

Leadership in Providing Personal Life Insurance Services as a Way to Increase the Sustainability of the Insurance Company's Business



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Abstract The particularities of building the models for actuarial services in Russia have been considered for the purposes of optimizing the business of insurance companies, with evaluation and forecast of numerous statistical factors against regional differentiation taken into account. In order to economically justify the insurance company's creation of a reserve fund for each personal life insurance contract, in the paper, an economic and mathematical model of calculation of the insurance contract cost is suggested that implies payment of insurance to a surviving spouse in the case of the other's death before the spouse's retirement age. The expression for the lump-sum net rate for such contracts has been obtained that depends on the interest rate, age of spouses, their remaining time to retirement, mortality rates, and maximum permissible ages. The calculations performed allow determining the rate of reduced redemption insurance sums, which enables insurance companies to adjust the insurance installments as soon as terms and conditions of life insurance contracts are amended. The suggested technique of calculation of insurance rates ensures the sustainable development of the insurance company in the market of insurance services, enhances its competitiveness, and provides for its achieving the leading positions in the segment of individual contracts of personal insurance. In the work, the actuarial insurance rate calculation technique is applied, using the methods of the theory of probabilities and mathematical statistics, as well as simulation modeling methods.

1 Introduction

Currently, the social and economic policy of the Government of the Russian Federation is aimed at accelerating the development of the personal voluntary and retirement insurance of citizens. With regard to this, there arises the problem of a correct calculation of the insurance rates that enable an insurance company to form

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the relevant reserves of the insurance liabilities. The insurance companies need a mathematically grounded cost of the equivalent payment for the case of an insured event occurring.

Voluntary personal life insurance and retirement insurance are an integral part of life in all the developed countries. This is associated with a highly developed culture of insurance in the countries and the people realizing its necessity. In Russia, there has not been much demand for this kind of insurance so far, with the market of such insurance products being still in the formation stage. Given the demographic and economic situation (the introduction of sanction and anti-sanction mechanism), numerous problems arise that touch on the majority of the population of the Russian Federation. Pension support of citizens is especially problem-haunted. Longer life expectancy and reorientation of values (in particular, having children at a later age due to women's opting for building the career) result in higher load on the employable working population. So for further economic development of Russia, new tools of ensuring the decent life conditions have to be made popular. Voluntary personal life insurance and retirement insurance promote solving the said "pension problem," which in turn helps reducing the load on the state, enhances the level of conscience of the population, and helps ensure a decent level of life after retirement. The object of the research is the Russian market of long-term life insurance in the section of the individual personal insurance.

2 Literature Review

The role of mathematical evaluation methods in various branches of the economy has been increasing from year to year. So, some authors focus their research on the mathematical background of the theory of life insurance and pension schemes (Barrows and Tsyganov 2016), on using the environmental audit procedure in the system of environmental insurance of agricultural enterprises in the polluted areas (Sukhorukova and Shved 2014) and methodological framework of housing insurance against environmental and technology-related risks (Sukhorukova and Serdyukov 2015), on economic optimization model for the case of centralized procurement management in subsidiaries of a state corporation (Sukhorukova and Likhachev 2016), and on the level of the information and communication technology development and accessibility for the public (Minashkin 2014). Currently, actuarial calculations are a part of the mathematical theory of insurance, and they are used not only for evaluating the rates but also for justifying the company's insurance reserves, franchise amounts, and liability limits and evaluating the financial stability of the insurance portfolio and address and some other problems (Gantenbein and Mata 2008; Boyle et al. 1998; Olivieri and Pitacco 2011; Portal "Actuaries: problems, information, events" 2017). The insurance operations feature the principle of equivalence that is expressed in the equality of financial liabilities of the assurator and the insurant (Kaas et al. 2001). Using the actuarial calculations, the

share of participation of each assurator in creation of the insurance fund is determined as well as the amount of tariff rates.

In order to determine the sufficient level of the insurance fund, the assurator has to have the information about how many objects will or will not suffer from the insured event (Zolotarev 2016). Based on the statistical data, the amounts of the expected payments can be calculated. For instance, having the information about the mortality of population, one can calculate the probability of survival and death for people of various ages. Proceeding from these data, mortality tables are drawn up that show any change in the dynamics of the number of people in certain age categories. Using the population mortality tables, the net rates for personal life and retirement insurance are calculated for certain age categories of people. Taking into account the long-term nature of these investments, the tariff rates are initially reduced by the size of income obtained as loan interest on the assurator's funds that are used as credit resources.

The actuarial calculations are also used for economic justification of the insurance company creating a reserve fund for each personal life insurance contract. Moreover, actuarial calculations allow determining the amount of paid off reduced insurance sums, which allows recalculating the insurance premiums when terms and conditions of life insurance contracts are amended.

Actuarial calculations are based on studying the financial schemes with the stochastic nature of the insured events considered. The need of actuarial calculations is due to the equivalent payment calculated by the precise mathematical methods in the case of the insured event occurrence. In actuarial calculations, the relevant probabilities and statistical models are created that are used in certain calculations for insurance contracts. Alongside with the event-related constituent, an actuarial contract is a financial tool where the input cash funds are invested for obtaining the income.

In Russia, building of models for actuarial support is quite a difficult problem. Vast statistical data are required, such as the level and structure of wages and salaries; demographic parameters (the working people number, that of pensioners of various groups), preferably with the regional differentiation taken into account; the future return on investments; and so on. However, in evaluating the parameters, the main problem is the lack of the existing statistical and forecast base.

The official Russian publications on statistics currently have no such data with the regional structure considered. The ways for solving the problem have been detailed by the authors in their work (Chistyakova and Sukhorukova 2017; Sukhorukova and Chistyakova 2017).

3 Results and Discussion

Personal life insurance is one of the main ways for long-term savings, so it occupies one of the leading places in the market of insurance services. In most European countries, the insurance companies rendering the services of life insurance are

exempt from other insurance service types at the legislative level, in order to enhance the sustainability and efficiency of their business.

In Russia, retirement insurance has its specific features (Burak 2015; Falin 1996). They are first of all associated with the age at which one will receive the guaranteed insurance payments. In the Russian Federation, the retirement age is fixed, and as of now it is 60 years for men and 55 years for women for all retirement insurance contracts. However, it should be emphasized that a net rate under the insurance contract will be markedly lower for a man than for a woman of the same age. The insurance rate will be different because on average men live 7 years after retirement and women live 20 years. Another important particularity of Russia's retirement insurance is the differentiated distribution of mortality for each separate subject of the Russian Federation. The calculated insurance tariff rates for two men of the same age category but residing in different regions of Russia will be different. This is associated with the fact that for each RF subject, its own mortality tables are drawn up that reflect the mortality features in this particular region. However, in spite of this quite a significant circumstance, almost all insurance companies mainly use the mortality rates for Russia in general when calculating the tariff rates for retirement insurance. So the insureds coming from other subjects of the RF get initially incorrectly calculated rates, due to which the principle of equivalent liabilities of the parties is not observed. The insurer shortly receives the insurance premium, which results in a higher probability of ruin for the company, or the insured pays an excessive price for the certificate, due to which financial losses are incurred by the insured.

Another important aspect of the retirement insurance problem is to identify the required technical interest rate which is used in calculating the commutation functions. In Russia, the leading insurance companies resolve this problem in one of the two ways:

1. They use the refinancing rate of the Central Bank of the Russian Federation as the technical interest rate.
2. They adopt the rate applied by Russia's Pension Fund as the minimum rate of accumulation of citizens' retirement savings in the Russian Federation as the technical interest rate.

In the Russian Federation, the activity of actuaries is governed by Federal Law of November 2, 2013 No. 293-FL "On the actuarial activity in the Russian Federation," by the international treaties ratified by the Russian Federation, as well as other federal laws and regulations of the Russian Federation pertaining to it.

The principles of calculation of the tariff rates for risk-loaded types of insurance that are recommended by the Federal Supervision Service for Insurance Activity of the Russian Federation are based on the economic and mathematical models developed by the authors.

In the work, the task of voluntary personal life insurance is considered, implying that both spouses are insured yet only one of them is the beneficiary. Namely, if one of the spouses fails to live up to retirement, the surviving spouse is paid the insurance support specified in the contract (it is conventionally equal to one). If both spouses

live up to their retirement, the insurance company does not pay them anything. Thus, this is a life insurance contract but one having a more intricate structure. In the work by Chistyakova and Sukhorukova (2017), within the given conditions, probabilities were found for the events of the insurance benefit to be paid and for the certain spouse to receive it. In this work, the cost of the contract will be found, i.e., the mathematical expectation of the assurator's expenses taken as of the point of signing of the contract. It is adopted that throughout the term of the contract, interests are accrued on the insurant's funds lodged as the payment for the contract, according to the complex interest rate of $i\%$ per annum.

The following necessary designations will be introduced for solving the set task. Let the point of the insurance contract being signed be considered as zero. Let the vector of ages (x, y) of the wife and the husband, respectively, as of the contract making point be considered and the time intervals until their retirement be designated as (T_1, T_2) accordingly. As life expectancy is an event value, alongside with the determined characteristics introduced, the event values of $\tau_1(x)$ and $\tau_2(y)$ are going to be considered – the remaining time of life of the wife aged x and of the husband aged y , respectively. Let it also be supposed that the following are given as initial data: vectors (x, y) , (T_1, T_2) , maximum possible age values (ω_1, ω_2) of the wife and the husband, accordingly, as well as the mortality rates of the wife μ_x and the husband $\tilde{\mu}_y$ that depend on their current age $x \in (0, \omega_1)$, $y \in (0, \omega_2)$.

It should be reminded that in the personal insurance theory, the mortality rate is called the function of $\mu_x = \lim_{\Delta t \rightarrow 0+} \frac{1}{\Delta t} P(\tau(x) < \Delta t | \tau(x) > 0)$, $0 < x < \omega$, where $\tau(x)$ is the remaining lifetime of a person aged x . Next, the conditional distribution density of a person's remaining lifetime, if the person has achieved the age of x ,

has the appearance of $f_{\tau(x)}(t) = {}_t p_x \cdot \mu_{x+t}$, $0 < t < \omega - x$, where ${}_t p_x = e^{-\int_0^t \mu_{x+u} du}$ is the probability to live up to the age of $x + t$ for a person aged x [12].

Let the probabilities of the wife and the husband living up to the age of $x + t$ and $y + t$, respectively, be designated as ${}_t p_x$, ${}_t \tilde{p}_y$ and their remaining lifetimes be considered as independent. Then, the joint density function of the vector of remaining lifetimes $(\tau_1(x), \tau_2(y))$ has the appearance of $f_{(\tau_1(x), \tau_2(y))}(t, s) = {}_t p_x \cdot \mu_{x+t} \cdot {}_s \tilde{p}_y \cdot \tilde{\mu}_{y+s}$, $0 < t < \omega_1 - x$, $0 < s < \omega_2 - y$ (Katsnelson et al. 1995; Zhmurko et al. 1997). So, for the insured event to take place, it is necessary and sufficient that one of two disjoint events occur: either the wife died before retirement and the husband has survived, i.e., $\{\tau_1(x) < T_1 \cap \tau_2(y) > \tau_1(x)\}$, or the husband died before retirement and the wife has survived, that is, $\{\tau_2(y) < T_2 \cap \tau_1(x) > \tau_2(y)\}$. In the first case, payment is due at the point of $\tau_1(x)$, in the second – at the point of $\tau_2(y)$. If the present value factor is designated by $\nu = \frac{1}{1+i}$, it is obtained that the up-to-date cost A of such a contract with the conventional value of payment equaling one is an event value that can be expressed as follows:

$$A = A(\tau_1(x), \tau_2(y)) = \begin{cases} \nu^{\tau_1(x)} & \text{under } \{\tau_1(x) < T_1 \cap \tau_2(y) > \tau_1(x)\} \\ \nu^{\tau_2(y)} & \text{under } \{\tau_2(y) < T_2 \cap \tau_1(x) > \tau_2(y)\} \\ 0 & \text{in all other cases} \end{cases}$$

Then, according to the principle of equivalence of liabilities of the assuror and the insurant, the lump-sum net rate for such a contract equals mathematical expectation A . This is how it is obtained.

$$\begin{aligned} MA &= \iint_{R^2} A(t, s) f_{(\tau_1(x), \tau_2(y))}(t, s) \\ &= \int_0^{T_1} \nu^t \cdot {}_t p_x \cdot \mu_{x+t} dt \int_t^{\omega_2 - y} s \tilde{p}_y \cdot \tilde{\mu}_{y+s} ds + \int_0^{T_2} \nu^s \cdot s \tilde{p}_y \cdot \tilde{\mu}_{y+s} ds \int_s^{\omega_1 - x} {}_t p_x \cdot \mu_{x+t} dt \end{aligned} \quad (1)$$

Now, for consistency, an example is going to be viewed with the following initial data. Let the vector of ages of the wife and the husband, respectively, as of the contract making point be $(x, y) = (52, 56)$ and the maximum possible values of the wife’s and the husband’s respective ages be $(\omega_1, \omega_2) = (90, 85)$. Therefore, under the retirement legislation of Russia, the time intervals up to their retirement are $(T_1, T_2) = (3, 4)$, accordingly. Alongside with this, in order to illustrate the calculations in a simple way, the de Moivre mortality model [2] is going to be considered below, according to which

$$\begin{aligned} \mu_x &= \frac{1}{90 - x}, x \in (0, 90), \quad \tilde{\mu}_y = \frac{1}{85 - y}, y \in (0, 85). \quad \text{Then [1]} \\ {}_t p_x &= e^{-\int_0^t \mu_{x+u} du} = e^{-\int_0^t \frac{1}{90-x-u} du} = \frac{90 - x - t}{90 - x}, \quad 0 < t < 90 - x, \\ f_{\tau_1(x)}(t) &= {}_t p_x \cdot \mu_{x+t} = \frac{1}{90 - x}, \quad 0 < t < 90 - x, \end{aligned}$$

i.e., the distribution density of the wife’s remaining time to live has a uniform distribution within $(0, 90 - x)$. Similarly,

$$\begin{aligned} s \tilde{p}_y &= \frac{85 - y - s}{85 - y}, \quad 0 < s < 85 - y, \quad f_{\tau_2(x)}(s) = s \tilde{p}_y \cdot \tilde{\mu}_{y+s} \\ &= \frac{1}{85 - y}, \quad 0 < s < 85 - y. \end{aligned}$$

Now, the following is obtained from (1):

$$\begin{aligned}
 MA &= \int_0^3 \nu^t \frac{1}{38} dt \int_t^{29} \frac{1}{29} ds + \int_0^4 \nu^s \cdot \frac{1}{29} ds \int_s^{38} \frac{1}{38} \\
 dt &= \left[\frac{1}{38 \cdot 29} \int_0^3 \nu^t (29 - t) dt + \frac{1}{38 \cdot 29} \int_0^4 \nu^s \cdot (38 - s) ds \right]
 \end{aligned}$$

As the two obtained integrals are of the same time, the first one of them is taken.

$$\begin{aligned}
 \int_0^3 \nu^t \frac{1}{38} dt \int_t^{29} \frac{1}{29} ds &= \frac{1}{38 \cdot 29} \int_0^3 \nu^t (29 - t) \\
 dt &= \frac{1}{38 \cdot 29} \left(29 \frac{\nu^t}{\ln \nu} \Big|_0^3 - t \frac{\nu^t}{\ln \nu} \Big|_0^3 + \int_0^3 \frac{\nu^t}{\ln \nu} dt \right) \\
 &= \frac{1}{38 \cdot 29} \left(29 \frac{\nu^3 - 1}{\ln \nu} - \frac{3\nu^3}{\ln \nu} + \frac{\nu^3 - 1}{\ln^2 \nu} \right)
 \end{aligned}$$

Then

$$MA = \frac{1}{38 \cdot 29} \left(29 \frac{\nu^3 - 1}{\ln \nu} - \frac{3\nu^3}{\ln \nu} + \frac{\nu^3 - 1}{\ln^2 \nu} \right) + \frac{1}{38 \cdot 29} \left(38 \frac{\nu^4 - 1}{\ln \nu} - \frac{4\nu^4}{\ln \nu} + \frac{\nu^4 - 1}{\ln^2 \nu} \right)$$

The lump-sum net rate values for the relevant annual interest rate values are given below.

It should be remembered that under insurance support amounting to units, the lump-sum net premium is calculated by multiplying by the net rate.

Other analytical models of survival can also be viewed. For some of them, the possibility of integration by analytical methods will remain, while others will require numerical methods, yet this is not a difficulty in principle. If the above functions are replaced with their interpolation and statistical evaluations, numerical integration methods have to be used too.

4 Conclusion

The analytical expression of the rate of insurance payment for the surviving spouse in the case of the other's death before retirement age has been obtained; it depends on the interest rate, age of partners, their remaining time to retirement, mortality rate, and maximum permissible ages (The Human Mortality Database 2010).

Table 1 Table of contractual net rates

Interest rate	Present value factor	Contractual net rate
i	v	MA
0.03	0.970874	0.195075
0.04	0.961538	0.191816
0.05	0.952381	0.188661
0.06	0.943396	0.185607
0.07	0.934579	0.182649
0.08	0.925926	0.179783
0.09	0.917431	0.177005
0.1	0.909091	0.174312

Source: the authors

The calculation allows economically justifying the creation of a reserve fund with an insurance company on each personal life insurance contract as well as determining the rate of reduced redemption sums.

The said circumstance enables insurance companies to adjust the insurance installments as soon as terms and conditions of life insurance contracts are amended.

The economic and mathematical model of actuarial calculation of insurance rates worked out allows ensuring the sustainable development of the insurance company business and helps achieve leadership positions in the market of voluntary personal insurance services. After analyzing the results of the self-assessment survey (Table 1), the authors concluded how well the teacher graduates acquired both hard and soft skills that will enable them to become educational leaders and which they are called upon to form in schoolchildren.

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