Chapter 27
A Primer on Multiple Degree of Freedom Vibration Test for Aerospace and Military Applications

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Abstract The aerospace and military communities have performed vibration tests for decades using a single axis or single degree of freedom (SDOF) approach. In recent years, military standards have recognized a multiple exciter or multiple degree of freedom (MDOF) approach for conducting vibration testing. This primer on MDOF vibration testing serves to introduce the topic to the IMAC community and fits with IMAC-XXXVI's theme: Engineering Extremes.

This presentation will review how SDOF and MDOF vibration environmental definitions are obtained. The concept of *cloud plots* will be reviewed and approaches for determining acceptable test levels will be discussed. The common approach in the MIL-STD-810 community for accelerating testing using the Palmgren-Miner Hypothesis will be reviewed.

Next, the presentation will shift from the actual real-world vibration environment to the laboratory environment with special consideration reviewed such as: non-linearity effects, boundary conditions, controllability and observability.

Analogies between the power spectral density for an SDOF test and the spectral density matrix (SDM) for an MDOF test will be reviewed. The author's previous work will be adapted for illustrating how the SDM can take on a truly random nature for MDOF environments.

27.1 Extended Abstract

27.1.1 SDOF and MDOF Environmental Definitions

The raw time domain data measured during a vibration survey where two triaxial accelerometers are used, A1 and A2, at some distance apart can be represented in Eq. (27.1). $\mathrm{DATA}_{\mathrm{SURVEY}}$ represents a 6 × 1 × n matrix.

$$DATA_{SURVEY}(t) = \begin{bmatrix} A1_X(t) \\ A1_Y(t) \\ A1_Z(t) \\ A2_X(t) \\ A2_Y(t) \\ A2_Z(t) \end{bmatrix} \tag{27.1}$$

The Fourier Domain representation of a time domain accelerometer signal at location 1 in direction X is given in Eq. (27.2):

$$A1_X(f) = \mathit{fft}\left(A1_X(t)\right) \tag{27.2}$$

The SDM is then computed by multiplying the acceleration column vector by its conjugate transpose as shown in Eq. (27.3):

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$$
\begin{aligned}
SDM(f) &= \begin{bmatrix} A1_X(f) \\ A1_Y(f) \\ A1_Z(f) \\ A2_X(f) \\ A2_Y(f) \\ A2_Z(f) \end{bmatrix} \left[A1_X{}^*(f) \;\; A1_Y{}^*(f) \;\; A1_Z{}^*(f) \quad A2_X{}^*(f) \;\; A2_Y{}^*(f) \;\; A2_Z{}^*(f) \right] \\
&= \begin{bmatrix}
A1_X{}^2(f) & A1_X(f)A1_Y{}^*(f) & A1_X(f)A1_Z{}^*(f) & A1_X(f)A2_X{}^*(f) & A1_X(f)A2_Y{}^*(f) & A1_X(f)A2_Z{}^*(f) \\
A1_Y(f)A1_X{}^*(f) & A1_Y{}^2(f) & A1_Y(f)A1_Z{}^*(f) & A1_Y(f)A2_X{}^*(f) & A1_Y(f)A2_Y{}^*(f) & A1_Y(f)A2_Z{}^*(f) \\
A1_Z(f)A1_X{}^*(f) & A1_Z(f)A1_Y{}^*(f) & A1_Z{}^2(f) & A1_Z(f)A2_X{}^*(f) & A1_Z(f)A2_Y{}^*(f) & A1_Z(f)A2_Z{}^*(f) \\
A2_X(f)A1_X{}^*(f) & A2_X(f)A1_Y{}^*(f) & A2_X(f)A1_Z{}^*(f) & A2_X{}^2(f) & A2_X(f)A2_Y{}^*(f) & A2_X(f)A2_Z{}^*(f) \\
A2_Y(f)A1_X{}^*(f) & A2_Y(f)A1_Y{}^*(f) & A2_Y(f)A1_Z{}^*(f) & A2_Y(f)A2_X{}^*(f) & A2_Y{}^2(f) & A2_Y(f)A2_Z{}^*(f) \\
A2_Z(f)A1_X{}^*(f) & A2_Z(f)A1_Y{}^*(f) & A2_Z(f)A1_Z{}^*(f) & A2_Z(f)A2_X{}^*(f) & A2_Z(f)A2_Y{}^*(f) & A2_Z{}^2(f)
\end{bmatrix}
\end{aligned}
\tag{27.3}
$$

The notation $A1_{X^*}(f)$ is used to represent the complex conjugate of $A1_X(f)$. When Eq. (27.3) is normalized by multiplying both sides by $1/\Delta f$, the auto-spectrums, which are found along the diagonal, assume the form of power spectral densities (PSD). A recommended sanity check is to compute the area under the spectral density function and compare to the root-mean-square of the original time signal to ensure they are equal.

The SDM will contain off-diagonal terms, which are referred to as the CSD terms. The CSD terms contain the relational information between the two measurements. From a modal analysis perspective, the CSDs can be viewed as the phase and coherence between respective signals. The CSDs found below the diagonal are the complex conjugates of the CSDs found mirrored above the diagonal.

In test specifications, generally one PSD is given to define a single degree of freedom (SDOF) test. This would be analogous to collapsing Eq. (27.3) to any one PSD, while ignoring all the information from other accelerometers. Furthermore, the one PSD given is rarely denoted as the mean value, 2 sigma value, maximum spectra, an envelope or other. The author recommends reporting test specification PSDs with respect to the greater environmental *cloud plot.* These details are essential to understanding the appropriateness of the test level. Traceability back to the environmental *cloud plot* is required, especially when the test level has been or will be accelerated using Palmgren-Miner's Hypothesis. Depending on the device under test and the device's lifecycle, the test level may need to be accelerated differently and without a traceable test specification, this change in test level is not possible. The rescaling equation from MIL-STD-810G Change Notice 1 requires this information.

27.1.2 Considerations for Laboratory Testing

Vibration tests conducted in a laboratory are rarely straight forward for the engineer or technician conducting the test and operating the vibration control system. This is especially true when test items are modally rich and/or large. Four common reasons why testing may be difficult to conduct are: non-linear effects, boundary conditions, controllability, and observability. Each of these should be carefully considered in test planning and setup.

Non-linearity can arise in two common ways. The first is when a vibration test is ramping up, for example, from – 9 dB to 0 dB in 3 dB increments; this is a common ramp up sequence. Amplitude non-linearity can arise, during ramp up, causing the transfer function estimate to drastically change. When this occurs, the vibration controller may need longer time at one of the steps to equalize or may not be able to equalize. The second non-linearity can arise from test items or test setups which exhibit frequency non-linearity. This behavior has been observed when excessive energy exists in one frequency band and this energy feeds into other frequency bands. The vibration controller assumes frequency linearity and without a seasoned vibration operator, this non-linearity can be uncorrectable. When this arises, often times the reference profile may need to be adjusted to help the vibration controller obtain better estimates of the transfer function.

Differences between the in-service boundary conditions of a test item and the laboratory boundary conditions can cause the test item to exhibit modal responses in the laboratory, which defer from in-service modal responses. Ideally, operational deflection shapes would be measured for a test item in its in-service condition and then those operation deflection shapes would be matched in the laboratory.

The test laboratory is sometimes limited in driving down vibration levels and is thus limited in controllability of a test within the reasonable tolerance and goodness of a test defined by the test plan or military standards. Two examples which can contribute to the minimum level of vibration are: (1) the dynamic range of the vibration controller and (2) the ambient noise present in the test laboratory.

A common problem when conducting laboratory vibration test is the inability to observe all the mode shapes or operation deflection shapes. This shortcoming is often due to limitations in the quantity of sensors available to survey the vibratory event.

27.1.3 Power Spectral Density and the Spectral Density Matrix

Power spectral densities (PSDs) are to single degree of freedom testing what the spectral density matrices (SDMs) are to multiple degree of freedom testing. As vibration test expand into the multiple degree of freedom domain, the notion of test tolerances for SDMs needs to be understood by the testing laboratories, sponsors, and structural analysts. This understanding will include allowing test tolerances to be random for regions within the SDM.