

Chapter 9

Mathematics Lesson Study Around the World: Conclusions and Looking Ahead



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Abstract Educators around the globe seek to emulate the success of Japanese lesson study. However, implementation of lesson study outside Japan has been met with varying rates of success and challenges. To address the challenges, lesson study researchers and educators, have gathered their reflections in this book. This concluding chapter discusses strategies for developing a theorization that can be understood outside of Japan and its specific cultural norms, adaptations of lesson study outside Japan, the contributions of lesson study for educational reform, the sustainability of lesson study and the challenges involved in establishing lesson study on a larger scale. As the summary of this book, this chapter proposes our concluding statement and suggests future goals for implementation and research.

Keywords Lesson study · International · Theoretization · Mathematics education

9.1 Introduction

Researchers and educators of lesson study around the world came together for the 13th International Congress on Mathematical Education (ICME-13) discussion group in Germany in 2016. This discussion group examined lesson study and mathematics education in various countries and settings, as well as methodological and theoretical tools for researching lesson study. Members of this fruitful discussion worked together to produce this book in order to share our findings. In this chapter, we explain Japanese lesson study and its impact on mathematics education in Japan. Outside of Japan, implementation of lesson study has been met with varying rates of success and challenges. To address the challenges, our group

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members contributed in-depth research of Japanese lesson study, case studies of lesson study projects conducted in various countries and settings, strategies for developing a theorization that can be understood outside of Japan and its specific cultural norms, and close examination of the challenges involved in establishing lesson study on a larger scale. We summarize these chapters and suggest in response the model of “Collaborative Lesson Research” (CLR) as a way to define what lesson study outside of Japan can be. Our concluding statement suggests future goals for implementation and research.

9.2 Japanese Lesson Study and Its Impact on Mathematics Education

In Japan, lesson study has been the primary method of professional development for teachers ever since public education became an institution (Lewis, 2000; Lewis & Tsuchida, 1998; Makinae, 2010; Murata & Takahashi, 2002; Takahashi, 2000; Takahashi & Yoshida, 2004; Yoshida, 1999a). A cycle of lesson study is composed of several elements. The first element is intensive research regarding classroom materials. Teachers must study the standards, read relevant research articles, and examine available curricula. They use this research to design a lesson structured around solving a particular problem (or problems) that also addresses a broader educational theme. This lesson is taught by a single teacher under the observation of other teachers and is called a “research lesson.” It is an opportunity for everyone involved to closely examine teaching practices and judge whether or not the lesson properly supported the students. Afterwards, everyone who observed joins in a discussion with the teacher who led the lesson about how the students responded. This discussion provides valuable insights into the teaching-learning process.

Japanese lesson study has many benefits. The research and discussion improves teaching and learning. It also helps determine practical and effective ways to implement the national curriculum (Murata & Takahashi, 2002). Japanese schools typically use lesson study to help them transition to a new national curriculum. Researchers have also shown how lesson study leads to the implementation of new teaching approaches in Japanese schools (Lewis, 2002; Lewis & Tsuchida, 1998; Stigler & Hiebert, 1999; Yoshida, 1999b). Stigler and Hiebert (1999), in particular, discussed how lesson study promoted major reform in Japanese mathematics classrooms: teacher-led lecture lessons were replaced in favor of student-centered, problem-solving lessons. Early research on Japanese lesson study focused on these problem-solving mathematics lessons, however, it did not clearly explain how the lesson study process itself is what prompted this major reform (e.g. National Council of Teachers of Mathematics, 1980, 1989). Understanding this shift is key to understanding the success of Japanese lesson study. Stigler and Hiebert’s (1999) research generated great international interest in lesson study and inspired researchers around the world. However, despite the fact that many researchers and

educators have attempted to replicate the success of Japanese lesson study (e.g. Hart, Alston, & Murata, 2011) the effectiveness of these projects is unclear (Takahashi & McDougal, 2016). The authors of this book examined this issue from various perspectives.

9.3 Lesson Study in Western Countries

A lack of comprehensive understanding of Japanese lesson study is in large part why it is difficult to establish sustainable lesson study outside Japan. The two chapters of this book by Fujii (2017) and Baba, Ueda, Ninomiya, and Hino (2017), respectively, describe the lesson study process in detail. Fujii (2017) points out that it is crucial for teacher educators to understand not only the most visible parts of lesson study, such as the research lesson and its post-lesson discussion, but to also understand the less visible parts of the process, i.e., designing the lesson and the theory behind it. Baba et al. (2017) report three Japanese mathematics education researchers' perspectives regarding lesson study. These reports by Japanese researchers are valuable for researchers and educators outside Japan who may not have first-hand experience of lesson study to help them develop a more comprehensive view.

Educators attempting to implement lesson study in Western countries face many challenges. Our group members have shared their research regarding developing a theory of lesson study and how lesson study has been adapted in countries outside of Japan. This book assembles several European and US case studies (Ni Shuilleabhain, 2017; Ponte, Quaresma, Mata-Pereira, & Baptista, 2017; Quaresma et al., 2017; Takahashi & McDougal, 2017; Winsløw, Bahn, & Rasmussen, 2017). These case studies all involve local scholars (either originally from the area or originally from Japan but now living in the area) who conducted lesson study in their area without any initial involvement or support from local authorities or ministries of education. These texts all examine the local conditions, how lesson study was actually implemented, and the projects' respective successes, failures, and challenges. From their research we can gain a better understanding of how to develop a theoretical model of lesson study, what adapting lesson study to Western countries entails, and how introducing lesson study to Western countries can inspire educational reform.

9.4 Theorizing Lesson Study

Winsløw et al. (2017) point out that outside Japan, educators' understanding of lesson study is based on individual interpretation rather than an explicit definition. This individual understanding is determined by what research educators have read on their own or from training by Japanese experts. However, Winsløw et al. (2017)

argue that even the knowledge of Japanese experts is “contingent upon cultural and institutional conventions,” and that since these conventions are not shared by Western culture, they are not readily understood by Western educators (p. 125). This situation runs against the nature of what lesson study should be, “to create shared and documented knowledge, rather than (just) private experience and wisdom” (Winsløw et al., 2017, p. 125). By collaborating on this book, our ICME-13 discussion group aims to rectify this conflict. We are opening a dialogue between mathematics education researchers working with or about lesson study, presenting theoretical models analysing some of these lesson study realisations, and presenting research-based definitions of what lesson study outside of Japan can be.

One way of contributing to this necessary theorization is to use already established theoretical models to analyse the lesson study process in order to characterize its essence. Several researchers in our IMCE-13 discussion group tackled this challenge (e.g., Ni Shuilleabhain & Clivaz, 2017; Runesson, 2015; Widjaja, Vale, Groves, & Doig, 2015). In their chapter, Winsløw et al. (2017), use the Anthropological Theory of the Didactic (ATD, Chevallard, 1999) and the Theory of Didactic Situations in Mathematics (TDS, Brousseau, 1997) on two Danish case studies. These two frameworks, ATD and TDS, offer interesting viewpoints on investigating the means and processes of Japanese mathematics lesson study, building on the research of Miyakawa and Winsløw (2009) and Clivaz (2015a). Winsløw et al. (2017) found that using TDS has the advantage of being quite close to the teachers’ perspective and allows for “a detailed analysis of how both students (in research lessons) and teachers (carrying out lesson study, observing students’ learning) develop their knowledge in various phases of lesson study” (p. 139). This development of teachers’ knowledge in a learning situation for the teacher highlights a parallel between lesson study for the teacher and the problem solving lesson for the students, making the lesson study a learning milieu for the teacher (Clivaz, 2015b) as schematised in Winsløw et al. (2017, Fig. 7.2). In contrast to this, Winsløw et al. (2017) find ATD to be the “most appropriate to a researcher perspective,” as it considers lesson study from a global viewpoint, offering an institutional perspective on how to “analyse the viability of lesson study in the presence of other paradidactic infrastructures” (p. 140). The analysis by Winsløw et al. (2017) of these two Danish case studies helps form a theory for lesson study.

Making the lesser known aspects of Japanese lesson study understood to the international community also contributes to the theorization of lesson study. In their chapter, Takahashi and McDougal (2017) examine what lesson study is not, elucidated by the “result of flawed attempts to use it” (p. 144). These observations point towards a “clear definitions to distinguish authentic lesson study from lesson study-like activities” (p. 151). Takahashi and McDougal (2017) define criteria for what lesson study can be outside of Japan and coin the term “Collaborative Lesson Research,” or CLR. CLR is defined as having: a clear research purpose, *kyouzai kenkyuu* (“the study of classroom materials”), a written research proposal, a live research lesson and discussion, knowledgeable others, and a process for sharing results. This definition derives from the comparison between Japanese lesson study and its less than faithful adaptations overseas. Defining CLR contributes to building

a general theory of lesson study, and as such, Japanese lesson study may be more successfully understood and implemented internationally.

9.5 Adaptations of Lesson Study Outside Japan

Lesson study exists outside of Japan in various adaptations. Educators around the globe sought to emulate the success of Japanese lesson study following the publication of *The Teaching Gap* (Stigler & Hiebert, 1999). Adaptations were made, sometimes to fit a limited time frame (Takahashi & McDougal, 2017), and sometimes because these adaptations were more well-suited to certain cultures and situations. In their chapter, Winsløw et al. (2017) quote Lewis's argument that successful adaptation requires "a deep understanding of what it [lesson study] is and why it has been useful to Japanese teachers, and how it can be adapted to the very different setting of the US" (Lewis, 2004, p. 134). This "importing of cultural routines" (Stigler & Hiebert, 2016) is discussed from several perspectives in this book. Ponte et al. (2017) follow Stigler and Hiebert; they insist that the transformation of a cultural practice such as lesson study is both inevitable and necessary. In this adaptation process, "many significant aspects of the original cultural practice may be lost, but other aspects may emerge, yielding robust and flexible practices in the new environment" (Ponte et al., 2017, p. 88). Their chapter explores what successful adaptation of lesson study can be.

All countries are different, this fact alone implies adaptations when conducting lesson study outside of Japan. The Portuguese case study serves as one such example. During teacher training, the research lesson was not taught by the prospective teacher but by another teacher involved in the teacher training process (Ponte et al., 2017). This is radically different from Japanese lesson study. Even the impetus for performing lesson study varies from place to place. Our IMCE-13 group discussion members shared case studies from Portugal, Ireland, Denmark, and the USA, in which the lesson study processes are initiated at the university level (Ponte et al., 2017; Ni Shuilleabhain, 2017; Winsløw et al., 2017; Takahashi & McDougal, 2017). In most of these case studies, the faculty member often had to play both the role of a facilitator and of a knowledgeable other. This is again very different from lesson study in Japan. To address this issue, Takahashi and McDougal (2017) propose in their definition of CLR that the roles of supporting professionals should stay separate, the same way they are in Japan, and also suggest having two different knowledgeable others. The right adaptations can determine the success of a lesson study project.

Several of the case studies related in this book are regarding teacher training (Ponte et al., 2017; Winsløw et al., 2017). For reasons elaborated by Winsløw et al. (2017), pre-service education is actually the best place to initiate lesson study as a new paradigmatic infrastructure in Denmark. We can hypothesise that this might also be the case in other countries as well. These particular settings of pre-service education, similar to Japanese schools in the 1900s (Winsløw et al., 2017), merit

particular attention since participants have not yet developed the more individual strategies for professional development which may be entrenched among experienced teachers. Ponte (2017) has also recently reviewed research concerning the use of lesson study as it applies to prospective mathematics teachers' education for secondary school. He also calls for further research regarding these adaptations. We advocate that such research would be of great interest to the mathematics education community.

One of the most controversial adaptations of lesson study is re-teaching. Fujii (2014) shows that re-teaching is generally never done in Japan. Fujii even considers it as a misconception. On the other hand, Winsløw et al. (2017) side with Stigler and Hiebert (2016), and hypothesize that while re-teaching is not necessary in a mature lesson study culture like in Japan, where lesson study is a well-established part of the paradigmatic infrastructure for all schools “[...] in the case of pre-service teacher education, in a culture unaccustomed to lesson study, the paradigmatic structure of lesson study needs to include several iterations of the same lesson, in order for fundamental changes to be undertaken” (p. 139). In fact, they show in a Danish case study how, “repeated cycles could be of particular significance in pre-service education” (p. 139). In the initial teacher training, this adaptation seems beneficial in the sense that repetition creates “cycles” of paradigmatic situations and that “the learning resulting from the PoS [post-didactic situations] of the previous cycle feeds into the PrS [pre-didactic situations] of a following cycle” (p. 131). In particular, observing the variations in the subsequent research lesson could make participants aware of the effect on students' learning in response to a change in the lesson design. Thus, “after the second run, what is contingent and what is not, stands out more clearly for them” (p. 139). More general research may uncover other kinds of adaptations which make lesson study in other countries more than a mere copy of what is common practice in Japan.

9.6 Lesson Study and Educational Reform

Two cases of specific local conditions, in Portugal (Ponte et al., 2017) and in Ireland (Ni Shuilleabhain, 2017), both presented in this book, relate to curriculum reform. In both cases, the reform involves a profound change in the teaching of mathematics. Ni Shuilleabhain speaks of a “dramatic shift in perspective on teaching and learning mathematics from a traditional, “transmission” approach” (p. 66). Ponte et al. show in their case study how local educators employed an approach similar to what Fujii (2016) describes as “inquiry-based mathematics teaching” and “structured problem solving.” In both cases, and in fact in all the cases presented in our ICME-13 discussion group and in this book, the lesson study focused on creating lessons that taught through problem solving. In the Portuguese and Irish case studies, the authors report how educators took into consideration both anticipated student thinking and actual student responses when designing tasks. The authors conclude that this is an important feature of the lesson study process and

crucial to the implementation of the program as both a professional development practice and for influencing curriculum reform.

Both Ponte et al. (2017) and Ni Shuilleabhain (2017) stress that in their respective countries traditional professional development programs are quite different from lesson study. Traditional professional development programs in these countries generally occur off-campus and last no longer than a day. In contrast to this, lesson study takes place in the school on a regular basis over a long period of time. Lesson study is seen as a solution to the “lack of alignment between research and practice” (Ni Shuilleabhain, 2017, p. 66). It gives educators a chance to collaborate, supports their development, and can create effective changes in teaching practices. Despite the cultural differences between Japan and Ireland, “the culturally based elements of lesson study, suggested that this model could be legitimately adopted for the Irish school system” (p. 67). Despite differences in culture and education traditions, lesson study seems a like valuable tool for education reform.

Local teachers are often surprised by how different lesson study is from familiar professional development programs. Ponte et al. (2017) report that lesson study “may also seem strange” and that teachers “begin by questioning why to dedicate so many hours to work in single topic or curriculum aim when there are so many topics in the curriculum that deserve attention” (p. 99). However, when involved in the process, teachers quickly appreciate the collaboration. Despite initial doubts, these teachers felt that, “Along the sessions we were seeing that this is really productive and that we learned to work in a collaborative way” (p. 93). In the Ireland case study, teachers felt that lesson study provides an “environment which allowed them to take risks in their classroom practices” (Ni Shuilleabhain, 2017, p. 78). This safe environment is another aspect of lesson study as a learning milieu for the teacher. Ponte et al. reports that, “instead of saying ‘how the teachers must act’, we strive to create situations in which, through their collective work, they are led to discover how to act. We also provide a clear structure, in which all the work develops” (2017, p. 102). Some of the surprising aspects of lesson study are what make it so effective.

The effects of lesson study as professional development are often reported from a qualitative point of view. Ni Shuilleabhain (2017) reports teachers’ stated emphasis on students’ communication and collaboration. This also manifested in concrete details, such as changing the “classroom layout from individual tasks to tables arranged for group work” (p. 72). Teachers also reported that scripting open questions better prepared them to anticipate students’ responses and respond to misconceptions. When reflecting on their time spent teaching the lesson study research lesson, teachers reported that they saw themselves as facilitators of learning, “There was very little teaching involved in it really. Just kind of facilitating—which is what it’s all about” (p. 76). In this way, research shows qualitative evidence for how lesson study impacts classroom teaching and learning.

Reports also show how lesson study addresses the analysis, modification and even creation of tasks. In Ireland, for example, teachers engaged directly with curriculum documents and materials (Ni Shuilleabhain, 2017). Winsløw et al. (2017) report that comparing task analysis and observations gathered during the

research lesson, combined with the repetition of modified research lessons, allowed teachers to understand “how students respond to small details in the milieu” (p. 134). This awareness allowed teachers to “discover not only the mathematical knowledge needed to design the lesson, but also their hypothesising didactical situations made them reject one inappropriate milieu and develop one potentially more appropriate” (p. 134). Mathematical knowledge and practice, the mathematical praxeology, is developed during the lesson study process. Ponte et al. (2017) also report how deep mathematical work was performed during a lesson study program. One of the teachers from their case study reflects that, “Sometimes I need a little more mathematics, and I think that these sessions helped us with that” (p. 93). Lesson study demands close examination of mathematical teaching and learning, helping teachers to craft effective tasks for their students.

9.7 The Sustainability of Lesson Study

The question of the sustainability of lesson study is one of the more difficult issues to resolve. In the Irish case study (Ni Shuilleabhain, 2017), among all the teachers who wanted to continue lesson study, only the ones at the school where they all shared a free double-period were able to do so. Those at schools where this opportunity wasn’t available were forced to abandon the project. In the Portuguese case study (Ponte et al., 2017), professional development is only open to teachers during their free time. There is no reward and teachers often have to pay fees to attend. These conditions seem to be particularly disadvantageous. Nevertheless, Ponte and his colleagues consider that:

these are not unsurmountable obstacles, but they are contextual conditions that we need to pay attention to. Unless these conditions change in a deep way—by some change in educational policy, notably regarding teachers’ career and teacher education—it is natural that they keep putting constraints on the development of this professional development practice in Portugal. (2017, p. 101)

Even with local limitations, lesson study is still a worthwhile practice, and may lead to improved conditions. Deeper understanding of local situations can improve its sustainability.

Underscoring this issue of sustainability are the two chapters by Lim, Teh, and Chiew (2017) and Takahashi and McDougal (2017). These chapters discuss cases of lesson study outside Japan which began as small, externally funded, ad hoc volunteer group projects. As such, these group projects struggled to establish and maintain positions within their existing school systems, despite that aim being one of the original goals. These projects found themselves constricted by grant periods and often ended before seeing vital impacts on either teacher or student learning (Lim, Teh, & Chiew, 2017; Takahashi & McDougal, 2017). Institutional and financial support are key to the success of lesson study projects.

Takahashi and McDougal (2017) expound upon lesson study conditions in the U.S., where lesson study projects have a longer history. They conclude that in order to be successful, lesson study must be schoolwide. To ensure its sustainability, they outline the following requirements:

1. Enthusiasm for lesson study from the school principal, clearly communicated to the faculty
2. A persistent lesson study advocate in addition to the principal
3. A compelling school-wide goal for teaching and learning
4. A commitment on the part of the school administration to provide time for lesson study, through use of funds, staff, and district-mandated professional development time (p. 146).

Lesson study cannot thrive in a vacuum; the greater its support, the greater its impact will be.

9.8 Implementing Lesson Study on a Larger Scale

The case study by Lim, Teh, and Chiew (2017) in Malaysia and the case study by Estrella, Mena-Lorca, and Olfos (2017) in Chile describe larger-scale lesson study projects within their school systems. Unlike most early attempts of lesson study pilot projects outside Japan, these two cases focus on schools who used lesson study to update their existing professional development program. The case in Malaysia began with a university research initiative involving a small number of schools. Because the initiative was a success, the Malaysian Ministry of Education became interested in adopting lesson study nation-wide. Based on their experience extending lesson study as a system-wide initiative, the authors offer the following suggestions:

1. Instill lesson study as a culture of professional learning for teachers
2. Enhance the knowledge and skills needed for effective lesson study: lesson planning, observation, reflection
3. Engage expert teachers to play the role of knowledgeable others in lesson study
4. Propose Collaborative Lesson Research (CLR) as an alternative form of lesson study.

These suggestions are particularly useful for lesson study projects outside Japan to help them establish sustainable lesson study in their school systems.

The case study by Estrella et al. (2017) from Chile reports a unique lesson study project: a large-scale government initiative that has strong support by a Japanese university. Leading researchers and educators of mathematics education were selected by the Chilean government to receive intensive training at a university in Japan. Japanese researchers and practitioners were then invited to work with schools and teachers in Chile to introduce the Japanese approach for teaching mathematics as well as lesson study. The authors reported the process of this unique top-down approach for implementing lesson study in their country.

9.9 Conclusion

Since the late 1990s, to help students become active constructors of mathematics, many researchers and school systems have tried to use lesson study to replicate Japan's success for shifting mathematics classrooms from teacher-led lectures to student-centered problem solving lessons. The case studies compiled in this book exemplify these efforts by researchers and educators around the world. These cases teach us a variety of ways to initiate lesson study in locations where no one has experienced such professional development before. We can learn a great deal from their successes and challenges. We believe that these pioneers' efforts will help researchers and educators who want to design projects to improve mathematics teaching and learning in their local schools. At the same time, we also believe that existing research on lesson study implementation may not provide enough information for us to judge if lesson study outside Japan can reach the same level of success that it has in Japan. In other words, it remains unclear if lesson study projects outside Japan can properly support schools and teachers. In order to answer this question, we need to quantify the impacts of international lesson study projects and analyze exactly how lesson study can be both effective and sustainable.

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