

Chapter 8

Wind Energy Investment Analyses Based on Fuzzy Sets



Cengiz Kahraman, Sezi Çevik Onar, Başar Öztayşi, İrem Uçal Sarı and Esra İlbarhar

Abstract Engineering economics deals with the investment decisions, where the investment parameters are very hard to estimate exactly. In the cases where we do not have the required data for parameter estimation, possibilistic approaches may be used. In this chapter, a brief literature review on wind energy investments is first presented. Later, the chapter gives present worth analysis (PWA) methods extended to fuzzy sets. The chapter introduces ordinary fuzzy PWA, type-2 fuzzy PWA, intuitionistic fuzzy PWA, and hesitant fuzzy PWA. A numerical application for each extension is presented.

8.1 Introduction

There is an increasing energy need in the world and carbon-based fuels are the main sources for fulfilling this need. Yet, these carbon-based energy sources damage the ecological environment and they are limited sources. Renewable energy sources are the best alternatives for carbon-based fuels since they are eco-friendly and can provide energy unlimitedly.

Wind energy can become an efficient energy source for many regions. The uncertainty in electricity prices and energy production levels of wind turbines limits the wind energy investments. Especially, the costs and benefits of the long-term wind energy investments are hard to calculate with the traditional engineering economic analysis since they need precise values of investment parameters (Çevik Onar and Kilavuz 2015).

Ordinary fuzzy sets and their extensions such as type-2 fuzzy sets, intuitionistic fuzzy sets, and hesitant fuzzy sets are exceptional tools for dealing with uncertainty in human thoughts and perceptions (Kahraman et al. 2016b). Ordinary fuzzy sets (Zadeh 1965) use membership degrees for representing vagueness and imprecise-

C. Kahraman (✉) · S. Çevik Onar · B. Öztayşi · İ.U. Sarı · E. İlbarhar
Industrial Engineering Department, Istanbul Technical University,
Macka, Istanbul, Turkey
e-mail: kahraman@itu.edu.tr

ness. Type-2 fuzzy sets introduced by Zadeh (1975) employ three dimensional membership functions. Type-2 fuzzy sets have grades of membership that are themselves fuzzy. Intuitionistic fuzzy sets introduced by Atanassov (1986) employ both membership and non-membership degrees for defining uncertainty. Hesitant fuzzy sets developed by Torra (2010) represent the hesitations in decision makers mind. Fuzzy net present worth analysis enables evaluating investment alternatives under vague and incomplete information. The extensions of fuzzy sets enable better defining the uncertainties inherent in investment parameters through their membership functions.

The wind energy investments involve uncertain, vague and incomplete parameters. Therefore, applying classical present worth analyses may create unrealistic results. Calculating present worth with vague and incomplete data may produce incorrect and misleading decisions. Therefore, this chapter shows the calculation of the fuzzy PW of a wind energy investment based on fuzzy parameters. Ordinary fuzzy PW, intuitionistic fuzzy PW and hesitant fuzzy PW are employed in wind energy investment problems.

The rest of the chapter is organized as follows: Sect. 8.2 summarizes the literature on wind energy investments. Section 8.3 presents the fuzzy present worth analyses based on extensions of fuzzy sets. In Sect. 8.4, a wind energy investment problem is analyzed with ordinary fuzzy PW, Intuitionistic fuzzy PW and hesitant fuzzy PW. Section 8.5 concludes the chapter.

8.2 Wind Energy Investments: A Literature Review

Much research on wind energy investments exists in the literature. The recent studies in this field will be further examined under two categories as classical techniques and fuzzy techniques.

8.2.1 *Classical Techniques*

Caralis et al. (2014) investigated the profitability of wind energy investments by employing a Monte Carlo approach to deal with the uncertainties. In their study, Monte Carlo simulation and a typical financial model were integrated to examine different cases of wind energy development. Uncertain parameters considered in the study of Caralis et al. (2014) are wind capacity factor, investment cost, interest rate, feed-in-tariff, absorption rate, grid accessibility. Kucukali (2016) utilized a scoring technique for the assessment of an onshore wind energy project. The proposed method enables decision makers to determine the most appropriate wind energy project by examining the risks of the alternatives. Site geology, land use and permits, environmental impact, grid connection, social acceptance, macroeconomic, natural hazards, change of laws, access road, and revenue are the risks considered in

the study of Kucukali (2016). Liu and Zeng (2017) used system dynamics approach to evaluate renewable energy investment risk, particularly wind power projects. After risks in renewable energy investment were analyzed in three categories as technical risk, policy risk and market risk, causal loop diagram for investment risk assessment was formed. The simulation results which are obtained using VENSIM software indicated that policy risk is more crucial in early stage of an investment whereas market risks become more significant with technological advancements and incentive policies improvement (Liu and Zeng 2017). Fazelpour et al. (2017) examined the wind resource and economic feasibility to assess investment risks. The Weibull distribution function was utilized to estimate the wind power and energy density. Windographer software was used to examine the wind direction. For the economic assessment, four types of wind turbines were taken into consideration. These wind turbines are different with respect to rotor diameter, variable rotor speed, nominal power output, cut-in wind speed, rated wind speed, cut-out wind speed, survival wind speed. Monthly capacity factor, energy output and cost of energy of the alternatives with these wind turbines were evaluated. Al-Sharafi et al. (2017) investigated the feasibility of solar and wind energy systems for power generation and hydrogen production and performed an economic analysis by using simulation software, Hybrid Optimization of Multiple Energy Resources (HOMER). Aquila et al. (2017) investigated wind power feasibility under uncertainty by employing Monte Carlo simulation and Value at Risk technique. The proposed framework is quite useful for potential investors because it is able to show the influence of the uncertainty on wind power and electricity prices. Kitzing et al. (2017) proposed a real options model to assess wind energy investments. The proposed model involves an upper capacity limit by considering investment timing and continuous sizing. Moreover, several uncertainty factors such as power price and wind speed are taken into consideration in a stochastic process in the study of Kitzing et al. (2017).

8.2.2 Fuzzy Techniques

Shamshirband et al. (2014) employed adaptive neuro-fuzzy optimization to maximize the net profit of a wind farm. While applying an intelligent optimization method based on the adaptive neuro-fuzzy inference system, net present value and interest rate of return were considered as the measures of net profit. Interest rate per year and unit sale price of electricity were utilized as inputs of optimization scheme whereas output was the optimal number of turbines which is an indicator of maximal net profit. In this study, while determining the optimal number of wind turbines, aerodynamic interactions between the turbines, as well as cost factors, are taken into consideration. In this way, both optimal solution with respect to the maximum net profit and the optimal layout for wind turbines were achieved (Shamshirband et al. 2014). Wu et al. (2014) investigated evaluation criteria considered in the process of wind farm project plan selection and proposed a

framework to select the best wind farm project. Criteria considered in this study are construction, resource, wind turbine, financial analysis, social risk, policy risk, technological risk, good influence, bad influence, the influence of project to the local society and stabilization, the influence of project to the local economy and employment, and the influence of project to resource utilization. Wu et al. (2014) employed intuitionistic fuzzy numbers, intuitionistic fuzzy Choquet operator, and generalized intuitionistic fuzzy ordered geometric averaging operator to reduce the probability of information loss and to stay away the independent assumption of multi-criteria decision making methods (Wu et al. 2014). Onar et al. (2015) utilized interval-valued intuitionistic fuzzy sets for the assessment of wind energy investments. Interval-valued intuitionistic fuzzy sets are employed because of its ability to cope with vagueness and impreciseness in a more comprehensive manner. The proposed approach provides an overall performance measurement for wind energy technology alternatives by considering the following criteria: reliability, cooperation, domesticity, performance, cost factors, availability, maintenance, and technical characteristics (Onar et al. 2015). Shafiee (2015) utilized fuzzy analytic network process to determine the most appropriate risk mitigation strategy for offshore wind farms by employing safety, added value, cost and feasibility criteria. Variation of offshore site layout, improvement of maintenance services, upgrading the monitoring systems, and modification in design of wind turbines are the alternatives considered in the study of Shafiee (2015). Petković et al. (2016) investigated the most influential factors on the net present value of a wind farm using adaptive neuro-fuzzy inference system. In their study, seven inputs, number of turbines, power production, cost per power unit, cost, efficiency, interest rate per year, unit sale price of electricity, are selected to analyze the wind farm net present value (Petković et al. 2016). Wu et al. (2016) proposed an inexact fixed-mix fuzzy-stochastic programming method for heat supply management in wind power heating system under uncertainty. In their study, uncertainties are presented as interval values, random variables and fuzzy sets. The proposed approach is a combination of interval-parameter programming, fixed-mix stochastic programming and fuzzy mathematical programming. The proposed approach enables decision makers to observe interval solutions and plausibility degrees of constraint violation in order to determine the best heat supply management strategies (Wu et al. 2016). Gumus et al. (2016) introduced a multi-criteria decision making method consisting of an intuitionistic fuzzy entropy method, an intuitionistic fuzzy weighted geometric averaging operator and intuitionistic fuzzy weighted arithmetic averaging operator for sustainable energy problems. The selection of V80 and V90 onshore and offshore wind turbines was investigated using the proposed method (Gumus et al. 2016). Cunico et al. (2017) proposed a mathematical model taking several uncertain parameters into consideration to analyze investments in the energy sector. It is aimed at covering both pessimistic and optimistic scenarios by integrating uncertain parameters in their decision making model. Therefore, a fuzzy approach and a set of possibilistic techniques were employed to handle the problem. The uncertain parameters considered in their study are uncertainty in the price of fossil resources, the trend in the growing demand and the variation in the

availability of fossil reserves (Cunico et al. 2017). Chang (2017) introduced a fuzzy score technique to optimally locate wind turbines. In this study, the proposed technique was utilized to measure the Euclidean distance between the achievement function and their aspirations (Chang 2017). Morshedizadeh et al. (2017) investigated the utilization of imputation techniques and adaptive neuro-fuzzy inference system to predict wind turbine power production. It was revealed that appropriate combinations of decision tree and mean value for imputation might enhance the prediction performance (Morshedizadeh et al. 2017).

There are various studies in the literature on wind energy investments. These studies have different objectives such as analyzing wind energy technology investments, maximizing investment profit, identifying optimal investment decisions, investigating suitability of a region, predicting energy output of a wind farm, and selecting a suitable site for investment. These studies utilize different methods such as Benefit/Cost analysis, real option analysis, adaptive neuro-fuzzy inference system, optimization, and AHP to achieve these objectives. Moreover, evaluation criteria or employed parameters may change with respect to the objective of the study. Table 8.1 shows some representative studies on wind energy investments in the literature.

8.3 Fuzzy Present Worth Analysis

Fuzzy logic is used to determine uncertainty occurred from linguistic assumptions. It is possible to represent linguistic definitions in a mathematical form using fuzzy sets. Fuzzy numbers have different types which determine the linguistic terms in different ways. In this section, present worth analysis is constructed using different types of fuzzy numbers such as ordinary fuzzy numbers, intuitionistic fuzzy numbers, type-2 fuzzy numbers and hesitant fuzzy numbers.

Especially in public sector projects such as highways, infrastructure, power generation facilities, project alternatives have very long expected useful lives. In such kind of projects, planning horizon could be taken as infinite to be effective. In this section, the present worth analysis for infinite time horizon is proposed using different types of fuzzy numbers.

8.3.1 Ordinary Fuzzy Present Worth Analysis

There are different types of ordinary fuzzy numbers such as triangular fuzzy numbers, trapezoidal fuzzy numbers, L-R type fuzzy numbers etc. The most used ordinary fuzzy numbers are triangular fuzzy numbers due to their easy calculations. Chiu and Park (1994) defined triangular fuzzy net present value (\widetilde{NPV}) formula as

Table 8.1 Some representative studies on wind energy investments

Authors	MCDM method	Classical method	Fuzzy	Objective	Evaluation criteria/parameters
Kahraman et al. (2016a)	None	Benefit/cost analysis	Interval-valued intuitionistic	To analyze wind energy technology investments	Present worth, annual worth
Ashkaboosi et al. (2016)	None	Bi-level optimization technique	None	To maximize the profit of investment and market clearing for the wind power	None
Petković (2015)	None	Adaptive neuro-fuzzy inference system	Type I	To propose a model to provide economically optimal layouts for wind farm	Wake effect, wind regime, cost factors
Panduru et al. (2014)	None	Fuzzy logic controller	Type I	To monitor the energy generated by a wind turbine, and to control its distribution	None
Sheen (2014)	None	Real option analysis	Type I	To evaluate economic effectiveness of wind power investment projects	Net present value
Yeh and Huang (2014)	DEMATEL, ANP	Goal/question/metric	Type I	To examine the important factors considered in selection of wind farm location	Safety and quality, economy and benefit, social impression, environment and ecology, regulation, policy
Baringo and Conejo (2013)	None	Multi-stage stochastic programming	None	To identify optimal investment decisions on wind power facilities	None
Ersöz et al. (2013)	None	Adaptive-network based fuzzy inference systems	Type I	To investigate the suitability of a region for wind plant investment	Average moisture, temperature and pressure
Soroudi (2012)	None	Scenario-based approach	Type I	To quantify the impact of distributed generation units on active loss and voltage profile	Active loss, technical risk

(continued)

Table 8.1 (continued)

Authors	MCDM method	Classical method	Fuzzy	Objective	Evaluation criteria/parameters
Lee (2011)	None	Real option analysis	None	To assess the value of wind energy investment opportunities	Underlying price, exercise price, time to maturity, risk-free rate, volatility
Madlener et al. (2011)	None	Fuzzy portfolio optimization with SMAD as risk measure	Type I	To show the implementation of fuzzy portfolio optimization on onshore wind power plants	Technical characteristics, economic characteristics
Onat and Ersoz (2011)	None	Adaptive-network-based fuzzy inference system	Type I	To analyze wind climate and wind energy potential	Average pressure, temperature, humidity
Aydin et al. (2010)	“and”, “or”, OWA	GIS	Type I	To provide a decision support system for site selection of wind turbines	Sufficient potential for wind energy generation, satisfaction of most of the environmental objectives
Lee et al. (2009)	AHP	Benefits, opportunities, costs and risks (BOCR)	None	To choose a suitable wind farm project	Wind availability, Site advantage, WEG functions, financial schemes, policy support, advanced technologies, Wind turbine, connection, foundation, concept conflict, technical risks, uncertainty of land
Sheen (2009)	None	Geometric moment fuzzy ranking algorithm, and cost-benefit analysis	Type I	To assess the feasibility of wind generation investment	interest rate, inflation rate, investment, and operating revenue and/or cost
Cavallaro and Ciraolo (2005)	NAIADE method	None	Type I	To investigate the feasibility of installing wind energy turbines	Investment costs, operating and maintenance costs, energy production capacity, savings of finite energy sources, maturity of technology, realization time, CO ₂ emissions avoided, visual impact, acoustic noise, Impact on ecosystems, social acceptability

(continued)

Table 8.1 (continued)

Authors	MCDM method	Classical method	Fuzzy	Objective	Evaluation criteria/parameters
Celik (2003)	None	Weibull wind speed distribution model	None	To derive the wind energy output for small-scale wind power generators	Climate, topography, wind speed measurement
Pinson and Kariniotakis (2003)	None	Adaptive fuzzy neural networks	Type I	To predict the power production of wind farms	Online SCADA measurement, numerical weather predictions
Sen and Sahin (1997)	None	Cumulative semivariogram method	None	To evaluate regional wind power	None

given in Eq. 8.1, where $\tilde{F}_i = (f_{it}; f_{im}; f_{ir})$, denotes net cash flows occurred in time period t and $\tilde{i}_t = (i_{it}; i_{im}; i_{ir})$ denotes the fuzzy interest rate.

$$\widetilde{NPV} = \left(\sum_{t=0}^n \left(\frac{\max(f_{it}; 0)}{\prod_{t'=0}^t (1 + i_{t'})} + \frac{\min(f_{it}; 0)}{\prod_{t'=0}^t (1 + i_{t'})} \right); \sum_{t=0}^n \frac{f_{im}}{\prod_{t'=0}^t (1 + i_{t'})} \right); \left(\sum_{t=0}^n \left(\frac{\max(f_{ir}; 0)}{\prod_{t'=0}^t (1 + i_{t'})} + \frac{\min(f_{ir}; 0)}{\prod_{t'=0}^t (1 + i_{t'})} \right) \right) \quad (8.1)$$

When the time horizon is infinite the fuzzy net present worth is calculated by Eq. 8.2:

$$\widetilde{NPV} = \left(\sum_{t=0}^{\infty} \left(\frac{\max(f_{it}; 0)}{i_{it}} + \frac{\min(f_{it}; 0)}{i_{it}} \right); \sum_{t=0}^{\infty} \frac{f_{im}}{i_{im}}; \sum_{t=0}^{\infty} \left(\frac{\max(f_{ir}; 0)}{i_{ir}} + \frac{\min(f_{ir}; 0)}{i_{ir}} \right) \right) \quad (8.2)$$

In this chapter Eq. 8.3 is used for the defuzzification of ordinary fuzzy sets:

$$Def(\tilde{F}) = \frac{f_l + 2f_m + f_u}{4} \quad (8.3)$$

8.3.2 Type-2 Fuzzy Present Worth Analysis

The concept of a type-2 fuzzy set was introduced by Zadeh as an extension of the concept of an ordinary fuzzy set called an ordinary fuzzy set (Zadeh 1974).

A type-2 fuzzy set $\tilde{\tilde{A}}$ in the universe of discourse X can be represented by a type-2 membership function $\mu_{\tilde{\tilde{A}}}$, shown as follows (Zadeh 1975):

$$\tilde{\tilde{A}} = \left\{ (x, u), \mu_{\tilde{\tilde{A}}}(x, u) \mid \forall x \in X, \forall u \in J_x \subseteq [0, 1], 0 \leq \mu_{\tilde{\tilde{A}}}(x, u) \leq 1 \right\} \quad (8.4)$$

where J_x denotes an interval $[0,1]$. In the literature review, it is seen that triangular interval type-2 fuzzy sets are the most preferred interval type-2 fuzzy sets.

A triangular interval type-2 fuzzy set is represented as $\tilde{\tilde{A}}_i = \left((a_{il}^U, a_{im}^U, a_{ir}^U; H(\tilde{\tilde{A}}_i^U)) \right), \left(a_{il}^L, a_{im}^L, a_{ir}^L; H(\tilde{\tilde{A}}_i^L) \right)$ where $\tilde{\tilde{A}}_i^L$ and $\tilde{\tilde{A}}_i^U$ are ordinary fuzzy sets, $a_{il}^U, a_{im}^U, a_{ir}^U, a_{il}^L, a_{im}^L$ and a_{ir}^L are the references points of the interval type-2 fuzzy

set $\tilde{A}_i, H(\tilde{A}_i^U)$ denotes the membership value of the element a_i^U in the upper triangular membership function $\tilde{A}_i^U, H(\tilde{A}_i^L)$ denotes the membership value of the element a_i^L in the lower triangular membership function $\tilde{A}_i^L, H(\tilde{A}_i^U) \in [0, 1], H(\tilde{A}_i^L) \in [0, 1]$ and $1 \leq i \leq 2$. Kuo- Ping (2011) gives detailed information on the basic algebraic operations of type-2 fuzzy sets.

Ucal Sari and Kahraman (2015) introduced type-2 fuzzy net present worth method. Triangular interval type-2 fuzzy net present value ($N\tilde{P}V$) is formulized in Eq. 8.5 where $\tilde{F}_t = (f_{il}^U, f_{im}^U, f_{ir}^U; H(\tilde{f}_t^U)), (f_{il}^L, f_{im}^L, f_{ir}^L; H(\tilde{f}_t^L))$ denotes the cash flow occurred at time t and $\tilde{i}_t = (i_{il}^U, i_{im}^U, i_{ir}^U; H(\tilde{i}_t^U)), (i_{il}^L, i_{im}^L, i_{ir}^L; H(\tilde{i}_t^L)), \forall i > 0$ denotes the discount rate at time t :

$$N\tilde{P}V = \left(\left(\sum_{t=0}^n \frac{f_{il}^U}{\prod_{r=0}^t (1 + i_{ir}^U)}, \sum_{t=0}^n \frac{f_{im}^U}{\prod_{r=0}^t (1 + i_{rm}^U)}, \sum_{t=0}^n \frac{f_{ir}^U}{\prod_{r=0}^t (1 + i_{rl}^U)}; \min(H(\tilde{f}_t^U), H(\tilde{i}_t^U)) \right), \left(\sum_{t=0}^n \frac{f_{il}^L}{\prod_{r=0}^t (1 + i_{ir}^L)}, \sum_{t=0}^n \frac{f_{im}^L}{\prod_{r=0}^t (1 + i_{rm}^L)}, \sum_{t=0}^n \frac{f_{ir}^L}{\prod_{r=0}^t (1 + i_{rl}^L)}; \min(H(\tilde{f}_t^L), H(\tilde{i}_t^L)) \right) \right) \tag{8.5}$$

When the time horizon is infinite triangular interval type-2 fuzzy net present worth is calculated by Eq. 8.6:

$$N\tilde{P}V = \left(\left(\sum_{t=0}^{\infty} \frac{f_{il}^U}{i_{ir}^U}, \sum_{t=0}^{\infty} \frac{f_{im}^U}{i_{rm}^U}, \sum_{t=0}^{\infty} \frac{f_{ir}^U}{i_{rl}^U}; \min(H(\tilde{f}_t^U), H(\tilde{i}_t^U)) \right), \left(\sum_{t=0}^{\infty} \frac{f_{il}^L}{i_{ir}^L}, \sum_{t=0}^{\infty} \frac{f_{im}^L}{i_{rm}^L}, \sum_{t=0}^{\infty} \frac{f_{ir}^L}{i_{rl}^L}; \min(H(\tilde{f}_t^L), H(\tilde{i}_t^L)) \right) \right) \tag{8.6}$$

In this chapter, realistic type reduction indices are used for the defuzzification of type 2 fuzzy sets. Realistic type reduction indices is calculated by Eq. 8.7 which transforms \tilde{A} into an ordinary fuzzy set where $\underline{\mu}_{\tilde{A}}(x)$ and $\overline{\mu}_{\tilde{A}}(x)$ are lower and upper membership functions of the \tilde{A} (Niewiadomski et al. 2006).

$$TR_{re}(\tilde{A}) = \frac{\underline{\mu}_{\tilde{A}}(x) + \overline{\mu}_{\tilde{A}}(x)}{2}, \quad x \in X \tag{8.7}$$

Equation 8.3 can be used to rank the ordinary fuzzy set which is obtained by type reduction indices method.

8.3.3 Intuitionistic Fuzzy Present Worth

Atanassov (1986) introduced triangular intuitionistic fuzzy numbers (TIFN) \tilde{A} . TIFN utilizes both membership value and non-membership value of a fuzzy number. Formulas of membership function ($\mu_{\tilde{A}}(x)$) and non-membership function ($v_{\tilde{A}}(x)$) are as follows:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-l}{m-l}, & \text{for } l \leq x \leq m \\ \frac{u-x}{u-m}, & \text{for } m \leq x \leq u \\ 0, & \text{otherwise} \end{cases} \tag{8.8}$$

and

$$v_{\tilde{A}}(x) = \begin{cases} \frac{m-x}{m-l}, & \text{for } l \leq x \leq m \\ \frac{x-m}{u-m}, & \text{for } m \leq x \leq u \\ 1, & \text{otherwise} \end{cases} \tag{8.9}$$

where $l \leq m \leq u$, $l \leq m \leq u$, $0 \leq \mu_{\tilde{A}}(x) + v_{\tilde{A}}(x) \leq 1$ and it is denoted by

$$\tilde{A}_{TIFN} = \left((l, m, u), (l, m, u) \right). \tag{8.10}$$

The sum of membership and non-membership values should be less than or equal to 1. The basic algebraic operations are determined by Mapatra and Roy (2009), Atanassov (2012) and Kumar and Hussein (2014).

In this chapter, TIFNs are ranked using the defuzzification method which is proposed by Kahraman et al. (2015).

The rank of a TIFN $\tilde{A} = \left((l, m, u), (l, m, u) \right)$ is determined as follows:

$$R(\tilde{A}) = \frac{1}{2} \left(\frac{l + 2m + u}{4} + \frac{l + 2m + u}{4} \right) = \frac{l + l + 2m + 2m + u + u}{8} \tag{8.11}$$

Triangular fuzzy number intuitionistic fuzzy weighted geometric ($TFNIFWG_w$) operator is used to aggregate triangular intuitionistic fuzzy sets (Chen et al. 2010):

$$TFNIFWG_w(\tilde{A}_1, \tilde{A}_2, \dots, \tilde{A}_n) = \left(\left(1 - \prod_{j=1}^n (1 - l_j)^{w_j}, 1 - \prod_{j=1}^n (1 - m_j)^{w_j}, 1 - \prod_{j=1}^n (1 - u_j)^{w_j} \right), \left(1 - \prod_{j=1}^n (1 - l_j)^{w_j}, 1 - \prod_{j=1}^n (1 - \acute{m}_j)^{w_j}, 1 - \prod_{j=1}^n (1 - u_j)^{w_j} \right) \right) \tag{8.12}$$

Kahraman et al. (2015) introduced intuitionistic fuzzy net present worth and intuitionistic fuzzy annual worth methods. The parameters used in the calculations are expressed by TFIN in Eqs. 8.13–8.18 where m evaluations are made for each of the parameter.

$$\widetilde{FC}_{T,I} = \left\{ \begin{array}{l} \langle fc_1, (TFN_1, T\acute{F}N_1), \dots, (TFN_m, T\acute{F}N_m) \rangle, \\ \langle fc_2, (TFN_1, T\acute{F}N_1), \dots, (TFN_m, T\acute{F}N_m) \rangle \\ \vdots \\ \langle fc_k, (TFN_1, T\acute{F}N_1), \dots, (TFN_m, T\acute{F}N_m) \rangle \end{array} \right\} \tag{8.13}$$

$$\widetilde{UAC}_{T,I} = \left\{ \begin{array}{l} \langle uac_1, (TFN_1, T\acute{F}N_1), \dots, (TFN_m, T\acute{F}N_m) \rangle, \\ \langle uac_2, (TFN_1, T\acute{F}N_1), \dots, (TFN_m, T\acute{F}N_m) \rangle \\ \vdots \\ \langle uac_k, (TFN_1, T\acute{F}N_1), \dots, (TFN_m, T\acute{F}N_m) \rangle \end{array} \right\} \tag{8.14}$$

$$\widetilde{UAB}_{T,I} = \left\{ \begin{array}{l} \langle uab_1, (TFN_1, T\acute{F}N_1), \dots, (TFN_m, T\acute{F}N_m) \rangle, \\ \langle uab_2, (TFN_1, T\acute{F}N_1), \dots, (TFN_m, T\acute{F}N_m) \rangle \\ \vdots \\ \langle uab_k, (TFN_1, T\acute{F}N_1), \dots, (TFN_m, T\acute{F}N_m) \rangle \end{array} \right\} \tag{8.15}$$

$$\widetilde{SV}_{T,I} = \left\{ \begin{array}{l} \langle sv_1, (TFN_1, T\acute{F}N_1), \dots, (TFN_m, T\acute{F}N_m) \rangle, \\ \langle sv_2, (TFN_1, T\acute{F}N_1), \dots, (TFN_m, T\acute{F}N_m) \rangle \\ \vdots \\ \langle sv_k, (TFN_1, T\acute{F}N_1), \dots, (TFN_m, T\acute{F}N_m) \rangle \end{array} \right\} \tag{8.16}$$

$$\widetilde{i}_{T,I} = \left\{ \begin{array}{l} \langle i_1, (TFN_1, T\acute{F}N_1), \dots, (TFN_m, T\acute{F}N_m) \rangle, \\ \langle i_2, (TFN_1, T\acute{F}N_1), \dots, (TFN_m, T\acute{F}N_m) \rangle \\ \vdots \\ \langle i_k, (TFN_1, T\acute{F}N_1), \dots, (TFN_m, T\acute{F}N_m) \rangle \end{array} \right\} \tag{8.17}$$

$$\tilde{n}_{T,I} = \left\{ \begin{array}{l} \langle n_1, (TFN_1, T\acute{F}N_1), \dots, (TFN_m, T\acute{F}N_m) \rangle, \\ \langle n_2, (TFN_1, T\acute{F}N_1), \dots, (TFN_m, T\acute{F}N_m) \rangle \\ \vdots \\ \langle n_k, (TFN_1, T\acute{F}N_1), \dots, (TFN_m, T\acute{F}N_m) \rangle \end{array} \right\} \tag{8.18}$$

where FC represents the first cost of the alternative, UAC represents uniform annual cost of the alternative, UAB represents uniform annual benefit, n represents project life, i represents interest rate, and SV represents salvage value.

The intuitionistic fuzzy present worth ($\widetilde{PW}_{T,I}$) of an investment alternative can be calculated by Eq. 8.19 or Eq. 8.20:

$$\begin{aligned} \widetilde{PW}_{T,I} = & -\widetilde{FC}_{T,I} - \widetilde{UAC}_{T,I} \left(\frac{P}{A}, \tilde{i}_{T,I}, \tilde{n}_{T,I} \right) \\ & + \widetilde{UAB}_h \left(\frac{P}{A}, \tilde{i}_{T,I}, \tilde{n}_{T,I} \right) + \widetilde{SV}_h \left(\frac{P}{F}, \tilde{i}_{T,I}, \tilde{n}_{T,I} \right) \end{aligned} \tag{8.19}$$

or

$$\begin{aligned} \widetilde{PW}_{T,I} = & -\widetilde{FC}_{T,I} - \widetilde{UAC}_{T,I} \left[\frac{(1 + \tilde{i}_{T,I})^{\tilde{n}_{T,I}} - 1}{\tilde{i}_{T,I}(1 + \tilde{i}_{T,I})^{\tilde{n}_{T,I}}} \right] \\ & + \widetilde{UAB}_{T,I} \left[\frac{(1 + \tilde{i}_{T,I})^{\tilde{n}_{T,I}} - 1}{\tilde{i}_{T,I}(1 + \tilde{i}_{T,I})^{\tilde{n}_{T,I}}} \right] + \widetilde{SV}_{T,I} (1 + \tilde{i}_{T,I})^{-\tilde{n}_{T,I}} \end{aligned} \tag{8.20}$$

where

$$\begin{aligned} \widetilde{FC}_{T,I} &= \bigcup_{j=1}^k TFNIFWG_w \left(\left\langle fc_j, (TFN_1, T\acute{F}N_1), \dots, (TFN_m, T\acute{F}N_m) \right\rangle \right) \\ \widetilde{UAC}_{T,I} &= \bigcup_{j=1}^k TFNIFWG_w \left(\left\langle uac_j, (TFN_1, T\acute{F}N_1), \dots, (TFN_m, T\acute{F}N_m) \right\rangle \right), \\ \widetilde{UAB}_{T,I} &= \bigcup_{j=1}^k TFNIFWG_w \left(\left\langle uab_j, (TFN_1, T\acute{F}N_1), \dots, (TFN_m, T\acute{F}N_m) \right\rangle \right), \\ \widetilde{SV}_{T,I} &= \bigcup_{j=1}^k TFNIFWG_w \left(\left\langle sv_j, (TFN_1, T\acute{F}N_1), \dots, (TFN_m, T\acute{F}N_m) \right\rangle \right), \\ \tilde{i}_{T,I} &= \bigcup_{j=1}^k TFNIFWG_w \left(\left\langle i_j, (TFN_1, T\acute{F}N_1), \dots, (TFN_m, T\acute{F}N_m) \right\rangle \right), \\ \tilde{n}_{T,I} &= \bigcup_{j=1}^k TFNIFWG_w \left(\left\langle n_j, (TFN_1, T\acute{F}N_1), \dots, (TFN_m, T\acute{F}N_m) \right\rangle \right). \end{aligned}$$

When the time horizon is infinite, triangular intuitionistic fuzzy present worth is calculated by Eq. 8.21:

$$\widetilde{PW}_{T,I} = -\widetilde{FC}_{T,I} - \left(\frac{\widetilde{UAC}_{T,I}}{\widetilde{i}_{T,I}} \right) + \left(\frac{\widetilde{UAB}_{T,I}}{\widetilde{i}_{T,I}} \right) \tag{8.21}$$

The defuzzified values of these parameters are needed for further calculations. For instance, the defuzzified value of $\widetilde{FC}_{T,I}$ is obtained by the following process:

$$\begin{aligned} TFNIFWG_w \left(\left\langle \left(fc_j, (TFN_1, T\acute{F}N_1), \right) \right\rangle \right) &= \widetilde{\mu}_{fc_j} \\ &= \left(\left(\mu_{fc_{jl}}, \mu_{fc_{jm}}, \mu_{fc_{ju}} \right), \left(\acute{\mu}_{fc_{jl}}, \acute{\mu}_{fc_{jm}}, \acute{\mu}_{fc_{ju}} \right) \right), \\ & \quad j=1, \dots, k \end{aligned} \tag{8.22}$$

Defuzzified value of $\left(\left(\mu_{fc_{jl}}, \mu_{fc_{jm}}, \mu_{fc_{ju}} \right), \left(\acute{\mu}_{fc_{jl}}, \acute{\mu}_{fc_{jm}}, \acute{\mu}_{fc_{ju}} \right) \right)$ is $Def \left(\widetilde{\mu}_{fc_j} \right)$ which is obtained by Eq. 8.11. The defuzzified value of $\widetilde{FC}_{T,I}$ is obtained by Eq. 8.23:

$$Def \widetilde{FC}_{T,I} = \frac{\sum_{j=1}^k fc_j \left(Def \left(\widetilde{\mu}_{fc_j} \right) \right)^2}{\sum_{j=1}^k \left(Def \left(\widetilde{\mu}_{fc_j} \right) \right)^2} \tag{8.23}$$

Other parameters could be defuzzified in a similar way.

8.3.4 Hesitant Fuzzy Environmental Economics Methods

Kahraman et al. (2015) introduced hesitant fuzzy net present worth and hesitant fuzzy annual worth methods. A hesitant fuzzy set (HFS) is another extension of fuzzy sets that aims to model the uncertainty originated by the hesitation that might arise in the assignment of membership degrees of the elements to a fuzzy set (Kahraman et al. 2017).

Triangular Fuzzy Hesitant Fuzzy Sets (TFHFS) are proposed in 2013 by Yu. In TFHFS several triangular fuzzy numbers are used to express the membership degree of an element.

A TFHFS \widetilde{E} on a fixed set X is defined in terms of a function $\widetilde{f}_E(x)$ that returns several triangular fuzzy values,

$$\tilde{E} = \{ \langle x, \tilde{f}_E(x) \rangle \mid x \in X \} \tag{8.24}$$

where $\tilde{f}_E(x)$ is a set of several triangular fuzzy numbers which express the possible membership degrees of an element $x \in X$ to a set \tilde{E} .

For a Triangular Fuzzy Hesitant Fuzzy Set (TFHFS), \tilde{f} , $s(\tilde{f}) = \frac{1}{l_{\tilde{f}}} \sum_{TFN \in \tilde{f}} \bar{X}(TFN)$ is called the score function of \tilde{f} with $l_{\tilde{f}}$ being the number of TFNs in \tilde{f} (Yu 2013). $h(\tilde{f}) = \frac{1}{l_{\tilde{f}}} \sum_{TFN \in \tilde{f}} \sigma(TFN)$ is called the deviation function of \tilde{f} . For \tilde{f}_1 and \tilde{f}_2 ,

- If $s(\tilde{f}_1) > s(\tilde{f}_2)$, then $\tilde{f}_1 \geq \tilde{f}_2$
- If $s(\tilde{f}_1) = s(\tilde{f}_2), h(\tilde{f}_1) = h(\tilde{f}_2)$, then $\tilde{f}_1 = \tilde{f}_2$
- If $s(\tilde{f}_1) = s(\tilde{f}_2), h(\tilde{f}_1) > h(\tilde{f}_2)$, then $\tilde{f}_1 < \tilde{f}_2$
- If $s(\tilde{f}_1) = s(\tilde{f}_2), h(\tilde{f}_1) > h(\tilde{f}_2)$, then $\tilde{f}_1 > \tilde{f}_2$

Let \tilde{f}_1 and \tilde{f}_2 be two THHFEs, then

$$\tilde{f}_1 \oplus \tilde{f}_2 = \{ (l_1 + l_2 - l_1.l_2, m_1 + m_2 - m_1.m_2, u_1 + u_2 - u_1.u_2) \mid TFN_1 \in \tilde{f}_1, TFN_2 \in \tilde{f}_2 \} \tag{8.25}$$

$$\tilde{f}_1 \otimes \tilde{f}_2 = \{ l_1.l_2, m_1.m_2, u_1.u_2 \mid TFN_1 \in \tilde{f}_1, TFN_2 \in \tilde{f}_2 \} \tag{8.26}$$

$$\tilde{f}^\lambda = \{ (l)^\lambda, (m)^\lambda, (u)^\lambda \mid TFN \in \tilde{f} \}, \quad \lambda > 0 \tag{8.27}$$

$$\lambda \tilde{f} = \{ 1 - (1 - l)^\lambda, 1 - (1 - m)^\lambda, 1 - (1 - u)^\lambda \mid TFN \in \tilde{f} \}, \quad \lambda > 0 \tag{8.28}$$

where $\widetilde{TFN}_1 = (l_1, m_1, u_1)$ and $\widetilde{TFN}_2 = (l_2, m_2, u_2)$.

For aggregating triangular fuzzy hesitant fuzzy sets, Triangular Fuzzy Hesitant Fuzzy Weighted Averaging (TFHFWA) operator is used. Let $\tilde{f}_j (j = 1, 2, \dots, n)$ be a collection of TFHFEs. $w = (w_1, w_2, \dots, w_n)^T$ is the weight vector of $\tilde{f}_j (j = 1, 2, \dots, n)$ with $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$, then a TFHFWA operator is a mapping TFHFWA: $F^n \rightarrow \bar{F}$ such that

$$TFHFWA(\tilde{f}_1, \tilde{f}_2, \dots, \tilde{f}_n) = \oplus_{j=1}^n (w_j \tilde{f}_j) \left\{ 1 - \prod_{j=1}^n (1 - L_j)^{w_j}, 1 - \prod_{j=1}^n (1 - M_j)^{w_j}, 1 - \prod_{j=1}^n (1 - U_j)^{w_j} \mid \widetilde{TFN}_1 \in \tilde{f}_1, \widetilde{TFN}_1 \in \tilde{f}_1, \dots, \widetilde{TFN}_n \in \tilde{f}_n \right\} \tag{8.29}$$

For the defuzzification of triangular hesitant fuzzy sets, the defuzzified value of a hesitant $\widetilde{TFN} = (l, m, u)$ can be defined as follows:

$$Def(\widetilde{TFN}) = \frac{l + 2m + u}{4} \tag{8.30}$$

In the hesitant fuzzy present worth analysis, investment parameters are expressed using triangular fuzzy hesitant fuzzy sets. The parameters used in the calculations are expressed by TFHFS in Eqs. 8.31–8.36 where m evaluations are made for each of the parameter.

$$\widetilde{FC}_{T,h} = \left\{ \langle fc_1, TFN_1, \dots, TFN_m \rangle, \langle fc_2, TFN_1, \dots, TFN_m \rangle, \dots, \langle fc_k, TFN_1, \dots, TFN_m \rangle \right\} \tag{8.31}$$

$$\widetilde{UAC}_{T,h} = \left\{ \langle uac_1, TFN_1, \dots, TFN_m \rangle, \langle uac_2, TFN_1, \dots, TFN_m \rangle, \dots, \langle uac_k, TFN_1, \dots, TFN_m \rangle \right\} \tag{8.32}$$

$$\widetilde{UAB}_{T,h} = \left\{ \langle uab_1, TFN_1, \dots, TFN_m \rangle, \langle uab_2, TFN_1, \dots, TFN_m \rangle, \dots, \langle uab_k, TFN_1, \dots, TFN_m \rangle \right\} \tag{8.33}$$

$$\widetilde{SV}_{T,h} = \left\{ \langle sv_1, TFN_1, \dots, TFN_m \rangle, \langle sv_2, TFN_1, \dots, TFN_m \rangle, \dots, \langle sv_k, TFN_1, \dots, TFN_m \rangle \right\} \tag{8.34}$$

$$\tilde{i}_{T,h} = \left\{ \langle i_1, TFN_1, \dots, TFN_m \rangle, \langle i_2, TFN_1, \dots, TFN_m \rangle, \dots, \langle i_k, TFN_1, \dots, TFN_m \rangle \right\} \tag{8.35}$$

$$\tilde{n}_{T,h} = \left\{ \langle n_1, TFN_1, \dots, TFN_m, n_2, TFN_1, \dots, TFN_m \rangle, \dots, \langle n_k, TFN_1, \dots, TFN_m \rangle \right\} \tag{8.36}$$

where FC represents the first cost of the alternative, UAC represents uniform annual cost of the alternative, UAB represents uniform annual benefit, n represents project life, i represents interest rate, and SV represents salvage value.

The hesitant fuzzy present worth ($\widetilde{PW}_{T,h}$) of an investment alternative can be calculated by Eq. 8.37 or Eq. 8.38:

$$\begin{aligned} \widetilde{PW}_{T,h} = & -\widetilde{FC}_{T,h} - \widetilde{UAC}_{T,h} \left(\frac{P}{A}, \tilde{i}_{T,h}, \tilde{n}_{T,h} \right) \\ & + \widetilde{UAB}_h \left(\frac{P}{A}, \tilde{i}_{T,h}, \tilde{n}_{T,h} \right) + \widetilde{SV}_h \left(\frac{P}{F}, \tilde{i}_{T,h}, \tilde{n}_{T,h} \right) \end{aligned} \quad (8.37)$$

or

$$\begin{aligned} \widetilde{PW}_{T,h} = & -\widetilde{FC}_{T,h} - \widetilde{UAC}_{T,h} \left[\frac{(1 + \tilde{i}_{T,h})^{\tilde{n}_{T,h}} - 1}{\tilde{i}_{T,h} (1 + \tilde{i}_{T,h})^{\tilde{n}_{T,h}}} \right] \\ & + \widetilde{UAB}_{T,h} \left[\frac{(1 + \tilde{i}_{T,h})^{\tilde{n}_{T,h}} - 1}{\tilde{i}_{T,h} (1 + \tilde{i}_{T,h})^{\tilde{n}_{T,h}}} \right] + \widetilde{SV}_{T,h} (1 + \tilde{i}_{T,h})^{-\tilde{n}_{T,h}} \end{aligned} \quad (8.38)$$

where

$$\begin{aligned} \widetilde{FC}_{T,h} &= \bigcup_{j=1}^k TFHFWA(\langle fc_j, TFN_1, \dots, TFN_m \rangle) \\ \widetilde{UAC}_{T,h} &= \bigcup_{j=1}^k TFHFWA(\langle uac_j, TFN_1, \dots, TFN_m \rangle), \\ \widetilde{UAB}_{T,h} &= \bigcup_{j=1}^k TFHFWA(\langle uab_j, TFN_1, \dots, TFN_m \rangle), \\ \widetilde{SV}_{T,h} &= \bigcup_{j=1}^k TFHFWA(\langle sv_j, TFN_1, \dots, TFN_m \rangle), \\ \tilde{i}_{T,h} &= \bigcup_{j=1}^k TFHFWA(\langle i_j, TFN_1, \dots, TFN_m \rangle), \\ \tilde{n}_{T,h} &= \bigcup_{j=1}^k TFHFWA(\langle n_j, TFN_1, \dots, TFN_m \rangle). \end{aligned}$$

When the time horizon is infinite, triangular intuitionistic fuzzy present worth is calculated by Eq. 8.39:

$$\widetilde{PW}_{T,h} = -\widetilde{FC}_{T,h} - \left(\frac{\widetilde{UAC}_{T,h}}{\widetilde{i}_{T,h}} \right) + \left(\frac{\widetilde{UAB}_{T,h}}{\widetilde{i}_{T,h}} \right) \tag{8.39}$$

For the defuzzification of triangular hesitant fuzzy sets, the defuzzified value of $\widetilde{FC}_{T,h}$ is obtained as follows:

$$TFHFWA(\langle f_{c_j}, TFN_1, \dots, TFN_m \rangle) = \tilde{\mu}_{f_{c_j}} = \left(\mu_{f_{c_{j1}}}, \mu_{f_{c_{jm}}}, \mu_{f_{c_{ju}}} \right), \quad j = 1, \dots, k \tag{8.40}$$

Defuzzified value of $\left(\mu_{f_{c_{j1}}}, \mu_{f_{c_{jm}}}, \mu_{f_{c_{ju}}} \right)$ is $Def(\tilde{\mu}_{f_{c_j}})$ which is obtained by Eq. 8.30. Other parameters could be defuzzified in a similar way.

8.4 An Application

Wind turbines have two major types based on their axis; the horizontal axis wind turbine (HAWT) and the vertical axis wind turbine (VAWT). In general HAWTs have greater capacities than VAWTs. Therefore, HAWTs are preferred for the industrial energy production. Mostly the useful life of HAWT is considered as 20 years. However the useful life of a wind turbine could increase by regular maintenances. In this chapter, a HAWT type wind turbine is analyzed for two scenarios that are (1) using the turbine without additional maintenances and reinvest at the end of its useful life, (2) using turbine with routine maintenances and take its useful life as infinite.

The economic parameter values of two alternatives are represented by different types of fuzzy numbers in Tables 8.2, 8.3, 8.4, 8.5 and 8.6.

Table 8.2 Parameters defined by ordinary fuzzy sets

Parameter	Scenario I possible cash flows (1000€)	Scenario II possible cash flows (1000€)
\widetilde{FC}	(630,650,670)	(630,650,670)
\widetilde{UAC}	(40,45,50)	(40,45,50)
\widetilde{UAB}	(300,350,400)	(300,350,400)
\widetilde{MC} (once in each five years)	-	(50,80,110)
\widetilde{SV}	(100,130,150)	
$\widetilde{i}\%$	(7,8,9)	(7,8,9)
nn	20	infinite

Table 8.3 Parameters defined by type 2 fuzzy sets

Parameter	Scenerio I possible cash flows (1000€)	Scenerio II possible cash flows (1000€)
\widetilde{FC}	(630,650,670;1) (640,650,660;0.9)	(630,650,670;1) (640,650,660;0.9)
\widetilde{UAC}	(40,45,50;1)(42,45,48;0.9)	(40,45,50;1)(42,45,48;0.9)
\widetilde{UAB}	(300,350,400;1)(310,350,390,0.9)	(300,350,400;1) (310,350,390,0.9)
\widetilde{MC} (once in each five years)	–	(50,80,110;1)(60,80,100;0.9)
\widetilde{SV}	(100,130,150;1) (110,130,140;0.9)	
$\widetilde{i}\%$	(7,8,9;1)(7.5,8,8.5;0.9)	(7,8,9;1)(7.5,8,8.5;0.9)
n	20	Infinite

Table 8.4 Experts' compromised membership degrees based on IVIFS

Parameter	Possible values	Experts' weights		
		E1	E2	E3
		0.3	0.4	0.3
FC	\$630,000	((0.3,0.6][0.2,0.4])	((0.4,0.6][0.2,0.4])	((0.3,0.5][0.2,0.45])
	\$650,000	((0.4,0.5][0.1,0.4])	((0.3,0.5][0.3,0.5])	((0.4,0.6][0.1,0.3])
	\$670,000	((0.3,0.7][0.2,0.25])	((0.2,0.6][0.2,0.4])	((0.4,0.5][0.3,0.5])
UAC	\$40.000	((0.2,0.5][0.3,0.5])	((0.4,0.7][0.1,0.3])	((0.7,0.8][0.05,0.1])
	\$45,000	((0.4,0.7][0.1,0.3])	((0.5,0.7][0.1,0.3])	((0.3,0.6][0.1,0.3])
	\$50,000	((0.5,0.7][0.1,0.3])	((0.5,0.7][0.1,0.2])	((0.3,0.5][0.3,0.4])
UAB	\$300,000	((0.4,0.5][0.3,0.4])	((0.6,0.8][0.1,0.2])	((0.5,0.8][0.1,0.2])
	\$350,000	((0.4,0.7][0.1,0.1])	((0.5,0.7][0.1,0.3])	((0.4,0.7][0.1,0.3])
	\$400,000	((0.6,0.8][0.05,0.1])	((0.4,0.6][0.2,0.4])	((0.3,0.6][0.2,0.4])
MC	\$50,000	((0.1,0.3][0.5,0.6])	((0.4,0.7][0.1,0.2])	((0.5,0.6][0.2,0.4])
	\$80,000	((0.4,0.6][0.1,0.2])	((0.4,0.8][0,0.1])	((0.4,0.6][0.1,0.3])
	\$110,000	((0.6,0.8][0,0.1])	((0.5,0.6][0.1,0.3])	((0.3,0.4][0.2,0.5])
SV	\$100,000	((0.3,0.5][0.2,0.4])	((0.5,0.7][0.1,0.3])	((0.4,0.5][0.2,0.5])
	\$130,000	((0.5,0.7][0.1,0.2])	((0.4,0.6][0.2,0.4])	((0.3,0.5][0.3,0.5])
	\$150,000	((0.6,0.7][0.05,0.1])	((0.3,0.4][0.3,0.5])	((0.1,0.2][0.5,0.7])
i	7%	((0.4,0.7][0.1,0.2])	((0.6,0.7][0.1,0.3])	((0.3,0.5][0.4,0.5])
	8%	((0.2,0.5][0.3,0.4])	((0.5,0.7][0.1,0.3])	((0.6,0.8][0,0.1])
	9%	((0.6,0.8][0,0.1])	((0.4,0.5][0.2,0.4])	((0.2,0.4][0.4,0.5])

Table 8.5 Aggregated and defuzzified matrix for IVIFS

Parameter	Possible values	Aggregated value	Defuzzified Value of membership	Defuzzified value of parameter
FC	\$630,000	([0.341,0.572][0.2,0.415])	0.803	649,867
	\$650,000	([0.361,0.532][0.186,0.446])	0.788	
	\$670,000	([0.294,0.607][0.231,0.392])	0.795	
UAC	\$40,000	([0.468,0.69][0.151,0.31])	0.962	45,074
	\$45,000	([0.415,0.672][0.1,0.3])	0.944	
	\$50,000	([0.494,0.702][0.165,0.294])	0.962	
UAB	\$300,000	([0.471,0.69][0.165,0.266])	0.973	348,840
	\$350,000	([0.442,0.7][0.1,0.245])	0.984	
	\$400,000	([0.443,0.675][0.157,0.322])	0.939	
MC	\$50,000	[0.358,0.578][0.271,0.403]	0.799	83,199
	\$80,000	([0.4,0.696][0.06,0.194])	0.984	
	\$110,000	([0.482,0.633][0.103,0.317])	0.952	
SV	\$100,000	([0.415,0.592][0.161,0.395])	0.864	123,716
	\$130,000	([0.405,0.607][0.203,0.380])	0.860	
	\$150,000	([0.361,0.468][0.306,0.532])	0.705	
i	7%	([0.465,0.65][0.203,0.341])	0.921	796
	8%	([0.461,0.69][0.138,0.279])	0.971	
	9%	([0.42,0.598][0.215,0.358])	0.866	

Table 8.6 Experts' compromised membership degrees based on triangular HFS

Parameter	Possible values	Experts' weights		
		E1	E2	E3
		0.3	0.4	0.3
FC	\$630,000	(0.3,0.4,0.6)	(0.4,0.5,0.6)	(0.3,0.4,0.5)
	\$650,000	(0.4,0.5,0.6)	(0.3,0.4,0.5)	(0.4,0.5,0.6)
	\$670,000	(0.3,0.5,0.7)	(0.2,0.5,0.6)	(0.4,0.4,0.5)
UAC	\$40,000	(0.2,0.3,0.5)	(0.4,0.5,0.7)	(0.7,0.8,0.9)
	\$45,000	(0.4,0.6,0.7)	(0.5,0.6,0.7)	(0.4,0.6,0.7)
	\$50,000	(0.5,0.6,0.7)	(0.6,0.7,0.8)	(0.3,0.4,0.5)
UAB	\$300,000	(0.4,0.5,0.6)	(0.6,0.7,0.8)	(0.5,0.7,0.8)
	\$350,000	(0.5,0.6,0.8)	(0.5,0.6,0.7)	(0.4,0.5,0.7)
	\$400,000	(0.6,0.7,0.9)	(0.4,0.5,0.6)	(0.3,0.5,0.6)
MC	\$50,000	(0.1,0.2,0.3)	(0.4,0.5,0.7)	(0.5,0.5,0.6)
	\$80,000	(0.5,0.7,0.8)	(0.7,0.8,0.9)	(0.6,0.7,0.8)
	\$110,000	(0.6,0.8,0.9)	(0.5,0.6,0.7)	(0.3,0.4,0.5)
SV	\$100,000	(0.4,0.5,0.6)	(0.5,0.6,0.7)	(0.4,0.5,0.5)
	\$130,000	(0.6,0.7,0.8)	(0.4,0.5,0.6)	(0.3,0.4,0.5)
	\$150,000	(0.5,0.7,0.8)	(0.3,0.4,0.5)	(0.1,0.1,0.2)
i	7%	(0.4,0.5,0.7)	(0.6,0.7,0.7)	(0.3,0.4,0.5)
	8%	(0.2,0.4,0.5)	(0.5,0.6,0.7)	(0.6,0.8,0.9)
	9%	(0.6,0.7,0.9)	(0.4,0.4,0.5)	(0.2,0.3,0.4)

8.4.1 Evaluation Using Ordinary Fuzzy Present Worth

Table 8.2 shows the values of parameters using ordinary triangular fuzzy sets.

In the present worth analysis, period is defined as least common multiples of the useful alternative lives. Therefore, in our analysis, the analysis period is taken as infinite. In scenario 1, there will be cash inflow series from the salvage values and cash outflow series from the reinvestment costs which occur once in each 20 years period. To calculate the fuzzy present worth for scenario 1, first effective interest rate for 20 years should be calculated as follows:

$$\begin{aligned}\tilde{i}_{20} &= (1 + \tilde{i}_1)^{20} - 1 = \left((1 + i_l)^{20} - 1, (1 + i_m)^{20} - 1, (1 + i_r)^{20} - 1 \right) \\ &= \left((1 + 0.07)^{20} - 1, (1 + 0.08)^{20} - 1, (1 + 0.09)^{20} - 1 \right) \\ &= (2.8697, 3.6609, 4.6044)\end{aligned}$$

Fuzzy net present worth of scenario 1 is calculated using Eq. 8.2 as follows:

$$\begin{aligned}\widetilde{NPV} &= -\widetilde{FC} - \frac{\widetilde{UAC}}{\tilde{i}} + \frac{\widetilde{UAB}}{\tilde{i}} + \frac{\widetilde{SV}}{\tilde{i}_{20}} - \frac{\widetilde{FC}}{\tilde{i}_{20}} \\ NPV_l &= -FC_r - \frac{UAC_r}{i_r} + \frac{UAB_l}{i_r} + \frac{SV_l}{i_{20r}} - \frac{FC_r}{i_{20r}} \\ NPV_m &= -FC_m - \frac{UAC_m}{i_m} + \frac{UAB_m}{i_m} + \frac{SV_m}{i_{20m}} - \frac{FC_m}{i_{20m}} \\ NPV_r &= -FC_l - \frac{UAC_l}{i_l} + \frac{UAB_r}{i_l} + \frac{SV_r}{i_{20l}} - \frac{FC_l}{i_{20l}}\end{aligned}$$

Using the formulas given above \widetilde{NPV} is calculated as (2702.8, 3020.46, 4345.59) for scenario 1.

To calculate the fuzzy present worth for scenario 2, the effective interest rate for 5 years should be calculated as follows:

$$\begin{aligned}\tilde{i}_5 &= (1 + \tilde{i}_1)^5 - 1 = \left((1 + i_l)^5 - 1, (1 + i_m)^5 - 1, (1 + i_r)^5 - 1 \right) \\ &= \left((1 + 0.07)^5 - 1, (1 + 0.08)^5 - 1, (1 + 0.09)^5 - 1 \right) \\ &= (0.4025, 0.4693, 0.5386)\end{aligned}$$

Fuzzy net present worth of scenario 2 is calculated using the following equation:

$$\widetilde{NPV} = -\widetilde{FC} - \frac{\widetilde{UAC}}{\tilde{i}} + \frac{\widetilde{UAB}}{\tilde{i}} + \frac{\widetilde{MC}}{\tilde{i}_5}$$

\widetilde{NPV} is calculated as (1903.545, 2992.033, 4388.634) for scenario 2.

Defuzzified values of \widetilde{NPV} for scenario 1 and 2 are calculated using Eq. 8.3 as 3272.328 and 3069.061, respectively.

8.4.2 Evaluation Using Type 2 Fuzzy Present Work

Table 8.3 shows the values of parameters using triangular interval type 2 fuzzy sets.

Effective interest rates for 5 and 20 years are calculated as follows:

$$\begin{aligned}\tilde{i}_5 &= \left(1 + \tilde{i}_1\right)^5 - 1 = \left(\left(1 + i_t^U\right)^5 - 1, \left(1 + i_m^U\right)^5 - 1, \left(1 + i_r^U\right)^5 - 1\right); 1 \\ &\quad \left(\left(1 + i_t^L\right)^5 - 1, \left(1 + i_m^L\right)^5 - 1, \left(1 + i_r^L\right)^5 - 1\right); 0.9 \\ &= \left(\left(1 + 0.07\right)^5 - 1, \left(1 + 0.08\right)^5 - 1, \left(1 + 0.09\right)^5 - 1\right); 1 \\ &\quad \left(\left(1 + 0.075\right)^5 - 1, \left(1 + 0.08\right)^5 - 1, \left(1 + 0.085\right)^5 - 1\right); 0.9 \\ &= (0.4025, 0.4693, 0.5386; 1)(0.4356, 0.4693, 0.5036; 0.9)\end{aligned}$$

$$\begin{aligned}\tilde{i}_{20} &= \left(1 + \tilde{i}_1\right)^{20} - 1 \\ &= \left(\left(1 + i_t^U\right)^{20} - 1, \left(1 + i_m^U\right)^{20} - 1, \left(1 + i_r^U\right)^{20} - 1\right); 1 \\ &\quad \left(\left(1 + i_t^L\right)^{20} - 1, \left(1 + i_m^L\right)^{20} - 1, \left(1 + i_r^L\right)^{20} - 1\right); 0.9 \\ &= \left(\left(1 + 0.07\right)^{20} - 1, \left(1 + 0.08\right)^{20} - 1, \left(1 + 0.09\right)^{20} - 1\right); 1 \\ &\quad \left(\left(1 + 0.075\right)^{20} - 1, \left(1 + 0.08\right)^{20} - 1, \left(1 + 0.085\right)^{20} - 1\right); 0.9 \\ &= (2.8697, 3.6609, 4.6044; 1)(3.2478, 3.6609, 4.1120; 0.9)\end{aligned}$$

$\widetilde{\widetilde{NPV}}$ s are calculated using Eq. 8.6, as (1983.983, 3020.458, 4345.592; 1)

(2288.598, 3020.458, 3846.05; 0.9) and (1903.544, 2992.033, 4388.634; 1)

(2223.783, 2992.033, 3862.259; 0.9) for scenario 1 and 2, respectively.

Defuzzified values are calculated using Eqs. 8.7 and 8.3 as 3,068.257 and 3043.294 respectively.

Table 8.7 Aggregated and defuzzified values of triangular HFS

Parameter	Possible values	Aggregated value	Defuzzified Value of membership	Defuzzified value of parameter
FC	\$630,000	(0.341,0.442,0.572)	0.449	650,347
	\$650,000	(0.361,0.462,0.562)	0.462	
	\$670,000	(0.294,0.471,0.607)	0.461	
UAC	\$40,000	(0.468,0.579,0.748)	0.594	45,020
	\$45,000	(0.442,0.6,0.7)	0.585	
	\$50,000	(0.494,0.597,0.702)	0.597	
UAB	\$300,000	(0.516,0.650,0.753)	0.642	346,515
	\$350,000	(0.471,0.572,0.734)	0.587	
	\$400,000	(0.443,0.571,0.736)	0.580	
MC	\$50,000	(0.358,0.424,0.578)	0.446	85,035
	\$80,000	(0.618,0.744,0.848)	0.739	
	\$110,000	(0.482,0.633,0.748)	0.624	
SV	\$100,000	(0.442,0.542,0.618)	0.536	124,043
	\$130,000	(0.443,0.546,0.652)	0.547	
	\$150,000	(0.317,0.449,0.562)	0.444	
i	7%	(0.465,0.569,0.650)	0.563	794
	8%	(0.461,0.633,0.748)	0.619	
	9%	(0.420,0.489,0.674)	0.518	

8.4.3 Evaluation Using Intuitionistic Fuzzy Present Worth

Table 8.4 shows the values of parameters using interval valued intuitionistic fuzzy sets.

Table 8.5 shows the aggregated and defuzzified values for IVIFS based on Eqs. (8.11) and (8.12).

Using the data shown in Table 8.4 NPVs for Scenario 1 and 2 are calculated as \$3,020.785 and \$2,987,552, respectively.

8.4.4 Evaluation Hesitant Fuzzy Annual Worth

Possible values of the parameters and their corresponding compromised membership degrees are given in Table 8.6 using triangular HFS.

Table 8.7 shows the aggregated and defuzzified values for Triangular HFS based on Eqs. (8.29) and (8.30).

Using the data shown in Table 8.6 NPVs for Scenarios 1 and 2 are calculated as \$2,996.775 and \$2,959.74, respectively.

8.5 Conclusions

Wind energy investments involve several uncertain parameters; each can be represented by linguistic terms or fuzzy numbers. The cost and benefit parameters of wind energy investments can be better represented by fuzzy sets. Thus, an investment decision report can be presented to the investor with a list of possible results and their membership degrees.

PW analysis is the most used investment analysis technique. However, applying classical PW analysis under vagueness may produce unrealistic suggestions. Taking all possibilities into consideration before an investment decision is given is extremely important. Fuzzy PW analysis exhibits all possibilities regarding the investment outcomes together with their membership degrees.

For further research, other extensions of fuzzy sets such as Pythagorean fuzzy sets can be used for analyzing the wind energy investments. Other renewable energy alternatives can be also examined such as biomass energy, solar energy, geothermal energy, hydroelectric energy, ocean energy, or hydrogen energy under fuzziness. Types of fuzzy numbers can be changed alternatively such as trapezoidal fuzzy numbers or LR-type fuzzy numbers.

References

- Al-Sharafi, A., Sahin, A. Z., Ayar, T., & Yilbas, B. S. (2017). Techno-economic analysis and optimization of solar and wind energy systems for power generation and hydrogen production in Saudi Arabia. *Renewable and Sustainable Energy Reviews*, *69*, 33–49.
- Aquila, G., Junior, P. R., de Oliveira Pamplona, E., & de Queiroz, A. R. (2017). Wind power feasibility analysis under uncertainty in the Brazilian electricity market. *Energy Economics*, *65*, 127–136.
- Ashkaboosi, M., Nourani, S. M., Khazaei, P., Dabbaghjamanesh, M., & Moeini, A. (2016). An optimization technique based on profit of investment and market clearing in wind power systems. *American Journal of Electrical and Electronic Engineering*, *4*(3), 85–91.
- Atanassov, K. (2012). *On Intuitionistic Fuzzy Sets Theory*. Berlin, Heidelberg: Springer.
- Atanassov, K.T. (1986), Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, *20*, 87–96 (1986).
- Aydin, N. Y., Kentel, E., & Duzgun, S. (2010). GIS-based environmental assessment of wind energy systems for spatial planning: A case study from Western Turkey. *Renewable and Sustainable Energy Reviews*, *14*(1), 364–373.
- Baringo, L., & Conejo, A. J. (2013). Risk-constrained multi-stage wind power investment. *IEEE Transactions on Power Systems*, *28*(1), 401–411.
- Caralis, G., Diakoulaki, D., Yang, P., Gao, Z., Zervos, A., & Rados, K. (2014). Profitability of wind energy investments in China using a Monte Carlo approach for the treatment of uncertainties. *Renewable and Sustainable Energy Reviews*, *40*, 224–236.
- Cavallaro, F., & Ciruolo, L. (2005). A multicriteria approach to evaluate wind energy plants on an Italian island. *Energy Policy*, *33*(2), 235–244.
- Celik, A. N. (2003). Energy output estimation for small-scale wind power generators using Weibull-representative wind data. *Journal of Wind Engineering and Industrial Aerodynamics*, *91*(5), 693–707.

- Chang, C. T. (2017). Fuzzy score technique for the optimal location of wind turbines installations. *Applied Mathematical Modelling*, 44, 576–587.
- Chen, D., Zhang, L., & Jiao, J. (2010). Triangle fuzzy number intuitionistic fuzzy aggregation operators and their application to group decision making. In *International Conference on Artificial Intelligence and Computational Intelligence AICI 2010* (pp. 350–357).
- Chiu, C. Y., & Park, C. S. (1994). Fuzzy cash flow analysis using present worth criterion. *The Engineering Economist*, 39(2), 113–138.
- Cunico, M. L., Flores, J. R., & Vecchiotti, A. (2017). Investment in the energy sector: An optimization model that contemplates several uncertain parameters. *Energy*, 138, 831–845.
- Ersoz, S., Akinci, T. C., Nogay, H. S., & Dogan, G. (2013). Determination of wind energy potential in Kırklareli-Turkey. *International Journal of Green Energy*, 10(1), 103–116.
- Fazelpour, F., Markarian, E., & Soltani, N. (2017). Wind energy potential and economic assessment of four locations in Sistan and Baluchestan province in Iran. *Renewable Energy*, 109, 646–667.
- Gumus, S., Kucukvar, M., & Tatari, O. (2016). Intuitionistic fuzzy multi-criteria decision making framework based on life cycle environmental, economic and social impacts: The case of US wind energy. *Sustainable Production and Consumption*, 8, 78–92.
- Kahraman, C., Çevik Onar, S., & Oztaysi, B. (2015). Engineering economic analyses using intuitionistic and hesitant fuzzy sets. *Journal of Intelligent & Fuzzy Systems*, 29(3), 1151–1168.
- Kahraman, C., Çevik Onar, S., & Oztaysi, B. (2016a). A comparison of wind energy investment alternatives using interval-valued intuitionistic fuzzy benefit/cost analysis. *Sustainability*, 8(2), 118.
- Kahraman, C., Oztaysi, B., & Çevik Onar, S. (2016b). A comprehensive literature review of 50 years of fuzzy set theory. *International Journal of Computational Intelligence Systems*, 9 (sup1), 3–24.
- Kahraman, C., Sarı, İ. U., Onar, S. C., & Oztaysi, B. (2017). Fuzzy Economic analysis methods for environmental economics. In *Intelligence Systems in Environmental Management: Theory and Applications* (pp. 315–346). Berlin: Springer.
- Kitzing, L., Juul, N., Drud, M., & Boomsma, T. K. (2017). A real options approach to analyse wind energy investments under different support schemes. *Applied Energy*, 188, 83–96.
- Kucukali, S. (2016). Risk scorecard concept in wind energy projects: An integrated approach. *Renewable and Sustainable Energy Reviews*, 56, 975–987.
- Kumar, P. S., & Hussain, R. J. (2014). A method for solving balanced intuitionistic fuzzy assignment problem. *International Journal of Engineering Research and Applications*, 4(3), 897–903.
- Kuo-Ping, C. (2011). Multiple criteria group decision making with triangular interval type-2 fuzzy sets. In *Proceedings of 2011 IEEE International Conference on Fuzzy Systems (FUZZ)*, Taipei (pp. 1098–7584), June 27–30, 2011.
- Lee, A. H., Chen, H. H., & Kang, H. Y. (2009). Multi-criteria decision making on strategic selection of wind farms. *Renewable Energy*, 34(1), 120–126.
- Lee, S. C. (2011). Using real option analysis for highly uncertain technology investments: The case of wind energy technology. *Renewable and Sustainable Energy Reviews*, 15(9), 4443–4450.
- Liu, X., & Zeng, M. (2017). Renewable energy investment risk evaluation model based on system dynamics. *Renewable and Sustainable Energy Reviews*, 73, 782–788.
- Madlener, R., Glensk, B., & Weber, V. (2011). Fuzzy portfolio optimization of onshore wind power plants. FCN Working Papers 10/2011, E.ON Energy Research Center, Future Energy Consumer Needs and Behavior (FCN), Revised Jul 2014.
- Mahapatra, G. S., & Roy, T. K. (2009). Reliability evaluation using triangular intuitionistic fuzzy numbers arithmetic operations. *World Academy of Science, Engineering and Technology*, 3(2), 422–429.
- Morshedizadeh, M., Kordestani, M., Carriveau, R., Ting, D. S.-K., & Saif, M. (2017). Application of imputation techniques and Adaptive Neuro-Fuzzy Inference System to predict wind turbine power production. *Energy*, 138(C), 394–404. Elsevier.

- Niewiadomski, A., Ochelska, J., & Szczepaniak, P. S. (2006). Interval-valued linguistic summaries of databases. *Control and Cybernetics*, 35(2), 415–443.
- Onar, S. C., & Kilavuz, T. N. (2015). Risk analysis of wind energy investments in Turkey. *Human and Ecological Risk Assessment*, 21, 1230–1245.
- Onar, S. C., Oztaysi, B., Otay, İ., & Kahraman, C. (2015). Multi-expert wind energy technology selection using interval-valued intuitionistic fuzzy sets. *Energy*, 90, 274–285.
- Onat, N., & Ersoz, S. (2011). Analysis of wind climate and wind energy potential of regions in Turkey. *Energy*, 36(1), 148–156.
- Panduru, K. K., Riordan, D., & Walsh, J. (2014). Fuzzy logic based intelligent energy monitoring and control for renewable energy. In *Irish Signals & Systems Conference 2014 and 2014 China-Ireland International Conference on Information and Communications Technologies (ISSC 2014/CICT 2014)*.
- Petković, D. (2015). Adaptive neuro-fuzzy optimization of the net present value and internal rate of return of a wind farm project under wake effect. *Journal of CENTRUM Cathedra: The Business and Economics Research Journal*, 8(1), 11–28.
- Petković, D., Shamshirband, S., Kamsin, A., Lee, M., Anicic, O., & Nikolić, V. (2016). Survey of the most influential parameters on the wind farm net present value (NPV) by adaptive neuro-fuzzy approach. *Renewable and Sustainable Energy Reviews*, 57, 1270–1278.
- Pinson, P., & Kariniotakis, G. N. (2003, June). Wind power forecasting using fuzzy neural networks enhanced with on-line prediction risk assessment. In *Power Tech Conference Proceedings, 2003 IEEE Bologna* (Vol. 2, 8 pp). IEEE.
- Şen, Z., & Şahin, A. D. (1997). Regional assessment of wind power in western Turkey by the cumulative semivariogram method. *Renewable Energy*, 12(2), 169–177.
- Shafiee, M. (2015). A fuzzy analytic network process model to mitigate the risks associated with offshore wind farms. *Expert Systems with Applications*, 42(4), 2143–2152.
- Shamshirband, S., Petković, D., Čojbašić, Ž., Nikolić, V., Anuar, N. B., Shuib, N. L. M., et al. (2014). Adaptive neuro-fuzzy optimization of wind farm project net profit. *Energy Conversion and Management*, 80, 229–237.
- Sheen, J. N. (2009). Applying fuzzy engineering economics to evaluate project investment feasibility of wind generation. *WSEAS Transactions on Systems*, 8(4), 501–510.
- Sheen, J. N. (2014). Real option analysis for renewable energy investment under uncertainty. In *Proceedings of the 2nd International Conference on Intelligent Technologies and Engineering Systems (ICITES2013)* (pp. 283–289). Cham: Springer.
- Soroudi, A. (2012). Possibilistic-scenario model for DG impact assessment on distribution networks in an uncertain environment. *IEEE Transactions on Power Systems*, 27(3), 1283–1293.
- Torra, V. (2010). Hesitant fuzzy sets. *International Journal of Intelligent Systems*, 25(6), 529–539.
- Ucal Sari, I., & Kahraman, C. (2015). Interval Type-2 fuzzy capital budgeting. *International Journal of Fuzzy Systems*, 17(4), 635–646.
- Wu, C. B., Huang, G. H., Li, W., Zhen, J. L., & Ji, L. (2016). An inexact fixed-mix fuzzy-stochastic programming model for heat supply management in wind power heating system under uncertainty. *Journal of Cleaner Production*, 112, 1717–1728.
- Wu, Y., Geng, S., Xu, H., & Zhang, H. (2014). Study of decision framework of wind farm project plan selection under intuitionistic fuzzy set and fuzzy measure environment. *Energy Conversion and Management*, 87, 274–284.
- Yeh, T. M., & Huang, Y. L. (2014). Factors in determining wind farm location: Integrating GQM, fuzzy DEMATEL, and ANP. *Renewable Energy*, 66, 159–169.
- Yu, D. (2013). Triangular hesitant fuzzy set and its application to teaching quality evaluation. *Journal of Information & Computational Science*, 10(7), 1925–1934.
- Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, 8, 338–353.
- Zadeh, L. A. (1974). Fuzzy logic and its application to approximate reasoning. *Information Processing*, 74, 591–594.
- Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning—I. *Information Sciences*, 8(3), 199–249.