

Chapter 1

Introduction

Abstract Voting is a common way to resolve disagreements regarding policies to be adopted or candidates to be chosen for various positions and is therefore a necessary ingredient of democratic government. Yet there are numerous voting rules that differ from each other in processing the ballots into voting results. In other words, it is possible that for a given set of voters having a fixed distribution of preferences among the competing alternatives, one would obtain the election of a different alternative as a result of using a different voting rule. We focus on the most obvious desiderata associated with voting procedures, viz., the avoidance of paradoxical outcomes.

Keywords Voting procedures · Berlin versus Bonn vote · Procedure–dependence of voting outcomes · Voting system desiderata

Voting is a common way to resolve disagreements regarding policies to be adopted or candidates to be chosen for various positions. Sometimes the actual balloting is preceded by a lengthy process of negotiation whereby various alternatives are being introduced, defended, opposed and evaluated. Once it is found that no unanimity about the policy or candidate to be chosen prevails in the community, voting is resorted to as the final arbiter of the disagreement. Sometimes the decision to take the vote is constitutionally or otherwise predetermined and no specific decision to resort to voting is needed. Even so, voting is, indeed, a very common way to make collective decisions. In the light of this, it is surprising to find that there are many different procedures that are used to achieve apparently the same goal, viz., to single out the collectively best alternative or candidate, “the will of the people”, as it is sometimes called.

In his *magnum opus* Riker (1982) demonstrated that in general it is not the case that the outcome of the voting—no matter which procedure is in use—would unquestionably be the “correct one”. Indeed, Riker tries to convince us that the notion of the will of the people lacks an unambiguous meaning. The reasons are three–fold:

First, all voting procedures are vulnerable to strategic misrepresentation of opinions in the sense that at least theoretically situations emerge where it is in the voters' interest not to vote according to their true preferences. So, says Riker, the outside observer or ballot return official can never be sure that the ballots submitted reflect the true opinions of the voters. Therefore, no matter how accurately the voters' ballots are aggregated into collective choices, there is no assurance that what has been aggregated is, in fact, the opinions of the people.

Second, when one must select one out of three or more alternatives the will of the majority of the people may turn out to be cyclical (intransitive): a majority of the people may prefer alternative A to B, a majority (composed of different people) may prefer alternative B to C, and a majority (composed of another set of voters) may prefer alternative C to A. In this case—which is known in the literature as *the Paradox of Voting*—the will of the majority of the people is unclear and the act of voting may be considered as meaningless.

Third, voting procedures aim at decisiveness no matter how the opinions are distributed among voters and yet there are opinion distributions (voting situations) where seemingly plausible voting systems result in different winners. Riker discusses several such situations, but let us illustrate this with the following purely fictitious example involving the election of the US president from the following set of candidates: Bloomberg, Bush, Clinton, Sanders, Trump. Let us assume the following distribution (profile) of opinions among 90 million voters (cf. Table 1.1 below).

The table indicates that 40 million voters order the candidates so that Trump is their favorite, followed by Bush, thereafter Sanders, then Bloomberg and finally Clinton, or slightly more formally, $Trump \succ Bush \succ Sanders \succ Bloomberg \succ Clinton$. The other opinions are indicated in the same manner. Now, suppose that only the opinions have been given, but the voting procedure is yet to be determined. Suppose, furthermore, that the one–person one–vote, or Plurality Voting, is being used. Then it is reasonable to assume that Trump gets 40 million, Clinton 30 million and Bloomberg 20 million votes, whereupon Trump wins.

Suppose that the constitution requires that whoever wins has to be supported by more than 50% of the electorate and that if this requirement is not satisfied by any candidate in terms of the one–person one–vote principle, there will be a second round of voting where only the two candidates with the largest number of votes can participate. Since the two largest vote–getters on the first count are Trump and Clinton, neither of whom gets more than 45 million votes, the 20 million voters whose favorite is Bloomberg now determine the winner (since we can assume that the Clinton and Trump supporters will vote for their favorite also on the second

Table 1.1 A fictitious preference profile over five candidates

| 40 million | 30 million | 20 million |
|------------|------------|------------|
| Trump | Clinton | Bloomberg |
| Bush | Bloomberg | Sanders |
| Sanders | Bush | Bush |
| Bloomberg | Sanders | Clinton |
| Clinton | Trump | Trump |

round). They prefer Clinton to Trump. Hence the former gets 50 million votes on the second round and emerges as the winner.

Looking at Table 1.1 from the angle of pairwise comparison of candidates, we could conduct all 10 ($5 \times 4/2$) comparisons involving different pairs of candidates and tally the number of victories of each candidate assuming that the winner of each comparison is the candidate that receives more votes than its contestant. The tally reveals that there is a candidate that defeats by a majority of votes every other candidate in such comparisons. This candidate is Bloomberg. All others suffer at least one defeat in those comparisons. In the theory of voting a candidate that defeats all others in pairwise comparisons is called the *Condorcet Winner*. As will be seen in the following there are several voting procedures that end up with a Condorcet Winner whenever there is one in the voting profile. These procedures are called *Condorcet extensions* or *Condorcet-consistent* procedures. In the example of Table 1.1 all Condorcet extension voting procedures elect Bloomberg.

Suppose that each candidate is given a number of points by each voter in accordance with the rank that the candidate occupies in the voter's preferences so that the last (lowest) ranked candidate receives 0 points, the penultimate candidate 1 point, the third lowest 2 points, and so on. Suppose, moreover, that the procedure elects the candidate with the largest sum of points given by each voter. In Table 1.1 this procedure, which is known as the Borda count, would yield Bush the winner (with 220 million points).

All candidates except Sanders have now been rendered winners by varying the procedure, while keeping the voter opinions fixed. With an additional *ad hoc* assumption we can make also Sanders the winner by using the Approval Voting procedure whereby each voter may vote for as many candidates as he/she wishes with the restriction that each candidate can be given either 0 or 1 vote. Making the *ad hoc* assumption that all voters in the left-most voter group of 40 million vote for their three highest ranked candidates, while all the other voters vote for their two highest ranked candidates, Sanders emerges as the Approval Voting winner in this example (with 60 million votes).

So, each candidate may become the winner in this fictitious profile. Admittedly this is a highly special profile and no suggestion is here made regarding its likelihood in practice. The point of the example, however, is to illustrate the oft-cited claim that voting procedures make a difference. In Table 1.1 the difference is, indeed, maximal.

The procedure-dependence of voting outcomes occasionally makes headlines in practical politics as well, although extreme cases akin to Table 1.1 have not been reported. The standard example is the discussion that followed the 2000 presidential election in the United States where the elected president received less popular votes than the runner-up candidate. The same kind of occurrence took place again in the 2016 US presidential election and has happened several times earlier in the electoral history of the United States.

A less known but very important case of demonstrating procedure-dependence is the parliamentary vote taken on 20 June 1991 concerning the location of the central governmental institutions—parliament and the highest level of the executive

branch—in Germany after the unification. The outcome, i.e., the re–location of both institutional bodies from Bonn to Berlin, was the result of a relatively complicated agenda of voting and arguably another outcome might well have resulted had a different and less complicated route been followed (see Leininger 1993; Nurmi 2002, pp. 68–71). The Berlin–Bonn example remains somewhat conjectural because, despite Leininger’s scrupulous analysis, we do not have complete information about the preference profile of all members of the Bundestag.¹ Yet, it seems quite likely that had the Plurality Voting procedure or Borda count been applied on all suggested decision alternatives, the outcome would have been Bonn as the site of both institutions, while the Condorcet Winner was Berlin, i.e., most systems based on pairwise majority comparisons would have elected Berlin. So, positional and binary voting outcomes would have been different. In practice neither of these procedures as such was followed but the outcome resulted from a mixture of binary systems complicated by the fact that some decision alternatives were withdrawn in the middle of the balloting sequence.

The fact that we have relatively few fully documented instances of downright discrepancy between voting outcomes in a fixed profile of opinions, is due to the paucity of information concerning voter preferences. In parliaments which are typical forums of voting, the full preference rankings of the parliamentarians are not reported. Instead, one has to infer them from the records on pairwise comparisons. The same is true *a fortiori* about voter preferences in general elections.

That different procedures may result in different outcomes motivates the research on the properties of various voting systems. Here the social choice theory has provided the major tools for analysis by suggesting desirable properties (*desiderata*) that a good voting system should always possess and by devising methods for analyzing preference profiles. Basically the theoretical literature on voting systems can be divided into two main bodies:

1. Research into the compatibility and incompatibility of various *desiderata*, i.e., into whether the satisfaction of one *desideratum* makes it possible to satisfy another *desideratum* under all profiles.
2. Research into whether various voting procedures always satisfy a given *desideratum*.

This book belongs to the latter genre by focusing on the most obvious *desiderata* associated with voting procedures, viz., the avoidance of paradoxical outcomes. The study of voting paradoxes is almost as old as the systematic study of voting procedures and many suggested voting procedures can be seen as attempts to avoid specific types of paradoxes. With the present work we aim at reminding social choice theorists, political scientists, as well as commentators, policymakers and interested laymen of the main social–choice properties by which voting procedures for the election of one out of two or more candidates ought to be assessed, and to

¹The agenda of voting was devised by the Bundesrat, the upper chamber of the German federal parliament, while the voters were members of the lower chamber, the Bundestag.

list, exemplify and explain the (in)vulnerability to various paradoxes of these voting procedures.

Thus this book should be regarded as an updated review by which to assess from a social-choice perspective the main properties of 18 known deterministic voting procedures for the election of a single candidate. As far as we know only three of the procedures (Plurality Voting, Plurality with Runoff, Alternative Vote) are actually used in general public elections, but many of the remaining procedures are used by various public and private organizations when selecting one out of several candidates or alternatives.

The book is organized as follows: In Chap. 2 we survey 13 paradoxes, several of which may afflict any of the 18 voting procedures that are described in Chap. 3. Chapter 4 deals with four voting procedures based on non-ranked voter input and determines their vulnerability or invulnerability to the various voting paradoxes. Chapter 5 assesses the performance of six ranked voting procedures that are not Condorcet extensions in terms of their vulnerability or invulnerability to paradoxes. Chapter 6 turns to eight ranked Condorcet extension procedures providing similar discussion on their performance with respect to the paradoxes. Chapter 7 summarizes and discusses the significance of the results for overall evaluation of voting procedures.

Exercises

Problem 1.1 You are looking for a new bike and, on the basis of extensive study of relevant journals, three models stand out: Bike 1, Bike 2 and Bike 3. There are three bike stores in your town: A, B and C. None of them has all these bike models. A has Bike 1 and Bike 2, B has Bike 2 and Bike 3 and C has Bike 1 and Bike 3. Suppose that you would prefer Bike 1 in store A, Bike 2 in store B and Bike 3 in store C. Does your choice behavior exhibit transitivity of underlying preferences? If it does, write down the ranking. If it doesn't, which changes are needed to make it transitive?

Problem 1.2 Use now three criteria assumed to be of equal importance to you: price, weight, outlook. Suppose that in comparing any two bike models, your preference is determined by the respective ranking of these two models on a majority of criteria. Can you form a set of rankings over the three bike models with respect to the three criteria so that the resulting preference ranking is intransitive?

Problem 1.3 Construct a 3-voter, 3-alternative Condorcet Paradox. Switch the ranking of any two adjacent alternatives in one ranking. Analyze the ensuing profile: is there still an intransitive majority preference relation? Is there a Condorcet Winner?

Answers to Exercises

Problem 1.1 The answer to the first question: No, it doesn't.

The answer to the second question: Choose Bike 1 in store C.

Problem 1.2 If the preference orderings for bikes (from top to bottom) in terms of the three criteria are as shown in the table below then the resulting ordering is intransitive.

| Price | Weight | Outlook |
|--------|--------|---------|
| Bike 1 | Bike 2 | Bike 3 |
| Bike 2 | Bike 3 | Bike 1 |
| Bike 3 | Bike 1 | Bike 2 |

Problem 1.3 Here's a profile constituting a Condorcet Paradox in which the social preference ordering is intransitive ($a \succ b \succ c \succ a$):

| No. of voters | Preference ordering |
|---------------|---------------------|
| 1 | $a \succ b \succ c$ |
| 1 | $b \succ c \succ a$ |
| 1 | $c \succ a \succ b$ |

Now switch a and b in the first ranking to get:

| No. of voters | Preference ordering |
|---------------|---------------------|
| 1 | $b \succ a \succ c$ |
| 1 | $b \succ c \succ a$ |
| 1 | $c \succ a \succ b$ |

Here the social preference ordering is transitive ($b \succ c \succ a$), i.e., b is the Condorcet Winner.

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