An Extended Integrated Assessment Model for Mitigation and Adaptation Policies on Climate Change

Willi Semmler, Helmut Maurer, and Anthony Bonen

Abstract We present an extended integrated assessment model (IAM) that explicitly solves for optimal climate financing policies. As with other IAMs, our approach ties economic activity with their externalities and feedback effects. We extend standard IAM methodologies to find the optimal allocation of infrastructure expenditure to carbon-neutral physical capital, climate change adaptation, and emissions mitigation. Optimal control solutions are obtained by discretizing the control problem and applying nonlinear programming methods. We demonstrate that the endogenously selected infrastructure shares out-perform fixed allocations by increasing consumption, private capital and tax revenue, while reducing public debt and CO_2 emissions. We find 92–95% of spending should be allocated to physical infrastructure with the remainder going to mitigation and adaptation, for which the major part is used for adaptation. Further, homotopic analysis is conducted on unobservable parameters. We show that adaptation expenditure increases with the

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productive efficiency of non-renewables and emissions mitigation rises as its effect becomes nonlinear. The homotopic results demonstrate that our main findings are stable.

1 Introduction

Balancing the competing yet often complementary needs of climate change mitigation, adaptation and development is a complex policy goal (Bernard and Semmler 2015; IMF 2014, 2016). This paper presents a modelling framework for prioritizing funding to these three policy areas. Building on Bonen et al. (2016), we develop an extended integrated assessment model (IAM) that explicitly solves for the public funding allocation problem for climate change policy in the decision framework of a developing economy. Climate change policy is operationalized as the share of government expenditures made in support of carbon-neutral productivity-enhancing infrastructure, infrastructure that helps people adapt to the negative effects of a changing climate, and infrastructure used to mitigate carbon emissions. Depending on the parameterization, we find that between 92 and 95% of infrastructure expenditure should be allocated to productivity-enhancing infrastructure, 5–8% should be spent on adaptation, and the remainder on emissions mitigation. Productivity-enhancing infrastructure is prioritized as it increases the overall wealth in the country, thereby increasing the total capital available for the climate change adaptation and mitigation while increasing consumption and reducing government indebtedness.

Leading IAMs typically assume the economy's carbon intensity falls over time because of an exogenous 'back stop' of green technology. Our approach endogenizes carbon intensity by linking emissions to the extraction of a non-renewable resource (e.g., fossils fuels), and shows how renewable energy can be phased in through public-sector investment. This allows us to combine contemporary 'social cost of carbon' IAM approaches with the resource extraction models due to Hotelling (1931) and Pindyck (1978) as extended by Maurer and Semmler (2011). Thus, the IAM presented here extends the recent modelling advances that allow agents to respond to climate change by combining mitigation and adaptation actions (e.g., Ingham et al. 2005; Tol 2007; Lecoq and Zmarak 2007; Bosello 2008; de Bruin et al. 2009; Bréchet et al. 2013; Zemel 2015).

Computationally, IAMs represent complex dynamic systems that do not lend themselves to standard, closed-form solutions. Early iterations developed workarounds such as forecasting economic growth trajectories in isolation and then using those output scenarios to generate emissions and/or temperature responses (Bonen et al. 2014). We avoid such simplifications by determining optimal control solutions for the full IAM system—a facet we believe to be crucial in accurately modelling economic-environmental interrelations. To this end, the optimal control problem is discretized on a fine grid which leads to a large-scale nonlinear programming problem (NLP) that can be conveniently formulated via the Mathematical Programming

Language (AMPL), *cf*. Fourer et al. (1993). AMPL can be linked to several efficient optimization solvers. In our computations, we use interior point optimization solver IPOPT (Wächter and Biegler 2006) that furnishes the control and state variables as well as the adjoint (co-state) variables. In this way, we are able to check whether we have found an *extremal solution* satisfying the necessary optimality conditions.

Employing AMPL enables us to advance the complexity—and thus realism—of the policymaker's action set. Under the initial parameterization, which is designed to match the stylized facts of a typical developing country, we find that 95% of funding should go toward productivity-enhancing investments, 5% to adaptation infrastructure, and none to emissions mitigation.[1] As expected, we show that allowing the optimizing policymaker to control the infrastructure expenditure allocations significantly improves social welfare relative to the case of fixed spending shares (a limitation other solution techniques would have to accept). Furthermore, we show that each constitutive element of social welfare is improved by the advancement: per capita consumption and private capital increase while public debt and CO_2 emissions fall relative to the fixed allocation scenario.

After demonstrating the superiority of expanding the policymaker's action set, we conduct a series of homotopic analyses to test both the model's stability and sensitivity of the main allocation results. First, the efficiency (viz. inverse of marginal cost) of fossil fuel energy is explored. We find that as fossil fuels become more efficient (cheaper for producers), the relative funding of productivity-enhancing infrastructure falls to 92% and the allocation to adaptation-focused infrastructure increases. Optimal mitigation, for the developing country, remains nil. Secondly, the concavity of the emissions effect of mitigation efforts is allowed to vary. Here we find that as mitigation's concavity increases, the impetus to reduce CO_2 emissions rises as the marginal return (at low mitigation levels) has a greater-than-proportional effect. Although allocations to emissions mitigation do not surpass 1.2%, social welfare monotonically increases with increased mitigation efforts. We also test welfare's sensitivity to intertemporal discounting. Our results here demonstrate the model conforms to the important theoretical insight that outcomes improve when policymakers reduce their discounting of the future. Crucially, improvements in terminal welfare are shown to flow from increased expenditure of emissions mitigating infrastructure.

The remainder of the paper is organized as follows. Section 2 presents the model and optimal control solution technique. Results are reported and discussed in Sect. 3. Section 4 concludes.

[1]We have also tested a specification in which these allocations are continuously updated in each time period, instead of being selected based on the initial expected social utility. There is little improvement in moving to this approach. In addition to reducing computational costs, the slight reduction in utility from optimally selecting a single set of allocations suggests that any loss of flexibility in guaranteeing long-term mitigation and adaptation funding is likely outweighed by the benefits of policy stability. Due to space constraints we do not present these results here.

2 Integrated Assessment Model as Optimal Control Problem

The integrated assessment model (IAM) has 5 state variables

$$X = (K, R, M, b, g) \in \mathbf{R}^5, \tag{1}$$

where K is private capital, R is the stock of the non-renewable resource, M is the atmospheric concentration of CO_2, b is the government's debt, and g is public capital. The dynamic system of the IAM is defined according to

$$\dot{K} = Y \cdot (\nu_1 g)^{\beta} - C - e_P - (\delta_K + n)K - u\,\psi R^{-\tau}, \tag{2}$$

$$\dot{R} = -u, \tag{3}$$

$$\dot{M} = \gamma\, u - \mu(M - \kappa \widetilde{M}) - \theta(\nu_3 \cdot g)^{\phi}, \tag{4}$$

$$\dot{b} = (\bar{r} - n)b - (1 - \alpha_1 - \alpha_2 - \alpha_3) \cdot e_P. \tag{5}$$

$$\dot{g} = \alpha_1 e_P + i_F - (\delta_g + n)g, \tag{6}$$

The control vector is given by

$$U = (C, e_P, u) \in \mathbf{R}^3, \tag{7}$$

where C denotes consumption, e_P is tax revenue, and u is the quantity of the resource R extracted each period.

The first dynamic $\dot{K}$ is the accumulation rate of private capital K that produces renewable energy and which drives output by the CES production function,[2]

$$Y(K, u) := A(A_K K + A_u u)^{\alpha} \tag{8}$$

where A is multifactor productivity, A_K and A_u are efficiency indices of private capital inputs K and (non-renewable) fossil fuel energy u, respectively. In (2), private-sector output Y is modified by the infrastructure share allocated to productivity enhancement $\nu_1 g$, for $\nu_1 \in [0, 1]$. This public-private interaction generates total output as $Y(\nu_1 g)^{\beta}$ from which the economy consumes C, pays taxes e_P, and is subject to physical δ_K and demographic n depreciation. The exponent β is the output elasticity of public infrastructure, $\nu_1 g$. The last term in (2) is the opportunity cost of extracting the non-renewable resource u, where ψ and τ are the scale and shape parameters that tie the marginal cost of u to the remaining stock of the resource à la Hotelling.

[2]For such a simplification of a production function see Acemoglu et al. (2012) and Greiner et al. (2014).

Equation (3) indicates the stock of the non-renewable resource R depletes by u units in each period.

The non-renewable resource emits carbon dioxide and thus increases the atmospheric concentration of CO_2 at the rate γ in Eq. (4). The stable level of CO_2 emissions is $\kappa > 1$ of the pre-industrial level $\widetilde{M}$, which is naturally re-absorbed into the ecosystem (e.g., oceanic reservoirs) at the rate μ. The last term in (4) is the reduction of per-period emissions $\dot{M}$ due to the allocation of $0 \leq \nu_3 \leq 1$ of infrastructure g to mitigation projects.

The last two dynamics are the accumulation of debt b and public capital g. In (5) public debt grows at the fixed interest rate $\bar{r}$, and is serviced with the share of tax revenue e_P not allocated respectively to capital accumulation α_1, social transfers α_2 or administrative overhead $\alpha_3 > 0$. Thus, $\alpha_4 \equiv 1 - \alpha_1 - \alpha_2 - \alpha_3$. Equation (6) says the stock of public capital, or total infrastructure, evolves according to the allocated tax revenue stream $\alpha_1 e_P$ and funds paid in from abroad, i_F. For developed countries $i_F = 0$, but may be positive for many developing countries. As with private capital, g depreciates by δ_g, and is adjusted for population growth n.

We assume throughout that the infrastructural allocations satisfy

$$\nu_k \geq 0 \quad (k = 1, 2, 3), \qquad \nu_1 + \nu_2 + \nu_3 = 1. \tag{9}$$

In later analyses, we either choose fixed values of ν_1, ν_2, ν_3 or we consider the allocations as additional optimization variables. All parameters in the dynamics (2)–(6) may be found in Table 1.

Using the state variable $X \in \mathbf{R^5}$ and control variable $U \in \mathbf{R^3}$, we write the dynamics (2)–(6) in compact form as

$$\dot{X}(t) = f(X(t), U(t)), \qquad X(0) = X_0. \tag{10}$$

The initial state vector X_0 will be specified later. To this system we add the terminal constraint

$$K(T) = K_T \geq 0, \tag{11}$$

the control constraint

$$0 \leq u(t) \leq u_{max}, \tag{12}$$

and the pure state constraint

$$M(t) \leq M_{max} \quad \forall\, t \in [0, T]. \tag{13}$$

The terminal constraint restricts the final level of the capital stock to a predetermined non-negative value, the control constraint prescribes an upper bound for the extraction rate, and finally the state constraint places a cap on the total level of CO_2 in the atmosphere in each period.

Table 1 Parameter values

Variable	Value	Definition
ρ	0.03	Pure discount rate
n	0.015	Population Growth Rate
η	0.1	Elasticity of transfers and public spending in utility
ϵ	1.1	Elasticity of CO_2-eq concentration in (dis)utility
ω	0.05	Elasticity of public capital used for adaptation in utility
σ	1.1	Intertemporal elasticity of instantaneous utility
A	$\in [1, 10]$	Total factor productivity
A_K	1	Efficiency index of private capital
A_u	$\in [50, 500]$	Efficiency index of the non-renewable resource
α	0.5	Output elasticity of privately-owned inputs, $A_k k + A_u u$
β	0.5	Output elasticity of public infrastructure, $\nu_1 g$
ψ	1	Scaling factor in marginal cost of resource extraction
τ	2	Exponential factor in marginal cost of resource extraction
δ_K	0.075	Depreciation rate of private capital
δ_g	0.05	Depreciation rate of public capital
i_F	0.05	Official development assistance earmarked for public infrastructure
α_1	0.1	Proportion of tax revenue allocated to new public capital
α_2	0.7	Proportion of tax revenue allocated to transfers and public consumption
α_3	0.1	Proportion of tax revenue allocated to administrative costs
$\bar{r}$	0.07	World interest rate (paid on public debt)
$\widetilde{M}$	1	Pre-industrial atmospheric concentration of greenhouse gases
γ	0.9	Fraction of greenhouse gas emissions not absorbed by the ocean
μ	0.01	Decay rate of greenhouse gases in atmosphere
κ	2	Atmospheric concentration stabilization ratio (relative to $\widetilde{M}$)
θ	0.01	Effectiveness of mitigation measures
ϕ	$\in [0.2,\ 1]$	Exponent in mitigation term $(\nu_3\, g)^{\phi}$

Let us now define the objective functional, the social welfare functional. We maximize (viz. minimize the negative) the following functional over a given planning horizon $[0, T]$, where $T > 0$ denotes the terminal time:

$$W(T, X, U) = \int_0^T e^{-(\rho-n)t} \frac{\left(C\,(\alpha_2 e_P)^{\eta}\left(M - \widetilde{M}\right)^{-\epsilon}(\nu_2 g)^{\omega}\right)^{1-\sigma} - 1}{1-\sigma}\, dt\,. \tag{14}$$

The felicity (utility) function in (14) is isoelastic with four input components all in per capita terms: (1) consumption C; (2) the share $0 \leq \alpha_2 \leq 1$ of tax revenue e_P used for direct welfare enhancement (e.g., healthcare); (3) atmospheric concentration of CO_2 M above the pre-industrial level $\widetilde{M}$; and (4) the share

$0 \leq \nu_2 \leq 1$ of infrastructure g allocated to climate change adaptation. Restricting the exponents $\eta, \epsilon, \omega > 0$ ensures social expenditures and adaptation are utility enhancing, and that carbon emissions directly reduce utility. This approach differs from other models that map emissions to temperature changes and then to reduced productivity-*cum*-output. We believe the direct disutility approach better captures the wide ranging impacts of climate change that may include health impacts, ecological loss and heightened uncertainty, in addition to reduced productivity. Finally, note that the discount factor adjusts for the population growth rate n from the pure discount rate ρ as all values are normalized by the population.

To summarize, the IAM gives rise to an optimal control problem $OC(p)$, where the social welfare (14) is maximized subject to the dynamic constraints (10) and the terminal, control and state constraints (11)–(13). In this problem $OC(p)$, the notation p denotes a suitable parameter in Table 1 for which we shall conduct a sensitivity analysis in the next section.

A detailed discussion of the necessary optimality conditions of the Maximum Principle for optimal control problems with state constraints (*cf.* Hartl et al. 1995) is beyond the scope of this paper and will be given elsewhere.

3 Results

3.1 Discretization and Nonlinear Programming Methods

We choose the numerical approach "First Discretize then Optimize" to solve the optimal control problem $OC(p)$ defined in (10)–(14). The discretization of the control problem on a fine grid leads to a large-scale nonlinear programming problem (NLP) that can be conveniently formulated with the help of the Mathematical Programming Language AMPL (Fourer et al. 1993). AMPL can be linked to several powerful optimization solvers. We use the Interior-Point optimization solver IPOPT developed by Wächter and Biegler (2006). Details of discretization methods may be found in Betts (2010), Büskens and Maurer (2000), and Göllman and Maurer (2014). The subsequent computations for the terminal time $T = 25$ are performed with $N = 1000$ to $N = 5000$ grid points using the trapezoidal rule as integration method. Choosing the error tolerance $tol = 10^{-8}$ in IPOPT, we can expect that the state variables are correct up to 6 or 7 decimal digits. The Lagrange multipliers and adjoint variables are computed *a posteriori* by IPOPT thus enabling us to verify the necessary optimality conditions.

3.2 Parameter Values and Initial Conditions

The parameter values in the dynamics (2)–(5) are reported in Table 1. We set the initial conditions to

$$K(0) = 1.5,\ g(0) = 0.5,\ b(0) = 0.8,\ R(0) = 1.5,\ M(0) = 1.5,$$

and choose the terminal time terminal constraint as

$$T = 25, \quad K(T) = K_T = 3.$$

Furthermore, we restrict the extraction rate to

$$0 \leq u(t) \leq 0.1,\ \forall\, t \in [0, T].$$

We have considered the following two strategies for the allocations:

Strategy 1: Choose fixed values ν_1, ν_2, ν_3 satisfying (9).
Strategy 2: Consider ν_1, ν_2, ν_3 as optimization variables satisfying (9).

It would be also possible to treat $\nu_k = \nu_k(t), k = 1, 2, 3$, as time-varying control variables. However, our computations show that this strategy improves only slightly on Strategy 2 and is computationally much more expensive. For that reason, we do not report those results here.

Strategy 1 selects the fixed values for the allocation of infrastructural investments, such that the majority of infrastructure enhances productivity and the remainder is evenly split between mitigation and adaptation. Specifically, we consider $\nu_1 = 0.6, \nu_2 = 0.2, \nu_3 = 0.2$. In the second and third strategies we endogenize these allocative shares as choice variables maximizing (14).

3.3 Fixed Versus Optimal Values of ν_1, ν_2, ν_3

Comparing state variable trajectories under Strategies 1 and 2 demonstrates the latter considerably improves on the former. In the first comparison we assume the economic efficiency of the non-renewable resource is low ($A_u = 50$)[3] and that CO_2 mitigation efforts exhibit constant marginal returns, $\phi = 1$. The trajectories for the three control variables (C, e_P, u) and five state variables (K, R, M, g, b) are plotted in Fig. 1. Under this parameterization, Strategy 2's optimal allocation is $\nu_1 = 0.95, \nu_2 = 0.05, \nu_3 = 0$. That is, no infrastructure expenditures are put

[3]By construction the efficiency index A_u should be larger than A_K as the former calibrates a flow input and the former a stock value.

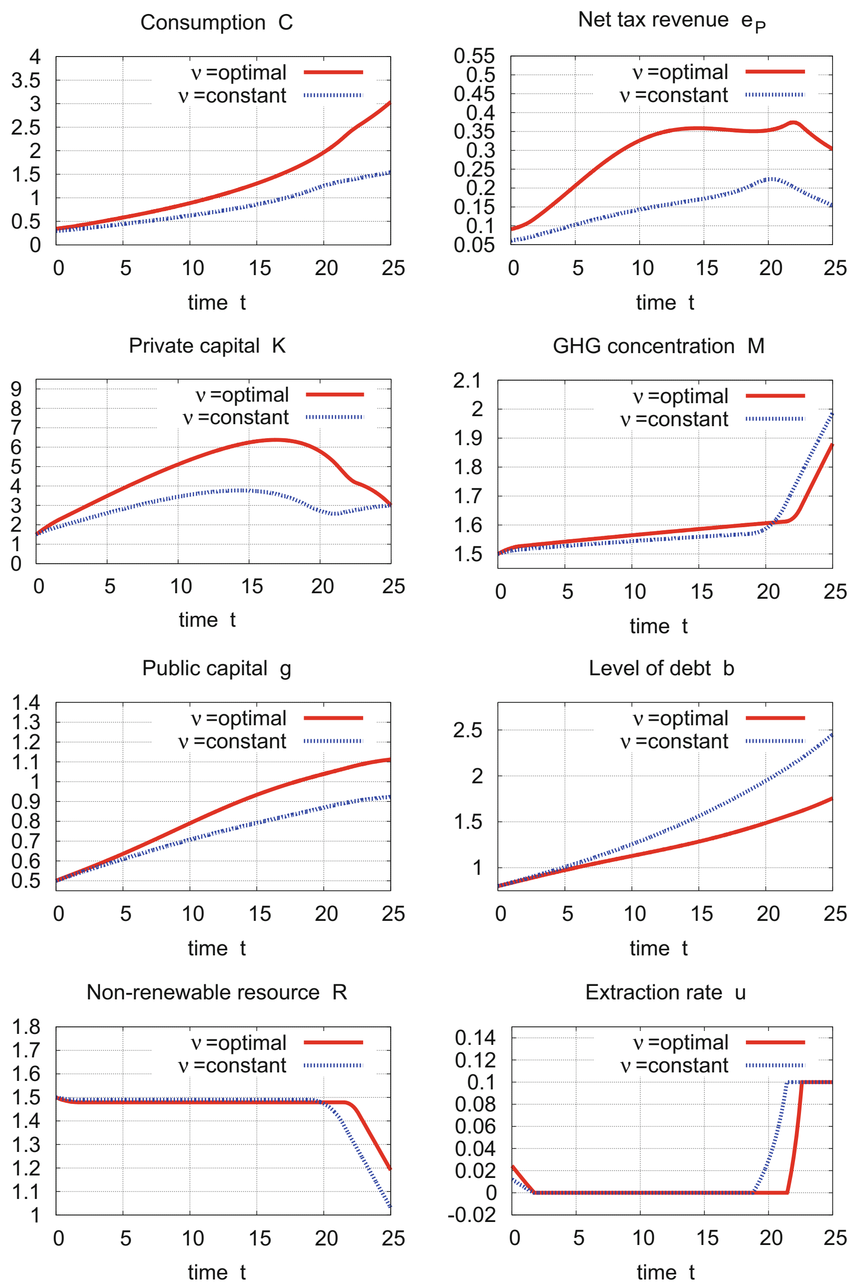

Fig. 1 Strategy 1 vs. 2, state and control variable trajectories. Strategy 1 (dashed blue) sets $\nu_1 = 0.6$, $\nu_2 = \nu_3 = 0.2$ and generates a final welfare value of $W(T) = -2.1006$. Strategy 2 (solid red) optimally selects $\nu_1 = 0.9534$, $\nu_2 = 0.04662$, $\nu_3 = 0$ and results in $W(T) = 5.1086$

toward mitigation and a mere 5% is allocated to adaptation.[4] The top four panels of Fig. 1 show this endogenous allocation, as compared to Strategy 1, results in higher per capita consumption, private capital accumulation and tax revenue in all periods, yet the final atmospheric CO_2 concentration is also lower. Although M is slightly lower under Strategy 1 through the first twenty periods, this abruptly reverses in the final periods when M grows exponentially. This seemingly odd result is explained by the trajectories in bottom four panels.

Under both strategies the per-period amount of non-renewable (and, here, inefficient) resource extracted is quickly pushed to zero so as to minimize the negative utility impact of CO_2 emissions. However, Strategy 1 over-allocates public infrastructure to mitigation efforts which generates suboptimal (climate-neutral) private capital accumulation. The low level of K in turn leads to less output and reduced tax revenue. Moreover, as the debt burden grows it begins to further dampen investment in K, which peaks in the fifteenth period. The falling per capita capital stock exhibits little impact until the terminal condition $K(t) = K_T$ begins to bite. From the twenty-first period onwards, preceding capital investment shortfalls are made up by shifting production to the inefficient non-renewable resource. The extracted amount u begins to ramp up from zero, reducing the stock R and generating CO_2 emissions.

Under Strategy 2 the peak in private capital comes at a delay and the terminal condition is not problematic since $K(t) > K_T$ for $3 < t < T$. Under this optimal allocation approach, overinvestment in mitigation infrastructure is avoided and the savings are put toward productivity enhancements. This generates a larger capital stock "buffer" allowing the economy to hold off the extraction of R. As in Strategy 1, maximum K is reached as the debt burden approaches 1.5, and tax revenue is redirected toward debt servicing. However, greater productivity and the lower stock of debt forestall this effect in Strategy 2. When extraction does begin in the twenty-second period, it merely reduces the *rate* at which K, the capital used for the production of green energy, falls toward K_T, rather than makes up for the previous investment shortfalls seen in Strategy 1. Again, the higher stock of private (green) capital has diminished the economy's reliance on the carbon-emitting non-renewable resource.

3.4 *Homotopic Analysis of A_u*

Many of the model parameters remain uncertain and/or unobservable. This limitation, common to all models, is particularly acute for IAMs due to the multifaceted feedback effects between economic decision-making and climatological impacts. To address the issue we apply homotopic parameter variation to $\mathbf{OCP}(p)$ for

[4] It is important to note that funding for renewable energy production is already captured through the variable K.

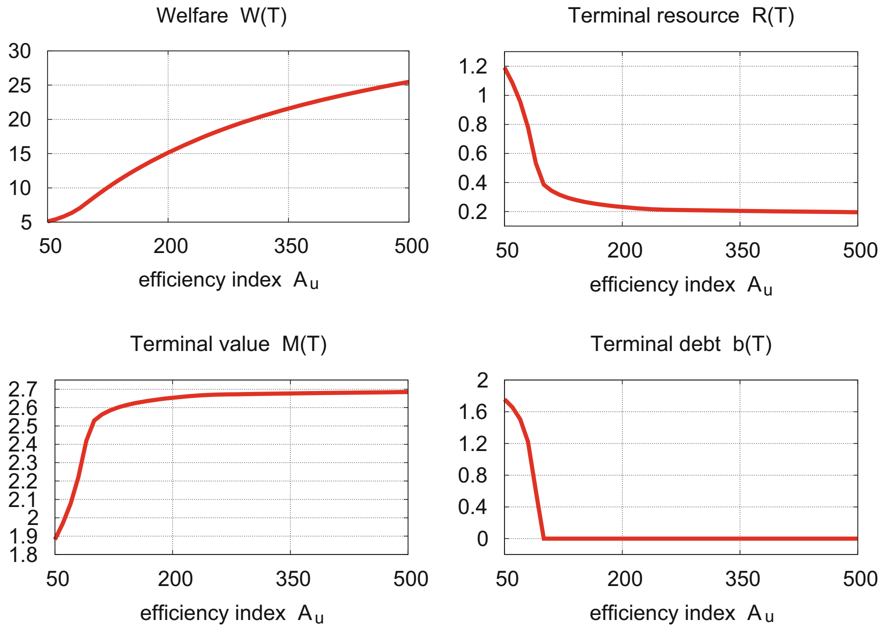

Fig. 2 Terminal states for homotopy $50 \leq A_u \leq 500$

several key parameters. In each case we use the optimal selection of infrastructure allocations ν_1, ν_2, ν_3 as they continue to outperform arbitrarily fixed values.

First, we consider scenarios in which the non-renewable resource—fossil fuel energy—generates output more efficiently than the generation of renewable energy by allowing A_u to range from a high of 500 down to 50 (as used in Sect. 3.3). Figure 2 plots the terminal values of welfare $W(T)$, CO_2 concentration $M(T)$, unextracted nonrenewable resource $R(T)$, and terminal debt $b(T)$. Unsurprisingly, welfare rises monotonically as the efficiency of this input is increased. Looked at the other way, welfare falls when fossil fuel energy becomes more costly to find and extract. The higher cost (viz. lower productive efficiency) of u decreases incentives to extract it, meaning the remaining stock of non-renewable resource rises from 0.2 for $A_u = 500$ to 1.2 at $A_u = 50$. At very low costs, the extraction rate is very inelastic, as shown by the slow increase in $R(T)$ between $A_u = 500$ and $A_u = 100$. After this point, the shift away from extraction rises rapidly as A_u halves from 100 to 50. This pattern of extraction maps inversely to CO_2 concentrations, which fall slowly as $A_u \to 100^+$, only to fall rapidly when extraction becomes sufficiently costly (which is calibrated here at $A_u = 100$).

The lower-right panel in Fig. 2 suggests why $R(T)$ rises in such a distinctly nonlinear fashion as A_u falls. At a low efficiency (high cost) of u, greater investment into K is supported through borrowed funds. For larger A_u, dependence on private capital K and productivity-enhancing infrastructure ν_1 is lower because the cheaper

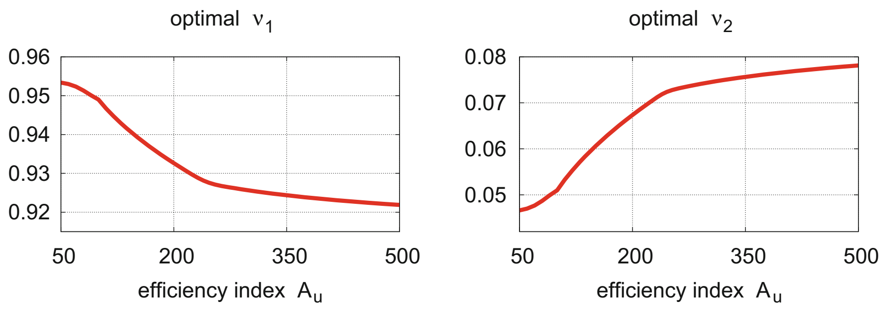

Fig. 3 Infrastructure allocations for homotopy $50 \leq A_u \leq 500$

non-renewable energy substitutes for carbon-neutral K. Figure 3 confirms this interpretation: the optimal allocation proportion ν_1 is 92% at $A_u = 500$ versus 95% for $A_u = 50$. In the former case, when extraction of the non-renewable resource is expensive, less infrastructure needs to be allocated toward adaptive projects: ν_2 falls from 8% to less than 5%. That said, the overall welfare outcome, is greater when A_u is large, in spite of the rise in M. Also implied by Fig. 3, $\nu_3 = 0$ for all values of A_u. Overall, the above case of $\nu_3 = 0$ is not likely to give realistic solutions since ν_3 enters the control problem linearly, which gives rise to the so-called 'bang-bang' problem.

3.5 *Homotopic Analysis of ϕ*

Since the result of no infrastructural investments put toward mitigation efforts is due to the linear relationship assumed by setting $\phi = 1$. Recall,

$$\dot{M} = \gamma\, u - \mu(M - \kappa \widetilde{M}) - \theta(\nu_3 \cdot g)^{\phi} \tag{4}$$

We now loosen this assumption of linearity to consider the mitigation exponent over the range $0.2 \leq \phi \leq 1$, which should be interpreted as the rate of diminishing returns to climate change mitigation efforts. Whereas $\nu_3 = 0$ for $\phi = 1$ (which is likely to be cause by the aforementioned 'bang-bang' problem), we obtain $\nu_3 > 0$ for $\phi \leq \phi_0 \approx 0.88$.

Figure 4 compares the optimal allocation of infrastructure expenditures toward productivity-enhancement ν_1, adaptation ν_2, and mitigation ν_3, as well comparing the final social welfare $W(T)$ at each value of ϕ. The results show that, as the rate of return to mitigation efforts diminishes, the impetus to reduce CO_2 emissions rises with ν_3 reaching 1.2% for $\phi = 0.2$. The rising mitigation share comes primarily at the (small) expense of traditional infrastructure, the allocation of g to which falls from 94% to just above 92.8%. The remaining difference ($\approx$ 0.1%) comes from reduced adaptation efforts. Note that as mitigation efforts are increased above nil,

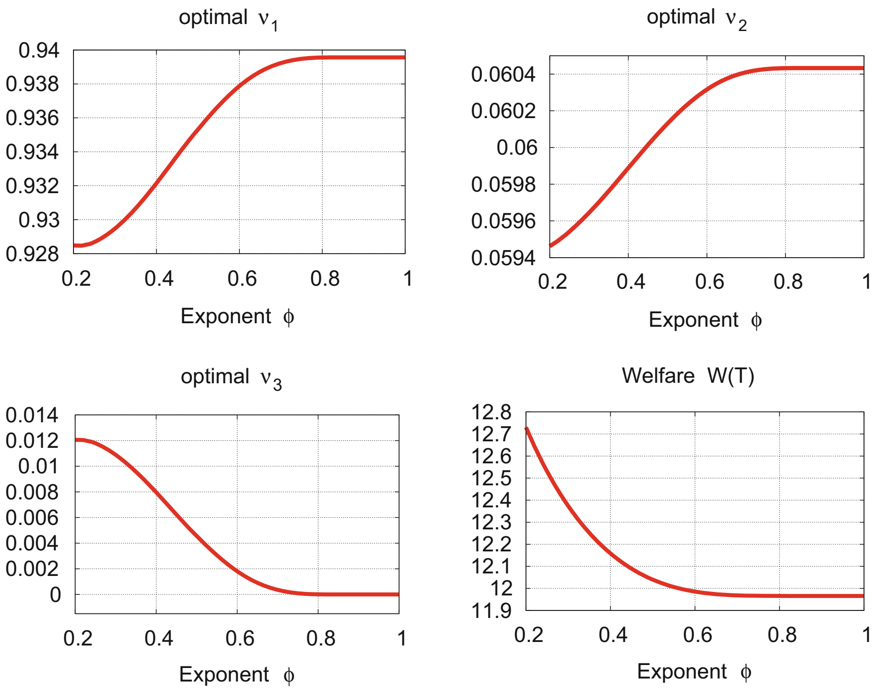

Fig. 4 Allocations and terminal welfare for homotopy $\phi \in [0.2, 1]$. The non-renewable resource's efficiency index is set at $A_u = 150$

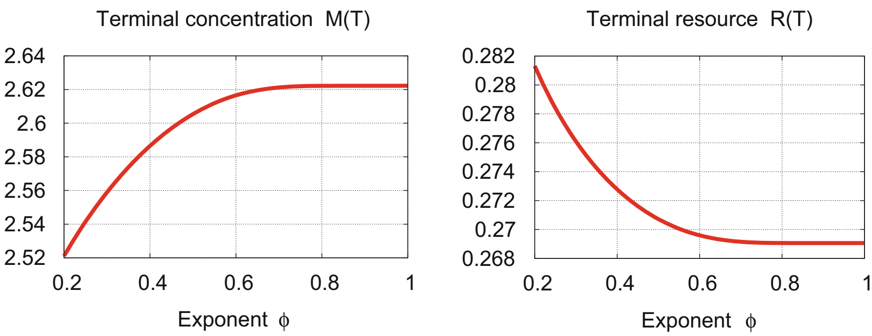

Fig. 5 Terminal resources and CO_2 for homotopy $\phi \in [0.2, 1]$. The non-renewable resource's efficiency index is set at $A_u = 150$

total social welfare increases by approximately 6%. Figure 5 confirms that as ϕ falls, the heightened mitigation effort helps reduce the final concentration of CO_2 in the atmosphere. Moreover, and corresponding to the latter result, the total amount of non-renewable resources extracted is lower ($R(T)$ higher) as ϕ falls.

3.6 Homotopic Analysis of A_u for $\phi = 0.2$

The unambiguous improvement to welfare and CO_2 concentration reduction for $\phi = 0.2$ found above assumed $A_u = 150$. To test whether the results from Sect. 3.5 were contingent on that efficiency index, we again perform a homotopy on A_u this time specifying a concave mitigation term in (4) at $\phi = 0.2$. As before we find that terminal welfare $W(T)$ increases when the efficiency of u falls (viz. the cost of extraction rises), infrastructural allocations to productivity ν_1 rise as adaptive efforts ν_2 fall (see Fig. 6). However, with $\phi = 0.2$ mitigation efforts ν_3 are no longer nil, although they remain between 1.0% and 1.7% of g. Interestingly, allocations mitigation are not monotonic over A_u. Over the 'high cost' range found in Sect. 3.4, $A_u \in [50, 100]$, ν_3 in Fig. 6 becomes increasingly desirable as extraction costs rise (A_u falls). For lower costs, $A_u > 100$, ν_3 falls as extraction costs increase (A_u falls) implying mitigation efforts must ramped up when fossil fuel energy is cheap in order to counter the increase in CO_2 emissions.

This interpretation of ν_3 is supported by the terminal states plotted in Fig. 7. The terminal atmospheric carbon concentrations rise rapidly over A_u (i.e., as extraction costs fall) and then stabilize above $A_u = 100$—aided in part by the increase in ν_3. Again, as the productive efficiency of u is increased, the extraction rate rises ($R(T)$ falls) nonlinearly and public debt becomes less relied upon as production shifts away from private capital toward non-renewable resources. Total infrastructure g

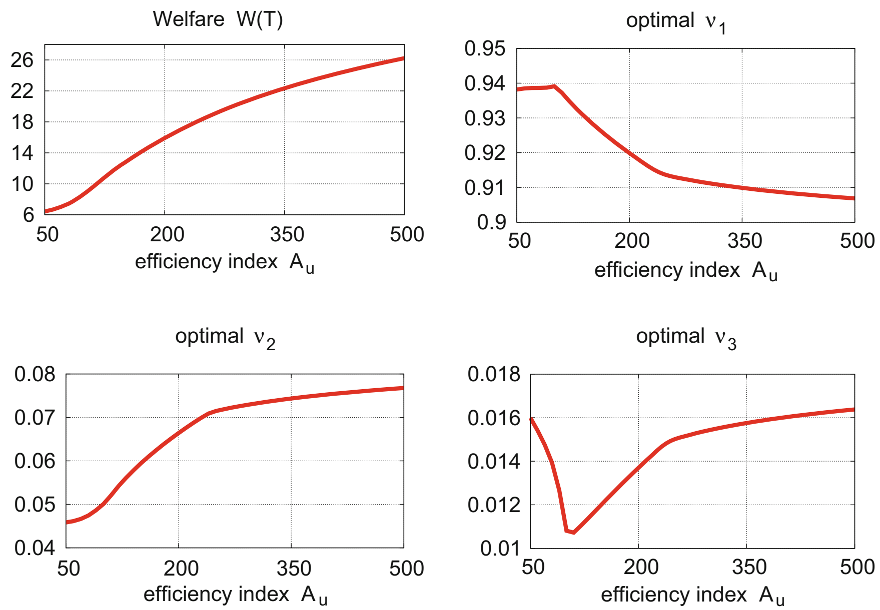

Fig. 6 Allocations and Welfare for homotopy $A_u \in [50, 500]$, $\phi = 0.2$

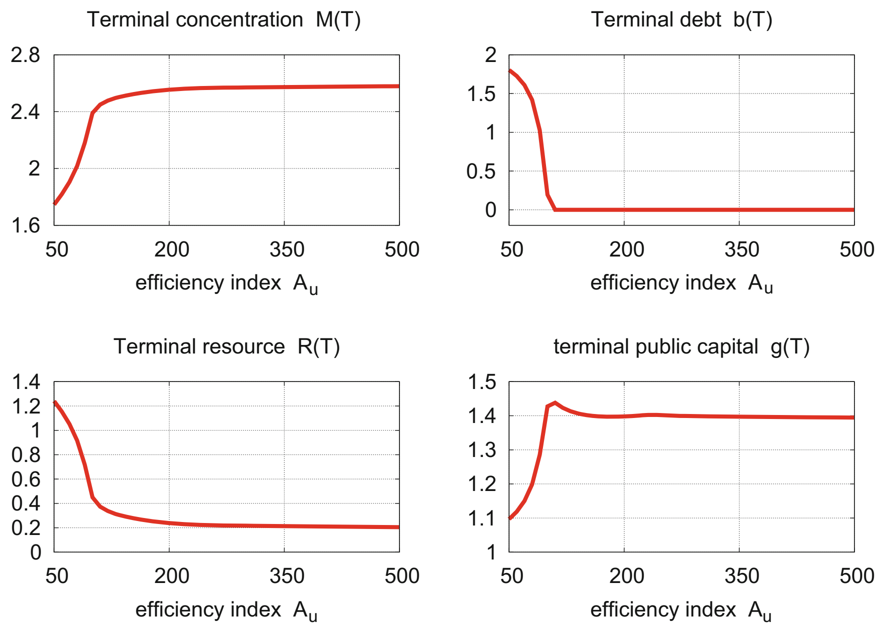

Fig. 7 Terminal states for homotopy $50 \leq A_u \leq 500$ for $\phi = 0.2$

also rises rapidly over the initial low range of A_u and then stabilizes for at values above 100.

Figure 8 shows the full trajectories of private capital K, consumption C, carbon concentrations M, the extraction rate u for three representative values of $A_u = 100, 200, 500$. In the extreme case of $A_u = 500$ private capital is driven to zero for the majority of periods between the initial and terminal points of K_0 and K_T, meaning production is driven entirely by the non-renewable resource. This result does not seem economically reasonable. The motivation to discard this parameterization is even stronger since the trajectories of M and u for $A_u = 500$ and $A_u = 200$ are nearly indistinguishable.

For an efficiency index of 150, K falls slightly from its initial value and fluctuates slightly before converging to K_T. Conversely, for $A_u = 100$, capital stock rises rapidly, peaks and then falls unevenly to K_T as was the case in Sect. 3.3 for $A_u = 50$, $\phi = 1$. As in Sect. 3.4, the extraction rate for $A_u = 100, 200$ reaches the maximal level near the end of the projection, with the less efficient scenario reaching the peak earlier. However, with $\phi = 0.2$ the lower efficiency index scenario now leads to a lower total and terminal CO_2 level as mitigation efforts are no longer held at zero.

Further trajectories for $\phi = 0.2$ are presented in Fig. 9. The total stock of infrastructure g is little changed under three A_u scenarios. As suggested by the trajectory of u in Fig. 8, the remaining stock of the non-renewable resource R is greatest for $A_u = 100$, but only by a small margin over the $A_u = 200$ scenario.

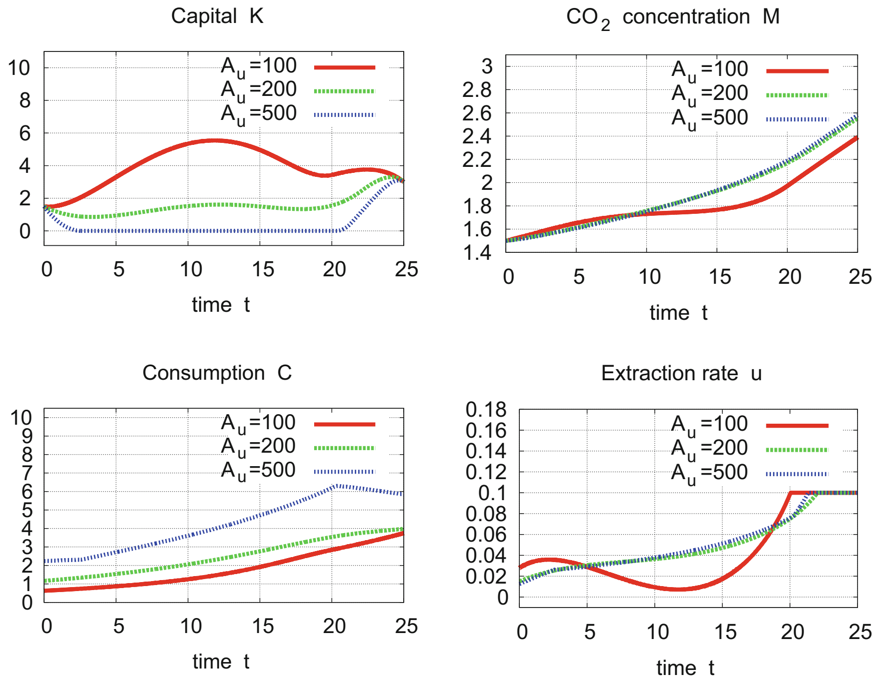

Fig. 8 Selected trajectories for $\phi = 0.2$ with $A_u = 100$, 200 and 500

Conversely, the tax revenue trajectory e_P fluctuates far more under $A_u = 100$ than the other scenarios. In the former case, e_P leads the fluctuations in u, falling before u rises and vice versa. This tendency supports the argument made above that greater reliance on the non-renewable resource reduces the need for fiscal deficits.

3.7 *Homotopic Analysis of* ρ *for* $\phi = 0.2$

Finally, we consider the homotopy of ρ, the pure discount rate. There has been much debate over the correct intertemporal discount rate that should be used in climate change economics (e.g., Stern 2007). While we do not weigh in on that debate here, it is informative to investigate the IAM results under various discount rate assumptions. Figure 10 shows that terminal welfare $W(T)$ falls smoothly as the discount on future outcomes rises. Although the falling allocation of infrastructure to mitigation ν_3 as ρ rises is expected, it is interesting to note that the shares of ν_1 and ν_2 move in opposite directions. In other words, the savings from ν_3 are not shared between productive infrastructure and adaptation. Instead, for higher discount rates, mitigation efforts are increased while ν_1 falls by a greater amount that ν_3.

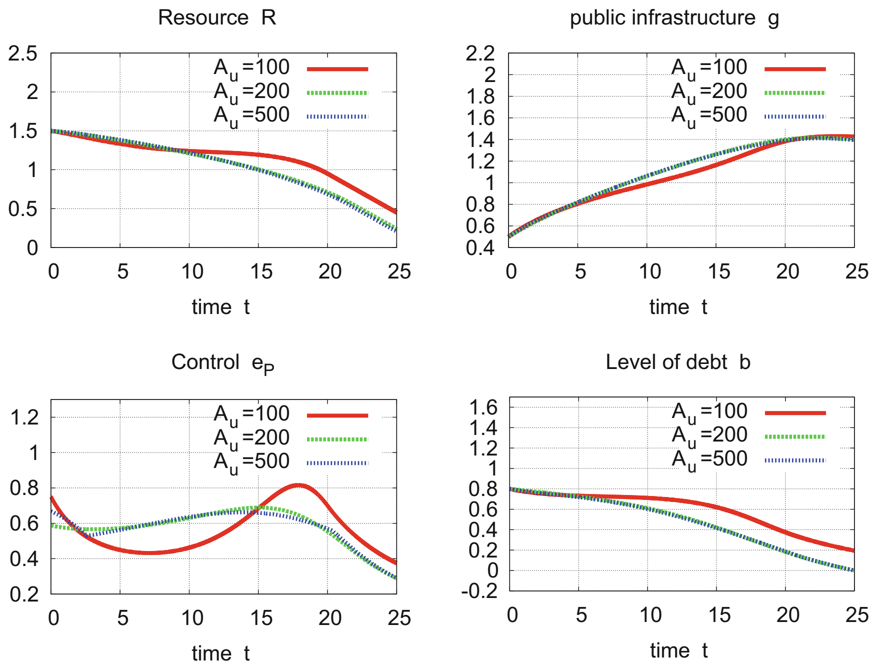

Fig. 9 Further trajectories for $\phi = 0.2$ with $A_u = 100$, 200 and 500

The reason for this behaviour is in Fig. 11. As the economy discounts future outcomes at a higher rate, the present cost of non-renewable resource extraction falls and thus the rate of extraction rises. The bottom panel in Fig. 11 indicates that indeed the remaining stock of non-renewable resource is driven down as ρ is increased. And, as in all other cases, when u rises the final stock of CO_2 concentration $M(T)$ rises. It is also notable that a higher discount rate is associated with a lower level of public infrastructure available to be used for any purpose. These results indicate that, indeed, the discount rate we choose to inform climate change policy can have a great effect on the trajectory ultimately followed.

4 Conclusion

Following a review of recent policy developments and modelling approaches to climate change economics, the paper developed an extended integrated assessment model explicitly accounting for the extraction of non-renewable resources and the phasing in of renewable energy. Another extension of the IAM framework is to include public sector policies concerning optimal decisions of both revenue and tax expenditures. Although the focus was on climate policy financing through

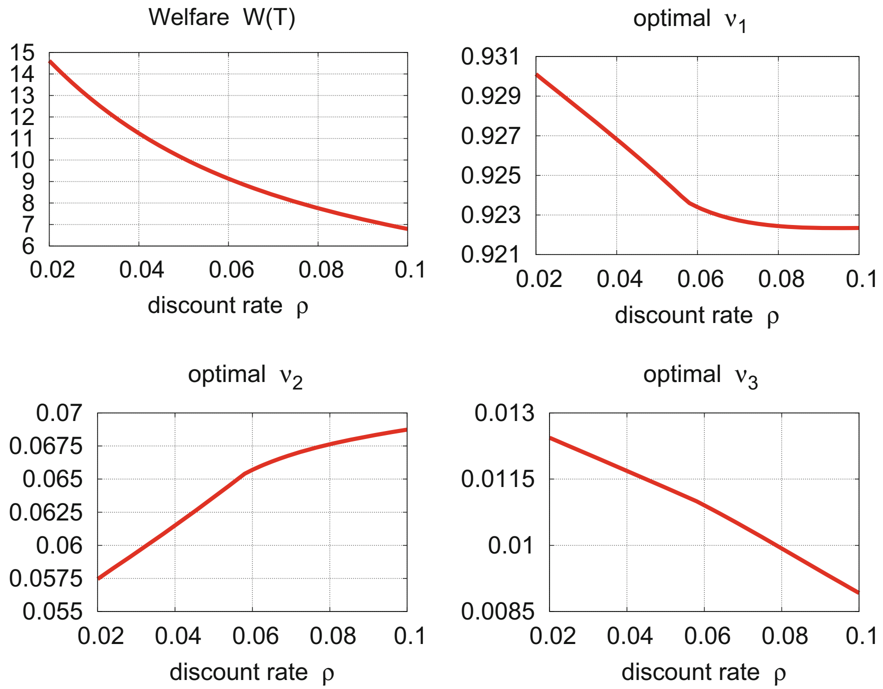

Fig. 10 Allocations and Welfare for homotopy $\rho \in [0.02, 0.1]$, $\phi = 0.2$

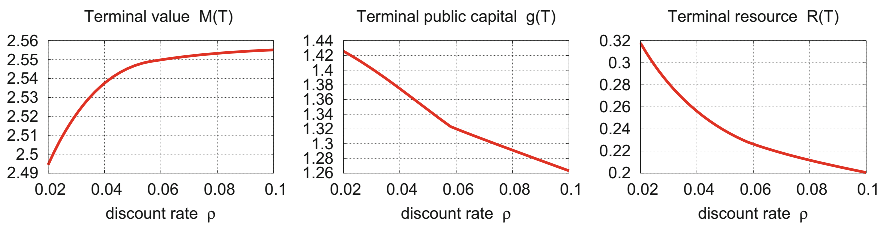

Fig. 11 Terminal states for homotopy $\rho \in [0.02, 0.1]$ for $\phi = 0.2$

taxation, future research could elaborate on the financing mechanisms through climate bonds.[5]

The IAM was solved using the AMPL algorithm which enabled us to maintain all of the system's nonlinearities and dynamic interactions. A particularly useful feature of this methodology is the ability to optimally determine the allocative variables ν_1, ν_2, ν_3 in order to indicate the best policy mix for addressing the challenges of climate change. In Sect. 3.3 we showed endogenously selected allo-

[5]In this context, a recent discussion of proposals for central banks to accept climate bonds as collateralizable securities is available in Flaherty et al. (2016).

cations consistently outperformed *ex ante* parameterizations. We then considered parameter homotopies under a strategy of optimally selecting the allocation shares to traditional, adaptive and climate change mitigating expenditures.

Given that green energy is already phased in through the accumulation of private capital, the model consistently found that over 90% of infrastructural investment should be geared toward productivity-enhancing investments. The phasing in of green energy is also supported by the fact that private capital enhancements $\nu_1 g$ are, by design, enhancements for carbon-neutral production. In other words, the model assumes that no public funds are used to directly support the extraction of CO_2-emitting resources.

Sections 3.4–3.6 consider the homotopy of A_u and ϕ, respectively the production efficiency index for the non-renewable resource and the exponent on mitigation efforts. The results demonstrated that greater efficiency of CO_2-generating resources incentivizes their use, thereby increasing carbon emissions. Increasing the input level of u also led to a reduced reliance on debt to finance ν_1. This result accords with the stylized fact that resource-dependent economies typically have large fiscal surpluses when primary products are in high demand. On the other hand as the efficiency of CO_2 generating energy declines, the results are reversed: more of this resource is left in the ground and cumulative CO_2 emissions are lower. The exponent ϕ proved to be crucial. As the concavity of mitigation efforts rose (lower ϕ), the level of mitigation efforts increased monotonically. One interpretation of this finding is that if mitigation is seen to be relatively inexpensive (i.e., fixed linear impacts on $\dot{M}$), then agents may continuously hold off on investing in mitigation.[6] We also considered the homotopy of ρ, the pure discount rate. As expected total social welfare was lower and CO_2 concentrations higher when, *ceteris paribus*, the discounting of future outcomes rose.

Overall, the IAM developed here is an advancement both in terms of the solution algorithm employed and in its use of novel dynamics. As mentioned, the modelling of non-renewable resource extraction and detailed public sector policies on climate change are new features in the IAM literature. In addition we have treated the damage function of climate change as impacting social welfare directly, as opposed to indirectly through reductions in the rate at which output is produced. While neither approach is perfect, we have employed the direct-utility impact version because we believe it is better able to capture the many physical, ecological and societal losses that may be induced by unabated climate change.

Finally, a necessary extension of the climate change policies studied here is consideration of the optimal financing sources, including policies for burden sharing. For example, standard IAMs place the cost and implementation burden of financing climate policies on the current generation. Indeed, the extended IAM developed here posits public sector financing of climate action through current tax revenues and expenditures. As an additional extension to the framework, we can

[6]Another issue is that when the control enters linearly, then the corresponding control variable (in this case mitigation effort) is driven to zero. This could be the result of a 'bang-bang' solution.

consider the extent to which climate policies can be funded by both a carbon tax and the issuing of climate bonds—the latter being repaid by future generations. For more specific work on this type of burden sharing between current and future generations, see Sachs (2014), Flaherty et al. (2016) and Gevorkyan et al. (2016).

References

L. Bernard, W. Semmler, *The Oxford Handbook of the Macroeconomics of Global Warming* (Oxford University Press, Oxford, 2015)

J.T. Betts, *Practical Methods for Optimal Control and Estimation Using Nonlinear Programming*, 2nd edn. (SIAM Publications, Philadelphia, 2010)

A. Bonen, P. Loungani, W. Semmler, S. Koch, Investing to mitigate and adapt to climate change: a framework model. IMF Working Paper, WP/16/164 (2016)

A. Bonen, W. Semmler, S. Klasen, Economic damages from climate change: a review of modeling approaches. SCEPA Working Paper 2014-03, Schwartz Center for Economic Policy Analysis (2014)

F. Bosello, Adaptation, mitigation and "green" R&D to combat climate change: insights from an empirical integrated assessment exercise. Working paper, Centro Euro-Mediterraneo Per I Cambiamenti Climatici (2008)

T. Bréchet, N. Hritoneko, Y. Yatsenko, Adaptation and mitigation in long-term climate policy. Environ. Resour. Econ. **55**(2), 217–243 (2013)

C. Büskens, H. Maurer, SQP methods for solving optimal control problems with control and state constraints: adjoint variables, sensitivity analysis and real-time controls. J. Comput. Appl. Math. **120**, 85–108 (2000)

K. de Bruin, R. Dellink, R. Tol, AD-DICE: an implementation of adaptation in the DICE model. Clim. Change **95**, 63–81 (2009)

M. Flaherty, A. Gevorkyan, S. Radpour, W. Semmler, Financing climate policies through climate bonds: a three stage model and empirics. Res. Int. Bus. Financ. **42**, 468–479 (2017)

R. Fourer, D.M. Gay, B.W. Kernighan, *AMPL: A Modeling Language for Mathematical Programming* (Duxbury Press, Brooks-Cole Publishing Company, North Scituate, 1993)

A. Gevorkyan, M. Flaherty, D. Heine, M. Mazzucato, S. Radpour, W. Semmler, Financing climate policies through carbon tax and climate bonds: a model and empirics. Manuscript, New School for Social Research (2016)

L. Göllman, H. Maurer, Theory and applications of optimal control problems with multiple time-delays. J. Ind. Manag. Optim. **10**, 413–441 (2014)

R.F. Hartl, S.P. Sethi, R.G. Vickson, A survey of the maximum principles for optimal control problems with state constraints. SIAM Rev. **37**, 181–218 (1995)

H. Hotelling, The economics of exhaustible resources. J. Polit. Econ. **39**(2), 137–175 (1931)

IMF, How much carbon pricing is in countries' own interest? The critical role of co-benefits. IMF Working Paper (2014)

IMF, After Paris: fiscal, macroeconomic, and financial implicatoins of climate change. IMF Staff Discussion Note (2016)

A. Ingham, J. Ma, A. Ulph, Can adaptation and mitigation be complements? Working Paper 79, Tyndall Centre for Climate Change Research (2005)

F. Lecoq, S. Zmarak, Balancing expenditures on mitigation and adaptation to climate change: an exploration of issues relevant for developing countries. Policy Research Working Paper 4299, World Bank (2007)

H. Maurer, W. Semmler, A model of oil discovery and extraction. Appl. Math Comput. **217**(13), 1163–1169 (2011)

R. Pindyck, Gains to producers from the cartelization of exhaustible reources. Rev. Econ. Stat. **60**(2), 238–251 (1978)

J. Sachs, Climate change and intergenerational well-being, in *The Oxford Handbook of the Macroeconomics of Global Warming*, ed. by L. Bernard, W. Semmler (Oxford University Press, Oxford, 2014), pp. 248–259

N. Stern, *The Economics of Climate Change: The Stern Review* (Cambridge University Press, Cambridge, 2007)

R. Tol, The double trade off between adaptation and mitigation for sea level rise: an application of FUND. Mitig. Adapt. Strateg. Glob. Chang. **12**(5), 741–753 (2007)

A. Wächter, L.T. Biegler, On the implementation of an interior-point filter line-search alogrithm for large-scale nonlinear programming. Math. Program. **106**, 25–57 (2006)

A. Zemel, Adaptation, mitigation and risk: an analytical approach. J. Econ. Dyn. Control. **51**, 133–147 (2015)