Chapter 10
Control Strategies for Sustainable Mobility in Freeways

10.1 Sustainability Concepts for Freeway Traffic Control

The concepts of *sustainability* and *sustainable development* are worldwide recognised as of primary importance for the growth of individuals and organisations, in order to meet the needs of present and future generations. Various definitions of sustainability have been provided in the last decades, highlighting different specific aspects and considering different indicators and goals (see e.g. [1, 2] and the references therein for an overview on these concepts).

Several areas of development and various objectives can be defined for achieving sustainability. For instance, the Sustainable Development Goals Report [3], issued by the United Nations in 2017, has fixed seventeen general goals towards sustainability, regarding health, education, safety of people, as well as careful management of natural resources. These goals represent an ambitious challenge for the entire society in order to achieve an equitable and sustainable progress.

All the definitions of sustainable development, even with different peculiarities, agree on a common point, related to the necessity of strengthening actions now that do not neglect the possible negative consequences that will occur in the ecosystem in the long period. In other words, sustainability means satisfying the present needs of individuals and organisations without compromising the possibility of *future generations* to meet their own necessities.

Another common point in the various definitions of sustainability is related to three main areas of interest, which should be properly integrated and balanced to achieve a sustainable development. These three dimensions are as follows:

- *environment*: environmental protection and ecological integrity should be guaranteed, maintaining a balance among all the natural resources;
- *economy*: the economic sustainability must be preserved to allow that all the human communities have access to the resources they need;
- *society*: healthy, safe and secure systems should be realised to ensure the wellness of people worldwide.

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In this very general and challenging framework, also *road transportation* has an important role. Indeed, it is undeniable that if, on the one hand, the increment in road transport systems allows the improvement of social and economic welfare, on the other hand, such increase produces several negative effects which have implications for the society in the form of social, environmental and, also, economic consequences.

Of course, when a driver is planning a travel or is moving on a road network, he takes his decisions considering his own and presently perceived costs, without estimating or forecasting more general costs related to the ecosystem and the impacts on the future generations. Nevertheless, a *traffic management and control tool*, which acts, instead, at a macroscopic level, should be devised to be *sustainable*, hence not neglecting these global factors and their future implications (see Fig. 10.1). A control tool designed for sustainable mobility should therefore regulate traffic in order to achieve system-wide objectives guaranteeing a high quality of life for citizens and ensuring environmental protection, but also taking into account the individual goals of travellers.

Despite this new sensibility for sustainability concepts, analysing the wide literature on freeway traffic control, it is worth noting that most of the research works are devoted to the sole reduction of congestion phenomena, i.e. to the minimisation of the total time spent by the drivers in the traffic network. However, in the last years, many other sustainability-devoted aspects have received attention, such as the reduction of pollutant emissions, as well as the increase of safety, and have been explicitly taken into consideration in the design of traffic control schemes for realising sustainable mobility systems.

Fig. 10.1 A road stretch in A4 freeway, close to Leiden, the Netherlands (courtesy of Rijkswaterstaat, Photo: Essencia Communication/Rob de Voogd)

In Sect. 10.2, the scientific literature on freeway traffic control explicitly addressing sustainable issues is revised. Then, Sects. 10.3 and 10.4 propose some possible freeway traffic control solutions which take into account, as control objectives, not only the reduction of congestion phenomena but also the mitigation of emissions in the environment, by distinguishing different typologies of vehicles. The control schemes described in Sects. 10.3 and 10.4 are of different types, from simple and easy-to-implement solutions to more sophisticated optimisation-based frameworks, and can constitute a basis for researchers to develop new traffic control strategies for sustainable mobility.

10.2 Overview of Traffic Control Schemes for Sustainable Freeways

Traffic control for sustainable mobility in freeway networks is a very recent research topic that is becoming more and more relevant within the scientific community of traffic control engineers. Sustainable issues can be taken into account in traffic control schemes in different ways. The most relevant directions followed so far to address sustainability-related factors are the following:

- considering *sustainable objectives* explicitly in the controller design;
- differentiating the traffic flow in different *vehicle categories*, so that it is possible to model them in a customised way (this is particularly relevant for instance for emission models) and to control them separately.

While some research works address these two aspects separately (see Sects. 10.2.1 and 10.2.2), some recent works consider them jointly, as discussed in Sect. 10.2.3 in general and addressed more in detail in Sects. 10.3 and 10.4.

10.2.1 Traffic Control with Sustainable Objectives

The idea of considering sustainable issues in the design of a traffic controller is rather recent and has been conceived, in most of the works, by including the reduction of traffic emissions among the objectives of the traffic control scheme. Another aspect that has been addressed explicitly is safety, generally expressed in terms of number of accidents expected to occur in the freeway.

The reduction of *traffic emissions* is explicitly considered as control objective in [4], where a receding-horizon parametrised traffic control strategy is proposed to jointly minimise travel times and emissions in the freeway through ramp metering and variable speed limits. In [5], a general framework is introduced to integrate the macroscopic METANET model with the microscopic emission and fuel consumption model called VT-micro, resulting in the so-called VT-macro model. The purpose of that modelling framework is to provide a prediction tool able to guarantee accurate estimates of the emissions and fuel consumptions in short computational times, as those required by freeway traffic controllers to be applied in real time. In [6], a model for the *dispersion* of traffic emissions along a freeway is proposed: such model should be adopted for control purposes in order to keep pollutant concentrations under legislation limits and, hence, it should require a low computational effort.

The VT-macro framework is adopted in [7], where a Model Predictive Control (MPC) scheme for a combined strategy of ramp metering and variable speed limits is proposed. In [7], in order to deal with an optimal control problem affordable from a computational point of view, the non-linear METANET model is approximated through a piecewise-affine formulation.

Besides ramp metering and variable speed limits, route guidance control has been investigated as well in order to reduce emissions in the freeways, corresponding to the so-called *eco-routing* strategies. For instance, in [8], the authors assess the environmental and energetic impacts produced by the route choice decisions using both a microscopic and a macroscopic tool, and they show that the faster routes preferably chosen by drivers are not always the best in terms of environmental issues and energy consumptions. In [9], a microscopic traffic assignment and simulation framework are proposed for setting eco-routing strategies for drivers.

In [10], game theory is applied for developing road pricing methods for routing drivers in urban and freeway traffic networks. Such methods should be used by traffic authorities to induce users to follow routes that are efficient from a system-optimum perspective, i.e. routes which minimise the total time spent by drivers in the network and reduce the total traffic emissions. In [11], an MPC scheme for real-time route guidance control is proposed, not only to improve traffic efficiency in terms of total time spent by drivers but also considering the reduction of emissions and fuel consumptions for all vehicles moving in the network.

Another very relevant issue addressed in freeway traffic towards sustainability is *road safety*. This aspect has been investigated in many papers and research reports, since it is undeniable that one of the major criticalities and consequences of congested roads is the high number of accidents, often serious or fatal, affecting many drivers every day. The causes of traffic accidents have been examined by researchers and are still under investigation. Many studies in this area rely on statistical analyses of real historical data of crashes, in order to correlate accidents with specific traffic states or conditions, as well as with other factors, such as road geometry, drivers' behaviours and environmental factors.

Among the works investigating the correlation between the safety level in a freeway and the present traffic conditions, it is possible to cite for instance [12], referring to the case of freeways in California, U.S. In that work, traffic data measured with loop detectors and detailed information about accidents, classified in different crash typologies, are used to highlight the relationships between traffic flow conditions and the likelihood of traffic accidents. Based on traffic and crash data from a Canadian case, the study developed in [13] aims at defining a relation between crashes and traffic data, such as flow and density, for both rural and urban freeway segments. A methodology to investigate the relation between traffic states and crash involvements in a freeway is discussed in [14].

Other researchers have focused their attention on analysing the impact on safety of the adoption of traffic control strategies in freeways. For instance, the benefits in terms of crash likelihood reduction due to the application of variable speed limits are analysed in [15, 16]. Analogously, the effects of the implementation of ramp metering strategies on safety are assessed in [17].

Very few research works are devoted to consider safety explicitly in the controller design. In [18], a freeway control algorithm adopting variable speed limits is defined to minimise the total crash risk in the system, while in [19] variable speed limits are applied specifically to reduce rear-end collision risks. In [20], a coordinated ramp metering strategy is proposed in order to both minimise the travel times for the drivers in the freeway and minimise the expected number of crashes in the system.

10.2.2 Multi-class Traffic Control

A relevant feature towards the definition of sustainable traffic control strategies includes the possibility of distinguishing different classes of vehicles, i.e. cars, trucks or other specific vehicles, since they generally present different dynamic behaviours and have different environmental impacts on the freeway system. Also, it is possible to distinguish vehicles in classes according to other aspects, e.g. one can distinguish among private vehicles, public means of transport, vehicles travelling for commercial uses and so on. A *multi-class traffic* framework consists not only in adopting multi-class traffic models but also in designing multi-class control strategies, so that specific control actions are defined for the different vehicle classes.

It is important to emphasise that the use of a *multi-class traffic model* allows to represent the traffic system behaviour more accurately than with a one-class model which assumes that the whole traffic is a homogeneous fluid (see Sects. 3.4 and 4.3 for a detailed discussion and some motivations for multi-class models). This is especially true for instance in case a high percentage of trucks is present in the freeway traffic system, since trucks have a strong impact on the overall traffic flow for many reasons, e.g. for their dimensions, low operating capabilities and so on.

The design of *multi-class traffic controllers* enables the adoption of specific policies for the different classes of vehicles, in order to assign them different priorities or different rules according to their characteristics. It is worth noting that multi-class control requires, from the *implementation* point of view, some specific features to be applied in the actuators. For instance, controlling separately different vehicle classes via ramp metering means that separate lanes and signals must be present at the on-ramps, while, for route guidance and variable speed limits, it means that specific indications must be given to the different vehicle typologies on Variable Message Signs (VMSs). Note that the increasing availability of on-board devices enables the communication of routing indications, as well as speed limits, directly to drivers, further motivating the development of multi-class control strategies.

The idea of proposing multi-class regulators is rather recent and has been developed in few research works. For instance, in [21], combined multi-class strategies relying on ramp metering and variable speed limits are investigated and an MPC control scheme is proposed. Multi-class ramp metering is also analysed in [22], also possibly considering different priorities for different vehicle classes.

10.2.3 Multi-class Sustainable Traffic Control

Some very recent works have combined the emission-related issues with the distinction of multiple vehicle types, leading to *multi-class sustainable control* frameworks. In [23], an MPC approach for multi-class coordinated ramp metering is developed, aiming at jointly reducing traffic emissions and travel times in freeway stretches. A two-class freeway traffic controller to reduce congestion and emissions is also presented in [24], while different multi-class traffic and emission models are compared in [25] for MPC schemes with end-point penalties in the objective function.

In [26], an optimal control scheme is proposed for reducing congestion and improving safety via multi-class coordinated ramp metering. The optimal control problem is solved with derivative-free solution algorithms.

Other multi-class sustainable control frameworks are for instance the local feedback control strategies, of ramp metering type, investigated in [27–29] and described in detail in Sect. 10.3.1. These control strategies are based on standard proportional–integral local controllers, extended to deal with a multi-class traffic flow and to reduce the emissions in the freeway.

These latter local ramp metering strategies were extended in [30, 31], leading to a supervisory coordinated ramp metering framework, in which local feedback controllers receive a communication from a supervisor about the control law to be applied. Specifically, a supervisor, acting at a higher level, receives measurements from the freeway network and periodically makes a prediction on the system evolution. At the lower level, local feedback controllers compute the control action on the basis of measurements in a given area close to the on-ramp and the parameters of the control law are communicated by the supervisor in real time, according to an event-triggered logic. This supervisory event-triggered control scheme for coordinated ramp metering is analysed in Sect. 10.3.2.

Optimal control techniques are adopted in [32, 33] for optimally reducing the total time spent by the drivers and the total emissions experienced by them in freeway systems, as discussed in Sect. 10.3.3. The optimal solution of this non-linear optimal control problem is obtained with gradient-based solution techniques and is used to verify if the reduction of traffic emissions and the reduction of congestion are conflicting objectives or not.

Finally, the combination of ramp metering and route guidance control strategies is exploited in [34, 35] to reduce the total time spent and the total emissions in a balanced way. Both the ramp metering and the route guidance controllers are of the multi-class type and are based on feedback predictive control laws, i.e. they compute the control actions not only on the basis of the measured system state but also on the basis of the prediction of the system evolution, in terms of traffic conditions and traffic emissions. This combined multi-class control framework is described in Sect. 10.4.

10.3 Multi-class Ramp Metering Strategies for Emission Reduction

This section describes three control schemes, having in common the multi-class nature, the adoption of ramp metering as control action, and the combined goal of reducing traffic emissions and mitigating congestion phenomena in a freeway stretch. The first control scheme is a simple local feedback control strategy (see Sect. 10.3.1). This feedback strategy is then extended to be included in a more sophisticated control framework, that is, the supervisory event-triggered control scheme described in Sect. 10.3.2. Finally, an optimal control approach is reported in Sect. 10.3.3, in which the solution found allows to reduce the traffic emissions and the total time spent by the drivers in the whole freeway.

10.3.1 Local Feedback Control

This section presents a *local feedback control strategy*, in which different classes of vehicles are considered, in order to better account for the fact that vehicles of different types present different dynamic behaviours and have different environmental impacts on the freeway system. Of course, the most practical and relevant example of multi-class traffic flow is the two-class case in which cars and trucks are distinguished. In the considered scheme, not only the macroscopic dynamic model is of the multi-class type but also the considered controllers are designed in order to define specific control actions for each vehicle category. More specifically, the adopted control strategy is ramp metering; hence, it is assumed that differently metered lanes are present at the on-ramps for each class of vehicles. It is straightforward that the implementation of *multi-class ramp metering strategies* is realistic with a small number of vehicle classes, surely for the two-class case of cars and trucks.

One of the main advantages of the present local ramp metering control scheme is that it is *simple* and *easily implementable* in real systems. The adopted regulator is a multi-class version of the well-known *PI-ALINEA* strategy, which has shown its effectiveness both theoretically and in practice [36], as discussed in Sect. 8.3.1. Generally speaking, PI-ALINEA is a feedback regulator of proportional–integral type, designed in order to track a set-point value of the density (or occupancy). If the goal of the controller is to reduce the total time spent by the drivers in the freeway system, the set-point is fixed equal to the critical density. Since we are dealing with multi-class control, PI-ALINEA is suitably extended to address the case in which different classes of vehicles are separately controlled.

Let us start by introducing the standard *one-class PI-ALINEA* in case the METANET model for a freeway stretch with on-ramps described in Sect. 4.2.2 is adopted. This is a discrete model, in which the freeway stretch is divided into N road sections (with index i indicating the generic road section of length L_i), while the time horizon is discretised into K time intervals (with k the index of the time step and T the sample time).

In that model, the generic ramp metering control variable is $r_i^{\mathrm{C}}(k) \in [r_i^{\min}, r_i^{\max}]$, representing the flow that should enter section i from the on-ramp during time interval $[kT, (k+1)T)$, $i = 1, \ldots, N$, $k = 0, \ldots, K$. The PI-ALINEA control law follows

$$r_i^{\mathrm{C}}(k) = r_i^{\mathrm{C}}(k-1) - K_P \left[\rho_i^{\mathrm{down}}(k) - \rho_i^{\mathrm{down}}(k-1)\right] + K_R \left[\rho_i^* - \rho_i^{\mathrm{down}}(k)\right] \quad (10.1)$$

where $\rho_i^{\mathrm{down}}(k)$ is the traffic density measured downstream the on-ramp, ρ_i^* is a set-point value for the downstream density, while K_P and K_R are regulator parameters.

Let us now describe the *multi-class PI-ALINEA* regulator, introduced for the first time in [37]. Let us rely on the multi-class METANET model for a freeway stretch described in Sect. 4.3.1, in which C classes of vehicles are explicitly modelled, with conversion factor η^c, $c = 1, \ldots, C$. Remind that $\rho_i^c(k)$ is the traffic density of class c in section i at time kT, while $l_i^c(k)$ is the queue length of vehicles of class c waiting in the on-ramp of section i at time kT. Moreover, the ramp metering control variable is referred to each class c. Specifically, the control variable is denoted as $r_i^{\mathrm{C},c}(k) \in [r_i^{\min,c}, r_i^{\max,c}]$, representing the flow of class c that should enter section i from the on-ramp during time interval $[kT, (k+1)T)$.

According to the multi-class PI-ALINEA regulator (see a generic scheme in Fig. 10.2), the on-ramp flow is computed by extending the control law (10.1) to the multi-class case and by taking into account that the set-point for the density ρ_i^* is still referred to the total density. Hence, the flow $r_i^{\mathrm{C},c}(k)$ at the on-ramp of section i, at time step k, for class c, is obtained as

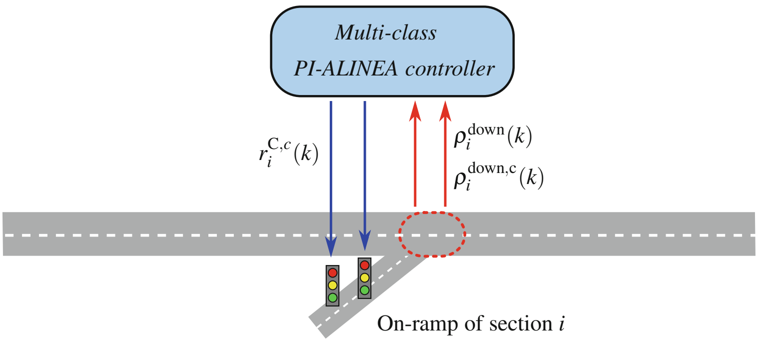

Fig. 10.2 Multi-class PI-ALINEA controller

$$r_i^{\mathrm{C},c}(k) = r_i^{\mathrm{C},c}(k-1) - K_P{}^c \left[\rho_i^{\mathrm{down,c}}(k) - \rho_i^{\mathrm{down,c}}(k-1)\right] \\ + K_R{}^c\, f_i^c(k) \left[\rho_i^* - \rho_i^{\mathrm{down}}(k)\right] \qquad (10.2)$$

where $\rho_i^{c,\mathrm{down}}(k)$ is the traffic density of class c measured downstream the on-ramp, while $K_P{}^c$ and $K_R{}^c$ are parameters of the regulator depending on class c. Note that the multi-class PI-ALINEA control law (10.2) is based on density measurements, both referred to the total density and to the density of specific class c. In particular, the total density measurement $\rho_i^{\mathrm{down}}(k)$ is used to compute the difference with the set-point value ρ_i^*, since this latter is a reference value for the total density.

In (10.2), the term depending on the difference between the total density and the set-point value is split, among the different vehicle classes, by means of the *ratio* $f_i^c(k)$. Specifically, this ratio computes, for road section i and time step k, the quantity of vehicles of class c over the total vehicles, and is given by

$$f_i^c(k) = \frac{\eta^c \left[l_i^c(k) + \rho_i^c(k) L_i\right]}{\sum_{h=1}^{C} \eta^h \left[l_i^h(k) + \rho_i^h(k) L_i\right]} \qquad (10.3)$$

The adoption of ramp metering control laws as the one expressed by (10.2) may cause the creation of long queues at the on-ramps, especially when the mainstream is congested. This phenomenon is often not feasible, because the on-ramps have physical limitations which impose a maximum queue limit. Nevertheless, also in the cases in which no restrictive physical limitations occur, too long queues are undesirable, for instance, because they can imply high concentrations of polluting emissions close to urban areas. Taking into account such motivations, the control law (10.2) can be extended in order to consider possible *maximum values of the queue lengths*.

Let us denote with $l_i^{\max,c}$ the maximum queue length for section i and class c. The limit on the queue length is activated in case the on-ramp flow computed by multi-class PI-ALINEA according to (10.2) creates a queue that is higher than $l_i^{\max,c}$. In this case, this on-ramp flow should be increased in order to reduce the queue to be not greater than its maximum value $l_i^{\max,c}$. The detailed algorithm for including the maximum queue lengths on the multi-class PI-ALINEA control law can be found in [27, 28].

It is worth noting that ALINEA and PI-ALINEA, in the standard one-class case, have been generally applied in order to reduce the Total Time Spent (TTS), i.e. to increase the system throughput, by fixing the set-point equal to the critical density, as deeply discussed in Sect. 8.3.1. In [27–29], different simulation analyses have been carried out to verify if these types of controllers, especially in the multi-class case, can be used also to reduce the Total Emissions (TE) of vehicles in the freeway. In particular, in those works, it has been analysed, first, which values of the downstream density set-point should be chosen in order to reduce the traffic emissions in the freeway and, second, if reducing the traffic emissions can also imply a reduction of the TTS by the drivers in the freeway or if, instead, the two control objectives are conflicting. The precise definitions and formulas for the TTS and TE can be found in Sects. 8.2.2 and 8.2.3, respectively.

In particular, this analysis has been carried out in [27, 28] by adopting the average-speed emission model COPERT for computing the emissions in the freeway, both in the mainstream and at the on-ramps (a detailed description of this model, as well as the mathematical formulation to adopt it within a freeway traffic model, is reported in Sect. 6.3). Note that the choice of this model is basically motivated by the fact that it is able to provide good estimations of the traffic emissions, while being simple to be applied. A similar analysis has been carried out in [29], where, instead, the VERSIT+ model is adopted to estimate the traffic emissions in the freeway system (see further details on VERSIT+ in Sect. 6.4).

All these tests have led to conclude that multi-class PI-ALINEA controllers represent an effective solution to reduce emissions and congestion in a freeway traffic system. In addition, the simulation analysis referred to many different traffic scenarios has shown that the reduction of emissions and the maximisation of the throughput are nonconflicting objectives, since both the total emissions and the congestion are reduced if this type of control actions is applied. Furthermore, the results reveal that the adoption of ramp metering control strategies may cause a high concentration of pollutants at the entering on-ramps that could be very critical, especially if the on-ramps are located in proximity of urban areas. As a consequence, the effect of these emissions should be expressly computed by means of models that calculate the emissions both in the mainstream and at the on-ramps, as done by both the COPERT and the VERSIT+ models, described in Sects. 6.3 and 6.4.

10.3.2 Supervisory Event-Triggered Control

The multi-class local feedback PI-ALINEA regulators described in Sect. 10.3.1 have shown to be effective in reducing congestion and emissions in freeways, since these two objectives have generally a nonconflicting nature. Nevertheless, as also discussed in Sect. 8.3.1, it is well known that the main weaknesses of ALINEA-like feedback regulators are due to their local nature, since they compute the control law only on the basis of measurements close to the on-ramp in which the control action is actuated. This aspect was addressed for instance in [36], where the authors analyse the application of ALINEA and PI-ALINEA in the presence of bottlenecks that are located far downstream the merge area.

The control framework described in this section goes further in the idea of considering distant bottlenecks, since it is based on *extended multi-class PI-ALINEA controllers* which compute the control law not only on the basis of the measurement downstream the on-ramp but also on the basis of measurements in a *neighbourhood* of the on-ramp. These further measurements refer to locations that are time-varying and are communicated to the local controllers by a *supervisor*, which acts according to an event-triggered nature, i.e. it changes the parameters of the control laws of the PI-ALINEA controllers only when suitable triggering conditions are met. The interested reader can find more details on event-triggered control, and its application to freeway traffic regulation, in Chap. 9.

The supervisory event-triggered control scheme based on extended PI-ALINEA controllers has been proposed for the first time in [30] for the one-class case, and then extended to the multi-class case in [31]. Note that the notation adopted here to describe the supervisory event-triggered control scheme is referred to a freeway stretch, to be more easily comparable with the other controllers described in this section, and then it is slightly different from the one used in [31], where a freeway network is instead considered.

The supervisory event-triggered control scheme is a hierarchical scheme composed of two levels (see a sketch in Fig. 10.3):

- at the *higher level*, the supervisor receives measurements from the network, periodically makes a prediction on the system evolution and, on the basis of this information, decides if the parameters of the control law for the lower level controllers should be updated or not, according to an event-triggered logic;
- at the *lower level*, local feedback controllers, specifically extended multi-class PI-ALINEA controllers, compute the control action on the basis of measurements in a neighbourhood of the on-ramp (the neighbourhood composition and the parameters of the control law are communicated by the supervisor).

A key point in this control scheme is the definition of the *neighbourhood* of a given on-ramp, from which measurements are taken. This neighbourhood is time-varying and is decided by the supervisor according to an event-triggered logic. It

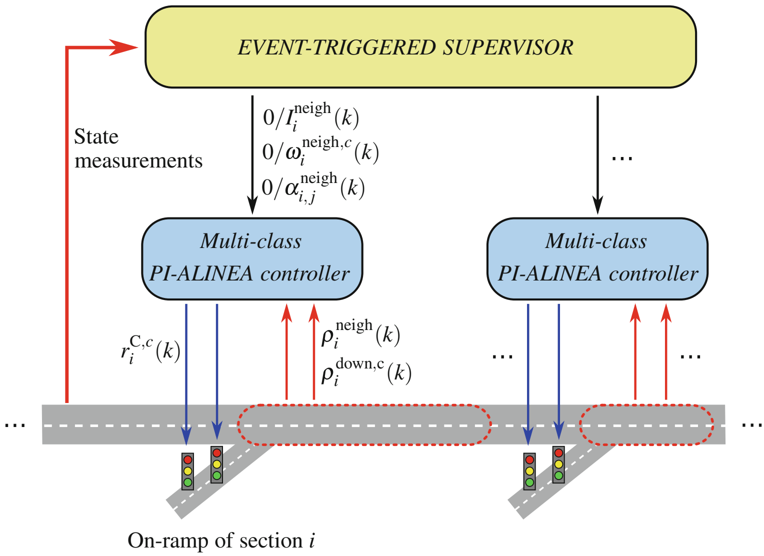

Fig. 10.3 Supervisory event-triggered control scheme based on extended multi-class PI-ALINEA controllers

is defined as a set of road sections downstream the on-ramp, i.e. more specifically, the neighbourhood associated with an on-ramp always starts from the section of the on-ramp and may last until a downstream section located before the next controlled on-ramp.

The two levels of the control scheme are described hereinafter.

Extended Multi-class PI-ALINEA Controllers As already done in Sect. 10.3.1, let us rely on the multi-class METANET model for a freeway stretch described in Sect. 4.3.1, in which the freeway stretch is divided in N road sections, the time horizon is discretised into K time intervals and C classes of vehicles are considered. According to this model, the ramp metering control variable is $r_i^{\mathrm{C},c}(k) \in [r_i^{\mathrm{min},c}, r_i^{\mathrm{max},c}]$, i.e. the flow of class c that should enter section i from the on-ramp during time interval $[kT, (k+1)T)$.

In the supervisory event-triggered scheme, the flow $r_i^{\mathrm{C},c}(k)$ at the on-ramp of section i, at time step k, for class c, is computed as

$$r_i^{\mathrm{C},c}(k) = r_i^{\mathrm{C},c}(k-1) - K_P{}^c \left[\rho_i^{\mathrm{down},c}(k) - \rho_i^{\mathrm{down},c}(k-1)\right] + K_R{}^c\, f_i^{\mathrm{neigh},c}(k) \left[\rho_i^* - \rho_i^{\mathrm{neigh}}(k)\right] \tag{10.4}$$

where, as in (10.2), $\rho_i^{\mathrm{down},c}(k)$ is the traffic density of class c measured downstream the on-ramp, ρ_i^* is a set-point value for the downstream total density, $K_P{}^c$ and $K_R{}^c$ are parameters of the regulator depending on class c. Differently from (10.2), the split ratio $f_i^{\mathrm{neigh},c}(k)$ now depends on the traffic state in the neighbourhood of the on-ramp, and the density to be compared with the set-point value is no more the measured total density $\rho_i^{\mathrm{down}}(k)$ but it is an 'extended density' $\rho_i^{\mathrm{neigh}}(k)$, again referred to the neighbourhood.

Let us explain these new terms more in detail. The *split ratio* $f_i^{\mathrm{neigh},c}(k)$ has a meaning analogous to the split ratio defined in (10.3) but it is now referred to the *neighbourhood*, i.e. to the set of road sections $I_i^{\mathrm{neigh}}(k) \subseteq \{1, \ldots, N\}$ from which measurements must be used to compute the control action to be actuated at the on-ramp of section i at time step k. More specifically, the split ratio $f_i^{\mathrm{neigh},c}(k)$ is a weighted ratio, at time step k, of the number of vehicles of class c over all the vehicles, which are present in the on-ramp of section i and in the sections belonging to the neighbourhood of section i. Such quantity can be computed as

$$f_i^{\mathrm{neigh},c}(k) = \frac{\omega_i^{\mathrm{neigh},c}(k)\eta^c \left[l_i^c(k) + \sum_{j\in I_i^{\mathrm{neigh}}(k)} \rho_j^c(k) L_j\right]}{\sum_{h=1}^{C} \omega_i^{\mathrm{neigh},h}(k)\eta^h \left[l_i^h(k) + \sum_{j\in I_i^{\mathrm{neigh}}(k)} \rho_j^h(k) L_j\right]} \tag{10.5}$$

where $\omega_i^{\mathrm{neigh},c}(k) \in [0, 1]$ is the weight associated with vehicles of class c in the neighbourhood of the on-ramp of section i at time step k.

Moreover, in (10.4), the value of the *extended total density* to be compared with the set-point is a weighted sum of the total densities in the neighbourhood, i.e.

$$\rho_i^{\mathrm{neigh}}(k) = \sum_{j \in I_i^{\mathrm{neigh}}(k)} \alpha_{i,j}^{\mathrm{neigh}}(k)\rho_j(k) \tag{10.6}$$

where $\alpha_{i,j}^{\mathrm{neigh}}(k) \in [0, 1]$, $j \in I_i^{\mathrm{neigh}}$, are parameters decided by the supervisor in order to properly weigh the measurements in the different sections belonging to the neighbourhood of the on-ramp of section i at time step k.

Event-Triggered Supervisor The supervisor receives measurements from the system state of the whole freeway stretch and makes periodic predictions. On the basis of the measured and predicted state variables and global performance indexes over the whole freeway stretch, the supervisor verifies specific triggering conditions and evaluates whether the parameters of the present control law of the extended multi-class PI-ALINEA controllers must be changed or not. The idea is that changes in the control laws are required if there are relevant variations in the system state and/or in the predicted system evolution, either locally or globally.

More specifically, at each time step $k = 0, \ldots, K - 1$ the supervisor receives *measurements* of the system state over the whole network, i.e. the traffic densities $\rho_i^c(k)$, the mean traffic speeds $v_i^c(k)$, and the queue lengths $l_i^c(k)$, $\forall c$, $\forall i$. Besides monitoring the single values of these state variables, at each time step k the supervisor also computes some *performance indexes* referred to the entire network and specifically defined for each vehicle class c. For instance, in [31], two global indicators have been defined, i.e. the instantaneous number of vehicles of class c in the network, denoted as $\eta^c(k)$, and the instantaneous emissions of vehicles of class c in the network, denoted as $\xi^c(k)$ (see [31] for the precise formula to compute these quantities).

The supervisor periodically makes a *prediction* of the system state evolution. In particular, the prediction of the system is computed at each time step $\bar{k} = nP$, where $n = 0, 1, 2, \ldots$ and P is an integer representing the number of time steps between one prediction and the next one. The prediction is realised over a given prediction horizon of K_p time steps. Note that different traffic and emission models can be used for the prediction; for instance, in [31], the multi-class METANET model for a freeway network described in Sect. 4.3.2 and the VERSIT+ emission model reported in Sect. 6.4 are used.

On the basis of the system state measured at time step $k = \bar{k}$ and using suitable traffic flow and emission models, the supervisor computes the *predicted state*, in terms of predicted traffic densities $\tilde{\rho}_i^c(k)$, predicted mean traffic speeds $\tilde{v}_i^c(k)$, and predicted queue lengths $\tilde{l}_i^c(k)$, $\forall c$, $\forall i$, $k = \bar{k} + 1, \ldots, \bar{k} + K_p$. With these predicted state variables, the supervisor also computes the *predicted* values of the considered *performance indexes*, e.g. the predicted instantaneous number of vehicles $\tilde{\eta}^c(k)$ and the predicted instantaneous emissions $\tilde{\xi}^c(k)$, $\forall c$, $k = \bar{k} + 1, \ldots, \bar{k} + K_p$.

If a change is required, the supervisor defines a new *neighbourhood* and the associated parameters, i.e. it properly communicates to the extended multi-class PI-ALINEA controller of the on-ramp of a generic section i the neighbourhood composition $I_i^{\text{neigh}}(k)$, the weights $\omega_i^{\text{neigh},c}(k)$, $c = 1, \ldots, C$, and $\alpha_{i,j}^{\text{neigh}}(k)$, $j \in I_i^{\text{neigh}}(k)$.

The event-triggered behaviour of the supervisor can be summarised as follows:

- at each time step $k \neq \bar{k}$, the supervisor verifies specific triggering conditions on the measured system state and on the global indicators;
- at each time step $k = \bar{k}$, the supervisor verifies specific triggering conditions on the measured system state, on the global indicators, as well as on the predicted system state and on the predicted global indicators;
- if at least one of the triggering conditions is met for the on-ramp of section i, the *neighbourhood* of section i is updated, i.e. the neighbourhood composition $I_i^{\text{neigh}}(k)$, the weights $\omega_i^{\text{neigh},c}(k)$, $c = 1, \ldots, C$, and $\alpha_{i,j}^{\text{neigh}}(k)$, $j \in I_i^{\text{neigh}}(k)$, are communicated to the extended multi-class PI-ALINEA controller in the on-ramp of section i, in order to compute (10.4), with (10.5) and (10.6);
- if none of the triggering conditions is met, the supervisor does not communicate any change to the extended multi-class PI-ALINEA controllers, which continue to apply the same control law (10.4) as before.

Different triggering conditions can be defined, and different logics to update the control law parameters can be formalised. The interested reader can find some examples for instance in [30, 31].

10.3.3 Coordinated Optimal Control

The combined reduction of traffic emissions and congestion in freeways is also the goal of the coordinated multi-class ramp metering control strategy described in this section, based on optimal control techniques. The control strategy is here sought by defining and solving an *optimal control problem* which turns out to be a finite-horizon non-linear optimal control problem with constrained control variables, that can be found also in [32, 33].

As introduced in Sect. 8.4.2, applying optimal control techniques for freeway traffic means that the control actions are computed by considering the dynamic evolution of the freeway traffic system over a given time horizon and by optimising its performance on the basis of specified control objectives. Hence, an optimal control problem is defined, being characterised by an objective function (the performance to be optimised), the state and control variables to be computed, and the constraints representing the dynamics of the system and bounds on the control variables. A sketch of the multi-class ramp metering optimal control strategy is reported in Fig. 10.4.

In the specific case considered here, the dynamic evolution of the system is expressed in terms of the multi-class METANET model for a freeway stretch described in Sect. 4.3.1 (in case the ramp metering control variables are the metering rates), which is a discrete-time non-linear model. The emission model COPERT described in Sect. 6.3 is used to compute the emissions in the freeway system.

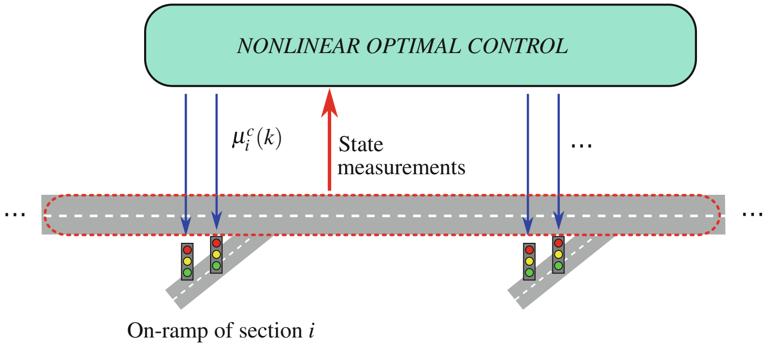

Fig. 10.4 Multi-class coordinated ramp metering optimal control strategy

The state variables are given by the traffic densities $\rho_i^c(k)$, the mean speeds $v_i^c(k)$, and the queue lengths $l_i^c(k)$ for each class $c = 1, \ldots, C$, for every section $i = 1, \ldots, N$, referred to time step $k = 0, \ldots, K$. The control variables are the ramp metering rates $\mu_i^c(k) \in [\underline{\mu}_i^c(k), 1]$, $c = 1, \ldots, C$, $i = 1, \ldots, N$, $k = 0, \ldots, K$.

The objective function takes into account the minimisation of the TTS and the TE, according to the definition provided in Sects. 8.2.2 and 8.2.3, respectively.

Let us report the formulation of the optimal control problem for finding the optimal multi-class ramp metering control strategy which minimises traffic emissions and congestions in a freeway stretch over a finite horizon of K time steps.

Problem 10.1 Given the system initial conditions $\rho_i^c(0), v_i^c(0), l_i^c(0), i = 1, \ldots, N$, $c = 1, \ldots, C$, given the estimated exogenous inputs $q_0^c(k)$, $v_0^c(k)$, $\rho_{N+1}^c(k)$, $s_i^c(k)$, $d_i^c(k)$, $c = 1, \ldots, C$, $i = 1, \ldots, N$, $k = 0, \ldots, K-1$, find the optimal control sequence $\mu_i^c(k)$, $c = 1, \ldots, C$, $i = 1, \ldots, N$, $k = 0, \ldots, K-1$, that minimises

$$J = \beta \, \Gamma \, TE + (1-\beta) TTS + J^{\mu} + J^{l} \tag{10.7}$$

with

$$TE = \sum_{k=0}^{K} \sum_{i=1}^{N} E_i^M(k) + \sum_{k=0}^{K} \sum_{i=1}^{N} E_i^R(k) \tag{10.8}$$

$$TTS = T \sum_{k=0}^{K} \sum_{i=1}^{N} \sum_{c=1}^{C} \eta^c \rho_i^c(k) L_i + T \sum_{k=0}^{K} \sum_{i=1}^{N} \sum_{c=1}^{C} \eta^c l_i^c(k) \tag{10.9}$$

$$J^{\mu} = \sum_{k=1}^{K-1} \sum_{i=1}^{N} \sum_{c=1}^{C} \omega_i^c \left[\mu_i^c(k) - \mu_i^c(k-1) \right]^2 \tag{10.10}$$

$$J^{l} = \sum_{k=0}^{K} \sum_{i=1}^{N} \sum_{c=1}^{C} \gamma_i^c \left[\max\left\{0, l_i^c(k) - l_i^{\max,c}\right\} \right]^2 \tag{10.11}$$

subject to the multi-class METANET model for a freeway stretch described in Sect. 4.3.1 and

$$\mu_i^{\min,c} \leq \mu_i^c(k) \leq 1 \qquad c = 1, \ldots, C, \ i = 1, \ldots, N, \ k = 0, \ldots, K-1 \quad (10.12)$$

□

The first two terms in the cost function (10.11) are the Total Emissions and the Total Time Spent, given, respectively, by (10.8) and (10.9), that are properly weighted by means of the parameter $\beta \in [0, 1]$, and reported to the same order of magnitude with parameter Γ. The third term in (10.11), i.e. J^{μ}, is introduced in order to prevent oscillations of the control variables over consecutive time steps, and ω_i^c, $c = 1, \ldots, C$, $i = 1, \ldots, N$, are suitable weights. Finally, the last cost term J^l is included to penalise the cases in which the queue lengths at the on-ramps exceed their limits $l_i^{\max,c}$, with proper weights γ_i^c, $c = 1, \ldots, C$, $i = 1, \ldots, N$.

Problem 10.1 is a constrained non-linear optimal control problem. Problems of this kind often arise in the freeway traffic control domain, as deeply discussed in Sect. 8.4.2. Their numerical solution may be attempted by direct use of available non-linear programming codes, but this can present difficulties, especially in case of large-scale applications, since the problem dimensions become very high. An efficient numerical solution can be obtained by use of the *feasible direction algorithm*, which is a gradient-based algorithm adopted within the optimal freeway traffic control tool AMOC [38, 39]. A very efficient algorithm to solve this problem is the version of the feasible direction algorithm which applies the derivative back-propagation method *RPROP* (see [40] for further details on this algorithm and [41] for a recent application of this algorithm to another traffic control problem).

The feasible direction algorithm applying the derivative back-propagation method RPROP has been used to solve Problem 10.1 in [32, 33], where a detailed simulation analysis has been also carried out, for the specific case of two classes of vehicles, cars and trucks. In particular, different traffic scenarios have been considered, with and without limits on the maximum queue lengths, and by varying the parameter β assigning different importance levels to the minimisation of the TE and the TTS, respectively. As aforementioned in the previous sections, the results reported in [32, 33] show that the TE and the TTS are largely non-conflicting objectives, since both the average travel times and the emissions are reduced if the control actions manage to reduce or eliminate traffic congestion.

10.4 Multi-class Combined Strategies for Emission Reduction

A multi-class control strategy to reduce congestion and traffic emissions is reported in this section, based on the control scheme presented in [35], a preliminary version of which can be found in [34]. The main difference with the control strategies discussed

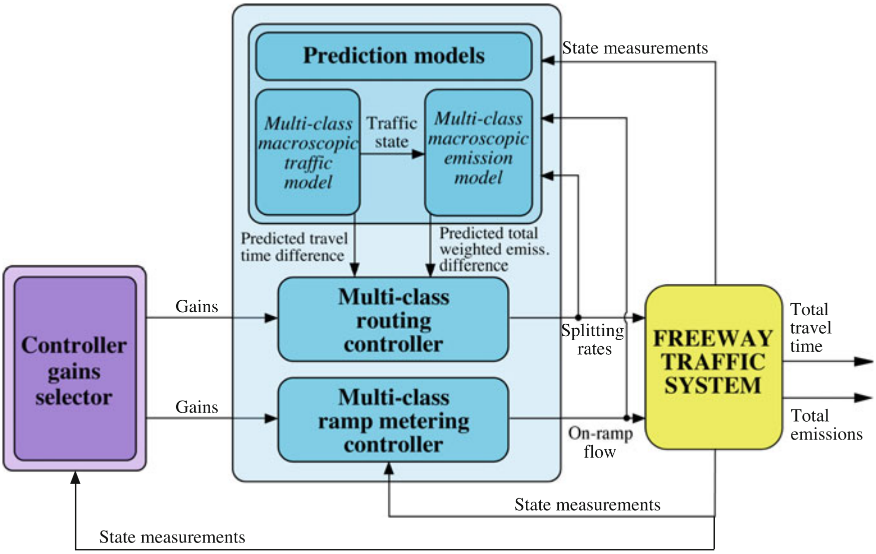

Fig. 10.5 Layout of the multi-class combined ramp metering and route guidance control framework

in Sect. 10.3, which rely on ramp metering for a freeway stretch, is that here a traffic network composed of interconnected stretches is considered and a combined control action given by ramp metering and route guidance is taken into account.

In particular, route guidance is actuated through VMSs (located near the freeway bifurcations) to inform the road users about alternative routes. These indications are assumed to be specifically differentiated for the different classes of vehicles. Moreover, ramp metering is applied in order to regulate the access of traffic to the mainstream through traffic signals installed at the on-ramps. Again, the ramp metering strategy is of the multi-class type, i.e. the different classes of vehicles have dedicated lanes and signals.

The layout of the proposed control framework is depicted in Fig. 10.5. The overall scheme consists of two main components:

- *multi-class controllers*: two types of controllers are adopted, a route guidance controller and a ramp metering controller;
- *gains selector*: this module computes the values of controller gains, according to a specified selection procedure.

The multi-class *routing* controller includes a traffic model and an emission model in order to make predictions of the system state evolution. In particular, the prediction models are run periodically and are initialised with the current system state. The multi-class METANET model for a freeway network described in Sect. 4.3.2 and the VERSIT+ model reported in Sect. 6.4 are here considered.

Let us briefly recall the main notation of the multi-class METANET model for a freeway network described in Sect. 4.3.2, in which the time horizon is divided into

K time intervals (with sample time interval T), C classes of vehicles are considered (with conversion factor η^c), the freeway network is composed of M freeway links, O origin links, and N nodes. Each freeway link $m = 1, \ldots, M$ is further divided into N_m sections with length L_m. Moreover, remind that $\rho^c_{m,i}(k)$ denotes the traffic density of class c in section i of link m at time instant kT and $l^c_o(k)$ is the queue length of class c at origin link o at time instant kT. The route guidance control variables are $\beta^{\mathrm{C},c}_{m,n,j}(k) \in [0, 1]$, i.e. the splitting rates representing the portion of flow of class c present in node n at time instant kT which should choose link m to reach destination j, while the ramp metering control variables are $r^{\mathrm{C},c}_o(k) \in [r^{\min,c}_o, r^{\max,c}_o]$, i.e. the flows of class c that should enter from the origin link o during time interval $[kT, (k+1)T)$.

The prediction models included in the controller module allow to compute the *predicted travel time differences* and the *predicted total weighted emission differences*, on the basis of which the routing control action is determined. Since the routing control action is not only based on local state measurements but also on traffic and emission prediction models, the considered controller can be defined as a *local feedback predictive* regulator.

The multi-class *ramp metering* controller computes the on-ramp flow on the basis of the local measurements obtained from the real system; hence, it is a *local feedback* regulator.

Both the controllers are characterised by some parameters, to be properly tuned, that are the *controller gains*. These gains are provided by the gains selector module (see Fig. 10.5), which includes a library of traffic scenarios (corresponding to specific traffic states and demand patterns), each of which has a set of associated controller gains. These gains are calibrated through a specific optimisation-based procedure which is applied offline. Moreover, the gains selector uses a classification algorithm which periodically chooses, on the basis of the present system state and the estimated demands, the most proper scenario and the corresponding controller gains.

The main components of the considered control scheme are described below, i.e. the multi-class routing controller, the multi-class ramp metering controller and the controller gains selector. The interested reader can find more details in [35].

The Multi-class Routing Controller The routing control strategy consists in informing the users about the preferred link to choose in a bifurcation, as deeply discussed in Sect. 8.3.3 on local route guidance strategies.

The routing controller relies on a prediction module, which runs the prediction periodically, and computes the routing control action with the same sample time. Specifically, let us denote with $\bar{k}$ the generic time step in which the prediction is run and the routing control law is updated. Such a prediction refers to alternative paths starting from bifurcation nodes, i.e. nodes having two outgoing freeway links. Therefore, it is carried out considering some *virtual test vehicles*, which leave the bifurcation node in order to reach their destination through alternative paths. The predicted behaviour of the virtual test vehicles is used to have information about such alternative paths.

Let us consider a generic bifurcation node n, from which it is possible to reach a generic destination j, and let us denote with m and m' the two exiting links (see also Fig. 8.9). Since the routing suggestion in node n is related to the choice of one of the two freeway links exiting the node, it is possible to gather the different paths connecting node n to destination j in two sets according to the freeway link exiting node n in each path. The two sets are denoted as the set of *primary* and *secondary* paths on the basis of the most common path choices made by the drivers: the primary paths having m as first freeway link and the secondary paths having m' as first freeway link.

For each pair of nodes (n, j), a number of virtual vehicles equal to the number of paths from n to j are introduced for each class of vehicles. The prediction at a generic time step $\bar{k}$ is realised assuming that the routing control actions are maintained constant for the whole prediction horizon, while the ramp metering control actions are computed with the multi-class PI-ALINEA control law.

The computations related to each virtual vehicle end when the vehicle itself reaches its destination. In particular, for each virtual vehicle, the following quantities are computed:

- the *predicted travel time* needed by the virtual vehicle to reach its destination;
- the *predicted total weighted emissions* experienced by the virtual vehicle to reach its destination.

The predicted travel time for the primary path and the one for the secondary path are then calculated as the minimum among all the predicted travel times of primary and secondary paths, respectively. Analogously, the predicted total weighted emissions are computed for the primary path and the secondary path. For each pair of nodes (n, j) and for each vehicle class c, it is possible to compute the *predicted travel time difference* at time step $\bar{k}$, denoted as $\Delta t^c_{n,j}(\bar{k})$, and the *predicted total weighted emission difference*, denoted as $\Delta e^c_{n,j}(\bar{k})$, being the difference computed between the secondary path and the primary path. These differences of travel times and total weighted emissions are used to calculate the control law, relying on equilibrium concepts.

Conditions of *Dynamic User Equilibrium* have been widely used in route guidance control schemes, by considering that traffic flows with the same origin and destination are distributed in the network so that the travel times on these routes are the same. At a generic time step $\bar{k}$ at which the routing control action at node n is computed, the conditions of Dynamic User Equilibrium relate the predicted travel time difference $\Delta t^c_{n,j}(\bar{k})$ with the splitting rates $\beta^{\mathrm{t,c}}_{m,n,j}(\bar{k})$, indicating the portion of flow of class c present in node n at time step $\bar{k}$ which should choose link m to reach destination j in order to reduce the travel times. Such conditions can be defined as

$$\Delta t^c_{n,j}(\bar{k}) > 0 \quad \Rightarrow \quad \beta^{\mathrm{t,c}}_{m,n,j}(\bar{k}) = 1 \tag{10.13}$$

$$\Delta t^c_{n,j}(\bar{k}) = 0 \quad \Rightarrow \quad 0 < \beta^{\mathrm{t,c}}_{m,n,j}(\bar{k}) < 1 \tag{10.14}$$

$$\Delta t^{c}_{n,j}(\bar{k}) < 0 \quad \Rightarrow \quad \beta^{\mathrm{t},c}_{m,n,j}(\bar{k}) = 0 \tag{10.15}$$

Analogously to the travel times, it is possible to consider a *Dynamic Emission Equilibrium*, aimed at balancing the weighted pollutant emissions along the suggested routes. The conditions of Dynamic Emission Equilibrium may be formulated as a relation between the predicted total weighted emission difference $\Delta e^{c}_{n,j}(\bar{k})$ and the splitting rates $\beta^{\mathrm{e},c}_{m,n,j}(\bar{k})$, indicating the portion of flow of class c present in node n at time step $\bar{k}$ which should choose link m to reach destination j in order to reduce the total weighted emissions, i.e.

$$\Delta e^{c}_{n,j}(\bar{k}) > 0 \quad \Rightarrow \quad \beta^{\mathrm{e},c}_{m,n,j}(\bar{k}) = 1 \tag{10.16}$$

$$\Delta e^{c}_{n,j}(\bar{k}) = 0 \quad \Rightarrow \quad 0 < \beta^{\mathrm{e},c}_{m,n,j}(\bar{k}) < 1 \tag{10.17}$$

$$\Delta e^{c}_{n,j}(\bar{k}) < 0 \quad \Rightarrow \quad \beta^{\mathrm{e},c}_{m,n,j}(\bar{k}) = 0 \tag{10.18}$$

The proposed feedback routing control strategy is based on PI-controllers, i.e. feedback controllers of the proportional–integral type. Let us consider the two PI-controllers control laws adopted at time step $\bar{k}$ to define the splitting rates $\beta^{\mathrm{t},c}_{m,n,j}(\bar{k})$ and $\beta^{\mathrm{e},c}_{m,n,j}(\bar{k})$, by taking into account the equilibrium conditions (10.13)–(10.15) and (10.16)–(10.18), i.e.

$$\beta^{\mathrm{t},c}_{m,n,j}(\bar{k}) = \beta^{\mathrm{t},c}_{m,n,j}(\bar{k}-1) + K^{\mathrm{t},c}_{P}[\Delta t^{c}_{n,j}(\bar{k}) - \Delta t^{c}_{n,j}(\bar{k}-1)] + K^{\mathrm{t},c}_{I}\Delta t^{c}_{n,j}(\bar{k}) \tag{10.19}$$

$$\beta^{\mathrm{e},c}_{m,n,j}(\bar{k}) = \beta^{\mathrm{e},c}_{m,n,j}(\bar{k}-1) + K^{\mathrm{e},c}_{P}[\Delta e^{c}_{n,j}(\bar{k}) - \Delta e^{c}_{n,j}(\bar{k}-1)] + K^{\mathrm{e},c}_{I}\Delta e^{c}_{n,j}(\bar{k}) \tag{10.20}$$

where $K^{\mathrm{t},c}_{P}$, $K^{\mathrm{t},c}_{I}$, $K^{\mathrm{e},c}_{P}$ and $K^{\mathrm{e},c}_{I}$, $c = 1, \ldots, C$, are controller gains. It is worth noting that the resulting splitting rates $\beta^{\mathrm{t},c}_{m,n,j}(\bar{k})$ and $\beta^{\mathrm{e},c}_{m,n,j}(\bar{k})$ should be truncated to the admissible interval [0, 1].

The route guidance control variables $\beta^{\mathrm{C},c}_{m,n,j}(\bar{k})$, i.e. the splitting rates representing the portion of flow of class c present in node n at time step $\bar{k}$ which should choose link m to reach destination j, are given by the following weighted sum:

$$\beta^{\mathrm{C},c}_{m,n,j}(\bar{k}) = \alpha^{c}\beta^{\mathrm{t},c}_{m,n,j}(\bar{k}) + (1-\alpha^{c})\beta^{\mathrm{e},c}_{m,n,j}(\bar{k}) \tag{10.21}$$

where α^{c} is a design parameter defined for class c, with $0 \leq \alpha^{c} \leq 1$. These parameters are fixed in order to apply specific control policies for each vehicle class, by properly balancing travel times and total weighted emissions.

The Multi-class Ramp Metering Controller The ramp metering control strategy is based on feedback controllers of the proportional–integral type, and in particular on the multi-class PI-ALINEA already described in Sect. 10.3.1 for a freeway stretch (the notation here is slightly different since it is referred to a freeway network instead of a freeway stretch). The control law is updated with a sample time T, which is equal to the model sample time, and allows to compute the ramp metering control variable $r_o^{C,c}(k)$, i.e. the flow of class c that should enter from the origin link o during time interval $[kT, (k+1)T)$.

In order to compute such flow, let us first of all introduce the variable $f_o^c(k)$ indicating the *ratio*, at time step k, of the number of vehicles of class c over the entire number of vehicles, which are present in origin link o and in the mainstream section immediately downstream link o (namely the first section of the downstream leaving link m). Such quantity can be computed as

$$f_o^c(k) = \frac{\eta^c[l_o^c(k) + \rho_{m,1}^c(k)L_m]}{\sum_{h=1}^{C} \eta^h[l_o^h(k) + \rho_{m,1}^h(k)L_m]} \tag{10.22}$$

Referring to a generic origin link o, the flow of class c that should enter at time step k is computed according to the following multi-class PI-ALINEA control law:

$$\begin{aligned} r_o^{C,c}(k) = r_o^{C,c}(k-1) - K_P{}^c \left[\rho_{m,1}^{\text{down,c}}(k) - \rho_{m,1}^{\text{down,c}}(k-1)\right] \\ + K_R{}^c\, f_o^c(k) \left[\rho_{m,1}^{*} - \rho_{m,1}^{\text{down}}(k)\right] \end{aligned} \tag{10.23}$$

where $\rho_{m,1}^{\text{down,c}}(k)$ is the traffic density of class c measured downstream the origin link, i.e. in the first section of the downstream link m, $\rho_{m,1}^{\text{down}}(k)$ is the total density measured in the same location, $\rho_{m,1}^{*}$ is the total density set-point of the first section of link m, K_P^c and K_R^c are gain parameters of the regulator.

The Controller Gains Selector The proposed controllers are characterised by some gains, which should be properly tuned. In particular, let us denote with $\mathbb{K} = \left\{K_P^{\text{t},c}, K_I^{\text{t},c}, K_P^{\text{e},c}, K_I^{\text{e},c}, K_P^c, K_R^c, c = 1, \ldots, C\right\}$ the set gathering these *controller gains*. In the considered control framework, these gains are selected according to a specific selection procedure.

In particular, a finite set Ξ of *traffic scenarios* is defined to take into account different traffic conditions. Each scenario $\sigma \in \Xi$ is characterised by a set of initial traffic conditions and a demand pattern and is associated with a set $\mathbb{K}^\sigma$ of controller gains. The controller gains to be used are chosen by the gains selector with a sampling time T^S [s], normally larger than T, i.e. every T^S seconds the selector identifies the most adequate scenario $\bar{\sigma}$ for representing the present traffic conditions using suitable classification techniques or clustering methods. On the basis of the chosen scenario $\bar{\sigma}$, the selector module feeds the controllers with the corresponding set $\mathbb{K}^{\bar{\sigma}}$ of gains.

The controller gains associated with each scenario are found by offline running an optimisation-based procedure. Specifically, the controller gains are found by solving an *optimisation problem* in which the minimisation of the TTS and the minimisation of the TE are explicitly taken into account in the objective function, while the system dynamics is included in the problem constraints, and the decision variables are represented by the gains. The statement of this optimisation problem and other detailed explanations can be found in [35].

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