Sixty Years (or so) of Language Data in Mathematics Education

David Pimm

Abstract This chapter, based both on pre-ICME-13 conference documents as well as on the author's actual panel presentation made at TSG 31, covers a range of themes concerned with the issues of 'language data' in mathematics education. It also addresses several instances from its history, including word problems, classroom language and transcription, in addition to the mathematics register, its syntax, semantics and pragmatics.

1 Introduction

My title contains a conscious hedge. While it is likely not a span of *exactly* sixty years that language data have been offered as central and focal for research efforts in mathematics education, it seems close enough as a temporal marker to consider what sorts of language data have counted and how they have been dealt with over such a period of time. But it also raises for me some interesting ('Patient zero') thoughts in terms of what the first reporting and analysing of such data in mathematics education was like (for what purpose, in what form, what was done with it, to what end and why). And, also, whether we, some sixty (or so) years later, might recognise and still acknowledge it as language data research, not to mention moving on to the ever-pressing question of what mathematics education language-data-based research might come to look like in the future. Here are a few possibilities from the past.

- Is it to be found in research work on arithmetic word problems, which was a popular US doctoral research topic in the 1960s, but also discussed by Thorndike (1922)?

D. Pimm (✉)
Simon Fraser University, Burnaby, Canada
e-mail: djp3@sfu.ca

J. N. Moschkovich et al. (eds.), *Language and Communication in Mathematics Education*, ICME-13 Monographs, https://doi.org/10.1007/978-3-319-75055-2_2

- Was it in an attempt to depict an element or aspect of a mathematics classroom setting or interaction (and almost certainly fragments of such accounts appeared in professional publications and journals prior to research ones)?
- Was it actually to be found in the research foci and accounts of psychologists who regularly, even persistently, decided that instances of mathematical thought offered the best example of 'pure' cognition to be found, so used mathematical prompts or probes in order to find out, not about mathematical thinking per se, but about 'thinking' *tout court*? And the common way they attempted to access thought, mathematical or otherwise, was through speech.

Mathematics education, even to this day, has not fully emerged from the shadow of academic psychology of a certain sort[1]—one surface trace of this can be found in the persistently lingering 'P' in the name of the PME annual conference, despite a likely majority of the papers presented there in any given year having very little, if any, connection to that field. Instead, papers regularly draw on other cognate disciplines, including sociology, socio-linguistics and linguistics (though even to this day it is not straightforward to have a solely linguistics-rooted mathematics education paper accepted by a mainstream mathematics education journal).[2] (For more on the 'turn to language', see Morgan, 2006.)

A further echo exists in the broad (and unquestioned) adoption of the genre and style (including section order and dull general headings) of academic psychology writing by most mathematics education journals. One consequent aspect of this is the overvalued mirroring of the quasi-scientific basis for a certain sort of empirical psychology study, one based on isolated experiments and statistical analysis, rather than, say, a classroom-based study of classroom phenomena (thus, the classroom as both setting and site for much mathematics education research). There is an interesting question as to whether the emergence of language as data relates more than simply chronologically to the field's move away from psychological approaches: in particular, did attention to language as data prompt a move away from psychology (or, indeed, was it vice versa)?

I recently came across Richard Nisbett and Timothy Wilson's (1977) piece entitled 'Telling more than we can know'.[3] This article addresses, among several other things, self-reports of action rationales (in response to 'how' and 'why' questions about

[1]For a previous ICME, held in Québec in 1992, I prepared a talk (which I was unable to give) entitled 'Another psychology of mathematics education' (see Pimm, 1994).

[2]The same is true, interestingly, of papers on mathematics education and technology (see Sinclair, 2017). However, while there are general technology and education journals (as there are for linguistics and education), there are not any specialist language and mathematics education journals comparable with *Digital Experiences in Mathematics Education* or STEMmy journals such as *The Canadian Journal of Science, Mathematics and Technology Education*.

[3]This article title contains a very nice play on the wording of Michael Polanyi's (1966/2009) startling claim-as-fact, in his book *The Tacit Dimension*, that, '*we can know more than we can tell*' (p. 4; *italics in original*). Curiously, the header throughout Nisbett and Wilson's article is the subtitle, 'verbal reports on mental processes', which contains more characters than the actual main title.

behaviour, for example)—a common-enough twentieth-century research practice in social psychology—coming into collision in the US in the 1960s with some other psychologists (such as George Miller or Ulric Neisser). The latter doubted we can have any direct or worthwhile access to high-level mental processes, "such as those involved in evaluation, judgment, problem solving, and the initiation of behavior" (p. 232), merely to their products. And this not to mention on the other side of things Sigmund Freud and his influence on the question, which includes the prospect that we actually do not or cannot awarely 'know' certain things, because we repress them.

Nisbett and Wilson note the widespread ease of interviewees' responses to such questions, and neatly turn the tables on the challengers by inviting them to account for such responses. Nevertheless, the article provides a consistent attempt to undermine and devalue self-report data. This widely cited article from the 1970s reflects a second-wave, anti-introspection challenge, perhaps echoing an earlier generation's attempts to require a focus on objectively observable data as the only data worth having (see Christopher Green's magnificent 1992 trace history of physicist Percy Bridgman's 'operation[al]ism' in psychology).

Another contested practice involved in generating language data is that of the 'clinical' interview. Two of its staunch defenders in relation to mathematics (and science) education have been Herbert Ginsburg (e.g. his 1981 article) and Andrea diSessa (e.g. his 2007 one), both writing about a language data source once again under challenge from psychologists of a certain stripe, albeit some twenty-five years apart. And even Ginsburg's actual title frames it as 'psychological research on mathematical thinking'.

In both of these examples, it is not so much the method of data generation that was under attack as the value and validity of the speech data generated that was/is being called into question. Behind this seemingly gentle enquiry is the question of when, where and how mathematical language itself, in some written, spoken or perhaps signed form, was made into an *object* of study, into a thing in its own right, rather than simply being seen as a (transparent) carrier of something else (meaning, significance, thought, emotion, …)? And all the transcription practices (I discuss this a little further below) make one realise that, whatever 'it' is, you can never fully capture it. Speech (as well as para-linguistic phenomena) is profoundly different from writing. I have offered these micro-synopses here to remind us all that it has not always been plain sailing in mathematics education with regard to the presentation and analysis of language data. There is a much deeper intellectual history of this area still waiting to be written.

2 Word Problems

Word problems form a stable linguistic target with a very long history, at least as a mathematical–pedagogical object (e.g. some problems in the Rhind Mathematical Papyrus, a text, claiming to be a copy of an earlier text, dating from nearly 2000 BCE). The very name 'word problem' seems to identify it by its linguistic aspects.

They are also primarily written, usually in ‘textbooks’, hence already a textual object of sorts. This feature is something which has made them a less transient phenomenon, even if they may frequently be read aloud in a classroom setting. (For a single instance of this, see, for example, Herbel-Eisenmann & Pimm, 2014.)

As I mentioned in passing above, they became subject to a strong doctoral study focus in the US in the 1960s (where I encountered them *en bloc* as a doctoral student myself at the University of Wisconsin-Madison in 1977–9, charged with producing a research review in relation to addition and subtraction word problems, work which in a small way fed into Carpenter, Moser, and Romberg 1982). Many of the doctoral studies that I read focused on aspects of syntax (occasionally semantics), features which were used to attribute an *a priori*, variably weighted, numerical degree of difficulty (based on a range of linguistic features such as active or passive voice and other measures of syntactic complexity, the number of words in the problem or per sentence, …) to arithmetic word problems, a purported ‘measure’ which was then compared empirically with students’ success with them (a different notion of ‘actual’ difficulty). An example of such an approach to arithmetic word problems can be seen in Jerman and Rees (1972), while an instance in regard to algebraic ones can be seen in Lepik (1990). Jerman and Rees claim, “A basic assumption of this approach is that the structure of the arithmetic problem itself, to a large measure[,] determines its difficulty level” (p. 306).

These early instances of (basic) linguistic notions used to engage with a phenomenon of mathematics education also reflect a style and genre of research in which raw data was immediately quantified and then discarded and the numerical measures then ‘became’ the data that was (parametric-statistically) analysed. The more contemporary style tends not to turn words into numbers (something which, for me, echoed the ancient practice of *gematria*), preferring to work with them more on their own terms.

In the 1970s, Pearla Nesher undertook a considerable amount of research in this area (e.g. 1972; Nesher & Teubal, 1975) and she was a central figure in broadening the focus of attention from lexico-grammmatical features to questions of semantics (e.g. Nesher & Katriel, 1977). However, one significant more recent example (involving both syntax and semantic elements) can be found in Susan Gerofsky’s extensive and sophisticated work (e.g. 1996, 2004) on word problems, not least where she draws on Levinson’s (1983) account of pragmatics (and of L-tense and M-tense of verbs in particular, in relation to time coding; see his chapter on deixis, pp. 54–95).

Linguistic verb tense (L-tense) refers to conventional grammatical tense in a language, while meta-linguistic verb tense (M-tense) signals a deictic category that encodes an event relative to the coding time (CT) of an utterance. Gerofsky (1996) writes:

> In an M-tense system, we distinguish the temporal location of events in relation to CT: *past* refers to events prior to CT, *present* to events spanning CT, *future* to events succeeding CT, *pluperfect* to events prior to *past* events (which are themselves prior to CT) and so on. […] I have found that determining M-tense in mathematical word problems is problematic. (p. 40)

As an illustrative instance, Phillips (2002) provides discussion of a grade-four, student-generated word problem:

> Jane and Lucy both weigh 35 kilograms. Lucy went on a diet and now she is 30 kilograms and Jane has gained 7 kilograms. How much more does Jane weigh than Lucy? (p. 254)

Attending to the verbs and their L- and M-tenses (as well as the temporal deixis of ‘now’ as marking the CT of the problem) reveals the students’ apparent unawareness of this feature of word problems.

The research topic of mathematics word problems and their characteristics has not gone away. Research attention is still in place, while certain analyses have gained in linguistic sophistication. (For an encyclopædia entry on research on word problems in mathematics education, see Verschaffel, Depaepe, & van Dooren, 2014, although there is no specific mention of linguistic analysis of word problems either in their keywords or in the body of their text.)

3 Classroom or Research Events and Their Records: Making a Thing of Things

Earlier, I made mention of the need to make an object out of language in order to study it. In this instance, language is no different from anything else: all events take place in time and vanish. Records are required in order to make time stand still (even if only temporarily), as well as to allow repeated entry over time, as far as is possible, via the record to the event itself. The records become proxies for the events themselves. (For a little more on this, see Pimm, 2018.)

Nowhere is this clearer than with speech data. With audio-taping and subsequent videotaping technology, real-time records (albeit records still made from particular points of view, not least depending on the location of the recording device[4]) could be made (and, in our digital era, made very ‘cheaply’). A conventional device for rendering speech into writing is the transcript (of a record of an event). And in different discourse analytic traditions, most particularly that of Conversation Analysis (CA), a great variance in sonic detail is or is not to be included. Gail Jefferson, a colleague of sociologist Harvey Sacks, both founding figures of CA, has produced what she has termed the ‘gold standard’ of transcription. In her 2004 chapter, ‘Glossary of transcript symbols with an introduction’, Jefferson writes:

> Although I’d probably rather transcribe than any [*sic*] do any other part of the work (analysing, theorizing, lecturing, teaching, etc.), the one thing I’d rather *not* do is talk about transcribing. It’s not a topic. You might as well talk about typewriting. Transcribing is just something one does to prepare materials for analysis, theorizing, etc. Do the best you can, but what is there to talk about? (p. 13; *emphasis in original*)

[4]Including the possibility of it being situated behind an ear of the teacher, thereby mirroring the teacher’s eye-line. Without intending this to be an instance of product placement, see *Looxcie*.

Of course, rest assured, being an academic, she does nonetheless find a number of things to talk (write) about! But I am struck that there is no mention of calibrating the degree or extent of transcriptive fidelity of the audio recording to one's research intents and interests, even bearing in mind that one often does not fully know the nature or aspects of a research phenomenon of interest before beginning one's analysis.

In her chapter in this very book,[5] Judit N. Moschkovich concurs on this point and goes further:

> Transcription and transcript quality are theory laden (Ochs, 1979; Poland, 2002). Researchers make many decisions about transcripts that are based on their theoretical framework and on the particular research questions for a study. Decisions regarding what to include in transcripts and which transcript conventions to use are informed by theory. Whether a transcript will include gestures, emotions, inscriptions, body posture, and description of the scene (Hall, 2000; McDermott, Gospodinoff, & Aron, 1978; Poland, 2002) will depend on whether these aspects of activity are relevant or not to the particular research questions. Similarly, selecting transcript conventions and deciding whether overlapping utterances, intonation, and pauses are included or not in a transcript depends on whether these aspects are relevant to the research questions and analysis that will be carried out with the transcript and video. (Moschkovich, 2018, p. 45)

Historically, early depictions of mathematics classrooms took the form of a short narrative account of a lesson or, possibly starting around 1960 (or so!), occasional brief transcripts of teacher–student or student–student exchanges began appearing both in professional and in research journals. Questions of the veracity or fidelity of such transcripts did not explicitly feature initially, but the form of such transcripts was much influenced by the antecedent genre of play script: identified speaker turns, non-overlapping turns, occasional para-lingual or prosodic indicators, conventionalised spelling, stage directions, etc. (For more on the notion of antecedent genre, see Jamieson, 1975; for much more on recent mathematics education research employing forms of scripting, see Zazkis & Herbst, 2018.)

Staats (2008, 2018) has helpfully provided access for mathematics education to a different transcription model (one from linguistic anthropology, based on poetry rather than prose), that among other things provides a way of depicting (hence highlighting) structuring elements of repetition, highly pertinent in regard to conversations about mathematics. She writes:

> While linguistic meaning is often considered a property of words, significant mathematical ideas – arguments, inference, and relationships – can also be expressed through discourse structure. At times, the form of a student's statement can convey meaning as much as the isolated definitions of the words themselves. (2008, p. 26)

Much of this structure is marked by prosodic elements (such as rhythmic emphasis and dynamic speech variation) which may not be conventionally transcribed. The technique Staats introduces to mathematics education brings these

[5] Ah, deixis—'this very book' refers to (points at) the book I presume you, dear reader, are currently reading, not the book Jefferson's chapter is in.

elements to the fore (and allows us to see once more that *prose* transcription is a presumption). This interesting transition (or at least enlarging of transcription options), still in its infancy with respect to mathematics education (somewhat ironically, given how structured mathematical speech is), reflects a shift of what counts as significant (something I mentioned at the outset of this chapter). Who would have thought that speech rhythm and repetition (see also Tannen, 2007) could be important?[6]

4 The ICME-13 TSG 31 Panel Talk Itself (July 26th, 2016)

For my talk, I had thought I would identify a few touchstone linguistic moments from the past sixty years, both from inside and outside mathematics education. Possibilities included: John Sinclair and Malcolm Coulthard's work on classroom language from 1975, including their identification of the Initiation—Response—Feedback sequence and how it differs from Mehan's (1979) Initiation—Response—Evaluation; Paul Grice on conversational implicature, also from 1975, not to mention exploring the regular violation of his conversational maxims by teachers in mathematics classrooms; Pearla Nesher's work on the semantics of word problems from the 1970s; Julie Austin and Geoffrey Howson's seminal literature review of language and mathematics education from 1979 and, prior to that, Lewis Aiken's review from 1972 (though his 'language factors' are not the same as 'language data'); Michael Halliday's talk from 1975, introducing the notion of the mathematics register; Michael Stubbs PME 10 plenary lecture on logic and language in 1986, as well as his subsequent work on corpus linguistics (1996, 2001); Zwicky's (2003) magnificent work on metaphor; Beth Herbel-Eisenmann and David Wagner's accounts of lexical bundles in mathematics classroom talk from 2010; Andreas Ryve's meta-analytic work, from 2011, on the full extent of the decidedly smudgy use of the term 'discourse' in a wide range of mathematics education research texts (108 of them, in fact).

In passing, I feel the same about 'text' as I do about 'discourse', namely its problematic use at times to refer both to the spoken and to the written (which are sufficiently different, I feel, not to support an overarching term). And so much of striking contemporary interest is to be found in Barwell et al. (2016). And then I planned to use these pieces as potential dipsticks to try to identify changes in how language data have been approached, perceived and manipulated in published research in mathematics education (and beyond).

However, events took another course, although not as in 1992 (see footnote 2) when I was actually unable to attend the sixth ICME conference. I was in Hamburg in July 2016 and turned up on the right day and at the right time for the opening

[6]This connects more widely to issues of metaphor (as potentially central to mathematics and not just poetry)—see Zwicky (2003, 2010) and Pimm (1987, 2010). After all, it was Goethe who proclaimed, "Mathematics is pure poetry".

panel in Topic Study Group 31 (entitled “language and communication in mathematics education”). Unfortunately, I had followed the conference suggestions not to bring my own computer and had somehow managed to copy onto a memory stick only an alias of my detailed *Powerpoint* presentation (in other words, just an echo of the file was on the stick, which basically contained nothing at all). So, now (CT), I have found myself here faced with a decision: do I now write a paper that would be a pseudo-record of my intended talk that had not taken place (wow, M-tense issues all over that sentence!) or do I produce an (unpolished) written version of what I actually (well, more or less) said? I have decided to go with the latter.

However, I have also decided to do so in the form of aphoristic notes. Partly this is due to a fondness for aphoristic texts (see, for instance, Jean Baudrillard’s five volumes of aphorisms, compiled at regular intervals over the space of twenty-five years, each entitled *Cool Memories*, or James Richardson’s *Vectors*), and partly for other reasons (e.g. memory and its erratic ageing). Here are two aphorisms, one from each author mentioned above. (For more on aphorisms and their strengths, see Pimm, 2017.)

> Greater than the temptations of beauty are those of method.
>
> (Richardson, 2001, p. 26)

> All terms with negative prefixes are already stereotyped language.[7]
>
> (Baudrillard, 1995/2007, p. 88)

The panel talk slot was fifteen minutes long and seems a long time ago. Many years ago, one of my university mathematics teachers, David Fowler,[8] offered me a conference presentation meta-thought: “to say one thing that is worth saying takes two-and-a-half minutes”. So, having said this aloud in Hamburg, it left me with just five more things to say.

One concerned the profound involvement of language in and with the shaping of the nature of mathematics itself, not just mathematics classroom language (see, for instance, Morgan & Burton, 2000). But there is also a curiosity in that there has been far less attention paid to the geography of mathematics as opposed to its history. For one instance of the former, concerned with the dynamic writing of fractions, see Bartolini Bussi, Baccaglini-Frank, and Ramploud (2014) and Pimm (2014a). For an extensive account of the staggering geographic and historical diversity in relation to number system notation, see Chrisomalis (2010). And Reviel Netz’s (1999) important work on what he terms the ‘archaeology’ of the mathematical diagram highlights the effects of the devaluing of the mathematical diagram (as opposed to the ‘text’, as if diagrams too were seen as not ‘written’) of mathematical proofs. This is all perhaps the result of a combination of the illusion of and the imposition of a sense of mathematical ‘universality’.

[7]What particularly comes to mind for me from this aphorism are such terms with negative definitions, like *irrational* or *non-recurring* or *discontinuous*.

[8]See Pimm (2004).

A second point involved how mathematics also leaves (potentially unaware) traces in us: gestures, rhythms, body counting, and so on—traces that may not be as contemporary as the words that accompany them. It is possible for one's hands to be in one century and one's vocal cords in another. (For an example of this, though not explicitly framed as such, see Núñez, 2004/2006.) And certain technology (e.g. touchpad dynamic geometry environments) might provide ways of returning one's hands to a previous century, such as by heading back to the seventeenth-century notion of variable in regard to creating diagrams/graphs (as opposed to graphs being an already-existing static set of points satisfying a relation). For more on the connection between mobility, gestures and diagrams in mathematics, see Châtelet (2000) and Ng (2015).

This is closely connected to my third point which was (of course) about the mathematics register. How it can mess with the mainstream grammar of the language: the tension between nominalisation (everything in formal, written mathematics ends up as a lifeless, timeless noun) and verbification (the return of time and human action—see Lunney Borden, 2011). But professional mathematicians and scientists talking informally to each other are much more informal and casual in their language use (see Barwell, 2013; Ochs, 1979). Mathematics is both static and dynamic (and dynamic geometry software is supporting the return of 'time and motion' studies[9] as part of mathematics) and this connects to a core distinction

[9]In the early part of the twentieth century, there was considerable interest in the notion of 'arrested motion' in sculpture, endeavouring to capture the dynamic in the static. In a *Paris Review* interview, writer William Faulkner asserted:

> The aim of every artist is to arrest motion, which is life, by artificial means and hold it fixed so that 100 years later when a stranger looks at it, it moves again since it is life. […] This is the artist's way of scribbling […] oblivion through which he must someday pass. (1956, pp. 49–50)

This contrasts interestingly with historian of science Catherine Chevalley's comments about her father Claude, a core Bourbaki mathematician:

> For him [Claude Chevalley], mathematical rigour consisted of producing a new object which could then become immutable. If you look at the way my father worked, it seems that it was this which counted more than anything, this production of an object which, subsequently, became inert, in short dead. It could no longer be altered or transformed. This was, however, without a single negative connotation. Yet it should probably be said that my father was probably the only member of Bourbaki who saw mathematics as a means of putting objects to death for aesthetic reasons. (in Chouchan, 1995, pp. 37–38; *my translation*)

There is also a significant sense in which language—not least mathematical language (especially the written)—provides a means for arresting motion:

> a point is an instance,
> arrested motion – the geometric,
> its unsigned art.

between definition by genesis and definition by property (see Molland, 1976; for a piece using this distinction within mathematics education, see, for instance, Chorney, 2017). Syntax matters. And it shapes our perception of mathematics, something which becomes indistinguishable from mathematics itself.

My fourth point concerned language data, which I have already addressed to a certain extent at the outset of this chapter.

The last one (which I did not get to say aloud) I had labelled *être et avoir*. This is, among other things, the title of a French documentary film about a year in a small rural elementary school with a very mixed-grade class (aged from four to twelve), but is also a binary classification (based on auxiliary verbs) of verbs in French, with respect to how the perfect tense is formed. In English, both the verbs 'to be' and 'to have' show up in mathematics a lot and convey different messages about the mathematical object or result under consideration. How does the copula 'to be' (which sounds so active) actually deep-freeze the mathematical world?

5 In Conclusion

A few years ago, I began a ZDM commentary piece as follows.

> Part of the opening paragraph of the description for a doctoral course I am teaching runs as follows:
>
> Almost every piece of mathematics education research is based on language data to some greater or lesser extent, where 'language' needs to be more or less broadly interpreted. Whether these data arise from oral interviews, transcripts of classroom video recordings, textbooks (ancient or modern), student written responses to tasks, mathematicians' writing or teacher study group recordings or …, you need to be able to work with and analyse language data at length and at depth. I am particularly interested in questions of *method* and the *manner* of data collection/generation/creation, their examination and analysis.
>
> As you can see, I am interested in the varied uses the field makes of language as data, and this is how I propose to open my remarks here. But before that, I start with a question. When did research articles in mathematics education start including elements of language as data? Some fragment of classroom language perhaps or possibly an extract of clinical interview data or maybe student written responses to test questions or even a textbook page in a curriculum study? I ask because it clearly does have an origin as a practice (as well as a possibly earlier genesis in professional journals) and I am curious about not just when but why. (Pimm, 2014b, p. 967)

Four years on, I am pretty much still where I was—interested, but not much further forward (and, of course, preparing to teach that same course in January 2018). An enormous amount of data in mathematics education actually is language data. (Work on gestures may be one contemporary exception, though by incorrectly labelling this phenomenon as 'para-linguistic' a certain monster-adjustment of my primary claim may be made. Gestures are certainly a key element of mathematical communication.) Consequently, how language data is conceived and handled is of the utmost significance to our field.

Acknowledgements I am grateful to David Wagner and Judit N. Moschkovich for comments on earlier drafts, as well as Nathalie Sinclair, my ideal reader.

References

Aiken, L. (1972). Language factors in learning mathematics. *Review of Educational Research, 42*(3), 359–385.

Austin, J., & Howson, G. (1979). Language and mathematical education. *Educational Studies in Mathematics, 10*(2), 161–197.

Bartolini Bussi, M., Baccaglini-Frank, A., & Ramploud, A. (2014). Intercultural dialogue and the geography and history of thought. *For the Learning of Mathematics, 34*(1), 31–33.

Barwell, R. (2013). The academic and the everyday in mathematicians' talk: The case of the hyperbagel. *Language and Education, 27*(3), 207–222.

Barwell, R., Clarkson, P., Halai, A., Kazima, M., Moschkovich, J., Planas, N., et al. (Eds.). (2016). *Mathematics education and language diversity*. Cham, CH: Springer.

Baudrillard, J. (1995/2007). *Fragments: Cool memories III, 1990–1995*. London, UK: Verso.

Carpenter, T., Moser, J., & Romberg, T. (1982). *Addition and subtraction: A cognitive approach*. Hillsdale, NJ: Lawrence Erlbaum Associates.

Châtelet, G. (2000). *Figuring space: Philosophy, mathematics and physics*. Dordrecht, NL: Kluwer Academic Publishers.

Chorney, S. (2017). Circles, materiality and movement. *For the Learning of Mathematics, 37*(3), 2–5.

Chouchan, M. (1995). *Nicolas Bourbaki: Faits et légendes*. Argenteuil, FR: Éditions du Choix.

Chrisomalis, S. (2010). *Numerical notation: A comparative history*. New York, NY: Cambridge University Press.

diSessa, A. (2007). An interactional analysis of clinical interviewing. *Cognition and Instruction, 25*(4), 523–565.

Faulkner, W. (1956). The art of fiction XII: William Faulkner (an interview with Jean Stein). *The Paris Review, 12,* 28–52.

Gerofsky, S. (1996). A linguistic and narrative view of word problems in mathematics education. *For the Learning of Mathematics, 16*(2), 36–45.

Gerofsky, S. (2004). *A man left Albuquerque heading east: Word problems as genre in mathematics education*. New York, NY: Peter Lang.

Ginsburg, H. (1981). The clinical interview in psychological research on mathematical thinking: Aims, rationales, techniques. *For the Learning of Mathematics, 1*(3), 4–11.

Green, C. (1992). Of immortal mythological beasts: Operationism in psychology. *Theory and Psychology, 2*(3), 291–320.

Grice, P. (1975). Logic and conversation. In P. Cole & J. Morgan (Eds.), *Syntax and Semantics 3: Speech acts* (pp. 42–58). New York, NY: Academic Press.

Halliday, M. (1975). Some aspects of sociolinguistics. In UNESCO (Ed.), *Interactions between linguistics and mathematical education* (pp. 64–73). Copenhagen, DK: UNESCO.

Herbel-Eisenmann, B., & Pimm, D. (2014). The one and the many: Transcripts and the art of interpretation. *For the Learning of Mathematics, 34*(2), 38–40.

Herbel-Eisenmann, B., & Wagner, D. (2010). Appraising lexical bundles in mathematics classroom discourse: Obligation and choice. *Educational Studies in Mathematics, 75*(1), 43–63.

Jamieson, K. (1975). Antecedent genre as rhetorical constraint. *Quarterly Journal of Speech, 61*(4), 406–415.

Jefferson, G. (2004). Glossary of transcript symbols with an introduction. In G. Lerner (Ed.), *Conversation analysis: Studies from the first generation* (pp. 13–31). Amsterdam, NL: John Benjamins Publishing Company.

Jerman, M., & Rees, R. (1972). Predicting the relative difficulty of verbal arithmetic problems. *Educational Studies in Mathematics, 4*(3), 306–323.

Lepik, M. (1990). Algebraic word problems: Role of linguistic and structural variables. *Educational Studies in Mathematics, 21*(1), 83–90.

Levinson, S. (1983). *Pragmatics*. Cambridge, UK: Cambridge University Press.

Lunney Borden, L. (2011). The 'verbification' of mathematics: Using the grammatical structures of Mi'kmaq to support student learning. *For the Learning of Mathematics, 31*(3), 8–13.

Mehan, H. (1979). *Learning lessons: Social organization in the classroom*. Cambridge, MA: Harvard University Press.

Molland, G. (1976). Shifting the foundations: Descartes's transformation of ancient geometry. *Historia Mathematica, 3*(1), 21–49.

Morgan, C. (2006). What does social semiotics have to offer mathematics education research? *Educational Studies in Mathematics, 61*(1/2), 219–245.

Morgan, C., & Burton, L. (2000). Mathematicians writing. *Journal for Research in Matheamtics Education, 31*(4), 429–453.

Moschkovich, J. (2018). Recommendations for research on language and learning mathematics. In J. Moschkovich, D. Wagner, A. Bose, J. Rodrigues, & M. Schütte (Eds.), *Language and communication in mathematics education: International perspectives* (pp. 37–47, this volume). Dordrecht, NL: Springer.

Nesher, P. (1972). *Transition from natural language to arithmetic language in the primary grades* (Unpublished Ph.D. dissertation). Harvard University, Cambridge, MA.

Nesher, P., & Katriel, T. (1977). A semantic analysis of addition and subtraction word problems in arithmetic. *Educational Studies in Mathematics, 8*(3), 251–269.

Nesher, P., & Teubal, E. (1975). Verbal cues as an interfering factor in verbal problem solving. *Educational Studies in Mathematics, 6*(1), 41–51.

Netz, R. (1999). *The shaping of deduction in Greek mathematics: A study in cognitive history*. Cambridge, UK: Cambridge University Press.

Ng, O. (2015). The interplay between language, gestures, dragging and diagrams in bilingual learners' mathematical communications. *Educational Studies in Mathematics, 91*(3), 307–326.

Nisbett, R., & Wilson, T. (1977). Telling more than we can know: Verbal reports on mental processes. *Psychological Review, 84*(3), 231–260.

Núñez, R. (2004/2006). Do *real* numbers really move? Language, thought, and gesture: The embodied cognitive foundations of mathematics. In R. Hersh (Ed.), *18 unconventional essays on the nature of mathematics* (pp. 160–181). New York, NY: Springer.

Ochs, E. (1979). Transcription as theory. In E. Ochs & B. Schieffelin (Eds.), *Developmental pragmatics* (pp. 41–72). New York, NY: Academic Press.

Phillips, E. (2002). *Classroom explorations of mathematical writing with nine- and ten-year-olds* (Unpublished Ph.D. dissertation). The Open University, Milton Keynes, UK.

Pimm, D. (1987; reissued 2016). *Speaking mathematically: Communication in mathematics classrooms*. London, UK: Routledge & Kegan Paul.

Pimm, D. (1994). Another psychology of mathematics education. In P. Ernest (Ed.), *Constructing mathematical knowledge: Epistemology and mathematics education* (pp. 111–124). London, UK: The Falmer Press.

Pimm, D. (2004). A case of you: Remembering David Fowler. *For the Learning of Mathematics, 24*(2), 16–17.

Pimm, D. (2010). 'The likeness of unlike things': Insight, enlightenment and the metaphoric way. *For the Learning of Mathematics, 30*(1), 20–22.

Pimm, D. (2014a). Unthought knowns. *For the Learning of Mathematics, 34*(3), 15–16.

Pimm, D. (2014b). Authority, explanation, contention and register: Language data and the surface search for essence. *ZDM—The International Journal on Mathematics Education, 46*(6), 967–976.

Pimm, D. (2017). Making a thing of it: Some conceptual commentary. In E. de Freitas, N. Sinclair, & A. Coles (Eds.), *What is a mathematical concept?* (pp. 269–283). Cambridge University Press: Cambridge, UK.

Pimm, D. (2018). Script and subscript. In R. Zazkis & P. Herbst (Eds.), *Scripting approaches in mathematics education: Mathematical dialogues in research and practice* (pp. v–xi). Cham, CH: Springer.

Polanyi, M. (1966/2009). *The tacit dimension*. Chicago, IL: University of Chicago Press.

Richardson, J. (2001). *Vectors: Aphorisms & ten-second essays*. Keene, NY: Ausable Press.

Ryve, A. (2011). Discourse research in mathematics education: A critical evaluation of 108 journal articles. *Journal for Research in Mathematics Education, 42*(2), 167–199.

Sinclair, N. (2017). Crossroad blues. In E. Galindo & J. Newton (Eds.), *Proceedings of the Thirty-Ninth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 100–108). Indianapolis, In: Hoosier Association of Mathematics Teacher Educators.

Sinclair, J., & Coulthard, M. (1975). *Towards an analysis of discourse: The English used by teachers and pupils*. London, UK: Oxford University Press.

Staats, S. (2008). Poetic lines in mathematics discourse: A method from linguistic anthropology. *For the Learning of Mathematics, 28*(2), 26–32.

Staats, S. (2018). The poetics of argumentation: The relevance of conversational repetition for two theories of emergent mathematical reasoning. *Research in Mathematics Education*, *19*(3), 276–292.

Stubbs, M. (1986). Language, meaning and logic: A case study of some children's language. In C. Hoyles, & L. Burton (Eds.), *Proceedings of the 10th PME Conference* (Vol. 2, pp. 59–74). London, UK: University of London Institute of Education.

Stubbs, M. (1996). *Text and corpus analysis: Computer-assisted studies of language and culture*. Oxford, UK: Blackwell.

Stubbs, M. (2001). *Words and phrases: Corpus studies of lexical semantics*. Oxford, UK: Blackwell.

Tannen, D. (2007). *Talking voices: Repetition, dialogue, and imagery in conversational discourse* (2nd ed.). New York, NY: Cambridge University Press.

Thorndike, E. (1922). *The psychology of arithmetic*. New York, NY: Macmillan.

Verschaffel, L., Depaepe, F., & van Dooren, W. (2014). Word problems in mathematics education. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (pp. 641–645). Dordrecht, NL: Springer.

Zazkis, R., & Herbst, P. (Eds.). (2018). *Scripting approaches in mathematics education: Mathematical dialogues in research and practice*. Cham, CH: Springer.

Zwicky, J. (2003). *Wisdom & metaphor*. Kentville, NS: Gaspereau Press.

Zwicky, J. (2010). Mathematical analogy and metaphorical insight. *For the Learning of Mathematics, 30*(1), 9–14.