Chapter 16
Early Childhood Mathematics Education: Reflections and Moving Forward

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Abstract This book brings together creative and insightful current research studies on teaching and learning mathematics in early childhood by scholars from different parts of the world. In this chapter we reflect on this work and discuss the major insights, conclusions, implications and future research directions for early childhood mathematics education. We focus on each of the five key themes of the book: pattern and structure, number sense, embodied action and context, technology and early childhood educators' professional issues and education.

Keywords Early childhood · Mathematics education · Reflections
Implications · Future research

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I. Elia et al. (eds.), *Contemporary Research and Perspectives on Early Childhood Mathematics Education*, ICME-13 Monographs,
https://doi.org/10.1007/978-3-319-73432-3_16

The early childhood period (up to 7 years of age) is probably the most crucial time for developing children's capacities and dispositions for learning. Consequently, high-quality early childhood education is essential in any educational system and research in this field can have a strong educational and social impact. It is well documented that children are capable of, and should engage in mathematics learning in the earliest years of their life (Sarama and Clements 2009), thus mathematics education should be a key component of high-quality early childhood education. These considerations have induced a growing impetus of research in early childhood mathematics internationally over the last years.

The work reported in this book records the advancement in international research and perspectives in early childhood mathematics education based on the work of TSG 1 at ICME-13, aiming to provide further and new insights on how to enhance children's mathematics learning and development and help support and improve pedagogy in early childhood mathematics.

From our perspective the book addresses significant and timely issues in the field of early childhood mathematics education which correspond to five key themes. In particular, the book starts with three chapters focusing on the meaning and importance of developing mathematical pattern and structure and how it can be promoted in early childhood (Theme 1). The next three chapters address the development of young learners' number sense competences in relation to various contributing factors, including picture book reading, the quality of early mathematics classroom practice and a measurement-oriented curriculum (Theme 2). The study of the vital roles of embodied action and mathematics-in-context within early childhood mathematics education, and particularly in geometry learning, is the central theme of the three chapters that follow (Theme 3). In the next three chapters the impact of, and opportunities provided by technology in early mathematics learning and teaching are examined and discussed (Theme 4). The following two chapters consider the critical role of early childhood educators, their roles and aspirations and the importance of professional development, as well as the challenges they encounter in engaging children in mathematics, and in light of new curricula, building effective environments for mathematics learning (Theme 5).

In what follows, a summary of the major insights as well as our conclusions, reflections, implications and questions for further research are discussed, based on the research and related studies reported in the preceding chapters.

16.1 Theme 1 Pattern and Structure

The studies on pattern and structure and patterning abilities were focused on both pre-school (the German kindergarten) and the early years of formal schooling. In the tradition of much research on early assessment of mathematical development these studies included both the assessment of individual's abilities and strategies and the impact of intervention programs situated in authentic classroom settings. An important outcome of the studies highlighting patterning and structural relationships is that the assessment tasks and forms of analysis, and the intervention tasks can be accessed, interpreted and adapted in the regular classroom. In most cases this could be achieved without a research team or specialist researcher conditions.

The detailed sequential account of the development of patterning strategies with pre-schoolers by Lüken (2018, this book) extends the research field by providing a new analysis of the very early stages of patterning concepts (Papic et al. 2011). Although Lüken's study is limited to six cases it does provide insight into the importance of tracking children's learning in explicit ways and providing fine-grained analysis of such learning. The results of the classroom intervention with first graders in the study by Lüken and Kampmann (2018, this book) may describe some limitations but again this is promising evidence that can be traced to the intervention strategies. The assessment and teaching strategies are at least provided with clarity and sufficient detail to be embedded in more effectively controlled studies with larger and more diverse samples of children. The impact on mathematics learning could then be ascertained through further evaluation studies.

Mulligan and Mitchelmore (2018, this book) describe the important interrelationships between structural groupings common to The Pattern and Structure Mathematics Awareness Program (PASMAP) approach: sequences, structured counting, shape and alignment, equal spacing and partitioning. An explicit sequence of pedagogical strategies scaffold these interrelationships with a view to promoting visualization, abstraction and emergent generalization. Further recommendations from this study and also from Lüken's and Lüken and Kampmann's studies might also provide guidance about how teachers can scaffold children's learning to make connections within and between concepts. At international level mathematics education curricula often silo concepts and processes which encourages teachers to disregard interrelationships.

Although there has been increased attention on the influence of teacher mathematical content knowledge and pedagogical knowledge, the research reported in this theme was limited to a focus on the child. What might have provided a broader view of the research context would be complementary analyses of the teachers' knowledge about patterns and structural relationships in mathematics. A structural approach to teaching mathematics may require teachers to first develop an Awareness of Mathematical Pattern and Structure (AMPS) in their own mathematical thinking. In the pre-school setting the inclusion of patterning in children's play and structured activities should not be assumed. The effective implementation of patterning tasks such as those developed by the Lüken studies points the need for teachers to gain insight into their own knowledge of patterning as well as their practice being informed by current research.

16.2 Theme 2 Number Sense

Number and arithmetic operations are arguably of primary importance for children's mathematics learning and development (Torbeyns et al. 2015; Verschaffel et al. 2007). Furthermore, mathematical development in this domain may be one of the most extensively researched fields in mathematics education, especially in the early primary grades and also in the pre-school years (4–6 years of age).

The two chapters related to number development turn attention to a new construct, the Spontaneous Focusing On Numerosity (SFON) showing that although numerical abilities are still in the focus of research concerning number sense, there is also an increasing interest into the dispositions for numerical competence. In the chapter by Rathé et al. (2018) a research review about SFON traces the development of this construct to the Finnish researcher Hannula-Sormunen (see Hannula-Sormunen et al. 2015). SFON refers to children's spontaneous (i.e., self-initiated) focusing of attention on the number of a set of items or incidents and using this numerosity in their actions. An important finding of research on SFON, which was mostly conducted in European and Western countries, is that young children's SFON tendency could be identified as a predictor of later mathematics achievement in primary school.

Rathé et al. report in their chapter that the research findings on the relation between SFON in experimental tasks and everyday situations for kindergarten children are inconsistent and contradictory. Although the chapter by Rathé et al. gives new insights into the relation between SFON in experimental tasks and everyday activities, it should be taken into consideration that some data revealed only partial empirical evidence for a possible association.

The study of Bojorque et al. (2018, this book) also provides new insights into the relation of SFON and children-related or classroom-related factors. In particular, children's early numerical abilities measured by a standardized numerical test in Ecuador is a predictive factor of children's SFON development, whereas the quality of early mathematics education, as measured by a standardized instrument, did not contribute to children's SFON development. Both chapters highlight the difficulty of measuring and analyzing children's mathematics competences in early childhood education in daily-life situations as they are often more spontaneous than in school. This will be an ongoing challenge for research in mathematics education.

The chapter by Cheeseman et al. (2018) shows that measurement could be the focus of research concerning number sense, rather than explicit numerical abilities. Here an approach of the Russian tradition of Davydov (1975), adopted earlier in the Measure Up project in Hawaii (Dougherty and Zilliox 2003), was enacted for a design research project in Victoria, Australia. In this project children started formal schooling at a school with a Reggio Emilia and socio-cultural approach to learning. The children did not start with a typical number-focused curriculum but with a measurement-focused curriculum which includes numbers. The case studies in this chapter revealed that when young children measure, they use numbers and can acquire number competencies. The qualitative analyses also revealed the richness of other learning possibilities in the project.

Here again a methodological challenge is raised about the use of reliable and valid instruments in early childhood education for analyzing educational and psychological aspects of mathematics learning. By using standardized assessment instruments often the children's competencies are only partly surveyed. Therefore especially in early childhood education, where a great amount of learning takes place in informal situations, even the well-explored domain of number sense remains an area for in-depth and explicit empirical research.

In this book only a few new aspects of research concerning number sense are addressed in the three chapters. But it is obvious that the development of number sense is influenced by various factors and connected to many other mathematical domains. Therefore there are chapters in other parts of the book, i.e., the themes of pattern and structure, the role of technology and embodied actions and context, which are connected to the research on number sense. A more holistic view of the contributions of these studies can contribute to enhancing a more coherent body of knowledge in this domain.

16.3 Theme 3 Embodied Action and Context

Each of the three chapters in Theme 3 (Anderson and Anderson 2018; Elia 2018; Thom 2018) offers important insights into the roles that embodied action and context play within early childhood mathematics education, and more specifically in geometry. As Elia's study readily indicates, hand and finger gestures do more than complement the kindergarten children's verbal descriptions, serving as a means by which children develop and communicate geometrical thinking under varied conditions. In turn, Thom's grade 1 children similarly build their understandings, in particular their spatial-geometric reasoning with two-dimensional and three-dimensional, from the hand and body movements they invoke when explaining and conjecturing about "what they see" in a photograph. Likewise, for Anderson and Anderson's families, the parent-child dyadic interactions (non-verbal and verbal) point to children enacting mathematics-in-context, including geometry, space and measurement, prior to formal school. In all three studies, it is evident that these young children's experiences of geometry are multimodal, are embedded within a vast array of activities and settings, are connected with other semiotic resources, and are inherently creative sense making. Likewise, while there is evidence in all three studies that acting-thinking-talking are tightly interconnected for many of these 4–7 year olds, we are also reminded that this is not always the case. For example, a child's embodied measurement was visible in the Water Sprinkler activity and was accompanied by minimal talk (Anderson and Anderson 2018). In Elia's (2018) study the children's 'silent' gestures appear to be interpersonally synchronized with their peers' talk. Both these examples remind us that children engaging with mathematics-in-context or embodied mathematics (acting-thinking) without speaking deserve our closer attention. And yet, as Thom indicated, not only is geometry underrepresented in most early years curriculum, the limited ways in which we invite children to engage with geometrical, spatial and measurement concepts in many classrooms undervalues the embodied, gestural, in-context nature of young children's engagement with mathematics.

What insights, then, might these case studies, both through their design and their findings, provide early childhood educators? First and foremost, in all three studies, young children's geometric and spatial reasoning comes into focus as they share their thinking with others. Indeed, while the contexts and concepts varied across the three studies, each activity provided space, time, and we would argue, motivation for children to overtly share their ideas. In turn, each child in these studies, rather than withdrawing from what some might seem challenging mathematics, is engaged in moment-to-moment in-context action. In this process the individual is thereby making sense of geometry (mathematics) through body and mind. Furthermore, each child enacted the mathematics using gestures and body movement voluntarily, e.g., pointing to shapes and where to place them, or moving them within a composite figure. Other pertinent examples were the children's constructing of an imagined cylindrical shape through hand and arm movements associated with a circle in a photo or positioning one's body—raised knee or bended torso—to measure changing water levels from a sprinkler, without being explicitly asked to do so. On one level, then, since all three studies report on data gathered in natural settings, the ecological validity of the actual activity descriptions suggest that literal translation of these 'research' tasks into early years' settings may prove unproblematic. That said, keeping in mind the nuanced nature of such activities (Anderson and Anderson 2018), what such case study work provides is not necessarily contexts or activities to be emulated but compelling evidence of children's funds of knowledge and ways of knowing (James et al. 1998) that we, as educators, need to leverage as we plan and carry out mathematics-related activities in our pre-school and primary classrooms.

While these authors (Anderson and Anderson 2018; Elia 2018; Thom 2018) outline important directions for further research based on their individual study, when we consider the three chapters together, several research directions are strengthened while other avenues for future research also become apparent. For instance, we see evidence that the adults (e.g., teacher, interviewer, parent) and peers (e.g., same age classmates, older siblings) are implicated in the ways in which individual children's gestures and bodies bring forth the mathematics (geometry). Such convergence adds strength to Elia's call for further research into the "others" role(s) in provoking, sustaining or diminishing children's embodied mathematical experiences. For example, as Thom suggests, how might the nature of the adult's initial prompt (e.g. "what is this?" versus "what do you see?") or the questions that follow support young children's generative responses and/or provide young children opportunities to explore possibilities in ways similar to those illustrated in these studies? Whether due to coincidence or implicated in classrooms where children are often seated when engaging in mathematics, most of the gestures and movement in the three studies were limited to finger, hand and arm movements (upper body). Further research seems warranted to explore what role the learning space and positioning of children plays with respect to their embodied ways of knowing. What occasions promote whole body action, lower body gestures, or gross motor movement when young children are trying to understand geometrical ideas? What might young children's facial and head gestures, as well as gaze, tell us about their mathematical thinking? Although not specified in the three studies, European and Euro-Canadian cultures seem prominent in this field of research. Thus as we continue to explore the affordances of embodied action and context in relation to mathematical teaching and learning in the early years, it is vital that we do so within contexts with children and teachers/parents from diverse cultural, linguistic, socio-economic backgrounds, inclusive and respectful of both indigenous and immigrant populations. Finally, while these three studies and those in the remainder of this book situate their research in early childhood mathematics, most do so with children aged 4–7 years. However, the study of embodied action and mathematics-in-context seems to offer ways of engaging with and learning about children's mathematics that more language-dependent analysis (e.g., Sfard 2008) does not, and thus seems particularly suited to future research with toddlers and younger (nonverbal) children (ages 1–3 years).

16.4 Theme 4 Technology

Theme 4 discusses the important issue of using technology in early childhood mathematics education, and, specifically addresses issues regarding its integration into mathematics teaching and learning that may take place both within schools and at home. First of all, in tackling this complex issue, we remind the reader of the many different types of digital technology for early mathematics education that have developed quickly thanks to touchscreen tablet-based applications, now available in many pre-schools and schools and relatively easy to use (in comparison with desktop software). Indeed the contributions to Theme 4 mostly focused on touch-screen tablet-based applications.

Integrating this kind of technology into the teaching and learning of mathematics can be rather straightforward if the applications replicate physical manipulatives (e.g., Cuisenaire rods, geoboards or tens-charts) and are designed with high developmental and curricular fidelity. However, as suggested by Sinclair (2018, this book), there are also applications that depart significantly from physical technologies (e.g., paper-and-pencil or manipulatives), and which may present significant challenges for teachers. In this respect, we can look at what has happened over the last thirty years to Dynamic Geometry Software (DGS). DGS has gained such widespread approval in the mathematics education community, however its integration into mathematics curricula around the world has taken many years and is not yet completely achieved. Moreover, most research has been carried out at the secondary level, focusing especially on teacher integration, task design and assessment (Sinclair and Yerushalmy 2016), while more research, some of which is provided in this book (Sinclair 2018; Fletcher and Ginsburg 2016; Sinclair and Baccaglini-Frank 2016), is needed at the primary and pre-school levels. Our intention is that the results of the research presented in this book can be used to guide the professional development of early childhood and primary school teachers who wish to use DGS in their classrooms. If it has taken this long for DGS to become integrated into mathematics curricula, we imagine that integrating new applications that share with DGS the characteristic of departing significantly from physical technologies, such as TouchCounts (see Sinclair 2018; Baccaglini-Frank 2018, this book) will be a long and non-linear process. Possibly, new forms of professional development could aid the process, helping educators identify and implement effective uses of such new applications, especially when there are no pre-designed tasks introduced by the software and the interactions afforded are not highly constrained.

The chapters discussed in Theme 4 also highlight the fact that even if the software in focus is not extremely distant from physical manipulatives or the paper-and-pencil environment (at least not to the extent of DGS or TouchCounts) there are still important issues that need to be further investigated. A first issue is the role of the educator (a teacher or parent) during (or in between) the child's interactions with the software. Indeed, analyses of the student-software-teacher interactions in Baccaglini-Frank's chapter (2018) shed light onto how the educator's short-term goal of helping the children experience success, and her narrow-sighted view of how to obtain this in the domain of numbers, actually limited the development of numerical abilities for many children in the study.

Also Ginsburg et al. (2018, this book) highlight the importance of parents being involved, as educators, in understanding and promoting children's mathematical thinking and learning in the context of Interactive Mathematics Storybooks (IMS), stressing the need for further research to explore the interactions between the children, the educators involved, and the IMS used. Indeed, for all these types of technology designed for early childhood mathematics education it makes sense to ask: What might effective forms of professional development be if the goal is to promote proper integration of such technology into early childhood mathematics education?

A second issue is the possibility offered today by more and more software that exposes young children to advanced mathematics early on. This issue is addressed both by Fletcher and Ginsburg (2016) and by Sinclair (2018). Indeed, Fletcher and Ginsburg find that through appropriate technology young children can learn more about symmetry than what they acquire naturally and what is currently taught in US schools. They conclude that teaching symmetry earlier than designated by the US's Common Core State Standards for Mathematics (NGACBP and CCSSO 2010) can provide a unique opportunity to utilize and build upon the skills and interests that young children bring to the classroom. Analogously, Sinclair reflects upon the ease of children's earlier exposure to advanced mathematical concepts through new multi-touch tablet-based applications. For example, interacting with TouchCounts allows children to encounter large numbers, also in symbolic form; moreover this application puts an emphasis on ordinality. This kind of interaction with numbers is not typical in classrooms where the meaning of number is usually associated with cardinal quantities represented by physical manipulatives such as tens' charts and Dienes' blocks, and the numbers encountered are typically below 20. Therefore, as Sinclair states, there is a challenging choice to be made, both by teachers and researchers, about the extent to which new digital technologies—those that significantly change mathematics—can and should be integrated.

We hope that the issues and perspectives introduced in this theme will help to develop positive and constructive, though critical approaches to the issue of using technology to promote mathematical learning in early childhood education.

16.5 Theme 5 Early Childhood Educators' Professional Issues and Education

Theme 5 focuses on three distinct but interrelated areas of research impacting on change in practice for early childhood educators influenced by several conditions at a macro, meso and micro level. First, new curricula and frameworks for early mathematics teaching and learning provide varying expectations about the scope and depth of the pre-school environment—informal learning (such as through play), content to learn and activities to experience. Second, early childhood educators' mathematical knowledge, pedagogical knowledge, understandings, beliefs, and perceptions influence how they enact these expectations. The importance of professional learning and the assessment of pre-service teacher mathematical competencies are discussed. Third, educational programs, resources and activities implemented in the pre-school environment impact on the mathematical opportunities children engage in at the micro level. The discussion at these three levels consider both the child's engagement with mathematics and the impact of the professional on this learning based on relevant contributions in TSG 1.

At a macro-level, the introduction of pre-school mathematics curricula or frameworks in several countries raises fundamental questions about the views of the early childhood educator. Cooke and Bruns (2018, this book) highlight the tensions raised by several papers about new curricula and frameworks that may impose mathematical content rather than allowing the child to develop mathematical concepts through play. Some research supports new directions on curriculum development for early childhood mathematics internationally. What could follow from these studies is further articulation of how the elements of each curriculum or framework are interpreted, implemented and evaluated within pre-school mathematics learning environments. Longitudinal research could consider how these elements may direct broader change over time such as how the curriculum is enacted, the need for increased mathematical content and pedagogical knowledge of the educator, and the need for tailored and increased professional development. The impact these policy documents and changes have on professional practice and the views of early childhood mathematical learning is another important issue for further research.

At the meso-level the focus is the early childhood educators' competence. All of the relevant studies in TSG 1 show consensus about the importance of fundamental understanding of mathematics by the educator as the basis for high-quality early mathematics education. However, different studies used different conceptualizations and instruments to measure the mathematical competence of educators, including pre-service teachers. Several papers used the Shulman framework (Bruns et al. 2016; Dunekacke et al. 2016; Jenßen et al. 2016; Tsamir et al. 2018). These studies found that pre-school educators' competence is related to a range of complex factors that incorporate mathematical content and pedagogical content knowledge, general pedagogical content knowledge and affective-motivational aspects as well as their perception of mathematical situations. Cooke (2016) identifies and measures similar facets of educators' math-related competence, indicating that these facets are subject to sustained investigation by different groups of researchers focused on improving teacher competence and confidence.

However, Cooke and Bruns (2018) point to a lack of consensus among the findings of many studies about these competency facets. This was seen in TSG 1 presentations: For example, a number of studies found no significant correlations between mathematics anxiety and mathematical content knowledge (Cooke 2016), mathematics pedagogical content knowledge or perceptions of mathematics situations (Dunekacke et al. 2016), whereas Tsamir et al. (2018, this book) found that self-efficacy for identifying repeating patterns and errors in repeating patterns was well-matched to the pre-school teacher understandings.

In particular, the chapter by Tsamir and colleagues draws attention to the importance of the educators' ability to identify and continue repeating patterns. Their study found that pre-school teachers were able to identify drawings which represent repeating patterns and identify the errors which preclude a drawing from actually being a repeating pattern. However, identifying appropriate continuations proved more difficult. The chapter provides an analysis of early-childhood educators' responses to conceptually-oriented tasks. This study could serve as an example of the need to develop rigorous assessment tasks that can identify and remedy misconceptions of early childhood educators from the outset.

While the contributions in TSG 1 on teacher-related issues incorporated a wide range of studies, and many reported on large scale projects such as KomMa, in light of the above, much more longitudinal and systematic research is needed as well as studies that provide internationally comparative insights. In line with this, Cooke and Bruns (2018) conclude that there is a need for further studies that use different measures and more representative samples, and studies that examine the relationship between early childhood educators' competences and the quality of the experiences they create to engage children in mathematics.

At the micro-level the focus is on the child as the centre of mathematics learning, although the research reported in the chapter by Cooke and Bruns (2018) address to some extent the effectiveness of program intervention and teacher knowledge. Several intervention programs are outlined, yet questions are raised about the practicality of their implementation processes. Fritz-Stratmann et al. (2016) demonstrate the need for high level professional training for a program to be effective. Another approach is to focus intervention programs on one aspect of mathematics such as number learning (in this case, arithmetic) that Wang et al. (2016) found to increase the likely effectiveness of their program. There are few other examples to draw upon within Theme 5, but there are other relevant examples presented through other chapters in this book. What could be gleaned from the discussion on intervention programs is the importance of designing and implementing carefully designed activities that are appropriate to the wide and developing needs of the children. However, it is very difficult to match these requirements to every learning context such that the creative mathematics learning of the students is not stifled.

Clements et al. (2013) reiterated the importance of the use of research-based programs, stating that developmentally-sequenced activities can enable teachers to "become aware of, assess, and remediate" (p. 10). Targeting children early may improve learning in both their first years of formal schooling and their later mathematical achievement (Watts et al. 2014), and development-oriented interventions can help children develop mathematical understandings (Fritz et al. 2013). However, there needs to be careful monitoring to ensure that the impact of the interventions does not fade away (Sarama and Clements 2015).

16.6 Further Perspectives and Concluding Comments

The international scope of the contributions in this book highlights the need to consider the above discussion by taking into account the diversity of early childhood mathematics education across different countries. All over the world pre-school educational systems and traditions in different countries vary and children begin primary school at different ages. Also the varying philosophies of how learning in early childhood education is supported should be acknowledged. Moreover, analogous variations apply for the education of prospective early childhood educators primarily regarding mathematics pedagogy. The very different conditions for early childhood mathematics education, from the perspective of the children or the educators all over the world, and the lack of international studies at this level highlight the need to design, implement and evaluate cross-cultural and comparative studies in this field.

The themes addressed in the chapters of this book reflect an impetus to develop broader and more integrated views of early mathematics learning rather than a traditional focus on counting and number concepts. Thus the role of early childhood mathematics education, as it is approached in this book, can be regarded from a new broader perspective, for example, as an opportunity to reflect on the contribution of this field to a recent educational direction, that is, early STEM (Science, Technology, Engineering and Mathematics) learning (McClure et al. 2017). Although this latter new approach to early learning is closely related to how young children explore and make sense of the world, very little research has been carried out on how concepts in STEM learning in early childhood are associated with young children's mathematical learning and development. In this book, research reported in many chapters concentrates on types and processes of mathematics which involve reasoning, structure, interconnections between topics, integration of technology, multimodal and embodied aspects of communication, thinking and learning, maths-in-context and in everyday activities. These features of early mathematics learning and teaching have a significant place in STEM topics and may contribute to a deeper understanding of relevant concepts, foster problem solving, and enhance the understanding of the application of concepts in real life. Thus, we hope that, even in subtle ways, this book may inspire reflective insights and new ideas for further investigation about how early childhood mathematics education may contribute to interdisciplinary research important for early and later STEM learning and development. Furthermore, investigating early childhood educators' roles and their knowledge for teaching mathematics, as well as the impact of the curriculum, from such an integrated approach could fuel new and critically important research directions.

Overall, we hope that the diverse international collection of studies in this book will provide powerful foundations for future research, professional learning and curriculum development in early childhood mathematics education.

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