Chapter 5
Disaggregating a Mathematics Teacher's Pedagogical Design Capacity

Moneoang Leshota and Jill Adler

Abstract The analytical approach for describing teachers' Pedagogical Design Capacity (PDC) adopted in this chapter is part of a larger study investigating mathematics teachers' use of a prescribed textbook. In this chapter, we describe teachers' PDC through (i) the type of use of the curricular resource, that is, whether use is deliberate or tacit; as well as, (ii) the type of a relationship the teacher forges with this resource. We find that a deliberate use of the textbook and an intimate relationship with the textbook reflects a high PDC. We argue that PDC is more than the degree of appropriation of the affordances of curricular resources by the teacher: it is also about the quality of opportunities for mediation of mathematics that the teacher creates.

Keywords Mathematics teachers' resources · Pedagogical design capacity
Omissions · Injections · Teacher-textbook relationship

5.1 Introduction

The present chapter derives from a study which investigated teachers' use of a prescribed mathematics textbook in South Africa. The motivation for the study arose while conducting preliminary classroom observations of teachers who were

M. Leshota (✉)
Department of Science Education, National University of Lesotho, P.O. Roma, 180, Roma, Lesotho
e-mail: moneoangleshota@ymail.com

J. Adler
University of the Witwatersrand, Johannesburg, South Africa
e-mail: jill.adler@wits.ac.za

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going to participate in a professional development programme.[1] While the programme was not on teachers' use of textbooks, the observations of how two teachers in two different schools interacted with their textbooks piqued our interest in teachers' relationship with their textbooks. For example, in one teacher's classroom, Grade 9 learners were instructed to do exercises from the textbook which required generating tables of values for the expressions, $H = (t-3)(t-3)$ and $K = t^2 - 6t + 9$, for different values of t. A question which followed the generation of tables of values compared the values of H and K for the same value of t to show learners that the two expressions were equivalent. However, after learners had completed the tables of values, the teacher told them to skip that particular question on comparing H and K, and moved on to the next set of exercises which was completely different from the ones learners had been doing. Our question was why the teacher omitted that particular question which in our opinion was critical to the understanding that the two expressions were equivalent. Ultimately, the exercise which was intended as an introduction to transformational algebra by the textbook authors ended up being about multiplying factors of H and a substitution exercise in the case of K.

In another lesson, on Variance at Grade 11, learners were given a set of data from which to calculate Variance whose formula the teacher wrote out as, $\text{Variance} = \sum \frac{(x_i - \bar{x})^2}{n}$. The set of data was written on the chalkboard by the teacher. We would later on observe that it was an exercise from a prescribed learner textbook, as learners started paging through the textbook for guidance on how to calculate Variance. The teacher complained that learners were taking too long and promised them a shorter method which he obtained from a workbook[2] in the cupboard. The formula the workbook used was given as, $\text{Variance} = \sum \frac{x_i^2}{n} - \bar{x}^2$. Some learners continued with the first formula while others utilised the latter formula. When learners called out their answers, the teacher observed that four different answers were provided by the learners, at which stage he asked the researchers which of those we thought were correct. We wish to point out that the two formulas are in fact correct and are both used to calculate the Variance. However our attention was once more called to the teacher's use of the textbooks: two seemingly different formulae for Variance whose difference was not explained by the teacher. Furthermore, the teacher had not done the exercise himself before assigning it as a class activity.

The two examples narrated above prompted an interest and consequently an investigation of the teacher-textbook relationships and the processes by which

[1]The Wits Maths Connect Secondary Project (WMCS) is a research and development programme funded by the FirstRand Foundation (FRF), and the Department of Science and Technology offering a school-based professional development and research programme for mathematics teachers.

[2]We adhere to terminology of Taylor and Vinjevold (1999) who distinguish between textbooks and workbooks by defining textbooks as providing a systematic learning programme while workbooks providing supplemental and revision materials in support of the textbooks.

seven teachers from three different schools mobilised the affordances of the textbooks they were using. A multilayered analytical process was used to determine how teachers incorporated their textbooks and other resources in designing effective classroom episodes. Brown (2002, 2009) named this capability, *pedagogical design capacity* (PDC), which shall be described in detail in latter sections. The present chapter however focuses on only one of these seven teachers, whom we gave a pseudonym, Mpho. Mpho's case is used as an illustration of the analytical process of describing the teachers' PDC in the study in an effort to understand better the relationship between the teachers and their curricular resource.

We suggest that through our analysis of Mpho's lessons, the process of describing her PDC illuminates our understanding of the teacher resource relationship, with possibilities to inform policy and professional development programmes on teachers' use of curricular resources. We are however cognizant of the complex nature of this undertaking as evidenced in research on teacher–resources interactions (Choppin 2011; Remillard 2005).

5.2 Teacher–Resource Relationships

Investigations of teachers' PDC fall within the larger field of teachers' use of curriculum resources and the impact of these resources on teaching. Within this field, there are studies that investigate the teacher–resources interrelationships looking into the factors that influence the teacher resources interactions. Among them is Remillard's (2005) synthesis of over 25 years of research on curriculum use in mathematics. In this synthesis, Remillard suggested four different conceptualisations of curriculum 'Use' as: 'Use' as following or subverting the text; 'Use' as drawing on the text; 'Use' as interpretation of text; and 'Use' as participation with the text. Studies on teachers' PDC align with the conceptualisations of use as participation with text and that of use as interpretation of text. These two conceptualisations of use emphasise the participatory interrelationship between the teacher and the resource, as well as the interpretative nature of teachers' interactions with the resource. Teachers and resources are viewed as engaging in a dynamic interrelationship in which the teacher shapes the resource, and the resource in turn shapes the teacher while they both shape the outcome of instruction (Stein and Kim 2009). On the other hand, research recognises that teachers interpret the intentions of the authors of the resource to suit their classroom goals (Ben-Peretz 1990; Chavez 2003).

Other than the 'what' of the teacher resource interactions, there is also research on the 'how' of the interactions. On one hand, Brown (2002) studied the degree of use of a given science resource by teachers and suggested a continuum of three differential ways through which teachers engaged with resources and called them: *offloading, adapting*, and *improvising*. At the far opposite extremes of the continuum are *offloading* and *improvising* with *adapting* nested in the middle. When offloading, the agency for the delivery of content lies with the resource, whereas

with improvising the agency lies with the teacher. Adapting reflects both teacher and resource agency equally in a lesson. Thus, the three processes illuminate the degree of appropriation (Wertsch 1998) of the affordances of the curriculum resource. However, a cautionary observation from Brown (2002) is that the categories do not necessarily correlate to teacher expertise or the quality of educational designs and therefore their occurrence may not be used as an indication of such.

In our study we have adopted these categories as we found them useful in the initial stages of analysis for providing information on how Mpho mobilised the resources available to her. Similar studies include those of Sherin and Drake (2009) in which the authors showed that teachers have three general approaches lying on a continuum too, when adapting the resource. They either *omit* components of a lesson, or *replace* one component with another, or completely *create* new components. Kim and Atanga (2014) studied teachers' decisions on whether to use, modify, omit, or make additions to the curriculum lesson, and concluded that these decisions have a bearing on opportunities for learning that teachers make available. The wider study from which this chapter has been drawn, examined the omissions from the textbook as well as the injections of content not available in the textbook (Leshota 2015; Leshota and Adler 2014), and concluded too that the types of omissions and injections teachers make have a bearing on the opportunities for mediation being opened up in the classroom.

The studies discussed above begin to illuminate teachers' pedagogic actions that are part of their PDC. Analysing these actions thus highlights important elements of the teachers' capacity to utilise their resources in ways that open up opportunities for mediation in the classroom. The next section provides a detailed description of the notion of teachers' PDC.

5.2.1 The Notion of Teachers' Pedagogical Design Capacity

Teachers' pedagogical design capacity (PDC) coined by Brown (2002, 2009) is a theoretical construct. It describes teachers' unique skill of perceiving the affordances of a resource, and reflects teachers' ability of creating "deliberate, productive designs" (Brown 2009, p. 29). In the teacher-curriculum interaction framework (Remillard 2005) illustrated in Fig. 5.1, PDC falls among the features which the teacher brings to the interaction.

It is worth noting from Fig. 5.1 that PDC is different from teacher knowledge, that is, from pedagogical content knowledge (Shulman 1986), or subject matter knowledge. PDC is not what a teacher "has", like knowledge, but characterises *a process* by which the teacher utilises their knowledge and other features together with features of the resource to design instruction for students. It is therefore more than teacher knowledge; it is about what teachers are capable of doing with that knowledge to 'craft' (Brown 2002, 2009) classroom episodes. Thus, teachers need to be able to recognise and understand the affordances and constraints of available

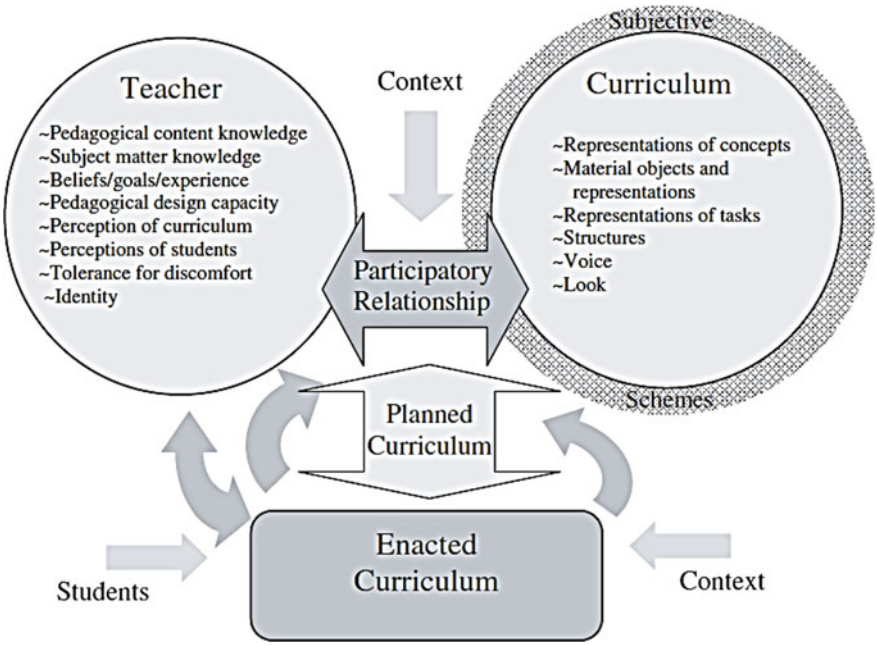


Fig. 5.1 Framework of components of teacher-curriculum relationship. Taken from Remillard (2005, p. 235)

resources, and weave these with their personal capabilities to generate episodes that create opportunities for mediation in the classroom.

Teachers' PDC hence, depends on two major aspects. Firstly, on the teacher's capacity to perceive needs and opportunities in their classrooms; and secondly, on the teachers' capacity for opening up opportunities for mediation with the available personal and external resources. Thus we expect each teacher's PDC to have specificity; reflecting her preferences, her context, and her understanding of different features of the resources. In describing Mpho's PDC in this chapter, our aim is to understand her pedagogic design capacity as reflected in her actions. We achieve this aim through identifying the patterns by which Mpho mobilises the affordances and constraints of available resources together with her personal capabilities to open up opportunities for mediation in her classroom. We operationalise Mpho's PDC by considering the content she omits from the textbook (or other curricular resources she utilises), the content she injects into the lesson that is not available from these curricular resources, and how these omissions and injections open or close the opportunities for mediation.

Before we move on to elaborating the overarching theoretical framing in the chapter, we have found it prudent to provide a background of textbook use and textbook availability in the South African mathematics education context, and thus some of the contextual conditions within which Mpho and other teachers in our study worked.

5.2.2 Mathematics Textbooks in South Africa

In South Africa, textbooks undergo a process of approval by the Department of Basic Education, with the major criterion for successful approval being alignment to the curriculum statement. This implies hence that a textbook approved for a particular grade represents the approved official curriculum in terms of the grade specific curriculum and the expected sequencing of topics. The context is such that teachers are under considerable pressure for curriculum coverage, and so usually align their goals with those in the prescribed textbook.

For the majority of teachers in South Africa, the print textbook is still the most accessible teacher resource, and in some cases, the only teacher resource. The state provides textbooks for learners and teachers in government schools even though in schools with large numbers of learners, it is not uncommon to find up to six learners sharing one textbook. A timetable would be drawn that showed who got the textbook, and when. Approved textbooks are highly regarded as a useful learning resource for school in South Africa. To illustrate this assertion, we recall an incident in 2012 where government was taken to court by nongovernmental organisations for failing to deliver textbooks to schools on time, in what was to become known as the "Limpopo Saga". In the Limpopo province, it was discovered that textbooks had not been delivered to schools six months into the beginning of the school year. An uproar by parents, teachers and NGO's prompted a parliamentary commission of inquiry with human rights organisations conducting their own investigations into the matter. The Minister of Basic Education had to make presentations to a parliamentary committee, and the matter ended up in the High Court where government was directed to make arrangements for a six-month 'catch-up' plan for the affected learners. The assumption here seemed to be that if there were no textbooks, then there was no learning. Textbooks delivery has since become a burning national issue and government is called upon to account for delivery of textbooks on a yearly basis.

The textbook package usually consists of a learner textbook and a teacher manual, and sometimes additional exercises on CD. The teacher manual typically only includes answers to questions in the learner book. This means that for purposes of teaching, teachers use the learner book as it includes mathematics explanations, tasks and exercises. In the schools where this particular study took place, the project provided some two hundred (200) textbooks to all learners and teachers in participating classrooms to curb the issue of unavailability. Furthermore, the present study took place during the period when a new curriculum was being implemented in South Africa, with new textbooks being developed. It was possible to obtain access to these new materials for teachers in the project. More details will be provided in later sections. At the same time, WMCS had begun the professional development programme for teachers and provided additional handouts for teachers through the programme's workshops. Some teachers also utilised other textbooks besides the prescribed textbook for teaching. Therefore while the study was about the teachers' relationship with the prescribed textbook, with respect to the teachers'

PDC it also considered the other resources which teachers mobilised for teaching. We shall see later in the chapter, that Mpho actually utilised the workshop materials in one of her lessons and not the prescribed textbook.

5.2.3 Theoretical Considerations

Vygotsky's (1978) Sociocultural theory wherein all humans are inherently social beings and grow from and through the use of tools, provides the overarching theoretical framework of this study. In this framework, cultural artefacts and tools mediate the relationship between subject and object, and as Wertsch (1991) argues, the cultural tools that are employed in mediated action are the key to understanding the relationship between sociocultural settings and human action. In appreciating this "agent-acting-with-mediational-means" (Wertsch 1998, p. 24) theory, we are forced to "go beyond the individual agent when trying to understand the forces that shape human action" (ibid) and to focus on the agent-instrument dialectic. In the teacher–resource interactions studies, whether following the theory of *documentational genesis* (Gueudet and Trouche 2009) or the *interpretation of and participation with resources* (Remillard 2005), analysis emphasises on one hand the affordances (Gibson 1977) of the resource, and on the other hand how the teacher appropriates (Wertsch 1998) these affordances and constraints (Norman 1999) of the resources.

We have already mentioned that our study aligns with the *interpretation of and participation with resources theory* which conceptualizes the teacher-curriculum relationship as a "dynamic interrelationship" involving participation of both the teacher and the curriculum, and where the teacher and resource shape each other and both shape the instruction (Remillard 2005). Remillard, in reviewing studies on teacher-curriculum relationships highlights the participatory relationship between the teacher and curriculum as a significant construct in greater understanding of these relationships. The participatory relationship views teachers as 'active' designers of the curriculum (Brown 2009; Pepin et al. 2013; Remillard 2005) and not just mere conduits of the authors. Curriculum materials are regarded as artefacts (Brown 2009; Vygotsky 1978; Wertsch 1998) with affordances and constraints on teachers' activities, while the activities are conceptualized as design processes (Brown 2002, 2009) where teachers use these resources in specific ways. This 'specificity' of use illuminates each teacher's design capacity in selecting the resource; in adapting it to suit the teacher's classroom needs, and in utilizing it 'as is' depending on her classroom needs. However, it does highlight the complex nature of studies such as this where there are many elements that influence each teacher's decision making process including the teacher's own beliefs.

How then is the teacher-curriculum materials relationship to be observed and described? In the next section we outline our methodology with a focus on what is pertinent for this chapter. Inevitably, we touch on how this methodology informed the larger study.

5.3 Methodology

5.3.1 Data Collection

The larger study involved seven Grade 10 teachers participating in a professional development programme. All teachers used the same prescribed textbook and were observed teaching a topic on functions. The textbook is one of the most popular in South Africa and a textbook of choice according to the teachers. The curriculum requirements as reflected in the prescribed textbook included three main components: (i) introducing graphs of the quadratic function, the hyperbola, the exponential function, and the three basic trigonometric functions, sine, cosine, and tangent; (ii) determining the properties of these functions; (iii) performing horizontal transformations and compressions $g(x) = af(x) + q$, and observing the effect of the parameters a and q on the parent graphs; and then (iv) interpreting functions, including determining equations of given graphs and sketching the graphs.

The study took place during an unstable phase of curriculum policy in South Africa. Specifically, as data collection was about to begin, a curriculum review process was concluded leading to the reformation of the existing NCS curriculum (National Curriculum Statement) into the present curriculum that came to be known as the CAPS curriculum (Curriculum and Assessment Policy Statement). The review and reformation was officially described as a refinement and not radical change, and in mathematics, textbooks were simultaneously being revised so as to be available for evaluation for approval by the department of basic education. The NCS aligned prescribed textbook which Mpho and the other teachers were using was still available in their class. The author who wrote the existing chapters on functions was also the author of the 'same' chapters in the new edition. Before commencing on the classroom observation we got hold of the author who organized a two hour workshop for the teachers, mainly to make visible her design rationale (Ball and Cohen 1996; Davis and Krajcik 2005; Stein and Kim 2009) in the new edition. Mpho and the other teachers were also provided with the draft copies of the new edition to use as they saw fit. Due to the high regard for the textbook series in the country, it was expected by teachers that the new edition would also pass the evaluation by the department of basic education, which it eventually did.

Data collection for the study also coincided with a workshop organized by the WMCS project on teaching functions for the teachers in the project schools. The teachers were furnished with handouts from this workshop to utilize as they saw fit. All these materials were collected for documentation and analysis in the study.

Three lessons on functions were observed and video-recorded in Mpho's class where all learners had been provided with the prescribed textbook by the WMCS project. As noted, Mpho had access to the latest edition of the textbook as well to use as she saw fit.

In addition to lesson recordings, a post interview with each teacher took place some months after the lessons were taught. The post interview was designed to probe teachers' interpretations of the affordances of their textbooks, with specific

questions related to the lessons they had taught on functions, but in an atmosphere that was distanced from the observations. Unfortunately, despite coaxing teachers' reflections on their use of the curricular resources, the post interviews did not move beyond general information. For example, the teachers said that they used the textbook for preparing lessons and assigning homework or class activity. They did not talk specifically about how they did this for specific lessons. The interviews thus did not provide post hoc insight into teachers' goals and intentions, and did not function as intended for further illuminating teachers' PDC in relation to their lessons. In the wider study, we only drew on the interviews to support our interpretations across teachers of their awareness of their textbook affordances. We return to this point when we discuss limitations of our study. We note this here to flag up for readers that the descriptions we build of teachers' PDC relied on their observable actions in their classroom lessons.

The data set for this chapter thus included the two editions of the prescribed textbook, the workshop handouts, and the video recordings of the three lessons observed. The lesson observations were all transcribed before the commencement of data analysis.

5.3.2 Data Analysis

The process of data analysis of the larger study entailed two main phases. In the first phase, the prescribed textbook was analysed for its affordances to the teachers' classroom. This process while it does not form part of the present chapter, became instrumental in the second phase of analysis when we were determining how the teachers mobilized these affordances. Each lesson was chunked into analysable episodes and the episodes were subjected to a process of coding for determining appropriate themes to determine how teachers mobilized their curricular resources.

5.3.2.1 Determining Textbook Affordances

The process of analysis for the larger study began with a thorough and detailed analysis of the affordances of the prescribed textbook in line with the *interpretation of and participation with resources theory* (Remillard 2005). In the analysis we looked for three main aspects: (i) content areas or subthemes covered under the topic of grade 10 functions; (ii) how the content was presented and sequenced; (iii) the embedded instructional approach of the textbook, and (iv) the conception of function that the textbook conveyed, that is, whether pointwise or global or a progression from pointwise to global strategies (Even 1998). This analysis does not form part of this chapter, and therefore we shall not elaborate deeply on it. However, it was important to note the four main subthemes expected to be taught and their sequencing as follows: *Introduction to function notation and terminology; determining properties of functions; interpreting functional properties;* and,

transformation of functions. These results made it possible for us to compare the content and sequencing of Mpho's lessons to those of the textbook to be able to determine how much of the textbook had been utilised in Mpho's lessons. The results would also enable us to determine what the teacher omitted from, or injected to the textbook content. We wish to reiterate the context for this study where an approved textbook is regarded as being representative of the official curriculum and therefore the goals of the textbooks can be expected to align with goals in the classroom.

5.3.2.2 Determining Degree of Appropriation

The first step in the analysis of the lessons was to utilise Brown's (2002, 2009) scale of artefact appropriation. By comparing Mpho's lesson content and its sequencing with that of the textbook we were able to determine whether the teacher was *offloading* or *adapting* or *improvising* the textbook content in each lesson. This provided an indication of whether or not the teacher utilised the prescribed textbook; and if they did, how much had been utilised. Where Mpho utilised curricular resources other than the prescribed textbook, we were also able to determine which those were.

5.3.2.3 Identifying Omissions and Injections

It is important to mention at this point that the main aim of the analysis of the lessons was to investigate how teachers used the prescribed textbook, especially in the classroom. However, after determining the degree of use as outlined above, we could not fully describe Mpho's PDC with the information at hand. At this stage we could only say how much or how little of the textbook Mpho had used, but the analysis could not help us say what it meant for the opportunities for mediation opened up in the classroom.

While we were determining the degree of textbook use by Mpho in the lessons, we were at the same time noting elements of content which were omitted by Mpho, and those which were not in the textbook but which Mpho inserted into her lessons. For example, the table for analysis of her first lesson was as follows:

The third column of Table 5.1 was used for any observations which we deemed important, and what started emerging was that there was textbook content which Mpho would omit, while at the same she would insert some content which was not available in the textbook. This led to the emergence of new analytical constructs. We named the content that Mpho omitted from her lessons as "Omissions". It was at this stage of analysis that we found we needed to make a distinction between content that was improvised and content that was newly inserted.

We described "Improvisations" as the content that was required at the grade level by the curriculum, and which therefore was available in the prescribed textbook, but which the teacher decided to bring from a different resource other than the

Table 5.1 An example of the process of analysis for lesson one

Episode	Degree of appropriation	Comments
Function notation	Offloaded	Mpho uses the CAPS textbook for definition
Worked examples	Offloaded	An omission: two other worked examples from textbook omitted in the lesson
The vertical line test for functions	Not in textbooks	An injection: the vertical line test for distinguishing between functions and non-functions inserted into the lesson
Practice exercises	Offloaded	Practice exercises do not include questions on the vertical line test

prescribed textbook. The "Injections" (or insertions) we then described as all content which the teacher introduced into the lessons but which was not specifically required by the curriculum at the particular grade level. This content would not be available in the prescribed textbook. In our analysis, the *omissions* and *injections* became the first indication of the teachers' ability to mobilise curricular resources and a demonstration of the "creative and constructive dimensions of teachers' instructional capacities" (Brown 2009, p. 29). We took the analysis to a new level where we further categorised the *omissions* and *injections*.

5.3.2.4 Categorising Omissions and Injections

Leshota (2015) building on our earlier work (Leshota and Adler 2014) categorizes injections into *robust* and *distractive*. *Robust injections* referred to injections which were enhancing the content while *distractive injections* led to erroneous mediation. Omissions were categorized into *productive*, for those that did not detract from the opportunities for mediation, versus *critical omissions*, that is, "those aspects of the object of learning that are critical to its mediation that teachers omit from the lessons" (Leshota 2015, p. 96). The question at this point would be how the researchers decided what was critical and what was not critical. We have already mentioned that there is a process of selection and approval of prescribed textbooks in South Africa by the Department of Basic Education. The textbooks have to align themselves with the curriculum statement, but more than that, they align with the expected teaching sequence in each grade as well. If a particular textbook is approved as a prescribed textbook in a certain grade, then it means the textbook has satisfactorily sequenced the topics in a way teachers are expected to teach in that grade, but most importantly, the textbook prescribes the minimum requirement on content expected in that classroom.

In the analysis hence, if the teacher omitted content that was outlined as a minimum requirement in the textbook, and coupled with our own knowledge of mathematics, we were able to determine whether such an omission would be critical to the opportunities for learning the particular aspect of the topic or not. We also

thought about what if the teacher omitted the particular aspect at that time but could still pick it up at a later date? We resolved that while this was a possible scenario, our focus was on the actions at the time in the classroom. We wanted to find out what was the space for learning the teacher was opening.

The constructs of *omissions* and *injections* thus emerged as the result of the interaction between theoretical resources and the empirical texts, that is, the lessons themselves, in this study. We used these constructs to develop an analytical framework that illuminates types of teacher–resource relationships at play. The relationships were then used to evaluate Mpho's PDC, and PDC for the rest of the teachers in the study.

We now elaborate on the teacher–resource relationships which we deduced from analysing the *omissions* and *injections* in Mpho's lessons.

5.3.2.5 Classifying Teacher–Resource Relationships

Our definition of *robust injections*, that is, injections that are enhancing, suggests an existence of a relationship between the teacher and the resource. This is a teacher who is able to mobilize the resource productively. Similarly, we infer that a teacher who omits content that does not detract from what is being learned has some prior transaction with the textbook. We argue that the presence of *robust injections* and *productive omissions* (omissions that are not distractive) indicate deliberateness in the teacher's use of the resource. We further argue that *deliberateness* reflects a *participatory* resource use, and we describe the relationship that is forged between the teacher and resource in this case as an *intimate relationship*. If the teacher–resource relationship is *intimate*, then the teacher's capacity for pedagogic design, her *PDC is high*.

On the other hand, the teacher who *omits critical* content that is available in the resource raises questions about her interaction with the resource. We argue that this is a reflection of a non-participatory and therefore non-deliberate interaction with the resource. We have referred to this kind of resource utilisation as *tacit* (Polanyi 1967), and argue that relationships of this kind lack intimacy and imply *low* levels of PDC.

We draw similar conclusions for *distractive injections*: injecting content that is not enhancing and which may lead to erroneous mediation, while it might say something about the teacher's subject matter knowledge (SMK) (Shulman 1986), is indicative of a *tacit* use of the resource. We consequently conclude that the presence of *critical omissions* or *distractive injections* in a lesson suggest low PDC.

The categorizations of omissions and injections, their implications for resource use, and subsequent teacher–resource relationships forged are summarized in Table 5.2.

Table 5.2 shows that an *intimate* teacher–resource relationship, and consequently high PDC is produced from the combination of *robust injections* and *productive omissions* only. The rest of the combinations result in tacit textbook use, teacher–resource relationships which are not intimate, and PDC levels that are not

Table 5.2 Relating omissions and injections with teacher-textbook relationships

	Robust injections	Distractive injections
Productive omissions	Deliberate and participatory textbook use	Tacit textbook use
	Intimate teacher–resource relationship	Teacher–resource relationship not intimate
	High PDC	PDC not high
Critical omissions	Tacit textbook use	Tacit textbook use
	Teacher-textbook relationship not intimate	Teacher-textbook relationship not intimate
	PDC not high	PDC low

high. The combination of *critical omissions* and *distractive injections* indicate low levels of teachers' PDC.

Thus, the constructs of *omissions* and *injections* as these emerged through the analytic process in the wider study provide this chapter with indicators for *deliberate* versus *tacit* resource use; and *intimate* teacher–resource relationships versus relationships that lack intimacy. In the next section, we outline the results from the analysis of Mpho's three lessons.

5.4 Results

5.4.1 Mobilization in Lesson One

5.4.1.1 Degree of Appropriation

In the first lesson, Mpho introduced the functional notation, $f(x)$, and how to evaluate function values if given a function, $f(x)$. The lesson began with Mpho copying notes from the textbook page to the chalkboard before she explained them. Figure 5.2 shows the notes Mpho had copied alongside the textbook page she was copying from.

Figure 5.2 shows that Mpho copied the notes from the textbook almost word for word. After explaining how to evaluate the function value, $f(-1)$ given that $f(x) = x^2 + 1$, Mpho gave learners an example which she worked on the chalkboard guiding the learners on the procedure for evaluating $f(3)$, $f(-3)$, and $f\left(-\frac{3}{2}\right)$ given the function $f(x) = 2x - 3$. This example is shown in Fig. 5.3 alongside a textbook page of exercises for evaluating function values.

Figure 5.3 shows that Mpho has used the same example from the textbook as well. So far, the two examples from the textbooks have been used by Mpho in her lesson. As learners continued to write in their books, Mpho wrote the following notes on the chalkboard, and this time she was not copying from a book or any other resource.

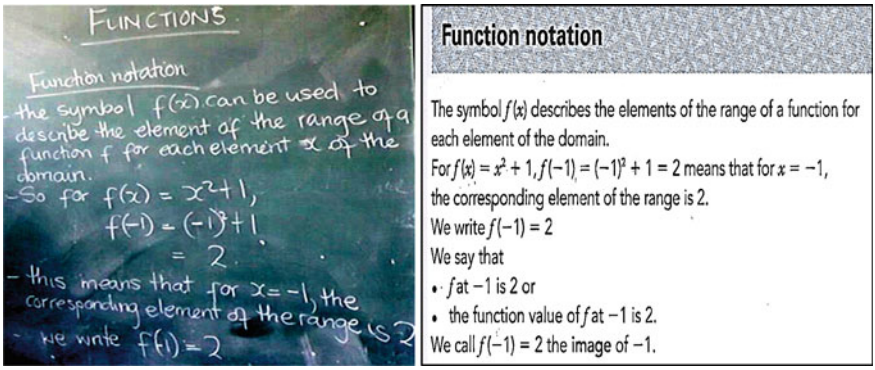


Fig. 5.2 Mpho's notes in lesson one alongside a textbook page

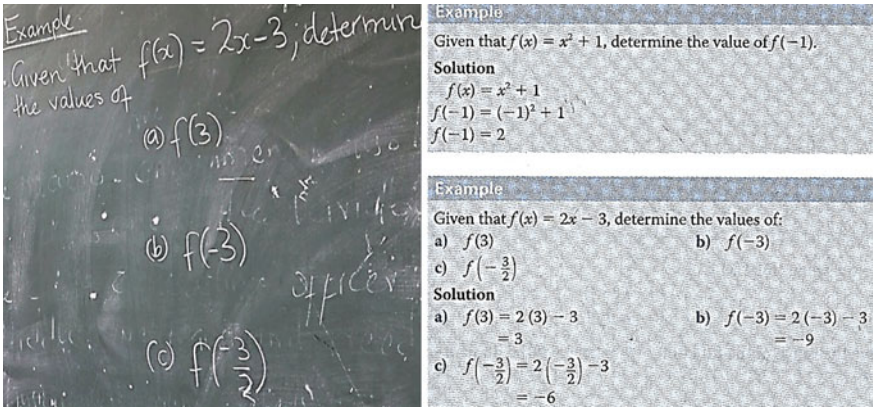


Fig. 5.3 An example in lesson one alongside examples from the textbook page

The information in Fig. 5.4 about the *Vertical line test* which is used for determining whether graphs in the Cartesian plane represent functions or not, is not available in the textbook. Mpho spent some few minutes demonstrating on the chalkboard to learners how the vertical line test was used, after which she assigned a class activity, shown in Fig. 5.5a. We noted that up to this point, the vertical line test had been the only aspect which Mpho had not extracted from the textbook.

Mpho wrote the class activity on the chalkboard for learners to work on individually as she moved around helping them.

The exercises from Fig. 5.5a above were extracted from two different exercises from the textbook, shown in Fig. 5.5b.

Figure 5.5b shows that Mpho took questions 1 a), b), and c) from Exercise 7.1 of the textbook and question 1 of Exercise 7.2 to design a class activity.

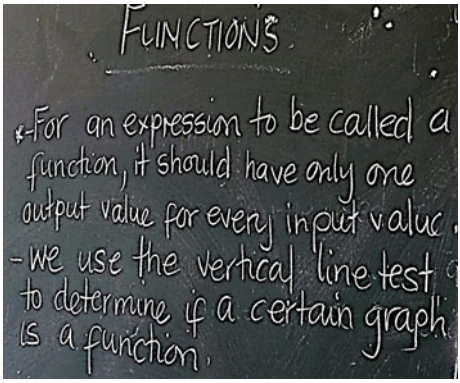


Fig. 5.4 Notes on the vertical line test

All in all, in lesson one, all content which Mpho used except for the vertical line test, she had extracted from the textbook. We concluded therefore that lesson one was an *offloaded lesson* because only a small part of that lesson did not come from the textbook.

5.4.1.2 Omissions and Injections

Injections

We begin with injections in this lesson because there were very few of them. The only injection in the lesson is that of the *vertical line test*, as we have already shown. It is considered as an injection instead of an improvisation because it is not stipulated in the textbook, and therefore its absence from the prescribed textbook indicate that it is not regarded as part of minimum requirements of the curriculum. However, the vertical line test is regarded as an important aspect of the teaching and learning of functions which is featured in many textbooks on functions. It is used as a visual means to distinguish between graphs in the Cartesian plane which represent functions and those which do not. In fact, it was featured in the workshop on functions organised and conducted by the WMCS for teachers in the project schools which Mpho also attended. We therefore regard Mpho's inclusion of the vertical line test as enhancing to the learning of functions, and categorise it as a *robust injection* hence.

Omissions

Even though this lesson was largely composed of content from the textbook, we observed that Mpho did not utilize all the content which the textbook had provided. As Fig. 5.5b shows, in Exercise 7.1 Mpho omitted questions 1 d), e), question 2 and question 3. In Exercise 7.2, she only used question 2 and omitted all the other seven

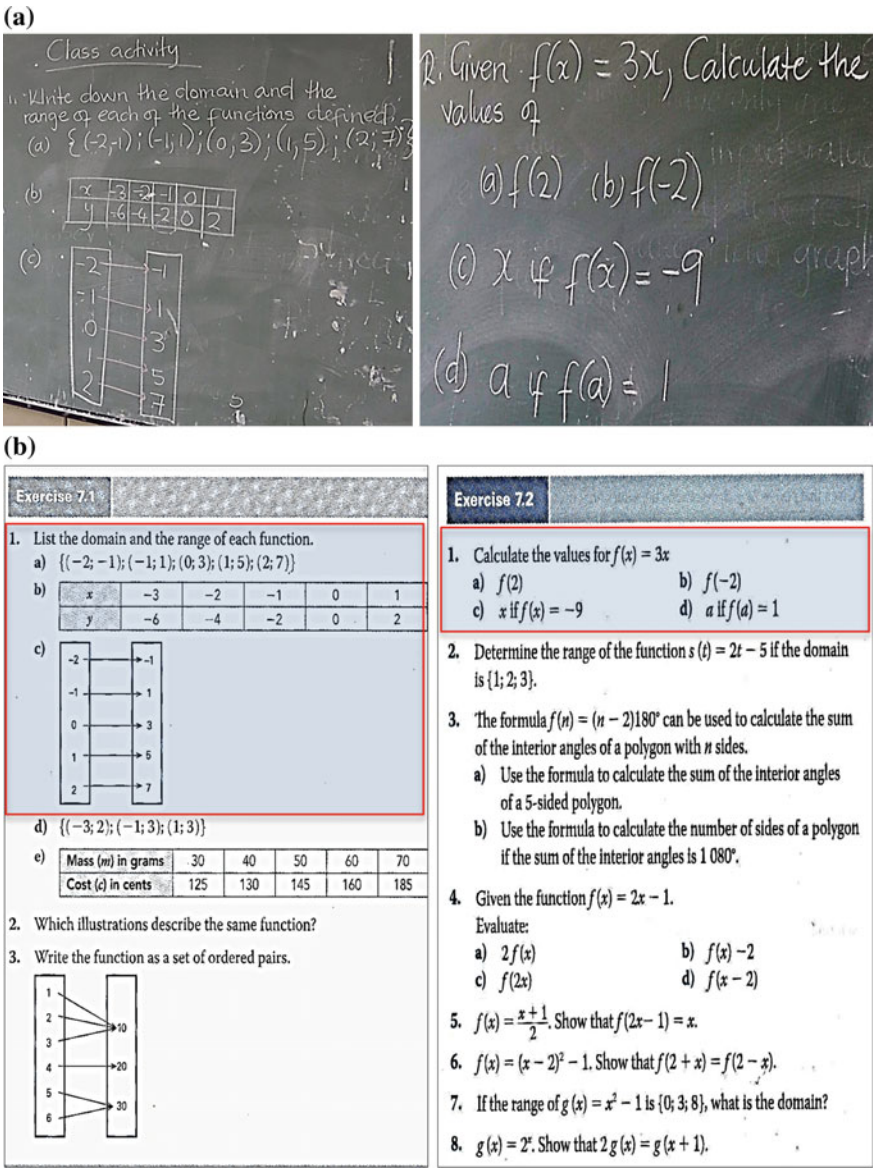
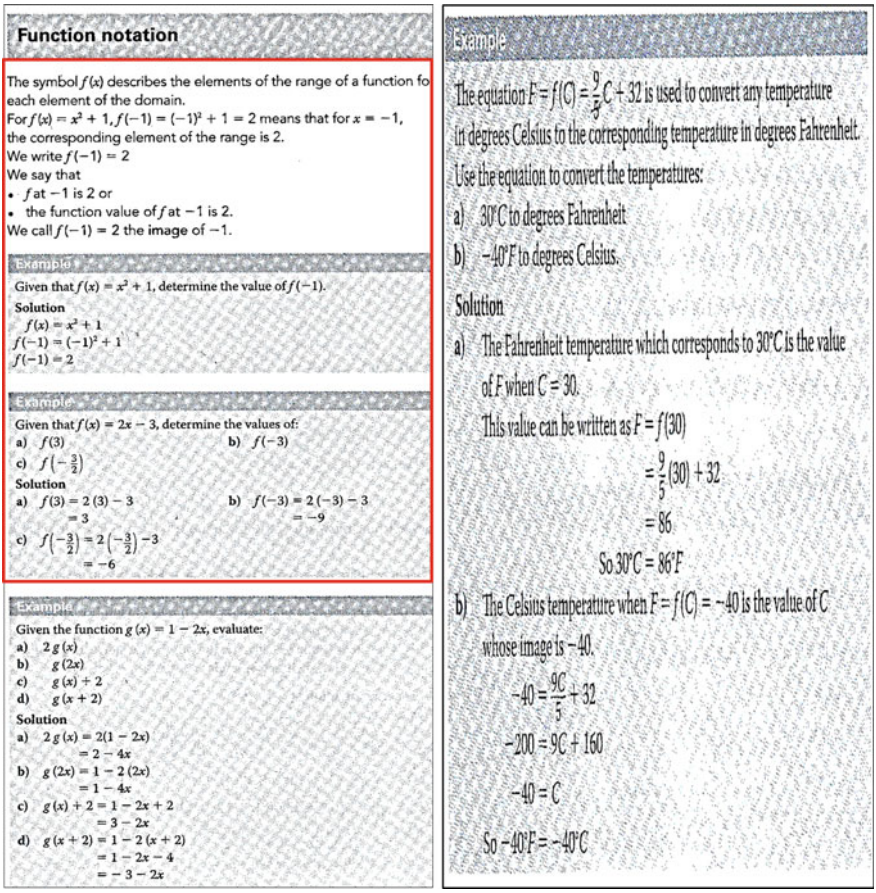


Fig. 5.5 **a** Class activity in lesson one. **b** Textbook exercises from where Mpho developed the class activity

questions. In fact, these were not the only omissions in the lessons as indicated in Fig. 5.6.

In the lesson, Mpho omitted the last example indicated in Fig. 5.6 on the left diagram on evaluating functions $2g(x); g(2x); g(x)+2;$ and $g(x+2)$ given that



Function notation

The symbol $f(x)$ describes the elements of the range of a function fo each element of the domain.
For $f(x) = x^2 + 1$, $f(-1) = (-1)^2 + 1 = 2$ means that for $x = -1$, the corresponding element of the range is 2.
We write $f(-1) = 2$
We say that
- f at -1 is 2 or
- the function value of f at -1 is 2.

We call $f(-1) = 2$ the image of -1.

Example

Given that $f(x) = x^2 + 1$, determine the value of $f(-1)$.

Solution

$f(x) = x^2 + 1$
$f(-1) = (-1)^2 + 1$
$f(-1) = 2$

Example

Given that $f(x) = 2x - 3$, determine the values of:
a) $f(3)$ b) $f(-3)$
c) $f\left(-\frac{3}{2}\right)$

Solution

a) $f(3) = 2(3) - 3$
$= 3$

b) $f(-3) = 2(-3) - 3$
$= -9$

c) $f\left(-\frac{3}{2}\right) = 2\left(-\frac{3}{2}\right) - 3$
$= -6$

Example

Given the function $g(x) = 1 - 2x$, evaluate:
a) $2g(x)$
b) $g(2x)$
c) $g(x) + 2$
d) $g(x + 2)$

Solution

a) $2g(x) = 2(1 - 2x)$
$= 2 - 4x$

b) $g(2x) = 1 - 2(2x)$
$= 1 - 4x$

c) $g(x) + 2 = 1 - 2x + 2$
$= 3 - 2x$

d) $g(x + 2) = 1 - 2(x + 2)$
$= 1 - 2x - 4$
$= -3 - 2x$

Example

The equation $F = f(C) = \frac{9}{5}C + 32$ is used to convert any temperature in degrees Celsius to the corresponding temperature in degrees Fahrenheit. Use the equation to convert the temperatures:

a) 30°C to degrees Fahrenheit

b) −40°F to degrees Celsius.

Solution

a) The Fahrenheit temperature which corresponds to 30°C is the value of F when $C = 30$.
This value can be written as $F = f(30)$

$= \frac{9}{5}(30) + 32$
$= 86$

So $30°C = 86°F$

b) The Celsius temperature when $F = f(C) = -40$ is the value of C whose image is −40.

$-40 = \frac{9C}{5} + 32$
$-200 = 9C + 160$
$-40 = C$

So $-40°F = -40°C$

Fig. 5.6 Worked examples from the textbook

$g(x) = 1 - 2x$. She omitted the question on evaluating functions based on temperatures on the right diagram of Fig. 5.6 as well.

We begin the categorization of the omissions with these worked examples.

Omission of $f(x)$ as an entity in its own right

In the worked example on the left diagram in Fig. 5.6 which Mpho omitted, the function $g(x)$ is regarded as an object that can be operated on in its own right, for example, as in adding a number 2 to the object, or multiplying the object by 2. This example is different from the other examples and exercises Mpho was doing with learners in the classroom. In the classroom, the input value was given as an integer, for example, -1, and through a process of substituting x for the given integer, a function value, which was another integer was obtained. With the omitted example came a new concept where there was no substitution of integers, and the outputs were no longer integers but other functions.

We observed further that Mpho actually omitted all similar questions in the practice exercises of Exercise 7.2 (see Fig. 5.5b).

Looking at what Mpho was doing in the lesson, that is, getting learners to substitute a value for the input given by x in the equation, and then obtaining another integer as the output, the omission of these examples did not detract from what was being learned in the lesson. In order to introduce this new aspect, for example, $2g(x)$, Mpho would need to bring in the aspect of transformations of functions, which actually, came later in the curriculum after determining the functional properties. We therefore regarded not using these examples in the present lesson as a productive omission. It is however worth noting that Mpho did not completely ignore these kinds of examples as they were part of her second lesson.

Omission of 'applied' functions

The worked example on the right diagram in Fig. 5.6 used a real life application of functions involving temperature. We termed functions of this type as 'applied' functions in the analysis. This worked example and a similar question (question 3, Exercise 7.2) in Fig. 5.5b were omitted from the lesson by Mpho. Question 3 in Exercise 7.2 expressed the sum of interior angles of a polygon, as a function of the number of its sides, n, that is, $f(n)$. We considered the 'applied' functions as particular types of questions which served a particular purpose of using mathematics to understand phenomena in the real world. In another 'applied' example (see Fig. 5.5b) the input of a function was expressed in terms of the Mass in grams, and the output as the Cost in cents. We argue that while the application of mathematics to real life is important, as is mostly desired and encouraged in the teaching and learning of mathematics, in this particular lesson of introducing function terminology and notation, it was not critical in that its omission did not detract from the opportunities for mediation. As in the case of function as an entity which Mpho omitted, we regarded this type of omission as well, as being *productive.*

Omission of questions with similar structure

At the same time, we observed that question 1 e) in Exercise 7.1 was similar in structure to 1 b) which was assigned in the class activity, the difference being that in 1 b), the inputs were represented by the variable, x and the outputs by the variable, y. Similarly, question 1 a) and 1 d) had the same structure, and therefore omitting 1 d) did not detract from the opportunities for mediation. Learners had done similar things in question 1 a). These omissions were once more, *productive* omissions as opposed to being distractive.

In summary, lesson one was an *offloaded lesson* in which *robust injections* occurred and the *omissions* made were all *productive.*

5.4.2 Mobilization in Lesson Two

5.4.2.1 Degree of Appropriation

In the second lesson, Mpho did not use the content from the prescribed textbook, but drew content from a workshop on grade 10 functions organized by the WMCS project for all teachers in the project schools. This particular workshop done once a week over a period of four weeks looked at the following key features of school functions:

- Key aspects of functions: *definition; domain and range; static points (intercepts and turning points); function behaviour (symmetry, gradient, concavity, end behaviour, continuity)*
- Transformation of functions: *vertical and horizontal shifts; vertical and horizontal stretches; reflections*
- Multiple representations of functions: *algebraic; verbal; graphical; and numeric*
- Ways of approaching functions: *pointwise; global; function as object*
- Applications of functions in the real world.

The content aligned with the curriculum specifications but was organized and packaged differently from the textbook. For example, lesson two began with Mpho drawing a table of values shown in Fig. 5.7 and asking learners to provide answers to the missing cells on the table. The given function for the activity is, $f(x) = x^2$. This activity had been part of the workshop activities for the teachers with two main foci: to determine the effect on the graph of $f(x) = x^2$ of the transformations of the form: $f(x) + a$, $f(x + a)$, $af(x)$ and $f(ax)$; and to contrast the forms $f(x + a)$ and $f(x) + a$, as well as the forms $f(ax)$ and $af(x)$. For Mpho's second lesson, learners

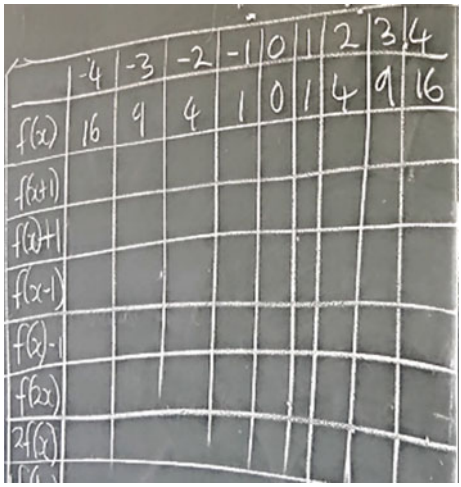


Fig. 5.7 Transformations of $f(x) = x^2$ in lesson 2

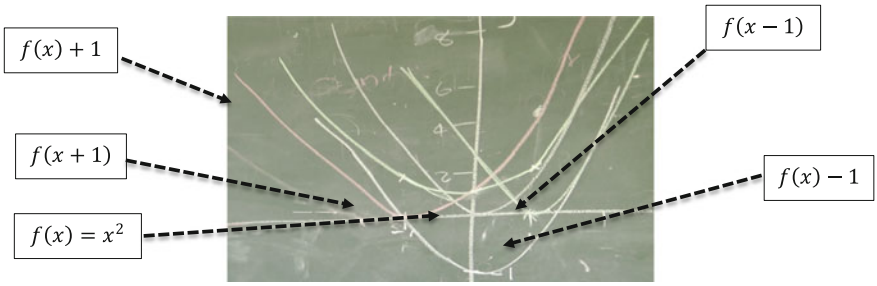


Fig. 5.8 Graphs of transformations of $f(x) = x^2$

had been assigned this table to fill out as homework and the process of completing it in the lesson went very quickly with learners calling out the answers and Mpho filling in the cells.

When the table of values had been completed, Mpho drew a sketch of the graph of $f(x) = x^2$ on the chalkboard and invited a few learners to come to the chalkboard to draw the rest of the functions on the same set of axes, using different colours. Up to the function $f(x) - 1$ the graphs looked as illustrated in Fig. 5.8.

When all graphs had been drawn, Mpho led a discussion with learners that concluded that functions of the form $f(x) + a$, resulted in vertical shifts of $f(x) = x^2$ while the functions of the form $f(x + a)$, resulted in horizontal shifts of $f(x) = x^2$.

The vertical shifts are a grade 10 curriculum requirement and would therefore be available in the textbook. In the textbook, however, the presentation is different as it does not include the contrasting of $f(x + a)$ with $f(x) + a$. In the textbook, the activities on transformations of the quadratic function are as shown in Fig. 5.9.

In the textbook, the forms $af(x)$ are dealt with separately from $af(x) + q$ as Fig. 5.9 shows. Mpho used the materials from the workshop instead of the textbook. Since the material Mpho used was available in the textbook, this was an *improvised* lesson.

5.4.2.2 Omissions and Injections

We noted that functions of the type $f(x + a)$ and $f(ax)$, that is, horizontal shifts and stretches respectively, have been included in the table of values in Fig. 5.7. These functions are not required at grade 10, and therefore their inclusion is considered an *injection* of content in this lesson. The question to ask is what purposes did they serve at this grade? We argue that the inclusion of these functions was significant in showing the differences in the shifts of the graph of $f(x) = x^2$. During the lesson, Mpho focused learners' attention on these differences, suggesting it was her intention that these shifts be observed. The transcript below shows how Mpho contrasted the graphs of $f(x + 1)$ and $f(x) + 1$.

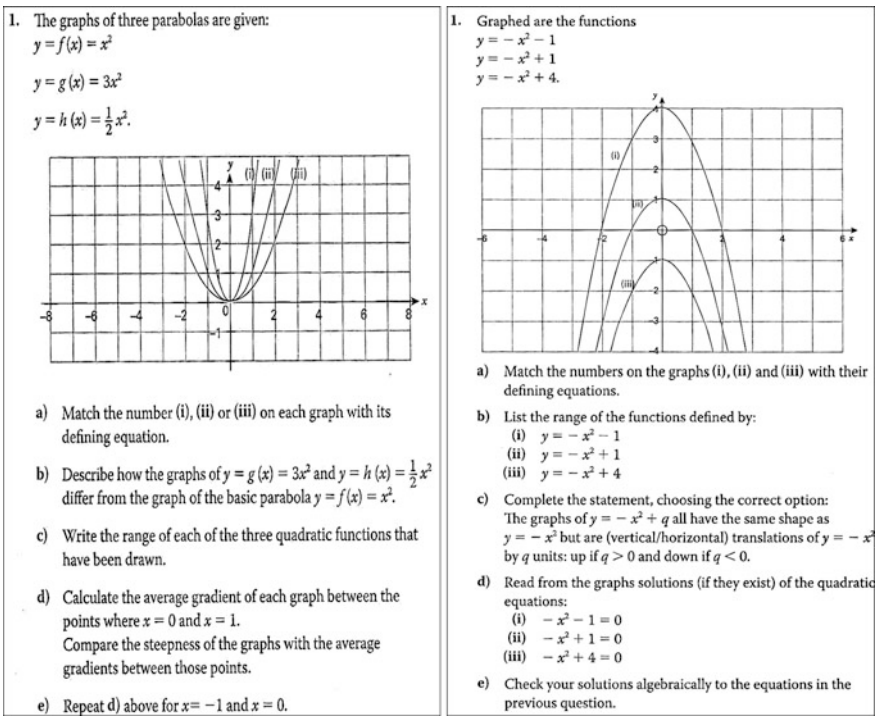


Fig. 5.9 An example of activities on transformations of $f(x) = x^2$ in the textbook

Mpho	Okay, so this is a shift in the *y*-axis and remember here we added one inside and now this one is being added to the function (*points to* $f(x) + 1$). Okay?
Students	Yes.
Mpho	So, when you add one inside of the bracket, the movement is being affected in which axis? (*Pointing to* $f(x + 1)$). In the *x*-axis. Now we added outside the function, in the function plus one the movement is going to be affecting the *y*-axis. Okay?
Students	Yes.

Thus, the inclusion of the horizontal shifts and stretches enhanced the lesson. Instead of waiting to do these shifts and stretches at grade 11, the learners were given an opportunity to 'see' the difference between a parameter forming part of an argument of a function and when it was not part of the argument. We thus considered these to be *robust injections*. With regard to transformations of the function, $f(x) = x^2$, we could not identify any omissions in this lesson.

In summary, the second lesson was an improvised lesson from the workshop handouts. There were no apparent *omissions* when we compared the lesson content to the textbook content, but Mpho made *robust injections* to the content nevertheless.

Draw sketch graphs of the following functions, showing clearly
- any intercepts on the axes
- the coordinates of one other point on the graph (the turning point or vertex where applicable)
- asymptotes and/or axes of symmetry should be shown where appropriate.

Fig. 5.10 An extract of the procedure for sketching graph

5.4.3 Mobilisation in Lesson Three

5.4.3.1 Degree of Appropriation

The third lesson began with the correction of homework questions where Mpho worked out answers to the questions on the chalkboard. In the homework learners had to find equations of given graphs using the information provided. An inspection of the textbook showed that all the questions came from an end of chapter exercise in the textbook. After the correction of homework, Mpho introduced the topic on sketching functions, and began this part of the lesson by reading out and explaining the procedure (presented in Fig. 5.10) to be followed when sketching functions as outlined in the textbook.

The procedure in Fig. 5.10 outlines features that are key in sketching graphs of functions, such as: *the intercepts, turning points, asymptotes, axes of symmetry.* In order to demonstrate the procedure to the learners, Mpho used one of the functions, $f(x) = \left(-\frac{3}{2}\right)x^2$, from the textbook, as an example. After the demonstration there was very little time left and Mpho used the few minutes left to assign homework questions to be discussed in the next lesson. The homework questions too were selected from the textbook.

This third lesson hence was an *offloaded* lesson as all content which was dealt with in the lesson came from the textbook.

5.4.3.2 Omissions and Injections

We did not observe any injections of content in this lesson and therefore the discussion in this section shall be about the omissions only.

We firstly checked the Exercise in the textbook from where the homework which Mpho and learners were correcting at the beginning of the lesson came. There were two questions in this Exercise. The first question contained six graphs and a general equation of each graph was given. For example, for the straight line graph, the equation was given as $y = a.x$, for the hyperbola as $y = \frac{a}{x}$, for the quadratic function graph as $y = a.x^2$, and so forth. There was additional information provided on the graph, for example coordinates of one or two points. Learners were to use the

information to find the value of a for each graph. To answer the question, learners needed to substitute a value of x and y in the given general equation and solve for a. Mpho had selected two graphs out of the six given for homework from this question. These were an upward facing parabola, and an increasing exponential graph.

The second question consisted of eighteen (18) graphs. Learners had to first decide to which of the given general equations the graph belonged. For example, the general equations provided included: $y = \frac{a}{x}$, $y = a.b^x$, $y = ax$, $y = ax^2$, $y = a\sin x$, $y = a\cos x$, and $y = a\tan x$. These general equations involve all the seven different functions learners are expected to learn at grade 10. Using the information provided on each graph, learners had to determine the equation of each graph. Mpho assigned four graphs in this question which included: a downward facing parabola, a decreasing exponential graph, a tangent function graph, and a hyperbola.

In the first place, there were numerous graphs, and thus too many for all to be assigned for homework. A selection was required. We observed that in the first question, an upward facing parabola was chosen and in the second question, a downward facing parabola. Similarly for the exponential function, in the first question, the chosen graph was increasing and a decreasing one chosen in the second question. This showed Mpho's deliberateness in the choice of questions for homework. The selection does not look haphazard but seems to have been deliberately worked out: The choices would illuminate the differences in the values of a in $y = a.x^2$ which depended on the direction the parabola was facing. Similarly for the exponential function, $y = a.b^x$. A trigonometric graph and a hyperbola were also chosen ensuring that the homework included all different functions studied and the homework was thus representative of key issues to be learned in determining equations of graphs at this grade. The selection of the homework exercises reflected a deliberate choice, where the omissions made were not distractive. We made similar observations regarding the homework which was assigned at the end of lesson three. There was no time left in the lesson for learners to practice the sketching of graphs. The homework was assigned quite hurriedly and Mpho called out the first four graphs out of the seven for homework. We regarded these omissions as a result of time constraints.

We made a further consideration of omissions in this lesson by looking at the demonstration and subsequent discussions of the procedure for sketching graphs of functions. We saw the four key features as outlined by the textbook when sketching graphs of functions as: *the intercepts, turning points, asymptotes,* and *axes of symmetry* (see Fig. 5.10). As Mpho illustrated the procedure to the learners, we observed that she guided them to determine the x- and y-intercepts of the graph by equating the function to zero and finding the x values which caused this, and substituting $x = 0$ in the equation, respectively. Mpho guided learners to determine that the graph would face downwards because the coefficient of x^2 was negative. However, we noted that Mpho never mentioned the axes of symmetry for the graph. This feature was explicitly mentioned in the procedure in the textbook as one of the

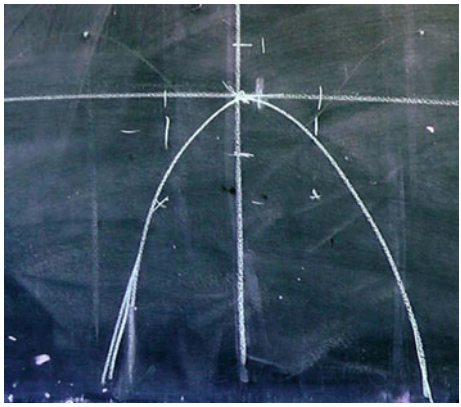


Fig. 5.11 Mpho's sketch of the graph of $y = -\frac{3}{2}x^2$

key characteristics in sketching graphs. What Mpho did was to guide learners to determine that the graph would be passing through the points $\left(-1, -\frac{3}{2}\right)$ and $\left(1, -\frac{3}{2}\right)$. This in our opinion had also provided an opportunity for learners to see that the one point was a reflection of the other on the *y*-axis, and therefore an opportunity to 'say' something about the axes of symmetry, but this did not happen. Mpho drew the sketch of the graph on the chalkboard as shown in Fig. 5.11 but still made no mention of the axes of symmetry.

The sketch of the graph in Fig. 5.11 is not quite symmetric. On the one hand, this is understandable considering that Mpho was using a free hand to sketch the graph. On the other hand, the fact that Mpho did not mention the property of the axes of symmetry for this graph made us wonder if she had paid attention to the symmetry in her sketch. This was a problem for us: If a learner were to sketch the graph of a hyperbola and did not mention an asymptote, we would have the same problem. We considered the omission of the feature of axes of symmetry as a *critical omission*.

One of the content areas for grade 10 functions has been mentioned as determining the properties of the functions being studied which include the quadratic function. The feature of symmetry is fundamental to quadratic functions and is one distinguishing feature of quadratic functions. We consider it distractive if the feature of symmetry is not mentioned in the procedure to illustrate how to sketch the graphs of functions. This is an activity that demands one to be conversant with the critical features of the different functions, as the opportunity to graph the function using point by point methods and be accurate in the graphing has been taken away. For these reasons, we regard the omission of determining the axes of symmetry in sketching the function $f(x) = -\frac{3}{2}x^2$ as a *critical omission*.

The analysis of Mpho's three lessons has been summarized in Table 5.3 and indicates the degree of appropriation, and the different types of omissions and injections made.

The summary in Table 5.3 shows that Mpho offloaded content in two out of the three lessons, and improvised in one lesson, meaning that for the three lessons, the

Table 5.3 A summary of Mpho's mobilization of curricular resources

Lesson	Mobilizing of the textbook	Omissions and/or injections
1	Offloaded	Robust injections Productive omissions
2	Improvised	Productive omissions Robust injections
3	Offloaded	Productive omissions Critical omissions

textbook remained Mpho's main resource. With respect to *omissions* and *injections,* the results show that in lessons one and two, Mpho produced a combination of *robust injections* and *productive omissions*, whereas in lesson three both types of *omissions*, *productive* and *critical* occurred. We further note that there were no *distractive injections* in all the three lessons.

In the next section we describe the implications for Mpho's PDC as determined through the analysis of the *omissions* and *injections* she made in the three lessons.

5.4.4 Determining Mpho's Pedagogical Design Capacity

What do we know about Mpho's mobilisation of the available resources from her lessons? We know that Mpho utilised available resources in the form of the textbook and the workshop handouts in her lessons, but mostly the textbook, from our observation that she offloaded in two lessons and improvised in one. We also know that in all the three lessons, there were no *distractive injections*, meaning that Mpho, while she was able to improvise content from other resources than the textbook, did not introduce in her lessons content that would reduce the opportunities for mediation. Instead, the injections in her lessons were *robust,* meaning that when she brought in content which was not yet required at grade 10 level, Mpho only brought in content that enhanced her lessons. As our analytical framework (Table 5.2) suggests, this is indicative of deliberateness in Mpho's mobilisation of the available resources, and therefore suggests an *intimate* teacher–resource relationship between Mpho and her available curricular resources.

Furthermore, when it came to omissions in the lessons, in all the three lessons, Mpho made *productive omissions* of content, meaning that in all her lessons, she had omitted content from the textbook in a way that did not detract from the goals of the lesson. We appreciate that the content made available in the textbook is important, the analysis process has also revealed that there are many valid reasons why any teacher would find themselves in a situation where they may need to cut on some content, including time constraints, for example. More indepth analysis may be needed including asking the teachers about their process of selection to understand why they omit content that is available in the textbook and therefore regarded as important. However, what we are saying about the *productive*

omissions in the lessons is that Mpho's selection of content to include or to omit did not seem haphazard as it did not detract from the goals of the lesson. This serves as a confirmation of a participatory way of interacting with the resources, and again, a suggestion of an intimate teacher–resource relationship. To reiterate, PDC reflects the teacher's ability to create "deliberate, productive designs" (Brown 2009, p. 29) for her classroom.

Up to this point, all the signs of a high level of capacity for pedagogic design are showing for Mpho: the *intimate* relationship with the resources forged through Mpho's ability to weave together affordances from different resources; deliberate and participatory resource use that ensures only *robust injections* and no *distractive injections;* and *productive omissions* which do not harm the opportunities created in the classroom.

However, in continuing about what we know about Mpho's mobilisation of the resources, we know that Mpho made a *critical omission* of content as well in lesson three. That is, Mpho omitted content that was considered to be critical in the learning of the particular object, but most importantly, this content was available in the textbook/resource she was utilising at that time in the classroom. We regard this *critical omission* as being indicative of a "break" in communication between Mpho and the resource, and in terms of our analysis, indicative of *tacit use*. However this shows some inconsistency in how Mpho transacted with the resources. The analysis shows that overall, Mpho's pattern of use across the lessons is one of participation, and *intimate* relationships with her resources, and therefore with respect to PDC, we conclude there are relatively high levels of PDC for Mpho. The occurrence of this unevenness suggests how important the interview could have been in probing this inconsistency.

5.5 Discussion and Conclusion

We set out in this chapter to disaggregate or 'unpack' Mpho's capacity to mobilise the resources available to her effectively in her lessons, and we have done so, even though not without challenges. The analysis of Mpho's lessons has shown that a teacher's PDC manifests itself in the kinds of teacher–resource relationships that the teacher forges with her curricular resources. We have demonstrated that *intimate* teacher–resource relationships occur as a result of deliberate and participatory use of the resources thus demonstrating high levels of PDC. We have presented the notions of *omissions* and *injections* as indicators for determining deliberateness in use and argue that these analytical constructs provide a lens for describing teachers' capacity and competence to mobilise their curriculum resources in productive ways.

However, we recognise that although the final product appears to be smooth, we have seen that the analytical process is not smooth and clean, but involves a fair amount of complexity. This implies that the theoretical base for these notions still needs tightening, but we reiterate that they have a potential in disaggregating teachers' PDC.

Teachers' PDC is defined as the teachers' capacity to perceive the affordances of the resources and to then mobilise them to create instructional designs that respond favourably to teachers' classroom needs. Due to challenges which we encountered when trying to determine the teachers' processes of perception of affordances through interviews, we missed out on this critical aspect of teachers' PDC. The implication is that our story in this chapter is not complete and suggests that there is more work that still needs to be done to complete the description of teachers' PDC. We have mentioned that we could not obtain the information which we needed from the interviews which we conducted with the participating teachers, and we suggest that one of the challenges with teachers in the South Africa context could well be the high level of prescription in the textbooks which has a potential to mask the individual teacher's goals and intentions.

Secondly, research has determined that curricular resources do not only provide the teacher's practice with content, but with instructional approach as well (Ensor et al. 2002; Leshota 2015). The *omissions* and *injections* as used in the chapter take the instructional approach of resources into the background. This is another limitation of the study which needs to be explored further.

In concluding this chapter, we wish to comment that while the world at large seems to be moving towards digital resources, the situation in countries such as ours remains that of a print textbook still being the most accessible resource for teachers and learners alike, and therefore a very precious resource. The need to re-source (Adler 2000, 2012) teachers and to help the teachers, teacher educators, policy makers, and textbook authors understand that availability of textbooks does not imply use (Adler 2000) through concerted efforts to research the teacher–resource relationships and conduct professional development on resource use remains as intense as it has ever been.

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Author Biographies

Moneoang Leshota currently holds a Postdoctoral Fellowship in Mathematics Education at the University of the Witwatersrand (WITS) in South Africa (2017–2019), working with Professor Jill Adler on teacher education and resources. Her research focused on teacher–textbook relationships especially the teachers' capacities to mobilise the affordances of their mathematics textbooks for effective classroom mediation. She is currently extending this focus to include other instructional resources that are being purposefully introduced by government into the mathematics classroom as teacher guides for instruction. Prior to pursuing her doctoral studies she was a member of the teaching staff in Mathematics at the National University of Lesotho (NUL) for over two decades, with her main teaching assignments being at first year for science-related fields. It was her experiences of teaching first year mathematics which made her want to learn more about what it took to teach and learn mathematics better and subsequently led her to changing her research interests from mathematics to mathematics education.

Jill Adler holds the SARChI Mathematics Education Chair at the University of the Witwatersrand, which focuses on research and development in secondary mathematics education, and is the President of the International Commission on Mathematical Instruction (ICMI). Jill has spearheaded several large-scale teacher development projects. The most recent one, within the Chair ambit begun in 2009, is called the Wits Maths Connect Secondary project. This work builds on her research on teaching in multilingual classrooms, and teacher professional development. Jill is a Visiting Professor of Mathematics Education at King's College London, UK. She is the recipient of numerous awards, the most significant of which are the 2012 Academy of Science of South Africa (ASSAf) Gold Medal for Science in the Service of Society, and the 2015 Freudenthal Award.