Chapter 3
A Comparative Study on the Presentation of Geometric Proof in Secondary Mathematics Textbooks in China, Indonesia, and Saudi Arabia

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Abstract This chapter presents a comparative study aiming to examine how geometric proof is treated in secondary school mathematics textbooks in China, Indonesia, and Saudi Arabia, and explore the similarities and differences revealed in these three countries' textbooks. The results show that, although all the selected textbooks from these countries introduced mathematics topics related to geometric proof, they differed considerably in three aspects: the number of examples, the distribution of contents and, to a lesser degree, the types of proof. The textbooks in China contain the highest percentage of geometric contents and pay the most attention to the topic of geometric proof itself. The national mathematics curricula are clearly a main factor for the differences revealed.

Keywords Mathematics teachers' resources · Mathematics textbook research Geometric proof · Chinese mathematics education · Indonesian mathematics education · Mathematics education in Saudi Arabia

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3.1 Introduction and Background

School textbooks have been the subject of research internationally for quite a long time. In the mathematic subject, school textbooks have also received increasing attention in the international research community over the last few decades (Fan et al. 2013). More recently, the inaugural International Conference on Mathematics Textbook Research and Development (ICMT) held in 2014 in Southampton, UK, and the second ICMT held in 2017 in Rio de Janeiro, Brazil attracted many researchers around the world (Fan and Wu 2015; also see www.im.ufrj.br/~ictm2).

In this paper, we report a part of a larger scale study that we have conducted recently comparing China, Indonesia, and Saudi Arabia mathematics textbooks. In a previous study, we examined the mathematics textbooks in those three countries at the primary schools level, aiming to document and analyse how the three countries introduce the equality and inequality of whole numbers. The results revealed a high level of consistency in the way of introducing the comparison of whole numbers in the textbooks across the three countries (Alafaleq et al. 2015). In a more recent study, we compared how Pythagoras' theorem was treated in secondary mathematics textbooks in these three countries. We found that it appeared overall highly similar across the three countries in terms of the total number of problems provided, when and how the theorem was introduced (i.e., all the countries introduced it in the first semester of grade 8 and all provided students' own exploration activities), though a higher percentage of real life problems were designed in the Chinese and Saudi Arabian textbooks than the Indonesian textbook. In addition, both Chinese and Indonesian mathematics textbooks introduced some historical knowledge about Pythagoras' Theorem, while this was not the case in Saudi Arabian textbooks (see more details in Fan et al. 2016).

In the present study, we focus on how proof, as a special kind of argumentation, is presented in the secondary mathematics textbooks in these three countries.

As is well known, geometry has traditionally played an important role in school mathematics curriculum in many countries. However, it has also proven to be one of the most difficult and challenging areas for both mathematics teaching and learning, and attracted considerable attention and controversial debate over the last century (especially during the new math movement in the 1960s), among mathematics education researchers, curriculum reformers, textbook developers, and school practitioners (e.g., see Burger and Shaughnessy 1986; Jones 2002; Kapadia 1980; Kilpatrick 1992; Usiskin 2014). In particular, it has been widely recognized that geometric proof is one of the most difficult parts in students' learning of mathematics and due to its difficulty, researchers have advocated different approaches to the teaching and learning of geometry and geometric proof in school mathematics (Boero et al. 2010; Fan et al. 2017; McCrone and Martin 2004; Kollar et al. 2014; Senk 1985; Usiskin 1972; Usiskin and Coxford 1972; Weber 2001). Given the history and debate of geometry in school mathematics curriculum reform and development, we are interested to know how geometry and geometric proof are presented in current school mathematics curriculum and textbooks in different countries.

We selected the textbooks from China, Indonesia and Saudi Arabia for comparison for several reasons: On the one hand, as a team, we have different members who have received education and worked in and hence have reasonable knowledge about all the three countries including their education systems; it is also practically feasible for us to access textbooks from these countries. Moreover, all these countries have a centralized education system, and they have the same system of grade levels in terms of student ages, namely, 6 years of primary education (starting at the age of 6), 3 years of junior high school (secondary) education, and 3 years of senior high school education.

On the other hand, these countries have different cultural traditions and social contexts. In terms of international student assessments, these countries also differ widely from each other. For instance, in the Programme for International Student Assessment (PISA) 2012, China (Shanghai) placed top while Indonesia placed second lowest out of the 65 participating countries or economies on the mean score table (OECD 2014). Saudi Arabia did not participate in PISA 2012, but according to TIMSS 2011 results, Saudi Arabia is regarded as a lower middle achiever in the assessment (Mullis et al. 2012). We are interested to know if curriculum and textbooks could be a factor contributing to the difference of student achievement as reflected in these international assessments. Having realised the challenge in establishing such a connection between textbooks and student achievement (e.g., see Fan et al. 2013), and as a first step in a sense, we wish to know how the textbooks in these countries represent mathematics topics, geometric proof specifically.

3.2 Literature Review

Researchers have offered a variety of definitions about proof and many linked it closely with argument, which appears more from the perspective of local organisation (for mathematics as an activity) instead of global organisation as Freudenthal (1971) once distinguished. For example, Conner (2008) defined proof "as logically correct deductive argument built up from given conditions, definitions, and theorems within an axiom system". Similarly, Clapham and Nicholson (2009) defined proof as "a chain of reasoning, starting from axioms, usually also with assumptions on which the conclusion then depends, that leads to a conclusion and which satisfies the logical rules of inference" (p. 638).

In recent years, many researchers have indicated the central importance of proof and proving to students' school mathematical learning in all content areas (not just in geometry) and at all grade levels (Knuth 2002; Stylianides 2007). This is so because proof and proving are fundamental to doing and knowing mathematics. Moreover, proof and proving are essential in developing, establishing, and communicating mathematical knowledge and understanding (Bartlo 2013; Hanna and Jahnke 1996; Hanna 2000).

Some researchers have investigated the nature of proof, justification and explanation presented in school mathematics textbooks. For example, Stylianides (2008) examined how proof is promoted in a popular US standards-based curriculum for middle grades. He found that about 5% of student tasks involved proof. Cabassut (2006) compared the reasoning presented in proofs in French and German school mathematics textbooks and found that deductive arguments often occur in conjunction with empirical arguments.

Nordstrom and Lofwall (2006) investigated proof in Swedish secondary mathematics textbooks. They revealed a very low occurrence of proof, and that proof was seldom made explicit in explanatory texts. Focusing on explanatory texts that introduced new mathematical rules or relationships, Stacey and Vincent (2009) examined the reasoning presented in seven topics in nine Australian eighth-grade textbooks. They classified explanations and found that textbooks generally did not distinguish between the legitimacies of deductive and other modes of reasoning.

Overall, it can be seen that the available research literature in this area has been scattered and focused more on western countries, and it merits a more systematic look at a wider international level.

3.3 Research Questions

With a focus on geometric proof, this study aims to investigate the presentation of proof in the school mathematics textbooks at the secondary school level, that is, from grade 7 to grade 9 in China, Indonesia, and Saudi Arabia.

More specifically, we are interested in the following three research questions:

1. When and where proof is first introduced in the textbooks concerned?
2. How the treatment of proof is distributed in the textbooks?
3. What types of proof are introduced in the textbooks?

Through addressing these questions, we hope to reveal and document the similarities and differences in the secondary school textbooks from these three countries in terms of grade levels, topics, and types of proof in relation to the treatment of proof, and explore possible reasons for the similarities and differences, and their implications for mathematics textbook research and development.

3.4 Methodological Matters

3.4.1 Textbook Selection

It is important to note that all the three countries selected have 6–3–3 school system, as mentioned earlier, 6 years of primary school, 3 years of junior high school, and 3 years of senior high school. Students are at about the same age for the same grade level, which makes it more comparable across the three countries.

In total, nine mathematics textbooks, that is, three textbooks from each country, were selected. For Chinese and Indonesian textbooks, there are a variety of mathematics textbook series being used in the secondary schools (Malizar et al. 2014; Xu 2013), while there is only one series being used in Saudi Arabia's schools.

For the Chinese textbooks, the latest series published by Beijing Normal University Press in 2014 were selected. As many authors and editors of this series of mathematics textbooks were also key members of the team who developed the national mathematics curriculum, this series was believed to largely reflect the ideas and purposes of the new national curriculum (Ma 2014).

In Indonesia, all the textbooks are published and distributed for free by the government on http://bse.kemdikbud.go.id/. There are five series of mathematics textbooks for each grade at the secondary level. It should be pointed out that for the same grade, all school mathematics textbooks in Indonesia contain the same mathematics topics and contents, as they must follow the national syllabus in which the progression of learning including learning objectives, topics and contents for each grade are clearly stipulated.[1] For this reason, we decided to choose the most popular textbook in terms of the number of users for each grade in the case of Indonesia, even though it could mean that they could be from the different series. As a result, the three textbooks we selected from grade 7 to grade 9 were indeed from three different series. Each was the most widely used textbook for the grade as indicated by numbers of downloader's revealed in the above website.

For Saudi Arabia textbooks, there is only one textbook series for the secondary schools and this series was developed and published by the Ministry of Education of Saudi Arabia (see Ministry of Education of Saudi Arabia 2008). Therefore we naturally chose the latest series developed by the government (Ministry of Education of Saudi Arabia 2014).

[1]Note this is not necessarily the case in other countries. For example, in China and England, the learning progression stipulated in the national syllabus (standards) is classified into different learning stages with each stage consisting of a few grades or years, and grade 7 to grade 9 (or year 7 to year 9 in England) are in the same learning stage (the third stage). Hence, textbook developers and authors might introduce different mathematics topics and contents for the same grade level, resulting in different sequencing of mathematics topics from grade 7 to grade 9.

3.4.2 *Coding Scheme*

Different researchers have established and used a variety of frameworks of proof levels or classifications for different purposes. For instance, Blum and Kirsch (1991) classified levels of proof into experimental, pre-formal, and formal proof. Harel and Sowder (1998, 2007) proposed a proof scheme which consists of three levels, namely external conviction, empirical proof, and deductive proof.

For this study, we used the following criteria to classify types of proof presented in the textbooks.

1. Direct proof ($P \rightarrow Q$: assume that P is true. Use P to show that Q must be true)
2. Proof by contradiction/indirect proof ($P \rightarrow Q$: assume P is true, and assume that $\neg Q$ is true. Use P and $\neg Q$ to demonstrate contradiction)
3. Counter examples (to prove that a property is not true by providing a counter example where it does not hold).

It should be noted that the above classification is based on the fact that this study focused on proof at the secondary school level, and hence from the perspective of school curriculum, it is more aligned with Freudenthal's conceptualization viewing proof being part of mathematics as an activity in the sense of local organization, as mentioned earlier (Freudenthal 1971).

In addition, in case that the above three categories do not cover all the types of proof introduced in the textbooks to be examined, we added a fourth type, that is, others, to cover all the other types of proof introduced. The result shows that it was in fact not necessary as later we found no proof introduced in the textbooks falling into this category (see Table 3.3).

Using the above classification, we examined all the main texts of the textbooks which contain proof with focus on geometric topics. Then the researchers first coded the results according to the grade levels, then topics of geometry, and finally the types of proof.

Furthermore, to ensure the reliability of coding, an external coder from each of the three countries was invited to code all of the examples independently and overall, the results of coding were highly consistent with an average agreement of 94.6%.

To end this section, we wish to point out that this study did not take into account the proof problems in the exercises in these textbooks. The first reason is pedagogical, that is, how teachers and students approach these proof problems is unclear to us given this is a textbook analysis study, and the second reason is mathematical, that is, there is often more than one way in mathematics to solve proof problems. Therefore, readers are reminded that the result of the study only reflects the intended experience of students' learning of geometric proof in these countries. Different research methods are needed to reveal the actual experience of students' learning in this area using these textbooks, which is a challenge for textbook research, as Fan (2013) argued.

3.5 Findings and Discussion

Before we report the results about the specific research questions, we first point out differences in the weight (or sheer exposure) of geometric contents in the textbooks we selected. For this purpose, the ratio of the number of pages which contain geometry contents/topics to the number of pages in the whole textbook was employed as an indicator.

The results showed that Chinese mathematics textbooks had the highest frequency. About 45% of the content pages are on the topics of geometry, while the figure is about 35% in Indonesia and 24% in Saudi Arabia textbooks.

3.5.1 The First Introduction of Proof in the Secondary Mathematics

In Indonesia secondary mathematics textbooks, proof is introduced for the first time in the first semester of grade seven in the topic of algebra about the properties of exponents. One of the examples was presented in the grade 7 textbook, as follows:

> Given m, n are positive integers, and p is a positive integer,
>
> prove that $(p^m)^n = p^{m \times n}$. (Nuharini and Wahyuni 2012, p. 29)

The textbook used the definition of p^m as the product of p multiplying itself for m times to deduce the property (which we consider as direct proof). However, all the three textbooks across the three grades do not mention any definition of proof in mathematics. Moreover, all the textbooks do not provide any explanation or a brief introduction of proof.

The same situation was also found in Saudi Arabian textbooks in which proof is introduced for the first time in an algebra chapter in grade seven, and the textbooks in all the three grade levels do not offer any explicit introduction about proof in mathematics (Ministry of Education of Saudi Arabia 2014).

On the other hand, Chinese mathematics textbooks have a separate chapter that introduces definition, statement and proof, which is not the case in the textbooks of the other two countries. The Chinese textbook defines formal proof as "the process of deduction". Considering the sequences of all mathematics topics, the author arranged the chapter of proof in the first semester in grade eight, connected to the Pythagoras' theorem (called "Gou Gu Theorem" in Chinese) and followed by parallel lines. The textbook firstly introduces that when proving a statement is a false statement, we usually use counterexamples. Then in order to prove that a statement is true, the textbook introduces nine basic facts as the foundation of proofs, which are based on the curriculum, and all of them are about geometry:

1. Two points determine a line.
2. The shortest distance between two points is a segment.
3. There is one and only one line that is vertical to a given line in a plane.
4. If two lines are intersected by a third line in a plane and the corresponding angles are equal, then the two lines are parallel to each other.
5. Given a point outside a given line, there is one and only one line passing through the point that is parallel to the given line.
6. If two sides and their included angle of one triangle are equal to the corresponding sides and angle of another triangle, the two triangles are congruent.
7. If two angles and their included side of one triangle are equal to the corresponding angles and side of another triangle, the two triangles are congruent.
8. If all three sides of one triangle are equal to the three sides of another triangle, the two triangles are congruent.
9. If two lines are intersected by a set of parallel lines, the segments obtained are in proportion.

All the proofs in this Chinese mathematics series take the nine facts as the basis and according to the facts, all the theorems introduced in the textbooks can be proved. Beyond the geometric curriculum, the textbooks mentioned one foundational concept related to algebra, which is the basis of proving equality and inequality. For example, if $a = b$ and $b = c$, then there must be $a = c$, which is known as "substitution of equal quantities". In the second semester of grade 8 and in the topic of triangles, the last proof technique is introduced: reduction to absurdity. The textbook offers only a simple example using that method and guides students to understand the process of its deduction.

From the above, it appears clear that the Chinese mathematics textbooks paid the most attention to the introduction of proof.

3.5.2 *Proof in Geometry of Secondary Mathematics*

Table 3.1 shows a summary of the distribution of texts (examples) introducing proof in the selected mathematics textbooks in these three countries. Overall, proof in geometry is presented in all the selected textbooks in the three countries.

Table 3.1 Distribution of texts (examples) introducing proof in the selected textbooks

Grade level	China		Indonesia		Saudi Arabia	
	Semester 1	Semester 2	Semester 1	Semester 2	Semester 1	Semester 2
7	0	0	0	9	0	1
8	9	29	0	1	4	3
9	11	5	2	0	0	8
Total	20	34	2	10	4	12

From Table 3.1, we can find that there exist two large differences among the three countries.

First, in terms of the total numbers of examples, China's textbooks present many more examples (54) than Indonesia (12) and Saudi Arabia textbooks (16).

Second, in terms of grade level, while Indonesia's textbooks present examples of proof at all the grade levels in the secondary school, most examples of proof were found in Semester 2 of grade 7.

In contrast, the Chinese textbooks do not introduce proof until grade 8, which also has the most intensive presentation of proof in all the three grade levels, while Saudi Arabia's textbooks almost introduce proof equally in grade eight and grade nine. Overall, we can see that the Chinese textbooks emphasize the proof the most.

The three countries also show large differences in the distributions of proof across the different topics in geometry. Table 3.2 summarizes the results.

In the Chinese textbooks, formal proof is presented in the topics of parallel lines, triangle, circles, and parallelogram including rhombuses, rectangles and squares, which appear to follow closely the Chinese National Mathematics Curriculum Standards for Compulsory Education. Furthermore, there are substantial examples in the topics of triangles and parallelograms since the textbooks take them as the focus of teaching on deduction and proof.

In Indonesia and Saudi Arabia's textbook series, no proof examples were found in parallel lines lessons. Moreover, it should be pointed out that proof examples are mostly concentrated in the topic of triangles in the three countries textbooks, particularly in the Indonesia and Saudi Arabia textbooks.

Regarding different types of proof, it was found that only the Chinese textbooks have more than one type of proof, that is, direct proof and proof by contradiction, while the textbooks in the other two countries only have introduced direct proof. In addition, no textbooks in all the three countries provided any example of proof by counterexample.

Table 3.3 summarizes the distribution of the numbers of different proof types in these textbooks. It seems apparent again that the Chinese mathematics textbooks set the highest requirement in this area of proof.

Table 3.2 Numbers of examples of proof across different topics of geometry

Topic	China			Indonesia			Saudi Arabia		
	G7	G8	G9	G7	G8	G9	G7	G8	G9
Parallel lines	0	9	0	0	0	0	0	0	0
Triangle	0	17	3	2	0	2	0	7	5
Parallelogram	0	12	0	1	0	0	0	0	3
Rhombus, rectangle and square	0	0	8	1	0	0	0	0	0
Circle	0	0	5	0	1	0	0	0	0
Angle	0	0	0	4	0	0	1	0	0

Table 3.3 Numbers of examples of different types of proof in the textbooks

Proof	China			Indonesia			Saudi Arabia		
	G7	G8	G9	G7	G8	G9	G7	G8	G9
Direct	0	36	16	9	1	2	1	7	8
Proof by contradiction	0	2	0	0	0	0	0	0	0
Proof using counterexample	0	0	0	0	0	0	0	0	0
Others	0	0	0	0	0	0	0	0	0

3.6 Concluding Remarks

In this comparative study, we aim to examine how geometric proof is presented in the secondary school mathematics textbooks from grade 7 to grade 9 in China, Indonesia and Saudi Arabia. Our comparison of the selected textbooks from the three countries revealed that the Indonesian and Saudi Arabian mathematics textbooks introduced proof earlier than the Chinese textbooks in the sense that the former introduced it in algebra in grade 7 while the latter in grade 8.

However, regarding the number, type, and distribution of proof in geometric contents, Indonesian and Saudi Arabian mathematics textbooks gave much less emphasis on proof compared with the Chinese mathematics textbooks. The textbooks in China presented many more proof examples at grade 8 and grade 9.

It is also important to notice that Indonesian and Saudi Arabian secondary mathematics syllabi do not explicitly mention the concept of proof, which is not the case in Chinese national curriculum, which might explain why the Chinese textbooks devoted a whole chapter to the topic of mathematical proof, while while Indonesianand Saudi Arabian textbooks did not pay significant attention to proof. As pointed out earlier, all the three countries have adopted a centralized education system, and all the textbooks are required to follow the national curriculum standards (or syllabus).

As mentioned at the beginning of this article, our earlier studies on the introduction of the comparison of whole numbers at the primary school level and on the presentation of the Pythagoras' theorem at the secondary school level in the mathematics textbooks across the three countries revealed a high level of consistency (Alafaleq et al. 2015; Fan et al. 2016). In contrast, this study reveals more inconsistencies or differences than similarities about the treatment of proof in the three countries, which are particularly evident in the three aspects: the number of examples, the distribution of contents and, to a lesser degree, the types of proof. We think these differences clearly reflect a lack of consensus in the international research community about the role and importance of proof in the mathematics curriculum and the way of teaching proof in school mathematics. It seems clear to us that more sound research is much needed in this area before any consensus can be truly reached.

Finally, we wish to emphasize that the development of textbooks is affected by a variety of factors including, for example, curricular, educational, social, and cultural factors, as researchers have argued (Fan 2013; Rezat 2006; Rezat and Sträßer 2012). Nevertheless, given the purpose, design, and scope of this study, it is less clear that, in addition to the national curricula and educational systems, how other factors have played a role in the development of the textbooks in these three countries in relation to the presentation of proof. To address these issues in the development of textbooks would require a different methodology, for instance, interview with the textbook developers, historical and longitudinal approach, which we believe in a large sense will be more challenging as well as interesting.

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