Investigating Dynamic Correlation in the International Implied Volatility Indexes

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Abstract. This paper investigates dynamic interaction among international volatility indexes, consisting of VIX, VSTOXX, VDAX, VFTSE, VNVIXN, VHSI and VKOSPI. This paper also extends the multivariate normal distribution and multivariate student-t distribution based dynamic conditional correlation (DCC) model to a multivariate skew distribution. We then apply this extended model to estimate the dynamic volatility and correlation in international volatility indexes. The empirical results of model comparison reveal the multivariate skewed student-t distribution based CGARCH-DCC model to perform the best in our real data analysis. This indicates that the time-varying conditional correlation coefficients as well as volatility are skewed and fat tailed or leptokurtic in characteristic.

Keywords: Implied volatility indexes · Dynamic correlation and volatility
Multivariate skew distribution

1 Introduction

Volatility indexes are the economic indicators of risk assessment in the financial markets and they are designed to measure the market's expectation of future volatility implied by options prices. Moreover, volatility indexes are able to estimate the expectation of the future volatility over the next 30 days. The implied volatility indexes were introduced and have been calculated and published by Chicago Board Options Exchange (CBOE) since 1993. CBOE proposed the volatility index or VIX methodology to minimize risk on the portfolio of investment. In addition, Badshah [3] stated that the volatility indexes and the stock market returns have negative relationship; therefore, VIX is advantageous for investors to manage their risks. There are several related studies and writings emphasizing the volatility indexes of various worldwide financial markets (see e.g., Kaeck and Alexander [11]; Bugge et al. [4]; Psaradellis and Sermpinis [15]; Huskaj and Larsson [10]).

Nowadays, there are many famous volatility indexes, for instances, VIX, VNX, VXD VSTOXX, VFTSE, VCAC 40, VSTOXX and VKOSPI. These indexes are computed and provided on a 60–s basis as an average of implied volatilities in at–the–money options with a residual time–to–maturity equal to 30 days. We can observe that all volatility indexes have similar fluctuation pattern and they changed over time from early 2008 until the early 2009, corresponding to the credit crunch and liquidity crises.

Badshah [3] found that the high volatility indexes can put high pressure on stock market and thereby reducing return of stock.

Many Researchers believe that the different volatility indexes are likely to have correlation. Therefore, it is very important to know the volatility indexes spillover from one market to another or others. In addition, studying volatility indexes spillover phenomenon across all markets will contribute a great benefit for risk management, international portfolios and options traders. Several studies have investigated volatility spillover using historical volatility indexes (e.g., Hamao et al. [9]; Badshah [3]; Gamba-Santamaria et al. [7]). Äijö [1] studied the relationship among various European volatility indexes (VDAX, VSTOXX, and VSMI) and found that these volatility indexes not only are highly correlated but also vary over time. In addition, Badshah [3] found that the volatility indexes (VIX, VXN, VDAX and VSTOXX) are positively correlated.

Furthermore, some studies suggested that other assets and securities in capital market can be the factor affecting the volatility indexes. Khositkulporn [12] revealed that the oil price hike has shocked the global economy. For example, when the oil price increased to above USD 114 per barrel, the global economy faced recession and the equity markets became volatile during global financial crisis in 2008. Moreover, Kumar [13] examined the return and volatility spillover between gold price and Indian stock sector by assuming that the error term followed the student-t distribution. Although this study could not found any significant spillover from gold to stock but it found a negative dynamic correlation between these two variables, especially during the crisis. We expect that we should consider the other factors that may affect volatility indexes. Finally, we add oil and gold price to further investigate the factors affecting volatility indexes.

In this paper, we employ a multivariate generalized autoregressive conditional heteroskedasticity (GARCH) with exogenous variables based dynamic conditional correlation to investigate the correlation among volatility indexes and also find the effect of oil and gold on conditional mean and variance of the volatility indexes. However, this study has a concern that the symmetric assumption of the multivariate normal and student-t distributions might not be adequate in reality. To tackle such unrealistic assumption, we extend symmetric based dynamic conditional correlation (DCC) model to a multivariate skew distribution. The aim here is to allow for possible departure from symmetry to produce more flexible and more realistic families of distributions. In this study, a multivariate skew-normal and skew-student-t distributions, presented in Azzalini [2], are considered to construct a likelihood function of DCC model. Consequently, our model will give more flexibility to embrace the skewed and fat tailed or leptokurtic characteristics of volatility index.

The next section briefly outlines the methodology. Section 3 is the empirical part presenting data description, model selection and the results. The last section provides the conclusion of this work.

2   Econometric Methodology

2.1   Brief Review of Generalized Autoregressive Conditional Heteroskedasticity (GARCH) Families

2.1.1   GARCH with Exogenous Variables

The generalized autoregressive conditional heteroskedasticity (GARCH) process is an econometric model developed in 1982 by Engle to describe an approach to estimating conditional volatility in financial markets. In this study, we aim to investigate the effect of exogenous variables on the volatility index return therefore we employ a general GARCH with exogenous variables which contain exogenous variables in both mean and variance equations. Our model reads

$$y_t = c + \sum_{k=1}^{K} \phi_k x_{kt} + \sigma_t \varepsilon_t, \tag{1}$$

$$\sigma^2 = \varpi + \sum_{k=1}^{K} \varphi_k x_{kt} + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i} + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2. \tag{2}$$

where y_t is the return, x_{kt} represents $K \times T$ matrix of exogenous variables and σ^2 is time varying volatility obtained from the GARCH process in Eq. (2). It is quite obvious the structure of GARCH(p, q) consists of two parts. It has a polynomial $\beta(L)$ of order p-the autoregressive term, and a polynomial $\alpha(L)$ of order q - the moving average term.

The parameter α_i and β_j are assumed to be less than 1 and their summation must be less than 1. In addition, parameter ϕ_k and φ_k are the coefficients of the exogenous variable k in mean and variance equation, respectively. Note that the mean equation is applied to every GARCH type model with exogenous variables.

2.1.2   The GJR-GARCH

The model was proposed by Glosten, Jagannathan and Runkle [8] to model an asymmetry in the ARCH process. The GJR-GARCH with exogenous variables model is represented by the expression

$$\xi_t^2 = \varpi + \sum_{k=1}^{K} \varphi_k x_{kt} + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i} + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{p} \gamma_i I_{t-i} \varepsilon_{t-i}, \tag{3}$$

where $I_{t-i} = \begin{cases} 1 \ if \ \varepsilon_{t-i} < 0 \\ 0 \ if \ \varepsilon_{t-i} \geq 0 \end{cases}$.

2.1.3   Exponential GARCH

The exponential GARCH (EGARCH) may generally be specified as

$$\sigma^2 = \varpi + \sum_{k=1}^{K} \varphi_k x_{kt} + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i} + \sum_{j=1}^{q} \beta_j \ln \sigma_{t-j}^2. \tag{4}$$

This model differs from the variance equation in GARCH structure because of the log of the variance. The following specification also has been used in the financial literature Dhamija and Bhalla [5].

2.1.4   Integrated GARCH

IGARCH model applies both autoregressive and moving average structures to the variance, σ^2. The IGARCH is specified as

$$\sigma^2 = \varpi + \sum_{k=1}^{K} \varphi_k x_{kt} + \sum_{i=1}^{p} \alpha_i \varepsilon_{t-i} + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2. \tag{5}$$

where the sum of coefficients (α, β) must be less than 1.

2.1.5   Component GARCH

The Component GARCH model (CGARCH) can be written as:

$$\sigma_t^2 = q_t + \sum_{k=1}^{K} \varphi_k x_{kt} + \sum_{i=1}^{p} \alpha_i (\varepsilon_{t-i}^2 - q_{t-i}) + \sum_{j=1}^{p} \beta_j (\sigma_{t-j}^2 - q_{t-j}),$$
$$q_t = \omega + \rho q_{t-1} + \phi (\varepsilon_{t-1}^2 - \sigma_{t-1}^2) \tag{6}$$

where effectively the intercept of the GARCH model is now time-varying following first order autoregressive type dynamics. The sum of coefficients (α, β) must be less than 1 and $\rho < 1$ (effectively the persistence of the transitory and permanent component).

2.2   Dynamic Conditional Correlation (DCC)

The DCC–GARCH model can be best understood by recalling the best fit GARCH type model in Subsects. 2.1.1, 2.1.2, 2.1.3, 2.1.4 and 2.1.5. The difference is that the DCC-GARCH model is for multivariate volatility modeling. The advantage of the DCC model is that we can examine the time-varying correlation between many dimensions of a time series instead of a constant correlation. We again consider a k-dimensional innovation ε_{it} to the asset return series y_{it}, $i = 1, \ldots, N$. Let $\boldsymbol{\eta}_t = (\eta_{1t}, \ldots, \eta_{it})$ be the marginally standardized innovation vector ($\boldsymbol{\eta}_{it} = \varepsilon_{it}/\sqrt{\sigma_{ii,t}}$). The DCC model can be formulated as the following statistical specification:

$$\mathbf{Q}_t = (1 - \theta_1 - \theta_2) + \theta_1 \mathbf{Q}_{t-1} + \theta_2 \boldsymbol{\eta}_{t-1} \boldsymbol{\eta}'_{t-1}, \tag{7}$$

$$\mathbf{R}_t = \text{diag}\{\mathbf{Q}\}_t^{-1} \mathbf{Q}_t \text{diag}\{\mathbf{Q}\}_t^{-1} \quad \text{and} \quad \mathbf{H_t} = \mathbf{J}_t \mathbf{R}_t \mathbf{J}_t. \tag{8}$$

Here, $\mathbf{R}_t$ is the correlation matrix, $\mathbf{H_t}$ is the conditional covariance matrix of returns, and this correlation matrix is allowed to vary over time. Moreover, $\mathbf{Q}_t \equiv \{\sigma_{it}\}$ is the conditional covariance matrix of $\mathbf{\eta}_t$, θ_i are non-negative real numbers satisfying $0 \leq \theta_1 + \theta_2 < 1$, and $\mathbf{J}_t = diag\{\sqrt{\sigma}_{1t}, \dots, \sqrt{\sigma}_{it}\}$.

2.3   Estimation

In this study, we are concerned that the large number of parameters in the model could bring a difficult optimization. Thus, the two-stage estimation method is used, following (Engle [6]). This method allows the model to be estimated more easily even when the covariance matrix is very large. The model is estimated in two steps: firstly, various GARCH type models are estimated and then dynamic conditional correlation parameters are estimated in the second step. In other words, the parameters to be estimated in the correlation and GARCH processes are independent Engle [6]. Under reasonable regularity conditions, consistency of the first step will ensure consistency of the second step (see, Newey and McFadden [14]). In the two-step method, we can maximize the DCC likelihood function conditional on the estimated parameters from GARCH-type models in the first step.

$$\widehat{\theta} = \arg\max(L_D(\theta|y, \widehat{\Theta})), \ \ \text{where} \ \widehat{\Theta} = \arg\max(L_G(\Theta|y)) \tag{9}$$

is the maximization of likelihood function of GARCH type models. In this study, we extend the multivariate normal and student-t distributions based dynamic conditional correlation (DCC) model to skew-normal and skew-student-t distributions. The study herein tries to propose a skew likelihood function in the DCC-GARCH type models with exogenous variables and their performances are compared based on Akaike and Bayesian information criteria. In addition, we are concerned about the consistency of the two-step estimator, hence the likelihood distribution of the GARCH-type model in the first step and DCC model in the second step are assumed to have the same distribution. In this estimation, we consider multivariate normal, skew-normal, student-t, and skew-student-t distributions function, which are justified in Azzalini [2], are employed to construct the likelihood function in DCC part.

In this paragraph, several likelihood functions are presented, giving simple consistent but inefficient estimates of the parameters of the model. Here, the likelihood functions of volatility part (GARCH types), $\widehat{L}_V(\Theta)$, and correlation part (DCC), $\widehat{L}_C(\theta|\Theta)$, are written as in the followings:

(1) Normal likelihood function

$$\widehat{L}_{Vn,i}(\Theta) = \prod_{t=1}^{T} \left(\frac{1}{\sqrt{2\pi(\sigma_{it}^2)}} \exp\left(-\frac{\sigma_{it}\varepsilon_{it}}{2(\sigma_{it}^2)} \right) \right), \tag{10}$$

for return $i, i = 1, \dots, N$ and

$$\hat{L}_{Cn}(\theta|\hat{\Theta}) = \prod_{t=1}^{T} \frac{\exp(-\frac{1}{2}(\boldsymbol{\varepsilon}_t)^{\mathrm{T}}\mathbf{H}^{-1}(\boldsymbol{\varepsilon_t}))}{\sqrt{|2\pi\mathbf{H}|}}, \tag{11}$$

where $\boldsymbol{\varepsilon_t}$ is $N \times T$ matrix of error term.

(2) Skewed normal likelihood

$$\hat{L}_{Vsn,i}(\Theta) = \prod_{t=1}^{T} \left(\frac{2}{\xi_i + (\xi_i)^{-1}} \cdot f_n(z_i/\Upsilon_i) \right), \tag{12}$$

$$z_i = \frac{\varepsilon_{it}}{(\sigma_{it}^2)} \sqrt{((1 - m_1^2)(\xi_i^2 + 1/\xi_i^2) + 2m_1^2 - 1)} + mu_i,$$

$$m_1 = 2/\sqrt{2\pi}, \ \Upsilon_i = \xi_i^{sign(z_i)}, \ mu_i = m_1(\xi_i - 1/\xi_i),$$

where ξ_i is skew parameter of return i, $f_n(\cdot)$ is the probability density function of the normal distribution for return i and $sign(\cdot)$ is a function that returns a vector with the signs of the corresponding elements of z_i (for example, the sign of a real number is 1, 0, or -1 if the number is positive, zero, or negative, respectively). Then, the correlation part is

$$\hat{L}_{Csn}(\theta|\hat{\Theta}) = \prod_{t=1}^{T} \left(\frac{2}{\boldsymbol{\xi} + (\boldsymbol{\xi})^{-1}} \cdot f_n(\mathbf{z}/\Upsilon) \right), \tag{13}$$

$$\mathbf{z} = \frac{\boldsymbol{\varepsilon_t}}{(\mathbf{H})} \sqrt{((1 - m_1^2)(\boldsymbol{\xi}^2 + 1/\boldsymbol{\xi}^2) + 2m_1^2 - 1)} + mu,$$

$$m_1 = 2/\sqrt{2\pi}, \ \Upsilon = \boldsymbol{\xi}^{sign(\mathbf{z})}, \ mu = m_1(\boldsymbol{\xi} - 1/\boldsymbol{\xi}),$$

where $\boldsymbol{\xi}$ is skew parameter of DCC likelihood.

(3) Student's t likelihood

$$\hat{L}_{Vt,i}(\Theta) = \prod_{t=1}^{T} \left[\frac{\Gamma\left(\frac{v_i + 1}{2}\right)}{\sqrt{(v_i - 2)\pi}\Gamma\left(\frac{v_i}{2}\right)} \left(1 + \frac{\varepsilon_{it}^2}{(v_i - 2)(\sigma_{it}^2)} \right)^{\frac{-v_i + 1}{2}} \cdot \left(\frac{1}{\sigma_{it}^2}\right) \right], \tag{14}$$

where v_i is degree of freedom of return i and Γ is gamma distribution. Then, the correlation part is

$$\hat{L}_{Ct}(\theta|\hat{\Theta}) = \prod_{t=1}^{T} \left[\frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{(v-2)\pi\Gamma\left(\frac{v}{2}\right)}} (1 + \frac{\varepsilon_t^2}{(v-2)(H_t)})^{\frac{-v+1}{2}} \cdot \left(\frac{1}{H_t}\right) \right], \tag{15}$$

where **v** is degree of freedom of DCC likelihood.

(4) Skewed student's t likelihood

$$L_{Vsstd,i}(\Theta) = \prod_{t=1}^{T} \left[\frac{2}{(\xi_i + 1)/\xi_i} f_{std}(z_i/\xi_i^{sign(z)}, v_i)F_i \right], \tag{16}$$

$$z_i = \frac{\varepsilon_{it}}{\sigma_{it}^2}F_i + \left[\frac{2\sqrt{v_i-2}}{(v_i-1)}\left(beta\left(0.5, \frac{v_i}{2}\right)\right)^{-1} \right]\left(\frac{\xi_i - 1}{\xi_i}\right),$$

$$F_i = \sqrt{1 - \left[\frac{2\sqrt{v_i-2}}{(v_i-1)}\left(beta\left(0.5, \frac{v_i}{2}\right)\right)^{-1}\right]^2 \left(\frac{\xi_i^2 + 1}{\xi_i^2}\right) + 2\left[\frac{2\sqrt{v_i-2}}{(v_i-1)}\left(beta\left(0.5, \frac{v}{2}\right)\right)^{-1}\right]^2 - 1},$$

where $f_{std}(\cdot)$ is a density of student-t distribution. $beta(\cdot)$ is beta distribution. Then, the correlation part **is**

$$L_{Csstd}(\theta|\Theta) = \prod_{i=1}^{T} \left[\frac{2}{(\xi + 1)/\xi} f_{std}(z/\xi^{sign(z)}, v)F \right] \tag{17}$$

$$z = \frac{\varepsilon_t}{H}F + \left[\frac{2\sqrt{v-2}}{(v-1)}\left(beta\left(0.5, \frac{v}{2}\right)\right)^{-1} \right]\left(\frac{\xi - 1}{\xi}\right)$$

$$F = \sqrt{1 - \left[\frac{2\sqrt{v-2}}{(v-1)}\left(beta\left(0.5, \frac{v}{2}\right)\right)^{-1}\right]^2 \left(\frac{\xi^2 + 1}{\xi^2}\right) + 2\left[\frac{2\sqrt{v-2}}{(v-1)}\left(beta\left(0.5, \frac{v}{2}\right)\right)^{-1}\right]^2 - 1}$$

3 Empirical Study

3.1 Data

This paper focuses on seven implied volatility indexes: the volatility index for S&P 500 stock index in the United States (VIX), the volatility index for Dow Jones EURO STOXX 50 stock index in Europe (VSTOXX), the volatility index for DAX 30 stock index in Germany (VDAX), the volatility index for FTSE stock index in United Kingdom (VFTSE), India's volatility index (INVIXN), the volatility index for HSI stock index in Hong Kong (VHSI), and the volatility index for KOSPI stock index in South Korea (VKOSPI). All of the 7 volatility indexes are calculated as the expectations of the future stock volatility index in the options. This study collected the data from

Bloomberg. We obtained the daily time series closing price data for VIX, VSTOXX, VDAX, VFTSE, VNVIXN, VKOSPI, and VHSI. In addition, the daily gold price and oil price are also collected from Thomson Financial DataStream. These data sets are collected from 1 November 2007 to 11 August 2017 covering 22,149 daily observations. The detailed description of variables and descriptive statistics for daily time series data are presented in Table 1.

Table 1. Data description

	VIX	VSTOXX	VDAX	VFTSE	VNVIXN	VHSI	VKOSPI	Gold	Oil
Mean	−0.0001	−0.0001	0.0000	−0.0002	−0.0005	−0.0006	−0.0004	8.7000	−0.0001
Median	−0.0025	−0.0009	−0.0004	−0.0008	0.0000	−0.0015	0.0000	0.0002	4.1900
Max	0.1761	0.1423	0.1327	0.1868	0.2158	0.1997	0.2116	0.0445	0.0712
Min	−0.1523	−0.1888	−0.1607	−0.1957	−0.2042	−0.0911	−0.1315	−0.0413	−0.0567
Std. dev.	0.0320	0.0278	0.0256	0.0314	0.0250	0.0237	0.0232	0.0052	0.0109
Skew	0.6916	0.4628	0.4801	0.2023	0.4593	1.5349	1.4267	−0.2543	0.1670
Kurtosis	6.6421	6.3000	5.9407	5.9820	13.8863	11.7211	13.3697	9.2134	7.5162
JB-test	1556.42	1204.55	981.335	928.629	12238.9	8765.40	11861.1	3985.34	2102.90
Prob.	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ADF	−54.2***	−38.1***	−36.8***	−38.0***	−39.8***	−28.4***	−28.4***	−48.9***	−52.5***
Sum	−0.1751	−0.2358	−0.0919	−0.4653	−1.3130	−1.4756	−1.0116	0.2142	−0.2822

Note: Table reports the descriptive statistics of the volatility indexes, gold and oil prices. The Jarpue-Bera (JB) test and the Augmented Dickey Fuller (ADF) test values are reported. ***, **, * denote rejection of null hypothesis at 1%, 5% and 10% significance respectively.

In this study, all raw data are transformed into log-returns. The table shows that the skewness and kutosis of volatility index returns and oil returns are positively skewed while gold returns show a negative skew. In addition, normality of all returns are rejected by the Jarque–Bera test, prob. = 0.0000. We also investigate the stationarity of our data. We find that all returns data are rejected at 1% significance level hence all nine returns are stationary.

3.2   Model Selection

3.2.1   GARCH Types and Distribution Selection
In this section we investigate and compare the performance of each model specification in the real data analysis. Tables 2 and 3 present the values of AIC and BIC, respectively. We observe that CGARCH with the skewed normal distribution is the best fit model in this study because the lowest values of AIC and BIC are obtained.

Table 2. Akaike Information Criterion (AIC)

	SGARCH	GJR-GARCH	EGARCH	IGARCH	CGARCH
norm	−89798.02	−90174.11	−91010.33	−88457.73	−89548.78
STD	−103990.1	−104290.3	−104776.1	−101971.5	−103965.7
SSTD	−100992.7	−101244	−101672.4	−99037.72	−101006.9
snorm	−378529.6	−360434.5	−337667.4	−346934.6	**−380235.7**

Source: Calculation

Table 3. Bayesian Information Criterion (BIC)

	SGARCH	GJR-GARCH	EGARCH	IGARCH	CGARCH
norm	−89420.48	−89755.92	−90592.13	−88080.19	−89089.92
STD	−103571.9	−103831.4	−104317.2	−101553.3	−103466.2
SSTD	−100533.7	−100744.5	−101172.9	−98578.86	−100466.7
snorm	−378111.4	−359975.7	−337208.6	−346516.4	**−379736.2**

Source: Calculation

3.3   Estimation Results

The estimated optimal parameters from the best model, DCC-CGARCH model with skewed normal student-t distribution, are presented in Table 4. We investigate the relationships between volatility indices. The most important parameters are sum of α_1 and β_1 in the model. They are close to 1 indicating a high degree of volatility persistence of transitory and permanent components. All volatility indexes exhibit high persistency,

Table 4. Estimation DCC-CGARCH with skewed normal distribution results

	VIX	VSTOXX	VDAX	VFTSE	VNVIXN	VHSI	VKOSPI
C	0.00039	-0.00003	-0.00008	-0.00003	0.00009	0.00004	-0.00006
	(0.0005)	(0.0005)	(0.0005)	(0.0006)	(0.0004)	(0.0004)	(0.0004)
ar1	-0.1011[a]	-0.01413	0.01158	-0.0376[a]	-0.1028[a]	-0.0868[a]	-0.0798[a]
	(0.0219)	(0.0220)	(0.0214)	(0.0218)	(0.0330)	(0.0201)	(0.0227)
$\varnothing_1$	0.05857	0.02216	-0.15981[a]	0.22820[a]	-0.06203	0.06329	0.04794
	(0.1143)	(0.1080)	(0.0969)	(0.1163)	(0.0871)	(0.0829)	(0.0794)
$\varnothing_2$	-0.68356[a]	0.08256	0.12489[a]	-0.71572[a]	-0.29274[a]	-0.20490[a]	-0.20135[a]
	(0.0574)	(0.0527)	(0.0476)	(0.0587)	(0.0471)	(0.0386)	(0.0413)
ω	0.000001[a]	0.000001[a]	0.000002[a]	0.000004[a]	0.000006[a]	0.000002[a]	0.000003[a]
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
α_1	0.15330[a]	0.07384[a]	0.06335[a]	0.06252	0.10971	0.08262[a]	0.13461[a]
	(0.0185)	(0.0147)	(0.0118)	(0.0162)	(0.0096)	(0.0162)	(0.0055)
β_1	0.70185[a]	0.79304[a]	0.85382[a]	0.62168[a]	0.59677	0.87675[a]	0.76803[a]
	(0.0408)	(0.0521)	(0.0222)	(0.1234)	(0.0738)	(0.0238)	(0.0259)
ρ	0.99914[a]	0.99928[a]	0.99748[a]	0.99523[a]	0.98899[a]	0.99647[a]	0.99464[a]
	(0.0002)	(0.0060)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
$\varnothing$	0.00201[a]	0.00017	0.00037[a]	0.01138[a]	0.02070[a]	0.00025	0.00755[a]
	(0.0006)	(0.0001)	(0.00)	(0.0030)	(0.0026)	(0.0006)	(0.0034)
φ_1	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	(0.0010)	(0.0002)	(0.0003)	(0.0012)	(0.0008)	(0.0006)	(0.0007)
φ_2	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	(0.0003)	(0.0002)	(0.0002)	(0.0005)	(0.00)	(0.0003)	(0.0003)
v	1.27469[a]	1.15575[a]	1.15627[a]	1.05636[a]	1.17309[a]	1.42484[a]	1.27925[a]
	(0.0304)	(0.0260)	(0.0262)	(0.0235)	(0.0233)	(0.0343)	(0.0291)
DCC	$\theta_0 = 0.0035^a, \theta_1 = 0.9943^a, \xi = 9.9986^a$						

Source: Calculation.

[a]denotes rejection of null hypothesis at 1%, 5% and 10% significance.

indicating a strong volatility of VHSI, VDAX, VKOSPI, VSTOXX, VIX and VNVIXN. In addition, the degree of asymmetry in volatility is highest in VSTOXX ($\rho = 0.99928$) and lowest in VKOSPI ($\rho = 0.99464$). Moreover, we also found that oil and gold have effects on return of volatility indexes. For example, the value of coefficient of gold price on VIX is 0.05857, meaning that the change in oil return by 1% will change the return of VIX about 5.857% in the same direction. To estimate DCC, the value of θ_0, θ_1 and ξ are parameters used to find integrated and mean reverting models. It presents dynamic correlation for volatility indexes. In this case it is fitted as the skewed normal distribution.

3.4 Dynamic Correlation from the Best GARCH Fit

3.4.1 Correlation Analysis

Due to the space limit, we only present the dynamic correlation results of some interesting pairs. We compute time series daily correlation between the changes in volatility indexes for instances, VIX, VSTOXX, VDAX, VFTSE, VNVIXN, VHSI and VKOSPI
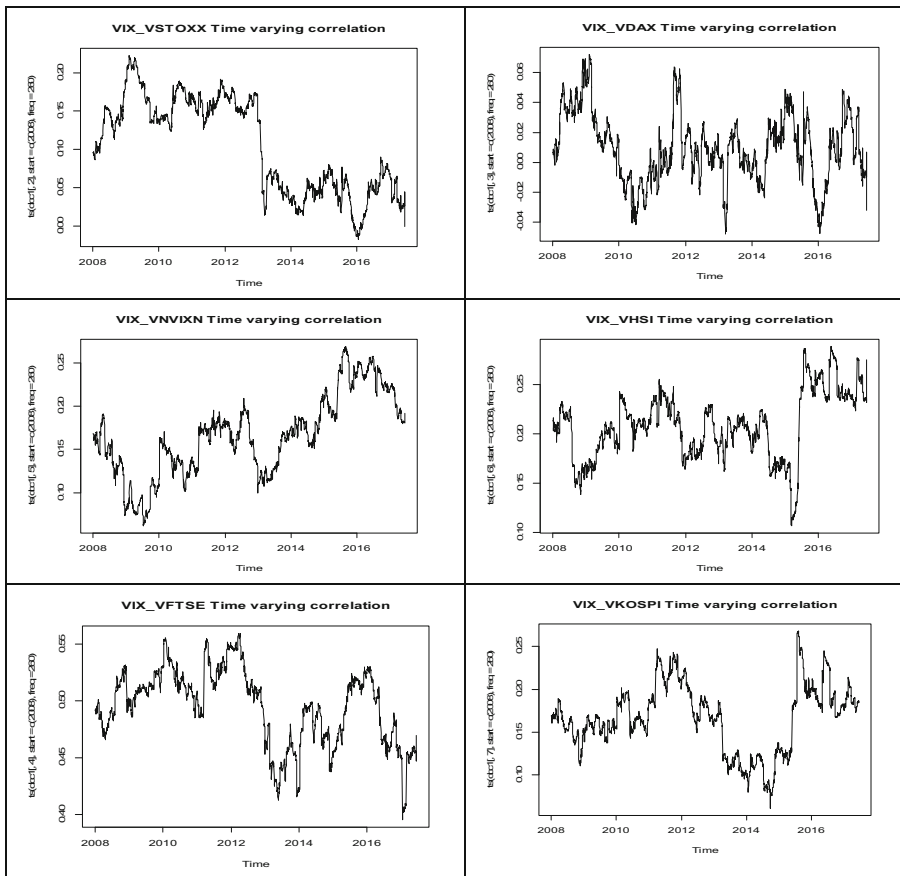

Fig. 1. Daily correlation between volatility indices from 1 November 2007 to 11 August 2017.

from 1 November 2007 to 11 August 2017 and only the pairs between VIX and other indexes are presented in Fig. 1.

There are a number of significant results and find that most volatility indexes have positive correlation. The correlation for VIX and VSTOXX is strongest in 2009 around 0.20, and then the value becomes close to zero after 2012, indicating a weak positive correlation. Similarly, the VIX and VDAX correlation reached the highest level in 2009 at 0.06 and it turned to be negative correlation around -0.04 in 2010, 2013, and 2016, and this indicates that there are both positive and negative correlations between them over time. Additionally, the correlation between VIX and VFTSE reached the highest level in 2012 around 0.55 indicating a high positive correlation. Meanwhile, the correlation for VIX and VNVIXN, VIX and VHSI, VIX and VKOSPI reached high level after 2014 around 0.25.

4   Conclusion

Based on the results, we suggest the best fit model for estimating the optimal parameters to be DCC-CGARCH based on the skewed student-t distribution. We observe that, U.S. volatility index (VIX) has the positive dynamic correlation with other volatility indexes (VSTOXX, VFTSE, VHSI, VKOSPI, VDAX, VNVIXN) meaning that if VIX increases, it will lead VSTOXX to increase. Additionally, this paper includes exogenous effects on volatility indexes consisting of oil and gold. We found that there is a significant effect of oil and gold on volatility indexes.

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