Optimal Academic Ranking of Students in a Fuzzy Environment: A Case Study

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Abstract Traditionally, academic ranking of students' performance is based on test score which can be interpreted in linguistic terms such as 'very good', 'good', 'poor', 'very poor' with varying degree of certainty attached to each description. There could be several students in a school having 'very poor' performance with varying degree of certainty. The authorities would certainly like to improve students' academic performance based on their ranking. The case study relates to the combination of Zadeh-Deshpande formalism with Bellman-Zadeh method to arrive at an optimal ranking of especially 'very poor' students based on well-defined performance shaping factors.

Keywords Students ranking • Academic performance • Goal Constraints • Decision • Zadeh-Deshpande formalism • Bellman-Zadeh fuzzy decision-making model

1 Introduction

Examination process tries to ensure students' abilities in any area of the academic program. Test scores, Grade point average (GPA) are widely used indicators of academic performance in educational systems to rank students [1]. Test Scores often reflect limited measures of some aspect of student proficiency. Many factors could act as barriers to students attaining and maintaining a high GPA that reflects their overall

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academic performance. Constraints (or factors) which can affect students' academic performance have been investigated in many studies in recent years [2–13].

The constraints analysed in academic performance of students relates to, motivation, personality, and psychosocial factors [14]. Students ranking using only achievement tests is, in our view is inadequate to assess the real performance as some other constraints as additional information might affect students' performance. These are *problem-solving skill*, *retention rate*, *the level of motivation*, *intellectual curiosity*, *test anxiety*, etc. Ranking of students' academic achievement with this additional information will be more meaningful for policy makers and educators to measure the real performance to distinguish differences among students [15]. Therefore, ranking students using cognitive as well as affective factors to define performance measure may be a realistic approach. Decision-making in a fuzzy environment [16] is one of the simplest kinds of an algorithm for optimal ranking strategy.

The objective of the study is to develop formalism for optimal ranking of students based on factors affecting academic performance in a Fuzzy Environment. Traditionally, only test marks are considered to rank the performance of a student. Therefore, in *Zadeh-Deshpande* (ZD) formalism only test-marks considered [17]. The academic performance of students as linguistic descriptions with varying *degree of certainty* (DC) attached to the report obtained from ZD formalism is the input to *Bellman-Zadeh* (BZ) formalism as a *goal* (G).

The paper is organized as follows: Sect. 2 relates to mathematical preliminaries of Bellman-Zadeh method while Sect. 3 presents the proposed methodology. The case study for ranking students considering the factors affecting students' academic performance using Bellman-Zadeh formalism is covered in Sect. 4. Results and Discussion of the case study are discussed in Sect. 5. A concluding remark is an integral part of Sect. 6.

2 Mathematical Preliminaries

Zadeh-Deshpande (ZD) formalism is shown in Fig. 1.

Brief Commentary on Bellman-Zadeh Approach

Bellman and Zadeh [16] proposed a fuzzy decision-making model in which decision are obtained by aggregate operations on goals and constraints which are expressed as fuzzy sets. The principal components of a decision process in this model are

(a) a set A of possible actions;

(b) a set of goals $G_i\ (i \in \mathrm{N}_n)$, each of which is expressed concerning a fuzzy set defined on A;

(c) a set of constraints $C_j\ (j \in \mathrm{N}_m)$, each of which is also represented by a fuzzy set defined on A.

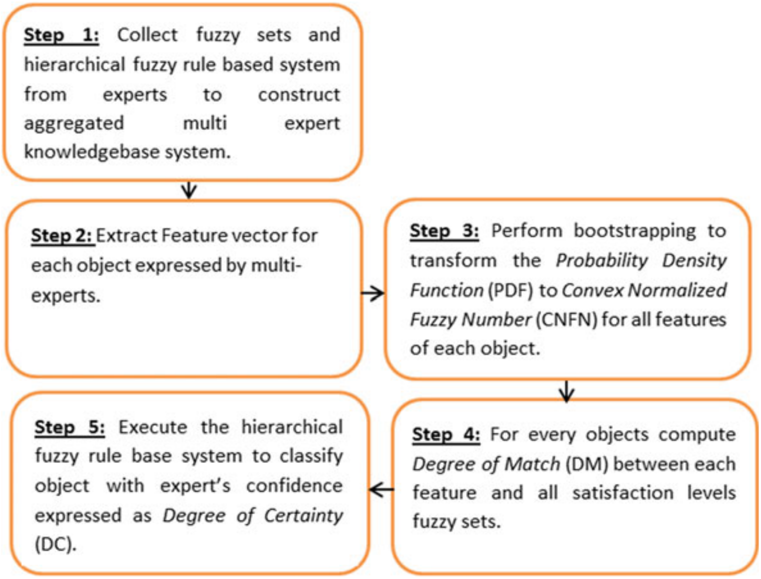

Fig. 1 Zadeh-Deshpande (ZD) formalism to classify objects using experts knowledgebase

Definition Assume that we are given a fuzzy goal G and fuzzy constraints $C_1, \ldots, C_n$ in a space of alternatives X. Then aggregation on G and C forms a decision D which is a fuzzy set resulting from the intersection of G and C. In symbols,

$$D = G \cap C \tag{1}$$

and correspondingly $\mu_D = \mu_G \wedge \mu_C$.

The relation between G, C and D is depicted in Fig. 2. Normalization is a major step which provides a common denominator for fuzzy goals and fuzzy constraints thereby making it possible to treat them alike. It is done as membership grade of individual constraint divided by sum of the same constraint across all alternatives or sites. Same is done for the fuzzy goals.

This concept explains why it is perfectly justified to regard fuzzy concepts of goals and constraints rather than performance function as one of the major component for decision making in a fuzzy environment. Normalization makes it possible to treat the fuzzy 'goals' and 'constraints' identically in the formulation of a decision.

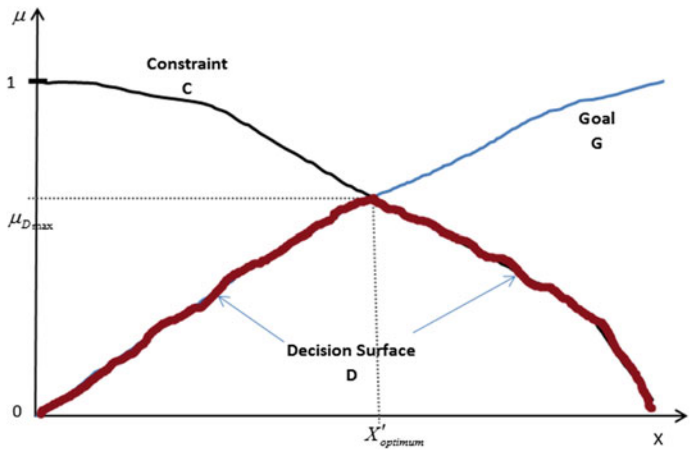

Fig. 2 Fuzzy decision model of Bellman-Zadeh formalism

3 The Proposed Study Methodology

The stepwise brief description of the activities is described below:

Step 1 Information of all constraints C_i are collected from academic records. Output of students test performance (Goal G) with expert's degree of certainty (DC) are obtained from ZD formalism for each student S_k, where $1 \leq i \leq m$, and $1 \leq k \leq n$.

Step 2 For each cluster of students P_j, normalize and transform goal G and all constraints C_i to either monotonic increasing or decreasing fuzzy sets depending on maximization or minimization objective function to obtain membership values of G and C_i for every student S_k.

Step 3 For each student S_{kj} of the cluster P_j, get the resultant decision as the intersection of the given goals $< G_{1j}, G_{2j}, \ldots, G_{tj} >$ and the given constraints $< C_{1j}, C_{2j}, \ldots, C_{mj} >$.

$$\mu_{D_{kj}} = \mu_{G_{1j}} \wedge \cdots \wedge \mu_{G_{tj}} \wedge \mu_{C_{1j}} \wedge \cdots \wedge \mu_{C_{mj}} \tag{2}$$

$$\mu_{D_{kj}} = \min(\mu_{G_{1j}}, \ldots, \mu_{G_{tj}}, \mu_{C_{1j}}, \ldots \wedge \mu_{C_{mj}}) \tag{3}$$

Step 4 Compute the optimal ranking of students (highest to lowest) in each cluster P_j for decision $\mu_{D_{kj}}$ obtained in step 3 in descending order. Select the student with maximum decision value as highest rank.

$$Highest\_Rank = \max(\mu_{D_{1j}}, \mu_{D_{2j}}, \ldots, \mu_{D_{kj}}) \tag{4}$$

Repeat this process to obtain next highest rank student by eliminating ranked students from the cluster. Repeat step 1 to 4 for all groups of students.

Step 5 Results obtained from Step 4 using optimal ranking strategy can be utilized by policy makers to take necessary corrective actions and decisions.

4 Case Study

The case study relates to optimal ranking of students' using test ranking results obtained from Zadeh-Deshpande (ZD) formalism [17] by considering all factors influencing academic performance. The examination answer script samples in subject 'Marathi' language were obtained from 237 secondary school students from three distinct institutions in Mumbai, India during the academic year 2013–2014. Each student wrote a total of 10–12 page solution for 12 subjective questions. Answers were evaluated using ten point rubric from the Secondary School (SSC) Board, Maharashtra. Twenty subject matter experts (teachers) from different schools were identified for the answer scripts evaluation. Total ten factors (i.e. constraints) influencing students' academic performance are measured for every student with evidence and survey carried out periodically throughout the school course by experts, while data collection for the test performance was conducted only on a sole day. The survey instrument consisted of a single page, back-to-back, with 30 items and questions to obtain descriptive data. Students test performance was expressed in a linguistic term like '*Very Poor*', '*Poor*', '*Average*', '*Good*', '*Very Good*' associated with evaluators *degree of certainty* (DC) using ZD formalism.

The factors (i.e. constraints) employed in Bellman-Zadeh method are *Attendance (C1)*, *Previous Test marks (C2)*, *Discipline (C3)*, *Problem Solving Skills (C4)*, *Motivation (C5)*, *Retention (C6)*, *Anxiety (C7)*, *Number of punishments and scolding received (C8)*, *Participation in extracurricular activities (C9)*, and *Accuracy (C10)*. These factors are normalized and transformed to either monotonic increasing or decreasing fuzzy sets. *Anxiety (C7)* and *Number of punishments and scolding received (C8)* have negative influence while rest of the factors have the positive impact on students' academic achievement. *Attendance*, *Previous Test marks*, *Number of punishments and scolding received*, *Participation in extracurricular activities*, *Accuracy* were recorded in quantitative form periodically in an academic term. *Discipline*, *Problem Solving Skills*, *Motivation*, *Retention*, *Anxiety* were measured using a 5-point Likert scale where 1—strongly agree, 2—agree, 3—neutral, 4—disagree, and 5—strongly disagree. The *degree of certainty* (DC) obtained from ZD formalism is input for BZ formalism as a goal (G).

5 Results and Discussion

Table 1 shows the students having '*very poor*' grades classified using ZD formalism and the factor affecting their scholarly performance. In BZ method, the Goal is *the* '*degree of certainty*' (DC) while constraints are '*C1*' to '*C10*'. Furthermore, Goal and Constraints are normalized into fuzzy sets using the standard procedure of normalization. The proposed methodology is applied, and student's optimal ranking is shown in Table 2.

For example student id-129 have an aggregation of goal and constraints to form fuzzy decision value 0.33. $\mu_{D_{129}} = \min(0.85, 0.74, 0.91, 0.62, 0.33, 0.8, 0.33, 0.5, 1, 0.78, 0.6, 1)$ $\mu_{D_{129}} = 0.33$ Student id-129 with maximum decision value is ranked top among the '*very poor*' category students.

Ranking of student id-87 with degree of certainty (DC) 0.565 with Membership Function (Goal) = 0, remains at the last position even with BZ method. The students with boundary marks have '*very poor*' academic performance as the DC is 0.565. In fact, in partial belief, the student is more likely in '*poor*' category than '*very poor*'. It is the reason why all other constraints, though with high membership value, does not alter his ranking and student id-87 remain last in '*very poor*' cluster. We can say that 0.565 is the cut-off value of membership function (MF) to decide the category of a student. He is belonging to '*very poor*' category, marginally and therefore ranks last. In summary, in such a case, it is not always the other constraints are significant—even if their membership value is high, but the test marks governs the category. If you have all other constraints having high membership value and do not score marks required to be in that category, then your large membership function-valued constraints will not take you to the higher rank. The purpose of the examination is to score high marks and not always score high MF in constraints. Table 3 is the outcome of the study. Ranking of '*very poor*' category students based on a single criterion of '*test score*' is computed using Zadeh-Deshpande formalism. Considering relevant *constraints* and using Bellman-Zadeh method, an optimal ranking of students has been worked out which might serve as a guideline to initiate suitable corrective actions to improve the performance of '*very poor*' category students. For example student id-93 with '*very poor*' category ranked 7th using ZD formalism while is rank 1st in BZ method and is considered as Optimal ranked. The management is expected to initiate corrective action to improve the performance of student id-87 more rigorously. The software implementation of this approach has been done in MATLAB R2008a and Microsoft Access 2010.

Table 1 Degree of certainty from ZD and parameters affecting students academic performance in '*Very Poor*' category

Stud_ID	Category	Confidence	DC	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
93	Very poor	Very high	0.9539	98	9.536	8	0	3	2	6	9	10	6
203	Very poor	Very high	0.9528	100	23.38	9	0	5	1	7	5	50	7
130	Very poor	Very high	0.9122	83	23.5	7	5	3	0	9	6	40	8
23	Very poor	High	0.8872	80	27.07	6	8	2	2	10	4	20	3
137	Very poor	High	0.8676	93	11.22	5	10	4	1	9	3	15	5
91	Very poor	High	0.8594	78	27.19	4	20	3	3	8	0	25	4
129	Very poor	High	0.853	98	25.37	3	40	3	4	5	2	30	8
133	Very poor	High	0.8487	100	20.91	1	50	4	1	3	3	10	5
85	Very poor	High	0.7994	100	37.08	5	34	2	3	5	1	35	4
237	Very poor	High	0.7895	93	36.04	6	25	1	6	7	5	40	3
84	Very poor	High	0.7583	85	16.08	7	10	2	8	6	2	45	6
134	Very poor	High	0.745	80	18.77	5	5	3	3	0	8	10	4
31	Very poor	High	0.7415	94	28.79	7	23	5	2	6	3	15	5
4	Very poor	High	0.7382	92	35.51	8	0	3	2	4	4	20	6
83	Very poor	High	0.7028	99	17.2	0	10	2	6	3	2	10	5
5	Very poor	High	0.6606	100	14.45	2	25	4	2	6	0	5	3
98	Very poor	High	0.6438	98	5.985	7	5	7	3	9	1	0	7
20	Very poor	Fair	0.5924	89	34.7	6	10	3	1	8	3	10	8
87	Very poor	Fair	0.565	88	23.59	4	25	2	4	6	4	15	5

Table 2 Optimal ranking of students using BZ technique

Stud_ID	Category	Confidance	DC	Goal	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	Decision
129	Very poor	High	0.853	0.74	0.91	0.62	0.33	0.8	0.33	0.5	1	0.78	0.6	1	0.33333
31	Very poor	High	0.7415	0.45	0.73	0.73	0.78	0.46	0.67	0.25	0	0.67	0.3	0.4	0.25
85	Very poor	High	0.7994	0.6	1	1	0.56	0.68	0.17	0.38	1	0.89	0.7	0.2	0.16667
84	Very poor	High	0.7583	0.5	0.32	0.32	0.78	0.2	0.17	1	0	0.78	0.9	0.6	0.16667
133	Very poor	High	0.8487	0.73	1	0.48	0.11	1	0.5	0.13	1	0.67	0.2	0.4	0.11111
137	Very poor	High	0.8676	0.78	0.68	0.17	0.56	0.2	0.5	0.13	0	0.67	0.3	0.4	0.1
134	Very poor	High	0.745	0.46	0.09	0.41	0.56	0.1	0.33	0.38	1	0.11	0.2	0.2	0.09091
20	Very poor	Fair	0.5924	0.07	0.5	0.92	0.67	0.2	0.33	0.13	0	0.67	0.2	1	0.0704
93	Very poor	Very high	0.9539	1	0.91	0.11	0.89	0	0.33	0.25	0	0	0.2	0.6	0
203	Very poor	Very high	0.9528	1	1	0.56	1	0	0.67	0.13	0	0.44	1	0.8	0
130	Very poor	Very high	0.9122	0.89	0.23	0.56	0.78	0.1	0.33	0	0	0.33	0.8	1	0
23	Very poor	High	0.8872	0.83	0.09	0.68	0.67	0.16	0.17	0.25	0	0.56	0.4	0	0
91	Very poor	High	0.8594	0.76	0	0.68	0.44	0.4	0.33	0.38	0	1	0.5	0.2	0
237	Very poor	High	0.7895	0.58	0.68	0.97	0.67	0.5	0	0.75	0	0.44	0.8	0	0
4	Very Poor	High	0.7382	0.45	0.64	0.95	0.89	0	0.33	0.25	1	0.56	0.4	0.6	0
83	Very Poor	High	0.7028	0.35	0.95	0.36	0	0.2	0.17	0.75	1	0.78	0.2	0.4	0
5	Very Poor	High	0.6606	0.25	1	0.27	0.22	0.5	0.5	0.25	0	1	0.1	0	0
98	Very Poor	High	0.6438	0.2	0.91	0	0.78	0.1	1	0.38	0	0.89	0	0.8	0
87	Very Poor	Fair	0.565	0	0.45	0.57	0.44	0.5	0.17	0.5	0	0.56	0.3	0.4	0

Table 3 Optimal ranking of students using BZ technique

Stud_ID	Ranking of students using ZD technique	Optimal ranking of students using BZ technique
93	7	1
203	13	2
130	9	3
23	11	4
137	8	5
91	5	6
129	12	7
133	18	8
85	1	9
237	2	10
84	3	11
134	4	12
31	6	13
4	10	14
83	14	15
5	15	16
98	16	17
20	17	18
87	19	19

6 Concluding Remarks

Multi-constraint optimization based on fuzzy membership is different than traditional optimization techniques. The authors have successfully demonstrated in deciding the optimal ranking of students' performance based on relevant performance shaping factors. Much more needs to be done.

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