactions* with surrounding vehicles; in this term the probability of overtaking is explicitly considered.

According to [100], the acceleration term depends on the desired speed distribution, denoted as $V_0(x, v, t)$, and can be written with the following expression:

$$\left(\frac{\partial \tilde{\rho}(x, v, t)}{\partial t} \right)_{\mathrm{acc}} = -\frac{\partial}{\partial v} \left(\tilde{\rho}(x, v, t) \frac{V_0(x, v, t) - v}{\tau} \right) \tag{5.26}$$

where τ denotes the acceleration time. This expression represents a collective relaxation towards an equilibrium speed dependent on the traffic composition, thus assuming that there is not a correlation between the speeds of slowing-down vehicles and the speeds of impeding vehicles.

For the interaction term in (5.25), the model by Prigogine and Herman is based on a set of assumptions, including the so-called *vehicular chaos assumption*, which are listed below:

- the length of vehicles can be neglected;
- the interactions affect at most two vehicles;
- if a fast vehicle moving with speed v reaches a vehicle moving with speed $w < v$, the fast vehicle either overtakes or reduces its speed to w and:
  - the speed w of the slow vehicle is not affected by the interaction;
  - the fast vehicle slows down immediately and overtakes immediately;
  - the speed of the fast vehicle after overtaking remains equal to v;
  - the overtaking event is associated with a probability π, while the slowing-down event is associated with probability $1 - \pi$.

To model the interactions between pairs of vehicles, the Prigogine–Herman models consider couples of vehicles located in the infinitesimal positions $[x, x + \mathrm{d}x)$ and $[x', x' + \mathrm{d}x']$, driving with speeds $[v + \mathrm{d}v)$ and $[v' + \mathrm{d}v')$, respectively, and introduces a two-vehicle distribution function $\tilde{\phi}(x, v, x', v', t)$. This function has the following meaning: $\tilde{\phi}(x, v, x', v', t)\mathrm{d}x\, \mathrm{d}v\, \mathrm{d}x'\, \mathrm{d}v'$ is the expected number of vehicle pairs located at the given infinitesimal areas and with the defined speeds. It can be noted that the previous assumptions and, specifically, the vehicle-chaos assumption, imply the following:

$$\tilde{\phi}(x, v, x', v', t) = \tilde{\rho}(x, v, t)\tilde{\rho}(x', v', t) \tag{5.27}$$

Then, the interaction is modelled with the so-called *collision equation* given by

$$\left(\frac{\partial \tilde{\rho}(x, v, t)}{\partial t}\right)_{\text{int}} = (1 - \pi) \int (w - v)\tilde{\phi}(x, v, x, w, t)\mathrm{d}w \tag{5.28}$$

which, by exploiting (5.27), becomes

$$\left(\frac{\partial \tilde{\rho}(x, v, t)}{\partial t}\right)_{\text{int}} = (1 - \pi)\tilde{\rho}(x, v, t) \int (w - v)\tilde{\rho}(x, w, t)\mathrm{d}w \tag{5.29}$$

This model has received some critiques regarding both the acceleration/relaxation term and the interaction term. Specifically, the acceleration/relaxation term has been criticised referring to the fact that the speeds of slowing-down vehicles and the speeds of impeding vehicles cannot be considered as uncorrelated quantities, meaning that individual relaxation terms in place of the collective one should be more suitable to be adopted. A way of overcoming this assumption was proposed in [101], where a quadratic *Boltzmann term* is used to represent slowing-down and speeding-up interactions. Suitable models for driver reaction and vehicular correlation are used to determine the adopted Boltzmann term.

Other approaches modelling the acceleration term in different ways have been proposed, by taking into account the individual desired speed v_0 and a class-specific acceleration time τ_0. Let us consider in particular the two following extreme cases:

1. all the vehicles can accelerate towards v_0 with an acceleration time equal to τ_0 (see, e.g. [102]);
2. only vehicles in free-flow conditions can accelerate towards v_0 with τ_0 as acceleration time. Vehicles which are constrained (possibly gathered in platoons) do not accelerate at all (see, e.g. [103]).

If the former assumption holds, it is

$$V_0(x, v, t) = v_0 \qquad \tau = \tau_0 \tag{5.30}$$

and these terms must be substituted in (5.26). In case, instead, the latter assumption is considered, the expected fraction θ of platooning vehicles is defined and the following relation holds:

$$V_0(x, v, t) = \theta v + (1 - \theta)v_0 \qquad \tau = \tau_0 \tag{5.31}$$

again to be inserted in (5.26).

As aforementioned, the Prigogine–Herman model received some critiques regarding the relaxation term, since it appears that the relaxation of the distribution function is a property of the road, it does not describe the behaviour of drivers, and it corresponds to discontinuous speed changes. Again in [102], it is discussed that also the collision term (as the acceleration term) proposed in the gas-kinetic model by Prigogine and Herman is only valid when vehicles are not platooning. By considering a scenario in which a free-flowing vehicle encounters a platoon, two cases are analysed in [102]:

1. the free-flowing vehicle overtakes the whole queue of vehicles constituting the platoon;
2. the free-flowing vehicle overtakes each single vehicle in the platoon as if it were alone.

In [102], it is shown that the Prigogine–Herman model is represented by the second case, while the real cases stand between these two extreme situations. Moreover, in [102], a new model is proposed, often known as the *Paveri–Fontana* model, which considers a phase-space density explicitly dependent on the individual desired speed v_0, i.e. $\rho(x, v, v_0, t)$, being

$$\tilde{\rho}(x, v, t) = \int \rho(x, v, v_0, t)\mathrm{d}v_0 \tag{5.32}$$

Moreover, the interaction term is expressed as

$$\begin{aligned}\left(\frac{\partial \tilde{\rho}(x, v, t)}{\partial t}\right)_{\text{int}} &= -\rho(x, v, v_0, t) \int_0^v (1-\pi)(v-\omega)\tilde{\rho}(x, \omega, t)\mathrm{d}\omega \\ &\quad + \tilde{\rho}(x, v, t) \int_v^{+\infty} (1-\pi)(\omega - v)\rho(x, \omega, v_0, t)\mathrm{d}\omega \end{aligned} \tag{5.33}$$

The overall Paveri–Fontana model can then be written in the following form:

$$\begin{aligned}&\frac{\partial \tilde{\rho}(x, v, t)}{\partial t} + v\frac{\partial \tilde{\rho}(x, v, t)}{\partial x} + \frac{\partial}{\partial v}\left(\tilde{\rho}(x, v, t)\frac{v_0 - v}{\tau_0}\right) \\ &= -\rho(x, v, v_0, t) \int_0^v (1-\pi)(v-\omega)\tilde{\rho}(x, \omega, t)\mathrm{d}\omega \\ &\quad + \tilde{\rho}(x, v, t) \int_v^{+\infty} (1-\pi)(\omega - v)\rho(x, \omega, v_0, t)\mathrm{d}\omega \end{aligned} \tag{5.34}$$

The complete Paveri–Fontana equation has not been solved in an analytical way, but it is numerically solved in some special cases. However, it is used as the starting point to construct macroscopic and mesoscopic models based on the gas-kinetic theory.

Another issue which is raised with reference to the basic gas-kinetic models is that the assumption that there are some drivers desiring to drive at any speed, no matter how small, seems somewhat unrealistic. The work in [104] addresses the case in which this assumption does not hold, by showing that at high densities it happens that a two-parameter family of solutions exist and, thus, continuously distributed mean speeds can be identified for each density value. This result also gives reason to the well-known scattering of observed data related to the relationship between speed and density for high density values.

An extension of the Paveri–Fontana model was proposed in [105], in which a multi-lane case is considered. Lane changing is explicitly modelled in the following way: indicating with j the lane index, a multi-lane phase-space density $\rho_j(x, v, v_0, t)$ is defined and the following expression holds:

$$\begin{aligned}\frac{\partial \rho_j(x, v, v_0, t)}{\partial t} + v\frac{\partial \rho_j(x, v, v_0, t)}{\partial x} &= \left(\frac{\partial \rho_j(x, v, v_0, t)}{\partial t}\right)_{\text{acc}} \\ &+ \left(\frac{\partial \rho_j(x, v, v_0, t)}{\partial t}\right)_{\text{int}} + \left(\frac{\partial \rho_j(x, v, v_0, t)}{\partial t}\right)_{\text{vc}} \\ + \left(\frac{\partial \rho_j(x, v, v_0, t)}{\partial t}\right)_{\text{lc}} &+ v_j^+(x, v, v_0, t) - v_j^-(x, v, v_0, t) \end{aligned} \tag{5.35}$$

where $v_j^+(x, v, v_0, t)$ and $v_j^-(x, v, v_0, t)$ are the rates of vehicles entering and leaving the road at place x, respectively. These rates are different from zero only for merging lanes at entrances and exits. As for the acceleration term in (5.35), it is assumed that vehicles are split into a set of vehicles that can move freely and a set of impeded vehicles that have to move slower than desired, since they are queued behind other vehicles. As in [103], a proportion of freely moving vehicles is, then, defined and the acceleration term is only related to the acceleration of these vehicles.

Moreover, the interaction term in (5.35) is similar to the one used in Paveri–Fontana model, whereas four terms have been added to that previous model. The first of these terms is a speed diffusion term expressed as

$$\left(\frac{\partial \rho_j(x, v, v_0, t)}{\partial t}\right)_{\text{vc}} \tag{5.36}$$

modelling individual fluctuations of the speed due to imperfect driving, while the second is a lane-changing term given by

$$\left(\frac{\partial \rho_j(x, v, v_0, t)}{\partial t}\right)_{\text{lc}} \tag{5.37}$$

representing the changes in the phase-space density of a lane due to vehicles moving to and from the lane itself. Finally, the third and forth terms are the rates of vehicles entering and exiting the road through merging lanes.

A further extension of the basic gas-kinetic models refers to the explicit representation of different vehicle classes belonging to a set $\mathscr{U}$. In [106], a multi-class phase-space density $\rho_u(x, v, v_0, t)$ is introduced, with the index $u \in \mathscr{U}$ related to the vehicle class. In this case, a relation analogous to (5.25) is considered for each vehicle class with

$$\left(\frac{\partial \rho_u(x, v, v_0, t)}{\partial t}\right)_{\mathrm{acc}} = -\frac{\partial}{\partial v}\left(\rho_u(x, v, v_0, t)\frac{v_0 - v}{\tau_u}\right) \tag{5.38}$$

where τ_u is the acceleration time of vehicles of class u. Moreover, the interaction term is defined by separately considering the interactions of vehicles of class u with vehicles of the same class and with vehicles of other classes. To this end, the two terms $I_{u,s}(x, t)$ and $R_{u,s}(x, t)$ are introduced and expressed respectively as

$$I_{u,s}(x, t) = \int_0^v (v - w)\rho_u(x, v, v_0, t)\rho_s(x, w, w_0, t)\mathrm{d}w\,\mathrm{d}w_0 \tag{5.39}$$

$$R_{u,s}(x, t) = \int_v^{+\infty} (w - v)\rho_u(x, w, v_0, t)\rho_s(x, v, w_0, t)\mathrm{d}w\,\mathrm{d}w_0 \tag{5.40}$$

The interaction term is given by

$$\left(\frac{\partial \rho_u(x, v, v_0, t)}{\partial t}\right)_{\mathrm{int}} = -(1 - \pi_u)\sum_s I_{u,s}(x, t) - R_{u,s}(x, t) \tag{5.41}$$

where π_u is the probability associated with an overtaking event for vehicles of class u. It can be noted that the presence of different classes of users results in an asymmetric slowing-down process for fast vehicles, i.e. fast vehicles are influenced by slow vehicles more frequently than vice versa.

In [107], a generic traffic model including multi-lane and multi-class aspects together with the presence of platoons is described. This model gathers all the features of existing gas-kinetic approaches for representing the traffic behaviour. Specifically, a phase-state density $\rho_{u,j,c}(x, v, v_0, t)$ is defined, depending on the vehicle class u, on the road lane j and on the possible belonging of vehicles to a platoon ($c = 2$) or not ($c = 1$). Several drawbacks of previous gas-kinetic models are tackled, since the model describes separately free-flowing and platooning vehicles instead of considering vehicles as independent moving entities. This overcomes the limitations due to the vehicular chaos assumption. Also, the acceleration term is determined in the model by the platoon leader, as it happens in real cases.

These mesoscopic principles present in gas-kinetic models have been also exploited to extend macroscopic models. For instance, in [108, 109], a macroscopic traffic model based on gas-kinetic logics is introduced for the case of multiple classes of vehicles. Analogously, gas-kinetic traffic flow modelling is the basis for a macroscopic model considering adaptive cruise control policies in [110]. Specifically, in [110], two approaches are considered, the former is adapted from [111], while the latter is a novel one and is based on the introduction of a new relaxation term which satisfies the time/space-gap principle of adaptive cruise control systems. The kinetic theory is also used in [112] to derive a new mathematical model of vehicular traffic, in which the assumption on the continuously distributed spatial position and speed of the vehicles is relaxed, consequently resulting in a discretisation of position and speed of the vehicles.

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