Chapter 15
Teacher Decisions on Lesson Sequence and Their Impact on Opportunities for Students to Learn

Ok-Kyeong Kim

Abstract When using existing resources to plan and enact a series of lessons, teachers make various decisions, one of which is whether to follow or modify the sequence of tasks and lessons presented in the resources. One important question to ask is how teacher decisions on lesson sequence affect the quality of instruction and opportunities for students to learn. I examined ways in which teachers, using three different curriculum programs, sequenced tasks and lessons, and whether these sequences provided opportunities for students to engage with mathematical points of the lessons and a mathematical storyline through a proper learning pathway. Findings of the study have implications for teaching, teacher education, and curriculum development.

Keywords Mathematics teachers' resources · Curriculum · Teacher decision Lesson sequence · Opportunities to learn

15.1 Introduction

When using existing curriculum recourses, teachers make a range of decisions for various reasons. One of the decisions teachers make is whether to use various elements in the resources and how to use them. Such decisions can influence lesson enactment significantly (Kim 2015; Kim and Atanga 2013, 2014). In this study, I examined ways in which teachers use existing curriculum resources to sequence tasks or activities within and across lessons and their impact on opportunities for students to learn important mathematical ideas and concepts.

Curriculum designers have specific intentions and mathematical goals to achieve through a series of lessons. These intentions are communicated through various kinds of support for teachers regarding how to enact the tasks and lessons, including

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a unit overview that provides the sequence of the tasks and mathematical ideas and concepts, and how these are connected to each other and developed across lessons. Moreover, each lesson outlines what is expected to take place so that students' learning of mathematical ideas and concepts can progress in a series of components of the lesson. This sequence of lessons and components provides a *curricular trajectory* to project the progression of a set of related mathematical ideas and concepts in student learning (Sleep 2009).

Teachers decide whether to follow or modify the sequence of components of the lessons provided in the curriculum. Such teacher decisions indicate various possible adaptations teachers can make as they use written lessons to design instruction. One important question to ask is how such decisions impact the quality of enacted lessons, or the quality of the transformation from the written to the enacted, and shape opportunities for students to learn the mathematics they are supposed to. Whereas teachers can improve student learning by making alternations in the sequence provided in the curriculum, modifying the sequence of tasks and lessons may be critical to the quality of the enacted lessons and student learning. Whether following or modifying the sequence in the resources, teachers need to make a well-developed plan for a proper trajectory for student learning.

15.2 Theoretical Perspectives

15.2.1 Teachers' Reasoning with Curriculum Resources

Researchers view that teachers actively engage in curriculum design through interactions with the curriculum resources that they use, rather than passively following them (e.g., Remillard 2005). When using existing resources to teach mathematics, teachers read, evaluate, and adapt the resources, and their reading and evaluation lead to various adaptations (Sherin and Drake 2009). Brown (2009) uses the notion of *pedagogical design capacity* (PDC) to explain the teacher capacity needed for productive curriculum use that helps achieve instructional goals. According to him, PDC is "a teacher's capacity to perceive and mobilize existing resources in order to craft instructional episodes" and "a teacher's skill in perceiving affordances [of the resources], making decisions, and following through on plans" (Brown 2009, p. 29). I argue that teachers are engaged in significant reasoning with curriculum resources in the process of reading and making sense of the resources, recognizing the affordances, and making decisions about what to use and how to use.

Researchers have articulated teacher knowledge actually used in teaching. Based on the assumption that teachers use any form of resources to teach a lesson and the view that teaching is a process of reasoning, Shulman (1987) elaborated aspects of *pedagogical reasoning and action*, which includes a cycle of comprehension, transformation, instruction, evaluation, and reflection. Following Shulman's approach to teaching and teacher knowledge, Rowland and his colleagues (Rowland 2013; Rowland et al. 2005) proposed *knowledge quartet* with a set of units (i.e., foundation, transformation, connection, and contingency) to describe ways in which teachers draw on their knowledge. Whereas the first unit describes knowledge base or propositional knowledge, the other three indicate situations in which teachers draw on various forms of knowledge to make instructional decisions. Also, Remillard and Kim (2017) conceptualized *Knowledge of Curriculum Embedded Mathematics* (KCEM, the mathematics knowledge activated by teachers when reading, interpreting, using mathematical tasks, instructional designs and representations in mathematics curriculum materials) to articulate the kind of knowledge teachers need to draw on in order to make sense of the mathematics presented in the written lessons to design instruction, and proposed four dimensions of KCEM: foundational mathematical ideas, representations and connections among these ideas, relative problem complexity, and mathematical learning pathways. All of the three notions mentioned above (i.e., *pedagogical reasoning and action*, *knowledge quartet*, and *KCEM*) illuminate the significance of teachers' reasoning with curriculum resources in designing instruction.

15.2.2 Mathematical Storyline and Lesson Sequence

Curriculum resources provide tasks and activities to support students' learning of mathematical points, and a proposed learning trajectory in their lessons, which can eventually help develop a coherent mathematical storyline—"a deliberate progression of mathematical ideas" (Sleep 2012, p. 954)—in a series of lessons. Individual tasks, lessons, and chapters are organized into a sequence to develop students' understanding of mathematical concepts and ideas, and build a mathematical storyline around a topic and across topics. Teacher decisions on whether to follow or modify the sequence in the curriculum can affect students' learning of mathematical points and the development of a mathematical storyline in the course of lessons.

In Shulman's notion of *pedagogical reasoning and action*, comprehending purposes and subject matter structures, and transforming them for students to learn are closely related to teachers' decision on the sequence of activities and lessons. Rowland and his colleagues (Rowland 2013; Rowland et al. 2005) also emphasized that teachers need to understand the mathematics that they teach and make necessary connections to design instruction. In particular, "Within a single lesson, or across a series of lessons, the teacher unifies the subject matter and draws out coherence" (Rowland et al. 2005, p. 265). Proposing principles for using curriculum in preservice teacher education, Drake et al. (2014) emphasized that teachers need to "examine multiple lessons and units in order to identify and understand the development of content over time" (p. 159). In addition, Sleep (2009, 2012) elaborated teachers' work of articulating learning goals of activities and lessons at the micro and macro levels to understand how they are connected and enact the activities and lessons toward the goals.

Two of the dimensions of KCEM, *relative problem complexity* and *mathematical learning pathways* are directly related to sequencing tasks and activities within and across lessons (Remillard and Kim 2017). Teachers need to carefully examine the proposed trajectory for student learning of the mathematical points in a series of lessons and how various tasks and activities within and across lessons support students' development of the mathematics in the lessons. Teachers certainly can decide to add new elements or omit existing components of the lesson to design instruction in order to better support the anticipated learning trajectory. Before making a decision, however, they need to examine whether the alterations affect the students' learning trajectory and, if so, whether they can enhance student learning through the revised learning trajectory. Remillard and Kim argue that "when using curriculum materials, being able to recognize learning pathways and their goals at different levels of focus allows teachers to find themselves at any moment on a broader curriculum map."

Using terms such as *mathematical purposing* and *focusing*, Sleep (2009) elaborated the complexity of teaching to the mathematical point, for which teachers have to attend to multiple learning goals and intentionally scrutinize a curricular trajectory in relation to a mathematical trajectory. Placing the importance and necessity of articulating the mathematical point from both mathematical and instructional perspectives, she described mathematical point as "a connected package of mathematical goals and instructional purposes, with depth and weight and time" and teachers' work of articulating the mathematical point as "articulating the intended mathematics and how the instructional activity is designed to engage students with it" (p. 14). She highlighted teaching for coherence, connections, and learning progression in the trajectory. Although there are other issues influencing the development of a mathematical storyline (Sleep 2009), properly sequencing activities within individual lessons and across lessons seems to be the starting point to develop a coherent mathematical storyline.

In this study, I define mathematical point as eventual goal(s) to achieve in the lesson (s), which may or may not be stated explicitly in the written lessons. Different curriculum programs use different terms (e.g., objectives, focus points, and math content) to indicate these goals, but reading the entire lesson including activities and guidance for teaching (i.e., articulating mathematical point) may illuminate something fundamental for student learning, but not explicitly stated as a goal or objective.

15.2.3 Teacher Decisions and Opportunities for Students to Learn

The National Research Council (NRC 2001) points out that opportunity to learn (OTL) is "the single important predictor of student achievement" (p. 334). Hiebert and Grouws (2007) explain that OTL depends on both teacher and curriculum materials. They further argue that creating moments in classrooms where students learn goes beyond exposing them to subject matter and learning goals. Stein et al. (2007) argue that curriculum materials can influence students' learning, as they contain different types of mathematical tasks that require various student engagement with the mathematics content embedded in them. They may also contain a well-developed sequence of tasks and lessons to support student learning. However, whether the tasks and the sequence are used as intended depends on the teacher. This may indicate that even though both curriculum materials and teachers are significant in creating opportunities for students to learn, the teacher's role seems even more critical. Many elements that can help create opportunities for the student to learn may be present in curriculum resources, and yet they may be inert if not deliberately pursued by the teacher (Kim 2015).

Teachers need to recognize the affordances of the resources they use in order to make a proper instructional decision (Brown 2009). It was observed that when teachers were not able to notice such affordances and made a poor decision, they created students' difficulty with learning the mathematical points of individual and multiple lessons (Kim 2015; Kim and Atanga 2014). For example, whereas the written lesson includes helpful intervention suggestions for struggling learners, the teacher, not using them, mainly repeated the same procedural explanations to the students in confusion (Kim 2015). Son and Kim (2015) also reported that two teachers enacted the same lessons from an inquiry-based curriculum program quite differently, which resulted in dissimilar learning opportunities for students. In enacting the lessons, the two teachers basically asked questions provoking different kinds of student thinking. Their goals of the tasks were different and one of the teachers failed to articulate the mathematical point of the tasks.

In this study, drawing on data from classroom teachers using curriculum programs with either a directing-teaching or student exploration model, I examine teachers' adaptions of lesson sequences and their impact on student learning. Research questions of the study are:

1. In what ways do teachers sequence lessons and activities from the existing resources?
2. What are the impacts of such decisions on opportunities for students to engage with the mathematical points and mathematical storyline of the lessons?

First, comparing and contrasting the sequence of written lessons with that of corresponding enacted lessons, I explored ways in which the participant teachers sequenced tasks and lessons from the curriculum resources they used. Then, I examined whether the sequence of the enacted lessons supported and enhanced opportunities for students to experience the mathematical points of the lessons and progress through a coherent learning pathway. The details of the methods are described below.

15.3 Methods

This study is part of a larger project investigating elementary teachers' use of mathematics curriculum resources in the United States, the *Improving Curriculum Use for Better Teaching* (ICUBiT) Project.

15.3.1 Participant Teachers and Curriculum Programs

I drew on data from 11 teachers in grades 3–5 who used three different curriculum programs (four, three, and four in each program, respectively). The participant teachers had at least three years of teaching experience (ranging from 3 to 25 years) and at least two years of using the current curriculum program (ranging from 2 to 14 years). One of the three programs, Scott Foresman–Addison Wesley *Mathematics* (Charles et al. 2008) was a traditional curriculum program with a direct-teaching model, which was commercially developed. One other program, *Math in Focus* (Singapore Ministry of Education/Marshall Cavendish International 2008), was also based on a direct-teaching model, but it emphasized conceptual foundations along with representations throughout the lessons. This program was developed in Singapore and had gained popularity in the United States. The lessons of the two programs with a direct-teaching model typically had components of teacher explanation/demonstration and student practice. Finally, a third program, *Investigations in Number*, *Data*, *and Space*, was a reform-oriented one with a student exploration model, primarily based on the recommendations by the National Council of Teachers of Mathematics (1989, 1991, 2000). The lessons of this program typically included components of group/pair work and whole group discussion after student work.

15.3.2 Data Sources

The data used in this study include Curriculum Reading Logs (CRLs), classroom observations data (videotapes, transcripts, and field notes), and teacher interviews (introductory and post-observation). Each participant teacher completed CRLs for a set of lessons that were observed; on a copy of the written lessons, using different colored highlighters, the teacher indicated which parts he/she read as he/she planned instruction, which parts he/she planned to use, and which parts that influenced his/her planning. CRLs helped me see teachers' plans for instruction and compare written and enacted lessons. Each teacher was observed for three consecutive lessons in each of two rounds. These enacted lessons were videotaped and transcribed. Also, each teacher was asked questions about his/her teaching experience and overall curriculum use at the beginning of the study, and then asked about specific teacher decisions in the observed lessons after each round of three observations. These interviews were audiotaped and transcribed.

15.3.3 Data Analysis

Data analysis began with identifying the sequence of the written lessons along with mathematical points (MPs) and the mathematical storyline. Sleep (2009) provided detailed examples of classroom episodes along with specific MPs she identified from written texts. I followed a similar process, but focusing on the development of MPs and mathematical storyline rather than individual MPs. By reading the entire individual lessons carefully, including objectives, key concepts, key ideas, tasks and activities, mathematical explanations, and instructional guidance, two researchers (including the author) identified the MPs within and across lessons and determined the proposed mathematical storyline in the sequence of multiple written lessons. In articulating the MPs for the purpose of analysis, the researchers attended to two separate but related aspects: conceptual and procedural goals. Next, I listed each teacher's sequence of tasks and lessons from the lessons observed, and compared the sequence of the written lessons with that of the enacted lessons. In comparison, the focus was given on whether the sequence from the observed lessons was significantly different from that of the written lessons in terms of the development of the MPs and mathematical storyline over the lessons, and if so, ways in which the sequence was modified. Then, I examined overall opportunities for students to engage with the MPs and mathematical storyline identified in the enacted lessons and whether the student learning opportunities were *enhanced* (better opportunities for student engagement with the MPs in the enacted mathematical storyline), *maintained* (the same level as in the written lessons or not much difference between written and enacted lessons in terms of student engagement with the MPs and mathematical storyline), or *reduced* (limited opportunities for student engagement with the MPs in the enacted mathematical storyline), compared to those proposed in the written lessons. Although single incidences, such as using an additional activity focusing on conceptual support to bridge a gap in student understanding, and omitting an important activity that is important in developing a proper mathematical storyline, were critical in the coding decision, the determination of *enhance*, *maintain*, or *reduce* was based on overall student learning opportunities in the course of the enacted lessons rather than discrete moments. Teacher interviews were analysed to see teachers' general approach to using their curriculum programs and their intentions and rationale for specific instructional decisions. These include explanations for why they omitted certain activities, added new elements, or made any other alterations to the proposed sequence. After examining individual teachers, I searched for patterns in teacher decisions on lesson sequence and their impact on lesson enactment within and across three curriculum programs. The patterns across programs were compared to account for characteristics of teacher decisions within each program. This was to search for a possible association between teachers' sequencing decision and the nature of the program.

15.4 Lesson Sequence, Mathematical Points, and Mathematical Storyline

In this section, I describe the participant teachers' sequencing of the lessons within each program in general and then two particular teachers' cases to illustrate specific ways they sequenced their lessons and how their sequences affected opportunities for students to learn mathematical points of the lessons and progress in the learning trajectory.

15.4.1 Patterns of Sequencing

Overall, teachers using the reform-based program, *Investigations in Number*, *Data*, *and Space*, made various decisions deviated from the curriculum in terms of sequencing lessons and activities. Teachers using the programs with a direct-teaching model showed different patterns; those who used Scott Foresman–Addison Wesley *Mathematics* seldom changed the sequence from the written lessons whereas those using *Math in Focus* altered the sequence of the written lessons significantly.

Four teachers using Scott Foresman–Addison Wesley *Mathematics* in this study added or omitted a short activity in a lesson occasionally, but this did not significantly alter the kind of opportunity for students to learn in terms of the content and the way they experienced the content. Lessons in Scott Foresman–Addison Wesley *Mathematics* had a typical format that included a short warm-up, teacher explanation of procedure or concept, and then a large set of practice problems for students. Often, the teachers omitted the warm-up activity but followed through the other two main parts of each lesson deliberately. Warm-up activities were composed of a small set of skill-based problems, omission of which seldom affected students' learning of the mathematics in the lesson because they were often not related to the main mathematics of the lesson. For example, the warm-up in a lesson whose objective was "tell time to the nearest 1 minute or 5 minutes using analog and digital clock, and identify times as A.M. or P.M." is "Write the number that is ten more than each number. 24, 56, 32, 98" (Charles et al. 2008, p. 190).

The four teachers also followed lessons as sequenced in the curriculum. Each lesson in this program had a narrow, focused content in a step-by-step order. For example, titles of lessons on division in grade 4 were as follows: dividing with remainders, two-digit quotients, dividing two-digit numbers, interpreting remainders, dividing three-digit numbers, zeros in the quotient, and the like. Since the focus of the lesson was narrow and each lesson had limited components (i.e., mainly teacher demonstration and student practice), teachers had little room to change the sequence of components and lessons, although they could have added student exploration or discussion, or combined lessons, such as those for “dividing with remainders” and “interpreting remainders.” Mostly, using Scott Foresman–Addison Wesley *Mathematics*, teachers determined what to show and explain, and then what problems to assign to students. No significant modification was evident in their sequence of lessons and components and mathematical storyline. As a result, the opportunities for students to learn the mathematical points and their progression in the projected learning pathway mainly remained the same as in the written lessons. Overall, conceptual aspects of the mathematical points were largely missing in enacted lessons, as these were not explicitly pursued in the written lessons.

Three teachers using the other program with a direct-teaching model, *Math in Focus*, had one additional lesson component to enact, compared to those using Scott Foresman–Addison Wesley *Mathematics.* As mentioned earlier, lessons in *Math in Focus* usually included a specific, explicit, deliberate component for conceptual foundation, which unpacks the mathematical concepts and ideas to be used in the subsequent procedural tasks and problems. Scrutinizing lessons from Scott Foresman–Addison Wesley *Mathematics* revealed that this program also had the potential and the necessity for such conceptual foundation, but that was not explicit in lesson components; especially, explicit students’ engagement in such conceptual foundation was usually not expected in the lesson segment. In contrast, lessons from *Math in Focus* began with conceptual foundation and then moved to procedures that students need to follow and practice. Therefore, enacting this component is critical in student learning because it affects students’ learning of the mathematical point and progression in the leaning pathway.

The three teachers using *Math in Focus*, however, dismissed lesson components for conceptual foundation in teaching their lessons. Two of the teachers explained procedures step by step, mostly using the practice problems only. They did not use base-ten blocks to illustrate multiplication or division, although these materials were explicitly used in the written lessons. Whereas the written lesson attended to place value in division (e.g., $810 \div 9 = 81$ tens $\div 9 = 9$ tens $= 90$), one teacher constantly made a comment, such as “add a zero at the end,” without using suggested terms including tens and hundreds. The other teacher asked many questions about “why” and attempted to support students’ understanding, but still without conceptual foundation components, she limited opportunities for students to make sense of the procedures they went through and do subsequent problems with meaning. In sum, the three teachers did not utilize the affordances of the written lessons (i.e., the conceptual foundation components) and focused on the practice problems for the procedures students were asked to do to find answers. The two teachers who were giving step-by-step explanations of procedures throughout the lessons did pick and choose practice problems from the written lessons and reorganized the lessons around the procedure practice. Their sequence of the lesson components was primarily related to what problems to provide and in what order. They also used practice problems outside the curriculum as they thought their students needed more practice with the procedures that they learned. They made the lessons even more teacher-centered than the original lessons.

Four teachers using the reform curriculum program, *Investigations in Number, Data, and Space*, adjusted the sequence provided in the curriculum significantly. The most common was omitting an activity or a lesson and combining multiple activities into one. Their rationale for the sequence change varied: redundancy or content similarity, student response, lack of time, assessment, and past experience. Apparently, the reform curriculum program placed a lot more demand on the teacher than the ones with a direct-teaching model. Although the lessons had usually two or three main activities, including individual or group work and whole-group discussion, the ways students were expected to work on the tasks/activities were not uniform in this program. Depending on the content explored and the representations or materials used, activity formats changed for student exploration. Some activities (e.g., a game as a choice for practice time) occurred in more than one lesson; also, often the same math focus points appeared in multiple lessons. For example, math focus points, such as "*finding fractional part of a rectangular area*," and "*identifying fraction and percent equivalents through reasoning about representations and known equivalents and relationships*," appear in several lessons on fractions, decimals, and percents in grade 5 (see Table 15.1 for more detail in the following section). These indicate that the foci of individual lessons were not as narrow as those in the other two programs with a direct-teaching model. Using this program required teachers to articulate mathematical points within and across lessons more carefully. When planning a lesson, however, the four teachers tended to focus on student pages for individual work to see the content of the lesson, which indicates that they prepared less for whole group activities and discussion that were important in the sequence of lessons and student learning through the anticipated trajectory.

The ways in which the teachers in this study altered the sequence of lessons in the resources include:

- Omitting a lesson component (activity/task)
- Omitting an entire lesson
- Combining lesson components within and across lessons
- Adding a new component or lesson.

Switching the order of lesson components or lessons was not observed in these teachers' lessons although they could have chosen to do so. Omitting and combining components and lessons reduced opportunities for students to learn the mathematics of the lessons. In the remainder of the chapter, I focus on two teachers' cases to illustrate these various ways in which the teachers modified the sequence of the tasks and lessons in their curriculum program, the reasoning behind their decisions, and how their decisions influenced the articulation of mathematical points (MPs) in student learning pathways and the development of a mathematical storyline.

Table 15.1 Sequence of the written lessons (Becca)

Lesson (MP)	Math focus points	Components
1.1 Everyday uses of fractions, decimals, and percents (Students understand everyday use of fractions and percents, and find fractional parts of a whole or of a group and a percentage of a whole or a group.)	• Interpreting everyday uses of fractions, decimals, and percents • Finding fractional parts of a whole or of a group (of objects, people, and so on) • Finding a percent of a group (or objects, people, and so on)	A. Uses of fractions, decimals, and percents (In the whole class, the teacher leads a discussion in which students talk about fractions, decimals, and percents used in everyday situations and their relationships.) B. What do you already know? (Students work on problems that relate fractions and percents, which helps the teacher assess students' prior knowledge.) C. Fraction and percent problems (Students share how they solved the problems in B, focusing on 2–3 problems.)
1.2 Relating percents and fractions (Students understand equivalents are fractions, percents, and decimals that represent the same amount, and identify percent equivalents of fractions and fraction equivalents of percents.)	• Finding fractional parts of a whole or of a group (of objects, people, and so on) • Finding a percentage of a rectangular area • Identifying fraction and percent equivalents through reasoning about representations and known equivalents and relationships	A. Introducing guess my rule (Students use fractions and percents to write statements about a group of students in front of the class [e.g., 50% are wearing buttons], and identify the characteristic of the students given a fraction or percent.) B. Writing equivalent percents and fractions (In the whole class, the teacher leads a discussion on what 50% means and its fraction equivalents and then other percents and their fraction equivalents.) C. Grid patterns as percents and fractions (Given shaded grids, students determine the percent and fraction of the shaded portion of each grid.)
1.3 Finding percents of an area (Students understand how percents and fractions are related, and find percent equivalents of fourths and eighths, by using area representations of fourths and eighths, and what they know about fraction relationships and equivalents.)	• Finding fractional part of a rectangular area • Finding a percentage of a rectangular area • Identifying fraction and percent equivalents through reasoning about representations and known equivalents and relationships	A. Percents for fourths and eighths (In the whole class, students share how they shade 1/4 of a 10 × 10 grid and determine the equivalent fraction with a denominator of 100 and the percent. Then, students individually use grids to shade 2/4, 3/4, 1/8, and 3/8, and write a fraction

(continued)

Table 15.1 (continued)

Lesson (MP)	Math focus points	Components
		with 100 as the denominator and its equivalent percent for each number.) B. What percent is 3/8? (Students explain how they found 3/8 of a grid and how they knew it equaled 37 1/2%.) C. Fraction and percent equivalents (Students individually or in pairs record the percent equivalent for each fraction for halves, fourths, fifths, eighths, and tenths, and get ready to explain how they figured them out.)
1.4 Percent equivalents for thirds and sixths (Students understand relationships between percents and fractions, and use these relationships, known equivalents, and representations to determine fraction equivalents of thirds and sixths.)	• Finding fractional parts of a rectangular area • Identifying fraction and percent equivalents through reasoning about representations and known equivalents and relationships	A. Reasoning about fraction-percent equivalents (Students share how they found the percent equivalents for halves, fourths, fifths, eighths, and tenths.) B. Finding thirds and sixths (Students find percent equivalents for thirds and sixths and show they figured them out by using 10 × 10 grids.) C. What percent is 1/3? (Students share how they found 1/3 = 33 1/3% by using a grid, and then percent equivalents of 2/3, and sixths.)

15.4.2 A Case of Becca with Investigations in Number, Data, and Space

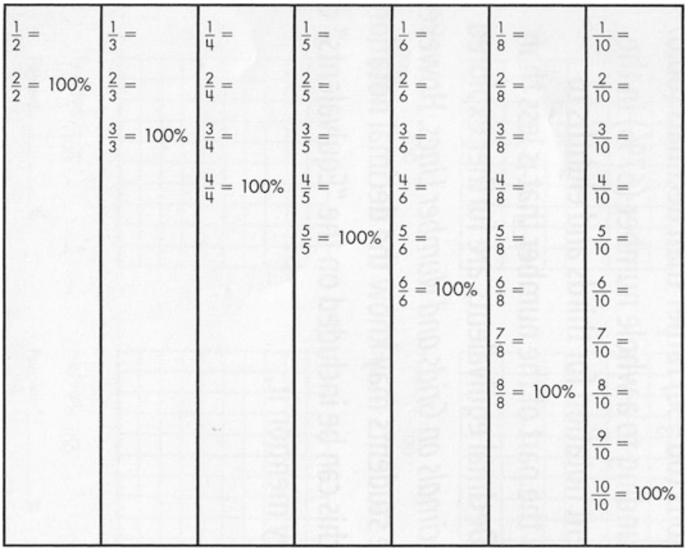
Becca had about 15 years of teaching experience and had used various curriculum programs. She had taught *Investigations in Number*, *Data*, *and Space* for 6–7 years by the time she was observed. She was confident in using the curriculum and had an established practice of using it. She also mentioned that using the curriculum helped her understand the mathematics she taught and made her gain confidence in teaching mathematics. Her sequence of the lessons, however, was far from articulating MPs of the lessons. Overall, her enacted lessons did not maximize opportunities for students to explore the MPs of the written lessons, let alone the mathematical storyline intended in the curriculum. In fact, Becca was the one who modified the sequence of tasks and lessons most drastically among the four teachers using the program in this study. She not only omitted tasks, but also added a new component and reorganized the tasks from multiple written lessons.

In the example described below, she taught three lessons on fractions, decimals, and percents by using four written lessons. Each written lesson was for 60 min, and all of the observed lessons lasted 60 min each as well. Table 15.1 presents details about the four written lessons, including specific lesson components and *Math Focus Points*, which is the term the curriculum program used to indicate objectives of lessons (TERC 2008). Please note that the content of the table is excerpted from a few pages of the curriculum, except for the MPs in the first column and the summary of lesson components in the last column in parentheses.

The very first written lesson (1.1) encourages students to think about everyday use of fractions, decimals, and percents. They review what they already know about fractions, decimals, and percents, and create a chart that lists how fractions, decimals, and percents are used in everyday situations. The second lesson (1.2) leads students to relate fractions, decimals, and percents, and introduces 10 × 10 grids, which represent fractions and percent equivalents (e.g., 1/2 = 50% = 3/6 = 10/20 = 25/50 = 50/100). Students also identify the percent and fraction of each 10 × 10 grid already shaded. The third lesson (1.3) has students use 10 × 10 grids to show fourths and eighths and find their percent equivalents. For this task, students use the area representation and what they know about relationships of fractions and equivalents to determine percent equivalents for fourths and eighths. The last lesson (1.4) finally extends to percent equivalents for thirds and sixths. Students discuss the fraction-percent equivalents they have found so far, find percent equivalents of thirds and sixths using the 10 × 10 grid, and explain the reasoning they used to find percent equivalents of thirds and sixths.

The MPs of the lessons are summarized in parentheses in the first column of Table 15.1. Examining the sequence of the lessons and their components reveals the progression of anticipated and projected student learning in the four written lessons. Students are expected to (1) activate their prior knowledge of fractions, decimals, and percents in the first lesson, (2) explore relationships among fractions, decimals, and percents with easy numbers, such as halves, fifths, and tenths, and start to use 10 × 10 grids in the second lesson, (3) extend to percent equivalents for fourths and eighths in the third lesson, and (4) finally move to percent equivalents for thirds and sixths. In this way, students could use what they know to develop a deeper understanding of relationships among fractions, decimals, and percents as lessons progress. By the end of the four lessons, students are expected to complete a chart for fraction and percent equivalents (see Fig. 15.1).

In contrast to the written lessons, Becca already asked students to use 10 × 10 grids to show percents in the first enacted lesson. In the subsequent lesson, she made students create and shade their own 10 × 10 grids and name the percents that the grids represented, by counting the number of shaded squares basically and



$\frac{1}{2}$ =	$\frac{1}{3}$ =	$\frac{1}{4}$ =	$\frac{1}{5}$ =	$\frac{1}{6}$ =	$\frac{1}{8}$ =	$\frac{1}{10}$ =
$\frac{2}{2}$ = 100%	$\frac{2}{3}$ =	$\frac{2}{4}$ =	$\frac{2}{5}$ =	$\frac{2}{6}$ =	$\frac{2}{8}$ =	$\frac{2}{10}$ =
	$\frac{3}{3}$ = 100%	$\frac{3}{4}$ =	$\frac{3}{5}$ =	$\frac{3}{6}$ =	$\frac{3}{8}$ =	$\frac{3}{10}$ =
		$\frac{4}{4}$ = 100%	$\frac{4}{5}$ =	$\frac{4}{6}$ =	$\frac{4}{8}$ =	$\frac{4}{10}$ =
			$\frac{5}{5}$ = 100%	$\frac{5}{6}$ =	$\frac{5}{8}$ =	$\frac{5}{10}$ =
				$\frac{6}{6}$ = 100%	$\frac{6}{8}$ =	$\frac{6}{10}$ =
					$\frac{7}{8}$ =	$\frac{7}{10}$ =
					$\frac{8}{8}$ = 100%	$\frac{8}{10}$ =
						$\frac{9}{10}$ =
						$\frac{10}{10}$ = 100%

Fig. 15.1 Fraction and percent equivalent chart

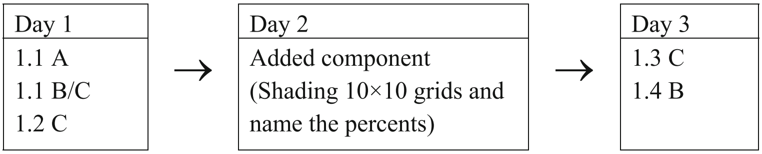


Fig. 15.2 Sequence of the observed lessons (Becca)

without necessarily relating percents with fractions. On the third observed lesson, the teacher had students find percent equivalents for fourths, eighths, thirds, and sixths all in one lesson. Figure 15.2 summarizes Becca's sequence of the lessons and components. She shortened components in Lesson 1.1 and squeezed in student work and sharing of Lesson 1.2 C to the first lesson. Then, she skipped Lesson 1.2 A and B; instead she did the grid shading activity in the second lesson. Again, she skipped most components of Lessons 1.3 and 1.4, and combined Lesson 1.3 C and Lesson 1.4 B in the last observed lesson. In other words, she combined the mathematical explorations of two lessons (Lessons 1.3 and 1.4) into one, in which she ended up showing the completed chart for fraction and percent equivalents of halves, thirds, fourths, fifths, sixths, eighths, and tenths at the end of the lesson because of a lack of time.

As she rushed through the lessons along with the added component on the second day, she limited opportunities for students to explore percent equivalents of fractions, such as 1/4, 1/8, 1/3, 1/6, and related fractions. The written lessons allotted one day for fourths and eighths, and another day for thirds and eighths,

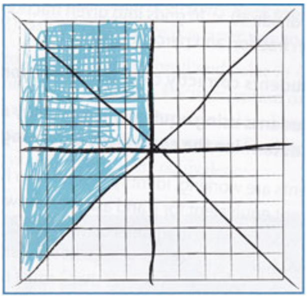


Fig. 15.3 Area representation of 3/8 on 10 × 10 grid

given the complexity of the fractions and their present equivalents. Students were expected to use 10 × 10 grids and relationships they already knew to find the percent equivalents for fourths and then move to other harder fractions. Figure 15.3 presents the grid use to find the percent equivalent for 3/8 as an example. She did not use the grid sufficiently for percent equivalents of fractions. In contrast, she spent one entire lesson for shading grids (Day 2), which was not related to the MPs of the lessons. Her students "designed" 10 × 10 grids on their own and determined percents and fractions of those grids (e.g., 78% or 78/100, and 43% or 43/100) by counting the number of shaded squares basically. Using the grid in this way was not related to the target fractions of the lessons, such as fourths and eighths. Moreover, the students were not asked to relate fractions with percents at all in the way the written lessons outlined.

Becca explained why she modified the sequence in the way she did. Her reason for skipping some activities and introducing the grid in the first lesson was: "I want them to make the grid and be comfortable with a fraction first, before I try to get them to jump into the percentages. … because those grids for them to color makes it easier then to figure the percentages." In contrast, the written lessons encouraged students to explore fractions and percents together to see their relationships, rather than one at a time. The focus of the written lessons was on the relationships between factions and percents whereas Becca treated them separately. She also explained why she did the activity of shading grids on the second day although it was not in the written lesson.

> I've noticed over the years, kids, because they enjoy that, they don't see it as learning. "Oh, I get to color in a grid!" And it's more fun for them and it helps them transition better into the other activities. … I go "Okay, do you guys remember the grids you made?" "Oh, yeah! Those were easy." "Okay, this is just like that, only—" So it's something to tie back to.

Spending the entire second day on the added activity and skipping important activities and discussions on halves, fifths, and tenths and their percent equivalents, however, she created a big gap in students' learning in her enacted lessons. She explained why she organized the third lesson in the way she did as follows:

> … kids know fourths because of quarters, and so I always relate fourths to quarters and most of them get that. Eighths, I saved until we had the percent equivalent chart, and then I had them go figure it out on those grids, what the percentage for eighths would be. … to

> stop a chart [for fraction and percent equivalents] like that and then have them come back to try and get them back in the mode of that thinking, actually takes more work than to just extend the lesson. It was a long lesson and they had to do a lot with it, but I like to get that chart done in one day.

It was evident that it was too ambitious to cover the mathematics content of two lessons in a one-hour lesson. Given the complexity of thirds and sixths, the curriculum deliberately saved those fractions and their equivalents for the last, separate day although students were expected to start filling in the fraction-percent chart (Fig. 15.1) in the second written lesson. The way Becca sequenced the lessons did not allow her students sufficient time to fully explore percent equivalents of eighths and sixths. It was evident that she did not clearly articulate the MPs of the lessons in the projected learning pathway.

To summarize, reorganizing tasks and lessons and adding a new activity, Becca limited opportunities for students to learn the MPs of the lessons (i.e., relationships among fractions, decimals, and percents) and develop the coherent mathematical storyline that the curriculum carefully laid out. The teacher created a sequence that students had difficulty following through. Without sufficient foundational and intermediate work, her students struggled with the task of finding percent equivalents for fourths, eighths, thirds, and sixths all in one day.

15.4.3 A Case of Kate with Math in Focus

As described earlier, *Math in Focus* is a curriculum program whose typical lesson format is teacher demonstration/explanation and student practice, which is similar to Scott Foresman–Addison Wesley *Mathematics*. Unlike Scott–Foresman Addison Wesley *Mathematics*, however, *Math in Focus* deliberately provides a conceptual foundation for procedures in every lesson, although this foundational work is primarily based on teacher demonstration/explanation. Building the foundation of the procedure in the lesson helps students know why they go through certain steps in particular problems. These foundations are usually built along with representations that illustrate the core mathematical idea embedded in a set of problems that follows. Therefore, using representations to build the foundational work in each lesson or a series of lessons is critical in using *Math in Focus*.

In the example below, I describe one third-grade teacher (Kate)'s case with lessons on fractions in grade 3. She enacted two two-day lessons (two lessons for four days) on improper fractions and mixed numbers (see Table 15.2). Throughout the lessons, conceptual components are prevalent. The MPs of the lessons are summarized in parentheses in the first column of Table 15.2. In the first two-day lesson (6.5), students are expected to understand the relationship between improper fractions and mixed numbers, and use multiplication and division to rename improper fractions and mixed numbers. Then, in the following two-day lesson (6.6), students use the relationship between improper fractions and mixed numbers to add two or three fractions to get a mixed number and subtract a fraction from a whole

Table 15.2 Sequence of the written lessons (Kate)

Lesson (MP)	Objective	Components
6.5 Renaming improper fractions and mixed numbers (Students understand the relationship between improper fractions and mixed numbers, and use multiplication and division to rename improper fractions and mixed numbers.)	• Use multiplication and division to rename improper fractions and mixed numbers	Day 1 A. Use models to rename improper fractions as mixed numbers or whole numbers (The teacher explains how to rename improper fractions as mixed numbers along with a representation and students do a "guided practice" problem.) Day 1 B. Use division to rename improper fractions as mixed numbers or whole numbers (The teacher explains how to rename improper fractions as mixed numbers by using "division rule" and students do "guided practice" problems.) Day 1 C. Roll and rename! (Students play a game in groups, where they roll two dice to form an improper fraction and rename it as a mixed number.) Day 2 A. Use multiplication to rename a mixed number as an improper fraction (The teacher explains how to rename a mixed number 3 3/4 as an improper fraction using the number line and introduce the multiplication rule for converting the mixed number to the improper fraction. Then, students do "guided practice" problems.) Day 2 B. Another way to use the multiplication rule (The teacher explains a shorten version of the multiplication rule with a representation and students do "guided practice" and practice problems.)
6.6 Renaming whole numbers when adding and subtracting fractions (Students understand the relationship between improper fractions and mixed numbers, and use the relationship to add fractions to get a mixed number and subtract fractions from whole numbers.)	• Add fractions to get mixed-number sums • Subtract fractions from whole numbers	Day 1 A. Add two fractions to get mixed numbers (The teacher explains how to add two unlike fractions and students do guided practice problems.) Day 1 B. Add three fractions to get a mixed number (Students explain how to add three fractions, such as 3/4 + 1/8 + 5/8, in teacher-led solution process and do guided practice problems.) Day 2 A. Subtract fractions from whole numbers (The teacher explains two methods for subtracting a fraction from a whole number with a bar model, as shown in Fig. 15.6 and students do "guided" practice problems.)

number. The conceptual components of these lessons are to support students' thinking in the procedural tasks.

The first lesson (6.5) is about renaming improper fractions and mixed numbers. On Day 1 of this lesson (6.5 Day 1 A) teachers are expected to "use fraction circles or pictures to show students how an improper fraction can be renamed as a whole number" (Singapore Ministry of Education/Marshall Cavendish International 2008, p. 243) including the following examples:

$$\frac{3}{3} = 3\,\text{thirds} = 1\frac{6}{3} = 6\,\text{thirds} = 2\frac{9}{3} = 9\,\text{thirds} = 3\frac{12}{3} = 12\,\text{thirds} = 4$$
$$\frac{5}{5} = 5\,\text{fifths} = 1\frac{10}{5} = 10\,\text{fifths} = 2\frac{15}{5} = 15\,\text{fifths} = 3$$

This component of the lesson (6.5 Day 1 A) also suggests the teacher "demonstrate how to rename 4/3 as a mixed number by separating 4/3 into a whole and a fractional part" (Singapore Ministry of Education/Marshall Cavendish International 2008, p. 243) and includes an illustration as seen in Fig. 15.4.

The explanations and the representation help students see what part of the improper fraction becomes a whole number part of the mixed number and why. A guided practice problem that follows also includes a similar representation with *fifths* to support the process to determine a mixed number for 13/5. Using words, such as 3 fifths and 5 ninths, instead of 3/5 and 5/9, is throughout Lesson 6.5 Day 1.

Day 2 of Lesson 6.5 (6.5 Day 2 B) also includes a conceptual explanation of a procedure ("the multiplication rule") using a representation (see Fig. 15.5), which unpacks the steps of "multiply the whole number by the denominator and add the product to the numerator" (e.g., $3\frac{1}{2} = \frac{3\times 2+1}{2}$) for renaming a mixed number as an improper fraction.

The teacher's guide includes the following elaboration of the multiplication rule: "First, multiply the whole number by the denominator. 1 whole = 2 halves, 3 wholes = 3 × 2 halves = 6 halves. Then add the product to the numerator (6 + 1 = 7)" (Singapore Ministry of Education/Marshall Cavendish International 2008, pp. 247–248). This component of the lesson conceptually supports students' sense-making of the rule for converting a mixed number to an improper fraction.

$$\frac{4}{3} = 4\text{ thirds}$$
$$= 3\text{ thirds} + 1\text{ third}$$
$$= \frac{3}{3} + \frac{1}{3}$$
$$= 1 + \frac{1}{3}$$
$$= 1\frac{1}{3}$$

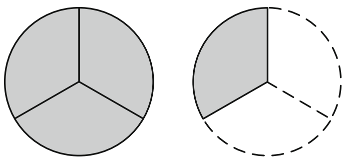


Fig. 15.4 Renaming an improper fraction as a mixed number

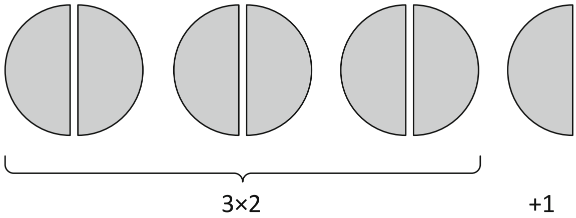


Fig. 15.5 "Multiplication rule" to rename a mixed number as an improper fraction

Day 2 of Lesson 6.6 also provides a conceptual support for subtracting a fraction from a whole number or a mixed number. The lesson introduces two distinct, but related methods for the operation seen below, including a representation for Method 1 (see Fig. 15.6).

$$\text{Method 1}: 3 - \frac{4}{9} = 2\frac{9}{9} - \frac{4}{9} = 2\frac{5}{9}$$

$$\text{Method 2}: 3 - \frac{4}{9} = \frac{27}{9} - \frac{4}{9} = \frac{23}{9} = 2\frac{5}{9}$$

As seen in Fig. 15.6, the written lesson uses a bar model to represent 3 − 4/9 visually and conceptually—what it means to subtract 4/9 from 3 and what is left as a result of the operation. If earlier conceptual approaches are employed, Method 2 is basically counting how many ninths are left after taking 4 ninths away from 27 ninths (=3 wholes): 27 ninths − 4 ninths = 23 ninths.

Overall, the lesson components for foundational work described above are to establish the relationship between numbers (improper fractions and whole numbers, or improper fractions and mixed numbers) that students will use later to solve problems and practice the procedures, and to support students' meaning making over four days of the lessons. Kate taught three lessons by using the two two-day lessons. She, however, did not use the conceptual lesson components in her instruction. Her sequence of the lesson components for three days is summarized in Fig. 15.7.

Kate skipped Lesson 6.5 Day 1 A, Lesson 6.5 Day 1 C, Lesson 6.5 Day 2 B, and the conceptual foundation portions of Lesson 6.6 Day 1 and Day 2. Basically, she

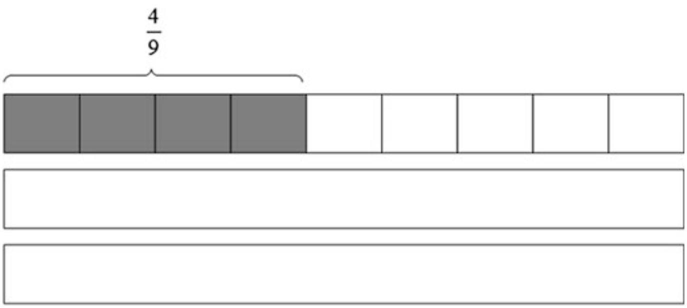


Fig. 15.6 Bar model used to illustrate Method 1

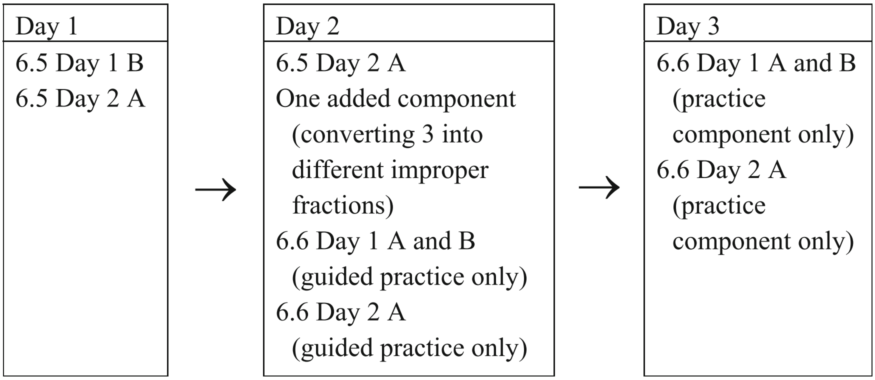


Fig. 15.7 Sequence of the observed lessons (Kate)

omitted lesson components that support for conceptual foundations for students' understanding of fraction operations involving whole numbers and mixed numbers. In contrast, she kept most of the teacher explanations (without conceptual component) and practice problems. Also, she added a short component, renaming 3 in different ways, such as 9/3, 15/5, 2 and 9/9, before introducing the procedure for subtracting a fraction from a whole number.

She assumed that previous work was sufficient for students to add or subtract fractions, and omitted lesson components to build conceptual foundations for procedures students needed to do. By eliminating the conceptual foundation work laid out in the written lessons, she reduced her students' accessibility to the mathematics in the procedures of adding and subtracting fractions. She repeated the practice portions of Lesson 6.6 on the third day observed, but her students still had difficulty adding two or three fractions to give the final answers in the form of mixed numbers, or subtracting a fraction from a whole number or a mixed number. Students were supposed to use what they learned from Lesson 6.5 (understand the relationship between improper fractions and mixed numbers, and rename improper fractions and mixed numbers) to do the operations. Without a solid foundation, however, her students struggled to follow through the procedures that the teacher explained.

In particular, without using the bar model (Fig. 15.6), Kate verbally explained renaming of 3 in different ways (e.g., 2 and 9/9, and 27/9) in order to subtract 4/9. The bar model clearly shows why they needed to change 3 to 2 9/9 (or 27/9) in order to subtract 4/9, but her explanations of renaming of 3 without the model kept the concept on an abstract level and did not help the students see the rationale for the procedures. The students still struggled to use the two methods in other problems during the lessons observed, and not being able to relate the two methods, many of them chose only one of the methods to solve other problems. In fact, some students suggested they use the bar model ("I can draw a picture on the board"), and yet the teacher was reluctant to do so ("No, that's okay. If somebody needs a picture, we will add that. I don't want to confuse anybody."). The teacher strongly believed that the model would confuse students rather than helping them see why the procedure works and explained the renaming repeatedly.

During the interview, Kate said, "I read this [lesson] to see if it is appropriate" to determine whether her "students will be able to make sense, or if I need to do something else, share another example." Her evaluation of the lessons led her to focus more on explanations with examples and remove visual representations. She said, "… sometimes if they start with the picture examples, those are too simplified and I just don't write through the number examples or the computation part because it tends to confused the kids sometimes if the pictures are involved." Kate was not in favour of using the representations (fraction circles or pictures, bars, and number line) the written lessons included, and did not see their mathematical significance and instructional affordances. Because of that, she omitted most of the conceptual components of the lessons. As many other teachers mentioned in this study, however, her rationale for omitting some lesson components was a lack of time. She said,

> They give us the time, the pacing, and then they give us a ton of activities, and like we've talked about, the games, the thinking, and the extra pages in the workbook that give you extra material. If you used all those you definitely wouldn't finish the lesson in a certain amount of time, but you have that option if you need it.

While removing lesson components for the reasons of limited time and student confusion, Kate added one lesson component (i.e., renaming 3 in different ways) that was not specified in the written lessons. She provided her rationale for this addition as follows:

> Well, I think the first time we did three as a whole number renaming if it came up as 9 I think. But then they kept using 3 with different denominators so you know if it was 10, how could you make 3 with a fraction with 10 in the denominator? And so I felt like it was important. First of all, fractions is something that they don't all grasp all the time. They look different and even with the picture representations early on in the chapter. They would look at 1/2 and 2/4 and not really think that they were the same thing. Equivalent fractions were just kind of out there, and I think it was important to show that a whole number could have different names according to what denominator you put it in. And that kind of goes along with multiplication and division and stuff too, how they get those equivalent fractions. So I thought it was important. And I'm big on connections with different topics. Fractions are not by themselves. You need to connect those with something.

As described earlier, the bar model could have helped students see why they needed to change 3 to 2 9/9 in order to subtract 4 ninths, without asking students to rename 3 in different ways and mechanically explaining that since "the denominator of 4/9 is 9" they needed to change 3 to 2 9/9, not 2 6/6, 2 12/12, or something else. Although during the interview she claimed that she emphasized connections, she did not see how the representations that she did not use could have helped students make the connections in the lessons.

To summarize, Kate created a lesson sequence quite different from the one laid out in the written lessons. She removed important lesson components for students' learning of the MPs (i.e., how improper fractions and mixed numbers are related, and how this relationship can be used in operations), which serve as building blocks in developing the mathematical storyline of the lessons. Her articulation of the MPs and learning trajectory did not accurately capture the affordances provided in the written lessons. As a result, her students had limited opportunities to learn the relationship between improper fractions and mixed numbers and do related operations with meaning.

15.5 Discussion

Analyzing a small set of teachers using each of the three elementary mathematics curriculum programs described above, this study explored teachers' decisions on lesson sequence and their potential impact on student learning. Although the patterns identified in the study cannot be generalized to all other teachers using the same programs, the ways the participant teachers enacted the lessons in the sequence are quite feasible in other teachers' classrooms and provide implications for teaching, teacher education, and curriculum development.

Teachers using Scott Foresman–Addison Wesley *Mathematics* tended to follow the sequence provided in the curriculum, whereas teachers using *Math in Focus* and *Investigations in Number, Data, and Space* often modified the sequence in the curriculum. It seems that the demand on the teacher is higher with programs incorporating conceptual support (ICUBiT Project 2011). Especially, using a program with a student exploration model requires more careful reasoning about the mathematics in instructional activities; teachers need to make sense, evaluate, and use various resources in the curriculum to sequence the lessons and tasks properly to support students' learning and development of the MPs and mathematical storyline over a period of time. It can be hard for teachers to see the connections in tasks/lessons, and it may be even harder to sequence them in a way that highlights the mathematical coherence (Sleep 2012). It is important for teachers to understand in the various given resources what MPs are addressed within and across lessons and in what ways the MPs are further developed to build a coherent mathematical storyline.

Then, is the curriculum program with a direct-teaching model easier to teach toward the MPs? We cannot answer just based on how lessons are sequenced alone. There are other aspects and elements of the programs that support or limit teaching to MPs. In fact, conceptual aspects of the MPs were not explicit in lesson components in Scott Foresman–Addison Wesley *Mathematics*, which led teachers to mainly focus on procedural aspects in their instruction. It is hard to claim that procedural aspects of the MPs alone can build a proper mathematical storyline. Noticing this limitation of the program, teachers may try to make up the gap, which is not an easy task.

In the cases of Becca and Kate, there was a significant gap between written and enacted lesson sequences in terms of the MPs and mathematical storyline within and across lessons. It seemed that both Becca and Kate failed to articulate the intended mathematics and how the instructional activities were designed to engage students with it. Using existing resources, teachers need to decide how to do so based on sufficient knowledge and capacity. It is likely that Becca and Kate lacked significant aspects of such knowledge and capacity, some of which were elaborated in the notions of *knowledge quartet* (Rowland et al. 2005), *knowledge of curriculum embedded mathematics* (KCEM, Remillard and Kim 2017), *pedagogical design capacity* (PDC, Brown 2009), *pedagogical reasoning and action* (Shulman 1987), and *mathematical purposing* and *focusing* (Sleep 2009). Teacher education and curriculum design need to support teachers' reasoning with resources and help them build a capacity required to enact lessons productively.

Teacher education should provide teachers with opportunities to use knowledge in various situations for decision making, in particular, articulating the mathematical goals of activities and lessons to develop a proper mathematical storyline. For this reason, lesson planning needs to be done in relation to multiple prior and subsequent lessons, and lessons in grades before and after. Teachers need to situate individual lessons in a broader context and understand how activities and lessons are weaved into mathematical pathways. Describing curriculum use for preservice teacher education, Drake et al. (2014) emphasized teacher learning about and from curriculum resources. They argued, "Learning to read and interpret the features of curriculum materials in ways that leverage the educative potential of those features seems particularly important" (p. 158). Teacher educators need to examine ways in which curriculum resources can be systematically used to support teachers' reasoning with the resources.

Reasoning *in* the resources (e.g., the intent of lessons and activities and a proposed mathematical storyline) needs to be transparent to teachers in order to support their reasoning with the resources. Curriculum programs have various ways to communicate the MPs of lessons to teachers, such as listing lesson objectives, describing activities and tasks, listing vocabularies and key content, and even explaining the MPs directly to teachers in a separate place (e.g., notes for teachers). However, it was observed that some written lessons failed to specify the core mathematical ideas of the lesson/tasks (ICUBiT Project 2011). In fact, the lessons on fraction and percent equivalents in *Investigations in Number, Data, and Space* could have made the MPs specific and clear in each lesson, rather than stating the same broad "math focus points" in multiple lessons. For example, instead of including, "Identifying fraction and percent equivalents through reasoning about representations and known equivalents and relationships" as one math focus point in three consecutive lessons (see Table 15.1), focused fraction and percent equivalents (e.g., fourths and their percent equivalents) can be specified in the math focus point of each lesson to support teachers to better understand how the math focus point can be met in a series of lessons. Moreover, the lessons from *Math in Focus* need to make conceptual aspects be part of lesson objectives so that teachers can attend to conceptual foundation components of the lessons. The examples suggest that curriculum designers attend to ways to make MPs and mathematical storylines explicit. It is notable that *Investigations in Number*, *Data*, *and Space* lists focus points of discussion segments explicitly, which will help teachers attend to the main ideas during discussion. Especially, reform-oriented curriculum programs that include various resources for teachers may bury MPs and mathematical storylines in those resources, rather than making them transparent. Curriculum designers need to provide a clear picture of how lessons are weaved to introduce and develop MPs and a mathematical storyline in a series of lessons, in a unit/chapter, within and across years.

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