



Democratization of Mathematics Through Cremona's Correspondence with Foreign Colleagues (1860–1901)

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Abstract

From the mid-nineteenth to the early twentieth century, many common traits were shared by national mathematical communities, which were not only separated geographically (from the Czech lands to Japan), and culturally (from northern to southern Europe), but which also varied from the point of view of the dynamism of original research (from Germany to the United States). Societies and journals were launched in the national languages, thanks to the widening of the social platform of mathematics and the emergence of a national leadership; the development of state school systems increased mathematical knowledge; and furthermore, mathematics played a role in and received encouragement from the processes of social and economic modernization and the evolution of state institutions. Intellectual competition among nations, very much a part of the spirit of the nineteenth century, seems to prevail over early Modern European universalism. A panorama of almost planetary dissemination of Western mathematics resulted from this evolution, leading eventually to a reinforcement of the international circulation of knowledge, which survived two world wars.

The collection of letters written to Luigi Cremona conserved at the Sapienza University of Rome casts light on several aspects of this evolution. The letters offer a “backstage” point of view, in contrast with official proclamations; they show the interplay between national leaders and the mathematical circles in the capitals as well as mathematicians working in isolation; moreover, they show a variety of connected activities—research, institutional commitments, and the fostering of culture, including translations and textbooks. International dialogue grew out of this hive of initiatives, driven by both national passion and philosophical and political convictions, in contrast with the present European trend of entrusting the

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circulation of ideas—and the production of knowledge—to initiatives governed from the top, standardized (design, funding and assessment) far beyond what is needed. The edition (in the Académie Internationale d’Histoire des Sciences series “De diversis artibus”) has been carried out by a European team directed by Giorgio Israel.

A collection of 1122 letters written to Luigi Cremona (1830–1903) from foreign colleagues is conserved at the Sapienza University of Rome Mathematics Department Library, in Italy. 1860–1901 represents 40 years of European scientific exchanges among mathematicians, a period in which a mathematical profession emerged and in which science, and technology, showed their potential in the creation of a modern society.

The collection offers a nice sample of the mathematicians in the second half of the nineteenth century, including their political feelings, mathematical interests (especially in the area of geometry) and cultural aims.

Several points deserve attention: who was Cremona? Who were his correspondents? What are the contents of the letters and the overall meaning of this archival source in regard to nineteenth century mathematics and science? An edition of this collection is forthcoming in the series of studies of the International Academy of History of Science “De diversis artibus,” so the contents will be available for further study. In this paper, we will discuss some aspects of the edition (two volumes, almost 2000 pp., see Israel 2017, CLC from hereon). Moreover, we would like to emphasize how these letters, aside from their mathematical contents, offer a point of view on the “backstage” of an evolution that can be considered to be a process of democratization of mathematics as a human enterprise, as a part and in the context of the evolution of nineteenth century liberal democracy.

An Italian PRIN national project in 2011 provided the financial support for the edition; in fact, in 2011, Italy commemorated the 150th anniversary of the unity of the country, and Cremona played an important role in the institutional building of the modern Italian state.¹

1 Luigi Cremona, the Geometer, the Senator

The first two letters of the collection, in chronological order, were written by Carl Wilhelm Borchardt (1817–1880), editor of Crelle’s Journal, and by Cremona’s peer, Alfred Clebsch (1833–1871); both date back to 1860, an important year in the life of Cremona and in the history of Italy. Cremona, who had graduated in 1853 from the university of his native town Pavia (in northern Italy, close to Milan) as a civil engineer and architect, after several years giving private lessons and working as

¹A first overall presentation of this edition was discussed by the first two authors with the title “Luigi Cremona’s network of foreign correspondents (1860–1901): a testimony to the evolution of the “Europe of science” in the late nineteenth century” at the *International Conference Mathematical Schools and National Identity (sixteenth to twentieth cent.)*, Turin, October 10–12, 2013.

a high school mathematics teacher, was called, in 1860, to a new chair of higher geometry at the University of Bologna (Brigaglia and Di Sieno 2011b). Bologna had just been liberated from the power of the Pope, a new step in the evolution that led to the unity of Italy: in fact, after a few months, in March 1861, the Parliament proclaimed Vittorio Emanuele II King of Italy. A new nation had achieved political unity and had impetuously entered nineteenth century Europe and the world stage in order to carve out a top-ranking position in the fields of industry and culture for itself.

In their letters, the two German colleagues show their strong appreciation for Cremona's early mathematical contributions to the field of geometry. Borchardt's letter, written in April, was still addressed to Cremona in Milan, as a teacher at the city's high school, Liceo Sant'Alessandro (from 1865 until now, Liceo Cesare Beccaria); he emphasized that Cremona was turning to pure (synthetic) geometry:

comme votre nom ne m'est point inconnu après les mémoires que vous avez fait insérer dans le recueil italien et dans lesquels vous avez montré tant d'habileté dans les questions de géométrie analytique je ne doute pas que ce nouveau mémoire qui semble prendre plutôt la voie de la géométrie pure ne formera un heureux enrichissement du Journal allemand [Borchardt to Cremona, Berlin, April 2, 1860].

As for Clebsch, his first letter (Fig. 1) started an intense exchange that lasted twelve years, until Clebsch's death in 1872:

Erlauben Sie Ihnen zunächst meinen herzlichsten Dank für die Uebersendung Ihrer ausgezeichneten Abhandlungen auszusprechen. Zugleich bin ich so frei, die beiden Abhandlungen beizulegen welche Sie die Güte hatten zu wünschen. Ich hoffe Ihnen bald Anderes über diese algebraischen Probleme zusenden zu können, welche nicht mit Unrecht die Mathematiker unserer Zeit so vielfach beschäftigen, und erlaube mir, Ihnen auch für die Zukunft einen Austausch unserer Arbeiten vorzuschlagen [Clebsch to Cremona, Karlsruhe, August 27, 1860].

Moreover, Cremona's intellectual and political-cultural figure as a whole marks the contents and meaning of the letters in the collection.² Cremona was a member of a generation of Italian scholars who shared a vision in which mathematics played a principal role in secular culture, because culture and education were intertwined with freedom and nationality; a generation of mathematicians with a radical attachment to national culture and progress. He had been a volunteer in the "Free Italy" battalion during the First Italian War of Independence (1848–1849), when he was only 18 years old. His introductory lecture to the higher geometry course in Bologna, held in November 1860, was an impassioned speech that reflects the strong feelings of a country, a large part of which had just been freed from the yoke of the foreigner (the "Austrian jailer," he said) and from the temporal power of the Church (the "livid Jesuit," in Cremona's words). In the face of oppression and obscurantism—this was Cremona's point of view—the new Italy offered reason and free thought, of which science was the model.

²For a biographical profile of Luigi Cremona, see Israel (2016) and the bibliography therein.

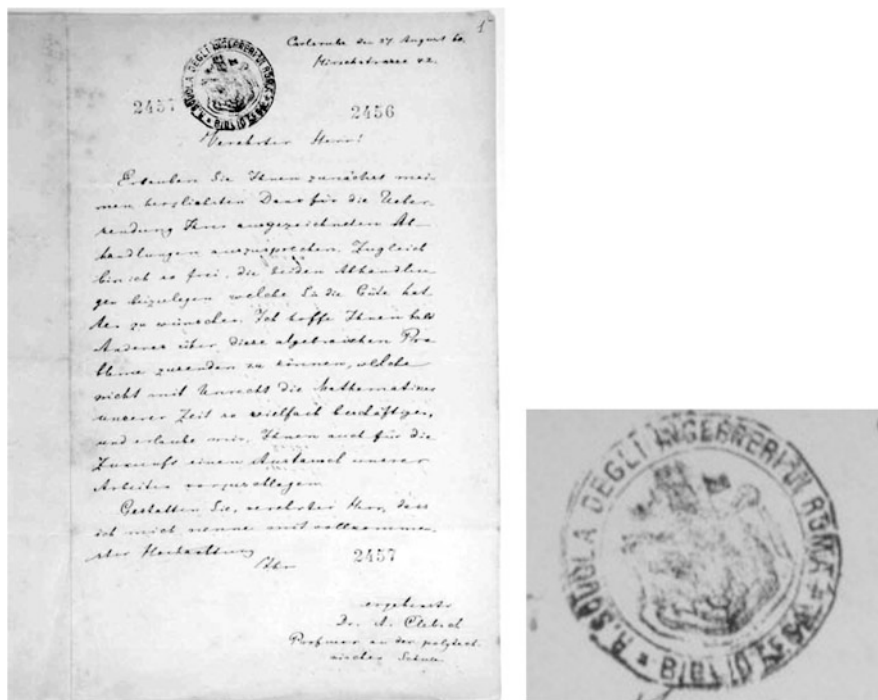


Fig. 1 Letter from Alfred Clebsch to Cremona, Carlsruhe, August 27, 1860. At the top, the stamp of the Library of the Royal School of Engineers of Rome and a catalogue number can be seen; the letters in the collection were stamped and numbered after Cremona's death, when his papers were acquired by the school library. Twenty-eight letters from Clebsch and two letters from his wife Minna Clebsch were found. The exchange and discussion of Clebsch's analytical methods and Cremona's synthetic methods is a *fil rouge* in the correspondence

The Bologna speech was very well received by his French colleagues. Eugene Prouhet (1817–1867), wrote, in May 1861, that Cremona's appeal to the patriotism of young people had moved him, as well as Terquem, Bonnet, Serret and Mannheim, and remembered at the same time the “common work shared by all the civilized nations” regarding modern sciences.³ Olry Terquem appears to have been well aware of Cremona's involvement in the tension in Italian politics between moderate, monarchist positions and radical republican ideas: Terquem wrote to Cremona three months before his death in 1862, when he was eighty years old.⁴ Political issues emerge in many letters, often entangled with tricky rivalries between nations and,

³Prouhet to Cremona, Paris, May 29th 1861; for further details, see Millán Gasca (2011, pp. 52 ff). Letters from Amédée Mannheim (1831–1906) can be found in the Genova Cremona Archive (see note 8).

⁴Terquem to Cremona, Paris, March 11, 1862. On Cremona's political evolution, see Brigaglia and Di Sieno (2009, 2010, 2017); see also Rossi 1984.

of course, even wars. Maximilian Curtze (1837–1903), a Gymnasium teacher in Thorn (now Torun, in Poland) and his main German translator, wrote, in February 1878, about the new German edition of Cremona's Introduction to a geometrical theory of plane curves (*Introduzione ad una teoria geometrica delle curve piane*, 1861), expressing his solidarity regarding King Vittorio Emanuele's death, and this is but one example of this kind of comment in letters from German correspondents (Knobloch 2013). As to the penetration of the pride of national identity into European intellectual and social life, François Furet wrote:

... none of the 19th century wars—in any case few in number—presented the fearful nature of those of the 20th century. Even in Germany, where it displayed most intensely the blindness and the perils involved in it, the national idea remains incorporated into that of culture. It does not propose as sufficient per se its pure substance, the particular election of the Germans, their superiority as human beings. It strongly enhances Germany's contribution to ethics, the arts, philosophy, culture (Furet 1995, p. 45).

Several letters come from places in Europe that have since changed their position relative to national borders, such as Torun, or Bromberg (now Bydgoszcz), where Rudolf Sturm (1841–1919) lived until 1872, or Breslau, now Wrocław, where Heinrich Schröter lived (1829–1892, born in Königsberg); many letters arrived from towns in the Austro-Hungarian or the Russian Empires, in areas such as Poland, Hungary and Bohemia, that were to become independent European countries—for example, Prague, where Cremona's research was translated into Czech.

Cremona's political and cultural evolution from his ardent mathematical and political youth to his moderate maturity was marked by the effort to develop mathematical and scientific institutions and culture in Italy. After 6 years in Bologna, starting in October 1867, he was professor of higher geometry at the Milan Royal Higher Technical Institute, directed by his mentor Francesco Brioschi (1824–1897), where he also taught graphical calculus and graphical statics, following the example of the Zurich ETH. In late 1873, he accepted an assignment to reorganize the old Pontifical School for Engineers so as to set up the third Italian polytechnic school in Rome (Fig. 2). Thus, he was appointed professor of graphical statics and Director of the Royal School of Engineers of Rome, and from that day—even though he received the Steiner prize for the second time in 1874—his institutional commitments made it difficult to carry through his scientific projects. Thus, his correspondence with Arthur Cayley (1821–1895), strictly regarding geometrical issues and characterized by a tone that is quite formal, stopped in 1872.

The new School of Engineers, where the letters comprising this collection were deposited and whose stamp they bear (see Fig. 1), was set up in the Roman convent of San Pietro in Vincoli, where the Faculty of Engineering of the “La Sapienza” University of Rome still has its premises.

In 1879, Cremona was nominated senator. His institutional work was intense in the field of educational reform, both in the schools and the university⁵; and

⁵He was offered the ministry of education twice: in 1881, by Quintino Sella, a request he turned down, and in 1898, this time accepting, although he only remained in office for the month of



Fig. 2 The venue of the Royal School for Engineers of Rome: the convent close to the church of San Pietro in Vincoli (with a Chiostro dating back to the sixteenth century); the library was set up in 1876 in the area of the former monks' refectory (Ippoliti 2012)

thus many of the letters include questions addressed to Cremona on educational organization, even on technical aspects of buildings.⁶

The ethos of research and national institutional commitment are both characteristics of Cremona's life and work; but to these should also be added his great attention to the European (mathematical) scene, as demonstrated by the numerous academies and foreign societies that selected him as one of their members, starting in 1867 with the Academy of Sciences of Lisbon⁷; and, of course, his correspondence with foreign colleagues is also noteworthy.

2 Cremona's Papers in Genoa and Rome

There are two main Italian archives containing Cremona's papers: the Rome Cremona archive at the Rome Sapienza University Department of Mathematics Library (Israel and Nurzia 1983) and the Genoa Cremona archive at the Istituto Mazziniano (Brigaglia and Di Sieno 2011a). Both of these are outstanding sources

June, owing to the political crisis within the government. In 1880, the minister Francesco De Sanctis appointed him government commissioner for the reorganization of the "Vittorio Emanuele" National Library in Rome housed in the Collegio Romano.

⁶For example, letters from Sturm initially addressed geometrical issues, but then turned towards institutional aspects; letters from German correspondents were edited in the CLC by Eberhard Knobloch and Karin Reich.

⁷Academy of Sciences of Lisbon (1867), Mathematical Society of London (1871), Society of Sciences of Bohemia (1872), Danish Academy of Sciences (1876), Cambridge Philosophical Society (1877), the Academy of Science of Munich (1878), the Royal Society of London (1879), the Society of Sciences of Göttingen (1880), the Dutch Academy of Sciences (1881), the Mathematical Society of Prague (1881), the Royal Society of Edinburgh (1883), the Prussian Academy of Berlin (1886), the Physico-Medical Society of Erlangen (1896), the Irish Academy of Dublin (1898), the Academy of Belgium (1899), the Institut de France (1899), the Swedish Academy (1901), and finally, the American Academy of Washington (1902).

on Cremona's scientific biography in the context of the evolution of science in Italy after the Risorgimento. The Rome collection came to its present location from the Library of the Rome School of Engineers, from which Cremona's library was acquired in 1909; the letters were transferred to the Sapienza University Mathematical Institute after it moved to its new premises (now Piazzale Aldo Moro) in 1935. Cremona's papers conserved in the Genoa Istituto Mazziniano—more than 6000 documents donated by Cremona's daughter Itala, probably in 1939—consist mainly of correspondences with Italian scientists and politicians or state officials, as well as correspondences with 34 foreign mathematicians.⁸

As Aldo Brigaglia and Simonetta Di Sieno have written, the Genoa archive is especially useful as a source regarding the history of science in Italy:

It was also during his time in Bologna that Cremona became acquainted with a large number of Italian and European mathematicians. The Archive of the IMG [Mazzini Institute of Genoa] contains many new documents about the early stages of these contacts and their subsequent development. Cremona's correspondence with the Italian mathematicians (e.g., Eugenio Beltrami, Enrico Betti, Francesco Brioschi, Felice Casorati, Placido Tardy . . .) is of particular importance, not only because of the clear description of Italian academic life and its problems, but also because of the discussions concerning two of the main organizational problems of the Italian scientific world: the problem of the development of the main Italian mathematical journal (the *Annali*) and the didactical problems relating to the programs and content of mathematical learning and teaching. The Archive of the IMG contains a large number of letters that shed light on these historical questions. Another important issue, strictly linked to the didactical problems (mainly at the university level), is the training of a new ruling group in Italy, a group no longer composed primarily of lawyers with a humanistic education, but one composed of engineers and technicians with sound scientific knowledge. The role played by Brioschi, Cremona, and many of the Italian mathematicians in this respect cannot be overstated (Brigaglia and Di Sieno 2011a, p. 101).

Moreover, letters in the Genoa archive from the period he spent in Milano, above all, the letters exchanged with Eugenio Beltrami (1835–1900), show “Cremona's efforts to keep pace with the rapidly changing face of modern mathematics. In

⁸In Genoa there are letters from 22 non-Italian correspondents included in the Sapienza Cremona Archive: Arthur Cayley (4 letters); Eugène Dewulf (2 letters); Lewis Carroll–Charles Dodgson (1 letter); James Glaisher (4 letters); Charles Hermite (1 letter); Thomas Archer Hirst (86 letters); Felix Klein (4 letters); Leopold Kronecker (1 letter); Ernst Eduard Kummer (1 letter); Sophus Lie (1 letter); Max Noether (1 letter); Emile Picard (1 letter); Eugène Prouhet (1 letter); Theodor Reye (1 letter); George Salmon (3 letters); Ludwig Schläfli (1 letter); Kyparissos Stephanos (1 letter); Rudolf Sturm (4 letters); James Sylvester (1 letter); Peter Tait (5 letters); Emil Weyr (1 letter); and Hieronymus Georg Zeuthen (1 letter). There are also letters from the following additional 12 foreign correspondents: James Booth (1806–1878) (4 letters); Maurice D'Ocagne (1862–1938) (1 letter); Morgan Jenkins (1 letter); Seligmann Kantor (1857–1902) (8 letters); Jacob Lüroth (1844–1910) (7 letters); Gösta Mittag Leffler (1846–1927) (2 letters); Amédée Mannheim (1831–1906) (55 letters); Henri Poincaré (1854–1912) (3 letters); Henry Smith (1826–1883) (9 letters); William Spottiswoode (1825–1883) (16 letters); J. Vanecek (7 letters); and Gustav Wolff (1834–1913) (20 letters, 1883). Data from Brigaglia and Di Sieno (2011a); the study of this collection is ongoing (see website www.luigi-cremona.it).

particular, he tried hard to fully understand Riemann's theory and to translate it into a more geometric language" (Ibid).

Other papers are conserved in Italy, for example, the correspondence with Domenico Chelini in the Rome Archive of the Piarist Order (*Archivio Generale delle Scuole Pie*). Some of this material has already been published.⁹ Further research will lead to the discovery of letters written by Cremona to his correspondents in various archives, mainly in Europe. Thus, the complete publication of Cremona's correspondence was not and is not a pursuable aim. Letters written to Hirst were published in Nurzia 1999; letters written to Darboux and Klein have been included in the CLC edition discussed in our paper. But apart from the few exceptions mentioned, the CLC contains only the letters sent to Cremona.¹⁰

3 The Letters in the Cremona Rome Archive and Their Authors

The bulk of Cremona's international correspondence was found in 1982 at the Mathematical Institute of Sapienza, the University of Rome, inside 28 envelopes, during a search organized by one of the authors of this paper, Giorgio Israel. Let us recall how he narrated this finding:

In November 1982, I was associate professor at the Istituto Matematico "Guido Castelnuovo" of Rome University. With the conviction that the Institute, which already had a rich library, probably contained other documents of historical interest, I planned a search of the building. The places to be explored consisted of the library and a local storeroom containing the duplicates of books, papers to be disposed of and many other kinds of objects, including broken furniture and an old bicycle used by the janitor in the 1950s. The search was carried out with the help of Laura Nurzia, then researcher at the same Institute. After a few days exploring this room, among the jumble of material, on the floor in one corner, under a pile of documents, I found twenty-eight envelopes containing letters that, at first view, had obviously been sent to the Italian mathematician Luigi Cremona. The many correspondents, more than 170, included the names of the most eminent 19th century European mathematicians (Israel 2016, p. 17).

The envelopes also contained two important archival documents. The first one was an autograph by Gauss, a small but accurate sheet of paper donated to Cremona by Alfred Enneper (1830–1885) in 1881, which Enneper had received in 1852 while attending a lesson by Gauss on the method of least squares. The second one was a group of four sheets of paper handwritten by Jean Victor Poncelet; on the envelope,

⁹See Carbone et al. (2001, 2002), Cerroni (2014), Cerroni and Fenaroli (2007), Enea and Gatto (2009) and Palladino et al. (2009). All of the publications are included on the web site www.luigi-cremona.it

¹⁰In fact, the overall research into Cremona's letters to every correspondent was so immense that it soon became obvious that the project would inevitably remain incomplete and that failure to acknowledge this fact would postpone the conclusion of the book indefinitely.

Cremona wrote that it was donated to him by Poncelet's widow¹¹ when he visited her on May 4th, 1884. Three drafts of mathematical notes by Cremona were also found, attached to the letters sent by Max Noether.

The envelopes contained letters from 176 mathematicians addressed to Cremona (among them, only three Italian colleagues),¹² and from representatives of three scientific societies (the British Association for the Advancement of Science, the Göttingen Königl. Gesellsch. der Wissensch., and the Société Mathématique de France).¹³ The *size* of the single correspondences varies. Fifty colleagues sent only one letter to Cremona (one of them a postcard). The longest correspondence was with his French translator Eugène Dewulf (1831–1896); the heftiest correspondences were those with other translators, with Thomas Hirst, Cayley, and George Salmon, and with many German-speaking correspondents: Clebsch, Sturm, Elwin Bruno Christoffel, Wilhelm Fiedler, Johann Nicolaus Bischoff, and Theodor Reye, as well as Carl Friedrich Geiser, Ludwig Schläfli, and Emil Weyr. In addition, a few letters from four correspondents were found that were not addressed to Cremona.¹⁴

The letters were sent mainly from European *places*, in Germany, France, and Great Britain. There are also letters from towns in the United States and Canada, and from British scholars in Calcutta (James B. Chalmers) and Adelaide (Horace Lamb (1849–1934), who, in 1885, returned to Manchester). The letters are written in seven different *languages*: German, French, English, Italian, Spanish, Portuguese, and Latin. In several cases, letters from a single correspondent are written in more than one language: for example, the first letter from Emil Weyr is written in French, followed by three letters in Italian, and the rest of the correspondence is written in German (Bečvář and Bečvářová 2006). A surprising inter-European linguistic facility emerges from the correspondences, if compared with current scientific exchanges among university scholars who use a single language—English—while only a few of them are able to read languages other than English and their own. Seven correspondences in the collections mainly regard the translation of mathematical works. First, the correspondence with Richard Baltzer (1818–1898) regards Cremona's Italian translation of Baltzer's second edition of *Elemente der Mathematik* (first published in Leipzig in two volumes in 1860 and 1862). Secondly, several research essays and three textbooks written by Cremona in 1872–1874 were translated into German, French, English, and Czech (see Millán Gasca 2011) by six different translators whose letters are included in the collection. The distinct

¹¹Louise Palmyre Gaudin (1813–1889).

¹²Ettore Caporali (2 letters), Valentino Cerruti (1 letter), and Carlo Saviotti (6 letters, referring to Louis Bossut's French translation of his 1872 essay *Le figure reciproche nella statica grafica*).

¹³See Table 1.

¹⁴Four letters from Martin Krause (1851–1920) to Eugenio Beltrami (written in 1898–1899); a letter from Rudolf Clausius (1822–1888) and 5 letters from a certain Heinrich Schramm to Francesco Brioschi dating back to the years 1867–1869; and three letters from Édouard Combesure (1824–1889) to an unidentified member of the editorial office of the journal *Annali di matematica*, written in 1871–1872.

impression that arises is one of intellectual richness: a widespread facility of exchange, together with great attention to the national language.

The scholars who wrote to Cremona belonged to different *generations*: the oldest, like the already mentioned Terquem, Borchartd, Prouhet and Baltzer, all of whom were born before 1820; his peers, born around 1830, like Clebsch, Fiedler and Dewulf; and younger scholars like Emil Weyr from Prague (born in 1848, a graduate student in Rome in 1870–1871), the Greek Kyparissos Stephanos (born in 1857, a graduate student writing from Paris in 1881), the English Carslaw (born in 1870, a graduate student in Rome in the late 1890s) and the American Julian Coolidge (born in 1873, a student in Hessen in the same period). Some of these mathematicians were at the forefront of research, some have not made many original contributions to mathematics, and among the latter were the scholars who took charge of translating Cremona's books, as well as scholars who were committed to developing mathematical culture in their own country (like Zoel García de Galdeano (1846–1924) from Zaragoza, in Spain).

Thus, the style of the letters (formal, friendly, deferential, and so on . . .) depends on the kind of correspondent; in each case, they wrote to Cremona with the confidence that they would be shown consideration, interest and cordiality. In correspondences spanning longer periods of time, a respectful, polite tone often evolves into a more informal or familiar one, as Eberhard Knobloch (2013) has underlined. There are letters dealing with problems regarding academia, especially problems of priority or lack of acknowledgement of results published or privately communicated, such as Dewulf versus Mannheim in a letter dated March 18, 1886. Others are light-hearted and even jocular. For example, the 22-year-old Coolidge wrote to Cremona about his difficulties in finding and buying his *Introduzione a una teoria matematica delle curve piane*:

Unfortunately, my knowledge of Italian is to be reckoned among the imaginary quantities, so I must have the work in an English, French or German translation [Coolidge to Cremona, Hessen, August 28, 1896].

But perhaps Cremona did not appreciate this very informal tone, if one were to judge from the second, more humble letter (in French):

Je vous prie de m'excuser si je n'ai pas très bien exprimé ce que j'ai voulu dire. Je suis depuis trois mois en Allemagne, et ainsi je le trouve plus difficile qu'ordinairement d'écrire bien le Français [Coolidge to Cremona, Hessen, September 27, 1896].

The few correspondents who were not mathematicians deserve some brief comment. Special mention should be made of Jean-Albert Gauthier Villars (1828–1898), the well-known French mathematics and science publisher, who was educated at the École Polytechnique and graduated as a telegraph engineer (Paul 1985). His letters were found together with the 85 letters from Dewulf, who translated Cremona's textbook on projective geometry; many comments on his industrial venture, both from the economic point of view—including competition with other European editors—and as a cultural project are included in his letters to both Cremona and

Dewulf. For example, in exchanges regarding the second edition of the above-mentioned textbook, he wrote:

La Géométrie est absolument délaissée en France ; elle n'est plus représentée à l'Académie et n'a pas une seule chaire où on la professe. J'ai pensé qu'un des meilleurs moyens de raviver, dans la limite du possible, le goût de cette Science, était de réimprimer l'ouvrage d'un maître, comme celui de M. Cremona.

Si j'agissais come un Editeur, uniquement préoccupé du côté industriel, je réimprimerais d'abord des ouvrages épuisés, s'adressant à un nombreux public, comme mes traductions de Tyndall. Mais, forcé de faire un choix, à cause de l'encombrement de mon Imprimerie, j'ai préféré la Géométrie de M. Cremona, qu'est appelée de rendre plus des services.

Je cherche toujours, dans la limite de mes moyens, publier des traductions pouvant développer certains courants d'études dans notre pauvre pays, qui ne lit rien de ce qui se fait à l'Etranger et qui a si grand besoin d'être tenu au courant des productions nouvelles. C'est ainsi qu'au lieu de publier des ouvrages à succès, j'imprime en ce moment les Quaternions de Tait, parce qu'on ne veut pas jusqu'à ce jour introduire ces nouveaux symboles dans notre enseignement ; c'est ainsi que je prépare des traductions d'ouvrages sur l'Electricité (Maxwell, Jenkins, Kempe etc.) parce que la dernière Exposition a montré notre infériorité dans cette branche de la Physique [Gauthier-Villars to Dewulf, December 27, 1882].

The Scottish industrialist Walter Macfarlane (1817–1885), an ironwork manufacturer, was also a correspondent: Cremona met him during his first visit to the UK when he attended the 1876 Glasgow meeting of the British Association for the Advancement of Science. There are no letters from women mathematicians in the archive, but only from relatives, such as Clebsch's second wife Minna Rays Clebsch, after the early death of her husband, and Clara, Karl Weierstrass' sister, who was in contact with Cremona during a stay in Italy in 1874. Nevertheless, the letters mention Cremona's and other mathematicians' support for Sofya Kovalevskaya's interest (as Charles Hermite put it, this meant defending "the interest of Science"); and Zeuthen recommended a pupil of his to Cremona, Miss Ellie (see Millán Gasca 2011, pp. 62–63).

We offer now a description of the distribution of the letters, which we obtained after the completion of the work. In Fig. 3, a chart of the chronological evolution over the years 1860–1903 is shown, elaborated from the chronological index included in the edition.

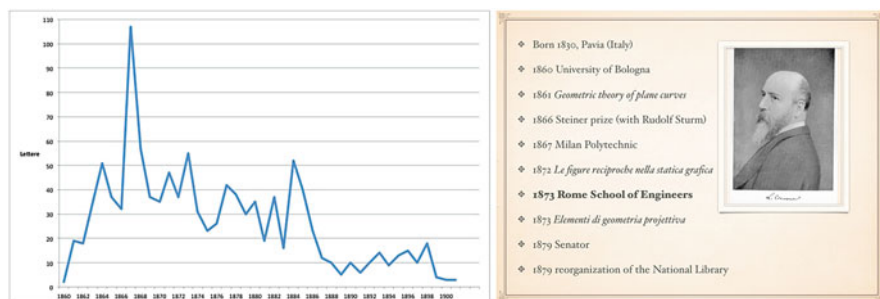


Fig. 3 The intensity of the exchange with foreign colleagues, as reflected by the number of letters in Cremona's Archive at the Dipartimento di Matematica, Sapienza University of Rome

Table 1 The geographical/national distribution of Cremona's correspondents outside Italy, as represented in Cremona's Archive at the Dipartimento di Matematica, Sapienza University of Rome

Nation	No.	Names of correspondents											
Germany	69	Bauernfeind	Becker	Bischoff	Boehm	Borchardt	Bremiker	Brill	Christoffel	Clausius~			
		Baltzer	du Bois-Reymond	Enneper	Fuchs	Gordan	Grosman	Grunert	Gugler	Gundelinger			
		Clebsch	Harnack	Helmholz	Hess	Hesse	Klein	Koenigsberger	Krause~	Krimphoff			
		Günther	Lampe	Lipschitz	Maser	Matthiessen	Mayer	Meyer	Neumann	Noether			
		Kronecker	Pflicker	Prüm	Reye	Rosanes	Röthig	Schell	Schering	Schilling			
		Ohrtmann	Schmitz-Dumont	Schoenflies	Schröter	Schwarz	Siebeck	Stammer	Stern	Sturm			
		Schlegel	Vahlen	von Lindemann	Weierstrass	Weissenborn	Wiedemann	Wöhler (K. Gesell.)	Zimmermann				
		Thomae	Timerding										
		France	36	Abakanowicz#	Autonne	Bossut	Boucher	Brisse	Chasles	Combescur~	Darboux	de Ficquelmont	
				de Jonquieres	de Longchamps	de Marsilly	Debacq	Dewulf	Fouret	Gauthier- Villars	Gerono	Grouard	
				Halphen	Hermite	Hotel	Jordan	Lemoine	Levy	Lucas	Painvin	Picard	
				Picquet	Prouhet	Salte1	Terquem	We11 (S. M. de France)					
				Beare	Chalmers	Chrystal	Cotterill	Dogson (Carroll)	Esson	Glaisher	Henrici		
Hirst	Leudesdorf			Macfarlane	Miller	Price	Sylvester	Tait	Thomson	Tucker			
Ireland	7	Crompton, Carpenter and Clarke (BAAS)											
		Anglin	Haughton	Malet	Roberts	Salmon	Townsend						
Belgium	4	Catalan	Folie	Le Paige									

Denmark	2	Petersen	Zeuthen										
Finland	1	Lindelöf											
Greece	2	Stephanos	Vitalis										
The Netherlands	2	Baehr	Schoute										
Norway	2	Bjerknes	Lie										
Portugal	2	Albuquerque	de Ponte Horta										
Romania	1	Urechia											
Russia	3	Boguslawsky	de Khanikof	Vassilief									
Spain	3	G. de Galdeano	de Moy	Vázquez Illá									
Switzerland	5	Affolter	Culmann#	Fiedler#							Schläfli		
Austria*	2	Härtenberger	Schramm~										
Czech Territories*	4	Durege	Houdek	Weyr Ed							Weyr Ern		
Hungary*	1	Schmidt F											
Poland*	1	Czarnowski											
Australia	1	Carlaw#											
Canada	1	Young											
United States	5	Coolidge	Dickson	Eddy							Elliott	Schwatt#	

National areas (Czech territories, Hungary, Poland) with national languages that were to become independent countries are marked with an asterisk. For mathematicians writing on behalf of a scientific society, the name of the society is included. Mathematicians who are authors of letters not addressed to Cremona are included with a ~. The symbol # indicates mathematicians working abroad.

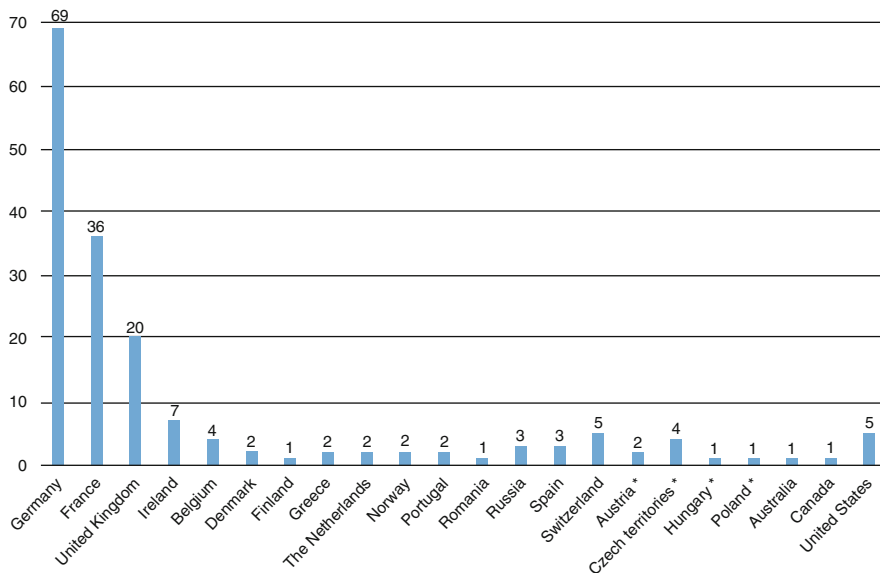


Fig. 4 Numbers of correspondents in European countries or areas, Australia, Canada, and the United States. It should be taken into account that subjects from the Austro-Hungarian and Russian Empires lived in areas such as Poland, Hungary or Bohemia, which were to become independent European countries

In Table 1, the correspondents are listed by country or European national area (Czech territories, Hungary, Poland, Romania, with national languages that would eventually become independent countries). The country where each mathematician mainly developed his professional activity has been chosen: thus, Isaac Joachim Schwatt (1863–1934) was born in Latvia, but emigrated to the United States; the Briton Horatio Scott Carslaw (1870–1954) had an important role in Australian mathematics [or in “taking mathematics to Ultima Thule,” as Michael Deakin has put it (see Deakin 1997)]; the Polish-Lithuanian Bruno Abdank-Abakanowicz (1852–1900) was in exile in Paris from 1881; Heinrich Durege (1821–1893), born in Danzig (now Gdansk), worked in Prague from 1869 on; the German Karl Culmann (1821–1881) and Fiedler were important figures in Zurich.

Figure 4 offers a chart comparing the numbers of correspondents in each area, showing the prevalence of Germany, and the presence of 1–3 correspondents in many areas with a developing mathematical community.

4 The Editing

Two main aspects had to be considered when planning the editing of this material: (a) the organization of the letters and (b) the critical apparatus.

The numbering of the letters marked with the stamp of the Royal School of Engineers goes from number 1742 to 2882, with some of the letters bearing the same number and others not being numbered at all. The first 1741 letters are missing, so a large part of the collection of Cremona's letters has been lost, perhaps becoming hopelessly disintegrated[?] in the storeroom over the years. Owing to its faulty and incomplete nature, this numbering was of no use, and was thus disregarded.

As to the organization of the edition, a first possibility was to publish the letters in chronological order. This criterion would perhaps have been useful if the main focus of the archival source had been either Cremona's scientific biography or the evolution of nineteenth century geometry. In fact, the collection offers insights into the evolution of nineteenth century geometry, because it contains a dialogue among a group of distinguished geometers.¹⁵

The second possibility was to order the letters by correspondent. In fact, the letters had been found divided into envelopes according to correspondent, so this organization was based on archival considerations; partial publications of letters in several booklets, starting in 1992, had already followed this mode of organization.¹⁶ Moreover, the contents of many of the single correspondences led us to consider the correspondents as individual scholars within their own national context, in parallel with the activity of Cremona himself. Besides their mathematical content, the letters also address political aspects and show that the participation of science in the process of modernisation was experienced by mathematicians all over Europe as a patriotic commitment (Millán Gasca 2011). The general editor decided—in agreement with the director of the series—to publish the letters in alphabetical order according to correspondent, and to include, at the end of the volume, a chronological index. A two-volume book has been produced. It is introduced by a foreword and an essay on Luigi Cremona by G. Israel, with a bibliography of Cremona's works edited by G. Israel and L. Regoliosi.

The critical apparatus was intended to be neutral, but capable of directing the reading; the main goal was to offer the letters for further studies regarding various historical problems. Each chapter is equipped with: (a) a short biographical note accompanied by biobibliographic references; and (b) a short introduction regarding the main topics and the meaning of the correspondence. The letters, published in the original language, are annotated, with cross-references to Cremona's works, which are listed in the bibliography, as well as full references to the books and papers mentioned in single letters and other helpful information. The critical apparatus in English has been translated or revised by Ian McGilvray.

A team of nearly 20 researchers from 6 European countries (Czech Republic, Germany, Greece, Italy, Portugal and Spain) have contributed to the editing of the collection: Martina Bečvářová, Aldo Brigaglia, Luca Dell'Aglio, Simonetta Di

¹⁵See Israel (2016).

¹⁶These booklets have been useful for obtaining a better understanding of the general historical meaning of this archival material and establishing the final criteria for the edition (Millán Gasca 1992a, b; Menghini 1994, 1996; Nurzia 1999).

Sieno, Paola Gario, Livia Giacardi, Angelo Guerraggio, Eberhard Knobloch, Marta Menghini, Ana Millán Gasca, Mara Monaldi, Pietro Nastasi, Efthymios Nicolaidis, Luigi Regoliosi, Karin Reich, Enrico Rogora, Lu s Ribero Sara va, Paola Testi Saltini, and Claudia Umani. One or two scholars in the group are the editors of each single correspondence.

The second volume includes an index of names and a chronological index.¹⁷ The names cited number more than one thousand. It was not always possible to identify them and obtain their date of birth and death; but the information available on the Internet was very helpful,¹⁸ if we compare it with the resources available in the 1990s. In some cases, identification was impossible, as they were minor figures of which all memory has been lost; in all cases in which only the family name was known, it was omitted from the index of names. In any case, it cannot be ruled out that more thorough research will lead to further information.

5 A European Network of Scientists

The letters encourage us to consider the set of correspondents as a *European network*, an evidence of the “European space,” the Europe of sciences (Blay and Nicolaidis 2001, Goldstein et al. 1996, Pepe 2013). The patriotic commitment returns in letters from every corner of Europe, so it may then seem a paradox to speak about a European network: the fresh desire that was spreading among European mathematicians to develop autonomous mathematical research in their own national languages might well have been detrimental to the universalistic ideal of mathematics and acted as a concrete obstacle to communication. This was not the case: one meaningful example is a letter from Weierstrass to an unknown mathematician, written in 1867, in which “he underlines the willingness of the German mathematicians to continue the fruitful cooperation with their Italian colleagues, just as the political alliance between their two countries had led to good results. The German mathematical achievements, he writes, are better understood and appreciated in Italy than in France or England.”¹⁹

In the preface to the essay on the “Europe of science” as a scientific space (*L’Europe des sciences. La constitution d’un espace scientifique*, 2001), Michel Blay and Efthymios Nicolaidis highlight the interest in approaching “the develop-

¹⁷Edited by G. Israel and L. Regoliosi.

¹⁸For example: at bbf.dipf.de (German Institute for International Educational Research (DIPF), Bibliothek f r Bildungsgeschichte Forschung, Germany), the BBF/DIPF/Archiv, Gutachterstelle des BIL—Personalb gen der Lehrer h herer Schulen Preu ens; at www.culture.gouv.fr/public/mistral/leonore_fr. The database *L onore* (L gion d’honneur), Archives Nationales (France).

¹⁹Knobloch, Reich, in CLC, pp. 1651–1652. This letter, written in Italian and dated March 25, 1867, in Berlin, was found together with two letters from Weierstrass to Cremona dated 1874 (and two letters from Weierstrass’s sister Clara). See Casorati’s letter published in Neuschwander (1978, p. 72 ff). We have already mentioned the letters from Prouhet, showing shared political feelings; also Neuschwander (1986).

ment of scientific knowledge as such along with its relations with the space in which it developed, as well as with the dialogue or conflicts those relations aroused." In fact, they add that, although there is a rich bibliography on the development of European scientific knowledge, the problem of "scientific Europe as an intellectual unit throughout the centuries" is an issue that deserves attention: the interest in that approach stems from the fact that it would be able to offer "[...] a global grasp of the origin and the development of scientific knowledge in its original space, as well as of the influence this knowledge had on the *homogenization of the societies that occupy this space*."

The evolution of the modern mathematical profession came about as a result of the growth in the number and size of national communities: societies and journals in the national languages were launched, thanks to the widening of the social platform of mathematics and the emergence of a national leadership; the deployment of the state school systems increased mathematical information; and mathematics played a role and received encouragement from the processes of social and economic modernization and development of state institutions. We know that many common traits were shared by national mathematical communities that were far apart geographically (from the Czech lands to Japan), culturally (from northern to southern Europe) or in regard to the dynamism of the original research (from Germany to the United States) (Grattan Guinness 1994, pp. 1427 ff.). The letters to Cremona offer a point of view on the backstage developments of this evolution that can explain their common traits: they show the interplay between national leaders and the circles in the capitals and mathematicians working in isolation (even in Germany and France); they show a variety of connected activities—research, institutional commitments, and cultural fostering, including translations and textbooks. International dialogue grew out of this nebula of initiatives, driven by national passion and philosophical and political convictions.

A new kind of communication developed in that period, communication stimulated by competition—typical of economical liberalism—which led the single nations to observe and imitate the successes, or the best practices, as we would say today, of other countries. These contacts driven by competition, combined with the traditional universal spirit of mathematics scholarship, helped to establish a new kind of international contact that contributed to the diffusion of ideas and the homogenization of the European scientific space.

6 Democratization of Mathematics and Science as a Secular Religion in the Nineteenth Century

As we have noted, Cremona was a member of a generation of Italian mathematicians with a profound attachment to the national ideal and the national secular religion, who also shared the view of the key role of science and mathematics in a liberal democracy. Science was viewed not only as a fundamental tool for the development of technology and industry at the national level, but also as a force that could liberate thinking from all dogmatic constraints and from the chains of backwardness.

The Italian model meant that mathematics was a patriotic activity, an element of technological and industrial modernization, but also a democratic activity, a universal element of culture inherited from the Greek world: the mathematical professions (math teachers, engineers, actuaries) were potentially open to everyone and were needed for both modernization and progress in regard to political and economic liberty.²⁰

In the years that have elapsed since the collapse of the Soviet political project and empire, Late Modern Age historians have investigated the creation of modern society in depth, spurred, above all, by the desire to identify the symptoms of its political malaise, the symptoms that could account for the First World War and the catastrophes and massacres caused by the ideologies of fascism and communism during the twentieth century.²¹ One of the greatest experts on the French Revolution, François Furet, with reference to what he calls the “revolutionary passion” that marked nineteenth century European society, wrote that “as the century advances, the Europeans no longer conceive of the political scene but through the death of God, as a pure creation of human will, intended at last to assure the liberty of all and the equality of each of us with the other” (Furet 1995, 44). A decisive contribution to the profound confidence in human will was made by the scientific revolution, which affirmed a human omnipotence that replaced that previously reserved for God in the mediaeval history of Christian Europe (Israel 2001); this substitution actually took place through a long process that has continued down to our times, to the era of biotechnology, far outrunning the intentions and convictions of the fathers of modern science. The Enlightenment disseminated this acquisition of modern science and introduced it into the eighteenth century political philosophy debate, which challenged the religious basis of society and opened the way to the revolution (Cassirer 1931). The prodigious development of science in the nineteenth century continued to feed the slow but gradual departure of Europeans from their traditional spiritual vision of society and its substitution for a materialistic vision of interactions among individuals.

Together with this philosophic contribution, science offered itself as a concrete and increasingly effective tool for the construction of this modern society, through the boost it gave to technological innovation and also thanks to its role in the democratic transformation of education. This philosophic and concrete contribution by science to the new bourgeois society inevitably led to the democratization of science itself. Indeed, the transformation of the network of European universities following the model of Berlin University, inspired by the ideas of the reformer Wilhelm von Humboldt, actually turned an ancient mediaeval European institution into a typically modern one based on the intellectual and teaching freedom of individuals emancipated from political and religious powers, as well as from

²⁰For its influence in Spain, see Millán Gasca (2012).

²¹The break-up of the Soviet Union brought to a close a cycle of development of modernity that began with the French Revolution and had as its guiding principle the development of democracy, enveloped as this was in the tension between universal aspiration and national dimension.

all other utilitarian servitudes (Turner 1971). A university professor was thus transformed into a researcher, where research was considered less as a form of study and the transmission of knowledge and more as an enterprise conducted within an intellectual environment, with the same drive towards originality and innovation as were present in other environments in European society. In this way, as the nineteenth century drew to a close, the scientific researcher became a professional figure who was no less important than the engineer in the industrial and economic development of the nation. Engineers, university professors, secondary school teachers, and later, other figures with a scientific background, such as actuaries and medical doctors: in every country, the number of “scientists” grew as the new bourgeois society advanced and, as is typical of a liberal society, scientists also organized themselves into numerous associations, publishing periodicals, organizing meetings and congresses, and interacting with the other economic and political organizations.

As András Gerö emphasized when examining the case of national sentiment in an area on the “periphery” of Europe, such as Hungary, national identity became a factor of social cohesion that replaced religion, and the traditional factors were gradually rejected to make way for the ideals of 1789 (Gerö 2006, p. 2):

Feudal Europe thought of itself primarily as a community of estates intent on safe-guarding the general value of Christianity. Identity was provided by the divergent legal status, the presence or lack of privilege, the commonality was provided by the religious culture. Therefore, the Middle Ages were the triumphal march not of the vernaculars, but of Latin. The marriage strategies of the ruling houses gave no consideration to the “nationalist principle,” and the same may be said of European aristocracy in general. This kind of universalism was seriously challenged by the Protestant Reformation, for the schism within Western Christendom created almost irreconcilable identities. Yet, the national dimension was far from dominant as yet; it remained without significance relative to the differences in religion. Nevertheless, feudal universalism had suffered its first setback, and it could not be mended or covered up by any religious peace.

The agony of universalism began with a process with roots in the eighteenth century. This was the tendency to contest the priority of the estates and of religion, two processes along parallel lines, although there were differences in pace: namely, the process of secularization and the development of national consciousness.

However, the association, science and the nation could not be too exclusive. Modern science was born as a universal intellectual undertaking. Indeed, the development of modern science followed on from the erudite mediaeval debate (in its themes, in the constant comparisons with the Greek classics) that took place across the frontiers, among educated men—many of whom were members of the clergy—and in the universal language represented by Latin. If anything, in modern science, even greater emphasis was laid on the universal character, since, in addition to the *lingua franca*—still Latin, and later French—the universal language of mathematics had been added. Mathematical universalism had its roots in the assimilation of Euclid's *Elements* and of the Greek mathematical corpus

among European scholars in the sixteenth to seventeenth centuries²² and has as its exemplars the correspondence of Father Marin Mersenne (1588–1648)²³ and that of Leonhard Euler (1707–1783),²⁴ as well the *Acta eruditorum* founded in 1682, in which Euler published his first works addressed to the “community of men of letters” so that they could be “subjected to careful scrutiny.” We have noted above the astonishing linguistic knowledge and flexibility shown by the nineteenth century Cremona and his correspondents. Two letters written in Latin by Hermann Weissenborn (1830–1896) are survivors of the classical universal tradition; and, as Zeuthen wrote, apologizing for the dispatch of a note in Danish, “[. . .] in the most essential part, the table, I use the universal language of Mathematicians” [Zeuthen to Cremona, Copenhagen, August 25, 1866].

The link-up between universalism and nationalism in the political and cultural worldview of nineteenth century mathematicians certainly represented a strong point that contributed to the development of the singular national communities. However, this interaction was only one aspect of the more general process of transformation that the world of mathematics was undergoing as a result of the rise of the modern bourgeois society. Cremona’s letters give us a picture of the world of mathematics in the second half of the nineteenth century, marked as it was by a strong dynamism, which was successful in coping with a harsh political and cultural challenge. Mathematics actually succeeded in passing through the deep cleft opened by 1789—as well as that of Jacobinism—without diminishing the value assigned to it by the preceding European tradition as an essential and universal element of culture inherited from the Greek world and reserved for the European intellectual aristocracy under the rigorous control of the Church. Indeed, the “progressive” sectors, ranging from the moderate liberal positions to those of the socialists, shared the same view of the role of mathematics in a modern society, a fresh and hybrid view stemming from both traditional and modern ideas. In this view, mathematics was to remain the main focus of education, to which every citizen had a right. Moreover, mathematics was to provide the intellectual platform on which to build technological innovation, as well as the future ruling classes required for the development of industrialization and the running of the State. Lastly, mathematics—and this was perhaps the idea most strongly resented by conservative thinkers—would be able to provide useful tools for the rational management of society. This view became widespread during the nineteenth century in all countries, East and West, even those lacking any democratic institutions but engaged in a

²²“The boundless number of editions, translations and reprints that followed each other throughout the sixteenth century bears witness to the circulation at all levels of Euclid’s works, the assimilation of which was to make a substantial contribution to a unitary mathematical culture, and thus to the formation of a universal scientific community.” (Giusti 1993, p. 2). Euclid’s Book V theory of proportions became the universal language of natural philosophy, “almost a metageometry, or better a *mathesis universalis*” (ibid.).

²³Fletcher (1996).

²⁴Euler, L. 1975. *Opera Omnia. Series quarta A, Commmerciumepistolicum*, vol. 1. Basel: Birkhäuser.

process of modernization, and also by virtue of the strong commitment made by professional mathematicians and their combined international efforts.

The role of mathematics in the construction of a modern society, the offspring of a liberal democratic view, is widely accepted and proposed in all latitudes as a pathway to development, with a much greater sense of conviction than the actual political democratization.²⁵ Perhaps even more noteworthy is the fact that, although mathematics was wholly an heir to the European tradition, the “importance” of mathematics was also accepted by those who continued to be inflamed by revolutionary and palingenetic passion, particularly in communist countries. Perhaps the principal explanation is linked to the scientism of Marxism and Marx's personal interest in both mathematics and its applications within economics. A more in-depth analysis of the penetration of mathematics into individual national cultures and, above all, of the political aspects that played such an important role among the men of the nineteenth century up until World War I is a task that essentially still remains to be carried out. As we have just seen, it would provide a deeper insight into issues that continue to be extremely topical today. Throughout the nineteenth century, in the case of several professional groups with a technical-scientific educational background, national passion—and democratic convictions themselves—found an outlet precisely through the establishment of national mathematical communities.

In this phase, the national spirit, according to Cremona's letters, did not run counter to the universal spirit.

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²⁵This is accurately summed up in the declaration of November 11, 1997, of UNESCO support for the IMU's decision to declare 2000 the International Year of Mathematics on the basis of the role of mathematics and its current applications in science, technology, communications and economy; of its ancient roots and universal character; and of the importance of a mathematical education for the development of rational thinking.

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