ICME-13 Monographs

Marisa Quaresma · Carl Winsløw
Stéphane Clivaz · João Pedro da Ponte
Aoibhinn Ní Shúilleabháin
Akihiko Takahashi *Editors*

Mathematics Lesson Study Around the World

Theoretical and Methodological Issues

Springer

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Introduction

Lesson study and learning study, as forms of teacher professional development, have been introduced and experimented with in many countries around the world, with teachers at all levels of education and in initial teacher education. In some countries, lesson study and learning study have become a regular part of teaching activities across the curriculum. However, these models of teacher collaboration have been most prevalent in the subject of mathematics—both in practice and in research.

While there are many descriptions of lesson study and learning study which have resulted in both teacher learning and student learning, the core dynamics of these models remain a subject of interest for many researchers. Within these elements of research, there remain many questions: What are the key elements of which a professional development activity can be considered as lesson or learning study? What are the central formative mechanisms of lesson and learning study which encourage teacher learning? Which conditions may affect a lesson or learning study and lead them to a loss of their formative potential? How might lesson and learning studies be adapted to suit the needs and the particular cultures of different countries or educational systems? What makes lesson study "viable" in some contexts, where the activity is regularly done by all teachers with no external initiative? While lesson study is often described as a form of research undertaken by teachers, can lesson study be utilized as a research method (not just research object) by mathematics education researchers?

The need to go beyond the mere sharing of experiences, to investigate and discuss the deeper issues related to the theorization of lesson and learning study, led to the constitution of a discussion group at ICME 13 in Hamburg. This group sought to discuss the specificities of mathematics lesson and learning studies, with regard to regional and national particularities. In addition, the group sought to discuss the methodological and theoretical tools which might be used to conduct research on and with this form of teacher education, from an inclusive international perspective. The discussion group thus had two main foci of work. The first was dedicated to presenting and discussing regional and national particularities and approaches of lesson and learning studies in mathematics in Japan and other Asian

countries, North and South America and Europe. The second was dedicated to discussing theoretical, methodological, and epistemological issues involved in organizing and carrying out research on and with lesson and learning study.

Given the interest in these discussions and value apportioned to the exchange among participants from around the world, the presentations were brief. The organizers of the discussion group, Marisa Quaresma and Carl Winsløw, therefore availed of the opportunity provided by the ICME brief series to challenge participants to further elaborate on their work and provide an account not only of their experiences with lesson and learning study, but also on the underlying theoretical and methodological issues in their cultural or educational context.

We are grateful to all authors who have participated in this endeavor and also wish to thank Stéphane Clivaz and Akihiko Takahashi who have provided a reflection of what has been achieved in this current collection of papers and outlined the potential next steps to be undertaken.

The organization of this book has been a collective activity, involving editors and authors in discussion of the content and involving additional external reviewers. The book will constitute a point of reference concerning where we, as a community of mathematics education researchers working with lesson and learning study, stand in relation to what we have learned to date about these forms of teacher professional development in 2016–17. We endeavor to contribute to further research on the underlying issues and processes within these models and hope this book will provide a stimulus to interested readers to reflect on the potential and the constraints of lesson and learning study.

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Chapter 1
Lesson Study and Teaching Mathematics Through Problem Solving: The Two Wheels of a Cart

Toshiakira Fujii

Abstract The international education community has taken an interest in lesson study, the Japanese approach to school education. Lesson study first came to the attention of educators outside of Japan primarily through the publication of The Teaching Gap (Stigler and Hiebert in The teaching gap: Best ideas from the world's teachers for improving education in the classroom. The Free Press, New York, 1999) and the TIMSS video study, which showed the typical structure of Japanese mathematics instruction in the classroom. However, while there has been tremendous international interest in using lesson study as a model for professional development, there has been less of a focus on applying the method of Japanese mathematics instruction, which is teaching mathematics through problem solving. This paper discusses the interplay between lesson study and teaching mathematics through problem solving and outlines suggestions for educators seeking to improve both teaching and lesson study outside Japan.

Keywords Japanese lesson study · Teaching mathematics through problem solving · Structured problem-solving · Designed tasks · *Kikan-jyunshi Neriage* · *Matome* · *Kyozaikenkyu*

1.1 Introduction

It seems that so far a quite a few versions of lesson study are practicing in the world. Although the author would like to distinguish them from Japanese lesson study, he simply uses the term "lesson study" in this Chapter with consideration of consistency in the book. The history of lesson study in Japan spans more than a century (Makinae, 2010). For Japanese educators, lesson study is like the air they breathe. It came to the attention of educators outside of Japan primarily through The Teaching Gap (Stigler & Hiebert, 1999), which described findings from the Trends in

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International Mathematics and Science Study (TIMSS) video study on eighth grade mathematics lessons in several countries. One chapter, "Japan's approach to the improvement of classroom teaching"[1], provoked enormous interest in lesson Study (Stigler & Hiebert, 1999). Independently, some educators such as Lewis also noticed the significance of Japanese lesson study and brought this to the attention of the international community (Lewis & Tsuchida, 1998). Since then, many mathematics teachers and teacher educators around the world have been involved in lesson study and many books and research papers have been written on its various aspects (Department for Children, Schools and Families, 2008; Doig & Groves, 2011; Hart, Alston, & Murata, 2011; Lewis, 2002; Lewis, Perry, & Hurd, 2009; Ono & Ferreira, 2010; White & Lim, 2008). However, some aspects of lesson study still seem to not be well understood outside Japan.

One crucial aspect of Japanese mathematics instruction is the method of teaching mathematics through problem solving. The international education community took a great interest in this when they were introduced to the typical structure of a Japanese mathematics lesson through watching a lesson in action in the TIMSS video study. Stigler and Hiebert (1999) refer to this method of instruction as "structured problem solving." However, teaching mathematics through problem solving has been largely overshadowed by interest in incorporating lesson study as a model for professional development. I assert that lesson study and teaching mathematics through problem solving are two wheels of a cart: one cannot succeed without the success of the other. In this chapter, I will discuss the close relationship between lesson study and teaching mathematics through problem solving.

1.2 Lesson Study Components and Process

The lesson study process is illustrated in Fig. 1.1 as having five steps. This figure differs from other, similar diagrams in other publications that only have four steps (e.g. Lewis, 2002; Lewis & Hurd, 2011). The inclusion of a fifth step more accurately portrays how Japanese teachers progress through the lesson study cycle. Borrowing from Lewis's (2002) and Lewis and Hurd's (2011) descriptions, each step can be summarized as follows:

1. **Goal Setting**: Teachers consider long-term goals for student learning and development. They identify gaps between these long-term goals and current reality and decide the research theme.
2. **Lesson Planning**: Teachers collaboratively plan a research lesson designed to address the goals. They prepare a "lesson proposal," a document that describes the research theme, content goals, connections between the current content and related content from previous and later grades, rationale for the chosen

[1]This chapter is based on Yoshida's doctoral dissertation (1999), which is now also available in book form (Fernandez & Yoshida, 2004).

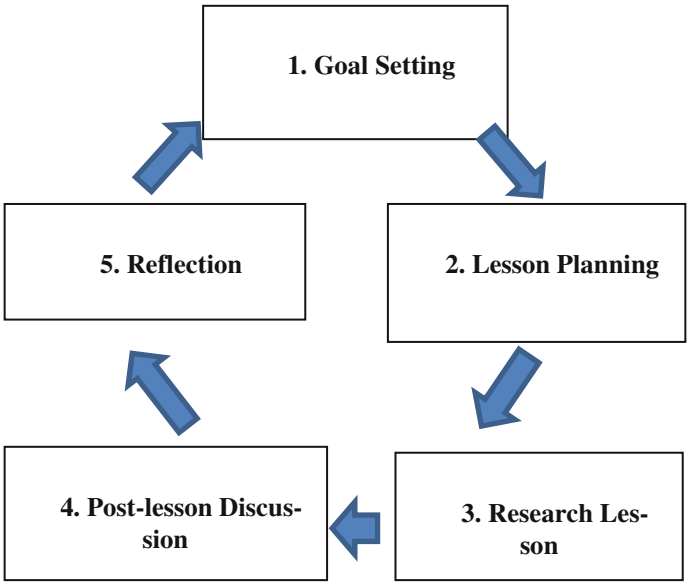

Fig. 1.1 Steps of a lesson study cycle (Fujii, 2014a, p. 113)

approach, a detailed plan for the research lesson, anticipated student thinking, data collection, etc.

3. **Research Lesson**: One team member teaches the research lesson while the other members of the planning team, staff members from across the school, and, usually, an external knowledgeable other observe and collect data.
4. **Post-lesson Discussion**: Observers share data from the lesson in a formal lesson colloquium, highlighting successes and issues concerning student learning, disciplinary content, lesson and unit design, and broader aspects of teaching and learning.
5. **Reflection**: Not only the instructor, but also other members in the school consolidate and document what was learned from the lesson study cycle so that it can be implemented in the future, and formulate new questions for the next cycle. They write a report or bulletin that includes the original research lesson proposal, student data from the research lesson, and reflections on what was learned.

Steps 3 and 4, the Research Lesson and Post-Lesson Discussion, are the most visible and, as such, are what has attracted the attention of the international education community. This leads to a typical misconception: that the Research Lesson and Post-lesson Discussion alone constitute lesson study. Lesson study is sometimes introduced as an open lesson by a veteran teacher "jumping in" to another teacher's classroom (Takahashi, 2013, p. 84). But the whole picture is much larger. Each step in the lesson study cycle closely relates to the others and Steps 1 and 2

provide a crucial foundation for the success of the more visible Steps 3 and 4. Step 3, the Research Lesson, is a lesson in which students work on their own to solve a given task for which they have not been shown solution strategies and critically analyze and reflect on their own and other students' solutions. This approach to mathematics instruction was described by Stigler and Hiebert (1999) as a "structured problem solving lesson." I will refer to the teaching method in this step as "teaching mathematics through problem solving."

1.3 Teaching Mathematics Through Problem Solving in Japan

1.3.1 The Structure of Teaching Mathematics Through Problem Solving

The meaning of "problem-solving" varies depending on the culture and historical time in which it is referred. We can see a clear progression of the changing meaning of "problem solving" in Japan after Word War II (Soma, 2000):

(1) (1945–1954)—The term "problem solving" appears as it applies to "life experience lessons" (In elementary school this could be participating school festivals, exploring the town, learning how to make bread, etc.).
(2) (1955–1964)—"Problem solving" appears as it applies to word problems.
(3) (1965–1974)—"Problem solving" appears as it applies to methods of instruction.

Japanese mathematics lessons have their own clear style, which often fascinates foreign observers and is often regarded as unique. Becker et al. (1990) identified "students' rising and bowing" as the first of eight components in a typical Japanese mathematics lesson. Stigler and Hiebert (1999) included "reviewing the previous lesson" as one of five major components. But these points of view are those of observers; Japanese teachers generally do not think about the structure of their lessons in these ways. What Japanese educators consciously strive to include in every problem-solving lesson are the following four activities, as seen in Table 1.1.

Table 1.1 The four phases of teaching mathematics through problem solving

1. Teacher presents the problem for the day. Students understand the problem	(5–10 min)
2. Students work to solve the problem	(10–20 min)
3. Teacher facilitates a comparison and discussion of students' solutions (This is called *neriage* in Japanese)	(10–20 min)
4. Teacher summarizes the lesson (This is called *matome* in Japanese)	(5 min)

These four activities or components are the framework of the lesson. Each component is 5–20 min long as part of a 45–50 min lesson. I will now describe each phase in detail.

1.3.1.1 Present the Problem for the Day

Some people may misinterpret "present the problem" as providing a detailed explanation or demonstration of the procedures involved in solving the given problem. However, this is not actually the case. In The Teaching Gap, Stigler and Hiebert (1999, p. 77) write, "the (Japanese) teacher presents a problem to the students without first demonstrating how to solve the problem," and, "we saw that a feature we hardly noticed before is perhaps one of the most important features of U.S. lessons—that the teacher almost always demonstrates a procedure for solving problems before assigning them to students."

In Japan, "present the problem" means to make students understand the context of a given task and the corresponding mathematical conditions that would be used to solve that task. Japanese educators distinguish between "teaching how to solve the task" and "problem solving based mathematics instruction," in which students learn how to solve the problem by solving it themselves. Therefore, it is important to choose the task carefully. If chosen well, the task allows for the important new mathematical ideas to emerge in classroom discussion.

Doig, Groves, and Fujii (2011) explain that there are four types of tasks typically used in lesson study:

(1) Tasks that directly address a concept
(2) Tasks that develop mathematical processes
(3) Tasks that are chosen based on a rigorous examination of scope and sequence
(4) Tasks that address a common misconception.

Typically, only one task is given in a lesson. People unfamiliar with this approach may feel that it is strange to only give students one problem per mathematics lesson. However, Japanese educators have studied and seen the success of this approach. Much thought is given to the selection and presentation of a lesson's task, so that the implementation of this single task can accomplish all the goals of the lesson.

Below are the following principles for an ideal task (Fujii, 2015):

A. It is appropriate and mathematically valuable in terms of the aims of the lesson.
B. It interests the students.
C. It is at the appropriate level of difficulty.
D. It can be solved in several ways.
E. It has a potential to elicit valuable basic wisdom.

1.3.1.2 Students Work to Solve the Problem

If the task principle "interests the students," is effective, students can start solving the task without any help from the teacher. While students work to solve the task on their own, the teacher in the classroom is extremely busy. The teacher must move among the students, practicing what we call in Japanese kikan-jyunshi, which means purposeful scanning or monitoring. Almost all teachers in other countries behave similarly, but what is consciously being done may differ from one country to another. To prove this point, Hino (2003) identified the result from the Learners Perspective Study to show how these conscious approaches were different between an Australian teacher and a Japanese teacher. An Australian math teacher was interviewed:

> Ms. M: Well I have to make sure that everyone's … involved and everyone's… participating in the lesson I guess - that I'm not excluding anyone … that um … that I'm encouraging everyone to have a go …

It seems that she is trying to consult with every student individually. So what do Japanese teachers consciously do during kikan-jyunshi? In Japan, the teacher:

A. Identifies how each student is solving the problem. Anticipated solutions, including typical incorrect solutions, are part of their prepared lesson plan.
B. Takes notes of the general atmosphere and how most students are solving the problem.
C. Gives hints to slower learners. If necessary, asks faster learners to come up with another way of solving the problem.
D. Plans what they should do during the compare and discuss (neriage) phase.

One of the most difficult aspects of kikan-jyunshi is the necessity to anticipate potential student solutions during lesson planning. This is particularly difficult for novice teachers. However, I think this practice of trying to prepare all possible solutions improves Japanese teachers' teaching abilities to see the mathematics through the eyes of the student and consider how best to facilitate their understanding of the underlying concepts or skills. Furthermore, it underscores how lesson planning, Step 2 of the lesson study cycle in Fig. 1.1, is a critical factor in determining the quality of mathematics teaching. Anticipating solutions enriches a teacher's knowledge on the topic and deepens their understanding of their students' cognitive nature and abilities. During kikan-jyunshi, if a teacher discovers an unexpected solution, they may ask the student about their thought process and record their answer. After the lesson, during the Post-Lesson Discussion, Step 4 of the lesson study cycle, teachers discuss how likely students actually were to attempt the anticipated student solutions listed in the lesson plan.

It is also important for the teacher to look beyond individual solutions and grasp how the class, as a whole, is approaching the task. While engaging in kikan-jyunshi, a teacher has to decide which student will present first and think about how to direct the class discussion. They must collect information and compose scenarios for the

next phase of the lesson, compare and discuss (neriage). Usually, student-teachers or novice teachers, or even experienced teachers, find kikan-jyunshi extremely difficult. During the Post-Lesson Discussion, Step 4 of the lesson study cycle, teachers discuss the instructor's kikan-jyunshi, supported by evidence or data collected by observers during the Research Lesson.

1.3.1.3 Compare and Discuss (Neriage)

The third phase of a lesson, called neriage in Japanese, assumes that students will arrive at different solution methods and therefore focuses on a comparison and discussion of those different solution methods. During the neriage phase o, students' solutions are typically written in order from "naïve" to "sophisticated" on the blackboard. Therefore, the teacher should be careful as certain students may realize why they are always the first to present. Teachers must also carefully plan how everything will be written on the blackboard in order to best compare and contrast students' solutions. The blackboard is used to organize both thought processes and results. Japanese teachers have been reluctant to use overhead projectors as they are small and therefore unsuitable for showing complete thought processes.

The neriage phase is difficult for novice teachers as it is closely connected to kikan-jyunshi. Novice teachers can typically give positive comments to each student, but struggle to elicit and clarify points that a student doesn't understand. Sometimes, students cannot distinguish between two similar solutions. In this case, the teacher has to explain how these two solutions are significantly different from each other. Or sometimes it's the opposite issue: two solutions appear to be different but are actually the same. Neriage provides an opportunity for teachers to foster mathematical ways of thinking in their students. In other words, it is a chance to focus on process. Mathematical ways of thinking cannot be demonstrated by simply reviewing the definition of concepts, they can only be truly experienced through problem solving activities. Therefore, Japanese teachers teach mathematics through problem solving. This kind of lesson encourages students to think mathematically and helps them become independent learners. This is one of the ultimate goals of school education.

Sometimes the neriage phase may appear to be implemented well, but is actually not. These cases typically are a result of a one-to-many correspondence between teacher and students describing their solutions during discussion. Takahashi (2008) described this phenomenon as "show-and-tell." The heart of neriage is reflective discourse (Doig, Groves, & Fujii, 2011). Although rather a classic citation, neriage should be an actualization of Vygotsky's zone of proximal development (Ohtani, 2014). Therefore, during neriage, teachers must see the students' potential as high as possible and help them move toward their potential from what they came up on their own, i.e., what they can do without help.

1.3.1.4 Teacher Summarizes the Lesson (Matome)

Fujii, Kumagai, Shimizu, and Sugiyama (1998) identified that American teachers tend not to summarize the lesson at the end of a class. Without this phase, the lesson ends with students only being able to take satisfaction in their individual work. By including a summary at the end of the lesson, every student can also feel satisfaction regarding the mathematical concepts explored.

Matome, or "summarizing the lesson," is brief, but critically important. It reflects the teacher's value system and helps impart this to students. Students may listen carefully to the teacher's final words. If the teacher concludes a lesson with a simply a review of mathematical procedure, students may feel that procedural knowledge is what matters most. However, if teachers sum up the lesson by reviewing how students constructed their ideas, it sends the message to students that how they think is also important. In other words, during matome, the teacher should review not only content but also process.

1.3.2 A Textbook Example Elucidating How to Teach Mathematics Through Problem Solving

Japanese teachers often say that they teach mathematics by using a textbook, but never teach a textbook. Therefore, it has been taboo for textbooks to explicate how to teach. Only one mathematics textbook out of six includes content showing how to implement the teaching mathematics through problem solving (Fujii & Iitaka, 2011). The textbook has been translated into English (Tokyo Shoseki Co., Ltd. 2011) and the first phase of a Japanese mathematics lesson, *Present the problem for the day*, can be seen in Fig. 1.2. The example problem asks students to find the area of an L-shaped figure. The page is on the right, so students cannot see the answer unless they turn to the next page. The next two pages, shown in Fig. 1.3, illustrate the third and fourth phases, *Compare and discuss (neriage)* and *The teacher sums up the lesson (matome)* (Tokyo Shoseki Co., Ltd. 2011). The textbook suggests three solutions to be shown on the blackboard for the *neriage* phase. However, in this case, a different student tries to understand their classmate's solution by interpreting their proposed mathematical expression. Other solutions are also possible if, for example, students count the unit squares to find the area of the L shape as.

Although the teacher's comment "we need to use what we have learned so far, don't we?" in Fig. 1.4 may seem rather general, it still clearly addresses the thought process needed for learning mathematics.

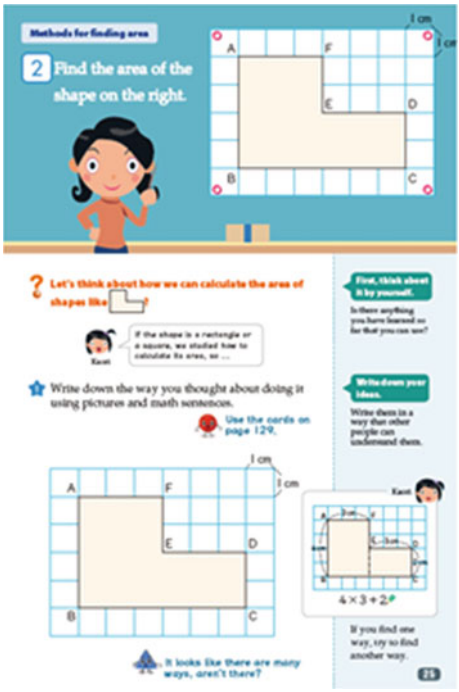

Fig. 1.2 The first page (Tokyo Shoseki Co., Ltd. 2011)—Present the problem for the day

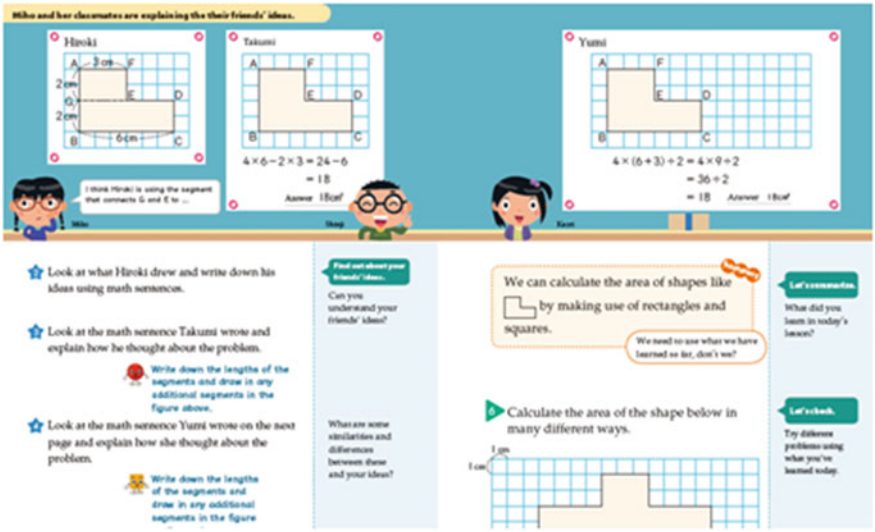

Fig. 1.3 The second and third pages (Tokyo Shoseki Co., Ltd. 2011)—Compare and discuss (neriage) and the teacher sums up the lesson (matome)

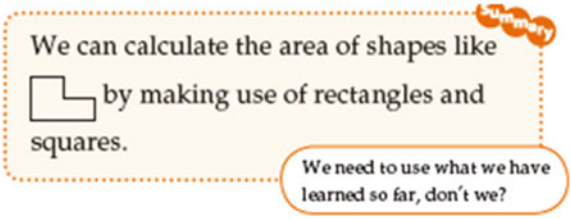

Fig. 1.4 Enlarged section of the teacher sums up the lesson (matome)

1.4 Unifying Lesson Study with Teaching Mathematics Through Problem Solving

1.4.1 Lesson Planning Is a Critical Step of Lesson Study

The Research Lesson is usually based on the "teaching mathematics through problem solving" structure, in which a whole lesson is taught on a single mathematical task. Choosing the correct task is critical, and is in part why Lesson Planning is so important to the lesson study cycle.

Watanabe, Takahashi, and Yoshida (2008) identified four core steps involved in constructing an instruction plan for a lesson from their case studies of Japanese teachers: (1) Understand the scope and sequence, (2) Understand children's mathematics, (3) Understand the mathematics, (4) During Lesson Planning, teachers plan the flow of the research lesson, based on the structure of teaching mathematics through problem solving and this is an important part of the planning process. In research conducted by the author in three schools in Japan, the average proportion of time spent on planning the flow of the Research Lesson, omitting meetings that did not involve the lesson plan, was 72%; if all meetings are included, the proportion becomes 63% (Fujii, 2016). The discussions specific to the flow of the research lesson during the planning meetings at each the three schools align with the four phases of teaching mathematics through problem solving (Table 1.1). For example, at the second meeting at school S (Fujii, 2016), teachers talked about the aim of the lesson and the concrete methods needed to achieve the research theme for only 3 min. A 15-min discussion ensued on how students might grasp the given task, which relates to phase 1, *Present the problem for the day*. Then there was a 14-min discussion about likely student responses, which relates to phase 2, *Students work to solve the problem*. Following that, there was a 15-min discussion about how to organize the comparison and discussion period, which relates to phase 3, Compare and discuss (*neriage*). Finally, 5 min were spent discussing how to conclude the lesson, which relates to phase 4, *The teacher sums up the lesson*. Out of these 49 min of discussion about the flow of the Research Lesson, the proportions of time spent on these four phases were about 31, 29, 31 and 10%, respectively. The other two schools showed similar proportions (Fujii, 2016).

Discussions by teachers during the Lesson Planning step (Fig. 1.1) can also be classified from another perspective: appropriateness of the task, plausibility of the anticipated student solutions, and quality of the *compare and discuss (neriage)* phase, which are now considered in further detail.

1.4.2 Appropriateness of the Task

Discussions about a proposed task for a research lesson can be classified into two types. The first type is a discussion about the task and unit from an advanced mathematical perspective, in which teachers clarify the scope and sequence of relevant topics and expansion of the content. In research conducted by the author (Fujii, 2016), I found that when teachers talked about the position of the unit within the curriculum, they carefully referred to the National Course of Study (NCS) published by the Ministry of Education, Culture, Sports, Science and Technology (2008).[2] For example, teachers at school M used their own diagram as they discussed why the unit was important and traced the students' learning path leading to the unit.

The second type of discussion involves the task itself. Teachers discuss how well the task suits the goal of the lesson, including detailed consideration of which numbers should appear in the task, the context of the task, etc.

This aspect of lesson study was also noted in The Teaching Gap (Stigler & Hiebert, 1999, p. 117), who reported that teachers would talk about the "problem with which the lesson would begin, including such details as the exact wording and numbers to be used." The research lesson described in The Teaching Gap (Stigler & Hiebert, 1999) covers the topic of teaching to first graders how to subtract a single-digit number from a two-digit number (less than 20) with regrouping. There are 36 possible problems that could be used to introduce this topic (for example, $18 - 9$, $17 - 9$, $17 - 8 \ldots 11 - 2$). This is regarded as an important area of content and which of these 36 subtractions should be the first for children to learn is hotly contested (Fujii, 2015). What is used in textbooks is typically $12 - 9$ or $13 - 9$.

The reason these are chosen is because the subtrahend 9 is close to ten. It is easier for students to separate, for example, 12 into 10 and 2, subtract 9 from 10, and then add the difference to 2. This can be written as the mathematical expression $12 - 9 = (10 + 2) - 9 = (10 - 9) + 2$. We call this approach the subtracting-adding strategy.

However for a task such as $12 - 3$, we can see that 2 and 3 are close to each other. Therefore, 3 can be broken down into 2 and 1, which gives us $12 - (2 + 1)$ and then $(12 - 2) - 1$. 2 is subtracted from 12 which gives us 10, and then one is deducted from 10, giving us a final answer of 9. We call this approach the subtracting-subtracting strategy. The teachers in The Teaching Gap (Stigler &

[2]The Japanese curriculum is national and revised every ten years or so.

Hiebert, 1999) decided not to use 12 − 9 from the textbooks because "it's not very interesting." One teacher suggested using 15 − 8 or 15 − 7 instead; another suggested using 11 − 6, "Because kids can conceptualize in their heads about up to the number 6 at this age" (Stigler & Hiebert, 1999, p. 118). Another teacher proposed 12 − 7, because "one of her students, who was a low achiever, happened to have seven family members. Everyone agreed that this was a good idea (Stigler & Hiebert, 1999, p. 118). Finally, the teachers decided to use 12 − 7, which seemed likely to provoke the subtraction-addition and subtraction-subtraction strategies equally, allowing for a discussion that would compare the relative merits of these two methods (Stigler & Hiebert, 1999). Such careful scrutiny of the sequencing of tasks is unusual by Western norms. Researchers have noted that, "Western observers are often astonished by the order of presentation being the subject of so much study and debate. However, Japanese lesson study is frequently used to investigate sequences of tasks that are different from those traditionally used" (Doig, Groves, & Fujii, 2011, p. 194). Such close attention to the specific numbers does not mean that teachers are focused merely on a concrete level of thinking and encouraging students to think about particular numbers. On the contrary: teachers consider the general aspect of a number as quasi-variable—deliberately used in a general way so that it serves as a representative of many numbers, just as a variable would be used (Fujii & Stephens, 2001, 2008). Numbers are often chosen based on their quasi-variable power, or on how well they demonstrate a general truth.

For instance, the tasks 12 − 9 and 13 − 9 are likely to lead students to the discovery of the subtraction-addition strategy. They are not mere calculation problems, but serve to introduce a particular general procedure for subtracting with regrouping in the base-ten system. Appreciating the base-ten system, the place value notation system and its benefit for calculation is more important than simply getting an answer and gaining skill at calculating 12 − 9. Therefore, the Japanese method of instruction is called "teaching mathematics through problem solving," and not "teaching how to solve the task."

1.4.3 Anticipated Student Solutions

In all three schools in this author's research (Fujii, 2016), teachers spent a significant amount of time discussing likely student responses to the task that would be given in the Research Lesson. These discussions usually began by considering what would be most likely response from the class as a whole. The discussion then moved on to consider likely responses from students who learned at a faster or slower pace. It is interesting to note that at each school teachers solved the task while role-playing as their students.

Teachers also discussed anticipated solutions in terms of the vertical curriculum and how students' prior learning was expected to be a resource for students to solve the task. For example, in the second meeting in School S, there was the following exchange (Fujii, 2016, p. 418):

Teacher A: Students learned how to arrange to get the same numbers for time or distance, didn't they?
Teacher B: Yes, I suppose. However, the idea of a common multiple was learned a long time ago from the students' point of view.
Teacher C: Probably they forgot the procedure to find the common multiple.
Teacher B: When they learned division of decimal numbers, they learned the idea of per-unit. It's the same thing here. However, the idea of per-unit was not learned in the context of comparing things.
Principal: The idea of per-unit quantity was applicable for comparing crowdedness. That is a mathematical way of thinking that could be applicable for Speed.

This kind of detailed and concrete consideration of previously-learned content was observed in all three schools.

1.4.4 The Compare and Discuss (Neriage) Phase

In order to gain an appreciation for which method or solution is better or best to use for solving a given task, students need to see alternative strategies. Therefore, a lesson teaching mathematics through problem solving includes the compare and discuss (*neriage*) phase for students to compare and interpret their classmates' methods and to discuss similarities and differences among strategies as a class. Each correct solution has equal value in terms of achieving an answer. However, the ideas involved may not have equal value. During the *neriage* phase, the teacher elicits these ideas and discusses the value of each solution. In Fujii (2016), a teacher at school S during the lesson planning meeting clearly stated, "Although each strategy is sure to get the correct answer, we should not end there…I want the students to know that getting the answer is not the final goal" (p. 419). It is important to note that the *neriage* phase is an opportunity to cultivate students' ways of thinking or attitudes towards arriving at a mathematical solution. Unifying lesson study with teaching mathematics through problem solving will cultivate students' habits of mind.

1.4.5 Tasks Designed During Lesson Planning Are Evaluated During Post-lesson Discussion

One of the critical features of teaching mathematics through problem solving in the context of lesson study, is the evaluation of the appropriateness of the task used in the research lesson during a Post-Lesson Discussion, Step 4 in the lesson study cycle (Fig. 1.1). The quality of a task is not judged based on an abstract determination about whether it is good or not for teaching a certain mathematical concept.

Its quality is judged based on concrete evidence collected during the Research Lesson evidencing how students responded to it.

The evaluation of a task is also discussed by teachers in terms of how well it promotes educational values. In fact, Japanese teachers often say that the aim of problem solving lessons is not for students to merely get an answer to the problem, but rather to teach them mathematical ways of thinking. During the Post-Lesson Discussion, Step 4 in Fig. 1.1, final comments usually by the knowledgeable other typically address educational values. For example, a final commentator is quoted in The Teaching Gap (Stigler & Hiebert, 1999, p. 182) explicitly addressing broad educational values, "He urged teachers to think carefully about what were the most important 'skills for living' that students should be learning from their mathematics instruction." Using, as an example, the formula for finding the area of a trapezoid, he says, "teachers should help students realize that moving from complicated to more simple forms is a convenient and a clever thing to do" (p. 183). This is an example of how Japanese teaching mathematics through problem solving lessons address both content and process, and how lesson study prioritizes such educational values. In this way, lesson study and teaching mathematics through problem solving are inseparable in terms of addressing educational values.

1.4.6 Why We Observe and Discuss Lessons in Lesson Study

When conducting "teaching mathematics through problem solving" during a Research Lesson, the aim is not only to develop or deepen students' content knowledge, but to also foster their mathematical ways of thinking and perspectives. In other words, the aim of the lesson is to unify content and thought process. However, thought processes are invisible. We cannot explicitly teach students how to think and this makes teaching this very difficult. What a teacher can do is anticipate student responses when Lesson Planning, Step 2 of the lesson study cycle in Fig. 1.1. Then, during the Research Lesson, Step 3 of Fig. 1.1, the instructor and fellow observers can note students' processes as they become visible in their work. Observers can also note how the instructor deals with students' thought processes. Following the Research Lesson, the instructor and observers can then use these notes as concrete examples of how to teach particular solutions and teach mathematical thinking. This is why the Research Lesson works best as a public lesson with the participation of many observers. Later, during the Post-Lesson Discussion, Step 4 in Fig. 1.1, all participants in the Research Lesson can discuss and evaluate the activities within the lesson using the concrete evidence they have collected. After the Post-lesson Discussion, comes Reflection, Step 5 in Fig. 1.1. During Reflection, participating teachers' reflections are written and later published as an annual school report.

Each step in the lesson study cycle is closely related. Lesson study is a system for teachers to learn how to teach students not only content but also thought processes to help them become independent thinkers. This is why it is essential for

any school that wants to incorporate the lesson study system to follow all 5 steps, ensuring unification with the method of teaching mathematics through problem solving.

1.5 Unifying Lesson Study with Teaching Mathematics Through Problem Solving

1.5.1 The Japanese National Course of Study

The research conducted by this author (Fujii, 2015) found that during Lesson Planning meetings, Step 2 in Fig. 1.1, teachers frequently referred to the National Course of Study (NCS) when they needed to confirm the role of the unit or Research Lesson within the whole curriculum. Sometimes, teachers talked about the placement of the content to teach in the lesson in the previous NCS. This is a more difficult conversation to have in countries which lack a clear curriculum. Lewis and Tsuchida (1998) argued that having a frugal, shared curriculum was necessary for implementing lesson study. With a clear curriculum sequence, teachers could identify the value of the Research Lesson or the unit within the curriculum. More importantly, the NCS, clarifying teaching and learning over the course of a student's experience in school, is a necessary component for implementing a teaching mathematics through problem solving lesson. Without the NCS, it would be difficult for teachers to anticipate students' idea or solutions. In other words, the teacher needs clear information regarding their students' learning history. By using the NCS to identify related content that students learned in previous grades, teachers can more accurately anticipate how students might solve a given task. The NCS makes it possible to clarify students' learning trajectory. It plays an important role in designing lessons which teach mathematics through problem solving. This point has not yet been articulated in academic papers or practical reports on lesson study in Japan, but it is an important one.

1.5.2 Kyozaikenkyu: An Essential Component of Lesson Study

The five steps of lesson study, shown in Fig. 1.1, are the core essentials of lesson study. However, there is another essential component which connects to each of these components, *kyozaikenkyu*. *Kyozaikenkyu* is known in English as "the study of instructional materials." When Japanese teachers begin their Lesson Planning they first engage in *kyozaikenkyu*. Watanabe et al. (2008) says "This practice is a central activity in teachers' everyday practice, but it plays a particularly important role in lesson study. In fact, one way lesson study contributes to the improvement of everyday instruction is through *kyozaikenkyu* (p. 133)." During the Post-Lesson

Discussion and Reflection, Japanese teachers will often remark, particularly when they are unsatisfied with the research lesson, that they should have done *kyozaikenkyu* more thoroughly.

I believe that teachers outside of Japan who want to implement lesson study need both to understand and successfully implement *kyozaikenkyu*. However the process of *kyozaikenkyu* is complex. Watanabe et al. (2008) list some essential questions which teachers need to ask in the process of *kyozaikenkyu* as follows:

- What does this idea really mean?
- How does this idea relate to other ideas?
- What is/are the reason(s) for teaching this idea at this particular point in the curriculum?
- What ideas do students already understand that can be used as a starting point for this new idea?
- Why is this particular problem useful in helping students develop this new idea?
- How can students solve this problem using what they already know, and how can their solution strategies be used to develop this new idea?
- What are common mistakes? Why do students make such mistakes? How should teachers respond to those mistakes?
- What new ideas are students expected to build using this idea in the future?
- What manipulatives and other materials should be provided to students?
- How do they influence students' learning?

Watanabe et al. (2008) warn that teachers should not take these questions as simply a checklist, but rather that they must understand why these questions are important when planning a lesson.

A concrete example of this is school M from the research conducted by this author (Fujii, 2016). School M made a school report based on their research which was originally written in Japanese and then translated into English by Project IMPULS in 2011. In the report, the school clearly shows the process of lesson planning as outlined below.

Figure 1.5. shows that the process starts with the term *Kyozaikenkyu No. 1 and No. 2*. *Kyozaikenkyu No. 1* is the actualization of the school's research theme into the Research Lesson. *Kyozaikenkyu No. 2* is the study of the NCS, teaching guides, textbooks published by other companies, and other reading resources. In Fig. 1.5, *kyozaikenkyu* seems to have a rather narrow meaning. However, this figure actually shows that *kyozaikenkyu* will continue into other activities such as developing teaching materials, improving classroom discussion, improvement of questioning, improving summarizing of the lesson, and improving blackboard organization. To sum up, *kyozaikenkyu* involves examining teaching materials and tasks from both mathematical and educational points of view as well as from the students' perspectives. Although *kyozaikenkyu* is recognized as a critical practice of lesson study by Japanese educators, teachers outside Japan often neglect it. This may be because the effort involved is almost invisible, the same way that 90% of an iceberg is hidden underwater, with all of our attention going to its visible tip (Doig et al., 2011, p. 182).

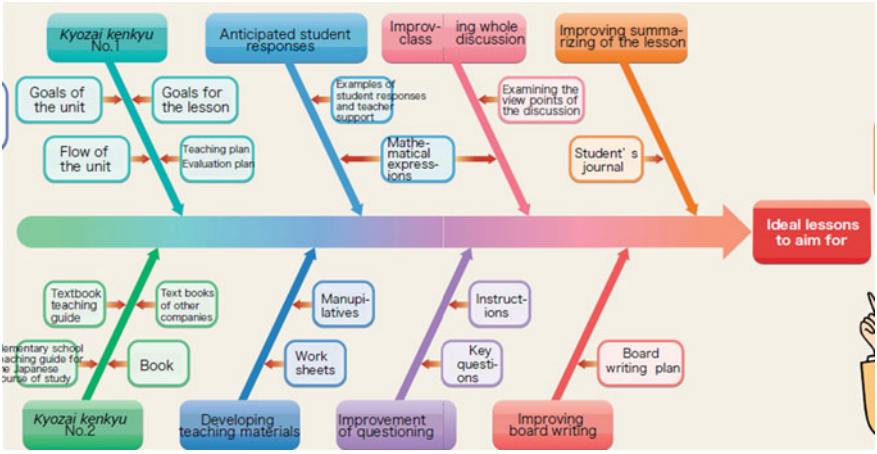

Fig. 1.5 The process of lesson planning at school M (Matsuzawa Elementary School, 2011)

1.5.3 Lesson Study as an Organic System

The Post-lesson Discussion, Step 4 in Fig. 1.1, provides a context for revising the task used during the Research Lesson, Step 3 in Fig. 1.1. However, this does not mean to imply that there is any value in re-teaching. Some countries seem eager to re-teach the Research Lesson. However, the most serious misinterpretation of lesson study is the question, "Should a research lesson always be re-taught?" (Fujii, 2014b).

The possible roots of this misconception might come from the lesson study steps as described in The Teaching Gap (Stigler & Hiebert, 1999, pp. 112–113). Stigler and Hiebert (1999) describe 6 steps, the last of which is called "Teaching the revised lesson." While it is true that, sometimes, Japanese teachers will perform trial lessons, this is different from simply re-teaching the whole lesson. Trial lessons are performed in Japan only in situations such as huge, nation-wide research lessons, in which observers come over from all over Japan. These trial lessons are taught just before the actual Research Lesson, because the instructor feels that it would be too risky to implement a lesson plan for the very first time in front of so many outside observers. The teacher will use this trial lesson to fine-tune their plan in light of students' actual responses. However, the teacher still needs to assume that students' responses during the actual Research Lesson will differ from those they encountered during the trial lesson. By misinterpreting this practice as plain re-teaching of the same lesson, student responses are de-emphasized, which goes against the core values of lesson study.

Re-teaching disrespects students' right to the best education one can provide them. Having the thought of re-teaching at the back of one's mind is like treating the first class as a pawn to be sacrificed in order to improve one's own teaching skills. It benefits teachers and lesson plan creators at the expense of their students.

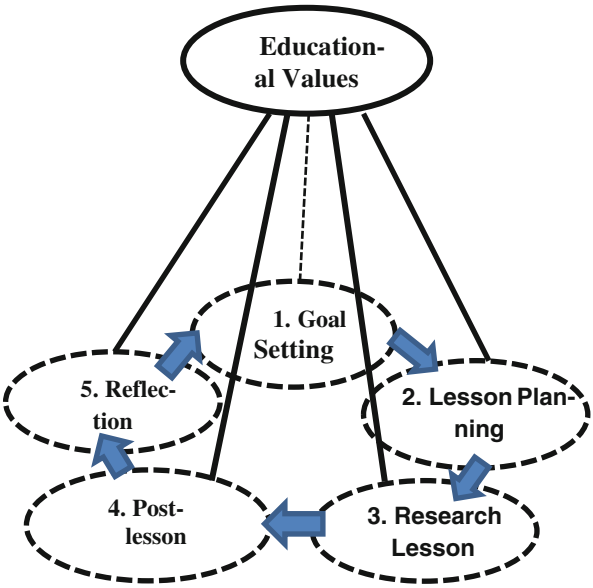

Fig. 1.6 An organic model of lesson study

An instructor leading a Research Lesson needs to feel that their lesson plan is the best one that they and their planning team could create. The lesson plan should be the result of extensive effort. In other words, Japanese teachers feel that the best lesson plan should be implanted during their Research Lesson, and the Research Lesson is the proving ground for teachers.

The misinterpretation of re-teaching suggests a practice of revising a faulty part and replacing it. An inorganic system is composed of parts that may be easily replaced. However, lessons and lesson study are organic systems, each part is systemic, not systematic. But what does it mean to state, "A lesson is an organic system" and "lesson study is an organic system?" Figure 1.6 shows a possible conceptual model for lesson study as an organic system in which the 5 steps of lesson study exist on a two-dimensional plane. Figure 1.6 differs from Fig. 1.1 in that these steps function as an organic system, each joined together by the additional component of Educational Values—the heart and most important factor of lesson study (Fujii, 2014a). This heart is shown in Fig. 1.6 as the third dimension of lesson study. When educational values are taken into account, lesson study can clearly be interpreted as an organic system, each component inseparable from the others.

1.6 Final Remarks

The Japanese NCS mainly covers content for each grade, from kindergarten to high school levels. However, at the symposium held in Tokyo in 2015 hosted by Project IMPULS at Tokyo Gakugei University (IMPULS, 2017), participants

argued that it is important for educators not only consider mathematical *content*, but also mathematical *processes*. They also discussed the relationship between *content* and *processes*. They described their dual nature as:

Content nests inside the processes.
Processes nest inside the content.

The NCS mainly covers *content*. How can we understand process from examining the NCS? In the 1958 NCS for mathematics, the term "mathematical ways of thinking" was listed for the first time as a goal of mathematical education. That was the first time not only *content* but also process was addressed in the NCS. Since then, "mathematical ways of thinking" has been one of the most important components of mathematics education in Japan.

On the other hand, at the symposium, we were pleased to recognize that the Japanese way of teaching mathematics, *mondai-kaiketsu-gata-jyugyou*, which is referred to here as "teaching mathematics through problem solving," is highlighted as reflecting what Stigler and Hiebert (1999) described as "structured problem solving." The Japanese style of teaching mathematics was considered as a reliable method of crystalizing the dual nature of *content* and *processes*. The results of the TIMSS 2002 support this view. The average percentages of TIMSS Mathematics Topics *taught* in Japanese schools, either prior to or during the year of the assessment, were 54% for grade 4, however, the *achievement* scores (average scale score) was 69% for these grade 4 students. These are remarkable findings; it shows that Japanese grade 4 students could solve tasks in the TIMSS which had not been taught to them in school. In other words, they could perform beyond what they have been taught. This fact supports the assertion that by "teaching mathematics through problem solving" we have been teaching not only *content* but also *processes*. We could say that Japanese educators have been implementing the "teaching mathematics through problem solving" approach to the curriculum without a defined theory or consciousness. However, this new insight demands that the concept of "teaching mathematics through problem solving" be fully described, along with lesson study, as the two concepts are two wheels of the same cart.

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Chapter 2
Mathematics Education Lesson Study in Japan from Historical, Community, Institutional and Development Assistance Perspectives

Takuya Baba, Atsumi Ueda, Hiroyuki Ninomiya and Keiko Hino

Abstract Japanese mathematics education is characterized by structured problem solving (Stigler and Hiebert in The teaching gap. Free Press, New York, 1999). Lesson study, where a group of teachers develops a lesson together, refines it furthermore. This learning opportunity has spread to various levels of the educational system during the last one hundred years, forming the lesson study practice and of today and becoming a part of our educational culture. This chapter addresses the culture, which Japanese mathematics education has nurtured from three perspectives such as history, community, and institution. These perspectives are discussed with a basis on Bishop in Mathematical enculturation: A cultural perspective on mathematics education. Kluwer, Dordrecht (1989). Besides these, a fourth perspective, developmental assistance, is also employed to compare the perspectives of Japan with those of developing countries.

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Keywords Lesson Study in Japan · Historical · Community, Institutional and Development Assistance Perspectives · International Cooperation

2.1 Introduction

The Japanese mathematics lesson is characterized as structured problem solving (Stigler & Hiebert, 1999), and lesson study, where a group of teachers develops a lesson together, is to refine it further. Here, structured problem-solving means a lesson where a teacher provides the students with a procedurally and conceptually demanding problem and allows students to invent their own procedures through a structurally designed problem-solving process (Stigler & Hiebert, 1999, p. 27). The teacher collaboration in lesson study develops the eyes of appreciating lessons—connoisseurship (Eisner, 1991)—through a discussion of possible alternatives, and illuminates the aspects which individual teachers often fail to recognize. Such an endeavor does not stop at school-based lesson study[1] (Stigler & Hiebert, 1999) but also extends to the in-service training at the municipal and prefectural levels and the exemplary lesson demonstration at the national level (Hashimoto, Tsubota, & Ikeda, 2003). These are interrelated and form the lesson study practice of Japan as a whole. In the long run, they have become part of the Japanese educational culture. In other words, the existence of lesson structure and the consistency of lesson study imply the existence of culture[2] at the background. On the other hand, it is not easy to see the culture by simply looking at a few lessons and/or the result of international comparative surveys such as TIMSS and PISA.

Hofstede (1997) categorized cultural components into two separate ones. One component lies at the core and is an invisible value, which characterizes culture. The other component is at more of a surface level and looks relatively visible, taking some forms such as lesson patterns. And the former value maintains such patterns. We must therefore pay close attention to the cultural values that we take as the convictions that an individual has internalized as being the things of importance and worth. What an individual values defines for her/him is the window through which s/he views the world around her/him. Valuing provides the individual with the will and determination to maintain any course of action chosen in the learning and teaching of mathematics. They regulate the ways in which a learner or teacher's cognitive skills and emotional dispositions are aligned to learning/teaching in any given educational context (Seah & Anderson, 2015, p. 169).

[1]School-based lesson study is commonly called *kounai-kenshu*, which literally means a training within school. This is one of the common lesson study practices in Japan.

[2]Cultural identity refers to the self-awareness of cultural ascription by means of life patterns, customs and languages, which characterize the culture that each member belongs to.

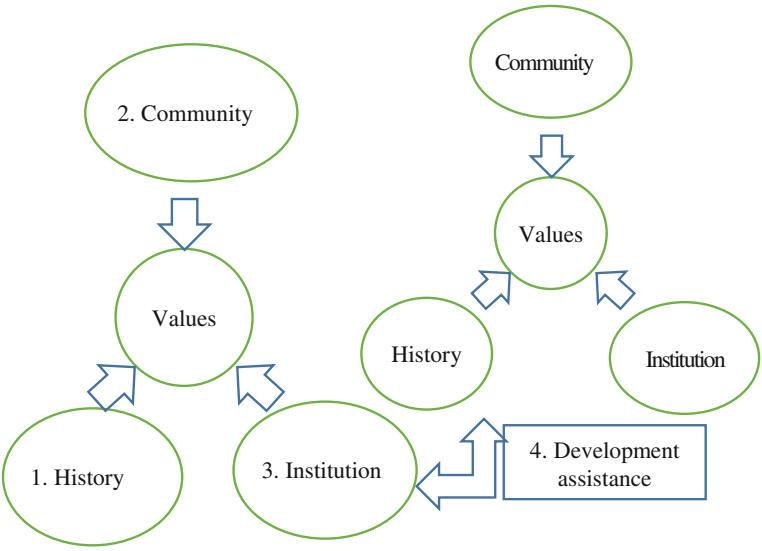

Fig. 2.1 Four perspectives to analyze values of Japanese mathematics education

Such values[3] are therefore related to actions and ideas of present day teachers and also those of the previous generations. They are regarded as providing consistency to the actions beyond time and space, as well as lesson study functions to transmit values through generations and preserve them. The patterns of the structured problem-solving lesson are polished and maintained through lesson study (Ueda, Baba, & Matuura, 2015).

The objective of this paper is to clarify the values that Japanese mathematics education has formed over a long period. As discussed later, lesson study is closely related to the historical development of Japanese mathematics education, and is also deeply connected to the values which the community as well as individual teachers have developed and internalized. To attain this objective, this chapter has chosen four perspectives, namely history, community, institution, and development assistance to developing countries (Fig. 2.1). The first three are based on Bishop's (1989) framework of culture. The last one is used to compare those three perspectives in different cultures.

Firstly, the historical perspective provides a description of how Japanese mathematics education has formed a central idea regarding "children's subjective

[3]Bishop (1989) categorized values into three types: general educational values, mathematical values, and mathematical educational values.

and extensive creation of mathematical ideas". Structured problem-solving for the treatment of mathematical ideas in present-day classrooms lies at the root of mathematics education in Japan. There must have been many practices and discussions in many classrooms before they were organized and crystallized into a complete idea and developed into a new educational philosophy and/or a new pedagogy. All of these efforts are engraved in the history.

Secondly, the community perspective is closely related to the historical perspective. Retracing the formation process of educational philosophy and pedagogical method provides us with the fact that there were many people involved in the process. The community perspective can be considered at various levels such as school, municipal, prefectural and national levels. All these different levels of communities are connected and form the whole community in Japan. The researches on teachers' community of practice (Wenger, 1999) focus more on the community of today. The above community in Japan is also connected to the one of previous times. For example, newly-recruited teachers participate in such communities peripherally in the form of school-based lesson study and lesson studies organized by prefectural organizations and voluntary study groups. They gradually acquire and reinterpret the values of senior members on how to develop lessons, views on mathematics education, and how to look at the teaching profession.

Thirdly, the institutional perspective provides an insight into how today's community functions. Here the institution is a rule which is established to control mathematics lessons and teacher education, and provides a regulation which manages each member at different levels. For example, it describes what to learn in the training, when to participate in the training, and how to learn. In this sense, the institutions reflect somehow the values that the community has formed, and may promote or control the community's activities. The institutional perspective is quite focused on current practice, compared to the history and community perspectives. However, when the reason for such practices is considered, they are connected back to those other perspectives. Thus, both community and institution influence each other through the sharing of values.

Lastly, the perspective of development assistance provides another important aspect. Here development assistance is the collaboration between Japan and developing countries with different cultural backgrounds. What is valued in Japan in terms of history, community and institution is made explicit through comparisons with other countries. What is natural in Japan such as community formation may not be constituted so easily in the developing countries, due to lack of historical conditions and institutions. The intended aim of development assistance may not be achieved by simply introducing the institutions. Here the above three perspectives such as history, community and institution are used to categorize and analyze previous studies.

2.2 Historical Perspective

2.2.1 Background and Objective

Historically speaking, mathematical ideas[4] as a philosophical stance in the national textbook *Jinjo-shogaku-sanjutsu* were used since 1935, preceding the notion of mathematical thinking, which would be developed later on Ueda (2006). This shows that there was a continuing aspiration of the Japanese mathematics education community, despite the temporal mutation during the Second World War, represented by a move towards mathematical thinking. It is possible to identify the roots of what can be observed today through this historical analysis. The objective of this section is to identify the historical development of mathematical thinking as a main focus of mathematics education. Since the period of consideration covers more than a life span, document analysis is employed with special focus on "mathematical thinking".

2.2.2 Emergence of Mathematical Thinking in the Course of Study

Just before the emergence of mathematical thinking, there was a former idea called the 'central concepts'. This term first appeared in the course of study for senior high schools in 1956. The characteristics of this course of study were the integration of Analysis I, Analysis II and Geometry into Mathematics I, Mathematics II and Mathematics III as mathematical subjects. The central concepts exemplified mathematical thinking to bridge all mathematical subjects with algebraic and geometrical contents. For example, the central ideas in Mathematics I were described as follows:

a. Expressing the concepts in symbols;
b. Extending concepts and laws;
c. Systematizing knowledge by deductive reasoning;
d. Grasping relations of correspondence and dependence;
e. Finding out invariance of equations and geometrical figures;
f. (Identifying) Relations between analytical and geometrical methods.

[4]Mathematical ideas are philosophical attitudes to love and enjoy mathematical philosophy in the pursuit and acquisition of mathematical truth, and to find and consider mathematical relations in daily events and to take actions based on them (Shiono, 1970).

Later, the notion of "mathematical thinking" first appeared in the objectives of the 1958 course of study. It is also called "mathematical thinking and mathematical treatment". Here mathematical treatment means to symbolize the patterns after finding out some patterns among the objects and simplify them. The objectives at the time were as follows:

(1) To enable students to understand basic concepts and principles about numbers, quantities and geometrical figures, and to let them develop more advanced mathematical thinking and understand how to treat the objects;
(2) To enable students to acquire basic knowledge and fundamental skills about numbers, quantities and geometrical figures, and to let them use those effectively and efficiently according to the purpose;
(3) To enable students to understand the significance of using mathematical terms and symbols, and to let them use expressions and think simply and clearly in situations involving quantitative events and relations using the terms and symbols;
(4) To enable students to extend the ability to set up an appropriate plan and think logically regarding quantitative events and relations, and to let them treat things more self-dependently and rationally;
(5) To enable students to develop attitudes favorable towards proactive mathematical thinking and understand how to treat the objects in daily life.
(Underlined by the authors.)

Here, mathematical thinking was expected to develop from acquiring fundamental concepts and basic skills to the more advanced level of knowledge and thinking, and to develop the attitudes to apply them extensively to daily-life situations.

The major concern of mathematics education research and practice at that time was the clarification of the definition and concretization through examples of mathematical thinking. Two researches are representative of many efforts by researchers and teachers. Katagiri, Sakurai, Takahashi, and Oshima (1971) published *Mathematical thinking and its instruction* and categorized characteristics of mathematical thinking into three groups: background to produce mathematical thinking, process orientation to form a flow of mathematical thinking, and content aspect of mathematical thinking. They also listed up some concrete examples of each category (addressing extension). Nakajima (1981) published *Mathematics education and mathematical thinking*. He interpreted mathematical thinking as abilities and attitudes of voluntary and creative activity, and sought to clarify its definition (addressing intention) by paying attention to the process of voluntary and creative activity. Katagiri (1988) reorganized the former categorization into mathematical methods, mathematical contents and mathematical attitudes. Later, Katagiri, Nakajima and many other researchers discussed over definition and examples of mathematical thinking and made this concept much richer.

2.2.3 Open Approach: From Evaluation Method to Nurturing Method

While these analytical researches were being developed to provide meaning to the notion of mathematical thinking, research on evaluation of mathematical thinking was developed almost simultaneously. This became "Developmental Research on Evaluation Method of Higher-Order Thinking Skills", which later led to "Proposal of Lesson Improvement by Open-ended Approach" (Becker & Shimada, 1997; Shimada, 1977). This research was funded by the JSPS (Japan Society of Promotion of Science) for six years from 1971. Researchers from NIER[5] (National Institute of Education Research), university researchers, primary, junior and senior secondary schools teachers formed a team to develop evaluation methods of higher-order thinking skills in mathematics education which were interpreted as mathematical thinking.

The team considered that one of the methods could be to give an incomplete problem to students. Here an incomplete problem means the type of problem which allows various answers and/or solutions. They set up a hypothesis that it would be possible to evaluate attainment of higher-order thinking skills from the analysis of the level of solutions and their expressions. The team conducted the research, based on this hypothesis. This developmental study was aimed not only at the development of an evaluation method but also at the development of effective instructional methods to nurture such higher-order thinking skills.

It is significant that the students' multiple solutions were raised and deliberation on them spontaneously occurred in the lesson. This is because it is not enough to have many ideas. It was recognized as necessary to systematize various students' solutions as mathematical activity and theorize them. In other words, the mathematization of the phenomenon was a part of mathematical thinking and it was assumed that the mathematization should not be just one way but allow for many possible ways. And it became necessary to revisit the significance of the diversity of students' solutions. The team understood this as an important research agenda and developed this study as a case of interaction between theory and practice. Gradually, the treatment of multiple solutions has been systematized and theorized.

2.2.4 Treatment and Summarization of Multiple Solutions

The teaching-learning method "open-ended approach" was proposed in 1977. The method, which deals with multiple solutions, has made an impact on textbook

[5]This is the national institute which has been in charge of national curriculum development and education research at the national level. It has been renamed into National Institute of Education Policy and Research but the English abbreviation remain the same as NIER. It is also now in charge of national assessment.

construction. This can be identified in the textbooks, published in 1980 and 1985, containing children's multiple solutions (Ueda et al., 2015). This format continues today.

Multiple solutions in the textbooks promoted research on how to deal with and summarize such various solutions and ideas during the lesson (Koto, 1990, 1998). Koto considered that multiple solutions would lead to the development of mathematical thinking and categorized those solutions into four types: independent, orderable, integrable, and structurable multiple solutions, according to the lesson objective and the relation among these solutions. Koto and others have influenced lesson patterns and made tremendous impact on the structured problem-solving lesson in Japan. These researches are regarded as a culmination of all efforts through lesson study focusing on higher-order thinking, open-ended approach and structured problem-solving.

2.2.5 Summary of the Historical Perspective

This section considered the development of mathematical thinking as an objective in the 1956 course of study through document analysis. It identified the following points:

- Analytical research on the extension and intention of mathematical thinking, development of evaluation methods of higher-order thinking skills and open-ended approach, change in textbooks, and change in teaching approach especially dealing with multiple solutions, are all intertwined in the development of mathematics education. In this development, mathematical thinking plays a central role.
- In the process, many lesson studies have been conducted. In fact, Shimada (1977) contained not only a theoretical framework but also many examples by teachers across the country. All of these are interrelated and have made a tremendous impact on mathematics teachers' shared values in Japan. In this sense, it also indicated how lesson study should be conducted.
- The teachers always consider how to utilize students' various mathematical ideas and solutions and how to let them develop mathematics subjectively and extensively, whether they use the open-ended approach consciously or unconsciously. This attitude has been developed through lesson study and the above historical development.
- This analysis proves that the historical perspective can provide a significant and potential research approach on values, which have been nurtured through the years and have been embedded in lesson study and mathematics lessons.

2.3 Community Perspective

2.3.1 Background and Objective

The historical perspective in the previous section focused on the national curriculum and the national level project, which is named as "Open-Ended Approach". This historical development was substantiated by many teachers and communities at the prefectural and municipal levels. It would not have made any impact on mathematics lessons through the country if it had remained only at the national level. There was no systematic research on the activities of teachers' community, but it is still possible to directly interview the community leaders of the critical period and to identify what happened from their perspective. So the objective of this section is to identify how the community of mathematics teachers at the local level perceived the national level movement. As for methodology, we take an interview approach to ask those leaders to reflect on how they thought and what action they had.

Here the Saitama prefecture[6] was chosen as a case because of two reasons. The first is that the community of mathematics teachers has been active for many years to the present time and the community leaders at that time could be approached. The second reason is the representativeness as Saitama is close to Tokyo but relatively quiet. This is regarded as a standard case.

Two types of the teachers' community here are considered, formal and informal. The formal community is organized within a school or between schools and supported by the Ministry, prefectural and/or municipal board of education. There is also an informal and voluntary community of teachers. In describing the case, the focus was on what measures the prefectural board of education took and how a formal and an informal community of teachers were formed in correspondence with the movement at the national level.

The target period is between the late 1960s and early 1970s, during which the modernization curriculum was introduced. It was the critical period to understand the characteristics of mathematics education in Japan. As mentioned in the previous section, this was the time when the higher-order thinking skills and the open-ended approaches were being developed. In this sense, the today's mathematics lesson has started around this time.

[6]Prefecture is an administrative unit in Japan. The whole country is divided into 47 prefectures. Saitama prefecture is adjacent to Tokyo. During the above period it was not yet urbanized so much.

2.3.2 Long-Term Training System[7]

After the Sputnik shock in 1957, it became a matter of concern to introduce new mathematical contents such as set theory and functions. In Saitama at that time, there was an urgency to develop an in-service training program on such newly introduced contents. In 1964, the Saitama Prefectural Board of Education started a new in-service teacher training system, in which the trainees were to learn these new contents. Ogawa Tadashi was the first long-term trainee, who was sent for one year to Prof. Wada at Tokyo University of Education. His research theme was "Teaching approach for introducing the idea of set". By that time, Ogawa had never taught sets. The interview in this section is directed at the seven leading teachers, including Ogawa.

Since then, the Saitama Board of Education selected a teacher for long-term training. The selection was done strategically from each area in turn so that he/she was expected to lead an in-service training course in the respective area. In this way, the modernization curriculum necessitated the system of long-term training courses and the formation of core members in the teachers' community.

2.3.3 Short-Term Training Course and Edition of Lesson Study Textbook

The course of study on modern mathematics was released in 1968 (primary), 1969 (junior secondary) and 1970 (senior secondary). Each prefecture started systematic teacher training courses. In Saitama, the training course started with the idea that "lessons matter most" and took the form of developing a lesson. The participants of the long-term training courses led these courses. This emphasis on lessons integrated developing good lessons and growing teachers' competence through the lesson study in-service training.

The lesson study was often conducted on Saturdays. Back then schools used to be open half-day on Saturdays. So the research lesson, which was conducted in the morning, was organized by the formal community. The participants of the long-term training as well as university professors participated as advisors in some cases. This lesson study was authorized by the prefectural and municipal board of education and thus was called a formal lesson study. In the afternoon of the same day, the research lesson conducted in cooperation with schools, parents and students, was organized by an informal community and was called informal lesson study. For example, Prof. Wada organized an informal study group of mathematics

[7]"Long-term training" means one year training. Many in-service training courses last only a few days or a week. So a one year training is called long term.

teachers and later Prof. Kikuchi in Saitama University developed it to an informal community at the prefectural level. The teachers who were indifferent at the beginning could have had a chance of participating in a research lesson and later become interested in the long-term training. All of this helped community formation.

Another measure which played an important role in modernizing the curriculum, was the development of training materials since 1970. The materials have been mainly edited by the long-term training participants and are being published even now. They were used to convey new mathematical contents to teachers across the Saitama prefecture so that they would learn how to teach the new contents effectively. The materials are called the "references for a post-lesson conference of mathematics lessons".

After publication, this lesson-based training course became popular. Teachers in each area gradually understood how the contents were introduced in the lesson. For example, in Higashi-Matsuyama City, Mr. Ogawa in Minami Junior High School and Mr. Ono in Matsuyama Junior High School collaborated to develop research lessons on new contents, with slogan of "lesson matters most". Teachers in the school as well as teachers in nearby schools participated the school-based lesson study. The Board of Education supported participation by the teachers in the nearby schools if it was part of a formal in-service training. They started to participate in formal lesson study and later also in informal lesson study.

2.3.4 Formation of Prefectural Level and Municipal Level Community

The Saitama Board of Education strategically sent teachers to the training course, as mentioned previously, and this made an impact throughout the prefecture. Later, all of the participants formed a community at prefectural level and simultaneously started a community at each area. The prefectural level community led mathematics education across the prefecture and developed an association of the long-term training course participants. The association holds an annual meeting and edits training materials. Its secretariat invites some promising teachers across the prefecture as the associate editors.

Both the teachers who participated the long-term training and those who worked as the associate editors, voluntarily led the formation of a teachers' community at the respective area. On one hand, these municipal level communities played a role in initiating an in-service training at each area, and, on the other hand, the members of these communities became members of the prefectural level community and supported the in-service training at the prefectural level. In this way, the communities at the prefectural and municipal levels promoted the whole training system.

2.3.5 Summary of Community Perspective: Values of Teachers' Community

Through the interviews, the following points are identified for the case of the mathematics teacher's communities at the Saitama prefecture which are in correspondence with the national level movement:

- The slogan "lesson matters most", which is set by the communities, represents the situation where lesson study has become integrated into daily teaching. Formal and informal communities were created through lesson study and, inversely, these communities developed lesson study.
- One of the notable characteristics at the Saitama prefecture is the blurred distinction between formal and informal communities. For example, the research lesson in the morning session was a formal one and that in the afternoon session was an informal one. Most teachers participated in both sessions. Such blurredness is related to the fact that the members of each community overlapped considerably and spent much time on *kyozai-kenkyu*[8] even if they were not officially requested. The efforts were supported and appreciated within an informal community. This made an impact on the values of the mathematics teachers who conducted the mathematics lesson study.
- It is notable that the in-service training, which started at the time of modernization curriculum, made an impact on mathematics education later on. During the period, the research lesson focus shifted from "contents" to "methods". Mathematical thinking was placed in the core of the course of study. Structured problem-solving and an open approach can be a necessary part of the lesson by these communities. Besides, the product of in-service training courses during the modernization period was shared and utilized in growing the next generation of teachers. This way, the values were handed down to the next generation within the community.

2.4 Institutional Perspective

2.4.1 Background and Objective

In Japan, many institutions were historically developed to make the whole mathematics education function smoothly. Some examples are curriculum, teacher education, in-service training, and staff management systems. As shown in the previous sections, the development of mathematical thinking and of mathematical

[8]The literal translation is "study on teaching-learning materials". However, it encompasses all educational aspects of the materials such as objective, questioning, possible learning paths, possible misconceptions, facilitation and so on.

teachers' community, as well as scrutiny of the mathematics curriculum and lessons reveal efforts by many people in the past. As a result, the institution reflects some shared values of the teachers and communities. The institutional perspective can be regarded as providing a cross-sectional view of the historical development in the preceding section.

The institution forms the foundation of lesson study in Japan. In the lesson study, the teachers become reflective through the comments from their colleagues. These colleagues are not only those within the school but also those who work in other schools in the municipalities and/or the prefectures. The collegiality and community become possible due to the transferring system of teachers, which reassigns teachers to different schools regularly. The lesson study is configurated on the basis of all these institutions.

Thus, the objective of this section is to clarify the institutional characteristics and the shared values or views through a questionnaire given to teachers. It is possible to describe the institution based on the documents but it is not easy to reveal the impact of the institution on mathematics education. The survey method was adopted to clarify the characteristics of shared views on professional growth and roles of different institutions among Japanese practicing teachers by comparing their responses with those of other groups of teachers.

2.4.2 Views on Mathematics Teachers' Growth

In Japan, teachers are required by law to conduct research and self-improvement efforts and are ensured of having an opportunity of in-service training. There are in-service training programs at national, prefectural and municipal levels. The training by the Board of Education at prefectural and municipal levels is classified as based on length of teaching service or for developing particular knowledge and skills of mathematics teaching (MEXT, 2016). The teachers acquire and maintain skills as well as knowledge and values through these in-service training and everyday practices. Besides, the regular change of the grades and the regular transfer of teachers among schools have an impact on how the teachers see children. Knowing the children in different grades and different school environments enable the teachers to grasp the education more holistically and thus widens their views on professional growth. In this section, teachers' views on professional growth refer to those on the growth of competence to teach mathematics.

Goldsmith, Doerr, and Lewis (2014) summarized six learning categories based on the models of teacher's growth by Clarke and Hollingsworth (2002). The "learning include changes in knowledge, changes in practice (it includes both classroom practice and out of school practice), and changes in disposition or belief which can influence knowledge and practice" (pp. 6–7). The categories are: teachers' identity, beliefs and disposition, teachers' instructional practice, teachers' collaboration/community, attention to student thinking, mathematics content knowledge, and curriculum and instructional tasks.

These values and views on professional growth are held by individual teachers and may vary from teacher to teacher. In this section, teachers' views on professional growth are identified by using the categories of Goldsmith et al. (2014). Ten items (A–J) were created by adding another item "J. Others". The respondents were expected to answer each item by self-evaluation according to the degree of importance. The answer is responded in the four levels of a Likert scale.

A. Views on mathematics
B. Views on mathematics instruction
C. Views on students' ideas
D. Mathematics lesson and instruction
E. Teachers' collaboration/community
F. Mathematics content knowledge
G. Curriculum and instructional tasks
H. Knowledge on teaching-learning materials
I. Research mind and inquiry attitudes

2.4.3 Improvement of Professional Competence

In this sub-section, results of a questionnaire on the above items are presented. In order to identify the teachers' views, two groups of Japanese teachers and a group of American practicing teachers were given a questionnaire. The first group, which was the Japanese practicing teachers, consisted of seven students in a professional development school, four long-term trainees, one teacher who completed a graduate school, and one teacher from a university affiliated school. The second Japanese group consisted of 19 Year-4 students: five who wished to be prospective primary school teachers, five for junior high school, four for senior high school, and five students who wished to be public servants (and who would not be teachers). The third group, the American practicing teachers, who had participated on lesson study, answered the questionnaire through the internet.[9] They consisted of 11 primary teachers, five middle school teachers, two junior high teachers, and one senior high school teacher.

The sum of the scores was calculated for each item and the percentage was then calculated by dividing it by the maximum score (i.e., (the number of respondents in the group) $\times$ 4). Among the ten items, "C. Views on students' ideas" got the highest value among the three groups of teachers. This meant that all groups commonly chose this item as the highest importance. The Japanese practicing teachers valued "B. Views on mathematics instruction", "D. Mathematics lesson and instruction", "F. Mathematics content knowledge" and "H. Knowledge on teaching learning materials" as the second importance. The "I. Research mind and

[9]The questionnaire was administered with help of Prof. Tad Watanabe.

Table 2.1 Top three important items in mathematics instruction

Item	Japanese practicing teachers	Japanese prospective teachers	USA practicing teachers
A. Views on mathematics	5	16	5
B. Views on mathematics instruction	57	22	0
C. Views on students' ideas	32	66	35
D. Mathematics lesson and instruction	28	39	31
E. Collaboration with colleagues	0	0	23
F. Mathematics content knowledge	8	15	17
G. Curriculum knowledge	0	3	12
H. Knowledge on teaching-learning materials	11	11	22
I. Research mind and inquiry attitudes	12	8	17
J. Others	0	0	18

Note Regarding the American teachers, the data is available only for 13 respondents, due to possible misinterpretation

inquiry attitudes" had some difference between Japan and USA, with higher value for the Japanese teachers.

In addition, the respondents were asked to choose the top three priority items. For each item, the sum of the scores was calculated by converting the "top priority" to 5 points, "second priority" to 3 points, and "third priority" to 1 point. The percentage was then calculated by dividing it by the maximum score (i.e., (the number of respondents in the group) $\times$ 5). Table 2.1 shows that the Japanese practicing teachers regarded "B. Views on mathematics instruction" as the highest priority, and "C. Views on students' ideas" as second. Both the Japanese prospective teachers and the American practicing teachers regarded "C. Views on students' ideas" as the highest and "D. Mathematics lesson and instruction" as the second. In comparison with the Japanese practicing teachers, the American teachers valued "B. Views on mathematics instruction" as low priority. The item "E. Collaboration with colleagues" was not recognized by the Japanese practicing teachers but was highly recognized by the American teachers.

2.4.4 Interpretation of the Above Results

The Japanese practicing teachers and Japanese prospective teachers regarded "C. Views on students' ideas" and "B. Views on mathematics instruction" as the most

important items. This suggests that they hold similar values, however there are some differences. The practicing teachers regarded more items as important than the prospective teachers did. This result showed that practicing teachers tend to acquire wider views through teaching experience and in-service training. Another difference is that "B. Views on mathematics instruction" was not very high at the preparatory stage of teacher training but became very high at the later stage. This is a unique characteristic of Japanese teachers' views on professional growth.

From the results of the prospective teachers, the values embedded in Japanese lesson study seem to start growing even during the preparatory stage. They regarded "C. Views on students' ideas" as the most important. It can be interpreted that they received lessons and practices which emphasize this point. For them, "D. Mathematics lesson and instruction" were also important. The prospective teachers wrote that they wanted to observe other teachers' lessons and implement the lesson effectively.

In summary, the Japanese practicing teachers hold wider views on instruction and teaching learning materials than the prospective teachers who tend to pay more attention to the practical part of the lesson. These results imply that the values that Japanese practicing teachers attach to the in-service training go beyond practical advancement of know-how and enhance views on instruction which support their practice.

2.4.5 Summary of Institutional Perspective

Through the questionnaires on the views of professional growth (above) and roles of different institutions, the following points are identified for institutional perspectives:

- In-service training opportunities influence the formation of values and views on teachers' professional development. A questionnaire confirmed that there are various types of training and the roles of the board of education and the educational center are crucial. Almost all Japanese practicing teachers mentioned the importance of conducting/observing lessons and this was consistent throughout. The American practicing teachers mentioned not only working around lessons but also different opportunities, such as learning new trends of curriculum or collaboration with outside teachers.
- The Japanese respondents tended to answer that they participated in in-service training only when they were in charge of lesson study in their schools. They voiced that it is not easy to participate in training unless they have more free time. Currently they are expected to play many roles in their schools and their learning opportunities are incorporated into such roles.
- The above point is regarded as a "systemic change", which is an Eastern perspective in the discussion on cultural differences in teachers' professionalism. Kaiser and Li (2011) state that "While in Eastern conception a change on a

systemic level is desirable, the Western conception refers to changes on the local level" (p. 349). While Western views of professionalism are limited to the interaction with students in the classroom, Eastern views grasp teachers' professionalism holistically by including role of the teacher as a researcher and a curriculum developer. The views imply that one of the Japanese teachers' tasks is to look for training opportunities to meet their individual needs and interests rather than to solely follow their duty and responsibility.

2.5 Development Assistance Perspective

2.5.1 Background and Objective

In the previous sections, mathematics education in Japan has been analyzed from the perspectives of history, community and institution. It is not easy to describe in a complete way the mathematics education in a country. So the fourth perspective is to provide a reflective description of such characteristics, through analyzing previous studies and comparing different education systems.

Isoda, Stephens, Ohara, and Miyakawa (2007) stated that there is a peculiar situation in Japan which supports the implementation of lesson study, and this situation has been historically formed. The Japanese government assists developing countries in Asia and Africa economically and technologically, and Japanese scholars and teachers (hereafter scholars) are involved in such an endeavor. Since the developmental assistance is a joint endeavor between the donor and recipient countries, the scholars who are involved in such an endeavor encounter different cultures and societies. Through realizing those differences, they also "encounter" their own society and culture as well.

In order to identify those differences in terms of three perspectives and the values embedded in them, the research method is the secondary analysis of research articles, which were written by different scholars involved in the developmental assistance. The assumption is that these scholars have internalized the values of mathematics education in Japan and those values become explicit when they encounter different cultures and values.

2.5.2 Analysis of Previous Studies

The previous studies were collected by the keywords "mathematics education" and "international (cooperation)"[10] on the site (CiNii), and 77 articles were identified. Five more articles were added onto the list later on. All of these were categorized

[10]Development assistance is commonly called an international cooperation in the Japanese context.

into the international cooperation (38 articles), the international comparative studies (16 articles), overseas field survey (20 articles), and others (8 articles). The target of analysis is only the category of international cooperation. These articles were obtained electrically except five articles which were unavailable.

Analysis from three perspectives

① History

Regarding history, we discuss two cases from Africa, Kenya and Ghana. Since they were both UK colonies, some similarities are expected such as the introduction of modernization curriculum and use of English as a medium of instruction.

Baba (2002) stated that Kenya used to have four different syllabuses according to different African, Arabic, Asian and European races. After independence, they unified those into one syllabus based on the African one. Later, it was revised into Kenya Primary Mathematics at the primary level and emphasized sets and topology under the influence of the modernization curriculum. In this sense, the government tried to balance local and international contexts. On the other hand, the medium of instruction has continued to be English, even after political independence. This is because there are many local languages and it was not easy to choose one language among them. Yoshida (2000) presented the case of Ghana. A prominent feature is that the textbooks there also contained a reminiscence of modernization in the 1970s containing sets and mappings.

In these papers (Baba, 2002; Yoshida, 2000), the Japanese case was also mentioned in comparison. Yoshida (1998, 2000) pointed out the importance of reinterpreting the modernization process of education and the adaptation of Western mathematics from the Meiji restoration in Japan in relation with the developing countries. As for the language when adapting Western mathematics, it is translated into Japanese language. Referring to Ghanaian case, Yoshida stated that our education largely owes to the efforts of great ancestors. Baba (2004) pointed out that there are two types of outputs from the endeavor of developmental assistance. One type is explicit and the other is implicit. He suggested that the latter is more important and is related to the values of the culture, which underlie the explicit outputs.

② Community

The notion of "community of practice" is being appreciated and experimented in some countries. In many cases, however, sustainability is an issue. On the other hand, the communities of this kind are prevalent all over Japan. As discussed previously, the teachers' community in Japan does not only refer to colleagues within a particular school but also to teachers in nearby schools, teachers within the prefecture and across Japan, and furthermore teachers of the previous generation. This was manifested in the Saitama case. Thus, the notion of community in Japan has characteristics of time-wise and place-wise connectivity. Through this connectivity, the values have been shared by the members and the community has become more solid. In this sense, sustainability does not matter in Japan. One of the

important shared values can be expressed by the word "観" (kan, the literal translation meaning "views"), representing a particular way of looking at the lesson.

In Japan, the attitudes and the way of looking at the teaching materials and lessons have been nurtured and shared within the group. The word "kan", used as lesson "kan" and teaching material "kan", indicates a way of looking at lessons and teaching materials. This is namely a cultural way of interpretation below the surface (Baba, 2014).

When Japanese scholars are engaged with development assistance, they focus on how the lack of such community causes difficulty in organizing any activity. So as a first step, it was regarded as important to form such a community in those countries.

Nagao (2003) mentioned patterns of mathematics lessons in Ghana: basic calculation exercises, explanation of key terms and definitions, demonstration of solution of today's problem, demonstration of solutions of similar problems, exercises and homework. Such patterns may have been acquired through their own education and teacher education system. If the patterns are to be changed, there must be a provision of opportunity in which the teachers are made to reflect on their practices and patterns. Community can play an important role in providing such an opportunity. Okubo and Tsuji (2005) observed and stated that teachers in Egypt might have a difficulty in introducing new views and methods and developing their capacities. While considering it, they reflect that most Japanese teachers have developed the capacity to overcome shortcomings of views and methods through participating in lesson study.

These reflections are made by Japanese scholars who were involved in the international cooperation endeavor. On the other hand, it is difficult for the scholars in those countries to become conscious about non-existence of such communities. It may be similar to the concept of "0". While Isoda (2010) discussed the issues of development assistance, one critical point is the difficulty of being properly able to appreciate the quality of lesson study and the scarcity of experts who can explain such quality.

Baba (2014) pointed out that in the developing countries the fragility of the professional foundation is related to the non-existence of academic associations, open research lessons, and so on. While these difficulties exist, school based in-service training and lesson study can serve as a first step to providing an opportunity of reflection over lessons. If these opportunities would function well, the participants should have common ways of looking at lessons and shared values for quality of lessons.

③ Institution

Many articles (Baba, 2002; Einaga, 2000; Nisikata & Nakajo, 2014; Watanabe, 2000) have stated that developing countries have a priority in industry development and thus have strong interests in mathematics and science education. For this purpose, it is necessary to improve an intended curriculum, the teacher education system (both pre-service and in-service) as well as the examination system.

As for the curriculum improvement, some countries still have an influence of modernization curriculum. At the same time, they have started discussing more advanced competences such as 21st century skills. While it still has a traditional teaching style, which has an examination orientation, an important task for those countries is to overcome the traditional teaching style and realize a new style of lessons to promote these advanced competences.

As for the teacher education, many scholars pointed out the weak capacity of teachers in the developing countries. Teachers in Laos are not conscious about the above trend of new abilities such as logical thinking and application to daily life (Akita & Saito, 2008). Ghanaian teachers attribute problems to the lack of teaching materials, but in reality they do not have enough capacity to utilize the teaching materials even if they are available (Nagao, 2003). This lack of capacity is caused by teacher education, which does not pay attention to such utilization. The teacher education system and environment in Japan are well developed to support lesson study practice and teachers' professional growth. For example, Japanese teachers conduct lesson study and open lesson within their working hours. They base their research lesson on the national curriculum and views on mathematics education and students (Isoda, 2010).

As for the examination and assessment system, in Ghana there is a national assessment system but the results are not utilized fully for the improvement of curriculum and lessons (Yoshida, 2000). The introduction of a new approach for new competences may be hindered or promoted by the examinations.

2.5.3 Summary of Developmental Assistance Perspective

Through the secondary analysis of academic papers, the following points are identified regarding developmental assistance perspectives:

- The three perspectives of Japan and of developing countries were discussed in the context of developmental assistance. The perspectives become explicit through interacting with the countries of different cultures and histories. They have been implicit in Japan and this may be one of the reasons why few articles on lesson study are found in this country. This fact itself may be indirect proof of how much we have internalized these in ourselves.
- Baba (2001) pointed out that reviewing the history of Japan and what to do in developmental assistance prompts us to think about the position of mathematics education in Japan in terms of the world trends. Yoshida (2000) also held the same view that revisiting the experiences in the process of modernizing education and thinking about what is useful for developing countries may provide universalization of Japanese peculiarity within the worldwide framework.
- The Japanese government values the principle of "self-help" in development assistance. It regards that the self-help is a crystallization of the modernization efforts from Meiji till today, and thus is a catalysis for developing countries to

engage in such modernization efforts. The above three perspectives are closely connected to self-help within the mathematics education development.

- Developing countries borrowed models from the colonial governments even after independence. Though Japan has not been colonized, it adapted the Western mathematics and education system into a traditional education practice. Of course they cannot be made exactly the same way (Baba, 2002). Model borrowing shows the externality of professionalism and gives rationales to the need for the formation of professional knowledge and a professional group for curriculum development (Baba, 2014). While in this situation, lesson study holds such characteristics as closeness to practice (Ruthven & Goodchild, 2008) and it can provide an alternative model to self-help.

2.6 Summary

Cultural characteristics of lesson study in mathematics may remain implicit as long as one stays inside the culture. Thus a comparative approach was employed to reveal these implicit characteristics. Three perspectives such as history, community and institution were used to shed light on cultural characteristics of mathematics education and lesson study in Japan.

First, as for comparative perspective in the development assistance, several developing countries were chosen as an object of comparison. Because they do not have some aspects, which are very obvious for us in Japan, we become conscious of the comparison of those implicitly "existing" aspects. On the other hand, if the developed countries are chosen as an object of comparison, then the comparison makes us find some aspects "not existing" in Japan. So we can learn from them. As for the historical perspective, we need to return to the root of those practices, when they were newly created, with a combination of development assistance and historical approach. Here explicit and implicit problems can be traced back to historical development. The third is to pay attention to the reason why those implicit features are being valued. This is related to the second issue. Tracing back to the origin gives us the reason behind the creation of such practices. This makes us conscious about the reason of creation and provides us with an opportunity to revisit whether it is still relevant to the present day or to other contexts.

In the above discussion, lesson study by the community with acquisition of voluntary and professional learning is an important and ultimate goal. Can such an ultimate goal be achieved simply through the introduction of a particular institution? Once an institution is established, its reason does not have to be questioned. Lesson study may sometimes look very superficial even in Japan if we only pay attention to the institutional aspect. Thus, institution is a necessary but not sufficient condition for the implementation of lesson study. If it is not accompanied by voluntary learning, it may remain an empty promise.

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Chapter 3
Promoting and Implementing Lesson Study in Malaysia: Issue of Sustainability

Chap Sam Lim, Kim Hong Teh and Chin Mon Chiew

Abstract This chapter discusses the experiences of promoting and implementing lesson study, with their related issues and challenges encountered in Malaysian schools. The Japanese model of lesson study was first introduced in 2004 to Malaysian mathematics teachers as small scale research projects and postgraduate student dissertations. It was in 2011 that lesson study gained the interest and attention of the Malaysian Ministry of Education (MOE) The Teacher Education Division of the MOE implemented professional learning communities and lesson study was identified as the core collaborative approach to improve teaching and learning. Although some Malaysian schools have been introduced to lesson study since 2004, there was a lack of tangible reports regarding the impact and success of lesson study implemented. Unofficial reports revealed that very few schools were able to sustain the lesson study practices. Thus, in this chapter, we highlighted the issue of sustainability, and analysed the possible challenges and constraints faced in an effort to sustain the lesson study practice. Based on our own experiences and reviews of related literatures, we proposed some feasible suggestions on how to sustain lesson study in schools, particularly in Malaysia and also schools in other parts of the world that face similar predicaments as Malaysia.

Keywords Lesson study · Malaysian studies · Sustainability · Issues and challenges · Promoting and implementing lesson study

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3.1 Introduction

As in many other countries outside of Japan that have adopted and implemented lesson study, sustainability remains a great issue to be resolved in Malaysia. Due to cultural and contextual factors, lesson study continuously faces inevitable challenges in Malaysia and probably needs modifications to suit local needs and context. As elaborated by Doig and Groves (2011), lesson study offers the potential for sustainable professional development because it provides teachers the opportunity to enquire, share, reflect and develop professionally and more importantly, get a sense of ownership of the improvement effort among colleagues. The principles of lesson study are also consistent to the contemporary model of effective teacher professional development in literatures. In our views, lesson study ought to be implemented and sustained due to its vast long-term benefits. It is hoped that analysing and discussing the constraints from our own experiences would shed light on some effective suggestions to implement lesson study in Malaysia.

This chapter begins with a brief history of how lesson study was introduced and promoted in Malaysia since 2004. Our discussion then focuses on the issue of sustainability, and analysing the possible constraints and challenges faced in an effort to sustain lesson study practice. Based on our own experiences and reviews from related literatures, we hope to provide some suggestions on how to sustain lesson study in schools, particularly in Malaysia and also schools from other parts of the world that face similar predicament as Malaysia.

3.2 A Brief History of Promoting Lesson Study in Malaysia

In 2004, lesson study was introduced in Malaysia for the first time as a research project and postgraduate study in two secondary schools in the district of Kulim, Kedah (Chiew, 2009; Chiew & Lim, 2005; Lim, White, & Chiew, 2005). The objective of the research was to investigate and explore the viability of lesson study as an innovative model of professional development for Malaysian mathematics teachers. Each school that engaged in this research formed a lesson study team consisting of eight mathematics teachers respectively, after obtaining consent from the school administrators. This lesson study project started off with an introductory lesson study workshop conducted by Professor Allan White from Australia. Professor White was also the head of evaluation of a lesson study project in New South Wales that involved 103 schools in 2001 (see White & Southwell, 2003). Over a period of one year on the lesson study project, the first school managed to carry out three lesson study cycles while the second school completed only two lesson study cycles. The findings from this study were indeed encouraging despite

the challenges faced by the researchers throughout the study. Overall, the participants gave positive feedback and encouraging responses about the lesson study process. They were in unison that lesson study had provided them the opportunity to engage in collaborative work and further enhance the professional relationship between colleagues; helping fellow teachers to enhance their pedagogical content knowledge through group discussions, self-reflection and peer teaching observations. The systematic process of lesson study enables the teachers to prepare explicit, well-focused lesson plans that are supposedly regarded as effective in enhancing students' learning (Lewis, 2002).

In view of the apparent impact of lesson study in promoting teacher collaboration and engaging good teaching practices, two more schools, one primary and one secondary, were engaged in two lesson study projects in 2006. The primary school group was led by the assistant principal, also a graduate student under the supervision of the first author, who was focusing on lesson study as her research study (see Goh, 2007). The secondary school group was headed by an *Excellent Teacher* who was also the head of the mathematics department of the school. In Malaysian context, an *Excellent teacher* is a promotional position for a teacher who shows extensive knowledge, skills, expertise; one who is dedicated and motivated to lead and carry out the duties and responsibilities of teaching and learning (Malaysian Ministry of Education, n.d.; Lim & Kor, 2012). Both of these teachers were very positive, passionate and keen on promoting lesson study. Being senior in teaching experience and playing the role of the group leader had probably and indirectly exerted some influences that contributed to their success and sustainability of the lesson study projects. They also shared much of their experiences about lesson study in international research conferences (Goh, Tan, & Lim, 2007). It is worthy to note that lesson study in Malaysia was introduced, initiated and promoted by researchers and teachers who were passionate about improving teaching, and also by some teachers who felt compelled to play active roles in promoting lesson study. This was inevitable for the latter when the understanding and knowledge of the lesson study processes among the participating teachers were still not well understood.

The positive feedback garnered from teachers involved in the lesson study projects had boosted and motivated the researchers to widen and encourage more participation and this has even encompassed the pre-service mathematics teachers (Lim, 2006). The spirit of collaboration and cooperation instilled through the lesson study process is viewed as an important factor to be cultivated among the pre-service teachers who could deepen their pedagogical content knowledge of mathematics. From 2006 to 2008, the concept and process of lesson study was expanded to pre-service mathematics teachers during their teaching practicum.

With a grant from Universiti Sains Malaysia (USM) during 2007–2008, a research team comprising of five USM researchers and three mathematics teachers from three secondary schools was initiated. Each school teacher led a lesson study

team in his/her home school to investigate and promote the application of *Geometer's Sketchpad* in mathematics teaching. At the end of the study, each lesson study group succeeded in encouraging and promoting the use of *Geometer's Sketchpad* among teachers and students who took part in the research study. However, the level of achievement for each group differed due to factors such as time constraint, school administrators' support, teachers' commitment and attitude. Based on that study, a book entitled 'Innovative use of Geometer's Sketchpad through Lesson Study Collaboration' (Lim & Kor, 2010) was published. In addition, several related papers were also presented at national and international conferences to share the lesson study experiences (see Kor, Tan, & Lim, 2009; Lim, Chew, Chiew, & Goh, 2009).

In 2008, a lesson study project on a larger scale was collaborated between the Southeast Asian Ministers of Education Organisation-Regional Centre for Education in Science and Mathematics (SEAMEO-RECSAM) and USM. The project involved 10 primary schools in Penang state. The main objective of this research project was to promote mathematical thinking and communication in teaching through lesson study collaboration among mathematics teachers. A monograph was subsequently produced based on the findings (refer Lim, Chiew, & Chew, 2011). In addition, a joint national lesson study conference was organised by USM and SEAMEO-RECSAM to share the lesson study experiences and it attracted a participation of more than 100 school teachers.

There was a significant development for lesson study in 2011. In searching for an effective programme to enhance teacher quality in teaching practices, the Teacher Education Division under the MOE officially adopted Lesson Study as a programme in selected schools. This nationwide project involved 289 primary and secondary schools that were identified as low performing schools based on academic achievement. There were four core subjects involved, namely Mathematics, Science, History and English. The project commenced in mid-April 2011 whereby six lesson study workshops were conducted. The participants for the workshops were the Head of Panel of the respective subject identified and officers from the district education office. Due to logistical constraints, only one subject for each of the 289 schools was accorded. Nevertheless, all of the lesson study group coordinators (teachers who attended the workshop are duly appointed) were encouraged to disseminate the concept and practice of lesson study to other subjects as well in his/her school. Prior to the workshop, a one-day briefing was conducted by the officer in charge of the project to the respective school principals involved and officers from the state education departments. The aim was to inform the principals about the lesson study initiative, as the support from the school administrators is a crucial factor for the success as highlighted in many of the lesson study related research studies (e.g. Gill, Jackson, & Wade, 2005; Liptak, 2005).

The programme was renamed as Professional Learning Communities (PLC) in 2012 when other collaborative tools of Peer Coaching, Learning Walks and Teacher Sharing Session were included alongside Lesson Study. This PLC programme involved 107 low-performing schools in 2012. As the evaluation of PLC by the

Teacher Education Division showed positive feedback, another 300 and 393 schools were engaged in 2013 and 2014 respectively.

Since the beginning of 2015 however, a new modus operandi was introduced by the MOE. As more schools needed to be engaged, the Teacher Education Division realised that it would not be able to cope with the long-term demands with just a mere 25 dedicated PLC master trainers. Consequently, a long-term strategy was launched whereby each district education office was assigned three schools within the district to monitor the implementation of PLC. In other words, the district education officers were empowered to promote lesson study within their capacity and jurisdiction. The duty of the national PLC master trainers was then reduced to merely providing support and consultation whenever the need arose. There were approximately 200 district education offices nationwide, and later on another 600 schools were introduced to lesson study. However, the lack of both experience and comprehensive understanding of the lesson study process among the practitioners (teachers, education officers and trainers) have indirectly contributed to the constraints in sustaining lesson study implementation in Malaysia.

3.3 Issues of Sustainability

Indeed, it may take a long time for the worth and benefits of promoting lesson study in Malaysia to become apparent. From 2003 to 2015 (as shown in Table 3.1), there were only 20 primary and six secondary schools involved in lesson study in the northern zone of Malaysia; these were led by some researchers and institution educators. However, when the Teacher Education Division launched and led the PLC project at the national level, 1089 schools in Malaysia were recruited to carry out lesson study from 2011 to 2014. The increasing number of schools involved was encouraging but sustainability remains the main concern. It was a remarkable start but the number of schools involved seemed to dwindle after a few years. For instance, Rahman and Jeffri (2013) revealed that out of the 289 schools involved in 2011 only 72 schools reported that they continued in 2012, and only 51 of them sustained lesson study in 2013. Nevertheless, the MOE and district education offices have been actively promoting lesson study as school-based teacher professional development in recent years.

To date, there are a couple of schools that are reported to be very active and doing very well in sustaining lesson study. These two schools are one secondary school in Kota Kinabalu, Sabah, and one in Penang state. When viewed on a micro scale, both schools managed to sustain lesson study as one of the professional development programmes, and this was attributed to the strong commitment and passion of the lesson study group leader as well as top-down directive from the district education offices. Besides, preliminary observations have suggested that the school administrative support, in particular from the principal, played a very significant role in supporting and sustaining lesson study practices in the schools.

Table 3.1 List of Lesson Study projects carried out in Malaysia from 2003 to 2015

Projects	No. of schools involved	No. of teachers involved	No. of pre-service teachers involved	Remarks
Conducted by research teams with grants	13 primary schools	51	–	
	3 secondary schools	23		
Conducted by research team without grant	5 primary schools	26	–	
Graduate studies	2 primary schools	13	–	
	3 secondary schools	21		
Pre-service teacher programme	1 secondary school	–	6 (2003)	During teaching practicum
		–	88 (2006)	Final year pedagogy course
	4 primary schools	–	8 (2006)	During teaching practicum
	–	–	94 (2007)	Final year pedagogy course
	–	–	107 (2008)	Final year pedagogy course
	4 primary schools	–	12 (2012–13)	During teaching practicum
Ministry of Education (Teacher Education Division)	289 (2011)	4356	–	90% secondary schools 10% primary schools
	107 (2012)			
	300 (2013)			
	393 (2014)			
Ministry of Education (District Office)	600 (2015)	2400	–	
Symposium, conference and open class on LESSON STUDY	–	570		
Total participants	1724	7465	361	

3.4 Constraints and Challenges in Sustaining Lesson Study Practices

Although there was no empirical data, based on our direct or indirect involvement and observations in these lesson study projects, we summarise the constraints and challenges in sustaining lesson study practices as follows.

3.4.1 Lack of Awareness on the Importance of Professional Development Among Teachers

In Malaysia, much initial effort has been invested by some academics and post-graduate students of local institutions, namely USM and SEAMEO-RECSAM in promoting lesson study among primary and secondary school teachers. Based on the statistics in Table 3.1, for the first 11 years (2004–2014) there were about 26 primary and secondary schools that engaged in lesson study projects supported by some university research grants. Based on our observations and experiences of direct involvement in these lesson study projects, we found that it was always a challenging task to get teachers to participate voluntarily. Most teachers needed some kind of persuasion or mild compulsion from their superiors to agree to participate in lesson study. In general, teachers who participated were identified and put forward by their principals. They were usually experienced and committed teachers who had dutifully carried out their daily teaching chores in school.

We nevertheless noted some teachers who showed reluctance while participating in the lesson study process. Two major factors identified were time constraint and the teachers' workload in school. As quoted by one of the participants:

> I think there will be no problem for lesson study if it is within the school time. If it's after school dismissal, various other things can happen. For some people, when the time comes, they will straight go home because every day we can leave at 2.00 p.m. (Interview: 11/08/04 in Chiew, 2009)

In the Malaysian school context, teachers are usually dismissed together with students unless there are additional activities that required them to stay back in school. Besides, teachers' workloads are heavy as working hours are confined to 7.30 a.m. to 2.00 p.m. There is very little room and opportunity for teachers to work together and collaborate as needed in the lesson study process. Another participant explained:

> First, the time. We don't have time because we have other things to do. Next, the workload, a lot of things to do. Sometimes when you come here for two hours of discussion, other teachers have something else to do. So, also cannot carry out lesson study and this is the problem. (Interview: 27/04/05 in Chiew, 2009)

However, even though the participants were aware of the long-term benefits of lesson study, not many were willing to commit and spend the time for the sake of their professional development.

In Malaysia, there is a common perception that students' excellent performance is indicated by their public examination results, with little regard to the actual happenings of the teaching and learning process in the classroom. In short, it is an exam-oriented education system. Generally, teachers feel they have to shoulder the burden of the considerable bulk of the teaching load and administrative chores. This heavy workload spares them little time in school to participate in any professional development activities. Under such circumstances, it is indeed not surprising that very few teachers would voluntarily participate in lesson study when approached by

the researchers. Nonetheless, due to the top-down directives or an obligation to maintain an expected status of school image, many teachers participate in lesson study with a tinge of unwillingness and without much intrinsic motivation.

It was evidenced in Chiew's study (2009) that the participating teachers, whether experienced or novice, acknowledged that lesson study practices influenced and improved their teaching practices, although they were skeptical of the engagement and sustained lesson study in the long-term due to heavy workloads and time constraints. One of the participants espoused:

> I think it helps. It helps to give ideas to all teachers especially for the novice teachers. This idea comes from five or six teachers, sure it's better than one. So, for me in the future, if we have something like this, for new teachers it's a very great advantage but for the senior teachers, I think it is also good. (Interview: 27/10/04 in Chiew, 2009)

The process of lesson study also encouraged the participants to reflect on their teaching weaknesses. A participant explained:

> So far when we teach any topics, we assume that we got the best way. We assume our students can understand it. Actually, when we discuss, there are so many shortcomings. Sometimes, we are also not confident with our content knowledge. We do not know a lot of things. (Interview: 27/04/05 in Chiew, 2009)

However, as alerted by Takahashi (2011) the impact of lesson study could only be visible after participating in lesson study over a period of time. Although the changes were small and gradual, the participating teachers would slowly accept the idea of lesson study. In brief, although time constraints and teachers' heavy workloads in school were cited in studies of lesson study in Malaysia (e.g. Chiew, 2009; Goh, Tan, & Lim, 2007), the lack of awareness on the importance of professional development in the teaching profession remains the main impeding factor (Chiew, 2009).

3.4.2 Lack of Experienced Lesson Study Practitioners to Play the Role of Knowledgeable Others

The successful implementation of lesson study needs quality support from a pool of experienced lesson study practitioners who can also play the role of knowledgeable others during the lesson study process. Yet, as alerted by Takahashi (2011) there was still "a lack of experienced lesson study practitioners outside Japan" (p. 80). This is indeed true particularly in the Malaysian context. As mentioned in the earlier part of this chapter, lesson study was introduced to the Malaysian school teachers in 2004 as a series of small grant research projects and postgraduate student dissertations. The researchers who initiated and carried out lesson study projects had no first-hand experience or professional training in lesson study. They were merely acquiring the knowledge of lesson study from reading the websites relating to lesson study, attending international conferences and observing open lessons (in

Japan). Thus, one of the major constraints faced was the lack of experienced lesson study practitioners to play the role of knowledgeable others.

We acknowledge that knowledgeable others play extremely important roles (Wang-Iverson & Yoshida, 2005) to incite, provoke and improve teachers' reflection when participating in the post-lesson discussion. The discussion was aimed to generate and enhance the content and pedagogical knowledge of teachers through deep and active self-reflection. However, many of the appointed master trainers who played the role of knowledgeable others (particularly in the MOE Lesson Study projects) were merely facilitating the post-lesson discussion without prior training as knowledgeable others. From our observations, due to lack of knowledge and inexperience of the appointed master trainers, the superficial discussions had resulted in little or minimal impact of lesson study upon the participating teachers.

Therefore, unless the concept and noble idea of lesson study implementation are fully understood and realised by the master trainers and teachers, lesson study would continue to face challenges in its implementation when little impact is realised. This is easily understood as Stigler and Hiebert (1999) proclaimed that lesson study is a cultural activity, and therefore there are always questions and doubts as to what extent lesson study can be replicated elsewhere outside Japan.

3.4.3 Lack of Zest in Lesson Study Implementation

The third challenge that impedes the sustainability of lesson study in Malaysia may be the lack of passion in lesson study implementation. Before setting up lesson study groups in each school, there would always be a three-day workshop designed as a comprehensive package for the participating teachers. The aim of this initial workshop was to introduce the concepts and the process of carrying out lesson study. Some hands-on activities such as collaborative lesson planning, and the observation of effective lessons on video and of making teachers' reflection based on the videos were also included in order for the participants to have a better knowledge of lesson study.

However, we observed that there were some differences between the lesson study groups facilitated by the institution researchers and those by the MOE master trainers. Due to the guidance and commitment of the researchers, the outcome of the lesson study implementation of the former was more favourable than the outcome of their counterparts facilitated by the MOE master trainers. Besides, the lesson study group leaders of the MOE projects who were duly appointed lacked experience in carrying out lesson study and convince his/her colleagues in school of lesson study. As a result, the lesson study implemented in some schools by the Teacher Education Division under the MOE was less encouraging, and was done mainly so that periodical reports of the lesson study implementation could be submitted. Perhaps this was also due to the lack of knowledge, logistical constraints and lack of monitoring by the education authorities.

Due to lack of understanding and appreciation of lesson study, it was observed that some teachers were carrying out lesson study merely for the sake of submitting reports as instructed by their superiors. The third author was actively involved as one of the master trainers of lesson study in the MOE projects. He noted that sometimes lesson plans were planned individually, rather than by teachers discussing and working together as a lesson study group. Consequently, he also observed that some participants were unable to give critical and constructive feedback to the teacher conducting the research lesson, as they were not fully involved in lesson planning from the beginning. Subsequently, the reflection session was rather superficial and improvements to the lesson taught were somehow limited. This pitfall might be the result of the inadequate knowledge of teachers in doing reflections and to clearly identify what to do in each stage of lesson study. There were also cases where the same research lesson was repeated in the same previously taught class just for the sake of demonstration in the presence of the knowledgeable others who were assigned to monitor the lesson study implementation. Thus, the close monitoring of lesson study by the appointed education officers should have been conducted seriously to ensure that the process of lesson study was not superficially implemented.

Besides the logistical constraints, some participating teachers did not fully participate in a complete lesson study cycle, thus further affecting the outcomes of the lesson study. Consequently, it was an expected outcome that the lesson study process produced little impact on the teacher's teaching and on his/her students' learning. As such, lesson study will not be sustainable as a practice among school teachers in Malaysia.

Indeed, the implications would be that the process should be carried out on a volunteer basis, with the Ministry of Education giving positive incentives to participating teachers, and that a lot of investment should be made in preparing leaders for this process, perhaps a task that should be played by the local universities.

3.4.4 Misconceptions About Lesson Study

After its onset by the Teacher Education Division in 2011, lesson study in Malaysia has taken a route that differs from most other countries. Lesson study was not viewed or perceived as an on-going school-based professional development in the teaching profession with the aim of developing and improving teaching practices. Rather, it was highlighted as a strategy to improve the academic achievement of the students in low-performing schools. This made some of the teachers sceptical and question whether engaging in two lesson study cycles was sufficient enough to uplift the examination results in such a short period of time.

As lesson study was only introduced to selected schools by the Teacher Education Division, many teachers at the other schools were not exposed to lesson study. Some teachers and teacher educators may have heard about lesson study, yet they lack the comprehensive knowledge and skills to conduct it. Some viewed the

procedures or steps of lesson study as similar to their practical teaching during the teacher education programme. There were also instances where lesson study was implemented differently or deviated from what was described in the literatures, and thus had been perceived as ineffective by the participating teachers. Due to such misconceptions, many school teachers were hesitant to be involved in lesson study.

Therefore, for lesson study to be an effective continuous professional development for teachers, it should be introduced as a process rather than an event, project or programme as mentioned by Guskey (2002). We believe that the concept of lesson study should be implanted in every teacher's mindset as a part of their continual professional development in teaching. Based on this perspective, the issue of sustainability of lesson study in schools may be addressed in a positive way.

3.5 Suggestions to Sustain Lesson Study in Schools

To sustain lesson study in Malaysian schools, we offer the following suggestions.

3.5.1 Instill Lesson Study as a Culture of Professional Learning for Teachers

In this chapter, we refer lesson study to the Japanese model of school-based lesson study. There are four major steps in this model of lesson study: (i) formulation of common goals related to students' learning and long-term development; (ii) collaboratively planning a research lesson addressing the goals; (iii) one member of the lesson study group teaches the research lesson while the others observe; and (iv) post-lesson meetings where all the lesson study group members and observers (may include outside experts to act as knowledgeable others) discuss and reflect on the observed lesson (Fernandez & Yoshida, 2004; Groves, Doig, Vale, & Widjaja, 2016; Lewis, 2002). Many of these lesson study processes are rather new or are seldom practiced by many Malaysian teachers, either in their daily teaching career or in teacher professional development programmes. Hence, lesson study presents a new perspective and culture for teacher professional development for most Malaysian teachers.

Culture is defined as "a system of shared knowledge, beliefs, procedures, attitudes and artefacts that exists among a group of humans" (Gill, 2013, p. 73). Instilling a new culture means getting a group of people to share a new system of beliefs, values and practices. If there is any conflict between the old and new culture, this will require a change (either adopt or adapt) in beliefs or values. Making a cultural change can be quite challenging, as a person's beliefs and values are usually deep-rooted and cannot be changed immediately. Moreover, one's beliefs and values greatly influence one's practices. Only teachers who believe and

value the benefits of lesson study will be expected to continue the practice of lesson study.

In accordance with the four major steps of lesson study, the Japanese model of lesson study emphasises four key features (Fernandez & Yoshida, 2004): (a) anticipating a pupil's answers; (b) opening up one's teaching for observation, critique and comment; (c) skills of observing and reflecting on the research lesson; (d) student learning is the main focus of lesson planning, observation and reflection instead of the teacher practice. These features are also mostly new or unfamiliar to the local teachers. For example, when planning a lesson, teachers usually plan it from their own views or perspectives of how to teach. Some teachers could not imagine or anticipate how the students will response when a question is asked. One of the participating teachers once argued that "Students can say anything they like! How to guess?"

It is a common practice in Malaysia that observing a teacher's teaching is merely for the purpose of evaluation, such as during teaching practice (in teacher education programme) or for promotion or appraisal purposes (by the seniors or promotion committee). Therefore, most teachers are shy in allowing their classroom teaching to be observed. In general, they are worried and afraid of being critiqued or commented on. However in lesson study implementation, observation is for the sake of professional learning and development, and not an evaluation of personal teaching. Hence, changing teachers' mindsets and perceptions about lesson study observation is pertinent in order to get them to accept the idea of lesson study, and thus be able to sustain their involvement in lesson study. In relation to the focus of observing, the skills of observing and reflecting are also crucial matters. Due to the lack of experience many teacher observers, even the knowledgeable others, may not able to know what to focus on in a research lesson, how to make critiques and comments that focus on student learning and consequently promote self-reflection among the participating teachers.

In brief, it is indeed a great challenge to convince the teachers to value these ideas and accept the differences, as well as how to adapt or adopt the beliefs of these practices. We acknowledge that this is one of the major challenges for the sustainability of lesson study. Only teachers who willingly receive the idea of lesson study and who fully believe in the effectiveness or benefits of lesson study process will continue to sustain lesson study in schools. Only when teachers are ready with these vast changes and accept that other teachers will come and observe them, also for different purposes or focus points, will lesson study be sustainable.

For long-term benefits, lesson study should be instilled by the educational authorities as a new culture as well as a voluntarily professional development programme. However, realising the culture and practice of top-down instruction in Malaysia, it may appear harsh when we propose that lesson study be made a compulsory requirement with top-down instruction from the MOE. In our opinion, overcoming the constraints and challenges of lesson study implementation should be made a priority. For instance, school administrators could arrange a stipulated time (such as 2 h every Wednesday) for teachers to carry out lesson study in school. Strategic planning such as raising teachers' awareness through immersion in lesson

study projects and participating in lesson study observations in neighbouring schools could also be encouraged. To be able to focus on making teachers feel that engaging in lesson study practices is a worthwhile endeavour and a long-term strategy in order to improve their teaching and their students' learning, rather than merely a compliant top-down directive or instruction, is the most important factor. Thus, bridging a top-down and bottom-up relation is pertinent.

3.5.2 Enhance the Knowledge and Skills of Carrying Out Lesson Study Effectively; Lesson Planning/Observation/Reflection

Attempting to make lesson study compulsory for teachers should not only be an intensive and extensive parallel expansion, but also a dissemination of the knowledge and skills of lesson study. The process of lesson study (Fujii, 2014) has to be clearly explained and elaborated through workshops. Due to educational differences and cultural context, we realise that the rationale and concept of lesson study would not be easily understood and accepted by most teachers. Malaysian educators and teachers are too familiar with the conventional approach of professional development programmes that were mainly driven by external experts. The tendency of communication flow is from the trainer to participating teachers, and the relationship between trainer and teachers are hierarchical (Liptak, cited in Lewis, 2002, p. 12). The shift towards a lesson study culture needs nurturing and time to develop and this is again not an easy task. Lesson study begins with a question from the teachers who wish to improve the teaching and learning of certain subject contents. Communication flow happens among teachers and there should be a reciprocal relationship among learners. There are precious and obvious skills to be acquired, namely setting a goal, lesson planning, teaching of research lessons, teaching observation, post-lesson discussion, and self-reflection. In order to support each step effectively, several skills need to be emphasised:

(i) Lesson planning skills: not just one lesson but a series of related lessons; including and considering previous knowledge;
(ii) Observation skills: providing guidelines and experiences of critical observation; focus observation; different teachers observing different groups of pupils (high performing, average and low performing pupils); and
(iii) Reflective skills: how to make effective reflection, what to reflect on and how to comment constructively; being able to transcribe the comments and reflect again.

Evidently, all of these skills require experiences over a long period of engagement in lesson study, and preferably facilitated and guided by an expert in lesson study.

3.5.3 Engage the Expert Teachers to Play the Role of Knowledgeable Others in Lesson Study

One of the shortcomings noted in lesson study implementation is the lack of experienced teachers or educators who can be recruited to play the role of knowledgeable others. This is indeed true as lesson study is a new concept to most Malaysian teachers. We therefore need to engage expert educators and lecturers who are specialised in teachers' reflections to conduct special courses and trainings for selected expert teachers to play the role of knowledgeable others. These expert teachers should be willing to shoulder the responsibility and the extra work for the sake of improving teachers' teaching practices.

Coincidentally, since 2013 experienced and expert teachers were duly selected and appointed as School Improvement Specialist Coaches (SISC), based at district education offices under the recent Malaysian Education Reforms. These SISC officers are given the responsibility of assisting and coaching teachers in schools, focusing on their teaching and student learning in the classroom. Initially, the SISC officers were sceptical of their roles in guiding and coaching the teachers in school. In our view, they are an excellent choice to shoulder the responsibilities of knowledgeable others in lesson study as they are able to reach a wide spectrum of schools within their respective districts. This would then help to promote and sustain the implementation of lesson study in Malaysian schools.

3.5.4 Proposing Collaborative Lesson Research (CLR) as an Alternative Form of Lesson Study

Fujii (2014) observed that many mathematics teachers and teacher educators outside of Japan learned lesson study based on literature, such as books and published research papers or through direct and cascading modes of participation in lesson study workshops. Many of these lesson study projects were carried out based on the model of school-based lesson study in Japan or on the research and experience of lesson study in schools in the United States. However, as argued by Takahashi and McDougal (2016), due to the lack of certain essential embedded structures in lesson study as practiced by the Japanese counterparts, the impact of these lesson study projects on teaching and learning was unclear. Thus, Takahashi and McDougal (2016) proposed an alternative form of lesson study called 'Collaborative Lesson Research' (CLR). CLR is a form of lesson study introduced and defined to encompass the structures and essential practices emphasised in the Japanese Lesson Study, which have been duly omitted in many lesson study practices outside of Japan. According to them, CLR should consist of components such as having a clear research purpose, *Kyouzai kenkyuu* (translated as the 'study of academic

content'), a written research proposal, a live research lesson and discussion, knowledgeable others, and the sharing of results. The components may appear similar to what has already been practiced by the school-based lesson study as discussed above, but the essentials of lesson study practiced by Japanese teachers are emphasised even more in this model. In particular, besides having a research theme and a clear focus, the lesson to be planned must pose various challenges to students in the learning. Kyouzai kenkyuu is an essential component in the CLR model. This stage involves a careful and detailed study of academic content and is analogous to a literary review in scientific research. It involves an investigation of the intended learning trajectory related to the topic from lower to higher grades, through a review of the standards and curriculum, as well as the research into teaching and learning issues such as typical misunderstandings around the topic (Fujii, 2016; Takahashi & McDougal, 2016). This process also includes the consideration of possible tools, manipulatives or materials that may be used, and in addition possible tasks that may be presented to students. Through this process of detail study of academic content, all the lesson study team member teachers involved will have acquired new perspectives of content knowledge related to their practices which many have been overlooked as common sense and knowledge. Instead of a research lesson plan as in a school-based model of lesson study, the teacher research team in CLR prepares a written document referred to as a research lesson proposal. The lesson plan is only a component of the research lesson proposal and will be used by a team member to teach in a class as a live research lesson. This will be followed up by a post-lesson discussion to consolidate ideas for addressing the research themes. The knowledgeable other is expected to summarise the learning and guide any future research. In this model, there is no re-teaching based on the lesson conducted. However, an elaborate lesson observation report is necessary to note student learning, the issues encountered and insights gained about the content knowledge and the pedagogy from the discussions. Thus, CLR is not just for the improvement of teaching and learning within the team, but also for the general improvement of teaching and learning in the community. In that aspect, there is a need to have a mechanism to promote CLR and disseminate what is learned from each research lesson to a larger community such as neighbouring schools within a district.

The steps in CLR may appear similar to what has been practiced by the local teachers; in our view, the much more well-defined structures of this model provide a very clear direction of the research lesson. Teachers who are involved will not lose sight of where they are heading and understand that the goal set is achievable. An accomplished study of the content is almost like winning half the battle in CLR. Teachers are expected to have acquired a very good grasp of the content knowledge regarding the topics concerned even before executing the anticipated lesson. This immediate accomplishment and improvement of knowledge in teachers will be a motivating factor to attract and encourage teachers to be involved in lesson study.

Apart from this, the sharing of teaching and learning practices will spin off from the lesson study research and will make the teachers more aware of the importance of lesson study and the participation in professional development. Such influences may be contagious and could bring about the teacher-led lesson study instead of a top-down compulsion.

In short, the local teachers who are often faced with time constraints may overcome the resistance of lesson study if they can see the whole process of lesson study as visible and well structured, and be able to bring about an improvement of their professional practices in the classroom.

3.6 Conclusion

Lesson study has certainly created a new dimension and perspective to teacher professional development. Conceptually, it is rather independent of external authorities and can be initiated within a school with just a few teachers involved. The modus operandi is simple; there is little cost involved and it can be conducted in a casual, relaxed and informal context. Therefore, a concerted effort should be gathered among all educators and researchers who are well versed in lesson study to develop and disseminate the concept and operations of lesson study to the school teachers. However, in order to sustain lesson study, the most important factor is ultimately the teachers' attitudes, passions, commitment and their awareness towards their professional development. The intrinsic motivations of the teachers should overcome the external top-down instructions by the educational authorities in any efforts to sustain lesson study in the long term. Perhaps at the initial stages, the state education departments and district education officers could provide adequate support to implement lesson study and accelerate the multiplier effect of lesson study workshops or courses among school teachers and teacher educators. When teachers begin to reap the benefits, and enjoy and realise the importance of lesson study for their professional development and career in teaching, voluntary participation would provide and pave the way for the next level of lesson study implementation.

Moreover, as suggested by Groves et al. (2016), "for sustainability, it will be necessary to establish a community of teachers, mathematics educators and researchers who can continue the process, including providing initial exposure to lesson study and acting as outside experts at post-lesson discussions" (p. 510). Likewise, administrative support and effective leadership provided by the senior management are also very crucial in making the teacher professional development programme a success. Senior management must value the professional growth of teachers as an important factor that contributes towards understanding how students learn, and thus be willing to make space and invest time for teachers to meet and discuss how best to conduct their professional growth (Ng, 2015) such as in lesson study.

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Chapter 4
Enacting Curriculum Reform Through Lesson Study in the Irish Post-primary Mathematics Classroom

Aoibhinn Ní Shúilleabháin

Abstract Lesson study was introduced in two secondary schools in the Republic of Ireland (ROI) at a time of mathematics curriculum reform. The research investigated lesson study as a model of professional development which might support teachers in implementing the revised curriculum. Twelve teachers participated in the research and data included transcripts of lesson study meetings, individual interviews, samples of student work, and research lesson plans. Findings suggest that, through their participation in successive cycles of lesson study, teachers were supported in: encouraging more student communication of mathematical thinking, developing their roles as facilitators of student learning, and incorporating contextualised mathematical tasks. The research indicates that lesson study can be utilised in the introduction of curriculum reform and provides evidence of the viability of school-based lesson study in the ROI.

Keywords Lesson study · Mathematics teacher education · Curriculum reform

4.1 Introduction

Over the past two decades, lesson study has grown in popularity as a form of teacher professional development and has been introduced as a model of teacher collaboration in a broad range of educational contexts around the world, from primary to third-level (e.g. Cerbin & Kopp, 2006; Huang & Shimizu, 2016; Leavy & Hourigan, 2016). Research on teacher learning in lesson study has demonstrated the potential of this model in developing teacher knowledge (e.g. Lewis & Perry, 2017; Ni Shuilleabhain, 2016) and impacting classroom practices (e.g. Olsen, White, & Sparrow, 2011). In this chapter, lesson study is investigated as a model of

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mathematics teacher professional development which supports the implementation of curriculum reform.

In 2010, the post-primary mathematics curriculum in the Republic of Ireland (ROI) was revised and changes were introduced both in the content and emphasis of teaching and learning approaches in the mathematics classroom (Jeffes et al., 2013; National Council for Curriculum and Assessment, 2012a, 2012b). This revised curriculum emphasised a sociocultural approach to teaching and learning mathematics, which highlighted the importance of students' sense-making and ability to apply their mathematical knowledge to non-procedural problems (National Council for Curriculum and Assessment, 2012a, 2012b). This reform demonstrated a dramatic shift in perspectives on teaching and learning mathematics, moving away from the traditional, 'transmission' approach commonly found in the Irish post-primary classroom (Lyons, Lynch, Close, Sheerin, & Boland, 2003).

As part of the introduction of this reform, centralised in-service professional development was made freely available to all post-primary mathematics teachers throughout the country. However, a report on the implementation of the curriculum found little change in the approaches to teaching and learning mathematics (Jeffes et al., 2013). In this research, lesson study was introduced to two case-study post-primary schools, as a potential alternative model to support teachers in the implementation of the revised curriculum. Twelve teachers across the two schools agreed to participate in the research and these two groups conducted successive school-based cycles of lesson study over one academic year.

Findings suggest that participating in school-based lesson study supported teachers in introducing new teaching and learning approaches related to the curriculum reform and impacted on their pedagogical practices outside of lesson study. The research also supports the introduction of lesson study as a viable, local model to support teacher learning in the ROI.

This work is grounded on the relationship between teachers' knowledge and beliefs about teaching and learning mathematics and their pedagogical practices (O'Shea & Leavy, 2013) and also recognises that classroom-level experiences are more influential than policy documents in the implementation of new curricula (Hopkins & Reynolds, 2001).

This chapter is related to findings reported in Ni Shuilleabhain and Seery (2017) and incorporates additional data from a second case study school.

4.2 Lesson Study in Ireland

Over the past 15 years, lesson study has been gradually included in research, policy, and practice in teacher education in the ROI. In 2003, Kelly and Sloane presented lesson study as a potential solution to an identified lack of alignment between research and practice in the Irish education system (Kelly & Sloane, 2003). In this initial reference to lesson study in the Irish context, Kelly and Sloane suggested that the incorporation of lesson study as school-based model of

professional development could provide opportunity for teachers to collaborate at county[1] (or district) level and, eventually, at a national level—thereby supporting the development of teacher knowledge in "an educational system that actively fosters professionalism by teachers in their own clinical development" (ibid., p. 34). In 2005, a report on international trends in post-primary mathematics education was published by the National Council for Curriculum and Assessment (NCCA), which highlighted the merits of lesson study as a model of teacher professional development in Japan. The authors, Conway and Sloane, noted that "the quality of instruction found in Japanese classrooms is something that should be emulated where possible" (2005, p. 68) and, while cognisant of the culturally based elements of lesson study, suggested this model be adopted in the Irish school system.

Lesson study began to be incorporated in mathematics teacher education in the ROI at a pre-service primary (see, for example, Corcoran (2007) and Leavy, Hourigan, & McMahon (2010)) and pre-service post-primary level. However, at the time of the curriculum reform (2010–2012) in-service teachers had little experience with school-based lesson study.

4.3 Revision of the Post-primary Mathematics Curriculum in Ireland

In Ireland, a centralised curriculum is written by a government education authority, the National Council for Curriculum and Assessment (NCCA), and distributed to teachers through syllabus documents (which are also available on-line). Following a review of international trends in post-primary mathematics education (Conway & Sloane, 2005) and discussions around the low numbers of students choosing to study mathematics at higher level (Oldham, 2010), the post-primary mathematics curriculum was revised. This new curriculum, colloquially known as 'Project Maths', explicitly encourages students' communication of their mathematical thinking, highlights contextual applications of mathematics, and emphasises the development of students' problem-solving skills (National Council for Curriculum and Assessment, 2012a, 2012b). This prominence of mathematical problem-solving is closely aligned with the primary mathematics curriculum, which also encourages students' communication of their mathematical thinking as part of classroom practice and highlights the role of the teacher as a facilitator of student learning (O'Shea & Leavy, 2013). This reform-oriented approach to teaching and learning mathematics contrasts with the more traditional, didactic approach prevalent in post-primary mathematics classrooms in the ROI (Lyons et al., 2003).[2] Instead, the revised approach emphasises classroom interactions where students

[1]The ROI consists of 26 different counties under one centralised curriculum system.

[2]Similar to the 'maths wars' in the U.S., this curriculum reform has also generated considerable debate in the Republic of Ireland (Lubienski, 2011).

learn to speak and act mathematically by developing core mathematical competencies (Goos, 2004; NCTM, 2010; Schoenfeld, 1992; Steele, 2001).

Although implementation of the new curriculum was supported with provision of extensive in-service professional development and complementary modular courses (National Council for Curriculum and Assessment, 2012c), many teachers remained concerned about the implementation of the revised curriculum (Irish Mathematics Teachers Association, 2013; National Council for Curriculum and Assessment, 2014). Furthermore, a study on the implementation of the curriculum reported continued widespread use of traditional practices, such as 'drill and practice' and students copying teacher work from the board (Jeffes et al., 2013).

Within this context, this research was conducted to investigate the impact of teachers' participation in school-based lesson study on enacting curriculum reform in the post-primary mathematics classrooms.

4.4 Lesson Study and Curriculum Reform

Across the globe, there has been a change in the emphasis of mathematics curricula from a strong weighting on mathematical procedures to an increased emphasis on the conceptual understandings related to those procedures (Prusak, Hershkowitz, & Schwarz, 2013). With this greater focus on the development of students' core mathematical competencies (NCTM, 2010), there has been renewed attention on classroom practices and approaches to teaching and learning mathematics. However, introducing curriculum reform is not a linear process and it is not often that standards set at policy level feed directly into teaching and learning experiences (Fung, 2000). In order to encourage and support educational change, teachers require opportunity to learn in new ways, to consider new ideas about teaching and learning, and to reflect on experiences of teaching and learning which may influence their beliefs and practices (Cohen & Hill, 2000; Remillard & Bryans, 2004).

Research has demonstrated that collaborative communities are a key element of successful educational reform (Lomos, Hofman, & Bosker, 2011), particularly in the implementation of reform curricula (Penuel, Fishman, Yamaguchi, & Gallagher, 2007). One of the challenges in supporting the enactment of curriculum reform is therefore the consideration that teachers often teach alone in isolated classrooms, without the opportunity to reflect on or observe other pedagogical practices. This is particularly relevant at post-primary level in the ROI, where levels of professional collaboration are low and where there is an absence of cultural and administrative school frameworks which support teacher learning (Gleeson, 2012). Further research on mathematics curriculum reform has found that the most significant learning for teachers occurs in processes of enacting the revised curriculum in the classroom (Remillard, 1999, 2000). In impacting teachers' beliefs and practices on teaching and learning mathematics, it is also important that teachers have opportunity to focus on student thinking (Mason, 1998).

The structure of lesson study provides teachers with opportunity to work collaboratively on teaching and learning, while also observing classroom practices and reflecting on student thinking (Lewis, Perry, & Hurd, 2009). This situationally contextualised professional development offers teachers the opportunity to work with colleagues and explicitly consider curriculum content, incorporate new and unfamiliar practices, and reflect on pedagogical strategies influencing student learning (Dudley, 2013). The sequence of phases within the lesson study cycle incorporates key elements of teacher professional development associated with mathematics curriculum reform (Ponte, 2012) and have been found to support reform in countries such as Japan (Lewis & Tsuchida, 1997; Takahashi, 2014).

In the ROI, teacher professional development is most often in the form of an "in-service" day held at an external location (Gilleece, Shiel, Perkins, & Proctor, 2009) and are often seen as an "add on" for teachers, who are neither incentivised nor formally acknowledged for their participation (Sugrue, 2006).[3] This structure of professional development has been criticised as diluting the autonomous role of the teacher as a professional with individual career preferences and aspirations, to that of a servant of the state (Kennedy, 2007). In contrast, lesson study provides teachers with a structure within which their professional knowledge, judgement, and autonomy are acknowledged in each phase of the cycle—affording teachers valuable opportunities to focus on and potentially refine their practices. In participating in lesson study, the challenge for teachers no longer emphasises changes to their practices, but rather focuses on identifying the kinds of innovation and reform that will improve students' learning experiences in the classroom. Lesson study therefore holds many key features of professional development which are likely to support teachers in implementing educational reform.

4.5 Methodology: Two Case Study Schools

This research was conducted in two case-study secondary schools: Doone and Crannog (all names used are pseudonyms). These schools were contacted through the school principal and, following an information meeting, a number of mathematics teachers within each school volunteered to take part in the research.

Participating teachers' years of experience ranged from 1 to 35 years and, as expected in the Irish post-primary system, a number of teachers reported themselves as "out-of-field" (i.e. teaching mathematics without full subject-requirements recognised by The Teaching Council (Ríordáin & Hannigan, 2011), see Table 4.1). In both Doone and Crannog, school management were supportive of teachers' participation in lesson study—an important factor in introducing this (as yet) unusual form of school-based professional development in the ROI.

[3]A policy on teacher education is due to be implemented by the Teaching Council, which will formally acknowledge teachers' participation in professional development.

Table 4.1 Participating teachers' years of experience

Doone		Crannog	
Name	Years of experience	Name	Years of experience
Kate	3 (out-of-field)	Dave	5
Lisa	7	Eileen	3 (out-of-field)
Michael	6 (out-of-field)	Fiona	31
Nora	30	Judy	19
Owen	1 (out-of-field)	Martin	27
		Stephen	9 (out-of-field)
		Walter	12

Table 4.2 Content and class focus of research lessons

School	Doone		Crannog	
Lesson study cycle	Lesson content	Student group	Lesson content	Student group
1	Introducing x-squared	Grade 7	Introducing quadratic patterns	Grade 8
2	Fractions—sense of measure	Grade 7	Factorising quadratic expressions	Grade 8
3	Exploring quadratic expressions	Grade 10	Factorising quadratic expressions (revised)	Grade 8
4	Introducing Pythagoras' Theorem	Grade 8	Introducing differentiation in Calculus	Grade 10

The research was conducted over the course of one academic year (2012–2013) and successive cycles of lesson study were conducted in both schools (four in Doone and three in Crannog) (see Table 4.2).

To determine if teachers' participation in lesson study supported them in their implementation of the revised curriculum, data was generated through a wide variety of sources including: transcripts of teachers' conversations and notes from all lesson study meetings, individual teacher interviews, research lesson plans, observation sheets and samples of student work from research lessons, and researcher field notes on observed lessons. The author, a former post-primary mathematics teacher, served as a participant-researcher on both sites (Bogdan & Biklen, 2007) and recorded a researcher log which guided, but did not form part of, the data analysis (see Ni Shuilleabhain (2015) for further details).

Each cycle of lesson study followed the structure of: formulating a goal and studying the curriculum, planning a research lesson with reference to literature and curriculum materials, conducting or observing the research lesson, and reflecting on the research lesson with opportunity to revise and re-teach the lesson if teachers felt that it did not meet the learning objectives of the lesson (Lewis, Perry, & Murata, 2006).

In both Doone and Crannog, teachers agreed to participate in the research as a way of supporting themselves in the implementation of the new curriculum and as a way of learning how to collaborate as a group of mathematics teachers. Unlike most in-service models of professional development presently available in the ROI, teachers had autonomy in choosing the subject matter focus of each of the research lessons. Teachers also had authority in deciding the number of research lessons which they would participate in over the course of the academic year. In both Doone and Crannog, teachers chose to plan lessons across a number of topics in the curriculum for a variety of student groups at both junior and senior post-primary level (see Table 4.2).[4]

Analysis of the data was conducted utilising a framework which reflected the goals of the revised curricula (National Council for Curriculum and Assessment, 2012a, 2012b) and aligned with pedagogical practices associated with sociocultural theories of learning (Goos, 2004; Schoenfeld, 1992; Steele, 2001):

1. Supporting students in communicating their mathematical thinking
2. Teachers acting as facilitators of student learning
3. Designing and incorporating contextualised content relevant to students' learning.

Analysis did not commence until all data had been generated and was triangulated through iterative analysis of teachers' collaborative conversations, materials generated in lesson study, and individual teacher interviews.

In establishing the base-line of participating teachers' participation in and implementation of the curriculum reform (Bubb & Earley, 2010), teachers were interviewed prior to their participation in lesson study. All teachers except Nora[5] had participated in the professional development courses offered as part of the curriculum reform. However, as can be the case with external professional development courses (Fung, 2000), teachers rarely utilised or trialled the curriculum materials and ideas introduced in these modules.

> They [referring to curriculum materials] kind of go in the pile and I'll say 'I'll come to it, I'll look at that again' and you just never do. (Kate, Interview 1)

Participating teachers' opinions of the revised mathematics curriculum varied. Nora (who taught remedial mathematics on a voluntary basis in the school[6]) was confident that her teaching already aligned with the new curriculum, but others remained unsure of how to incorporate new practices in their teaching. As an out-of-field teacher, Michael was wary of the reform and did not feel confident in his ability to teach mathematics. Others, such as Owen and Judy, were opposed to

[4]Grade 7 relates to 'first year' in the Irish system. Students in grades 9, 11 and 12 are preparing for important national examinations and teachers therefore chose not to include these year-groups as a focus of their research lessons.

[5]Nora was officially retired from teaching and was therefore not offered opportunity to participate in these courses.

[6]This is not common practice in the ROI.

the reform and the pedagogical practices espoused by the curriculum. None of the participating teachers were familiar with lesson study but, as reported in their initial interviews, they hoped that their participation would support them in their implementation of the revised curriculum and support them to collaborate with their colleagues.

> I think that we would have a better understanding of how to collaborate on what we are teaching and how to teach it. (Walter, Interview 1)

4.6 Findings

Three key findings related to the impact of teachers' approaches to teaching and learning mathematics are presented here, as relevant to the curriculum framework outlined above. These findings serve to demonstrate the opportunities provided to participating teachers to incorporate new and unfamiliar practices in research lessons, as a result of their participation in successive cycles of lesson study. The findings also document teachers' reported changes to their teaching and learning practices outside of lesson study.

Excerpts of qualitative data are used to provide the reader with examples of teacher learning during the research.

4.6.1 Emphasising the Social Dimension: Supporting Students in Communicating Their Thinking

A key finding of the study were the changes to teachers' approaches in supporting students to communicate their mathematical thinking. In Doone, at the commencement of the research the teaching and learning of mathematics had been of a direct nature, where students often worked individually and silently at their desks and where content was often introduced through exposition and repetition of tasks (Boaler, 1998). While teachers had explicitly wanted to incorporate more student communication in their first research lesson, neither the structure of the lesson nor tasks within the lesson provided students with opportunity to consider and communicate their mathematical thinking to one another or to the teacher. In their post-lesson discussion, teachers were disappointed with the lesson, noting a lack of engagement from the students and reflecting on the underlying structure of students' interactions in the lesson. Following this cycle, the conducting teacher, Lisa, changed her classroom layout from individual tasks to tables arranged for group work and shared her positive reflections on this new classroom environment with her colleagues during a planning meeting in the second cycle:

> They can all see each other's work. "You're not doing that right" [mimicking student]. I love it. I would never go back. (Lisa, meeting 3, cycle 2)

This change to Lisa's practice, relating directly to her teaching of the first research lesson, encouraged her colleagues to explicitly focus on encouraging more student communication in subsequent research lessons and, by the end of the year, all teachers within this group reported a new emphasis on student communication in their own teaching. This was most notable for Owen, an out-of-field teacher who, at the beginning of the research, described himself as "anti-Project maths" and described his mathematics classroom as very traditional. In his final interview, he noted the importance of students discussing their mathematical thinking:

> I would be conscious to have a chat about whatever [maths topic] we were doing. Even though it might waste twenty minutes, I don't think it's a waste. (Owen, interview 3)

In Crannog, teachers embraced the curriculum goal of incorporating more student communication of mathematical thinking from the initial lesson study cycle. However, while teachers were positive about incorporating this practice in their teaching, they did not feel confident in how best to encourage students' communication in class and wanted to utilise lesson study as an opportunity to trial this unfamiliar practice.

Fiona The class and myself both have to develop in improving group work techniques.

Walter What questions to ask, what tasks to set them and how to sort of facilitate that best.

Fiona Just to develop group teaching with group work.

Dave It's a different classroom culture. It's a very hard thing to change because we've had the same culture in so many of the classrooms in Ireland I think for so long.

The collective and collaborative exercise of planning research lessons provided the group with the opportunity to attempt this new practice and the teachers devised a mathematical task where students would work in groups to identify a quadratic pattern from a sequence of numbers. The teachers treated the research lesson as an experiment in how students would engage with and react to working in small groups and, in their post-lesson discussion, were excited to reflect on students' communication of their mathematical thinking during the lesson.

Fiona It worked! Those guys over there [referring to one group of students] I was watching, they really helped each other. They hugely helped each other, they were listening to each other. They had great discussions.

Walter That was all on task. Nobody, as far as I could see, nobody was talking about -

Martin They were really engaged in what they were doing. That dynamic is great to watch how they are hitting off each other and when they have their own ideas – it is good.

Judy They were all going "It's not linear". You know what I mean? They were delighted with themselves that they actually saw that -
Stephen They got there themselves.

Following this initial research lesson, the teachers were more positive about encouraging student communication during lessons and noted that, when given time to work on a collaborative activity, students stayed on-task in their mathematical work. The group continued to highlight and incorporate students' communication of their mathematical thinking in subsequent research lessons through pair-work, small group, and whole-class activities and discussions. At the end of the academic year teachers reflected on the impact their participation in lesson study had had on student learning and felt that students' experiences of sense-making would benefit their learning.

> Because even when they're having to work in groups and they're talking about maths, that has to help with those language-y questions, those wordy questions where they've been actually putting it into words themselves. (Fiona, final teacher meeting)

In both Doone and Crannog, teachers' participation in school-based lesson study provided them with a structure within which they could trial and refine the incorporation of students' communication of their mathematical thinking during research lessons. Over successive cycles of lesson study teachers integrated new ways to encourage students to verbally make sense of their mathematical thinking, in line with the goals of the curriculum reform. In their concluding individual interviews teachers also identified these practices in their teaching outside of lesson study.

4.6.2 Teachers as Facilitators of Student Learning

Incorporating more student communication of their mathematical thinking requires the role of the teacher to shift from that of a knowledge-provider to a facilitator of students' learning—moving from 'show and tell' to facilitating mathematical discussions (Stein, Engle, Smith, & Hughes, 2008). This was a new departure for almost all teachers participating in this research, who were more comfortable and familiar with a direct approach to teaching and learning mathematics through teacher exposition and repetition of tasks. By incorporating more organisational forms in the mathematics classroom, the role of the teacher began to evolve and, in both schools, the planning of research lessons began to explicitly incorporate the teacher's role as that of a facilitator who guides and sequences student learning.

From the first research lesson in Doone, teachers began to note the import of the teachers' role in responding to students' questions and utilising students' prompts to guide a discussion towards the learning goals (Webb et al., 2009). In their planning of a research lesson introducing x^2, the teachers had not anticipated a student query related to the exponent: "*Is the number [of the power] always 2?*". The conducting

teacher, Lisa, was unsure of how to respond and ignored the student's question during the lesson. In their post-lesson discussion, Kate noted that this question would have been a valuable opportunity to discuss the representations of indices. The teachers realised that facilitating discussions related to students' thinking was a valuable element of introducing new concepts (Stein et al., 2008) and they began to focus more on how teachers could guide students in their understanding. As part of their planning, they began to anticipate questions which students might ask during a lesson and incorporated potential teacher responses in their planning.

Similar teacher learning occurred during the first lesson study cycle in Crannog where, in their post-lesson discussion, teachers realised that they had not anticipated all of the strategies students might consider in identifying a quadratic pattern.

Dave I think two or three of the groups got the thing the wrong way around, the levels 1, 2, 3, 4. Your group did and yours as well, yeah.
Eileen Mine did, yeah.
Walter I don't think we spotted it as a potential.
Dave No we didn't.

In subsequent cycles, teachers paid explicit attention to considering all possible strategies students might have in solving a mathematical task, in order to ensure the conducting teacher would be equipped to respond to students' work during the research lesson. In their second research lesson, teachers spent a considerable amount of their planning sessions thinking about various ways of factorising a quadratic expression (see Fig. 4.1). Due to these conversations, teachers in the group felt more knowledgeable on various factorising strategies and better equipped to deal with students' misconceptions.

Eileen noted that, in an attempt to incorporate more student group work into her own teaching, she was also engaging in anticipating students' responses to mathematical tasks outside of lesson study.

> I probably would ask myself a bit more like "How would they react to this?" or "What questions will they have?" Like, pre-empt their questions or pre-empt their confusion. Em, yeah, I would think about that a little bit more. (Eileen, interview 3)

Over successive lesson study cycles, teachers in Doone and Crannog began to incorporate their knowledge of student learning into the planning or refining of research lessons and, in anticipating student responses to mathematical tasks, began to refine their roles as facilitators of student learning (Stein et al., 2008).

As well as incorporating ideas on students' anticipated responses, both groups also began to pay more attention to the questions asked by the conducting teacher in research lessons. Research lesson plans began to include scripted questions as a tool to support the conducting teacher in their role as a facilitator of student learning. This element of planning required teachers to write questions which deviated from the closed questions often asked as part of traditional classroom teaching (Lyons et al., 2003). In the post-lesson discussion of research lesson 3 in Doone, where

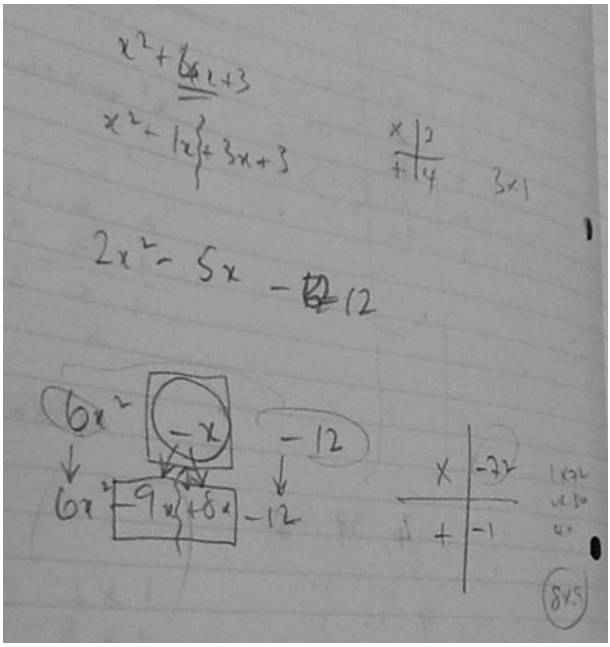

Fig. 4.1 Photograph of Crannog teacher planning notes—trialling various strategies of factorising a quadratic expression

students had worked in groups on exercises and shared their responses as a whole class discussion, Kate (the conducting teacher) commented that:

> There was very little teaching involved in it really. Just kind of facilitating – which is what it's all about. (Kate, post-lesson discussion 3)

She later noted that she wanted to continue to work on the skill of facilitation: "*That pure questioning will be going into my repertoire now*". Teachers reported that these experiences impacted on their practices outside of lesson study. In his final interview, Owen, who at the beginning of the research noted that it was "*hard to let go*" of his direct approach to teaching, noted that his participation in lesson study encouraged him to focus on his role of guiding student learning.

> From these meetings, as well, I'm conscious of open-ended questions rather than "this is what it is. This is the answer". (Owen, interview 2)

Walter, in Crannog, acknowledged that acting as a facilitator of learning required him to allow students more time to work by themselves and he considered this new practice to be much more beneficial to students' learning.

Over successive cycles of lesson study, individual teachers began to evolve in their roles as facilitators of learning and encouraged more "reasoning and sense-making opportunities" in teaching and learning mathematics, as outlined in the reform curriculum (National Council for Curriculum and Assessment, 2012a, p. 10).

4.6.3 Designing and Incorporating Contextualised Content

As part of their role as facilitators of student learning, teachers chose to incorporate mathematical tasks which would engage students in their learning and highlight the application of mathematics in the world around us. Task development is a critical element of lesson study (Doig, Groves, & Fujii, 2011) and, throughout the course of this study, teachers in both schools explicitly focused on incorporating tasks which were relevant to the mathematics, but also held meaning or had context for students (Lampert, 2001).

Prior to the introduction of the curriculum reform, textbooks in the ROI had been found to emphasise abstract and procedural tasks, with little prominence of situated problem-solving tasks (O'Keefe & O'Donoghue, 2011). As part of the lesson study planning phase, teachers rarely found appropriate mathematical activities in the text-books and were therefore prompted to modify tasks from other sources (for example from teacher magazines or alternative textbooks) or design their own tasks. This was a new departure for teachers since, in the ROI, textbooks often act as a mediator between curricular intention and implementation (Looney, 2004). This *kyozai-kenkyu* element of planning the research lesson (Doig et al., 2011) broadened the range of sources which would usually be utilised by teachers in their planning.

In Doone's first research lesson, tasks were based on a rural farmer calculating the area of a field and held little meaning for grade 7 students in an urban school. In their post-lesson discussion teachers resolved that, in order to further motivate students to engage with mathematical content, tasks should be relevant and have context for learners (Stanic & Kilpatrick, 1989). In subsequent research lessons teachers enjoyed collaboratively designing mathematical tasks which would be of interest to students, see Fig. 4.2 as an example.

In their post-lesson discussions, teachers noted how students were prompted to further engage with the mathematical content of the activity, due to the context.

Michael They were talking about the temperature rising and then they were saying, well, they were joking, "she's going to explode!"… But that shows that they understand what they are being asked.

Kate Even one group were saying like "Ah, this couldn't be true"… He [the student] was like "the temperature would clearly rapidly increase and then slowly come down".

Teachers reflected that explicitly providing context for mathematical activities provided students with meaningful representations for mathematical content, which benefitted their engagement in the lesson.

In each successive research lesson in Crannog, teachers also purposefully designed tasks which would provide relevant activities for students learning new mathematical content. In their final cycle, teachers devised a series of lessons to introduce the concept of differentiation through experiments and simulations.

After years of research and clinical trials, 2 biologists have shown that the temperature of a person suffering from the flu can be modelled by the function:

$$T = -0.1t^2 + 1.2t + 98.6$$

Where T is the temperature in degrees Fahrenheit, at time t, in hours.

Una is feeling unwell and visits Dr Nolan's surgery. Dr Nolan diagnoses her with the flu.

a) Predict her temperature after 1 hour.

b) Predict her temperature after 2 hours.

c) After how many hours will her temperature peak?

Fig. 4.2 Application of quadratic functions—contextualized activity from Doone research lesson 3

Modifying a task provided in curriculum materials (which none of the teachers had yet incorporated in a lesson), teachers constructed a task where students could develop a conceptual understanding of calculating an instantaneous rate of change, before being introduced to the methodology of differentiation by first principles (see Fig. 4.3).

Over the course of the academic year, teachers in both Doone and Crannog enjoyed the opportunity to design and develop their own mathematical tasks as relevant to the revised curriculum. Through this collaborative work teachers devised tasks which were directly relevant to their own students and demonstrated the application of mathematics in context, as opposed to a solely abstract or procedural approach.

4.7 Discussion: Incorporating Lesson Study in the Irish Post-primary System

Participating in successive cycles of school-based lesson study supported the teachers in Doone and Crannog to implement the revised post-primary mathematics curriculum. By taking part in this collaborative work, teachers engaged directly with curriculum documents and materials and were provided opportunity to devise research lessons in an environment which allowed them to: take risks in their

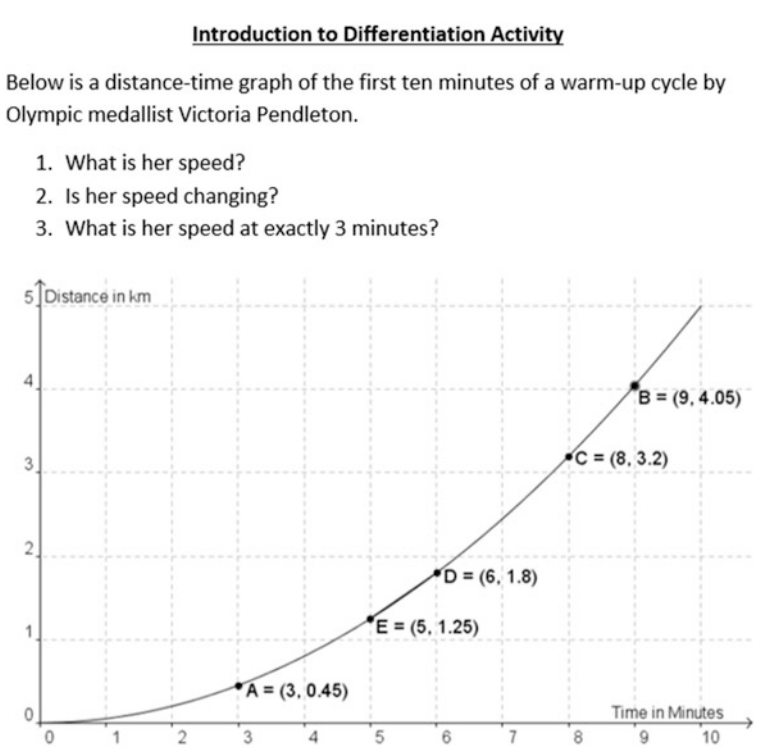

Fig. 4.3 Introducing differentiation—contextualised activity from Crannog research lesson 3

classroom practices (Dudley, 2013), collectively design lessons related to the curriculum, and reflect individually and co-operatively on teaching and learning. Over successive cycles, teachers in both schools began to develop research lessons which aligned with the reform curriculum by encouraging student communication of mathematical thinking, supporting the role of the teacher as a facilitator of learning, and emphasising contextual mathematical tasks as relevant to their students' learning.

By providing teachers with opportunity to work with their peers in a situated, school-based model, these participants felt supported in their efforts to implement the new curriculum.

> I think getting together like this formally is kind of, you know, it's vital. We talk all the time about how we're teaching and what we're teaching… If you're putting in place the opportunity for people to work together, I think it's going to be a good thing. (Dave, final meeting)

In their final interviews, all participating teachers felt they had gained from their participation in lesson study and felt more confident in implementing the revised curriculum. All teachers noted that they would like to continue to participate in lesson study. However, without official recognition or structural support, they did

not consider they would be able to do so. Aligning with other literature on collaborative professional development, each of the participating teachers highlighted the voluntary and autonomous nature of their participation in lesson study as an important factor of their buy-in and engagement (Robutti et al., 2016). Teachers valued that, unlike traditional in-service professional development, they chose the focus of mathematical content and decided which class groups they would like to plan a research lesson for. In addition, they could independently consider what pedagogical approaches they might adopt in planning the research lesson. Teachers' reflections on the import of their autonomy in participating in this form of professional development may be relevant in future policy-making on teacher education in the ROI.

In addition to systems around teacher professional development, school management and structures are key elements of teachers' meaningful participation in lesson study (Groves, Doig, Vale, & Widjaja, 2016). In Crannog, school management supported teachers in pursuing lesson study for the following academic year by ensuring mathematics teachers had a common double free-period where they could conduct meetings around pedagogy. This was, unfortunately, not possible in Doone and, while teachers continued to collaborate on an informal basis, school-based lesson study was not sustainable without policy provision on incorporating and supporting collaborative forms of school-based professional development.

A wide-scale introduction of lesson study is now being introduced across the ROI by the Department of Education, with over 250 teachers from 110 post-primary schools taking part in school-based and district-based groups (Maths Development Team, n.d.). Similar to Groves et al. (2016), this particular introduction of lesson study emphasises the implementation of structured problem solving in the mathematics classroom and support from the Mathematics Development Team is provided in the form of an induction programme with follow-up support from lesson study associates (similar to Maths Coaches in the USA). Since 2015, an annual conference has taken place where teachers participating in lesson study can showcase their work and observe live research lessons (Maths Development Team, n.d.). In contrast to the introduction of lesson study as a top down professional development initiative (Kusanagi, 2014), this national initiative is intended as a grass-roots programme for teachers to voluntarily participate in lesson study.

4.8 Conclusion

Educational systems require alignment and inter-connection of curriculum, assessment, teaching and learning practices, and policy in order to achieve the shared goal of improving students' learning experiences and outcomes. However, curriculum reform is often introduced through top-down policy mechanisms, which are not always paralleled in classroom practices (Fetters, Czerniak, Fish, &

Shawberry, 2002). In this chapter, teachers' participation in lesson study in two case studies in the ROI demonstrated the potential of this model to support the implementation of a mathematics curriculum reform. By participating in lesson study, teachers had opportunity to enact reform practices in classrooms to encourage students' communication of their mathematical thinking, develop the role of teachers as facilitators of student learning, and design tasks which provided context to relevant mathematical content. Each phase of the lesson study cycle provided teachers with opportunity to incorporate and utilise curriculum materials, plan content as relevant to their own students, observe the enactment of the curriculum in the research lesson, and reflect on how the goals of the research lesson were met as relevant to the reform. The swan-like nature of lesson study, with its deceivingly simple structure on the surface hiding a depth of knowledge and work incorporated and undertaken in a cycle, provides a model of professional development which supports teachers to introduce and enact curriculum reform.

As well as demonstrating the potential of lesson study to support teachers' engagement with curriculum reform, in this chapter lesson study has been confirmed as a practicable and feasible model of teacher education at post-primary level in the ROI. With pending policy changes to teacher professional development (The Teaching Council, 2011), lesson study can provide teachers in the ROI with sustainable opportunities of school-based collaboration within the continuum of teacher education.

Related to findings discussed in this chapter, further research is required to investigate the potential of this form of professional development in smaller schools in the ROI, where only a small number of teachers are available to engage in lesson study groups. In addition, an evaluation of the introduction of lesson study as a large-scale, national initiative on both teacher and student learning will be of value to both educational research and policy in the ROI.

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Chapter 5
Fitting Lesson Study to the Portuguese Context

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Abstract The adaptation of lesson study to different national contexts raises problems that are attracting the attention of researchers. In this chapter, we show how we have conducted lesson study in Portugal, taking into account the national educational context and our perspective on the teaching of mathematics, the exploratory mathematics curriculum approach, with particular attention on the development of students' mathematical reasoning. We present two examples of lesson study, one conducted with in-service teachers of grades 5–6 and another in initial teacher education with prospective teachers of grades 7–12. We reflect on what we have identified as the main obstacles to the realization of lesson study in these contexts due to cultural factors and teachers' concerns. We conclude with a summary of the adaptations that we have made in this teacher education process, with a view to the suitability and potential of lesson study in our country.

Keywords Lesson study · Professional development · Initial mathematics teacher education · In-service mathematics teacher education

5.1 Introduction

There is, nowadays, an active international movement concerning lesson study, centering around WALS (World Association of Lesson Studies), its well-attended international meetings (the last one at Exeter, United Kingdom, in September 2017)

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and its specialized journals (notably the *International Journal of Lesson and Learning Studies*). Originating in Japan, where lesson study assumes an important role in professional development and in the introduction of curriculum innovations (Takahashi & McDougal, 2014), this model has become well known in the United States and all over the world since the publication of *The teaching gap* (Stigler & Hiebert, 1999).

The positive discourse about lesson study tends to hide the fact that this is a demanding process. In fact, its dissemination across different countries has shown that planning and conducting lesson study may become problematic in many situations. Japanese authors such as Fujii (2014) claim that the introduction of lesson study in a number of countries has been based on misconceptions of how it is practiced in Japan. However, we must note that even in Japan there are quite diverse practices regarding lesson study, depending on the entities that promote them (schools, regional, national bodies, associations) and on its purpose (professional development of a group of teachers or demonstration "jumping in" lesson by a veteran teacher) (Fujii, 2016). From Japan to other countries, many transposition difficulties derive from deep differences in educational systems and in the teachers' professional culture. In addition, in Japan lesson study is a generalized practice, carried out in a large scale with the support of the educational authorities, whereas in many other countries it is a marginal practice, undertaken only in small scale, and mostly in an exploratory way.

Without downplaying the import of investigating the ways in which lesson study is carried out in Japan, we are mostly concerned in knowing how this professional development process may be adapted to the Portuguese context in a productive way, especially in mathematics education. As Stigler and Hiebert (2016) note, a given cultural practice (in this case, a practice of teacher education), that originated in a given culture, may enjoy a wider diffusion in other cultures, but must undergo an inevitable process of transformation and adaptation given the specific features of the different cultural contexts. In this process, many significant aspects of the original cultural practice may be lost, but other aspects may emerge, yielding robust and flexible practices in the new environment. This is our concern in Portugal, were the first lesson study experience was carried out by our group in 2011. In this paper, we seek to provide an account of the main adaptations that we have undertaken within lesson study that we conducted, taking into consideration the concerns and interests of Portuguese teachers, as well as our own perspectives about mathematics teaching.

5.2 Lesson Study as a Professional Development Process

5.2.1 Main Features of Lesson Study

Lesson study has been the object of many descriptions, often non coincident (Fuji, 2014; Lewis, 2002; Lewis, Perry, & Hurd, 2009; Murata, 2011). In a general way,

in lesson study, a group of teachers work together, beginning by identifying difficulties that students usually have in a given topic or issue related to a curriculum aim. Then, they prepare a lesson that may provide an important contribution to overcome those difficulties. In order to do this, they read the curriculum guidelines and review the available teaching strategies and materials regarding the chosen topic or issue.[1] They also make a diagnosis, as precise as possible, of students' prior knowledge and foreseeable difficulties and seek to know the results of related research on this topic. Based on these activities, the group prepares, in great detail, a lesson on the given topic or issue. The lesson, termed a "research lesson", is taught by one of the teachers and observed by the remaining teachers who focus their attention in the work of the students, their strategies and difficulties, and not in the activity of the teacher. Afterwards, the group of teachers carries out a joint reflection about the way the students worked, the processes that they used in solving the tasks, the responses that they gave to the teachers' questions, and any difficulties that they displayed. Sometimes, the teachers write their reflections and share them with other teachers in the school or at professional meetings. In this process, the teachers define an issue to inquire, work with curriculum documents and materials, collect data from their students, carefully analyze and interpret quite diverse information that is relevant to teaching and learning the chosen topic, and seek to systematize and report on their conclusions. Therefore, this is a process very close to a small-scale research carried out on teachers' professional practice, in a collaborative context (Ponte, 2008). At first glance, this may resemble the "classroom observations" carried out in teacher education during the practicum or in teacher evaluation processes. However, there is a substantial difference that in lesson study the main focus of observation is not the teacher's actions, but the work of the students. This is in line with the lesson study aim to understand how students' learning develops.

5.2.2 *Lesson Study in the Portuguese Context*

Our team at the Instituto de Educação (IE) of the Universidade de Lisboa, whose core group is the four authors of this chapter, has pioneered lesson study in our country. Our work as mainly a research purpose. We seek to understand what teachers learn in this professional development process and also to identify the characteristics of lesson study that may underpin its value in providing learning opportunities for teachers. Usually, we invite a group of teachers of a school to get involved in a lesson study. The first two groups with whom we conducted lesson study were made of teachers who participated in the Project "More School Success" of the Ministry of Education, from two schools in the region of Lisbon, one in a

[1]How this preparation may be carried out is presented in Ponte, Quaresma and Mata-Pereira (2015).

nearby suburb and another in a rural region 70 km away. We already had an ongoing working relationship with the teachers from these schools which was important for them to trust our proposal. One group comprised grade 4 teachers and another group grade 7 teachers. Two years later we made another invitation to three more groups of teachers from another school in Lisbon, one for grade 3, another for grade 5 and another for grade 7. This invitation was made in response to a request of support from the principal of the school, to support their own project of improving mathematics learning. Later we carried out a lesson study in initial mathematics teacher education, with activities undertaken in another school in Lisbon. In every case, the leadership of the lesson study group was assumed by our team, albeit seeking to value the role of the local leaderships that already existed in the participating groups of practicing teachers. Usually, one of the members of our team had the responsibility to conduct and facilitate the lesson study sessions. The other members of the university team participated as observers and, sometimes, they also provided contributions for the development of the session. The whole team prepared the structure and content of each session and carried out an ongoing reflection. We made video or audio recordings of all sessions and transcribed them. A member of our team kept a research journal describing the main aspects of each session as well as some initial reflections, with the entries receiving comments from the other team members. During the conduction of the research lesson, all teachers in the lesson study group, of our university team, and a special guest (for example, the deputy principal of the school) also participated as observers. In this chapter, we refer to two cases of lesson study, whose participants we will present later, drawing on data from working sessions and participants' interviews.

Common to each of the lesson studies that we carried out, was the strong attention that we gave to the planning phase, including the preparation of the research lesson. The lesson study group analyzed the key concepts involved in the topic identified for study, as well as the tasks suggested to be proposed to the students, seeking to identify all their possible difficulties in solving them. This practice involves the undertaking of significant mathematical work in solving tasks that, sometimes, involve some degree of challenge for the teachers. It also involves significant work in terms of mathematics teaching, with in-depth discussion of teaching and learning issues. As it is usually done in lesson study, this planning takes into account the curriculum guidelines and the results of previous research concerning students' learning in the given topic. An important element of this planning is the diagnosis of students' prior knowledge and difficulties, notably in concepts related to the topic. The final part of this preparation phase is the actual construction of the research lesson, with strong attention to the formulation of the task that will be proposed to students. This construction involves the detailed planning of the flow of the lesson, including the identification of students' foreseen responses, teachers' actions, and indications for assessment.

Underlying each lesson study that we have carried out, as the main curriculum perspective, is an exploratory approach (Ponte, 2005), widely disseminated in Portugal with the former 2007 Mathematics Curriculum (Ministério da Educação, 2007). In this approach, students work on tasks in which they have to construct their

own solution strategies, using, with flexibility, different mathematical representations. Different from the usual classroom, in which the teacher begins by presenting mathematical concepts, representations, and procedures, showing examples and assigning exercises for practice,[2] in the exploratory approach the teacher proposes the students tasks that lead them to develop mathematical knowledge. Usually, the exploratory lesson develops in three phases: (i) launching the task, which may involve some negotiation of meanings and discussion of contextual elements, (ii) students' autonomous work, individually, in pairs, or in small groups, and (iii) discussion, with students presenting their solutions, contrasting with each other, and ending with a summary of the most important aspects to learn from the work carried out.[3]

This approach has two main features: the choice of appropriate tasks and the establishment of stimulating a classroom communication environment. Such environment must support students' participation and reflection, both during students' autonomous work in which they interact with their colleagues and in the whole class discussions, and value negotiation of meanings and argumentation. This approach emphasizes the construction of concepts, the modeling of situations and, also, the use of definitions and properties of mathematical objects to arrive at conclusions. It pays attention to computational aspects of mathematics, but does not neglect the conceptual aspects. The message to students is that it is important to achieve a correct mathematical answer, but even more important is to understand general strategy that was used and its justification. Given the key role that it gives to students in the process of knowledge construction, this approach shares some resemblance to what some authors call "inquiry-based mathematics teaching" (Artigue & Blomhøj, 2013), "guided reinvention" (Gravemeijer, 2005), "landscapes for investigation" (Skovsmose, 2001), "reform mathematics" (Cobb & McClain, 2006) or "structured problem solving" (Fujii, 2016).

Another important aspect of teachers' participation in these lesson study groups is their attention to students' mathematical reasoning (Lannin, Ellis, & Elliot, 2011), especially in what concerns the realization of conjectures and generalizations (inductive and abductive reasoning) and justifications using mathematical properties, definitions and representations (deductive reasoning) (Mata-Pereira & Ponte, 2012). This attention is enacted during lesson study planning and reflection sessions, through focusing on and analyzing the tasks and the examples of students' work (solutions of tasks and transcripts of discussion episodes) that involve reasoning processes.

[2]This is often termed as "traditional teaching" (as in Skovsmose, 2001). However, this expression conveys the idea that all past teaching followed this approach, which is an undue oversimplification. That is why we prefer, aligning with Fitzgerald and Bouck (1993), to term this as "direct teaching".

[3]This is similar to what some Japanese, such as Fujii (2016), describes as a typical problem solving lesson, with the difference that we include the discussion and synthesis, since often there is no clear division between these two lesson elements.

5.3 A Lesson Study with In-Service Teachers

Our first example of a lesson study carried out in Portugal involved grades 5–6 in-service teachers of a school in Lisbon in 2013–2014. From these teachers, three taught at grade 5 and two at grade 6. Three of the teachers (which we name with the pseudonyms Francisca, Inês, and Maria) had a very large experience (more than 35 years of teaching practice) and were tenured in the school. The other two teachers (Luísa and Tânia) had also a significant experience (about 10–15 years of teaching practice) but had just annual contracts. This lesson study involved 12 sessions and was certified as an official professional development program accredited by the national agency for teacher education with 25 h of face to face work.[4] We strived to adjust the lesson study to these national requirements so that the teachers could have an official certificate stating that they completed a recognized professional development program.

The year in which this lesson study took place corresponded to the introduction of a new mathematics curriculum in grade 5. This grade was selected for the lesson study. In session 1, despite the fact that they had been informed about the purpose and nature of lesson study, the teachers showed concern about this format of professional development. Finally, they accepted to participate. After considering several possibilities, the teachers chose to address the topic of comparing rational numbers. This topic was quite familiar to several members of our team, so it was possible to easily locate many tasks and materials to explore in the working sessions. Therefore, solving mathematical tasks and analyzing students' responses to those tasks (available from previous research) was an important activity in several sessions. The diagnosis of students' previous knowledge was also an important part of the work undertaken by the lesson study group, and was tackled in two sessions, first in preparing it, designing a small test (session 3), and after in the analysis of the students' responses in the classes of the three grade 5 teachers who were involved in this lesson study (session 4).

The decision of who would conduct the research lesson was made in session 5 and was another rather difficult moment in the group. Two teachers could not be considered for this role since they did not teach grade 5. The two tenured teachers that taught this grade refused that role. One of them argued that her pupils had not a proper behavior and the other that, as she said, she did not want to assume the condition of a teacher "under evaluation". After some negotiation in the session, it was decided that the class would be conducted by Luísa, a contracted teacher, who was the only other grade 5 teacher remaining. In the session, Luísa expressed some uneasiness with the situation, but her unease was in part resolved in this session and the next as the detailed planning of the class progressed. The student task incorporated in the research lesson was prepared under the leadership of Luísa, and ended up as a worksheet with three questions, each one with several sub-questions.

[4]Additional information about this lesson study, with focus on participant teachers' professional learning may be found in Ponte, Quaresma, Baptista and Mata-Pereira (2014) and Ponte, Quaresma, Mata-Pereira and Baptista (2015a).

After six planning sessions, the research lesson was conducted in session 7 and the post-lesson discussion was held in session 8.

An interesting aspect of this lesson study were the follow-up sessions (sessions 9–12). These sessions were first established to comply with the 25 h of duration required for officially accredited professional development programs, but turned out to be very interesting working moments. In these sessions, the five teachers prepared new tasks according to the ideas discussed during the former sessions of the lesson study to take to their classes, proposed these tasks to their students, and reported their reflections about the experiences carried out. As a consequence, in these final sessions there was wide opportunity to review and systematize many issues of professional practice identified in the initial sessions.

At the end of the last session, the participating teachers and the members of our team made jointly a global reflection regarding all the activity. We recalled the work carried out in each session and asked the teachers what they thought about it, in particular if they found it should be maintained or dismissed in a future lesson study. The teachers spoke of their experience, justifying their opinions.

The first point addressed was teachers' initial difficulty in understanding this model of professional development. All of them reported that they were expecting something quite different. Maria indicated that she did not understood how they would "lose all that time, to prepare a single lesson." Luísa said that she expected that "several topics could be tackled." Francisca stated that she would have preferred to work on geometry. Tânia indicated her initial disappointment because, contrarily to what was proposed, she felt the need to analyze the new curriculum guidelines for several topics.

After the lesson study, we asked the teachers if their investment in this professional development activity had been worthwhile. All of them responded in an affirmative way and mainly highlighted the joint work that they had carried out. Regarding this, Francisca said:

> Throughout the sessions we were seeing that this is really productive and that we learned to work in a collaborative way. For me, that was sufficient. This is so, that the teachers of our mathematics group, in this moment, are able to speak with each other with no barriers. I think that is added-value. For me, this aspect was the important one, we begun working as group.

We also questioned the teachers about how they viewed the activity of solving mathematical tasks. Maria indicated that she enjoyed being challenged and solving tasks that required her to think and have a living relationship with mathematics:

> Sometimes we ended up quite limited and if there is a new thing that forces us to think a little further, to go beyond 2×3, that is motivating. For me, it is motivating. I still design the grade 9 exam papers … Sometimes I need a little more mathematics, and I think that these sessions helped us with that.

Afterwards, we addressed the importance of the undertaking of the diagnosis of students' prior knowledge about the topic. The teachers indicated that this phase of the lesson study led to an important discussion about students' knowledge and also referred to their surprise about what the students could do, which was quite beyond

their initial expectations. Maria also reported that they were doing less of these diagnostic activities than they did in the past and that the lesson study allowed them to reflect on their need.

We also asked the teachers about the work carried out in analyzing the nature of tasks and students' mathematical reasoning. Tânia considered that this activity was important. She recognized that many students are able to carry out reasoning processes, such as generalizing and justifying, and criticized teachers for focusing too much on students with difficulties and putting unnecessary limits to the degree of challenge in questions posed to studens:

> I think this is useful, since … We have to enlarge the scope and, it is what we say, we have students that will be doing the exercises, exercise after exercise, but we also have to think that there are students that are able to do much more than that, and we do not have to just think about those that have more difficulties. Often we have classes in which students can do generalizations and justifications.

We also asked the teachers to reflect on the planning, the research lesson, and the post-lesson reflection. The teachers indicated that research lesson was a bit disappointing since the students' responses did not match their expectations and there was no time for the students to carry out all the questions proposed in the task. However, they indicated that there were also interesting student solutions to reflect on. The teachers focused particularly on these in the post-lesson reflection:

> It was helpful to all of us, since we could verify that when we are in a classroom situation sometimes things do not run as well as we expect. And in other issues, the kids surprise us with the solutions that they present … I think it was an interesting class… Different from usual… (Francisca)

Francisca highlighted the unforeseeable nature of the lesson events and mentioned the way that the students often surprise teachers with their solutions. We must note the positive way in which she speaks of the work of the students, which is quite different from her discourse in session 4, when she analyzed the results of the diagnostic test and focused mostly on their difficulties and mistakes.

For Francisca and Luísa, recognizing the value of whole class discussion was a major learning event which occurred during lesson study. This was echoed by their colleague Maria:

> We did not have the time to do everything in the board, but when they explained how they had got there… I think that it is always… Moreover, this was one of the things that I learned in this professional development: We need to bore the kids to death to make them explain how they thought … Now they explain everything bit after bit, and explain in detail. I think that was what this professional development brought to me…

Concerning the four follow-up sessions after the research lesson, the teachers reported that they could put into practice the learning discussed in the lesson study sessions in their own classes:

> I think that is the logical way. Because … This was not just rational numbers, we had rational numbers in the background and we were working more things and we applied now what we learned in the research lesson. We were seeing the possible difficulties, each one

> chose the expressions more adjusted to her classes to verify something… What I think is essential and that we all learned was the discussion. The discussion with our students is the key point of all of this. (Francisca)

Tânia referred that the joint preparation of the classes with her colleagues in the follow-up sessions allowed them to diversify tasks and Maria highlighted the importance of group work and sharing:

> Anyway, just to finish, I wanted to say one more thing: I think that all teachers should participate in a lesson study. Once in their life they should have the opportunity to prepare a lesson in this way, because it is really a different experience and leaves many small things that we take from one session, from another session, still from another session. And then, other thing from these sessions, is listening to other people … I like this interaction with people. Therefore, lesson study for all teachers!…

In their reflection, the teachers put a special value on whole class discussions. They also highlighted the collaborative work and the opportunity to become a working group that included the new teachers at the school, Luísa and Tânia. They pointed out solving mathematical tasks and exploring themes such as the nature of tasks and students' reasoning processes as the main factors that led them to get involved in the work.

In summary, despite some difficult moments in the beginning of the lesson study and in choosing the teacher to conduct the research lesson, the participants recognize its value as a professional learning experience. This includes knowing about the value of different kinds of tasks, looking at students' prior knowledge and difficulties with important concepts, valuing students' strategies for solving problems, generalizations and justifications. The teachers especially note the value of conducting the classroom following an exploratory approach, with emphasis on whole class discussions in fostering students' learning.

Other lesson study experiences that we carried out with in-service teachers followed a similar format, except for the follow-up sessions that only occurred in the three lesson study carried out in the Lisbon school. A very important characteristic of our work with in-service teachers is its collaborative nature (Hargreaves, 1998). As we address teaching and learning issues in lesson studies, focusing on students' learning, we strive to encourage teachers to collaborate with one another both in the working sessions and in small tasks carried out between sessions. We also strive to establish a collaborative relationship between the members of our team, especially the person responsible for leading the sessions, and the participating teachers, and this is recognized by the participating teachers in their reflections.

5.4 A Lesson Study in Initial Teacher Education

We now turn to lesson study carried out in initial teacher education, for prospective mathematics teachers of grades 7–9 and 10–12 that was carried out as part of a fieldwork course of the second semester. The participants in this study were seven

prospective teachers undertaking a Master's of teaching program,[5] the cooperating teacher from a school,[6] and three members of our team. Taking into account the school activities of the cooperating teacher, the lesson study took place in a grade 7 class composed of weak and very weak students.[7] The topic for this lesson study was 'similarity of triangles', which was chosen to take into account the general planning that our team had made for the different phases of the process, the annual subject planning of the cooperating teacher, the pace of her classes, and also the fact that this is a topic in which the students usually face serious learning difficulties.[8]

Taking into account that there were seven prospective teachers involved in this lesson study, it was decided to carry out a process which would involve two research lessons, taking place as consecutive lessons of the cooperating teacher. The lesson study sessions were led by one of the members of our team, who was also the instructor of the university fieldwork course in which the lesson study took place. The two research lessons were taught by the cooperating teacher. This was decided on the grounds that with course time constraints, it was not possible for all prospective teachers to conduct a separate lesson and this would provide an unfair opportunity for some of them. Another reason to make that decision was due to the fact that these prospective teachers were still in an initial stage of their teacher education program, and we considered that they were not adequately prepared to assume the role of conducting the class. So, the prospective teachers were involved in all stages of the planning and reflection process and played the role of observers in the lesson. The prospective teachers were informed that they would participate in the lesson study as part of this course. They had a positive stance but also showed some concern in informal conversations just before or at the end of the session that the time spent in the lesson study could prevent them from doing other activities that could be important in their preparation. As the lesson study progressed they met with all assignments that were established from one session to the next and at the end of the course, but sometimes complained that they were getting too much work to do and that they had several other courses also to attend to.

After an initial presentation of the whole process, the first sessions of the lesson study were dedicated to a deep analysis of the current curriculum guidelines for the study of similar figures and, most especially, of similar triangles. The prospective teachers compared different approaches from textbooks and solved several

[5]This is a professionally oriented degree, that prospective teachers may apply to after they completed a three year bachelors' degree in mathematics or with a strong mathematics component (at least equivalent to two years of study in mathematics).

[6]In fieldwork activities in initial teacher education, the school cooperating teacher receives the prospective teachers in his/her class and carries out activities with them according to a plan established with the university supervisor. This supervisor often participates in the fieldwork activities together with the prospective teachers.

[7]Most of these students had failed one or more years and were behind the expected grade level for their age.

[8]Additional information about this lesson study, highlighting professional learning of the participating prospective teachers may be found in Ponte, Quaresma, Mata-Pereira and Baptista (2015b).

mathematical tasks involving similar triangles, many of which were selected by them and brought to the working sessions. They discussed the characteristics of those tasks and analyzed the possible students' difficulties. As it happens with all our lesson studies, this corresponded to a rather deep mathematical work (analyzing several definitions and approaches to similarity and identifying connections with other topics) as well as didactical work (analyzing different kinds of task that may be proposed to students). The planning was carried out jointly by all participants, in sessions that took place in the university and also in the school, with strong participation from the cooperating teacher.

Two consecutive research lessons were then planned—we called this feature a "double research lesson". The first aimed to introduce the concept of similarity to students and to encourage them to formulate the criteria for similarity of triangles by themselves. The second lesson aimed to promote students' capacity to solve problems using these criteria. The prospective teachers were divided into two groups and all of them observed the two lessons, with each group responsible for analyzing one lesson for the post-lesson reflection. Both the cooperating teacher and the prospective teachers had active roles in planning the two research lessons and in the two post-lesson reflections. The cooperating teacher, who was responsible for the class, took the most important decisions regarding the adaptation of the mathematical content for her students. So, in the preparation phase, she decided to not present the formal definition for similar figures indicated in the official curriculum, as she considered this to be beyond her students' ability to understand. Instead, she presented a mathematically simpler definition which she considered more intuitive for students. She also decided that the criteria for similar triangles, which are the main tools that students have to solve problems on this topic, should be presented in an exploratory way. This contrasts with the usual approach of deducing these criteria from other mathematical theorems about relationships among line segments in parallel lines, as established in the curriculum, that she felt would be very difficult to the students.

The teacher took these decisions based in her knowledge of her students and in her commitment to make the curriculum adaptations that she felt necessary so that students could learn the most important ideas on this topic. The prospective teachers found it strange that the cooperating teacher had so much flexibility in interpreting the official curriculum guidelines but later, as they reflected on the research lessons, they recognized that these decisions were very appropriate to this particular class. In their final written reflections, the prospective teachers recognized that this work helped them to develop a better understanding of the processes of identifying students' difficulties and planning a lesson that strives to promote a strong student involvement.

In the last session of the lesson study, we sought to know how this was perceived by the prospective teachers. One of the aspects that they most valued was the rather intensive work carried out during lesson study in selecting and analyzing tasks. One prospective teacher considered that such work should have gone even further:

> I think it was necessary to work on the tasks. I think it was interesting, yes, I even think that we should have considered more of our tasks [the tasks that the prospective teachers selected], since that part was a little quick. We only underlined some exercises and maybe others had also interest and were put aside. (Cristina)

Another prospective teacher referred to her learning from her participation in lesson study and reflected on the focus of how to teach the topic of similarity. She considered that she knew well this topic, in terms of content, so she did not learn much about it. But she indicates that she learnt a lot regarding its teaching:

> Maybe it was the way of introducing the task more as an exploration… I think that it was more that … Because normally this is not the way we use to work. [It was not the way] we worked even when we were at school. (Amélia)

The prospective teachers highly valued their learning as a result of their participation in lesson study, particularly concerning the types of tasks which should be proposed to students. They also highlighted what they learned concerning the planning process:

> Concerning planning, so, [my view] changed very much. The part of planning was very good for me. And we all discussed, paid attention to small things, and included these in the plan, and prepared what we would do when that difficulty arose, if it were to, I found that very interesting. I was never was taught to planning as we did in this process. So for me, I found that part more interesting. (Amélia)

> That was the first time we constructed a lesson plan. So, I found that sometime we need to make one like this, detailed. And… And, for me, I think it was fantastic, but perhaps this was because it was the first lesson plan that we did. But it enabled me to see all the details that we have to think about. I never thought that our lesson plan would have five pages, not such, I thought that it would be one page. (Cristina)

The prospective teachers also valued the work undertaken during lesson study in order to figure out what different strategies could be used to solve the tasks and to identify the students' difficulties in learning the topic:

> We already knew something, isn't it, but I found interesting the way we made it as group … Anticipated the students' difficulties. And we thought of different strategies. As this was the first time that we did this, since generally we design an exercise, and we design it our way. And in this case there was a confrontation [of different strategies], and I think that we learned a lot, a lot in that confrontation. Because different ideas arose. (Cristina)

> What I found more important for me, in that phase, was not solving the tasks. It was to understand the general difficulties of students… (Amélia)

The exploratory approach, in which learning is based on students' work, was also valued by the prospective teachers that participated in this lesson study. For example, one of them stated:

> Because… Generally, all the books, what they say, is to present the criteria. And I think that… They [the students] learned better by arriving at the criteria, even if there was not much time. If there was more time… I think that this is a topic that in my future I will teach with the exploratory method. So that they are able to get it… (Cristina)

Another prospective teacher referred, in a positive way, to the organization of a lesson in three phases (launching, autonomous work, and discussion), which is characteristic of the exploratory approach:

> I can speak before and after the master's degree. Before, I never had thought how the moments that we now consider were important and did not see how the definition of these moments was very important to manage the lesson, the classroom, and the work that is carried out and the students' learning. I think… That is quite important. (Amélia)

The creation of a collaborative environment is more difficult in the case of lesson study carried out in initial teacher education than in lesson study with practicing teachers, given the differences in status among the participants (prospective teachers, cooperating teacher, university instructor) (Ponte, 2017). With regards to this issue, it is interesting to note the reflection of a prospective teacher:

> In the first session, we were a little quieter, because it was very strange, to get several tasks in front of us, and just have to discuss them. That was a little different. I think that afterwards, as time went on, we began to forget who were the teachers were. Sincerely, I forgot about the camera, it was more a large working group … I think that we then were more productive and better. Therefore, I think that it evolved in a positive way. (Cristina)

In summary, despite some reservations that they could have about this activity, the prospective teachers developed a sense for the importance of tasks in planning a lesson and in anticipating possible students' difficulties in solving them. They become much more aware of adjusting the curriculum orientations to the characteristics of the students and of the value of following an exploratory approach, taking into account the nature of the tasks and the features of classroom communication.

5.5 Portuguese Teachers' Concerns Regarding Lesson Study

For researchers and teacher educators, the features of lesson study make this a very interesting process of professional development. However, for in-service teachers, this process is very different from what they have seen in professional development and can initially seem quite awkward. The practicing teachers who participated in our lesson studies questioned why so many hours are dedicated to working on a single topic or curriculum aim, when there are so many topics in the curriculum that deserve attention—and this was especially strong when they felt pressured to know about recent extensive curriculum changes. The prospective teachers showed a similar concern. Concentrating all of the attention on a very specific issue goes against the usual way teachers look at professional development. In fact, lesson study is not suited to a wide coverage of many topics and curriculum aims, but rather for an in-depth study, taking into account the different aspects that are important in preparing and teaching a topic or curriculum aim—due to its relevance to goals for mathematics education, to the articulation with other topics and aims, to

the curriculum materials and assessment of learning processes. If teachers understand the value of such in-depth work, lesson study may suit as a process of professional development. However, if wide coverage of topics or issues is the major concern, lesson study is not the option to follow.

Other issues that raise teachers' concerns are related to the conduction of the research lesson. Many participants ask if the students get distracted with the presence of so many observers. Some teachers, used to have to deal daily with misbehavior, fear that the students will use the opportunity to show off in front of the observers. In fact, our experience has shown that, generally, after a brief initial moment of adaptation, students quickly get used to the presence of observers and completely ignore them. That does not mean that there may exist cases in which the class—or only one or another student—acts in a way to disturb the work, and so this is an issue that is not problematic in most cases but, of course, requires attention.

Teachers also raise the question of the role of observers being limited, without having the possibility to intervene in helping students. Teachers are used to interacting with students and the role of observer does not fit their professional identity. This role is defined in this way in lesson study, given the purpose of understanding the learning processes and the difficulties of the students in a natural environment. Such analysis requires a collection of data and careful reflection regarding what happens in the classroom during the research lesson and not an immediate intervention during class that, if made by different observers, would break the unity and coherence of the class.

However, the deeper concern that teachers tend to raise has to do with the evaluation of the teacher that conducts the research lesson by their colleagues. Despite the fact that lesson study assumes that the focus of attention is not the work of the teacher but the learning of the students, the activity of the conducting teacher is necessarily highly visible during the research lesson. For the teachers who have had negative experiences in their initial teacher education or in teacher evaluation processes, this is an emotional obstacle that is naturally difficult to overcome. For teachers who have had former positive experiences or feel more self-confident, the problem is not so complicated. This issue tends to be solved based on the confidence that teachers have (or develop) with one another and with the team that facilitates the lesson study.

For prospective teachers several other concerns also exist. In addition to the extra work that participating in a lesson study may require, this activity may also seem strange, especially if they still are in a very initial stage of their professional preparation, since the actual planning, observing and reflection of the research lesson may pose a quite overwhelming challenge.

Besides these issues, particular to lesson study, there are other reasons that tend to reduce the involvement of Portuguese teachers in professional development processes. In fact, in our country, contrary to others, professional development is usually carried out outside their regular teaching schedule, that is, in the free time of the teacher. Also contrary to what happens in other countries, teachers do not receive any reward for their participation in professional development activities,

neither of material nature nor for development of their careers. Frequently, they even have to pay the fees of the professional development. And, finally, the professional development providers, who are university teachers, are often seen as too "theoretical" and not knowledgeable of the concrete realities of the schools, and so what they do in professional development is very limited in its scope and applicability in practice. As we have shown in the two cases presented above, these are not unsurmountable obstacles, but they are contextual conditions that we need to pay attention to. Unless these conditions change in a deep way—by some change in educational policy, notably regarding teachers' career and teacher education—it is natural that they keep putting constraints on the development of this professional development practice in Portugal.

5.6 Conclusion

The most salient feature of lesson study that we have undertaken, in terms of curriculum, is the emphasis on the exploratory approach, with particular attention paid to the development of students' mathematical reasoning (Lannin, Ellis, & Elliot, 2011). In terms of organization of lesson study in the Portuguese context, the follow-up sessions in the lesson study with in-service teachers and the double research lesson in initial teacher education stand out as important factors in introducing this model to a new cultural context. Teachers and prospective teachers alike recognized the value of the exploratory approach, based on tasks that create opportunities for students to make generalizations and justifications and that underlines the value of whole class discussions for knowledge development. The follow-up sessions, despite the fact that they had a circumstantial origin, constituted important learning moments for teachers, leading them to take initiative and support one another in putting new ideas into practice. The practice of revising and introducing tasks in their own classrooms, that took place at this phase, deepened teachers' reflections about the issues tackled during the initial phase of the lesson study. The organization adopted in initial teacher education, in a fieldwork course, proved to be adequate in achieving the aims for these pre-service teachers' learning and has been also used in initial teacher education of physics and chemistry teachers.

In our approach to lesson study, we regard each group as a small collective carrying out research on their professional practice (Ponte, 2008). We begin with the formulation of a research question, carry out a systematic work to prepare an experience (the research lesson), reflect on the outcomes, and strive to apply the knowledge that results from this process to other situations. We do this through combining participants' experiential knowledge (from their past experience and their lesson study experience) with research knowledge (elicited from the activities and discussions carried out in the sessions). We seek to do this work with the teachers also in an exploratory and collaborative way, that is, instead of saying "how the teachers must act", we strive to create situations in which, through their

collective work, they are led to discover how to act. We also provide a clear structure, in which all the work develops, first towards the organization of the research lesson, and then to summarize the conclusions based on it. In summary, lesson study allows to combine exploratory work with a clear structure and to combine knowledge developed by research and teachers' and prospective teachers' experiential knowledge. As such, it stands as a formative format of professional development with high potential to promote participants' learning regarding issues related to students' learning, the interpretation of the curriculum, and also, in an indirect way, regarding issues related to their own teaching practice.

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Chapter 6
Lesson Study in Chile: A Very Promising but Still Uncertain Path

Soledad Estrella, Arturo Mena-Lorca and Raimundo Olfos

Abstract Lesson study was introduced in Chile in 2006. After a specific educational agreement between the ministries of education in Japan and Chile, some of the Chilean ministry's specialists and academics involved in teacher training programs in Mathematics received specific training on Japanese methodologies at the University of Tsukuba. Upon their return, the trainees became involved in reformulating in-service teachers' development programs. Also, a number of renowned specialists from Japan offered public lessons in many places in Chile, so that local teachers could observe lesson study and the open approach for problem-solving strategies. There were promising results, but a change in the nation's government translated into a rather abrupt lack of funding and most of the activities faded away. However, some institutions are still working with the methodology and are expecting new opportunities for a new national strategy to develop.

Keywords Lesson study · Chilean education · Educational policies
Professional development of teachers · Statistical education · Open ended approach

6.1 Introduction

In some countries, lesson study groups have formed spontaneously in different places. The evolution of such groups is varied and is shared and evaluated in ad hoc meetings. In those countries that are comparatively less developed in education, the introduction and development of lesson study tends to be the fruit of a specific cooperation of the Japanese Ministry of Education (MEXT) with the ministry or

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secretary of education of the given country. This is so that a direct intervention in the educational system can be carried out with the collaboration of the Japanese International Cooperation Agency (JICA). This usually includes curriculum modification, specific training for local teachers with the support of Japanese experts, and the elaboration and distribution of textbooks (Isoda, Arcavi, & Mena-Lorca, 2012).

For the last 10 years a somewhat different path has been followed in Chile. In this chapter we report on the introduction of lesson study in Chile and the work of the institutions that stand out the most in this effort. We also review what has been learned in these years, as well as the possible future development of lesson study in Chile.

6.2 Lesson Study in Chile

Lesson study in Chile began in 2006. Chile is a lengthy country, over 4000 km long. It has 17 million inhabitants and over 20 universities that grant teaching degrees (Agencia de Calidad de la Educación, 2016).

Due to its interest in education progression and in joining the OECD, the country had solicited a comprehensive study of Chile's educational situation from the OECD; the results were officially provided in a report (OECD, 2004, 2009). The report provided relevant information in various respects: (1) initial teacher training was too general, lacking in sufficient knowledge of the disciplines that would be taught; (2) the disciplinary and pedagogical aspects were conducted along considerably disconnected paths with the hope, disproven by the data, that future teachers would make a harmonious synthesis of both; (3) the induction of new teachers into the educational system frustrated the eventual intents at innovation that their initial formation invited them to try, both because of the establishment culture into which they were being incorporated and because of the isolation in which they found themselves with respect to their peers; (4) educational research was scarce and did not have a great impact in educational policies; and (5) mathematics and science in the first half of secondary education were taught by teachers who didn't know the material well and lacked confidence in their own ability. It is interesting to consider already how the lesson study methodology allows for dealing with each of these elements of the diagnosis.

The following year the Global Forum on Education, organized by the OECD, took place in Santiago, Chile. On this occasion, representatives from the Japanese Ministry of Education (MEXT) and the Chilean Ministry of Education (MINEDUC) met and signed a collaboration agreement regarding the issues in the OECD Report. The agreement established the program of collaboration 'Improving Mathematics Education in Chile, with the Support of Japan' (with Mathematics considered especially delicate in the Report). Over the course of three years, this would allow for a total of three groups of 10 mathematics professors involved in initial and continuing teacher education from a total of 11 universities, and two

officials from the MINEDUC Center for Pedagogical Improvement, Experimentation, and Research (CPEIP) to travel to Japan to attend intensive courses of training in Japanese methodologies of lesson study, the Japanese open approach to problem resolution as well as other aspects of the Japanese educational system (Mena-Lorca, 2008). Once they returned to Chile, CPEIP would draft terms of reference in harmony with what was learned in Japan for post-graduate diplomas in continuing education in mathematics, and the participating academics would design corresponding programs for their respective universities according to the terms of reference. MINEDUC would finance the work of those teachers who had participated in the approved post-graduate programs.

The program was financed by JICA and carried out at the University of Tsukuba Center for Research on International Collaboration in Educational Development (CRICED); and in Japan Prof. Masami Isoda was in charge. The courses included the observation of classes at different levels, participation in public classes and in lesson study cycles; visits to continuing education centers for teachers, textbook publishers, and producers of teaching support materials; and meetings with representatives from diverse areas of the Japanese educational system. The kindness of the hosts also allowed the Chilean guests to learn about other aspects of Japanese culture.

The participants learned Japanese teaching methods, in particular the open approach to problem-solving (Nohda, 2000). Additionally, they learned about the general system of education in Japan and its intricate, participative, and well-regulated structure, which includes continuing teacher education, curriculum development, and the publishing of textbooks (Isoda et al., 2012). The participants were able to observe that the professional development of teachers is the fruit of a collaborative effort in which the teachers play an active role.

After the program participants arrived back in Chile, they began to carry out diverse initiatives related to lesson study. First, as planned, the CPEIP professionals made suggestions for modifying the terms of reference for post-graduate diplomas financed by MINEDUC, including lesson study in particular, while the participating academics became involved in their respective universities' projects and in the implementation of these post-graduate diplomas. Additionally, each year of the program and every subsequent year, Dr. Isoda and expert teachers from Japan (K. Tsubota, Y. Hozomisu, T. Seiyama, and others) would visit Chile and teach Mathematics public classes to Chilean children at the universities of the participating academics throughout the country.

CPEIP also developed additional activities, one being a series of county workshops for primary school teachers which focused either on Language or Mathematics. The other would be the beginning of a life-long learning program dedicated to creating teacher leaders who are also working teachers and who can give workshops for preschool and early primary school teachers in their educational establishments. These workshops were aimed at designing, carrying out, observing, and analyzing a class following the lesson study methodology. About 300 schools worked on these methodologies.

Additionally, during the four years there were at least a couple of meetings every year for the evaluation and monitoring of the collaboration program. This involved all participants including in situ or virtual participation by Prof. Isoda, the director of the program from CRICED, and members of JICA—in whose Santiago office the meetings were held. There was a formal closing of the program in September 2009, a ceremony that was attended by the Chief Representative of the JICA office in Chile, other office personnel from JICA, the Chief of the General Education Division of MINEDUC, the director of CPEIP, and Prof. Isoda.

The internal evaluation of the program was communicated and commented on during the closing meeting. Both the CPEIP and the participating universities valued the contribution of the program to the primary mathematics education in Chile.

There was also a program evaluation carried out by an external organization, with equally positive results. Its recommendation was to continue the lesson study initiatives in Chile and maintain the academic exchanges with the University of Tsukuba. It proposed that the CPEIP assume greater leadership and take advantage of instances such as county workshops, the network of 'teacher teachers' (leading teachers) created by MINEDUC, and its e-learning platform in order to expand lesson study in the country. Its recommendation to the universities was to systematically employ lesson study and an open approach to problem-solving in initial and continuing teacher education, to establish agreements with local school boards, and to incorporate lesson study and the open approach to problem-solving in its research and extension programs.

With the positive evaluation of the program together with the OECD diagnosis mentioned earlier, it was deemed appropriate to continue with the continued education diplomas and even to increase their number, as there were only around 20 teaching institutions involved that provided classes for a significant percentage of a total of 63,000 teachers (cf. Sanchez et al., 2013). However, in March 2010 the government was replaced by another with a new orientation and the implementation of the post-graduate diplomas was suspended. In their place, the Singapore method (Lee, Goh, Fredriksen, & Tan, 2008) was developed (apparently without the consideration that both methodologies could be used together).

The activities at most of the participating universities became progressively less, including at some of the other institutions that had joined in the implementation of the methodologies due to the program's influence. Two institutions stand out because of their permanent development of the Japanese methodologies and development of local theory: the Pontifical Catholic University of Valparaiso (PUCV) and the Catholic University of Maule (UCM), both with their headquarters outside of the capital. A characteristic that both these institutions share is that when the program of collaboration with Japan began, they were the only institutions in the country that offered master's programs in Didactics of Mathematics.[1] We report in greater detail on the PUCV below.

[1]Currently, there are half a dozen such programs in Chile.

In the UCM, the team of researchers in Didactics of Mathematics elaborated and implemented successful post-graduate programs with a great impact in the region. Additionally, in 2007 it created the 'Lesson study Days' and later organized several of these events, which were carried out jointly with several other universities and in collaboration with the CPEIP. At these events, MINEDUC professionals and academics who had participated in the program of collaboration gave conferences and workshops and reported on ongoing research.

Separately, a group of academics from the University of Antofagasta incorporated various problems and documents regarding lesson study centered on problem-solving into their web pages. Also, the Center for Advanced Research in Education (CIAE)[2] proposed two lesson study projects to the National Fund for Scientific and Technological Development (FONDECYT), which were accepted by FONDECYT. One of these was the work of a researcher from the University of Chile titled 'Integrated Public STEM Classes', which also included work from a Chilean lesson study researcher and invited Japanese experts. Additionally, Santo Tomás University, a private university with campuses in 14 major cities in Chile not participating in the program of collaboration, received training from an academic from the PUCV, and disseminated the methodology and incorporated it into both its program of initial teacher training and into the professional development of its pedagogy academics.

After the change in government, professionals leaving the CPEIP who received training in Japan during the program of collaboration led various projects that included the implementation of the lesson study strategy in classes from kindergarten to fourth grade in language and mathematics, in order to improve classroom practices and positively contribute to students' learning. To do so, workshops were developed with teachers where the teachers met in groups, with each group in charge of designing a class with relevant material. One group member was chosen to implement the class, observing the class in situ or, if necessary, remotely, and analyzing the class in order to determine difficulties and make adjustments to redesign it.

6.3 Lesson Study Based in the PUCV

Four academics from the PUCV Mathematics Institute participated in the program of collaboration between Chile and Japan. They have all used this training in their work within the institute and also in their relationships with schools. Two of the academics, authors of this chapter, continue to be active in this work. Additionally, various other academics from the PUCV have joined the effort based on their respective activities.

[2]The CIAE is an institution created by the University of Chile, the PUCV, and the University of Concepción (located in the southern part of the country). The authors belong to the institution.

The goals are: that lesson study be the object of research; to document the impact of lesson study in teachers' professional development and more generally in the country's mathematics education; that the methodology be part of initial and continuing teacher education; and that its use be disseminated throughout the country and where possible throughout Latin America.

It begins with an explanation of how the institute approached lesson study, it continues with a general overview of what has been done, and ends by showing two classes prepared using this methodology.

6.3.1 Lesson Study in the Institute

The philosophy of the PUCV Mathematics Institute in education comes from its development of didactics of mathematics (DM), as it offers the only doctoral program in DM in Chile and neighboring countries. The institute tries to pay attention to both the development of science and to international experiences, but its focus is always on the need to acquire one's own experience and to develop theory as a way of addressing the country's problems adequately and autonomously. A natural consequence of this philosophy is that the PUCV academics who participated in the program tried to connect lesson study with the research they were developing.

The institute understands DM as an experimental discipline, with explicit theories and methodology—such as the Theory of Didactic Situations and the Didactical Engineering (Chevallard, 1992; Douady, 1995)—that come out of reflection, study, and the search for evidence based on the phenomena that occur when people teach and/or learn mathematics. This has allowed for uncovering and dealing with a series of phenomena which had remained hidden to general mathematics education research (Douady, 1995). It also considers DM as a solid bridge that in a disciplined way unites the general pedagogical and mathematical aspects in play in learning and in teacher training, such as in the Mathematics Work Space frame (Montoya, Mena-Lorca, & Mena-Lorca, 2015).

The institute had studied and utilized the Didactic Engineering (DE) methodology of French origin, which appeared in DM in the 1980s (Douady, 1995). DE is considered fundamental to DM (Artigue, 1995) and has its origins in the consideration that the habitual methods of research in mathematics education are insufficient for dealing with the complexity of the problems that occur in a classroom (Artigue, 1995). Additionally, these methods are part of a determinist logic that runs the risk of ignoring the concrete functioning of a 'didactic system' constituted by students, teachers, school, mathematics (Chevallard, 1985), and whose validation is external based on the statistical comparison of the performance of experimental and control groups (Artigue, 1995). On the contrary, the validation of DE is essentially internal: the hypotheses formulated in research are based on the confrontation between a priori and a posteriori analysis, as defined by Artigue (*ibid.*). DE considers four stages (Artigue, 2009). 'Preliminary analysis' includes the epistemological study of the contents under consideration, its habitual teaching

and its effects, students' conceptions, the difficulties and obstacles they face, and the field of restrictions in which effective didactics will be carried out, etc. In 'conception and a priori analysis' of the didactic situations (Brousseau, 1997), the researcher decides to act upon the variables the teacher controls and that the researcher perceives as pertinent in relation to the problem being studied. The next phase is 'experimentation'. In the 'analysis a posteriori and evaluation' stage, the data collected in the experimentation stage is reviewed and often completed with others obtained through the utilization of external methodologies.

The researchers at the institute have reflected that DE, as well as some theories of DM, have what they call an 'epistemic vigilance device', which is there to ensure that the study is carried out without falling into unfounded speculation. As it is easy to see, lesson study possesses an analogous device with stages similar to those of DE (Miyakawa & Winslow, 2009), and the most noticeable difference between them is the purpose of research on one hand and the purpose of professional development on the other.

It is based on this perspective of studying and articulating DE and lesson study that the PUCV undertook its participation in the activities of dissemination and development of lesson study (also Clivaz, 2015). This participation aims at taking advantage of the theoretical and practical perspective that is part of the habitual instruction in undergraduate and graduate studies in education in the institute, in such a way that lesson study can also feed off of these elements. Thus, institute alumni could integrate both theoretical-experimental elements of Didactics of Mathematics and methodological elements of lesson study, so that in their in-service performances they would be able to use them in their own professional development and also to offer them to their peers.

6.3.2 Programs of Study

Since 2006, in the post-graduated diplomas of professional development—which were related to the program of collaboration between Chile and Japan and financed by the CPEIP until 2010—some elements of DM and lesson study have been offered. The teachers also work collaboratively in classes following the lesson study methodology.

At the same time, the master's degree in DM that the institute offers was changing its orientation to be less focused on research and more focused on professional development. It was decided that the students' final projects should use the lesson study method. In the third semester (of four), they are trained in lesson study and they work collaboratively in a lesson study process. During the fourth semester, they work on their individual final projects utilizing the lesson study experience and going into greater depth in some theoretical aspects, whether in mathematics, professional development, educational extension efforts, student learning, or some other topic.

In Chile, each teacher's training program is decided by the university (or sometimes other institutions) that offers it. In the case of Mathematics, the proportion of general educational and mathematical subjects varies. At the institute, students also receive training in DM. Since 2012, the curriculum includes elements of lesson study in combination with the last semester graduation activities; the 'Mathematics education workshop' and the professional practicum have been based on lesson study.

Lesson study is also a topic of study in the doctoral program in DM at the institute, which has existed for six years and has approved three theses that consider lesson study as a tool for study and intervention.

6.3.3 Research and Development

Several academics at the institute either direct or participate in projects financed by the government that utilize lesson study as a strategy for research and development. Their topics deal with teaching statistics, teaching mathematics in preschool and primary school, and tools for initial teacher training.

Currently, there are two doctoral theses in process regarding lesson study: one related to improving initial training in integral calculus, and another related to the professional development of working teachers.

Since ending their stay in Japan, the authors have also been participating in the successive instances of a long-term project financed by APEC and directed by the University of Tsukuba in Japan and the University of Khon Kaen in Thailand.

Recently, the project has dealt with emergencies and preparations for future disasters, which has led to the institute elaborating classes that make reference to earthquakes, tsunamis and fires. We comment on one of these efforts below.

6.3.4 The School System

The transfer of lesson study to the school system has been tried in a number of diverse ways. In 2007, academics from the institute presented the annual project 'County Workshops' to MINEDUC, which financed it. MINEDUC had been carrying out a project in which it highlighted the leading teachers (the 'teacher teachers') in counties throughout the country. The PUCV project consisted of providing lesson study training for more than one hundred of these teachers, from all over the country. They were convened for three periods of training in Valparaiso and Santiago, where they learned elements of DM and worked collaboratively on lesson study. On returning to their home counties, they disseminated the methodology and led the preparation of classes in teams. There was also an online support platform. Once the training periods were completed, the academics who led the workshops visited the teachers trained in the methodology, who were spread out

over 3500 km, to observe the progress of the classes whose preparation they had led in their respective counties. The work was being carried out as planned but it ended up abruptly with the change in government.

Lesson study groups still continue in schools where they began several years ago. This has occurred in three different ways. One is that a lesson study group has been formed spontaneously in a school but receives support from the institute. Another is that graduates from the post-graduate diploma have solicited support to start a lesson study group in a school in another county, a group that expands to a group of schools and eventually to specialties outside of mathematics—a primary school teacher graduated at the institute leads a multidisciplinary group at her school. The third is that an academic has introduced lesson study in a school and leads a group of teachers in that establishment.

6.3.5 Dissemination Activities

Thanks to generous invitations from Prof. Masami Isoda, the authors up to now have been visiting scholars at the University of Tsukuba and have written several books. One such book describes lesson study and the open approach to problem-solving from various perspectives (Isoda et al., 2012). It is a commented version of an original Japanese text that continues with experiences carried out principally in Chile and in other Latin American countries (which were accumulated over the course of three successive editions) and analysis of public classes taught by Japanese teachers in Chile. Another is about teaching under an open approach to problem-solving (Isoda & Olfos, 2009), and the third is about teaching multiplication (Isoda & Olfos, 2011). These books have been distributed in various Latin American countries and have been used in courses for introducing teachers to the lesson study methodology.

Additionally, together with another researcher from the CIAE, a book on mathematical thinking and its development in the classroom (Isoda & Katagiri, 2014) was translated. The authors also collaborated on Spanish versions of Japanese primary school textbooks published in Mexico (Isoda & Cedillo, 2012).

Beginning in 2006, a dozen public classes were organized in the region with visiting Japanese teachers, and later more public classes were elaborated by Chilean teachers and academics from the institute.

Additionally, the authors as a group have presented conferences about lesson study in more than 20 universities in Chile and in half a dozen other Latin American cities. Another important activity is the permanent presence in national and Latin American conferences, from Argentina to Mexico, in which the methodology is presented and research advances are reported on.

6.4 Lesson Study and Statistics Education in Valparaiso

Here we focus on one of the lesson study groups formed around the Mathematics Institute, which develops statistics classes in preschool and primary school, and classes prepared by Valparaiso school teachers and academics from the PUCV.

The teaching proposal adopted by the teachers participating in lesson study groups in Chile has been in concordance with the recommendations of the *Statistical Education of Teachers* (SET) report of the American Statistical Association, Franklin et al. (2015) and others that preceded it (Franklin et al., 2007; Ben-Zvi & Garfield, 2004), which are to develop *statistical reasoning*, promoting the knowledge of statistical contents and pedagogical content knowledge necessary for teaching statistics. As Franklin et al. (2015) indicate, ideally, statistics education should propose the development of statistical thinking and conceptual understanding through active learning and the exploration of real data. The report recommends that the fundamentals of statistical literacy should begin in the first years of school, from pre-K to fifth grade, when student begin to develop data sense—the understanding that data are not simply numbers, categories, or images, but entities in context that vary and that can be useful for answering questions about the world around them.

Various experiences of teaching statistics have been carried out in different schools in Valparaiso with the lesson study methodology. In these experiences, the groups of teachers have integrated teaching models specific to statistics content, such as that proposed in Tukey's (1977) *Exploratory Data Analysis* (EDA) and the investigative cycle known as PPDAC (Wild & Pfannkuch, 1999), in a lesson study cycle that contemplated three implementations of the lesson over a period of two months with weekly meetings. All the lessons have been planned for a duration of 45 min and the courses in which they were implemented have an average of 35 students.

The groups of teachers and researchers have worked together in a lesson study PPDAC cycle in which they collaboratively formed the goals for the students' learning and development. The researchers have encouraged the inclusion of cross-curricular goals (healthy eating, physical activity, sleep, among others) and the awareness of Chile's climatic, geographic, and geologic characteristics, according to general recommendations given by the Chilean Ministry of Education (MINEDUC, 2009).

Each of the lesson study groups specifically planned a lesson to put the learning goals into practice. A school teacher then administered and implemented the lesson as planned, and evidence was gathered about the students' learning and development. After the lesson had been implemented, the group of teachers reflected on and discussed the evidence gathered during the lesson, using this evidence to improve the lesson and their teaching practices and to implement the lesson again (Estrella & Olfos, 2013; Isoda et al., 2012; Isoda & Mena-Lorca, 2009; Isoda & Olfos, 2011).

6.4.1 Integrating Teaching Models in Lesson Study Cycles

Exploratory data analysis and the PPDAC (problem, plan, data, analysis, and conclusions) investigative cycle were the teaching strategies that would allow the teachers to involve themselves and the students in learning experiences with data in order to develop number sense and data sense. The goal of EDA is the exploration without restrictions of the data, in search of interesting and unsuspected regularities; the conclusions obtained are informal as they are based in what is perceived in the data and are only applicable to the specific subjects and circumstances which provide the data. EDA uses numerical representations and summaries to describe the variables of a data set and their relations (Moore, 2000). Figure 6.1 shows the lesson study cycle as actually formulated (as a variation of the one given in Isoda et al., 2012), and used by the group of teachers who supported each other in their considerations regarding learning statistics and mathematics and made professional decisions in order to design their lesson plan.

The lesson's learning goals concerned the statistics domain of the mathematics subject in the Chilean curriculum for primary school, which were related to the construction of representations from data and decision making based on data analysis. As part of the process foreseen by the teachers, the students began with the exploration and ended with the presentation and explanation of their representations of the data to their classmates. All the lessons implemented by the teachers in the lesson study group have activated the students' modeling competences, which include the capacity to identify the relevant question, the variables in play, relations in the real context and the translation, interpretation, and validation of the solution with respect to the variables' context.

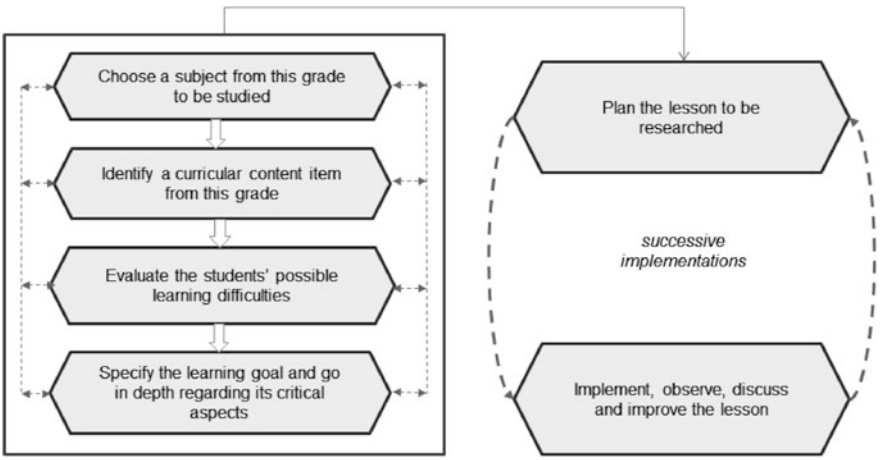

Fig. 6.1 The lesson study cycle implemented

The following details two of the statistics lessons studied. One carried was out in 2015 on the physical activities in kindergarten, and the other in the fifth grade in 2012 on the tsunami that affected Chile in 2010.

6.4.2 *Preferred Recreational Physical Activities Lesson in Kindergarten*

The preparation of the lesson was carried out by four teachers, who were all from the same school, and four researchers. The lesson's central question was, "How can we organize the data about our preferred recreational activity in order to know which are the preferred physical activities of kindergarten B?"

The lesson, integrating EDA and the PPDAC cycle, was designed by the teachers. They also created a problem situation that incorporated some of the key ideas in statistics education: real and motivating data and representations, with the particularity of being able to see the data as an aggregate (from the individual data to the group data). The students were grouped in teams of three children who organized the data, constructed representations, and argued for their conclusions with respect to the aggregate data, managing to understand data as numbers in context. This showed that some children are able to achieve statistical reasoning (Ben-Zvi & Garfield, 2004) early in the preschool classroom.

The teacher provided an environment of free work and investigation, and although not all the children in the kindergarten related the data to the question, as they could only represent part of the data set in a disaggregated manner and did not achieve the step from the individual data to group data, two of the eight groups of students were able to see the data as an aggregate and correctly graphically represent it. These six students (from a total of 27) were able to create a representation and compare the data, make a count, write the frequency and indicate the cardinal of a variable category as the frequency in order to respond to the central question and communicate it to the whole class. In Moment 1 (see Fig. 6.2) the students give evidence of categorization of the variables and therefore of implicit comprehension of the variable (the class's favorite sports activity) by showing their data representation through six activities classified by type (jumping, running, skating, bike riding, and playing both basketball and football).

Communicating to the other students of their findings and reasoning based on the representation and frequency allowed the other students in the class to be able to give meaning to numbers and data. The representation and the frequency allowed them to answer a question that they were interested in with their own preferred activities, and in doing so recognize the class's favorite recreational activity and allowing them—as previously offered—to have a special recess at class.

This experience of teaching statistics in preschool allowed the students to learn significant issues about working in groups, as well as be able to organize data and represent it, and to progressively acquire more tools for representing data. The

Fig. 6.2 Preschool pupils' data representation, and the moment a boy calculate and record frequency

participating teachers indicated that this way of working allowed them to carry out an excellent class, as one of them stated, "The students were able to organize and interpret data in diverse ways, and the vast majority participated with their own ideas and presented them to the course." They considered that the successive implementations of the lesson, two in first grade and two in kindergarten, "allowed us to improve the class proposal"—as another teacher put it. They stated that in order to "improve [the lesson plan], we planned to achieve the participation of all the children, and we changed some things [materials, question] so that they were all able to organize the data and display them in the manner they chose." This is how the idea of creating a prior class to present the question and motivate the children was born among the teachers. Also, the teachers concluded that the lesson study process allowed them, as educators, to "open their minds to mathematics", to "work as a group", and that "you can always improve what you've already worked on". One of the teachers concluded this experience of planning a class in a lesson study group as "a professional challenge that motivates me to try to carry out more classes like this, to relate the subject more with subjects that are more interesting and significant for the children, to not give them the answers and not tell them what they should do."

6.4.3 Tsunami Lesson

In February 2010 Chile was badly affected by a powerful earthquake, one that is considered to be the second largest in the history of the country and one of the five most powerful on record in the world. About 35 min after the earthquake there was a tsunami. Due to errors and indecision by those organizations in charge of sounding the corresponding alarm, the country's population was not alerted to this event. The lack of (cultural) preparation in dealing with earthquakes of such

magnitude led to more deaths being caused by the tsunami than by the earthquake. Some collective Chilean myths were that 'a tsunami is just one big wave' and 'there is a fixed period between one wave and another', which led some inhabitants to return prematurely to their homes on the coast, contributing to more deaths caused by the successive waves.

Faced with the importance of education in the face of natural disasters, a lesson study group chose the topic of the tsunami with three goals: (1) that the children 'do statistics' with representations of real data, (2) that the children learn knowledge that helps their community [helps to inform to save lives], and 3) that the teachers modify or consolidate their beliefs with respect to the notion that teaching statistics is not exactly 'teaching mathematics' but rather working with data as numbers in context (Ben-Zvi & Garfield, 2004). The lesson centered on an image (see Fig. 6.3) from a news article (Diario El Mercurio, 2012) about the tsunami that affected Chile in 2010 (Estrella, Olfos, & Morales, 2014). The preparation of the tsunami lesson was carried out by three classroom teachers and three researchers with the goal of tearing down the myths cited above. During the two month period, the lesson study group met weekly to prepare the lesson plan, test and improve it, and implement it again. The meetings of the teachers participating in this lesson study group were explicitly focused in a framework that promoted planning a class centered on a real statistical problem, fomented statistical argumentation, facilitated a space for discussion and communication of results, and favored the development of a deeper and more meaningful understanding of statistics. This was proposed with clarity and drove the teachers to maintain high cognitive demands so that the students would achieve statistical reasoning, 'doing statistics'. The researchers promoted the EDA paradigm so that the teachers became aware that a statistical analysis begins with an exploration open to discovery and ends with the presentation of the results through representations of data. This exploration involved delving deeper into the context, the knowledge of the data sources, and the inspection of the data, and in an objective public communication of the results in the real context.

In the third implementation of the tsunami lesson (detailed below), the teacher began by familiarizing the fifth grade students with the tsunami and its effects. He presented the problem, saying, "I found this image in a newspaper about the natural disaster of 2010; it has a lot of data, so I'm asking you all to organize it in a simpler way." Then the professor presented and wrote about the challenge of "extracting data and organizing it to communicate to the cities that were affected and cities that could be affected by tsunamis."

The competencies of statistically modelling the presented situation allowed both the teachers involved in the lesson study group and the students to develop the ability to identify relevant questions and the variables in play, their scale and measure to establish relationships with the real world, to translate and interpret representations, and to validate the information found in the data in relation to the situation in order to communicate it as a solution to the central challenge.

To confront the myths that had cost lives in the tsunami, the teacher handed out a copy of the image shown in Fig. 6.3 to each student. He then managed the moment of discussion and observation of the representation of data through the questions: Is

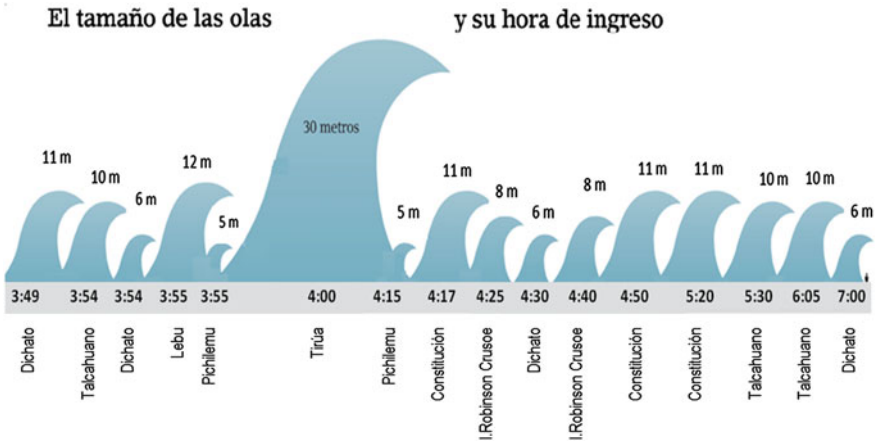

Fig. 6.3 "Size of the waves and their hour of entry" (up), by city (names down); image from Diario El Mercurio (2012), a national newspaper after the 2010 Tsunami in Chile

the number of waves per city predictable? Is the time between one wave and another predictable? Is it possible to save lives with this data? (These are open questions; the data shown in Fig. 6.3 suggest that prediction would not be accurate.).

During the lesson, all of the students constructed graphic and tabular representations of different types, explained their ideas to their classmates and argued for them. The data analysis that the students provided based on the data in the image showed that the number of waves per city was between one and five, and that the minimum time between one wave and another was five minutes, while the maximum was 180 min. With this lesson focused on tearing down myths, the teacher managed to promote informed decision making for protecting people in the face of natural disasters.

6.5 Results

6.5.1 What We Have Learned

In the PUCV we have researched what the lesson study methodology means to the teachers who participate in it and also to what degree lesson study is a response to the general diagnosis of Chilean education made by the OECD in its report (OECD, 2004). To do this, we base our work both on the testimony–either as results of questionnaires and clinical interviews—of teachers who have learned the methodology and applied it in their schools according to their possibilities, and on our own analysis of teachers' and students' learning in classes such as the ones reported on above. The relation that we make below explicitly includes the principal aspects of this report and of the Chilean experience.

- *Excessive general teacher training, without sufficient knowledge of the disciplines they teach*: The teachers declare that they learn mathematics through lesson study and that it is interesting to learn mathematics with their peers.
- *Lack of connection between discipline-specific and pedagogical aspects*: The teachers especially value that some of them know more about the discipline while others know more about the methodology of classroom management, pedagogical content knowledge, students' reactions, etc.; and they appreciate the opportunity to have discussions that do not only deal with administrative aspects of their work.
- *The induction of new teachers in the system frustrates their initiatives of innovation*: This variable is difficult to perceive directly and of course it would be necessary to have broader data. However, the experienced teachers who were consulted considered that this difficulty lies in the isolation in which a teacher works with respect to his or her peers, and they are in agreement that a strategy of collaborative work such as lesson study is the scenario that will allow for innovation and peer learning, especially in a circumstance in which there is not an evaluation they have to respond to.
- *Scarce educational research and without great impact in educational policies*: Chile has advanced significantly in educational research, and the government has financed a Center for Advanced Education in Education (CIAE), under the direction of three important universities. Although in a somewhat more limited perspective, those teachers who have had greater training in lesson study invariably ask how they can publish their experiences and collective findings to benefit the community.
- *In the first cycle of secondary education (grades 6–8), mathematics and statistics are taught by teachers who do not know the material and who lack confidence in their own performance*: While the number of teachers with lesson study training is comparatively small, they all (even those who are more proficient in Mathematics) declare that they know more mathematics after having done lesson study, independently of the level at which they work (preschool, primary, or secondary school) and that they feel more confident regarding the knowledge they are responsible for teaching.

6.5.2 Future Perspectives

The authors are convinced that allowing groups of teachers to use a few hours per week in lesson study is an alternative for professional development that greatly surpasses other strategies of continuing education for teachers, not only because of the more comprehensive and long-lasting professional learning that it brings, but also because of the horizontality of the methodology—they do not have to learn something that a supposed expert suggests, but rather create their own collaborative path among peers—and also because it generates greater reflection, confidence,

protagonism, and commitment. This is what they have made known in the evaluations of the program of collaboration between Chile and Japan, and they have insisted on it during every occasion in which the issue has been debated. Of course, they are not the only ones convinced of this, and they have the expectation, not unfounded, that the educational authorities will once again privilege the lesson study strategy for professional development of teachers in Chile.

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Chapter 7
Theorizing Lesson Study: Two Related Frameworks and Two Danish Case Studies

Carl Winsløw, Jacob Bahn and Klaus Rasmussen

Abstract Lesson study refers to certain well-established professional development practices for teachers in Japan. Over the past 30 years, the phenomenon drew the attention of scholars in other countries, and their writings have inspired several 'movements' of lesson study implementation. As scholars observe both successes and difficulties in these endeavors, the need arises for finer methods to characterize and monitor the processes and objects which go into what is broadly referred to as lesson study. This brief presents an overall characterization of lesson study in terms of the notion of paradidactic infrastructure, in relation to specific adaptations of two related theoretical frameworks. We argue that the use of these frameworks can help sharpen researchers' understanding of lesson study as a phenomenon. We exemplify the use of these tools with cases from our own work on pre- and in-service teacher development in Denmark.

Keywords Lesson study · Theory of didactic situations · Anthropological theory of the didactic

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7.1 Why and How Should Researchers Theorize Lesson Study?

In this section, we argue for the necessity of increased attention to theoretical precision in research related to 'lesson study', including more precise identifications of what practices the term itself refers to, and not least to describe and analyze essential parts of what happens in a lesson study. We also present two theoretical perspectives which we have found useful in this respect.

7.1.1 Why Theorize Lesson Study?

'Lesson study' was introduced into the Anglophone educational literature in the second half of the 1990s as a translation of the Japanese notion *jugyou kenkyuu*. Stigler and Hiebert's (1999) bestseller *The Teaching Gap* hugely increased the impact of an idea which had otherwise mainly appeared in scientific papers just a few years before. Among these, Lewis and Tsuchida (1997) do not use the term 'lesson study' (corresponding to the Japanese *jugyou kenkyuu*) but instead report on the phenomenon of 'research lesson', a translation of the Japanese term *kenkyuu jugyou*. They underline and exemplify that "research lessons take several distinct forms" and subsequent scholarly work—beginning with the volume by Fernandez and Yoshida (2004)—have presented case studies of some of these. Lewis (2016) further discusses various types of lesson study in Japan.

Soon after the publication of *The Teaching Gap*, efforts to introduce lesson study in the United States and other (mainly) English speaking countries began to emerge, and with these efforts more researchers (outside of Japan) became involved in the development and research associated to lesson study. Widely used handbooks on 'how to do lesson study', such as Stepanek, Appel, Leong, Mangan and Mitchell (2007), present detailed instructions for the different 'phases' of lesson study, usually as elements of a four or five step cyclic process of preparatory study and planning, observing, discussing and revising a lesson. Such publications evidently helped support the increasing number of lesson study groups in many countries, and it also made lesson study accessible as a phenomenon and experimental device to a much larger number of researchers. At the same time, the efficient marketing and dissemination of a phenomenon as foreign as *jugyou kenkyuu*, together with the further adaptations which are often made by novice practitioners, inevitably lead to a certain distance between the resulting practices and the Japanese original. At the same time, lesson study was certainly not promoted as something which could take several distinct forms, but rather as a definite type of teacher-led activity which should follow the steps indicated and is explained in the guidelines.

It did not take more than a few years of American experiments with lesson study before Lewis (2004, p. 132) noted that "many emerging examples of lesson study in

the US diverge substantially from lesson study as it is practiced by Japanese" and pointed out that:

> The graveyards of U.S. educational reform are littered with once-promising innovations that were poorly understood, superficially implemented, and consequently pronounced ineffective. If lesson study is to be any different, it will require a deep understanding of what it is and why it has been useful to Japanese teachers, and how it can be adapted to the very different setting of the US. (Lewis, 2004, p. 134)

For researchers and lesson study promoters alike, this evidently raises the questions of what this deep understanding would consist of, what it would be an understanding of, and who should have it. Given the claim of a manifest distance between lesson study in the US and in Japan, it seems natural to interpret the quote in such a way that the deep understanding would concern lesson study as it is practiced in Japan; however, most teachers and researchers abroad could only read about this in the sources already outlined, not observe and participate in the original (model) activity themselves. Indeed Lewis (2004, p. 134) considered that "the likelihood of success [*of lesson study*] would be increased by the participation of Japanese educators who could help to figure out the essential qualities of lesson study". There is no doubt that the involvement of US based Japanese born experts has had an impact on parts of the implementation of lesson study in that country (Fernandez, Cannon, & Chokshi, 2003), and possibly also elsewhere. But in general, promoters of lesson study are not likely to be satisfied with this solution, though in most countries outside of Japan lesson study would remain a very marginal activity if it required the frequent or even occasional participation of Japanese experts; and even if available, the needs for adaptations and explicitness of what lesson study is or could be would remain.

Additionally from a research point of view, there is something fundamentally unsatisfactory about knowledge remaining simply individual 'understanding' linked to people and their personal experience (or even national background), rather than being explicitly and accessibly described. It also seems to be a bit paradoxical, given that one of the often cited features of lesson study is to create shared and documented knowledge, rather than (just) private experience and wisdom. It has even been claimed that:

> Lesson study also provides mechanisms for teachers to move squarely into Popper's World 3 -developing knowledge that is intended for public discussion and examination. (Hiebert, Gallimore, & Stigler, 2002, p. 10)

If teachers' 'understanding' of teaching should be amenable to move from individual skills and beliefs to public and accumulating knowledge, and lesson study supplies important avenues to facilitate that move, it seems hard to accept that these 'mechanisms' could not also be subject to public and transparent analysis and description.

The various activities called lesson study (or, depending on the country, *jugyou kenkyuu*, or *lektionsstudier*, etc. …) are clearly phenomena which are contingent upon cultural and institutional conventions. There is obviously no straightforward 'model' which applies to them, but on the other hand, without explicit models we are left with

practices that are at best understood by the participants themselves, but remain more or less opaque to outsiders. Simplistic models, such as the cycle or step-by-step descriptions, clearly miss many essential aspects of lesson study including its purposes in terms of promoting teacher and student learning at various stages of the study. More importantly, a theoretical framework (with explicitly defined categories and terms) is needed to move the analysis of mechanisms and principles of lesson study away from the culturally contingent narratives about lesson study, with which the literature abounds. Also, research on lesson study needs to strive towards a more international and explicit stance (or, if you want, "move squarely into Popper's world 3"). In scientific terms, this requires more precise models of what lesson study is and is about—based on theoretical frameworks which are shared and developed by researchers. As in all modeling, the point is not to achieve complete or non-reductive descriptions and analyses, but to make assumptions and *foci* explicit and open to scrutiny. Thus, we need to get beyond 'lesson study' as if it is one well-defined package of activity, and as if we (or at least someone) understand what it consists of without having to describe it further than the few overall stages it involves.

In the sequel, we refer to lesson study as the (broad class of) teacher-led activities, which are referred to by this name in the Anglophone literature, or by practitioners themselves. Certainly this literature contains a number of 'local' (ad hoc) models of what lesson study is and how it works (e.g. Lewis, 2016). But we consider it an important and largely outstanding task for researchers to capture its elements, boundaries, typology and prerequisites within well-established theoretical frameworks for research in mathematics education, which could also help relate the phenomena to other forms of teacher professional development. This task should not be confused with the (equally important) efforts to make use of general theoretical frameworks, such as variation theory, as a tool *within* lesson study (e.g. Huang, Gong, & Hang, 2016).

7.1.2 Types of Research Related to Lesson Study

A quick search on the database MathEduc (https://www.zentralblatt-math.org/matheduc/) shows that in the mathematics education journals referenced by that database, about 165 papers (March 2017) have been published with 'lesson study' as part of the title (while broader searches suggest a total number close to 300). Many of the most recent papers on the topic are from a special issue of the German journal ZDM (vol. 48, no. 4). Browsing through these papers, we can identify three types of research:

1. Papers describing and analyzing what lesson study is in Japan;
2. Papers describing and analyzing what lesson study is or could be in other countries;
3. Papers reporting on experimental research using lesson study as a method to investigate specific questions related to mathematics education.

Table 7.1 Four kinds of research related to lesson study, with sample papers reporting on it

Examples of work	OBJECT: Lesson study in Japan	OBJECT: Lesson study (and the like) outside Japan
Descriptive research: lesson study as a research object	Lewis and Tsuchida (1997) Fernandez and Yoshida (2004) Lewis (2016)	Lewis (2004) Rasmussen (2016)
Intervention research: using lesson study as a method	Mostly in Japanese; some translated papers were presented in JSME (2000)	Bradshaw and Hazell (2017)

Some papers can be hard to classify, but it is clear that the second type is by far the most common. The first appears mainly before 2010, while English language papers of the last type are rare and appear only very recently (this could of course be due to 'lesson study' not appearing in the title of papers where lesson study is just the method, not the object of research). Notice that teachers' research (*modulo* all the debates there could be about what constitutes 'research' in the academic sense) would fall into the third point as above, and is more commonly published in Japan than in other countries. Looking at the literature more broadly, we propose a rough classification of the research field as shown in Table 7.1.

In the descriptive research strand, the researchers are not necessarily initiators or participants of the lesson study activity. Such studies are basically about what lesson study *is* and *how it happens*—for instance, what lesson study consists of, how it works in particular contexts, and what it takes to implement (the latter, of course, mostly in studies focusing on non-Japanese contexts). From a scientific viewpoint this clearly leads to a need for developing precise *models* of lesson study; this necessitates a theoretical framework to answer questions not only about lesson study as an activity, but also about the *objects* of this activity. Intervention research using lesson study as method may draw on such a model in order to explain and justify the methods used, but will otherwise focus on theoretical models related to the didactical research questions investigated (e.g. the study by Bradshaw and Hazell (2017) of students' different approaches to solving mathematical problems). Nevertheless, in contexts where lesson study is not a well-known working method for teachers, some kind of theoretical description will be needed in this case too.

7.2 Two Theoretical Frameworks for Research on Lesson Study

We now describe how two related theoretical frameworks (both originating in French didactics of mathematics) can be used to model lesson study: the theory of didactic situations, which allows researchers to analyze the fine mechanics of a lesson and (in our adaptation) also those of the teachers' learning from studying it;

and the anthropological theory of the didactic, which proposes a more global viewpoint on the institutional conditions and constraints which affect the realization of lesson study as a 'paradidactic' activity.

7.2.1 A Global Theoretical Perspective: The Anthropological Theory of the Didactic (ATD)

An ATD approach to lesson study takes as a point of departure two key notions of the theory, originally formulated by Chevallard (1999) and outlined as far as needed for this chapter below:

- *praxeologies* used to model human practice and knowledge
- *institutions* as ecologies (habitats) of praxeologies, formally structures of positions which human may occupy relatively to the exercise and development of praxeologies.

Here, a praxeology consists of *praxis* (the types of tasks, needs or questions which the practice aims to deal with, along with the techniques or actions used to do so), and of *logos* (the discourse about the practice and similar practices—roughly, 'theory'). The most relevant categories for mathematics education are *mathematical praxeologies* (where the tasks are 'mathematical') and *didactic praxeologies* (where the tasks are about teaching specific mathematical praxeologies). Focusing on the latter, the *institutions* in which didactical praxeologies are carried out are basically what we call schools (with the well-known variety), and it is also clear how different positions—most notably *teacher* and *pupil*—exist in these institutions, relative to didactic praxeologies, and are fundamental to their workings.

Lesson study is not simply didactic practice, but it is a *study* of didactic praxeologies—including a specific didactic *praxis*, the lesson which the participants plan, observe and reflect on. But these three fundamental *tasks*—planning the research lesson, observing it, and reflecting on it, are *not* didactic practice—they are *about it*. Similarly, any teacher can engage in praxeologies which are very closely related to specific teaching tasks (and more broadly didactic praxeologies), but which are not themselves didactic (the didactic ones being teaching tasks, techniques etc.); for instance, planning lessons, evaluating students' results, etc. We call these praxeologies of the teacher *paradidactic praxeologies*. Following Miyakawa and Winsløw (2013), we distinguish the different paradidactic praxeologies which teachers may engage in and develop in relation to a specific didactic practice, according to whether they occur *before* or *after* the didactic practice, or *at the same time*. These are called, respectively: *predidactic praxeologies*, *observational praxeologies* and *postdidactic praxeologies*. Notice that these notions are very general and do not only concern lesson study or mathematics teaching.

In lesson study, the predidactic praxeologies sometimes involve a group of teachers, and possibly an adviser. It is characteristic for lesson study that predidactic

praxeologies involve the study of a number of materials, formulating precise didactic tasks, techniques and hypotheses about how they will affect the students' development of mathematical praxeologies in the lesson. It is also characteristic of lesson study that observation praxeologies involve other teachers rather than the person who is engaged as teacher in the didactic praxis itself, and that these teachers participate in a specific postdidactic praxis where observations are shared in order to build the participants' didactic knowledge about the didactic tasks which the lesson has tried to solve, as well as more broad forms of didactic knowledge. All of these paradidactic praxeologies involve different positions and have tasks which are *relative* to the didactic praxis observed in the lesson (along with students' and teachers' mathematical praxeologies). Therefore, we talk of a paradidactic *system* (participants and praxeologies considered together) and of a pre- and post didactic system, when needed (Winsløw, 2011). The institutionally given conditions and constraints for the function of paradidactic systems are jointly called the *paradidactic infrastructure*. Therefore, lesson study furnishes a specific kind of paradidactic infrastructure, which itself may depend on other conditions and constraints on teacher's paradidactic praxeologies (e.g. time resources for their work to prepare and evaluate teaching, material conditions for teachers' work at the school, etc.).

The overall goal of lesson study as such is to improve the participants' knowledge about didactic praxis (both in terms of explicit knowledge about techniques, and in terms of didactical theory). A central focus is a specific set of didactic tasks, which in turn are more or less specific to a mathematical praxeology to be taught; this mathematical praxeology is indeed a very central part of what the participants work with in lesson study (both in the preparation and reflection parts), and what they learn about (in relation to the didactic praxis). At the same time, broader *theoretical* didactic knowledge may be drawn on and developed from this concrete basis (see e.g. Miyakawa & Winsløw, 2013).

7.2.2 *The Dynamics of the Lesson Studied, and of Lesson Study: The Theory of Didactic Situations*

The Theory of Didactic Situations in Mathematics (TDS), whose foundations were established by Brousseau (1997), models teachers' knowledge about mathematics and its teaching in terms of *didactic situations* (DS), which is a dynamic triad unfolding in time consisting of an interaction between teachers, students and a didactic *milieu* (see Fig. 7.1). Here, the didactic milieu typically contains a problem

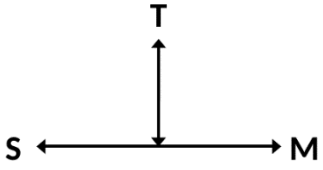

Fig. 7.1 Didactic situations as interplay between a teacher (T), students (S) and milieu (M)

and the resources related to solving it; the students interact with the milieu and their attempts to adapt their knowledge to the milieu (solving the problem using the resources it offers) is the main mechanism of learning in situations. This interaction is initiated and, in various ways, regulated by the teacher.

In order for the students to be able to interplay and accept to interplay with the milieu, this must not only be appropriate in itself but the teacher has to *devolve* it—hand it over—to the students appropriately. The teacher may also institutionalize the knowledge that the students' have obtained through this interaction.

In lesson study, teachers plan, observe and reflect on a didactic situation. Lesson study may be viewed as a sequence of *paradidactic situations* where the teachers interact with the didactic situation as a kind of milieu. Here they formulate, experiment and validate certain hypotheses about students' learning in the didactic situation (Fig. 7.2). *The paradidactic system* thus consists of the teachers participating in the lesson study, together with the various stages of the didactic situation itself. Considering the dynamic interaction of the system in the phases considered above, we may talk of *predidactic situations* (PrS), *observational situations* (ObS) and *postdidactic situations* (PoS).

During our observation and analysis of nine instances of lesson study, the term *didactical idea* emerged. A didactical idea is the hypothesis that the devolution (i.e. the 'handing over') of a certain milieu with further specified regulations will lead the students to a certain point (of knowledge, awareness etc.) through corresponding didactic situations. Didactical ideas may be(come) more or less explicit.

Analysis of realized *didactic situations* provides us with insight into students' actual interplay with the milieu and the ways in which it supports their learning. This is useful not only to a researcher but also to the teachers, and results from a feedback from the milieu of the teachers (i.e. the *didactical situation*). Furthermore, the teachers' explicit analysis of the didactical situation, in pre- and postdidactic situations, gives researchers insight into the development of teachers' knowledge as it develops from interaction with their milieu. This means that the researcher can investigate three levels of knowledge: the students' mathematical knowledge, the teachers' didactic knowledge, and the teachers' paradidactic knowledge (e.g. on how to generate knowledge through lesson study, how and what to hypothesize, and what to observe and analyze). Following the teachers' work with lesson study over time (multi-cycled lesson study and/or several lesson studies) allows us to

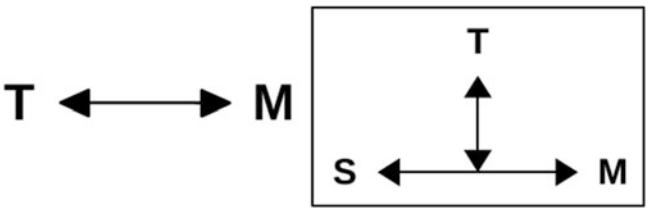

Fig. 7.2 The learning situation of lesson studies: the interplay between the teachers and their milieu, the didactical situation

analyze the evolution of such knowledge. This is particularly interesting when teachers in PrS are anticipating situations, which is one of the core ideas of lesson study: "Anticipating student responses … is one of the crucial processes while planning a lesson" (Takahashi, 2011, p. 79).

The collaborative approach in lesson study 'forces' teachers to express, discuss and operationalize their didactical knowledge when discussing objectives and proposals for the research lesson. The teachers put their didactical knowledge to the test when proposing milieus and anticipating situations in PrS, and when subsequently analyzing their observations in PoS. As demonstrated by Miyakawa and Winsløw (2009), TDS provides a strong tool to compare situations. With a special focus on the properties and roles of the milieus proposed by the teachers, we can then qualitatively identify the teacher knowledge that is put into these milieus; how it develops, and what drives this development.

When we talk about didactical situations, we assume a teacher is present, with the intention to teach specific target knowledge. In paradidactic situations such as those involved in lesson study where the teachers are the learners, there is no teacher with the intention to teach a specific knowledge. In this case we can use TDS to analyze how teachers adopt their didactic knowledge to the milieu (the didactical situation, cf. Fig. 7.2). This model allows us to observe the relationships between the teachers' actions on the milieu, the didactic situation, and the milieu's feedback to the teachers. It is especially interesting to study the relationships (and possible gaps) between:

(i) PrS and PoS (reflecting teachers' didactical learning)
(ii) PrS and DS (reflecting potential and realized didactic situation)
(iii) PrS and PrS' (reflecting the planning of different versions of a lesson).

Notice that (i) and (iii) are particularly interesting for the study of how teachers' didactic knowledge evolves, and hence the effects of lesson study. When using lesson study as a research method rather than as an object of study, (ii) is the essential mechanism offered by a TDS-based analysis.

7.3 Case 1: Lesson Study with Experienced Danish Teachers

This case study is based on the situational approach offered by TDS. Outside of Japan, the consecutive situations of research lesson, post-lesson reflection and revision of the lesson plan are often iterated, leading to 'cycles' of paradidactic situations. This was also the case in the intervention which we consider in this section. In a cycle following a previous one, the learning resulting from the PoS of the previous cycle feeds into the PrS of a following cycle. The model shown in Fig. 7.3 illustrates a lesson study with three cycles.

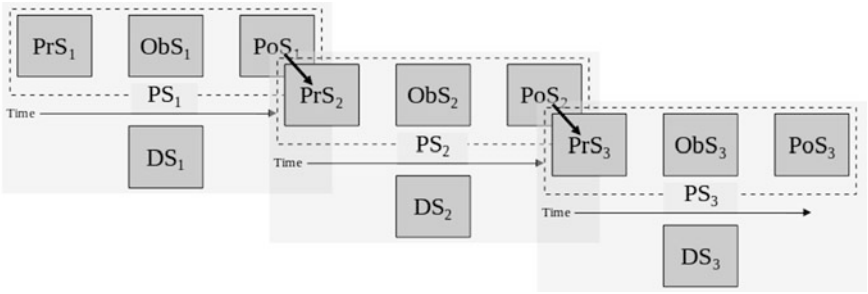

Fig. 7.3 A lesson study with three cycles. Here DS, PrS, ObS and PoS refer to didactic, predidactic, observational, and postdidactic situations, indexed according to the cycle

We now analyze a segment of observations and results from one lesson study (Bahn, in press), which was part of a research project where a team of three or four teachers at three different Danish schools conducted three lesson studies within one school year. That is, each team of teachers conducted three lesson studies each, with a total of nine research lessons. The lesson studies were structured as illustrated in Fig. 7.3, i.e. with three cycles comprising research lessons in three different classes. PrS_1 consisted of a start-up meeting of two hours and lesson (plan) preparation for five hours, a total of seven hours over two different days. PoS_1/PrS_2, PoS_2/PrS_3 and PoS_3 lasted approximately one hour each. The participants all had ten or more years of experience as teachers. During lesson study the facilitator (a researcher) was present at all times and guided the teachers using questions and suggestions to notice and analyze didactical mechanisms so that the teachers would be able to construct didactical ideas. The analyses are based on observational notes and video recordings.

The lesson study addressed here was the first of the three, and for the illustrative purpose only elements of the two first cycles are included. The lesson study showed that from the teachers' experience their grade three students tended to choose inappropriate strategies of subtraction. The research question therefore addressed how students can be expressly taught the different strategies and the properties of each, i.e. their advantages and disadvantages. The mathematical topic addressed in the lesson study in question was that of subtraction by counting, using a number line. Van den Heuvel-Panhuizen and Treffers (2009, p. 108) outline two models of subtraction, *taking away* (TA) and *determining the difference* (DD). TA always implies the use of *counting down* strategy, i.e. starting at the minuend (in this case being the larger number). DD may be performed by either *counting up* from the subtrahend to the minuend or *counting down* from the minuend to the subtrahend.

There are essentially three ways of performing subtraction when counting on a number line. In Table 7.2, 'TA' refers to a problem (as recognized by the student) of the TA model solved by counting down strategy, 'DD_{up}' a problem of the DD model solved by counting up strategy, and 'DD_{down}' a problem of the DD model solved by counting down strategy.

Table 7.2 Paradidactic situations in two lesson study cycles, as viewed by teachers and researchers

Paradidactic situation	Teacher perspective	Researcher perspective
PrS_1	The discussion of proposals for the lesson design leads to new knowledge of three ways of subtraction by counting on a number line (epistemological condition). The a priori analysis also convinced the teachers that each of the strategies would appear in the lesson, since each strategy would be known or found by at least some students (cognitive condition). Furthermore, the analysis led the teachers to think that rather than hearing the 'truth' from the teacher, that seeing actual application by and hearing explications from peer students in a controlled setting (e.g. not too large numbers) would hold a larger potential for students to actually watch, listen and understand (didactic assumption). Didactical Idea: If various students present and explain their strategies, all students will then know the existence and properties of all three strategies.	Without knowing which strategies for subtraction that exist, teachers first proposed a milieu in which problems of subtraction were written on colored cards, the color representing which of the (unknown) strategies is 'easiest' to use when solving the problem. In addition to discovering the three strategies and their properties, the a priori didactical analysis and discussions led the teachers to realize that the proposed milieu may cause students to try to guess what the teacher expects, rather than further their mathematical thinking (since the 'easiest' to use strategy depends, among other things, on prior knowledge). These discoveries (and other reflections) led the teachers to propose milieus in which students were to consider and explain their strategy when solving a problem of subtraction, and for students to show their strategy on a number line on the black board (for everybody to see), while explaining it.
ObS_1	When students explained their strategies and when they presented it on a number line on the black board, only TA came into play.	
PoS_1	From the a posteriori didactical analysis, teachers suggest that the teacher uttering 'minus' and problems presented as '7–5', for example, leads students to think only of TA.	A crucial feedback from the ObS_1 was that no student mentioned DD_{up} or DD_{down}. Previously, the teachers were convinced that all would come into play and they immediately started to discuss aspects in the milieus that could be possible reasons for this. From analyzing the observations it was suggested that (1) due to verbalizing 'minus' when devolving the milieu, students were led to think that the tasks were of TA type since this represents the 'standard' algorithm of subtraction, and (2) the way in which problems were written on the blackboard only represents TA and therefore reminds students of this strategy, leading them to think that the problems should be solved as such.

(continued)

Table 7.2 (continued)

Paradidactic situation	Teacher perspective	Researcher perspective
PrS_2	The above leads teachers to suggest not using the word 'minus' but instead to only talk about 'difference'. On the blackboard, problems should be presented, for example, as 'find the difference between 5 and 7'.	The teachers suggested altered milieus, where students were to consider 'finding the difference' instead of 'minus', and where problems on the blackboard were also formulated as finding the difference between two numbers, even changing the order of the minuend and subtrahend for some of them to further dissociate the problem from 'prescribing an operation'.
ObS_2	All three strategies were presented, illustrated and explained.	
PoS_2	The analysis of DoS_2 strengthen the teachers' hypothesis that verbalizing 'minus' and presenting problems of subtraction as 'minuend-subtrahend' (e.g. '7–5') lead students to solve the problem as they read or hear it.	Regarding the verbal and written formulations representing subtraction problems the teachers formed the hypothesis, based on the a posteriori analysis of ObS_2, that particular words and formulations lead the pupils to understand the problem differently from what was intended by the teachers.

In summary, the infrastructure of lesson study made it possible for teachers to realize two key points:

1. First, they discovered crucial correlations between *how students respond to small details in the milieu and the form of the devolution*, as observed in ObS_1 and analyzed in PoS_1. In PrS_2 the milieu and devolution were adjusted in accordance with the analysis in PoS_1, and in ObS_2 the desired student activity was observed. This led to a conclusion—a strengthened hypothesis—about the correlations in PoS_2.
2. With the aim of proposing a lesson plan for DS, the didactical analysis of PrS_1 led teachers to discover not only the mathematical knowledge needed to design the lesson, but also their hypothesizing didactical situations made them reject one inappropriate milieu and develop one potentially more appropriate. In ObS_1 the teachers observed unwanted and counterproductive student activity, which in PoS_2 they analyzed to be related to details in milieu and devolution. Aiming at a new DS, these details were altered in PrS_2 in accordance with the analysis of PoS_1. In ObS_2 students responded as hypothesized in PoS_1, and in PoS_2 the teachers consolidated their new hypothesis.

It is interesting to observe how the analysis of dynamics within a paradidactic situation, between didactical situations, and between DS and paradidactic situations, can generate insight into how teachers dynamically develop didactical

knowledge. This provides us with tools to examine the role of each paradidactic situation. For instance, there are different opinions as to whether lesson studies should comprise more than one cycle (e.g. Huang & Shimizu, 2016). In this respect, the analysis in question informs that discussion as it appears—at least with lesson study novices—that working with more than one cycle may be beneficial.

As has been illustrated above, the model of paradidactic situations supports the analysis of the details and interdependence of the various phases of lesson study. The notions of TDS provide tools to analyze how the details of one situation are conditioned by and determine other situations. TDS also allows us to analyze hypothetical or potential situations; looking at lesson study in terms of paradidactic situations enables us to analyze the evolution of teachers' knowledge in interaction with the didactic situation.

7.4 Case 2: Lesson Study in Danish Teacher Education

This section is about the endeavor to use lesson study in Danish pre-service teacher education. This case study is based on the institutional approach offered by ATD. As explained in the theory section above, we view lesson study as a paradidactic infrastructure, and as such it has similarities and notable dissimilarities to existing paradidactic infrastructures in the Danish institutional context. A number of conditions work against an easy incorporation of lesson study in Denmark.

When lesson study is to be conducted within the auspices of pre-service education in Denmark, it can take place in connection to either (1) practicum or (2) coursework.

Regarding (1), practicum takes place at schools that have a contract with one of the University Colleges (UCs) where the pre-service teachers do their coursework. The institutional documents and agreements between schools and the UCs do not hold any (official) information or requirement about lesson study as part of practicum. Furthermore, practicum is defined at the UCs as a discipline in its own right with its own curriculum, textbooks etc., so it is not subsumed under any subject matter course. When pre-service teachers are in practicum, they are guided by a regular teacher employed at the school, who—except for very rare cases—has no knowledge of lesson study. Therefore, for lesson study to take place it has to happen in some 'experimental setting' where special arrangements are made. Otherwise practicum is primarily controlled by the school with rather tenuous connections to the school subjects. It should be noted here that Danish primary and lower secondary schools teachers are educated as a generalist, having to teach at least three different subjects. The coursework related to specific school subjects and their didactics occupies only half of the total educational load, while the other half is first and foremost general pedagogics. To our knowledge, there have been only three Danish projects using lesson study in connection with practicum for pre-service teachers (Jørgensen, Rostgaard, & Mogensen, 2016; Rasmussen, 2015; Østergaard, 2016).

Regarding (2), lesson study has been tried out as a part of the coursework specific to the school subjects, under the guidelines for so-called '*associated practice*' (Danish: *praksistilknytning*) which can be many forms of cooperation between teachers from the schools, and educators or students at the UCs. The particular framework for associated practice is unique for each UC, and there is a lot of leeway for individual initiatives, hence also for lesson study (e.g. Nyboe & Rasmussen, 2015). So far the initiative to do lesson study in the context of *associated practice* has come from educators at the UCs, having their students prepare elaborate lesson plans, and then 'borrow' a class of pupils, e.g. from their previous years practicum school. The first example of such an initiative was reported at ICME13 in poster form: 'Consecutive cycles of "whole class" lesson study: A format for development of shared teacher knowledge in pre-service teacher education' (Rasmussen, 2016). In that reported initiative, it is uncovered that the immediate challenge of incorporating lesson study in associated practice is that coursework traditionally takes place during activities where whole classes of pre-service teachers attend to the same things, without necessarily being split into smaller working groups. Thus, if no particular changes are made to the paradidactic infrastructure of lesson study for small groups of in-service teachers, it becomes obvious that only a marginal fraction of pre-service teachers truly engage in the lesson study process; the rest opting for a much more peripheral involvement. This has led us to develop a much more explicit paradidactic structure for pre-service lesson study which involves parceling out *kyouzai kenkyuu* tasks, conferences (sessions where ideas are shared among groups), rotation of groups responsible for each iteration (cycle), and a lottery to decide on the pre-service teacher who will teach the research lesson.

We take our case from Rasmussen (2016), where the notion of praxeology is used to precisely pinpoint the changes in mathematical tasks proposed to the students, as well as the techniques that students were anticipated or even suggested to employ. And most significantly, we mapped the reasons (the didactic logos) which pre-service teachers gave for choosing to employ and forecast these tasks and techniques. The analysis is straightforward but quickly becomes rather extensive as it mirrors the great complexity of what goes on in just one (essentially repeated) lesson.

The lesson study analyzed in this case initially centered on the theme "Who is going to pick up school milk for the class?" (see Table 7.3) This alludes to a context which is familiar to students in Danish lower secondary school. Denmark is a dairy producing country, and school milk is made available to all children at a low cost. At school, the box with the milk for each class has to be picked up from a central refrigerated storage. The student teachers' didactic idea (in the sense of Sect. 7.2) is to have the students play a game of flipping coins to determine who has to go to fetch the box, and investigate if someone is most likely to 'lose'. The main *task* facing the students in the first research lesson was to play a game using two coins in

Table 7.3 Praxeological analysis of initial lesson study cycle

Initial task type (T) put to the pupils, anticipated techniques (τ)
T_1: Who in the class is most likely to be selected to pick up milk?
T_2: Of three pupils, investigate who is most likely to be the one to pick up milk, if they use two coins
T_3: Make a combinatorial argument to answer T_2
τ_1: Peer/class discussion based on prior experiences
$\tau_{2.1}$: Perform physical simulation, take count
$\tau_{2.2}$: Perform large number of simulations using a mobile phone app
τ_3: Draw a schematic of sample space
Reasons (θ) for tasks and techniques
θ_1: Pupils should become aware of subjective beliefs about probability
$\theta_{2.1}$: Pupils become aware of statistical probability
$\theta_{2.2}$: Pupils become aware that statistic probability variation decreases
θ_3: Pupils become aware of combinatorial probability
Associated didactic techniques ($\tau^\dagger$)
$\tau_1{}^\dagger$: Use of familiar context engages the pupils
$\tau_{2.2}{}^\dagger$: Watch video instruction about how to do simulation

groups of three students. The first technique to be deployed was to take count of who won and lost in a number of consecutive games. But to increase the number of experiments, which was to be realized within the relatively short time span of one lesson, the pupils would also simulate the game using a mobile phone app. The pre-service teachers employed a self-paced video instruction to help the students do the simulation on the app. This we characterize as associated *didactic* techniques. From a didactical point of view, the main justification of these techniques is to make pupils aware of statistical probability as a means to decide the long-term outcome of a stochastic situation. However, what is most explicit in the justifying discourse among the pre-service teachers is that using the familiar context of the theme (picking up school milk) could somehow engage the pupils.

The following conclusions emerged from the post-lesson reflection: The simulation using the app was difficult for the pupils, leading to the suggestion to use a computer spreadsheet which the students might be more familiar with. Moreover, in the research lesson, the work with the app simulation was carried out by students individually; the post-lesson discussion led to suggesting it to be done in groups.

Going on to the next (revised) research lesson (see Table 7.4), we see that nearly all tasks and techniques are the same.

The reasons are identical, and only the didactic techniques are altered (in accordance with the conclusions from the post-lesson reflection). This second research lesson progressed to a very high degree as the first; the students apparently still engaged poorly and reluctantly with the simulation task. At the second reflection session, the talk is ripe with concerns for more fundamental challenges in the lesson design, particularly it is now questioned if the students realized the

Table 7.4 Praxeological analysis after first revision of lesson plan (Note the slight change to $\tau_{2.2}$)

Task type (T) put to the pupils, anticipated techniques (τ)
T_1: Who in the class is most likely to be selected to pick up milk?
T_2: Of three pupils, investigate who is most likely to be the one to pick up milk, if they use two coins
T_3: Make a combinatorial arguments to answer T_2
τ_1: Peer/class discussion based on prior experiences
$\tau_{2.1}$: Perform physical simulation, take count
$\tau_{2.2}$: Perform large number of simulations using ICT (Excel spreadsheet)
τ_3: Draw a schematic of sample space
No new reasons for tasks and techniques (θ)
Associated didactic techniques ($\tau^\dagger$)
$\tau_{2.1}{}^\dagger$: Provide a table in which to record the results of physical simulation
$\tau_{2.2}{}^\dagger$: Step-by-step video and written instruction of how to do simulation

Table 7.5 Praxeological analysis after second revision of lesson plan

Task type (T) put to the pupils, anticipated techniques (τ)
T_1: Given one crooked dice, who should do the dishes? What are the rules to make a fair game?
T_2: What happens if two or more crooked dice are used? What are the rules to make a fair game in this situation?
T_3: Make combinatorial arguments to answer T_2
$\tau_{1.1}$: Peer/class discussion based on 'intuition' about how an irregular dice will perform
$\tau_{1.2}$: Perform physical simulation, sample frequency as probability
τ_2: Perform physical simulation with two dice
τ_3: Draw sample space (with unequal probabilities)
Reasons (θ) for tasks and techniques
$\theta_{1.1}$: Pupils should be able to make subjective inferences about probability on the basis of prior experience
$\theta_{1.2}$: Pupils realise the value of statistical probability to determine probability
θ_2: Investigate a more complex situation using statistics
θ_3: Make a connection to combinatorial probability
Associated didactic techniques (τ†)
$\tau_{1.1}{}^\dagger$: Use of crooked dice to make lesson more oriented towards problem solving
$\tau_{1.2}{}^\dagger$: Use of crooked dice generates a true need for statistical probability

purpose at all—the *raison d'être*—of the simulation task. It is suggested that the lesson is revised into a lesson with a problem situation that calls more urgently for the statistical view of chance: the investigation of a game using asymmetrical dice whose probabilities cannot be determined by combinatorics (see Table 7.5).

The third revision of the research lesson exhibits mathematical tasks where coins are exchanged with the asymmetric dice but are otherwise quite similar in surface

structure, whereas the didactic techniques and the logos are much different, and naturally the didactic technique of using the asymmetrical dice is prominent.

The third revision of the research lesson exhibits mathematical tasks where coins are exchanged with the asymmetric dice, but otherwise quite similar in surface structure. However, the didactic techniques and the logos are radically different, and naturally the didactic technique of using the asymmetrical dice are prominent. Furthermore, after the third iteration, it was concluded that the lesson now worked well for the use of one dice (T_1), but difficulties were encountered at the introduction of the second dice (T_2). It was also realized that every pupil should have dice of the same crookedness, in order to make the pooling of individual results for greater frequency stability of the sought probability possible. ICT should not be used at all.

The analysis in terms of praxeologies allows us to follow in detail the incremental changes in pre-service teacher knowledge. It also suggests to us that in the case of pre-service teacher education, in a culture unaccustomed to lesson study, the paradidactic structure of lesson study needs to include several iterations of the same lesson, in order for fundamental changes to be undertaken.

In this case, we used ATD to analyze paradidactic praxeologies and associated didactic praxeologies, looking for changes appearing with each cycle of the research lesson. The overall conclusion is that pre-service teachers are hesitant to draw conclusions from the research lesson and post-lesson reflection. They are reluctant to change their initial lesson plan based on just one experience. They believe the outcome of the lesson is much too contingent on the students, the performance of the pre-service teacher who teaches the lesson, and specific conditions at the particular school. Pre-service teachers are initially unable to identify from observations in the research lesson what could be more fundamental consequences of the didactic choices made in the lesson plan in relation to how students learn mathematics. After the second run, what is contingent and what is not stands out more clearly for them. One might even be pleased that pre-service teachers want the stronger evidence provided by a second cycle, which could be seen as a sound research principle. Our experiences therefore suggest that teachers need to have considerable systematic experience of lesson study (and more generally, a knowledge of teaching and learning) in order to benefit from standalone research lessons. Such experience and knowledge is naturally not available to pre-service teachers; and thus repeated cycles could be of particular significance in pre-service education, particularly when the practicum teachers are also more or less new to lesson study.

7.5 Conclusion

We have presented and showcased two related theoretical perspectives for research on lesson study. As illustrated by Case 1, TDS focuses on the dynamic interplay between a milieu and (individual) learners adaptation to the milieu as it unfolds in time. This allows for a detailed analysis of how both students (in research lessons)

and teachers (carrying out lesson study, observing students' learning) develop their knowledge in various phases of lesson study. On the other hand, ATD offers an institutional perspective on mathematical and didactical knowledge, which can be modelled in any desired detail using the notion of praxeology, and used to further analyze the viability of lesson study in the presence of other paradidactic infrastructures. We outlined how this works in Case 2 on practicum systems within initial teacher education. The ATD perspective appears particularly useful for research on the extent to which lesson study as a paradidactic infrastructure 'fits' into institutional frameworks already in place, such as teacher education. The TDS approach was historically closely linked to the researchers' infrastructure of didactical engineering (cf. Miyakawa & Winsløw, 2009) but when viewed as a series of paradidactic situations, this approach can also be adapted to research employing lesson study as a methodology, whether initiated by teachers themselves or by a researcher as the first case study (Sect. 7.2) exemplifies. For such research, we hypothesize that the TDS approach also has the advantage that the classical elements of the theory—such as didactic milieu, and the typology of didactic situations (devolution, action, formulation, validation and institutionalization)—have the advantage of being quite close to teachers' perspectives, while the analysis of students' and teachers' knowledge in terms of mathematical and didactical praxeologies seems to be most appropriate to a researcher perspective. But it should be noted that ongoing research by Carlsen (2017) also experiments with the operationalization of certain elements of the ATD framework for teachers.

The two theoretical frameworks are compatible in the sense that they consider teaching and learning relative to explicit models of the knowledge to be taught, and for the analysis the knowledge actually developed by students when implementing a research lesson. Unlike many commonly used frameworks they do not only model students' learning but also the teaching and the knowledge that is taught. TDS and ATD have slightly different affordances, with TDS being more close to the perspective of teachers (constructing, devolving, and observing the effects of a milieu) and ATD focusing on the institutional perspective which are so visible to researchers initiating and studying lesson study as a new element of paradidactic infrastructure.

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Chapter 8
Collaborative Lesson Research (CLR)

Akihiko Takahashi and Thomas McDougal

Abstract Lesson study (*jugyou kenkyuu*) is a form of professional development that has been credited for supporting profound changes in teaching in Japan, but its effectiveness outside of Japan has been uneven. From the research on and experience with lesson study in schools in the U.S., the authors hypothesize that certain institutional structures and practices are important for maximizing its impact. The authors introduce a new term: *Collaborative lesson Research* (CLR), defined to include those structures and practices. A three-phase model of school-based CLR, focused on implementation of the new learning standards for mathematics, is being piloted at 15 urban schools in three major school districts in the U.S. This chapter is based on some of the key findings from the Project IMPULS at Tokyo Gakugei University and the Chicago lesson Study Group Project. The full report has been published as "Collaborative lesson Research: Maximizing the impact of lesson study" (Takahashi & McDougal, 2016).

Keywords Lesson study · Mathematics · Problem solving · Teacher education
Teacher collaboration · Lesson design

8.1 *Jugyou Kenkyuu* Versus Lesson Study

Jugyou kenkyuu, the primary form of professional development in Japan for over a hundred years, was introduced outside of Japan in the late 1990s, translated as "lesson study" (Stigler & Hiebert, 1999; Yoshida, 1999). The early research articles

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that introduced lesson study described what Japanese teachers do to improve teaching and learning based on case studies (e.g. Lewis & Tsuchida, 1998; Stigler & Hiebert, 1999; Yoshida, 1999). Based on those reports, researchers, educators, and teachers around the world have attempted to use lesson study to improve mathematics teaching and learning. Some of these projects faithfully followed the descriptions of lesson study, and some adapted the process to fit the limited time that most schools were willing to offer. Nevertheless they hoped to replicate the success of lesson study in Japan at transforming traditional teacher-centered instructional practice to student-centered instruction that focuses on mathematical thinking and problem solving (e.g., Hart, Alston, & Murata, 2011).

8.2 Understanding Lesson Study

For Japanese teachers, lesson study is an integral part of teaching, "like the air" as one teacher put it (Fujii, 2014), and, as with the air, it has been hard to see what lesson study is really made of. But some aspects of Japanese lesson study have become visible as a result of flawed attempts to use it elsewhere and as a result of recent studies of *jyugyou kenkyuu* in Japan. Fujii (2014) examines how lesson study is practiced in some of the African countries supported by Japanese educators, and notes that many aspects of lesson study as practiced in Japan are left out. The same occurs in the U.S. For example, many projects omit the first crucial phase of lesson study, *kyouzai kenkyuu*, that helps teachers gain knowledge and insight into mathematics and student thinking (Lewis, Perry, & Friedkin, 2011; Takahashi, Watanabe, Yoshida, & Wand-Iverson, 2005).

The author of this chapter, who himself practiced lesson study as a teacher in Japan, has had plenty of experience observing activities referred to as "lesson study" which, in his eyes, looked very different from what he used to do. One school district, for example, decided to fit an entire lesson study cycle into one day. In the morning, a team of teachers came together to spend 30 min planning a lesson. They taught the lesson to students and reported what they observed. That afternoon, they modified the lesson plan in 30 min and taught the revised lesson. On the surface, this one-day process included all the components of lesson study that are described in most journal articles and resources. But the typical duration of one lesson study cycle in a Japanese elementary school is more than 5 weeks (Murata & Takahashi, 2002) it is certainly never done in just one day. Another example from a different district shows a profound misunderstanding of the purpose of lesson study. There, a team conduced multiple research lessons on the same topic, thinking that the purpose of lesson study was to create a perfect lesson plan. The true purpose of lesson study, however, is to gain new knowledge for teaching and learning, not to perfect a lesson plan. In fact, re-teaching a research lesson even once is not a common practice in Japan (Fujii, 2014). Thus the selection of the topic for conducting lesson study is crucial. Often teachers and educators who are now to lesson study choose a topic from their favorite lesson plans. One of the reasons may

be the team do not want to take any risk in front of their colleagues at a research lesson. As a result, lesson-planning team often become defensive during the post lesson discussion when participants raise some critical issues from their observation of the lesson such as "the students are more engaged in everyday lesson but not today because of observers." "This lesson usually go well but not today due to ….". Since the purpose of lesson study is to study ways to improve teaching and learning, selecting from well developed lesson plans may not be a good idea. Choosing a challenging topic to address issues in teaching learning and find a way to overcome the issues is the major purpose of the authentic lesson study.

In addition to these examples of lesson study being misinterpreted, and thus shedding light on what lesson study is and is not, researchers have recently investigated how and why Japanese teachers use lesson study and how the process of lesson study helps Japanese teachers build their knowledge and expertise of mathematics teaching and learning. An important finding is that lesson study in Japan is most often conducted as part of a highly structured, school-wide project, involving all or nearly all of a school's staff, aimed at addressing a common teaching-learning challenge (Takahashi, 2014b; Takahashi & McDougal, 2014). This contrasts with most lesson study projects outside of Japan, which are done by enthusiastic volunteer teachers independent of their school professional development activities. Another important finding from recent research concerns the role of supporting professionals (*koushi*), often referred to in English as "knowledgeable others." A knowledgeable other is someone from outside of the planning team with deep expertise in the content, often deep expertise in teaching, and much experience with lesson study. Many lesson study projects in the U.S. are done by teachers without a knowledgeable other, but lesson study in Japan almost always includes a knowledgeable other who provides final comments at the post-lesson discussion, and sometimes a different knowledgeable other who may draw attention to key issues during the planning phase (Watanabe & Wang-Iverson, 2005). Based on a study by Watanabe (2005) and Takahashi (2014a) conducted a case study that looked at three experienced knowledgeable others in Japan in order to better understand their role, and noted many ways in which their final comments helped participants connect the lesson with larger issues in mathematics and pedagogy.

Lesson study has been the primary mechanism of professional development for both prospective teachers and practicing teachers since the Japanese public education system started (Lewis, 2000; Lewis & Tsuchida, 1998; Makinae, 2010; Murata & Takahashi, 2002; Takahashi, 2000; Takahashi & Yoshida, 2004; Yoshida, 1999). The forms of lesson study vary depending upon its purpose; the most common form of lesson study takes place within a single school as a school-based professional development program (Yoshida, 1999). A very common purpose of school-based lesson study is to seek practical ideas for the effective implementation of the Japanese national curriculum, or Course of Study (Murata & Takahashi, 2002).

Japanese teachers begin lesson study by carefully reading the Course of Study, reading relevant research articles, and examining available curricula and other

materials, a process called *kyouzai kenkyuu*, or "study of materials for teaching" (Takahashi et al., 2005; Takahashi & Yoshida, 2004). Based on their *kyouzai kenkyuu*, they then design a lesson focused on a problematic topic while also addressing a broader research theme related to teaching and learning. This lesson, known as a "research lesson" (*kenkyu jugyou*), is taught by a teacher from the planning team while the other team members—and other educators who are not on the planning team—observe. The planning team and observers then conduct a post-lesson discussion (*kenkyuu kyougikai*) focusing on how students responded to the lesson in order to gain insights into the teaching-learning process and into how the Course of Study should be implemented (Lewis & Tsuchida, 1997).

From the "lesson study" activities that so clearly deviate from lesson study as practiced in Japan, and from the research that specifically analyzes the nature of lesson study in Japan, the following features emerge as likely to be important for lesson study to be most effective:

1. Participants engage in lesson study to build expertise and learn something new, not to refine a lesson.
2. It is part of a highly structured, school-wide or sometimes district-wide process.
3. It includes significant time spent on *kyouzai kenkyuu*.
4. It is done over several weeks rather than a few hours.
5. Knowledgeable others contribute insights during the post-lesson discussion and during planning as well.

8.3 What We Learned from Lesson Study in US Urban Schools

The authors have been working with five public elementary schools where the administrations not only support lesson study, but want to make it a routine component of professional development for all teachers. All of these schools are high-poverty schools whose students face many challenges. Expanding lesson study to all teachers can be challenging, but our experience with these schools has shed valuable light on what it takes to make it happen.

Based on our work with US schools, the following elements seem to be important catalysts of school-wide lesson study:

- enthusiasm for lesson study from the school principal, clearly communicated to the faculty;
- a persistent lesson study advocate in addition to the principal;
- a compelling school-wide goal for teaching and learning;
- a commitment on the part of the school administration to provide time for lesson study, through use of funds, staff, and district-mandated professional development time.

8.4 Collaborative Lesson Research (CLR): A Powerful Form of Lesson Study

Lesson study is not an end in itself, but a process for accomplishing specific teaching-learning goals. From the first author's experience, from research on lesson study in Japan (e.g. Fujii, 2014; Takahashi, 2011, 2014a, b; Takahashi & McDougal, 2014), and from our experience with the schools described above, we hypothesize that certain institutional structures and practices are important for maximizing the impact of lesson study. In order to differentiate these collective structures and practices from other, less-effective implementations of lesson study, we have coined a new term: *Collaborative lesson Research* (CLR). The term is drawn from Catherine Lewis's original translation of *jugyou kenkyuu* as "lesson Research" in the late 1990s, which we revive in order to emphasize the research purpose of *jugyou kenkyuu*. As a form of lesson study, CLR is an investigation undertaken by a group of educators, usually teachers, using live lessons to answer shared questions about teaching and learning. We define Collaborative lesson Research (CLR) as having the following components:

1. A clear research purpose
2. *Kyouzai kenkyuu*
3. A written research proposal
4. A live research lesson and discussion
5. Knowledgeable others
6. A process for sharing results.

8.4.1 A Clear Research Purpose

One of the *jyugyou kenkyuu* counterexamples in earlier section above involves a lesson that the team had refined through multiple trials. In that instance, the team was not trying to learn anything new; the lesson was more a demonstration of what they had developed. In contrast, CLR is research, a search for a solution to a teaching-learning problem.

The research focus of CLR usually has two layers. One layer involves the teaching of specific content: how can we design a lesson so that students learn such-and-such concept or skill better than they have in the past? Thus the topic of the research lesson should usually present some challenge for students or teachers. The second layer involves a broad teaching-learning goal that is shared by the CLR community, and that goes beyond any particular topic or grade level and may even be cross-disciplinary. This second layer is referred to as the *research theme*.

A good research theme describes (a) a desired outcome for students, and (b) an entry point for achieving that outcome. At two schools we are working with, for

example, teachers seek to improve their students' ability to give a viable argument and to critique the reasoning of others; their entry point is teaching students to use journals to record their own ideas and the ideas of others. A complete statement of the theme should be short enough to be memorable, such as "For students to be able to clearly explain their thinking and consider the ideas of others through the support of their own journals."

A compelling research theme is an important motivator. As teachers become conscious of a gap between the outcomes they desire for their students and what they have been able to achieve, they become eager for the opportunity to work together to close that gap. When the research theme is relevant to all grade levels, the teachers see how they can benefit by observing research lessons with students older or younger than their own, and engaging in lesson study allows them to contribute to their professional community.

8.4.2 *Kyouzai Kenkyuu*

Kyouzai kenkyuu, the careful study of academic content and teaching materials, is integral to lesson study as practiced in Japan (Takahashi et al., 2005; Takahashi & Yoshida, 2004); it is analogous to a literature review in scientific research. It involves an investigation of the intended learning trajectory related to the topic from lower to higher grades, through a review of the standards and curriculum, and research into teaching and learning issues such as typical misunderstandings around the topic. *Kyouzai kenkyuu* also includes consideration of possible tools, manipulatives, or materials that may be used, and possible tasks that may be presented to students. Thorough *kyouzai kenkyuu* helps avoid "reinventing the wheel", making it more likely that CLR will contribute new knowledge to the education community.

One obstacle for teachers in the U.S. is the quality of available materials to support *kyouzai kenkyuu*, especially compared to the materials available to Japanese teachers (Lewis et al., 2011). This was especially true when new standards were introduced, and no curricula were available that aligned with them. Many of the teachers we work with have used a translation of a popular Japanese textbook series.

8.4.3 *A Written Research Proposal*

A CLR planning team creates a written document, called the lesson research proposal, to communicate what the team learned from their *kyouzai kenkyuu*, and to explain their instructional thinking. It includes learning goals for a unit, an overview of the unit, a detailed teaching-learning plan for one particular lesson within

the unit (the *research lesson*), a rationale for the design of the unit and research lesson, and a clear statement of how the research lesson aims to address the research theme and the learning goals.

Although many lesson study program outside Japan use the term *lesson plan*, it may be useful for teachers and educators to use a different term to distinguish a plan for research lesson in CLR from their everyday use of lesson plan. In fact some teachers who try lesson study first time often confuses "Why does lesson study require teachers to develop a lesson plan for several weeks? We usually develop lesson plan for a lesson in 15 min." In order to avoid such confusion CLR suggests to use the term *lesson research proposal* as a plan of the research lesson in order to distinguish a typical plan that many teachers outside Japan use for everyday lesson planning. The plan for CLR research lesson is a proposal for seeking effective ideas for improving mathematics teaching and learning and asking research lesson participants to discuss based on the data they collect by observing research lessons.

In our experience, a thorough lesson research proposal may be 9 pages long. The authors developed a template document to guide CLR teams in organizing their lesson study work and in writing their proposal. The template document includes the following.

- Title of the lesson research proposal
- Grade and the topic
- Date of the lesson
- Teacher's name: [name of the teacher who teach the lesson]
- Lesson designed for: [name of the school and the class]
- Lesson plan developed by: [names]

1. Title of the Lesson:
2. Brief description of the lesson
3. Goals of the Lesson:
4. Relationship of the Unit to the Standards
5. Related prior learning standards (topics/objectives)
6. Background and rationale of teaching the topic
7. Consideration for Designing the Unit (Findings from your research)
8. About the Unit and the Lesson
9. Flow of the Unit
10. Detailed Lesson Plan from the Unit
    a. Introduction
    b. Posing the Task
    c. Anticipated Student Responses
    d. Comparing and Discussing
    e. Summing up
11. Evaluation.

8.4.4 A Live Research Lesson and Post-lesson Discussion

Based on the lesson plan in the research lesson proposal, one member of the team teaches the research lesson, observed by the entire planning team and by additional members of the CLR community. Observers are responsible for collecting data on how the lesson impacts the students, relative to the research theme and the learning goals. A video recording of the lesson can be useful for some purposes, but CLR requires observations from multiple viewpoints, so video does not substitute for live observation.

As soon as practical after the research lesson, observers share data and discuss implications, especially with respect to the learning goals of the lesson and the research theme. The primary goal of the discussion is to gain insights into teaching and learning and to inform the design of future lessons, not to revise the lesson plan. These discussions generally benefit from a moderator, someone not on the planning team, who helps focus the discussion on important issues and keeps the conversation grounded in data.

8.4.5 Knowledgeable Others

As discussed above, "knowledgeable others", persons with both extensive knowledge of the topic and extensive experience with CLR, are invited by the team to help them go beyond what they know. Ideally a CLR community needs two knowledgeable others: one for supporting proposal development and another for providing the final comments at the end of the post-lesson discussion.

During planning, a knowledgeable other may help the team identify instructional examples to review, valuable resources in the form of articles or results from other CLR work, and may give feedback on the proposal. Besides having extensive knowledge of the subject matter and the topic, this knowledgeable other should be familiar with the school's curriculum and students. An experienced teacher or a content coach who often works at the school may play the role of this kind of knowledgeable other.

Another knowledgeable other is needed at the research lesson. At the end of the post-lesson discussion, he or she is expected to highlight important events from the research lesson that were not discussed, and make connections between the lesson and new knowledge from research and standards. The knowledgeable other also provides suggestions to the CLR community of possible steps they could take toward accomplishing their research theme (Takahashi, 2014a; Watanabe & Wang-Iverson, 2005).

8.4.6 A Process for Sharing Results

CLR is not just for the improvement of teaching and learning within the team, but also for improving teaching and learning more broadly. Thus CLR should include a structure or process for disseminating what is learned from each research lesson to a larger community. Simply inviting people from outside of the planning team to observe and discuss the research lesson is one valuable way that CLR teams contribute to the learning of other educators while benefiting from the additional eyes and expertise that the additional observers bring. In addition, the team may distribute their research lesson proposal, which encapsulates the team's research and their instructional ideas, and can be useful to other educators. This document is made more powerful through the addition of a written reflection by the team, completed within a few days after the research lesson, which describes what they learned from the live lesson observation and post-lesson discussion about their research hypothesis, mathematics, student thinking, teaching, etc.

We chose these six defining characteristics of CLR based on findings from research on lesson study outside of Japan and *jugyou kenkyuu* in Japan, and our own experience of working with schools. As defining characteristics, we consider them required elements of CLR: if any of them is missing, then the activity cannot be called CLR. In addition, we note that although CLR does not have to be done more than once, it almost always is, because the research theme is usually difficult to accomplish and is broader than any single topic. Lessons learned from one CLR cycle lead to revised theories about how to address the theme, or adjustments in the theme itself, which lead to another CLR cycle involving a different topic perhaps at a different grade level.

8.5 Conclusion

In the summer of 2002, a joint US/Japan seminar entitled "The professionalization of teachers through lesson study" was held at Park City, Utah. Although one of the major goals of the seminar was to clarify the mechanisms and operating principles of lesson study, Japanese mathematics education researchers and teachers did not then have clear definitions to distinguish authentic lesson study from lesson study-like activities. After more than a decade of attempts to use lesson study outside of Japan, important mechanisms and operating principles of effective lesson study are becoming clear. By coining the term "Collaborative lesson Research" and clearly defining it, the authors hope to guide educators who wish to use lesson study to improve teaching and learning.

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Chapter 9
Mathematics Lesson Study Around the World: Conclusions and Looking Ahead

Stéphane Clivaz and Akihiko Takahashi

Abstract Educators around the globe seek to emulate the success of Japanese lesson study. However, implementation of lesson study outside Japan has been met with varying rates of success and challenges. To address the challenges, lesson study researchers and educators, have gathered their reflections in this book. This concluding chapter discusses strategies for developing a theorization that can be understood outside of Japan and its specific cultural norms, adaptations of lesson study outside Japan, the contributions of lesson study for educational reform, the sustainability of lesson study and the challenges involved in establishing lesson study on a larger scale. As the summary of this book, this chapter proposes our concluding statement and suggests future goals for implementation and research.

Keywords Lesson study · International · Theoretization · Mathematics education

9.1 Introduction

Researchers and educators of lesson study around the world came together for the 13th International Congress on Mathematical Education (ICME-13) discussion group in Germany in 2016. This discussion group examined lesson study and mathematics education in various countries and settings, as well as methodological and theoretical tools for researching lesson study. Members of this fruitful discussion worked together to produce this book in order to share our findings. In this chapter, we explain Japanese lesson study and its impact on mathematics education in Japan. Outside of Japan, implementation of lesson study has been met with varying rates of success and challenges. To address the challenges, our group

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members contributed in-depth research of Japanese lesson study, case studies of lesson study projects conducted in various countries and settings, strategies for developing a theorization that can be understood outside of Japan and its specific cultural norms, and close examination of the challenges involved in establishing lesson study on a larger scale. We summarize these chapters and suggest in response the model of "Collaborative Lesson Research" (CLR) as a way to define what lesson study outside of Japan can be. Our concluding statement suggests future goals for implementation and research.

9.2 Japanese Lesson Study and Its Impact on Mathematics Education

In Japan, lesson study has been the primary method of professional development for teachers ever since public education became an institution (Lewis, 2000; Lewis & Tsuchida, 1998; Makinae, 2010; Murata & Takahashi, 2002; Takahashi, 2000; Takahashi & Yoshida, 2004; Yoshida, 1999a). A cycle of lesson study is composed of several elements. The first element is intensive research regarding classroom materials. Teachers must study the standards, read relevant research articles, and examine available curricula. They use this research to design a lesson structured around solving a particular problem (or problems) that also addresses a broader educational theme. This lesson is taught by a single teacher under the observation of other teachers and is called a "research lesson." It is an opportunity for everyone involved to closely examine teaching practices and judge whether or not the lesson properly supported the students. Afterwards, everyone who observed joins in a discussion with the teacher who led the lesson about how the students responded. This discussion provides valuable insights into the teaching-learning process.

Japanese lesson study has many benefits. The research and discussion improves teaching and learning. It also helps determine practical and effective ways to implement the national curriculum (Murata & Takahashi, 2002). Japanese schools typically use lesson study to help them transition to a new national curriculum. Researchers have also shown how lesson study leads to the implementation of new teaching approaches in Japanese schools (Lewis, 2002; Lewis & Tsuchida, 1998; Stigler & Hiebert, 1999; Yoshida, 1999b). Stigler and Hiebert (1999), in particular, discussed how lesson study promoted major reform in Japanese mathematics classrooms: teacher-led lecture lessons were replaced in favor of student-centered, problem-solving lessons. Early research on Japanese lesson study focused on these problem-solving mathematics lessons, however, it did not clearly explain how the lesson study process itself is what prompted this major reform (e.g. National Council of Teachers of Mathematics, 1980, 1989). Understanding this shift is key to understanding the success of Japanese lesson study. Stigler and Hiebert's (1999) research generated great international interest in lesson study and inspired researchers around the world. However, despite the fact that many researchers and

educators have attempted to replicate the success of Japanese lesson study (e.g. Hart, Alston, & Murata, 2011) the effectiveness of these projects is unclear (Takahashi & McDougal, 2016). The authors of this book examined this issue from various perspectives.

9.3 Lesson Study in Western Countries

A lack of comprehensive understanding of Japanese lesson study is in large part why it is difficult to establish sustainable lesson study outside Japan. The two chapters of this book by Fujii (2017) and Baba, Ueda, Ninomiya, and Hino (2017), respectively, describe the lesson study process in detail. Fujii (2017) points out that it is crucial for teacher educators to understand not only the most visible parts of lesson study, such as the research lesson and its post-lesson discussion, but to also understand the less visible parts of the process, i.e., designing the lesson and the theory behind it. Baba et al. (2017) report three Japanese mathematics education researchers' perspectives regarding lesson study. These reports by Japanese researchers are valuable for researchers and educators outside Japan who may not have first-hand experience of lesson study to help them develop a more comprehensive view.

Educators attempting to implement lesson study in Western countries face many challenges. Our group members have shared their research regarding developing a theory of lesson study and how lesson study has been adapted in countries outside of Japan. This book assembles several European and US case studies (Ni Shuilleabhain, 2017; Ponte, Quaresma, Mata-Pereira, & Baptista, 2017; Quaresma et al., 2017; Takahashi & McDougal, 2017; Winsløw, Bahn, & Rasmussen, 2017). These case studies all involve local scholars (either originally from the area or originally from Japan but now living in the area) who conducted lesson study in their area without any initial involvement or support from local authorities or ministries of education. These texts all examine the local conditions, how lesson study was actually implemented, and the projects' respective successes, failures, and challenges. From their research we can gain a better understanding of how to develop a theoretical model of lesson study, what adapting lesson study to Western countries entails, and how introducing lesson study to Western countries can inspire educational reform.

9.4 Theorizing Lesson Study

Winsløw et al. (2017) point out that outside Japan, educators' understanding of lesson study is based on individual interpretation rather than an explicit definition. This individual understanding is determined by what research educators have read on their own or from training by Japanese experts. However, Winsløw et al. (2017)

argue that even the knowledge of Japanese experts is "contingent upon cultural and institutional conventions," and that since these conventions are not shared by Western culture, they are not readily understood by Western educators (p. 125). This situation runs against the nature of what lesson study should be, "to create shared and documented knowledge, rather than (just) private experience and wisdom" (Winsløw et al., 2017, p. 125). By collaborating on this book, our ICME-13 discussion group aims to rectify this conflict. We are opening a dialogue between mathematics education researchers working with or about lesson study, presenting theoretical models analysing some of these lesson study realisations, and presenting research-based definitions of what lesson study outside of Japan can be.

One way of contributing to this necessary theorization is to use already established theoretical models to analyse the lesson study process in order to characterize its essence. Several researchers in our IMCE-13 discussion group tackled this challenge (e.g., Ni Shuilleabhain & Clivaz, 2017; Runesson, 2015; Widjaja, Vale, Groves, & Doig, 2015). In their chapter, Winsløw et al. (2017), use the Anthropological Theory of the Didactic (ATD, Chevallard, 1999) and the Theory of Didactic Situations in Mathematics (TDS, Brousseau, 1997) on two Danish case studies. These two frameworks, ATD and TDS, offer interesting viewpoints on investigating the means and processes of Japanese mathematics lesson study, building on the research of Miyakawa and Winsløw (2009) and Clivaz (2015a). Winsløw et al. (2017) found that using TDS has the advantage of being quite close to the teachers' perspective and allows for "a detailed analysis of how both students (in research lessons) and teachers (carrying out lesson study, observing students' learning) develop their knowledge in various phases of lesson study" (p. 139). This development of teachers' knowledge in a learning situation for the teacher highlights a parallel between lesson study for the teacher and the problem solving lesson for the students, making the lesson study a learning milieu for the teacher (Clivaz, 2015b) as schematised in Winsløw et al. (2017, Fig. 7.2). In contrast to this, Winsløw et al. (2017) find ATD to be the "most appropriate to a researcher perspective," as it considers lesson study from a global viewpoint, offering an institutional perspective on how to "analyse the viability of lesson study in the presence of other paradidactic infrastructures" (p. 140). The analysis by Winsløw et al. (2017) of these two Danish case studies helps form a theory for lesson study.

Making the lesser known aspects of Japanese lesson study understood to the international community also contributes to the theorization of lesson study. In their chapter, Takahashi and McDougal (2017) examine what lesson study is not, elucidated by the "result of flawed attempts to use it" (p. 144). These observations point towards a "clear definitions to distinguish authentic lesson study from lesson study-like activities" (p. 151). Takahashi and McDougal (2017) define criteria for what lesson study can be outside of Japan and coin the term "Collaborative Lesson Research," or CLR. CLR is defined as having: a clear research purpose, *kyouzai kenkyuu* ("the study of classroom materials"), a written research proposal, a live research lesson and discussion, knowledgeable others, and a process for sharing results. This definition derives from the comparison between Japanese lesson study and its less than faithful adaptations overseas. Defining CLR contributes to building

a general theory of lesson study, and as such, Japanese lesson study may be more successfully understood and implemented internationally.

9.5 Adaptations of Lesson Study Outside Japan

Lesson study exists outside of Japan in various adaptations. Educators around the globe sought to emulate the success of Japanese lesson study following the publication of *The Teaching Gap* (Stigler & Hiebert, 1999). Adaptations were made, sometimes to fit a limited time frame (Takahashi & McDougal, 2017), and sometimes because these adaptations were more well-suited to certain cultures and situations. In their chapter, Winsløw et al. (2017) quote Lewis's argument that successful adaptation requires "a deep understanding of what it [lesson study] is and why it has been useful to Japanese teachers, and how it can be adapted to the very different setting of the US" (Lewis, 2004, p. 134). This "importing of cultural routines" (Stigler & Hiebert, 2016) is discussed from several perspectives in this book. Ponte et al. (2017) follow Stigler and Hiebert; they insist that the transformation of a cultural practice such as lesson study is both inevitable and necessary. In this adaptation process, "many significant aspects of the original cultural practice may be lost, but other aspects may emerge, yielding robust and flexible practices in the new environment" (Ponte et al., 2017, p. 88). Their chapter explores what successful adaptation of lesson study can be.

All countries are different, this fact alone implies adaptations when conducting lesson study outside of Japan. The Portuguese case study serves as one such example. During teacher training, the research lesson was not taught by the prospective teacher but by another teacher involved in the teacher training process (Ponte et al., 2017). This is radically different from Japanese lesson study. Even the impetus for performing lesson study varies from place to place. Our IMCE-13 group discussion members shared case studies from Portugal, Ireland, Denmark, and the USA, in which the lesson study processes are initiated at the university level (Ponte et al., 2017; Ni Shuilleabhain, 2017; Winsløw et al., 2017; Takahashi & McDougal, 2017). In most of these case studies, the faculty member often had to play both the role of a facilitator and of a knowledgeable other. This is again very different from lesson study in Japan. To address this issue, Takahashi and McDougal (2017) propose in their definition of CLR that the roles of supporting professionals should stay separate, the same way they are in Japan, and also suggest having two different knowledgeable others. The right adaptations can determine the success of a lesson study project.

Several of the case studies related in this book are regarding teacher training (Ponte et al., 2017; Winsløw et al., 2017). For reasons elaborated by Winsløw et al. (2017), pre-service education is actually the best place to initiate lesson study as a new paradidactic infrastructure in Denmark. We can hypothesise that this might also be the case in other countries as well. These particular settings of pre-service education, similar to Japanese schools in the 1900s (Winsløw et al., 2017), merit

particular attention since participants have not yet developed the more individual strategies for professional development which may be entrenched among experienced teachers. Ponte (2017) has also recently reviewed research concerning the use of lesson study as it applies to prospective mathematics teachers' education for secondary school. He also calls for further research regarding these adaptations. We advocate that such research would be of great interest to the mathematics education community.

One of the most controversial adaptations of lesson study is re-teaching. Fujii (2014) shows that re-teaching is generally never done in Japan. Fujii even considers it as a misconception. On the other hand, Winsløw et al. (2017) side with Stigler and Hiebert (2016), and hypothesize that while re-teaching is not necessary in a mature lesson study culture like in Japan, where lesson study is a well-established part of the paradidactic infrastructure for all schools "[…] in the case of pre-service teacher education, in a culture unaccustomed to lesson study, the paradidactic structure of lesson study needs to include several iterations of the same lesson, in order for fundamental changes to be undertaken" (p. 139). In fact, they show in a Danish case study how, "repeated cycles could be of particular significance in pre-service education" (p. 139). In the initial teacher training, this adaptation seems beneficial in the sense that repetition creates "cycles" of paradidactic situations and that "the learning resulting from the PoS [post-didactic situations] of the previous cycle feeds into the PrS [pre-didactic situations] of a following cycle" (p. 131). In particular, observing the variations in the subsequent research lesson could make participants aware of the effect on students' learning in response to a change in the lesson design. Thus, "after the second run, what is contingent and what is not, stands out more clearly for them" (p. 139). More general research may uncover other kinds of adaptations which make lesson study in other countries more than a mere copy of what is common practice in Japan.

9.6 Lesson Study and Educational Reform

Two cases of specific local conditions, in Portugal (Ponte et al., 2017) and in Ireland (Ni Shuilleabhain, 2017), both presented in this book, relate to curriculum reform. In both cases, the reform involves a profound change in the teaching of mathematics. Ni Shuilleabhain speaks of a "dramatic shift in perspective on teaching and learning mathematics from a traditional, "transmission" approach" (p. 66). Ponte et al. show in their case study how local educators employed an approach similar to what Fujii (2016) describes as "inquiry-based mathematics teaching" and "structured problem solving." In both cases, and in fact in all the cases presented in our ICME-13 discussion group and in this book, the lesson study focused on creating lessons that taught through problem solving. In the Portuguese and Irish case studies, the authors report how educators took into consideration both anticipated student thinking and actual student responses when designing tasks. The authors conclude that this is an important feature of the lesson study process and

crucial to the implementation of the program as both a professional development practice and for influencing curriculum reform.

Both Ponte et al. (2017) and Ni Shuilleabhain (2017) stress that in their respective countries traditional professional development programs are quite different from lesson study. Traditional professional development programs in these countries generally occur off-campus and last no longer than a day. In contrast to this, lesson study takes place in the school on a regular basis over a long period of time. Lesson study is seen as a solution to the "lack of alignment between research and practice" (Ni Shuilleabhain, 2017, p. 66). It gives educators a chance to collaborate, supports their development, and can create effective changes in teaching practices. Despite the cultural differences between Japan and Ireland, "the culturally based elements of lesson study, suggested that this model could be legitimately adopted for the Irish school system" (p. 67). Despite differences in culture and education traditions, lesson study seems a like valuable tool for education reform.

Local teachers are often surprised by how different lesson study is from familiar professional development programs. Ponte et al. (2017) report that lesson study "may also seem strange" and that teachers "begin by questioning why to dedicate so many hours to work in single topic or curriculum aim when there are so many topics in the curriculum that deserve attention" (p. 99). However, when involved in the process, teachers quickly appreciate the collaboration. Despite initial doubts, these teachers felt that, "Along the sessions we were seeing that this is really productive and that we learned to work in a collaborative way" (p. 93). In the Ireland case study, teachers felt that lesson study provides an "environment which allowed them to take risks in their classroom practices" (Ni Shuilleabhain, 2017, p. 78). This safe environment is another aspect of lesson study as a learning milieu for the teacher. Ponte et al. reports that, "instead of saying 'how the teachers must act', we strive to create situations in which, through their collective work, they are led to discover how to act. We also provide a clear structure, in which all the work develops" (2017, p. 102). Some of the surprising aspects of lesson study are what make it so effective.

The effects of lesson study as professional development are often reported from a qualitative point of view. Ni Shuilleabhain (2017) reports teachers' stated emphasis on students' communication and collaboration. This also manifested in concrete details, such as changing the "classroom layout from individual tasks to tables arranged for group work" (p. 72). Teachers also reported that scripting open questions better prepared them to anticipate students' responses and respond to misconceptions. When reflecting on their time spent teaching the lesson study research lesson, teachers reported that they saw themselves as facilitators of learning, "There was very little teaching involved in it really. Just kind of facilitating—which is what it's all about" (p. 76). In this way, research shows qualitative evidence for how lesson study impacts classroom teaching and learning.

Reports also show how lesson study addresses the analysis, modification and even creation of tasks. In Ireland, for example, teachers engaged directly with curriculum documents and materials (Ni Shuilleabhain, 2017). Winsløw et al. (2017) report that comparing task analysis and observations gathered during the

research lesson, combined with the repetition of modified research lessons, allowed teachers to understand "how students respond to small details in the milieu" (p. 134). This awareness allowed teachers to "discover not only the mathematical knowledge needed to design the lesson, but also their hypothesising didactical situations made them reject one inappropriate milieu and develop one potentially more appropriate" (p. 134). Mathematical knowledge and practice, the mathematical praxeology, is developed during the lesson study process. Ponte et al. (2017) also report how deep mathematical work was performed during a lesson study program. One of the teachers from their case study reflects that, "Sometimes I need a little more mathematics, and I think that these sessions helped us with that" (p. 93). Lesson study demands close examination of mathematical teaching and learning, helping teachers to craft effective tasks for their students.

9.7 The Sustainability of Lesson Study

The question of the sustainability of lesson study is one of the more difficult issues to resolve. In the Irish case study (Ni Shuilleabhain, 2017), among all the teachers who wanted to continue lesson study, only the ones at the school where they all shared a free double-period were able to do so. Those at schools where this opportunity wasn't available were forced to abandon the project. In the Portuguese case study (Ponte et al., 2017), professional development is only open to teachers during their free time. There is no reward and teachers often have to pay fees to attend. These conditions seem to be particularly disadvantageous. Nevertheless, Ponte and his colleagues consider that:

> these are not unsurmountable obstacles, but they are contextual conditions that we need to pay attention to. Unless these conditions change in a deep way—by some change in educational policy, notably regarding teachers' career and teacher education—it is natural that they keep putting constrains on the development of this professional development practice in Portugal. (2017, p. 101)

Even with local limitations, lesson study is still a worthwhile practice, and may lead to improved conditions. Deeper understanding of local situations can improve its sustainability.

Underscoring this issue of sustainability are the two chapters by Lim, Teh, and Chiew (2017) and Takahashi and McDougal (2017). These chapters discuss cases of lesson study outside Japan which began as small, externally funded, ad hoc volunteer group projects. As such, these group projects struggled to establish and maintain positions within their existing school systems, despite that aim being one of the original goals. These projects found themselves constricted by grant periods and often ended before seeing vital impacts on either teacher or student learning (Lim, Teh, & Chiew, 2017; Takahashi & McDougal, 2017). Institutional and financial support are key to the success of lesson study projects.

Takahashi and McDougal (2017) expound upon lesson study conditions in the U.S., where lesson study projects have a longer history. They conclude that in order to be successful, lesson study must be schoolwide. To ensure its sustainability, they outline the following requirements:

1. Enthusiasm for lesson study from the school principal, clearly communicated to the faculty
2. A persistent lesson study advocate in addition to the principal
3. A compelling school-wide goal for teaching and learning
4. A commitment on the part of the school administration to provide time for lesson study, through use of funds, staff, and district-mandated professional development time (p. 146).

Lesson study cannot thrive in a vacuum; the greater its support, the greater its impact will be.

9.8 Implementing Lesson Study on a Larger Scale

The case study by Lim, Teh, and Chiew (2017) in Malaysia and the case study by Estrella, Mena-Lorca, and Olfos (2017) in Chile describe larger-scale lesson study projects within their school systems. Unlike most early attempts of lesson study pilot projects outside Japan, these two cases focus on schools who used lesson study to update their existing professional development program. The case in Malaysia began with a university research initiative involving a small number of schools. Because the initiative was a success, the Malaysian Ministry of Education became interested in adopting lesson study nation-wide. Based on their experience extending lesson study as a system-wide initiative, the authors offer the following suggestions:

1. Instill lesson study as a culture of professional learning for teachers
2. Enhance the knowledge and skills needed for effective lesson study: lesson planning, observation, reflection
3. Engage expert teachers to play the role of knowledgeable others in lesson study
4. Propose Collaborative Lesson Research (CLR) as an alternative form of lesson study.

These suggestions are particularly useful for lesson study projects outside Japan to help them establish sustainable lesson study in their school systems.

The case study by Estrella et al. (2017) from Chile reports a unique lesson study project: a large-scale government initiative that has strong support by a Japanese university. Leading researchers and educators of mathematics education were selected by the Chilean government to receive intensive training at a university in Japan. Japanese researchers and practitioners were then invited to work with schools and teachers in Chile to introduce the Japanese approach for teaching mathematics as well as lesson study. The authors reported the process of this unique top-down approach for implementing lesson study in their country.

9.9 Conclusion

Since the late 1990s, to help students become active constructors of mathematics, many researchers and school systems have tried to use lesson study to replicate Japan's success for shifting mathematics classrooms from teacher-led lectures to student-centered problem solving lessons. The case studies compiled in this book exemplify these efforts by researchers and educators around the world. These cases teach us a variety of ways to initiate lesson study in locations where no one has experienced such professional development before. We can learn a great deal from their successes and challenges. We believe that these pioneers' efforts will help researchers and educators who want to design projects to improve mathematics teaching and learning in their local schools. At the same time, we also believe that existing research on lesson study implementation may not provide enough information for us to judge if lesson study outside Japan can reach the same level of success that it has in Japan. In other words, it remains unclear if lesson study projects outside Japan can properly support schools and teachers. In order to answer this question, we need to quantify the impacts of international lesson study projects and analyze exactly how lesson study can be both effective and sustainable.

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Japanese Vocabulary—A Proposal for Standard Transcriptions

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A Proposal for a Standard Use of Japanese Lesson Study Vocabulary

In the wake of an increasing interest in Japanese lesson study, the amount of research literature covering it has grown significantly during the last twenty years. Since most authors and readers are not familiar with Japanese, one challenge is how to transcribe Japanese vocabulary into Western languages. There is no agreement to one of the various transcription systems, and in some cases, a mix of these is used. In this brief comment on the use of Japanese vocabulary in research literature, the author suggests the use of one common system of transcription, namely modified Hepburn *without* contraction of vowels and splitting words.

Written Japanese comprises four types of characters: *Kanji*, *hiragana*, *katakana* and Roman (Latin) letters. Kanji bear meaning and are often complex. Hiragana and katakana are sets of syllables and carry no meaning, comparable to an alphabet. The two sets are phonetically identical, i.e. they represent identical sounds, but have different purposes. Roman letters are used as well to a minor degree. Numbers are usually written with Arabic numerals (also known as European digits) but are also written with kanji. The process of transcription is often referred to as *romanizing*.

Most words we adopt from Japanese are originally written in kanji, but—a complicated matter made simple—it suffice here to say, that any word in kanji can be meaningfully written with hiragana syllables. Since we cannot transpose the meaning of a kanji into Latin script, we can (in a simplified way) consider transcription as an act of 'translating' a Japanese word's 'spelling' with hiragana into Roman letters.

Transcriptions take compromises and for the sake of communication a common standard is necessary (as for any language). There are a number of systems for

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transcribing Japanese in use. In many cases, one person will consistently use one of these systems to transcribe, but in other cases the systems are mixed. For readers and writers not familiar with Japanese, the different transcriptions may be confusing (and occasionally that goes for Japanese writers and readers too).

There are a number of historical and linguistic reasons for the several systems, and hence good reasons for each of them. Still, especially for readers and writers not familiar with Japanese, it is inconvenient with different transcriptions. This is the sole motivation to suggest a standard of transcription.

Commonplace Cases

Hiragana (and katakana) comprise 46 *monographs*, i.e. single characters, of which some can be combined into *digraphs*, two characters combined to function as one. Most of the monographs cause no confusion, but some do. For instance, the hiragana つ can be romanized as *tsu* or *tu*. Likewise し is written as *shi* or *si*. To distinguish digraphs from two monographs, the second character is smaller, e.g. きや, *kiya*, (monographs of equal sizes) and きゃ, *kya*, (digraph, one normal and one small). Confusing cases of transcribing digraphs include じゅ (*ju* and *jyu*) and しょ (*sho* and *syo*).

One of the differences is whether the transcription differentiates between short and long vowels. Written with hiragana, the length of a vowel is directly represented with the number of characters. For examples, the sound きょ has a short vowel, which can be represented as *kyo*, whereas きょう has a long vowel, often represented as *kyou* or *kyō* (with a macron) but often as *kyo* as well.

Contrary to Western languages, space between words is not used in Japanese. When applying Japanese vocabulary to Western writing, this calls for some choices regrading readability. As a result, 授業研究 (lesson study) romanizes as *jugyoukenkyuu, jugyou kenkyuu* or *jugyou-kenkyuu*. Combined with the above illustrated multiple variations regarding romanization of each character, this allows for a myriad of ways to transcribe.

There are other not so frequent cases of confusion, but for the moment the main difficulties summarize to

- General romanization
- Long vowel spelled out or contracted (with or without a macron)
- Combined or split words.

The following examples—taken from recent literature—illustrate the problem. These are not examples of an author using a wrong transcription, but solely examples of the use of different transcription systems, resulting in different 'spellings'. In the headline of each example, the word in question is romanized by the suggested system, spelling out vowel and splitting words. Emphasis is added to the Japanese word.

Jugyou Kenkyuu - じゅぎょうけんきゅう - 授業研究

> Lesson study (jugyou kenkyuu) is a form of professional development that has been credited for supporting profound changes in teaching in Japan... (Takahashi & McDougal, 2016, p. 513)

Comments: Split words, long vowel spelled out.

> LESSON study (jugyoukenkyuu) is a Japanese professional development process… (Fernandez & Chokshi, 2002, p. 1)

Kyouzai Kenkyuu - きょうざいけんきゅう - 教材研究

> In Japan, teachers consider kyozaikenkyu inherent in a teacher's life so they are actively involved in this endeavour in the hope of improving their level of teaching (Fujii, 2014, p. 7)

Comments: One word, contracted vowels [o] and [u] (unidentifiable)

> Kyouzai kenkyuu, the careful study of academic content and teaching materials, is integral to lesson study…. (Takahashi & McDougal, 2016, p. 520)

Comment: Split words, long vowel spelled out.

> …addressed and the material developed for it –that is, the result of kyōzaikenkyū (study of teaching materials)… (Miyakawa & Winsløw, 2017, p. 7)

Comment: One word, contracted vowels (identifiable).

Kenkyuu in Other Contexts

> … in the setting of teacher study meetings (in Japanese, kenkyu-kai). (Miyakawa & Winsløw, 2013, p. 186)

Comment: Hyphened word, contracted long vowel (unidentifiable)

> Here, "practice research" (jissenkenkyū) is a broader term that denotes the study and research on teaching practices, carried out mainly by an individual teacher or a group of teachers… (Miyakawa & Winsløw, 2017, p. 2)

Comments: One word, contracted long vowel (identifiable).

The examples presented here illustrate various ways of spelling the same word, different approaches to splitting words or not and a variety of writing specific hiragana.

Suggested System for Transcription

There are already a number of systems in use, but—without going into further detail—they each hold some difficulties regarding ease of use and readability for readers and writers not familiar with Japanese. The suggestion presented here is, based on the author's experience with the initial encounter with Japanese and—years later—as a teacher of basic Japanese, believed to be a good balance between simplicity and precision. One of the major systems of transcription of Japanese is called Modified Hepburn System. From its widespread use and from the author's experience, this system to a high degree represents the sounds of each syllable well. In Hepburn, long vowels are contracted though, represented with a macron (e.g. ō). For the sake of simplicity and consistence, it is suggested to omit this and instead spell out each hiragana (which too will dissolve the issue of *oo* vs. *ou*).

Table A.1 Examples of Japanese lesson study vocabulary transcribed with the suggested system

Meaning	Transcription	Japanese
Lesson study	Jugyou kenkyuu	じゅぎょうけんきゅう (授業研究)
Research lesson	Kenkyuu jugyou	けんきゅうじゅぎょう (研究授業)
Research meeting/conference	Kenkyuu kai	けんきゅうかい (研究会)
Open lesson	Koukai jugyou	こうかいじゅぎょう (公開授業)
Study of topic, curriculum, learning, learning progression and related teaching materials	Kyouzai kenkyuu	きょうざいけんきゅう (教材研究)
Practice research	Jissen kenkyuu	じっせんけんきゅう (実践研究)
Instruction and/or guidance including formative assessment at the pupils' desks	Kikan shidou	きかんしどう (机間指導)
Lesson plan	Gakushuu shidou an	がくしゅうしどうあん (学習指導案)
Knowledgeable other	Koushi	こうし (講師)
Attached school	Fuzoku gakkou	ふぞくがっこう (付属学校)
Elementary school mathematics	Sansuu	さんすう (算数)
Mathematics	Suugaku	すうがく (数学)
Calculate/calculation	Keisan	けいさん (計算)
Education	Kyouiku	きょういく (教育)

In addition, it is suggested to split but not hyphen words. This is in accordance with English and other Western languages.

The list of words presented (Table A.1) serves as an example of how given words are romanized, following the suggested system.

The list is not exhaustive and should be expanded.

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