Chapter 7
Teachers as Designers of Digital Educational Resources for Creative Mathematical Thinking

Chronis Kynigos and Angeliki Kolovou

Abstract This chapter focuses on the design of digital C-books, c for creativity, incorporating dynamic constructionist artefacts that aim to induce creative mathematical thinking (CMT). We studied the design of C-books by mathematics teachers in collectives of educational professionals with a diversity of expertise. The analysis of the design process of a C-book on Curvature shows that the interactions fostered allowed mathematics teachers to learn from diverse practitioners. The C-book was developed as a collective document combining a variety of shared resources. The C-book technology allowing for the design of diverse malleable and improvable resources supported the infusion of constructionist and creative elements in the C-book resulting in more innovative approaches to teaching curvature.

Keywords Mathematics teachers' resources · Constructionist design
Social creativity · Creative mathematical thinking · Curvature

7.1 Introduction

Technology was first introduced into mathematics education as holding the potential to bring about fundamental changes in the experience of learning mathematics or even the power to transform mathematics teaching and learning. However, research has shown that the impact of technology on students' learning is less significant than initially anticipated (Clark-Wilson et al. 2014) and technology still plays a marginal role in mathematics classrooms. Since teachers have a key role in the uptake of digital technologies in classrooms, several researchers have focused

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on investigating teachers' interactions with digital resources (Pepin et al. 2013) or teacher expertise related to the integration of digital technologies into everyday teaching practice (Mishra and Koehler 2006; Ruthven 2014). Others (Fuglestad et al. 2010; Laurillard 2012) have stressed the importance of involving teachers in all stages of the design process of learning activities involving the use and design of digital tools. In this way, teachers can become aware of the complexities involved in the use of digital tools in the classroom and become "active agents in the process of creating new cultures of practices capitalizing on the possibilities of digital tools" (Fuglestad et al. 2010, p. 309).

Acknowledging the complexity of technology integration into mathematics teaching thus induced a shift towards supporting teachers not only by providing them with resources, but by helping them to design their own resources individually and collaboratively. The main avenue for providing such support is through teacher professional development courses placing teachers in communities of practice (CoP, Lave and Wenger 1991) where resource production becomes an experience over which technological, pedagogical, and content knowledge (TPaCK, Mishra and Koehler 2006) is shared and cultivated. This support entails, among other things, conceiving of new media for ongoing exchanges (e.g., via platforms) and taking into account student and classroom interactions, as well as teachers' interactions (Trouche et al. 2013). Trouche et al. (2013) show how resource production and perpetual reformulation (they aptly use the term 'a living document' for a resource under continuous improvement) is an integral part of the teacher's professional life and an important vehicle for manifesting reflexive teaching practice. In this wake, some in-service teacher education programs have placed the teacher at the centre of the design process (Kynigos and Kalogeria 2012; Trouche et al. 2013), and have supported their ensuing exchanges in CoP to focus on the sharing of resources and even their collaborations in joint resource productions. So, individual and joint production of digital resources becomes a natural professional development activity involving the sharing and discussing over this kind of resource and its use in the classroom.

In this chapter we take the idea of joint construction of digital resources for teacher professional development further by means of placing the mathematics teacher in hybrid communities where she/he collaboratively designs with colleagues of diverse expertise. We ask the question 'how can this experience inspire creativity in the design process and product'?

We do acknowledge that much more research is needed in order to understand the complex interplay of several mechanisms that are being put into action during a collaborative design processes (Kynigos and Kalogeria 2012; Pepin et al. 2013) and, in particular, to clarify the role of the mathematics teacher as designer of digital resources in collectives.

The design of digital resources is not a neutral process, in the sense that it reflects certain views on teaching and learning, which may be affected by the new opportunities technology offers (Trouche et al. 2013). Take for example the case of expressive digital media for mathematical meaning-making. Hoyles and Noss (2003) use this term to signify customizable tools that offer learners various semiotic registers with which mathematical knowledge can be explored, such as dynamic geometry systems. Expressive media for mathematics education afford dynamic manipulation, interlinked representations, simulations of phenomena and situations embedding mathematical rules. They are thus designed to enable knowledge construction through creative engagements with (advanced) mathematical concepts. In fact, meaning-making through the process of tinkering with digital media has been perceived as a creative activity (Kafai et al. 2009). Likewise, interactions with such media challenge teachers' epistemology and perceptions of teaching and learning mathematics (Kynigos 2007) and might induce novel pedagogical approaches and teaching strategies. Designing for students to engage in creative activities with digital media is, thus, a challenging endeavour, which also brings the issue and study of creativity in the design to the fore. In collaborative design activity, process and product are inextricably linked, in the sense that the former draws its very existence on the pursuit of the creation of some novel product, and that a creative product acquires its substance as the end-result of some design processes. However, the role of creativity in instructional design (or specifically in educational resource design) has been only recently acknowledged (Clinton and Hokanson 2012).

In this chapter, we look for ways to enhance this kind of creativity by means of addressing it as a social phenomenon in collaborative design (Daskolia et al. 2016). In particular we are interested in the question of how can the mathematics teacher be inspired to think creatively when working in a group of colleagues with diverse expertise. How can this diversity generate creativity in the design process and product? We thus look at mathematics teachers working with researchers, designers of digital tools, teachers educators, teachers of other domains and creative writing specialists.

To make this activity meaningful and challenging, we provided our diverse groups with a new genre of resource, a particular type of authorable e-book which we called the 'C-book' which was used to encourage the re-considering of the structure of mathematics curricula in the quest for innovative conceptual fields (Vergnaud 2009) enabled by the new medium and aiming to generate creative mathematical thinking in students. From a technical point of view, a 'C-book' allows for text to intertwine with any kind of expressive media. A mathematical story or problem may easily involve different digital widget instances for students to dynamically manipulate, construct and tinker with (Kynigos 2015). In this way, a medium meshing narrative with dynamic constructionist widget instances may enhance student creativity in mathematical thinking and meaning-making. Before moving on to how we viewed collaborative design of C-books, let us focus for a moment on how we view creative mathematical thinking as it constitutes a main design target for the C-book medium.

Our perception of creativity is aligned with the view of Silver (1997) who sees creativity as a disposition toward mathematical activity that can be broadly fostered in the general school population through engaging students with ill-structured, open-ended problems. Mathematical creativity is susceptible to instruction and can be supported with tools that integrate open, real-life problems amenable to multiple solutions, offer students a multiplicity of representations, promote inter/intra-disciplinarity and foster constructionist activity (Papadopoulos et al. 2016). So, narrative and stories embedding opportunities for mathematization through the use of digital widget instances seemed an appropriate medium to encourage mathematical creativity in students.

We began from the premise that collaborative design by educational professionals with diverse expertise would generate a socio-technical environment likely to produce creative ways to enhance creative mathematical thinking (CMT) for students. In addition, by involving mathematics teachers in all stages of the design process, we aimed to induce their reflection on the affordances and pedagogies incorporated in the tools, the tasks and narratives under development, as well as the changes to the mathematical content and the classroom practices that the presence of technology brings about.

Addressing this problem is a complex venture. Not surprisingly, we did not find a single theory or framework which would enable us to make sense of creativity emerging in collaborative design interactions and production versioning. Based on our experience with forging connections between frames for mathematical learning with digital media (Artigue and Mariotti 2014), we engaged the employment of four theoretical constructs which we felt would critically inform our study of collaborative design process and production: Social Creativity, Constructionism, Boundary Crossing (BC), and the Documentational Approach (DA). Instead of adopting the metaphor of a theoretical landscape where priority is given to ad hoc connections between fragmented theories (Artigue and Mariotti 2014), we found it more useful to think of these four theories in a nested structure. We thought of social creativity as being our focal point and constructionism enhanced by the other two constructs, BC and DA as being the tools to think of the former. Our connections thus had a sense of directionality and complementarity, as shown in the subsequent sections (Kynigos et al. 2016). BC and DA fed our understanding of constructionism in collaborative design which in turn helped us make sense of emergent social creativity in the design groups. In the following sections, we explicate how these frameworks were employed in an integrative approach to teachers' collaborative design activity. In particular, the first, social creativity, is delineated in Sect. 7.2 of this chapter, while the others are explained in Sect. 7.3.

7.2 Social Creativity in the Collaborative Design of Digital Educational Resources

In order to form diverse communities in which teachers would be prone to be creative and produce out-of-the box but relevant ideas, we found Fischer's "community of interest" (CoI) to be directly relevant and useful (Fischer 2001; Kynigos and Daskolia 2014). A CoI is a collective of practitioners from diverse disciplinary and professional domains and differs from a Community of Practice (CoP, Lave and

Wenger 1991)—defined as a community of practitioners in a particular domain who undertake a similar work—in that CoI members are representatives of more than one CoP who were brought together to solve a particular problem of common concern. A CoI collaboratively designing a C-book should comprise professionals representing diverse CoP, each with a distinct disciplinary and knowledge background, field of professional practice and area of expertise. At the same time, many CoI members bring overlapping fields of experience by simultaneously participating in more than one CoP (e.g., a teacher with design expertise). Thus, each CoI member carries one main CoP membership but also potentially a subsidiary working knowledge and practical experience background from participating in more than one CoP in parallel. Inside the CoI, these members participate as representatives of the CoP they choose as most appropriate, according to their personal statements for their everyday professional activity, their personal interests and the task at hand. In the current study, the CoI consisted of teachers from different/diverse educational domains with a shared interest in designing digital resources for CMT.

The social dimension in collectives of educational designers and its role in enhancing both individuals' and the group's creative capacity has gained, so far, little attention in mathematics education research. Social Creativity (e.g., Fischer 1999) offers an appropriate theoretical frame for understanding and fostering the creativity of teachers working in collectives operating within particular technological environments. It addresses creative performance both as a process and through its outputs, as they grow out of the interaction taking place among the individual members of a specific group or community and between them and computational media, technologies and artefacts, all of them constituting and acting as a context to this end.

Social creativity can be fostered by innovative digital environments that enable users to frame and solve problems collaboratively. Meta-design is a methodology that addresses the design of socio-technical environments that help users "to be creative by allowing them to go beyond the explicitly described functionality of any artefact, to use it in new ways, to evolve it, and to explore its potential for new processes" (Fischer 2002). This is a perspective allowing socio-technical environments to stay open to modification and customization to new needs and opportunities that emerge during use. As such, meta-design addresses critical design challenges, including coping with ill-defined problems, supporting reflective practitioners and design as a collaborative process (Giaccardi and Fischer 2008) thus, eventually, design as a constructionist activity.

In the context of the research presented in this chapter, social creativity in the design of digital educational resources is addressed, on the one hand, through specially formed Communities of Interest. In addition, a constructionist digital environment (the C-book environment) was specially developed to allow CoI members to coordinate their efforts in designing CMT resources. To analyse the creative interactions taking place during collaborative design of digital media fostering CMT, three other relevant constructs were used, namely constructionism, boundary crossing and community documentational genesis. In the following section, we elaborate on these constructs and attempt to draw connections between them.

7.3 Constructionism and Its Intersection with Boundary Crossing and Documentational Appoach Frameworks

Constructionism (Harel and Papert 1991) emerged through the study of ways in which technology can be designed and used to enhance mathematical learning. It addresses the design for and the study of constructivist learning, in particular, the process of generating meanings and expressing them through interacting with the structural affordances of artefacts, mainly digital artefacts (Kynigos 2015). A key notion here is that of 'half-baked microworlds' coined by Kynigos (2007). Half-baked microworlds are digital artefacts intentionally designed and given to students as malleable and improvable asking of them to engage in discovering faults and shortcomings and changing them. It is through this process of exploration, bricolage and construction with expressive digital media that students—individually and collectively—are enabled to construct sound mathematical meanings.

Apart from providing analyses of designs affording constructionist learning, constructionism also provides analyses of design processes with an emphasis on understanding the meanings that designers generate through and with the changes they implement to the artefact under construction.

7.3.1 Constructionist Artefacts as Boundary Objects Facilitating Boundary Crossing Between Designers

In the process of communal design of digital resources, the diversity in collectives of professionals causes difficulties in communication and collaboration, but it also provides unique opportunities for the creation of new shared knowledge. According to Akkerman and Bakker (2011) boundaries are defined as the "socio-cultural differences that give rise to discontinuities in action and interaction" (p. 139), which can be overcome through *boundary crossing processes*, i.e., efforts made by individuals or groups 'at boundaries' to establish or restore continuity in action or interaction across practices. Hence, social creativity can be viewed as located in and nurtured by the boundary crossing encounters among the CoI members, in the mechanisms and strategies employed and as an outcome of them. In fact, Akkerman and Bakker (2011) distinguished between four mechanisms of learning at the boundary: (a) *identification* processes through which boundaries are reconstructed without necessarily overcoming discontinuities, leading to a renewed sense-making of different practices, (b) *coordination* which entails processes such as communicative connection between diverse practices, leading to the overcoming of boundaries, facilitating effortless movement between different sites, (c) *reflection* on the differences between practices leading to an enrichment and new construction of identity, and (d) *transformation* leading to profound changes in practices and the emergence of new in-between practices.

Boundary crossing efforts are facilitated by *boundary objects* that fulfil a bridging function between 'intersecting social worlds' (Star and Griesemer 1989). They come in the form of artefacts (such as specially designed computer tools), discourses (as a common language), or processes that allow the coordination of actions. In the course of designing a C-book, designers negotiate and collectively implement changes to the artefacts under construction. The malleability of constructionist artefacts permits their fine-tuning so they are appropriate for the practices of specific communities of designers working together in a CoI as well as across these communities (Healy and Kynigos 2010). In this way, these artefacts play the role of improvable boundary objects that help to establish and maintain a common ground supporting communication and shared understanding between different communities of practice involved in a CoI.

7.3.2 Constructionist Design for Collective Documentational Genesis

For analysing teachers' interactions with constructionist media, the *instrumental approach* (Guin et al. 2005) is highly relevant. A key notion of this approach is *instrumental genesis*, a process through which an artefact is associated with a set of schemes resulting in the transformation of artefacts into instruments (Gueudet and Trouche 2009). This is a reciprocal process, since the tool shapes the user activity (a process called *instrumentation*) and, vice versa, the tool is shaped by the user activity (a process called *instrumentalization*). The latter process is particularly enabled when interacting with incomplete, malleable, improvable constructionist tools (Kynigos and Psycharis 2013).

Stemming from the instrumental approach, the *documentational approach of didactics* (Gueudet and Trouche 2009) focuses on the interactions between mathematics teachers and resources and on the implications for teachers' professional development. Gueudet and Trouche (2009) coined the term *documentational genesis* to describe the ongoing process through which a new resource and a scheme of utilisation of this resource are jointly generated. In relation to collectives instead of individual teachers, *community documentational genesis* describes the process of gathering, creating and sharing resources to achieve the teaching goals of the community. The result of this process, the community documentation, is composed of the shared repertoire of resources and shared associated knowledge (Gueudet and Trouche 2012). A collaborative design activity involving teachers as designers of creative educational resources, is thus a process that is expected to trigger collective documentational genesis.

7.4 Design of a C-book Around Curvature

Curvature has hardly been addressed as a conceptual field (Vergnaud 2009) with respect to its potential to generate environments rich in opportunities for mathematical meaning-making by students. In traditional curricula, it lies disparately in Euclidean Geometry sections, in Algebra and Calculus depicting systematic co-variation, in 3D Geometry but in simple applications like conic sections. The dynamic and diverse representational repertoire provided by digital media allows us to approach curvature anew with a disposition to re-structure (to use Wilensky and Papert's term, 2010) the ways in which mathematics is conceptualized in education in the quest to make it more attractive to students, affording meaning-making, creative mathematical thinking and engagement. Previous research has indeed shown that dynamic digital environments and especially 3D spatial environments support students in constructing meanings about challenging conceptual fields (Kynigos and Psycharis 2003; Latsi and Kynigos 2011; Zantzos and Kynigos 2012). The study presented here focuses on the collective design of the 'Curves in space' C-book which eventually revolved around a story involving comparisons between Archimedes' and exponential spirals.

7.4.1 The CoI

The recruitment of CoI members was based on former collaborations developed as part of educational innovations related to the design of digital resources initiated by the Educational Technology Lab, the coordinating institute of this research study. In addition, CoI members had to exhibit particular traits, such as professional responsibility and sense of commitment, 'playful' attitude to work and sense of satisfaction out of professional collaboration with others, being open to accepting or acknowledging the perspectives projected by other communities, and willingness and readiness to cross own 'boundaries', critical, argumentative, and creative stance. The seven CoI members that were invited to participate in the joint design of the C-book 'Curves in Space' were graduate or in-service teachers in different levels of education (from primary to tertiary education) specialized in mathematics, mathematics education, creative writing, computer mediated communication and the design of digital tools for mathematics education. Among them, four had a strong mathematical background. This diversity in knowledge domains, perspectives and cultures as well as complementarity of expertise, were important factors for boosting social creativity in the design of the C-book.

7.4.2 The Computational Environment

The C-book environment provides the 'CoICode workspace', a tool for asynchronous online discussions allowing designers to choose between a threaded forum discussion organised in a tree-like structure (see Fig. 7.1) and a mind map view. When posting a contribution, CoI members have to state its nature (i.e., alternative, contributory, objecting, off task, or management) by using a specific icon, and can attach and refer to objects like online resources, texts or widget instances that reside in the C-book under construction. Furthermore, the CoICode tool provides the designers with the possibility to rate any idea or digital artefact in the form of CoICode contribution against three criteria[1]: (a) novelty, (b) appropriateness, and (c) usability of the contribution, on a yes/no basis. A creativity score per contribution is then automatically calculated as the aggregate score from the votes received on each criterion and across raters. Based on this score all generated ideas per C-book can be classified in terms of creativity, as well as in terms of their degree of perceived novelty, appropriateness, and usability.

In addition, the environment contains a platform which is the space for authoring (the C-book authoring tool) and the space where students interact with the C-book (the C-book player). The platform is designed to incorporate pages with dynamic and configurable widget instances accompanied by corresponding narratives (see Fig. 7.2). The authors can write text, attach links, files, or widget instances out of a set of widget factories (e.g., Geogebra is a 'widget factory', and a microworld of this factory incorporated in a C-book page is a 'widget instance'). In the C-book page depicted in Fig. 7.2, MaLT Turtlesphere, a 3D Logo-Based Turtle Geometry tool affording dynamic manipulation of variable values was used (http://etl.ppp.uoa.gr/malt2). With this tool, spirals can be generated by either constant or incremental curve and torsion changes to a turtle respectively repeating very small displacements.

7.4.3 Analysis of the C-book Design Process

Our data were: (a) the 124 contributions uploaded in the 'Curves in Space' workspace from the outset of the design process (6/4/2015) until the final version of the C-book was released (23/7/2015), (b) the actual C-book produced in terms of structure (pages) and contents (the 'script', the widget instances and the respective narratives per page), and (c) the data automatically collected and analyzed by CoICode, such as time series interaction data and/or creativity scores extracted per idea posted as contribution in CoICode workspace. The level of analysis employed

[1]These three criteria were developed throughout several cycles of work of a larger project focusing on the communal design of digital resources for fostering mathematical creativity (www.mc2-project.eu), of which the current study constitutes a part.

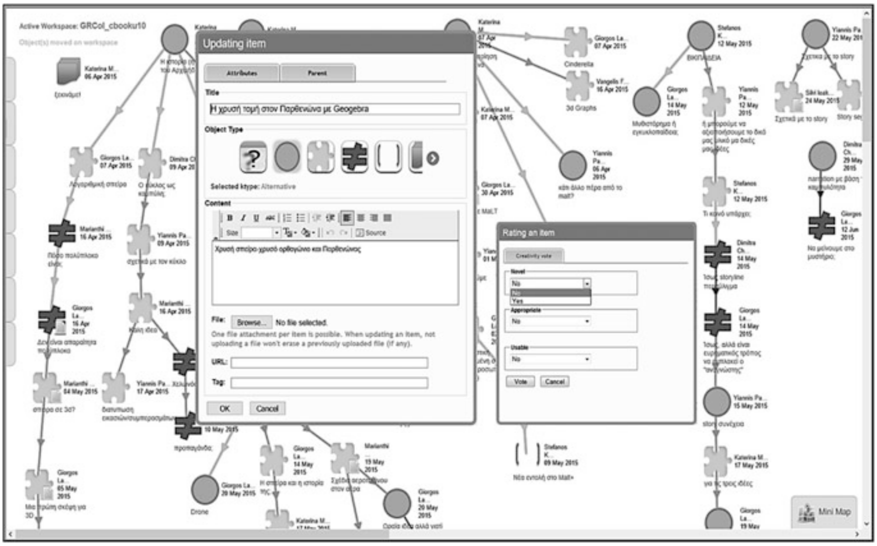

Fig. 7.1 The ColCode collaborative workspace of the 'Curves in space' C-book

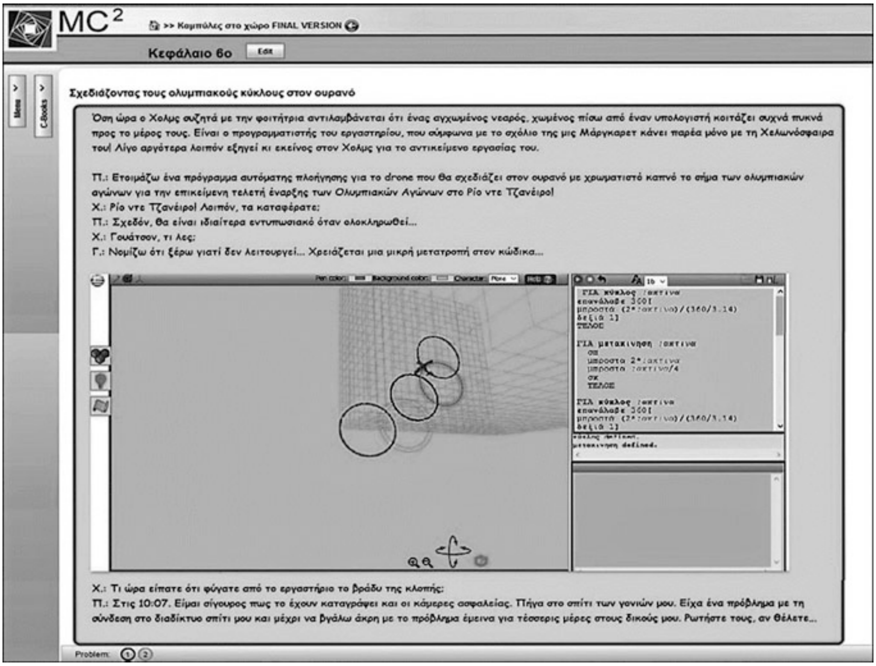

Fig. 7.2 A 'Curves in Space' C-book page asking students to fix the code for programming a drone to imprint circular traces

in this study involved the selection and analysis of critical episodes, i.e., relatively brief and uninterrupted periods in CoICode discussion, shedding light on some important aspect of the social creativity processes and/or products developed, by focusing on the interactions among the CoI members and with the C-book technology. The identification of each critical episode was guided and supported by the data automatically collected and analysed by CoICode. Furthermore, we traced paths of socially creative ideas, which stretch over longer periods of time and include several critical episodes, in terms of the critical moments in their evolution from the initial to the final idea (i.e., an idea implemented and incorporated into some part of the C-book). Again, the identification of creative ideas was guided and supported by the CoICode data. The analysis of the path of creative ideas enabled us to highlight conditions of the emergence of creative interactions among the CoI members, such as the role of specific resources functioning as boundary objects, or the complementarity of background of the C-book unit designers. This qualitative approach to understanding social creativity in the design of C-books was chosen to shed light on the social nature of the processes involved in the development of ideas and in the examination of the C-book environment features which added to the formulation, elaboration, and cross-fertilisation of the CoI members' ideas.

7.4.3.1 Critical Episode: The Design of a Widget Instance as a Result of the Creative Interaction Between CoI Members with Diverse Expertise

The episode selected (see Fig. 7.3) started one month after the outset of the design process, it lasted 8 days (14/5–20/5/15), and the participants in it were three CoI members: George, Mathematics teacher and graduate student in Mathematics education, Dimitra, Literature teacher and graduate student in ICT in Education, specialised in creative writing, and Marianthi, MA ICT in Education graduate and developer. At that time an exchange of resources was taking place on the mathematical idea of Helix-Spiral between Stefanos, a senior figure in the CoI from the Mathematics CoP and teacher educator) and George. The resources shared in the CoICode workspace triggered a discussion among all CoI members, which revolved around the history of curves going back to Ancient times. The episode is initiated by Dimitra (14/5) who, inspired by the airplane functionality in MaLT, suggests that students calculate the distance covered by airplanes performing spiral movement during air shows. She points out that this could be an engaging activity, though it does not relate to the historical perspective on curves, discussed so far. George (14/5) responds enthusiastically, stating that the 'airplane idea' is a powerful one, elaborates on it and provides a Wikipedia link on jets streams. He also wonders about how Dimitra's idea could be joined together with the history of curves, suggesting, thus, the coordination of the different ideas discussed so far. Marianthi then puts forth a suggestion on a half-baked widget instance:

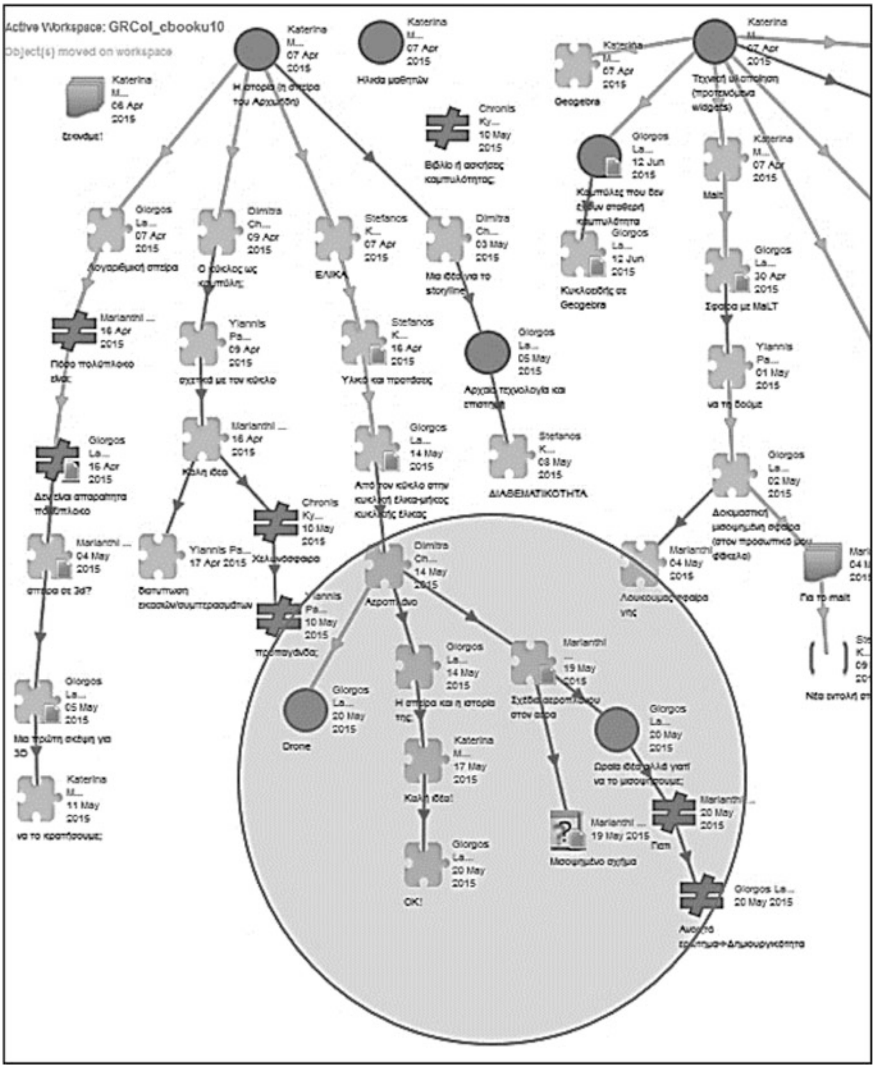

Fig. 7.3 Excerpt from the CoICode workspace depicting the critical episode

Marianthi (19/5): […] in MaLT + I created a procedure where the airplane movement forms the Olympic rings. I am sending you the complete code so that we can half-bake it, e.g. it can turn by a 45-degree angle in the last two turns so that the rings do not come out straight (attaches 'Olympic_correct.txt')

Marianthi (19/5): I am sending you the one I wrote with the wrong angles (attaches 'Olympic_wrong.txt')

George (20/5): I like it a lot! I suggest not to half-bake it, but ask students to create it by themselves by looking at an image of the Olympic rings […]

Marianthi (20/5): […] I think if it's half-baked it will be more challenging for students to correct it than create it from scratch. Also, we can focus on specific mathematical topics like the turn angle or the distance of cycles.

George (20/5): […] Since the unit addresses senior students it would be more creative to allow them work without such restrictions. If we half-bake it though, wouldn't it be better to use variables for the angles?

George (20/5) refers to drones as a more innovative alternative to airplanes and designs two alternative versions of the widget instance in which he adopts Marianthi's proposal. Finally, one of his versions was incorporated in the C-book including his suggestion of imprinting the traces of a drone instead of a plane (see Fig. 7.2).

This episode shows how the collective resource system is enriched through the sharing, reflection and transformation of individual resources to boundary resources. What is more, boundary crossing interactions between CoI members allowed the cross-fertilization of diverse perspectives: mathematics, digital tools development, and creative writing. Dimitra, having studied existing resources, is inspired to articulate the airplane idea stating in what ways it deviates from what has been heard before. Marianthi turns Dimitra's idea into a 'tangible' object, i.e., a widget instance, while George expresses considerations initiating an interesting exchange on the pedagogical affordances of different types of activities. He brings CMT to the fore and poses the challenge to other members to directly argue on specific pedagogical and technical affordances of the proposed activities. The final version of the instance appears in the C-book as a result of the coordination of George's and Marianthi's ideas. Social creativity is thus enhanced by exchanging, discussing on and modifying half-baked curve designs acting as boundary objects, allowing the communication and coordination of diverse perspectives. Mathematical resources thus take a mediational role between diverse perspectives undergoing several transformations and revisions until they are reified as widget instances in the C-book. As mathematics teachers negotiated over an emergent mathematical construction, they were challenged to reflect on and reconsider their beliefs and practices related to what constitutes a sound, challenging and creative mathematical activity, thus expand their learning.

7.4.3.2 The Evolutionary Path of the Narrative

The path presented below is related to the evolution of the narrative of the C-book. The respective path includes in total 52 contributions and stretches along the entire workspace. Early on in the design process, CoI members were concerned with devising a narrative that, together with appropriate widget instances, would provide opportunities for mathematization and meaning-making around curvature. The mathematical affordances of various digital tools also became an early topic of discussion so that tools, narrative and mathematical concepts were interrelated in the design of the C-book. Below we provide an excerpt of the path (see Fig. 7.4) including decisive contributions from individual CoI members and stress the social nature of the processes involved in the development of the scenario from its first appearance to its incorporation in the C-book. At that time a number of widget instances designed to afford creativity and meaning-making in curvature took the role of boundary objects by evolving through multiple cycles. However, a cohesive narrative that would incorporate and join together these elements was pending, despite the fact that some interesting ideas had been already suggested. The path sheds light into how the CoI members' conceptions about their productions in terms of didactical design (widget instances and corresponding learning activities) intertwined with their ideas about the narrative of the C-book.

Stefanos (22/6) presents a -rather loose- synthesis of his own and other members' ideas on the C-book narrative integrated in a new version of the C-book: the history of curves, two detectives working to solve a crime, a 3D printer laboratory, and solving riddles related to spirals. Stefanos had worked extensively in the past on curvature and had initially disposed to the CoI his own resources related to the history of curves, which directly affected the first two story versions.

George (24/6) reacts enthusiastically and attaches an elaborated version of Stefanos' story incorporating Sylvie's comments on enhancing the story: two renown detectives (Hercule Poirot and Sherlock Holmes) try to solve a mysterious robbery in a laboratory, which is connected to constructions related to curvature. Sylvie (25/6), a teacher and creative writing specialist who joins the discussion at that time, posts a new, radical version of the narrative relying on contemporary characters, which fuels an intense debate. Stefanos (25/6) objects to Sylvie's suggestion on the grounds that the storyline should blend with the widget instances so that students follow a learning trajectory working with tools of gradual increase in complexity. He also posts a document in which he justifies his rationale for building his own version in which mathematical concepts are presented in a coherent and meaningful way. George and Katerina (computer mediated communication specialist) react to Stefanos' post:

George (26/6): Very insightful comments, especially in relation to the way the current narrative supports the smooth integration of the learning sequence on curvature. […] Wouldn't it be better to make some corrections without discarding what we've done until now? […] (He attaches 'What a strange morning in the laboratory.doc' where he expands his previous version to include logarithmic spirals).

Katerina (27/7): I understand your argument related to mathematical coherence […] but I don't think that the new version rejects previous constructs and ideas […] but rather promotes them by organically binding them with a fresh, creative story.

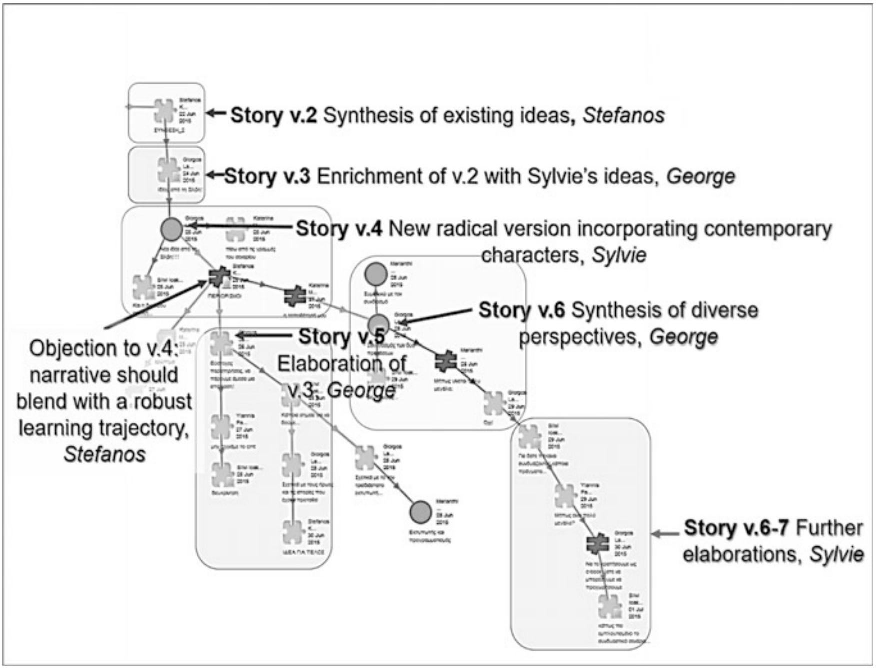

Fig. 7.4 Excerpt of the narrative path

At that moment, Ioannis, a senior researcher in mathematics education initiates a discussion on the C-book's CMT affordances with regards to students' involvement, both at the level of the narrative and at the level of activities (dealing with open problems, conjecturing, generalising, constructing, etc.).

Up to this point there are two opposing views on the scenario; a mathematics oriented strict, structured and robust learning scenario mainly supported by a senior member who is an experienced Mathematician, teacher and researcher (Stefanos) and a more innovative one which embodies a set of characters and situations of contemporary culture, aiming at increasing the readers' motivation and the narrative power of the story. This tension is released when George replying to Katerina, posts a new synthetic version of the scenario attempting also to address Ioannis's preoccupations regarding students' active involvement:

George (28/7): Here is a proposal for synthesizing the two alternative suggestions keeping the excellent work we've done till now […]

In the next two versions of the scenario Sylvie and George collaborate so that Sylvie presents a more robust synthetic version that organically integrates the designed widget instances.

The coordination of the two prevalent perspectives in the design of the C-book, i.e., the mathematics and the creative writing perspective, made possible the infusion of creative elements in the narrative, while not losing sight of its mathematical focus on curvature. It is noteworthy that these two perspectives are not only gradually reconciled after the catalytic intervention of George, they also enrich each other; on the one hand, the senior mathematician deviates from his original strictly structured storyline and proposes two additional ideas much more innovative than his initial ones. On the other hand, the creative writing specialist, after closely collaborating with mathematicians, comes to adjust her story in a synthetic version. In addition, both Sylvie and George reconsider their previous practice in the light of the rich exchange on CMT affordances. These reflective processes are essential for the interweavement of widgets instances with the narrative into a concise whole. Social creativity is thus facilitated by the meshing of the Sherlock Holmes narrative with curvature, which can only have emerged because of the diversity in the CoI. Furthermore, the process of story versioning boosts social creativity as it allows for the generation of new ideas which capitalize on, object to, and finally synthesize previous ones. It is an ongoing process where ideas are adjusted, adapted and combined to produce new documents.

7.5 Conclusions

In this study we tried to show how creativity might emerge in mathematics teachers' design processes and products under particular circumstances, that is, as they acted as members of a distinct socio-technical environment. The characteristics of the environment were based on the principle of integration. The technology afforded the meshing of narrative text and diverse constructionist widget instances. The collaboration was based on integrating expertise in a designer CoI, where our mathematics teachers worked with Dimitra, a language teacher, Sylvie, specialised in creative writing and story-telling, and Marianthi, a developer of constructionist widgets. Our analysis of social creativity in the design process of a C-book on curvature focused on the boundary crossing interactions between the CoI members and the role of the narrative in conjunction to widget instances as a key resource for the development of social creativity. Furthermore, to understand the interactions amongst the CoI members, we drew from diverse theoretical constructs. We employed BC and DA to elaborate on constructionist collaborative design processes so that we could illuminate the potential for generating social creativity in our mathematics teachers. We saw creativity as connected to the growth of their TPaCK within a teacher professional development capacity but also as a strategy to generate meaningful uses of digital media in the classroom based on the encouragement of CMT in students.

Two important boundary crossing processes, those of coordination and reflection, enhanced social creativity establishing communication between our mathematics teachers and colleagues from different communities of practice, i.e., from language teaching and computer science. Our mathematics teachers thus came to appreciate and appropriate the ideas of connecting a helix to ancient monuments and artefacts, of embedding mathematical concepts in a theft mystery storyline, and of affording digital modifiable representations of spirals for students to generate meanings of constant and systematic turn and torsion change. Reflection was a process which gave ground to the fertile synthesis of different views. Moreover, the story and its versions as a key resource was paramount to social creativity within the CoI. The story versioning process allowed for heated debate and idea exchange to take place. It created common ground for all CoI members to unfold their expertise, as well as the meshing of narrative with constructionist artefacts-widgets on curvature. As a result, a collective document, that is the C-book, was developed, associating various shared resources, i.e., activities, widget instances, text, and CMT representations and a scheme for interweaving all these elements in a coherent whole.

Along this process the CoI members were engaged in instrumentalization which in a constructionist frame can be seen to emphasise the collective modifying of the widget instances and the narrative. The issues that emerged during the construction of successive C-book versions challenged the mathematics teachers' perceptions with respect to the teaching and learning of curvature resulting in innovative approaches fostering creativity and meaning-making. Embedding the comparison of constant to incremental turn and torsion changes to generate Archimedes' and exponential spirals in space within a Sherlock Holmes 'who dunn it' story was not in any traditional curriculum section or chapter. The process challenged the mathematics teachers to step out of curriculum structures for curvature and to make a new conceptual field available to students connecting curvature with functions and 3D geometry.

By involving mathematics teachers in the collective design of digital tools we recognized them as active agents in the process of integrating technology in the mathematics classroom and not only practitioners involved in top–down integrations of technology in the classroom as suggested by other frameworks, such as TPaCK (Mishra and Koehler 2006). Socio-technical environments, like the one employed in this study, can serve as settings to study teachers' collaborative resource design and as a driving force for classroom innovations.

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