Chapter 13
Resourcing Teachers in Transition to Plan for Interactions with Students' Ideas

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Abstract We explore how resources support teachers' work broadly, and their preparation for interactions with students' ideas specifically. We draw on data from two professional development design experiments aimed at supporting teachers in making students' mathematical reasoning central to their instructional decisions. The forms of support that traditional resources and expectations provided were no longer present when teachers transitioned to proactively planning for classroom interactions. We identify new forms of support that designed instructional sequences can provide for teachers by (a) specifying simple initial goals for students' reasoning, (b) supporting teachers' design for classroom interactions, and (c) increasing the likelihood that these designs would do useful work in classrooms.

Keywords Mathematics teachers' resources · Instructional sequence
Learning trajectory · Ambitious teaching practices · Teacher support

13.1 Introduction

Supporting teachers' development of classroom practices that aim at ambitious goals for students' mathematical learning is a complex undertaking (Lampert et al. 2010). These practices place student mathematical reasoning at the center of a teacher's decision-making and foreground classroom interactions in formats such as project-based, inquiry-based, or problem-based learning. These practices are currently not typical, and involve substantial teacher learning (Maaß and Artigue 2013).

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Material resources such as online and printed textbooks, and instructional sequences are necessarily a piece in a puzzle of facilitating teachers' transition to ambitious teaching (e.g., Remillard 2012). The question of how material resources can and should support such transition has captured the interest of researchers, resource developers, and professional development (PD) practitioners. Davis and Krajcik (2005) advanced a vision of educative curriculum materials that should ground teacher learning "in specific instances of instructional decision making" while helping teachers "develop more general knowledge that they can apply flexibly in new situations" (p. 3). These and other researchers have since explored ways in which educative curriculum materials are used and how they could be designed and continually improved to support teachers' engagement in ambitious teaching (Davis et al. 2014; Stein and Kim 2009). Others have further theorized teacher-resource relationship and specified characteristics of both teachers and resources that come to play as teachers use resources to design their teaching (Brown 2009; Gueudet and Trouche 2009, 2012; Remillard 2005, Chap. 4). The work reported in this chapter confirms that material resources, indeed, have a potential to productively shape teaching of individual teachers and teacher groups, especially if ample time and adequate support is provided. We also show that even when materials are carefully constructed to communicate specific instructional rationales, and portray and justify a particular vision of teaching and learning mathematics, teachers' interpretations and uses of these materials will vary.

Teaching practices within which instructional decisions are based in how students reason mathematically had long been of interest to mathematics education research community. Teachers are envisioned to anticipate, understand, and respond to their students' ideas, thus proactively scaffolding the reasoning that occurs in their classrooms (Fennema et al. 1996; Smit et al. 2013). Analyses show that effective teachers indeed support, elicit, and extend their students' ideas (Fraivillig et al. 1999) and that learning to do so requires support (Franke and Kazemi 2001). There is a recognition that educative curriculum materials need to aim to support teachers' work with students' reasoning (Davis and Krajcik 2005) and they most frequently do so by providing examples of student solutions and snippets of classroom conversations. While these examples are practical and teachers recognize them as useful, they, inevitably, provide only limited guidance to organizing productive classroom interactions. They help teachers in transition to recognize the broad strokes and envision some possible classroom interactions. However, teachers often feel lacking the specific guidance, in particular when students' responses do not match those in provided examples, or when mathematical goals remain implicit (Grant et al. 2009).

Our aim in this chapter is to explore some of the *functions* that resources need to fulfill in supporting the teachers in transition as they work on changing the nature of interactions in their classrooms. We approach this task through our work as design researchers, oriented by the design theory of Realistic Mathematics Education (RME; Gravemeijer 1994). We draw on two PD design experiments, in which instructional sequences, previously developed in classroom design experiments, were used and revised (Gravemeijer 2004). We draw on data from the first design experiment to document the uncertainties the participating teachers voiced and use these to indicate the forms of support that 'traditional' curricula, textbooks, and expectations provide, but that are typically not sufficient when teachers transition to working with innovative teaching approaches and materials.

We then document our learning about supporting teachers' reconstruction of the rationales that underpin instructional sequences. We identify, in particular, the supports that the instructional sequence can provide for the teacher by (a) specifying relatively simple initial goals for students' mathematical reasoning, (b) supporting teachers' design of means of supporting students' reasoning, and (c) increasing the likelihood that the designed means would do useful work in classrooms. We propose that, with appropriate support, the demanding in-the-moment decisions about guiding classroom interactions can become more manageable for teachers, making ambitious teaching centered in students' reasoning a reality.

13.2 A Design Perspective on Resources and Teacher Learning

Design experiments, as developed by Cobb, Gravemeijer and colleagues (Cobb et al. 1997; Gravemeijer and Cobb 2006), are a research methodology of mathematics education that involves developing both instructional designs to support particular forms of learning, and explanatory theoretical constructs to account for how this learning was supported. We adopt this version of the design experiment methodology, and formulate our instructional designs following the RME design theory (Gravemeijer 1994) and its more recent adaptations (Cobb et al. 2008), some of which are in particular relevant to establishing our theoretical background for this chapter.

Within the RME theory, instructional design is guided by a set of positive heuristics, which initially aimed specifically at supporting ambitious student learning. Two adaptations to RME were later introduced, acknowledging the mediating role of the teacher in students' learning. First, the instructional designs began to aim for supporting teachers in achieving the envisioned instructional agendas. In other words, researchers' design decisions about means of supporting students' learning now take into account whether and to what extent the designed tasks and tools would be usable by teachers, and in particular by *teachers in transition*, as they organize for their students' learning.

For example, attention is not paid solely to designing tasks and tools that would open up possibilities for student reasoning so as to enable productive classroom discussions led by a researcher or by an expert teacher. It is considered equally important that the designed tasks and tools *productively* constrain (cf. Wertsch 1998) the number of mathematically different ideas that students might propose, so that the teacher in transition is aided in anticipating a reasonably small number of possible directions for classroom discussions that would advance their instructional agenda.

Second adaptation to RME theory is aligned closely with the notion of educative curriculum materials and concerns the potential contribution of designed instructional resources as a means of supporting teachers' learning. In particular, designed resources aim to provide opportunities for teachers to reconstruct the rationales for instructional sequences, which often differ considerably from the rationales that underpin instructional decisions of teachers in transition. Once instructional sequences are developed and refined in classrooms, we engage with teachers in a variety of PD and classroom teaching activities grounded in the instructional sequence, aiming to make visible, interrogate, and refine over time, the rationales that underpin the instructional decisions (Cobb and McClain 2001). Insights form experimenting in PD context further shape both the instructional sequence specifically and design heuristics for supporting teacher PD more broadly. In this chapter, we illustrate the sources of several such insights and combine these into a retrospective story, and a conjecture, of one way in which instructional sequences could be designed to provide a feasible starting point for teachers' transition.

13.3 Background to Design Experiment Illustrations

Illustrations for our discussion come from two rather different PD design experiments, in which both authors were involved. The two studies share the specific design research methodology (Cobb et al. 2008), and conceptualization of both goals for and means of supporting teacher learning (Visnovska et al. 2012). They allow us to explore the trajectories of teachers in transition as they worked to reconstruct design rationales of an instructional sequence, and plan for interactions with their students' mathematical ideas.

Data for the discussions of the complexities related to teachers' *expectations* for support come from years 3 to 4 of a 5-year PD design experiment with a group of 12 middle school mathematics teachers, conducted in the southeastern USA.[1] One of the primary goals was to support the teachers' development of instructional practices in which they would induct their students into the ways of reasoning of the discipline by building systematically on their current mathematical activity. At the beginning of the study, the teachers' practices were rather homogeneous and could be characterized as traditional (Dean 2005).

The PD group met for 6 full-day sessions dispersed through the school year and for a 3-day summer workshop every year. A statistics sequence designed in two prior classroom design experiments (Cobb et al. 2003; McClain and Cobb 2001) was a primary means of supporting teachers' learning. In years 1 and 2, the aim of

[1]The research team included Paul Cobb, Kay McClain, Chrystal Dean, Teruni Lamberg, Qing Zhao, Melissa Gresalfi, Lori Tyler, and the authors.

PD activities had been to foster the teacher community, and to deepen teachers' understanding of central statistical ideas. In years 3 and 4, PD activities focused increasingly on the teachers analyzing their own and others' pedagogical practices with particular attention to student learning opportunities. The teachers usually participated in a statistical activity as learners during PD session, collectively planned a lesson based on the same activity, taught it to their students, and brought students' work or classroom video back to the next PD session.

The data for our discussion of means of support that an instructional sequence can provide for designing classroom interactions come from a collaboration with Irene, an experienced Mexican teacher who agreed to conduct a classroom design experiment to test a fractions sequence (Cortina et al. 2014) with her fifth grade students. At the time of the experiment, Irene was enrolled in a Master's degree program on educational development at a local public university. She also worked as a full-time teacher in an urban elementary school that serves children living in unfavorable social and economical conditions, with parents' irregular access to employment. Prior to this collaboration, Irene's teaching could be characterized as traditional. In a series of six one-hour meetings with the second author, Irene first became acquainted with the instructional sequence, including its rationale, and how this sequence was developed and used in prior design experiments. She then worked with the instructional sequence during 18 dedicated weekly classroom sessions, about 35 min each. After each session, she met with the second author to analyze the classroom events, and collaboratively plan for the upcoming session. Throughout the collaboration, the two authors analyzed and planned for supporting Irene's learning. Gravemeijer and van Eerde (2009) characterized this type of research design as *dual design research*, because both the teacher's and her students' learning are the goal of systematic classroom experimentation.

Instructional sequences with which we engage the teachers in our studies instantiate how learning in a specific mathematical domain can develop in a classroom setting, and how the outlined developments can be supported. The design rationale for these sequences is grounded in analyses of how this process unfolded in actual classrooms, and is expressed in the form of a conjectured learning trajectory (Simon 1995). In it, the designers specify the key shifts in classroom mathematical practices (Cobb et al. 2001) and how each of those shifts might be supported at a classroom level. The supports include establishing productive norms of classroom interactions, engaging students in specific instructional activities, making particular issues the focus of whole class discussions, and the use of particular tools and inscriptions. These kinds of instructional sequences are different from collections of instructional tasks that address a particular mathematical construct, but do not specify (a) the mathematical insights students are expected to develop by engaging in the tasks, (b) how might those insights be related to each other in a learning process and, importantly to our present discussion, (c) how to instructionally support the emergence of a new insight from the prior emergence of another.

Our goal as we worked with the teachers was that they would examine issues of teaching and learning statistics and fractions, respectively, as they adapted, tested, and modified the sequences in their classrooms. We did not focus on specific teacher moves, but instead pressed the teachers to justify the moves and actions that they chose by considering opportunities for student learning. Many of these PD discussions specifically attended to classroom interactions, as the nature of classroom discourse and organizing whole-class discussions are among the key means of support within the instructional sequences.

13.4 Methodology

The data in the 5-year PD design experiment included video-recordings and field notes of all PD sessions, transcripts of the key episodes, copies of the teachers' work, and a debriefing and planning research log. Additional data included student work from the teachers' classrooms, the video-recordings of the statistics lessons taught by two of the teachers, and annual modified teaching sets (Simon and Tzur 1999) comprising a video-recorded lesson in each teacher's classroom and a follow-up audio-recorded teacher interview that focused on issues that emerged in the course of the lesson.

The data from the collaboration with Irene who conducted a classroom design experiment included video-recordings and field notes of all classroom sessions and copies of students' work. In addition, two logs were produced, first by the teacher. Prior to each classroom session, the teacher recorded the goals she pursued, the activities she planned to conduct, and her expectations for classroom interactions and outcomes. After each session, she reflected on what actually happened in the classroom, and outlined what needed to be done next. This log played a central role during the debriefing sessions with the second author.

The second author produced a *research log*, which included design conjectures and notes related to both students' and Irene's learning. First, the log documented the second author and Irene's conversations during weekly debriefing and planning meetings, in which they relied on Irene's notes, classroom video, and copies of student work to understand students' learning progress. Second, this log documented weekly to bi-weekly debriefing sessions between the two authors, which focused on Irene's teaching and planning, and on the ways in which her work was supported.

In both cases, we analyzed the data using an adaptation of constant comparative method described by Cobb and Whitenack (1996) that involves testing and revising tentative conjectures while working through the data chronologically. As new episodes are analyzed, they are compared with conjectured themes or categories, resulting in a set of the theoretical assertions that remain grounded in the data. The thorough analysis of the PD study is available in the author's dissertation (Visnovska 2009) and the analysis of the dual design experiment on fractions learning is ongoing.

When we collected and analyzed data in statistics PD design experiment, we conceptualized learning as participation in communities of practice (Wenger 1998). Our focus was on documenting the development of the teachers' views and uses of students' reasoning (Visnovska 2009). These analyses provided insight into the teachers' learning as related to work with the instructional sequence over time. For this chapter, we were in particular interested in how the teachers in transition actively pursued questions of *their pedagogical interest*, irrespective of whether these were aligned with our PD agenda. We therefore analyzed the data corpus to document the pedagogical concerns the teachers in the PD group voiced in years 3 and 4, when the PD activities focused on pedagogical practices. This involved searching the retrospective analysis log for the questions the teachers brought up that diverted from the theme of PD group conversation at the time.

We categorized these questions as *pedagogical* or *other*. We aimed to gain a sense of frequency of teachers' focus on pedagogy, as opposed to focus on non-pedagogical issues such as their own mathematical learning, or institutional context of their work. The two latter foci were ongoing topics of PD conversations, and were prevalent in the first two years of PD interactions (Dean 2005). Within pedagogical questions, we noted whether teachers inquired about enactment (the *how*) or rationale (the *why*) of teaching. We use the summative view of these unsolicited teacher contributions to speculate about the nature of support that teachers sought and what this reveals about supporting the needs and learning of teachers in transition.

In the fraction dual design experiment, we conducted a retrospective analysis using as a guide the log produced by the second author. We then checked the formulated conjectures about the evolution of the teacher's learning against the teachers' log and the rest of the collected data, looking for inconsistencies, and refining the conjectures whenever necessary.

13.5 Uncertainties and Needs of Teachers in Transition

Requirements frequently cited for educative curriculum materials are that they include features that help teachers "recognize both the rationales for recommendations and the ways in which they can productively adapt the recommendations in their own classrooms" (Davis et al. 2014, p. 26). Indeed, teachers are much more likely to productively engage with teaching materials when they have opportunities to understand the underlying rationales and can use these to design meaningful adaptations. However, evidence abounds to suggest that it is non-trivial for teachers to recognize designers' rationales, even where explicit efforts were made to make such rationales visible in the teaching materials (e.g., Chval et al. 2009; Stein and Kim 2009).

We draw on the statistics PD design experiment to illustrate difficulties in supporting teachers in transition to reconstruct the rationales for instructional decisions. We first introduce the instructional sequence and structure of PD sessions relevant to our discussion. We then follow the teachers' agendas, and illustrate the questions and clarifications they independently brought up in PD sessions in years 3 and 4, when the focus was primarily on pedagogical issues related to teaching exploratory data analysis. Finally, we outline the teachers' learning in relation to our PD agenda and illustrate what was involved in supporting the group in the reconstruction of the design rationales that underpin the sequence.

13.5.1 Statistics PD Design Experiment

The focus of the *statistics sequence* was on supporting students to reason about univariate and bivariate distributions of data, often in activities that involved making recommendations based on comparing two or more sets of data. The intent of the instructional activities was that students would conduct genuine data analyses in order to address problems that they considered significant. The initial phase of the classroom activities, in which the problem situation was introduced and students discussed how useful data could be generated, was of considerable importance. The data was then introduced as being generated by this process. Three computer tools provided the students with a variety of options for organizing data sets with data represented graphically on a computer screen (for descriptions and analyses of these tools see e.g., Bakker and Gravemeijer 2003). In the classroom design experiments in which the instructional sequences had been developed, students compared their recommendations in classroom discussions and justified them by explaining how they had analyzed data.

To organize classroom activities productively, the teachers needed to monitor how their students reasoned about the data as they came to understand the problem scenario and conducted data investigations. They needed to plan classroom discussions of their students' analyses and make decisions about subsequent tasks and statistical learning goals.

The PD goals in years 3 and 4 included supporting teachers in making sense of individual students' statistical interpretations and solutions, and in adapting the statistics sequence to the needs and constraints of their classroom situations (Cobb and McClain 2001). Our role during the PD sessions was to link the teachers' insights, comments, and questions to the bigger pedagogical issues that were the focus of particular activities (e.g., how to make decisions about when to "move on" in instruction).

13.5.2 Teachers' Focus on Enactment

In both conducting and analyzing PD sessions, we were aware of teachers' strong inclination to focus on the enactment of the statistics sequence activities. While our attempts at bringing students' reasoning to the fore were met with a degree of confusion, teachers periodically steered the conversation to issues such as *How do we know when to move on (within an activity, to the next type of task,* etc.*)?* and *When do we introduce vocabulary?* This led us to search PD data for instances where, in our view, teachers' questions diverted from the theme of conversation at the time. In some cases, a teacher introduced an unexpected theme. In others, teachers asked for clarifications within a theme that, we assumed, had already been resolved and concluded. The summary of the results is presented in Table 13.1.

To interpret the table it is important to first clarify that the overall participation patterns in years 3 and 4 were largely similar (Visnovska 2009). What seemed to differ is the degree to which the teachers shaped the themes for the PD conversation in year 4.

Enactment. While the *When to move on?* and *When to introduce (vocabulary, median, box plots, calculations)?* questions periodically emerged during this period, some enactment questions in year 4 became more refined and started to relate to specific elements within the sequence. For instance, teachers asked about how to conclude a lesson where students proposed incompatible problem resolutions, and how to find out what students understood.

Rationale. Typical questions in this category included teachers seeking clarification for purposes of pedagogical actions such as *launch* and *re-teaching*. In the second half of year 4, three questions were genuinely related to the rationale of the instructional sequence. The teachers discussed where would a specific activity best fit within the sequence, asked about purposes for moving on to the next computer tool, and elaborated on a specific example from their teaching, explaining that the sequence cannot be a set collection of tasks but must reflect what students do in the classroom.

Other. Illustrative examples of questions that did not have specifically pedagogical focus included asking whether the median always indicates where most of the data are, or whether materials created within PD sessions will be used by other teachers in the school district.

The analysis supported two observations. First, the teachers, unprompted, asked substantially more pedagogy-related questions in year 4. At the time, the teachers were more familiar with the sequence and began to appreciate its impact on their students' motivation and engagement. Elsewhere (Visnovska and Cobb 2013; Visnovska et al. 2012) we conceptualized the developments of the teacher group as a process of community documentational genesis (Gueudet and Trouche 2009, 2012), highlighting how both teachers and resources they used were transformed in

Table 13.1 Distribution of different types of questions brought up by the teachers in PD sessions in years 3 and 4

	Pedagogy		Other (mathematics, institutional context)	Total
	Enactment	Rationale		
Year 3	3	5	3	11
Year 4	32	7	8	47
Total	35	12	11	58

the course of their interactions. We illustrated how the instructional sequence came to have significantly different meanings in the activities of PD group over time. Present analysis suggests that the teachers' initial interactions with the sequence and their use of instructional activities in their classrooms supported teachers in taking a greater initiative and actively pursuing pedagogical learning during the later PD sessions.

Second, in the majority of the unprompted questions, the teachers sought enactment advice centered in issues such as timing of specific teaching moves. In most of these instances, our efforts at re-orienting teachers to consider their students' reasoning when deciding about enactment were unsuccessful. For instance, we prompted teachers to consider whether their students had created new meanings that called for new vocabulary. Suggestions of this kind were not constituted as adequate or relevant to the question at hand. Instead, the teachers seemed to seek recommendations that were absolute and independent of the messy details of what was happening in the classroom. Only towards the latter part of year 4, some teachers started to pursue clarifications of rationales for pedagogical decisions that underpinned the sequence, and took student reasoning in consideration.

It is important to consider that the teachers in the PD group had long histories of participation within instructional practices that center on their enactment of lessons and on the students' completion of specific tasks. When the lesson objectives are stated in terms of content coverage, or as mathematical concepts and relationships that a teacher needs to explain to their students, the teacher has a rather good control over whether or not these objectives are addressed in the classroom. When the focus is on teacher's actions, it is also relatively easy for the teacher to assess whether specific lesson objectives were met, and therefore, whether it is in order to move to the next lesson.

When teachers aim to transition to working with objectives that are stated in terms of specific forms of students' reasoning that are expected to 'emerge' in their classrooms, the relationship between teachers' actions and whether or not a lesson objective is met becomes a lot more complicated. What teachers do still shapes students' learning, but it no longer determines whether any particular lesson will be deemed successful. Determining whether lesson objectives were met becomes non-trivial, largely because there is no longer a set of 'the right things to do' that would guarantee meeting the objective.

Re-occurrence of enactment-focused questions illustrates some of the expectations that teachers in transition have of the supports provided by new teaching materials, and of their learning about a new pedagogical approach. We contemplate that these are the issues for which teachers would read selectively in written instructional resources and guidance materials. The question then remains whether and how could such materials bring to the fore the rationales for pedagogical decisions so that these would, over time, become a relevant element in teachers' search for an enactment advice.

Let us now further illustrate how the reconstruction of design rationales is shaped by teachers' existing interpretations and is thus in principle a non-trivial matter. We then move to the more recent dual design experiment on fractions as measures and describe some progress we made with addressing the teacher's concern of assessing the success of a lesson, or *how to know when to move on*.

13.5.3 Supporting the Reconstruction of Design Rationales

Our work in the statistics PD design experiment illustrates the demands of reconstructing designers' rationales even when the designers can talk to the teachers directly and provide PD activities to proactively support such reconstruction. In years 3 and 4, we engaged the teachers in a variety of activities that focused on different aspects of teacher's role in the classroom, including how to conduct data generation discussions (see also Visnovska and Cobb 2013). Data generation discussions serve to introduce the context of the problem in the classroom, establish its relevance, consider what data could be collected that would allow for developing insight into the question under investigation, and consider specific details of how this data could be collected. These discussions play a significant role in shaping the ways in which students interpret data.

Early in our collaboration, the teachers recognized that data generation discussions were an important aspect of instruction. However, it became apparent that, from their perspective, effective instructional activities involved a scenario that was immediately interesting and personally relevant to students. For example, the teachers considered that activities that involve soft drinks or roller coasters were instructionally more promising than those that focused on issues of broader social significance (e.g., driving safety). They therefore understood the importance of data generation discussions primarily in terms of capturing students' interest and enticing students' engagement in instructional activities. It was also apparent that, from the teachers' perspective, data generation discussions made little if any contribution to the ways that students interpreted and analyzed data. These understandings were shaping significantly teachers' planning decisions and how they guided classroom discussions when they introduced new activities.

During our ensuing collaboration with the teachers, we engaged them in a number of activities that focused on data generation discussions. For instance, we introduced a statistical activity, in which the teachers were to act as students. We described the context of the problem, but we did not press the group to propose or clarify the process in which relevant data could be generated. When the teachers received data sets to analyze, they raised a number of questions related to the meaning of the data and its suitability as a basis for making claims about the situation at hand. Instead of responding to teachers' questions, we asked them to create a list of these questions and suggest whether and how these could have been addressed in the initial data generation discussion. We also asked the teachers to trial problem scenarios that they did not initially see as exciting for their students (e.g., addressing speeding on a local highway) and monitor how their students responded.

This approach was reasonably successful in that the teachers came to view a broader range of problem scenarios as potentially productive and saw it as their responsibility to develop the significance and relevance of problem situations with students, rather than limiting problem scenarios to those that they judged as familiar and exciting to students. Towards the end of our collaboration with the teachers, there were strong indications that the teachers had become aware that the students' understanding of the process by which the data were generated influenced how they interpreted and analyzed the data. In particular, they explicitly linked the issues that they addressed (or failed to address) while conducting data generation discussions to students' subsequent analyses.

This development would have been unlikely had we not made the rationale for the instructional sequence an explicit focus of professional development activities. We had to exert considerable effort at shaping teachers' planning decisions, including which problem scenarios were worth trialing in classrooms. Given the ongoing PD relationship, the teachers on occasion gave us benefit of doubt and tried the suggested activities in their classrooms, but they often did so *against their better judgment*. Nevertheless, these trials, and reflections on what happened in classrooms, were instrumental in allowing teachers to start questioning their instructional rationales and reconstructing those that underpinned the instructional sequence.

Supporting teachers' reconstruction of instructional rationales that their resources aim to advance is even more important in light of other contributions to this edited book. Remillard (Chap. 4) explores how teachers read resources in order to use them, while Kim (Chap. 15) and Leshota and Adler (Chap. 5) look into the patterns related to teachers' adaptations, omissions, and injections of classroom activities and their sequences. It would appear that without a sustained support, teachers in transition are rather likely to omit, or substantially alter those parts of resources that do not align with their current, possibly largely implicit instructional rationales.

13.6 Designing Resources to Support Teachers' Transition

Some of the illustrations from the PD design experiment we reported in the previous section can be easily seen as portraying teachers in transition as being captive to the very instructional practices they aim to abandon. This appears to be the case even when the PD activities are designed with an explicit goal of exploring instructional rationales, and situated within the overarching goals of the sequence. Such interpretation, of course, is inaccurate given that some teachers, indeed, have been known to accomplish the transition.

The illustrated complexities inspire us to look for understanding of the trajectories of transition and identifying elements that made these successful. Could we, for instance, provide teachers in transition with initial, *local*, well-defined goals and foci that would meaningfully orient their planning and classroom decision making? Could we do it in ways that bring students' reasoning into the picture and harness teachers' interest even when the big picture of ambitious practices is still somewhat elusive? What might such goals and foci look like? What are the mathematical contexts in which this can be done most productively?

Our analysis of teachers' questions suggests that supporting teachers to judge the progress within the sequence, and to make decisions about ways to continue would be of particular importance. These were some of the background considerations that guided the dual design research project in which a fifth grade teacher Irene collaborated with us to further develop the instructional sequence on fractions as measures. We illustrate how we engaged Irene in discussion of different ways to judge progress within the sequence, and how co-planning to help students' problematize an established strategy or idea was central to her transition.

13.7 Transition as Co-participation

13.7.1 Fractions Dual Design Experiment

The fractions as measures sequence (Cortina et al. 2015) is set within a narrative about the ways in which a group of ancient peoples, the Acajay, measured. Before the students encounter a situation, in which they explore lengths as related to the notions of unit fraction and proportion, measurement is approached more broadly. Within the narrative, Acajay people initially measure with body parts. The students engage in measurement problems using the same technology until they—like the Acajay—realize that this is at times problematic. Once they develop a need to standardize the measurement unit, a traditional measurement tool, the wooden *stick*, is introduced.

While the stick is initially a solution to an earlier problem, its use is subsequently made problematic by engaging students in situations where they develop the need for measuring lengths more accurately than what the stick alone allows. At this point, students learn that Acajay elders solved this problem by introducing smaller length measures, *smalls*, lengths of which represent unit fractions of the length of the stick (Fig. 13.1). The overarching aim of the sequence is that students develop understandings of fractions as quantities (Thompson and Saldanha 2003), in particular in linear measurement situations.

Similar to working with the statistics sequence, to organize classroom activities productively, a teacher needs to monitor how their students reason about measurement, and later about relative lengths of different smalls. They need to plan classroom discussions in which measurement methods that were previously acceptable would become problematic from the students' point of view. To accomplish this, the teacher constantly makes decisions about suitable tasks, and the foci and goals for the classroom sessions, based on the actual reasoning of their students.

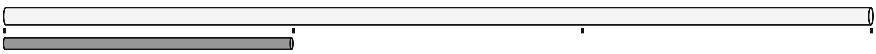

Fig. 13.1 *Small of three* rod with such a length (1/3) that three iterations of the rod cover the same length as the *stick* (reference unit)

13.7.2 Supporting Irene's Reconstruction of Design Rationales

When testing the sequence in her classroom, Irene initially followed a rationale in which she tried to be as faithful as possible to the instructional activities, in the way she understood they had to be enacted. At this point, Irene was much more focused on what she considered she had to do, than on how her students were reasoning. This became apparent after the first classroom session in which Irene engaged her fifth graders in an activity that entailed measuring lengths of different items with their body parts. She noticed that her students engaged in the activity enthusiastically, measuring and recoding the measures they took in their notebooks. Irene then tried to orchestrate a whole class conversation in which the advantages and disadvantages of measuring with body parts were to be discussed. Such a conversation was contemplated in the planed activity. However, when Irene asked her students about the disadvantages of measuring with body parts, none of them regarded measuring in this way to be problematic.

Irene knew that the ensuing activities in the instructional sequence involved using the stick (the standard unit of measure). She also knew that, within the sequence rationale, it was expected that students would regard using this tool as a means to overcome the limitations of measuring with body parts (i.e., inconsistent measures produced for the same length). She was therefore unsure about why none of her students saw measuring with their body parts as problematic. Irene considered whether the way she guided the activity was to blame, or even the students themselves, as they came from low-income families and performed poorly on standardized tests. Even though she had a clear sense that something had not gone according to her plan, the most reasonable course of action, in her view, was to move on to the next classroom activity. She planned to introduce the stick to students, ask them to measure with it, and hoped for the best.

In the debriefing that followed this first classroom session, the unexpected situation that Irene had faced became an opportunity to discuss how progress made in a classroom can be assessed. In particular, the second author used Irene's experience to contrast two ways in which one can make instructional decisions: trying to faithfully enact an instructional activity on the one hand, and progressively supporting students to reason about specific issues, in particular ways, on the other hand. The major difference between the two, they agreed, was in deciding when the instructional goals for a classroom session had been accomplished. Irene recognized that she was attempting to enact the first activity faithfully and when she completed the enactment, she considered herself, and the classroom, to be ready to move to the next type of activity. Seeing that her mentor had a different way of proceeding in mind, she agreed to instead explore whether she could help at least some of her students to realize that measuring with body parts had some limitations.

Irene and the second author designed several problem scenarios aimed at helping her students recognize how measuring with body parts could be unreliable, as it would render different numbers for the same length. In one of them, different students were to be asked to use their hands to measure a paper strip that was placed on the whiteboard. A conversation would follow in which the students would discuss why everyone did not obtain the same number and whether this meant that some of them made a mistake.

In the following debriefing with the second author, Irene commented that she had been successful in helping students recognize the different complications that measuring with body parts could cause. However, she was not satisfied with this result and wanted to make sure that *all* her students were aware of the limitations. From the sequence design point of view, developing such awareness was important so that all the students would see the standard unit of measure, once introduced, as a meaningful innovation. Irene thus decided to design additional problem scenarios and use them in the following teaching session.

Planning in this way represented an important shift in the rationale Irene employed for making instructional decisions. She no longer focused on which activities she needed to enact, when, and how. Instead, she now focused on the mathematical issues she wanted her students to discuss and understand, and viewed the problem scenarios as the means that could support the students in doing so. By and large, Irene kept focusing on learning goals, in terms of forms of students' reasoning she aimed to elicit, throughout the rest of the classroom design experiment.

Retrospectively, a number of issues have been critically influential in helping Irene shift her perspective. Several of them are related to the nature of the instructional sequence that was being tested, in which the learning goals were clearly specified and sequenced. First, the learning goal that was pursued at the beginning of the sequence was specific and relatively simple; to help students recognize the shortcomings of measuring with body parts. Had the sequence started directly with creating and comparing unit fraction lengths, we imagine the teacher would have been more inclined to follow a (perceived) enactment script.

Second, the problem scenarios Irene and the second author co-designed to help students uncover the problematic nature of measurement with body parts were also relatively uncomplicated. These built rather directly on Irene's awareness of situations in which measuring with body parts breaks down. The co-planning made her aware of a possibility of re-creating such situations in her classroom, thus creating opportunities for her students to notice and discuss the problematic results.

Third, and most importantly, these relatively simple means of support had been immediately effective in helping some of Irene's students develop forms of reasoning for which she was aiming. Rather than being incidental, these elements are products of prior classroom design experimentation, during which the capacity of the designed resources to support the work of teachers in transition was a primary consideration. We believe that without these supports, it is unlikely that Irene would have shifted her focus for making instructional decisions with such ease.

Nonetheless, we do not think that Irene would have made this shift had she been introduced to the sequence and then left to her own devices. It was critical that she had opportunities to regularly discuss the developments in her classroom with someone well acquainted with the sequence, and to receive feedback and support in the form of co-planning. She needed support in both understanding *that* she could proactively help students reason in specific ways, and developing images of *how* she could go about it in her classroom. Her initial planning ideas suggest that without this support she would have resorted to the 'covering the content' strategy.

13.8 Conclusions

Teachers have a need to know where they are going with their teaching: What is it that they are trying to achieve? Often, they solve this issue by focusing on what *they* need to teach, and base their instructional decisions on the content that needs to be covered, the activities that need to be implemented, and the work that students need to produce. The supports available in many curriculum materials, textbook resources, and teachers' workplaces often encourage, or at the very least, align well with this particular view of teaching goals and aims.

Transition to practices where students' reasoning is central entails a huge change for teachers. This goes beyond having to engage in a new kind of teaching, where problem solving plays a central role, and where students work collaboratively and share their ideas. Teachers also need an alternative way to keep track of progress and to guide their teaching, in issues such as "What comes next?" and "How much time should I spend on this?"

In this chapter, we first illustrated that teachers whose transition to ambitious teaching we facilitated in the statistics PD design experiment indeed felt somewhat under-supported around these issues. Importantly, this was the case even once the teachers' own mathematical understandings were reasonably strong and even in the PD program where care was taken to provide them with supports in responsive manner. We were eventually reasonably successful in supporting the PD group's reconstruction of instructional rationales for the sequence (Visnovska et al. 2012), but this success relied heavily on our ongoing co-participation in planning of and reflecting on the learning that ensued in the teachers' classrooms.

The case of Irene's participation in dual design experiment on fractions as measures allowed us to zoom in on several aspects of support that the sequence provided for her transition. First, it specified relatively simple initial goals for students' reasoning (i.e., coming to view measuring with body parts as insufficient), while at the same time creating opportunities for PD conversations in which two different rationales for deciding a specific course of teacher's action could be compared and contrasted. We recognize that holding such conversations is not the answer to supporting teachers' transition and that PD activities in which such comparisons would become meaningful to specific groups of teachers remain a non-trivial design challenge. However, targeting teacher's enactment-related decisions—an issue that is among their primary concerns—seems to be a direction worth further testing and development. We now plan PD activities so that they would allow us to intentionally initiate similar conversations with teachers early in their classroom work with instructional sequences, while grounding this work in relatively simple mathematical contexts.

The initial goals within the instructional sequence also directed Irene's attention to a more general view of *her role* in instigating the progress within the sequence, by helping her students problematize previously established classroom practices. This, however, would be likely less successful had Irene not been supported to *design* her own means of supporting student learning (in this instance tasks) that she tailored to the specific circumstance of her classroom. Indeed, if we accept that "teaching by design" is teachers' inevitable reality, not their choice (Brown 2009, p. 19), then the resources at teachers' disposal have to adequately support them in the design aspect of their work.

Related to this is the observation that the new tasks Irene co-designed and later designed independently for her classroom use, did useful work for her. This can be seen as a result of extensive prior experimentation in classrooms that the sequence reified. But equally importantly, Irene interpreted students' work with the designed activities as successful because her instructional agenda now oriented her to the specific forms of student reasoning she knew to expect, elicit, and reinforce during classroom interactions.

In discussions of educative material resources, teachers are often viewed as relatively independent learners, even when positive contribution of collaboration with other teachers or more experienced mentors is acknowledged. Our current view of a useful material resource inherently involves co-participation, but we find it equally worthwhile to explore ways in which some of the supports highlighted in this chapter can be embedded in material resources and thus support both teachers in transition and PD facilitators.

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