Chapter 12
Instructional Activity and Student Interaction with Digital Resources

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Abstract This chapter examines selected recent studies of the design and use of digital curriculum programmes and dynamic mathematical tools in school mathematics. The examples chosen bring out diversity both in the types of digital resources which are being adopted for teaching school mathematics and in the ways in which these are being taken up in instructional activity. These examples also show how any particular resource can be used in very different ways, and in ways quite different from those espoused by its advocates or intended by the designer. Digitised versions of traditional textbooks are cautiously innovative while individualised learning designs promote more 'personalised' instruction. Use of dynamic digital mathematical tools can support exploratory patterns of mathematical activity, underpinned by feedback from students' interaction with these tools.

Keywords Mathematics teachers' resources · Curriculum programmes
Digital resources · Dynamic tools · Instructional activity · Interaction patterns
School mathematics · Task environments

12.1 Introduction

The development of digital technologies is changing the media employed in doing, learning and teaching school mathematics. Although non-digitaltools and resources continue to be widely used, there is a shift towards their digital counterparts whether—by way of example—that be from ordinary to interactive whiteboards, or from graph paper to graphing software. These new media do not simply replicate the functionality of the old with increasing efficiency (although that is often how users initially view them); they make possible qualitatively different forms of interaction between user and medium, based—for example—on the introduction of

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new types of user interface or on the provision of instantaneous feedback on user actions.

This chapter examines some key current exemplars of instructional activity in school mathematics mediated by digital resources, focusing, in particular, on the types of interaction in which students are involved. Again, while digital resources are often, at least initially, assimilated to established patterns of instructional activity, they have the potential to reorganise such activity in significant ways. For example, the idea of the 'flipped classroom' proposes an inversion of a widely used pattern of instructional activity. Rather than starting with a lesson in school involving teacher exposition of new material through a whole-class presentation, followed up by some form of student practice of that material under teacher supervision through an exercise to be finished off after the lesson and typically at home, the idea of the 'flipped classroom' proposes a pattern of instructional activity which starts with students viewing a video-recorded exposition of new material at home (or otherwise outside lesson time) and continues with some form of class discussion and/or supervised practice during lesson time. It should be noted, however, that the two forms of exposition do not afford the same opportunities for interaction. On the one hand, a student can pause, review or advance the video-recorded version but not pose a question; on the other hand, the teacher can make their in-lesson exposition to the class more interactive and responsive, but while students can in principle pose questions, in practice this opportunity is limited and an individual student has little control over the pace and direction of the exposition. Equally, different forms of, and locations for, practice afford different opportunities for interaction. A student completing a paper-based exercise on their own at home can review material in the textbook, perhaps check answers against those given at the end of the text, and—*in extremis*—consult a family member or phone a friend. In class, as the 'flipped classroom' model recognises, there are possibilities for interaction with peers and the teacher that are not available at home. And, whether at home or in class, if the exercise is being undertaken on some kind of responsive and/or adaptive digital system, then the provision of automated guidance, feedback and customisation greatly changes the task environment and so the potential for student interaction.

12.2 Instructional Activity with Digital Curriculum Programmes

As its privileged mention in the title of this book acknowledges, the textbook has generally been the pre-eminent type of curricular resource for mathematics teaching. It is natural, then, for this chapter to give first consideration to the still evolving digital analogues of the printed textbook, commonly referred to as e-textbooks or digital curriculum programmes. Proponents of such programmes draw attention to the affordances they provide not just for production and distribution costs to be reduced and for material to be updated regularly, but—more significantly in pedagogical terms—for multimedia resources to be incorporated, for instruction to be customized, and for users to connect to virtual communities (Choppin et al. 2014).

While such developments are taking place around the world, it is in the United States that they have acquired a particular impetus in recent years which has led them to be subject to relatively extensive research. For that reason, I draw on two recent studies conducted in the US which have examined the emerging characteristics of digital curriculum programmes and their patterns of use in schools. In line with the aim of this chapter, my focus will be on the types of instructional activity associated with such programmes, including student interaction with and through their resources: interaction between student and programme system, interaction between students while using the system, and interaction between student and teacher in association with use of the system.

12.2.1 Choppin et al.'s Study of Digital Curriculum Programmes

This recent study examined the range of digital digital curriculum programmes emerging in the US: programmes designed to substantially supplement or entirely replace traditional printed textbook series (Choppin et al. 2014). The researchers found that such programmes were broadly of two types. Characteristic of the major educational publishers were what the researchers termed 'digitized versions of traditional textbooks', having structure and content similar to existing textbooks but taking a digitized rather than printed form, and intended to be used in much the same way as traditional textbooks, under the direction of a teacher. What the researchers termed 'individual learning designs' were designed to be used more directly by students as individualised study programmes, largely independent of the teacher, often with built-in assessments used to adjust the pacing and sequencing of content to the individual student user. These programmes can be seen as seeking to bring into the educational mainstream the type of approach pioneered by earlier traditions of paper-based programmed learning (Gagné and Paradise 1961) and individualized instruction (Hirsch 1976), followed by computer-based intelligent tutoring systems (Wenger 1987) and integrated learning systems (Becker 1992).

The researchers selected six programmes for more intensive study, mostly of the latter type (because these appeared to vary more in their characteristics) but including one programme of textbook type (representing what appeared to be many programmes with very similar characteristics). A substantial sample of the curriculum materials for each of these programmes was analysed in terms of themes derived from prior literature review. The theme which is of particular interest for this chapter concerned factors affecting student interaction with these programmes, conceptualised in terms of the types of learning experience (i.e. instructional activity) provided; the mechanisms provided for individualization and differentiation; and features aimed at virtual communication between students, teacher and others.

The study found that digital digital curriculum programmes of both types tended to emphasise what it characterised as ‘passive’ types of learning experience, such as viewing recorded presentations, following model demonstrations, and then completing related examples. Typically, the use of multimedia did not extend beyond videotaped presentations or narrated PowerPoint files. Thus the term ‘passive’ seems intended to highlight the limited possibilities of interaction with the system available to students. Nevertheless, a minority of programmes did take greater advantage of the learning potential of multimedia by employing interactive applets to introduce mathematical ideas with students assigned a more ‘active’ role in manipulating representations of a scenario so as to solve mathematical problems set in that context.

In terms of individualization or differentiation of activity, programmes ranged from those with some form of adaptive assessment built in, automatically assigning new tasks to students on the basis of their prior performance, to those which tracked student performance but, rather than using the results to set new tasks, either made suggestions to students about suitable tasks or provided the teacher with reports intended to inform their decisions about assigning tasks. At its most sophisticated, automated adaptivity introduced a high quality of interactivity between system and student, responding not just to the accuracy of students’ performance on tasks but to the speed and facility of their handling of virtual manipulatives. In this respect, then, some of these e-textbooks introduced an important degree of responsiveness to the student, lacking from conventional textbooks.

As regards facilities for virtual communication, some programmes provided a facility for teachers to comment on student work or offered a messaging or mailbox system enabling one-way (teacher to student) or two-way (student to teacher as well) communication. Other programmes did not make any provision of this type. Indeed, in most of the programmes, the emphasis appeared to be on individualised learning activity, involving each student working independently on the system at their own machine, with little interaction with the teacher or other students envisaged in the design of the materials themselves. The researchers also commented on the absence from the programme systems of discussion boards to enable students to exchange ideas about tasks. It seems that mechanisms for reciprocal interaction between students were rarely engineered into the systems themselves, although the way in which such systems were used in practice could introduce such interaction externally.

This study provides a useful overview of the types of digital curriculum programme currently available, based on studying their materials. In particular, it identifies the kinds of instructional activity and the forms of student interaction anticipated by these designs. However, mindful that designers’ intentions are not always reflected in users’ implementations, it will be useful to complement this study with one examining the use, adaptation and development in practice of one of these digital curriculum programmes.

12.2.2 *Murphy et al.'s Study of School Use of Khan Academy*

Murphy et al. (2014) examined the evolving use of one of the digital digital curriculum programmes included in Choppin et al.'s study: Khan Academy. Khan Academy originated as a website providing short videos showing the process of solving standard types of mathematical problem on a blackboard with a voiceover explaining each step. A natural extension was to provide associated sets of problems suitable for practicing the procedures demonstrated, interactive to the extent that users can check an answer or request a hint. Further facilities were added to generate reports on users' coverage of, and performance on, problem sets, as well as to introduce game-like features allowing users to gain points and badges.

Recently, the developers of Khan Academy have sought to strengthen its capacity for use in schools, undertaking a project in which they worked with researchers from SRI (who conducted this implementation study) to explore use of the product in a number of volunteer schools, refining the design responsively and developing professional guidance. The study treated Khan Academy as a generic example of a much wider class of similar digital learning tools and resources intended to support *personalized learning* of mathematics, tailored to the student user. The study took place over 2 consecutive school years and across 9 sites (school districts, charter management organisations, or individual schools). In each school year, data were collected—using both structured and semi-structured methods—through site visits, classroom observations, interviews with organization and school leaders as well as teachers, parents and students, surveys of teachers and students, and students' user log files. The approach to analysing the resulting data corpus could broadly be described as combining systematic survey and multiple-case study methods, including various forms of triangulation. Again, my focus here will be on those aspects of the study bearing on instructional activity and student interaction.

Khan Academy has become associated in the popular imagination with a 'flipped classroom' model in which the teacher assigns students to view, as homework, a video covering new material in advance of a follow-up classroom lesson. However, this study found that few teachers asked students to watch the Khan Academy videos, either inside or outside of school lessons. Considerations influencing teachers were not just logistical ones such as the feasibility of students being able to access the videos out of class, but pedagogical ones such as students being unable to raise questions—and more generally interact with a teacher—when meeting new material through viewing videos. Rather, teachers preferred to themselves continue to introduce the class to new mathematical material through conventional teacher-led classroom instruction. Nevertheless, the teachers did make extensive use of another component of the Khan Academy system, assigning its problem sets to give students practice relating to material that had either recently been introduced by the teacher or that the teacher had identified as requiring revision on the part of some students. The study reported that both teachers and students particularly appreciated the student-system interaction associated with these problem sets whereby the system provided immediate feedback when the user entered an answer to be checked. The study also reported that sites and teachers varied in whether they encouraged student-student interaction while working with the programme.

When students did choose to access videos, this was to view material relevant to the problem set they were currently working on. Nevertheless, the study found that the overwhelming majority of the time that students spent logged on to Khan Academy was devoted to working on problem sets rather than viewing videos. Equally, the study noted that when students were having difficulty with a problem they tended either to seek help from teacher or peers or to use the hint and step-by-step features in Khan Academy rather than viewing or reviewing the related video. Consequently, to support use of the videos as a resource by students, the developers made changes to the system: positioning links to relevant videos so as to make it easier for students working on a problem set to access them; adding a facility to fast-forward videos during playback so as to locate information more efficiently.

This study, then, highlights a two-way process of adaptation. First, teachers and students selectively appropriated those components of the original Khan Academy system that they perceived as enhancing existing forms of instructional activity. In particular, they embraced the use of problem sets, attracted by the supportive forms of feedback that the online system made available, and the consequent enhancement of student-system interaction within the established instructional activity of working on practice exercises. Then, the designers modified the system to improve its appropriability, particularly that of underused core features. In particular, the curriculum alignment and user interface of the video resources were improved in ways intended to make them more readily appropriable by, and valuable to, school users. At the same time, further modifications appear to be intended to encourage a shift in instructional activity towards the designers' vision of a personalised approach to learning mathematics. In particular, features were added to support more independent student use of the system: a search capability to quickly find videos and problem sets by topic; and a goal-setting feature to allow students to specify specific videos and/or problem sets to view and complete.

12.2.3 Discussion of Instructional Activity with Digital Curriculum Programmes

These studies suggest that the perceived quality and additionality of student-system interaction is likely to be a key factor influencing teacher and student decisions about whether to embrace innovative features that current digital digital curriculum programmes bring to a traditional instructional model of exposition and practice. In the case of Khan Academy, for example, where exposition of material was concerned, the student-system interaction associated with video presentations was generally viewed as inferior to that available through teacher-student interaction, and so was not embraced; conversely, where practice of material was concerned, the student-system interaction associated with checking answers to problem sets was viewed positively, and so was embraced.

In terms of the three aspects of interaction set out at the start, it seems that development to date of digital digital curriculum programmes has placed greater emphasis on, and had greater success with, the first of the aspects of interaction highlighted earlier—interaction between student and programme system—rather than the other two—interaction between students while using the system, and interaction between student and teacher in association with use of the system. Indeed, it seems that many current digital curriculum programmes aspire to individualise instruction and ‘personalise’ learning. Particular strengths of such programmes are their use of adaptive assessment to tailor the content presented by the system to the response history of the student, and/or their provision of reports through which the teacher can monitor student progress and adjust provision accordingly.

A particular weakness, however, of many of these programmes is lack of attention to peer interaction between students. Generally, facilities for such interaction are not engineered into the delivery system, nor are curricular tasks designed with it in mind. This interpersonal dimension is, then, an important topic for future research on the design and use of digital digital curriculum programmes. Bearing in mind, too, the risk that students come to see their responsibility as getting schoolwork done efficiently rather than as learning mathematics deeply, one can easily envisage ways in which overly instrumental use by students of a system’s provision of hints and checking of answers could lead to degeneration in quality of learning. Indeed, such phenomena bedevilled previous generations of individualised learning systems (Erlwanger 1975; Hativa 1988).

The study by Choppin et al. (2014) also examined the extent to which digital digital curriculum programmes provided students with learning experiences in which they could change parameters in figures or equations to explore dynamic relationships between quantities, or choose or manipulate tools or representations to solve problems. It found such provision in only a minority of the programmes, and commented on the absence from any of the programmes of resources which exploited the dynamic coordination of graphical, numeric and symbolic representations. This points to a further important way, now to be examined, in which digital resources potentially modify instructional activity and student interaction through changing the task environment for mathematical activity.

12.3 Instructional Activity with Dynamic Mathematical Resources

Just as the provision of hints and checks changes the task environment for undertaking practice exercises, the use of digital mathematical tools changes the task environment for tackling tasks which require the construction, manipulation and coordination of mathematical representations. In recent years, digital tools for such purposes have started to be taken up in school mathematics, typically to support the inclusion in the curriculum of more challenging investigative and problem-solving tasks, and often with the intention of developing a more inquiry-based approach to instruction. Thus, introduction of digital technologies may also influence instructional activity and interaction between teacher, students and resources through change in the classroom working environment and the mathematical tool system in play. To examine this issue, I draw on recent studies which illuminate features associated with the use, first, of dynamic mathematical tools, and then also of networked classroom technologies.

12.3.1 Ruthven et al.'s Studies of Instructional Activity with Dynamic Mathematical Tools

Recent years have seen considerable interest in, and increasing use of, various forms of dynamic mathematical software—either computer- or calculator-based—in mainstream school mathematics. My research team has conducted a number of collective case studies of teaching practices incorporating the use of such tools. These studies have gathered data through lesson observation and teacher interview, and employed both emic and etic modes of thematic analysis to analyse the teaching practices and the thinking behind them. A general finding is that teachers regard the use of such tools as supporting more investigative classroom approaches by enabling mathematical processes to be carried out more easily and efficiently, making them more open to replication and revision, and so supporting a more experimental style of working mathematically.

One study examined teaching practices involving use of dynamic geometry software (Ruthven et al. 2008). All the teachers involved indicated that they valued dynamic geometry for the contribution it could make to guiding students to discover mathematical properties for themselves, but the practical expressions of this idea were very varied. Correspondingly, while all of the teaching practices observed in this study exploited the dragging of figures to identify mathematically significant properties, beyond that there were important differences of approach to instructional activity. First, teachers differed in the degree and type of interaction with the software that they saw as being valuable for students. At one extreme, the software, projected to the whole class, was used only by the teacher as a presentational tool; more typically, students, working individually or in pairs at their own machine, were given the opportunity to manipulate prepared figures; but only occasionally did teachers see value in students learning to construct their own dynamic figures. One factor influencing these choices was the teacher's view of how best to manage mathematical complexities associated with the software: while some teachers sought to avoid exposing students to what they saw as unhelpful difficulties, other teachers welcomed opportunities to interact with students to help them recognise and resolve what they saw as challenges capable of generating mathematical insight. Another important factor was whether teachers saw students' own use of software as providing experience of mathematically disciplined interaction: teachers who took this view saw interacting with students to debug their constructions as a productive way of supporting learning.

Indeed, in one case, the teacher emphasised the way in which the dynamic software created a distinctive task environment for geometric work, enhancing opportunities for interaction both between student and system and between teacher and student. First, he saw as a key characteristic the way in which getting his students to make use of the software required them to develop the capacity to give clear instructions in mathematical terms. As students worked on their constructions, the teacher could help them to analyse and overcome difficulties they encountered and to express these in suitably mathematical terms. This teacher also noted the crucial part that he played in making key mathematical properties notable to students by prompting them to drag figures. Finally, he had identified how getting students to make use of text boxes to accompany their dynamic figures could help to sharpen the precision with which they expressed their procedures and findings in writing, because the provisionality of digital text made revision much easier.

Another study examined teaching practices involving use of graphing software (Ruthven et al. 2009). Here, one interesting common feature was the emergence of types of task structure, dependent on use of the graphing software by students to generate new information which they then had to find ways of interpreting so as to throw light on the fundamental mathematical question being addressed. In particular, this type of task structure and environment supported the teacher in taking on roles as co-enquirer with, or coach to, students.

To take the example of one lesson, the use of graphing software was crucial in underpinning the two related task formats in play. The focus of the lesson was on the graphs of quadratic algebraic forms. The first task that students were given was to use the graphing software to explore the effect of altering each of the coefficients of a quadratic form on the shape and location of the resulting curve. The second task, referred to by the teacher as "target practice", was to find equations for quadratic curves which would pass, in the first instance, through a single specified point, and then through a pair of specified points. Although it would clearly not be impossible for students to use other strategies, the intention was that they should tackle the first task by using what might be termed a *vary-and-infer* strategy based on finding a relevant pattern linking variation of the particular coefficient in the quadratic form to change in some property of the corresponding curve, and the second task using a *trial-and-improve* strategy based on iteratively trialling some speculative quadratic form and then successively refining it in the light of the fit of the resulting curve to the target point(s). What is fundamental for both strategies is the interaction between student user and graphing software; in particular, the information that the graphing software provides through displaying the graph of the expression that the student has entered; information which the student then needs to interpret mathematically to provide feedback relevant to the particular task.

12.3.2 Clark-Wilson's Study of Instructional Activity with Networked Dynamic Mathematical Tools

Over recent years there has been a progression in the working environment for making use of digital resources in school mathematics. During a period when students and teachers typically had access only to handhelds or workstations designed for personal use, instructional activity with digital resources was largely restricted to working individually or in pairs or small groups. The introduction of data projection and interactive whiteboards made whole-class activity with digital resources much more viable. Ideally, however, a mathematics classroom would provide scope, not just for activity at both scales, but for ease of switching from one to the other and of sharing the results of work. Thus, linking the various forms of digital technology in play through a digital network helps to create a more integrated working environment which facilitates the storage, retrieval and exchange of information, and its collective organisation and analysis.

Clark-Wilson (2010) examined development in teaching practices over an initial period of a few months following the introduction of a networking facility to mathematics classrooms where teachers and students were already experienced in using hand-held devices providing a range of dynamic mathematical tools. The networking facility linked students' individual hand-held devices to a central computer providing network management software, connected in turn to a classroom data projector or interactive whiteboard for public display. This central management software enabled the teacher to project the screen displays of all or some of the student handhelds as well as to distribute resource files to and from the handhelds.

The study employed data collected from a wide range of sources including teachers' own records and lesson logs, lesson observations, teacher interviews and questionnaires, student interviews, and e-mail correspondence. This was then analysed by combining two waves of coding: the first focusing on features of the technology, the second on teachers' descriptions of 'desirable' features and 'enhanced' student engagement and achievement. Clark-Wilson reports on the three main functionalities of the networked system which were taken up by teachers.

The Screen Capture facility allowed the teacher to display the state of all the handheld screens. This was the facility most widely used by teachers, and associated with a wide range of pedagogical purposes and forms of instructional activity. These included "monitoring students' activity during the lesson; supporting teachers to know when to intervene; promoting and initiating whole-class discourse; promoting and supporting peer and self-assessment; privileging mathematical generalization; … and enabling mathematical sorting" (pp. 752–753).

In particular, it seems that use of the Screen Capture facility led to further adaptation of the form of instructional activity which Drijvers (2011) has termed Work-and-walk-by. Drijvers argues that changes occur in the process whereby a teacher circulates observing different students at work as a result of that work being displayed on their computer screens rather than in exercise books. In particular, he suggests that the greater visibility of the screens makes it easier for the teacher both to establish a global view of work across the class and to follow up the work of particular students. Use of the Screen Capture facility brought this accessibility to bear on the smaller (and so less visible) screens of handheld devices, and made it still easier for the teacher to monitor a range of individuals and form an overview because of the simultaneous availability of the screens of all students, either displayed on the central computer or projected at the front of the class.

Equally, use of the Live Presenter facility, which allowed the teacher to select one handheld device for public projection, showing to the whole class the key presses and screen action from that device, appeared to support a form of instructional activity, termed Spot-and-show by Drijvers (2011), in which one student's work is demonstrated to, and then possibly discussed by, the whole class. This Live Presenter facility was used reasonably regularly by participating teachers, and again associated with a range of pedagogical purposes and forms of instructional activity: "teacher and student use to support the use of the … handhelds; teacher use to introduce and develop mathematical tasks; teacher use to generate data for use by the class; and student use to share mathematical observations, outcomes and insights" (p. 753).

The third facility used reasonably regularly by teachers was Quick Poll. This enabled the teacher to interrupt activity on all the students handhelds with a pop-up question accompanied by a forced choice of answers. The class set of responses could then be publicly displayed, with or without the students' names, and analysed in several ways. The reported range of uses covered: "as a focusing act to initiate the start of lesson activities; the generation of data for use during the lesson; prompting class discussion on a particular mathematical feature, concept or fact; and checking students' understanding of a particular mathematical feature, concept or fact" (p. 753).

Clark-Wilson concludes that exploiting these networking facilities supported change in instructional activity and patterns of interaction between teacher, students and resources. In particular, these facilities enabled development of teaching practices which enhanced formative assessment and were mathematically innovative. In terms of formative assessment, the public sharing of responses and screens through use of these facilities promoted more thoughtful teacher intervention and student discussion. These developments, in turn, increased opportunities for purposeful self and peer assessment by students. In terms of innovative mathematical tasks and approaches, amongst the examples offered are the use of a range of results from multiple handheld screens accessed through Screen Capture "to support mathematical generalizations; … as objects that can be sorted according to mathematical criteria; and … to increase the sample space of data or ideas" (p. 758).

12.3.3 Discussion of Instructional Activity with Dynamic Mathematical Resources

In terms of the aspects of interaction highlighted earlier, the first group of studies of dynamic mathematical resources emphasises forms of instructional activity in which distinctive types of interaction between student and digital system underpin complementary forms of interaction between teacher and student: here, the feedback that students gain from the system in response to their actions lies at the heart of instructional activity, allowing the teacher to focus on supporting students in interpreting this feedback and deciding how to act on it.

The second study focuses particularly on changed forms of collective interaction between teacher and students, underpinned by new forms of interaction between both teacher and student users and networked systems, and associated with a pedagogical shift towards practices of formative assessment. Clark-Wilson summarises these new forms of interaction with networked systems as promoting purposeful classroom discussion through which the teacher's awareness of students' current mathematical reasoning was enriched; providing teachers with fresh insights enabling them to provide thoughtful interventions during lessons; and supporting strategies for peer assessment and self-assessment by students.

Bearing in mind the dangers noted earlier of students treating mathematical tasks as work to be done rather than problems to be thought through, this interpersonal dimension seems crucial to avoid the risk of the more experimental approach often associated with the use of digital tools degenerating into unreflective trialling. What all these studies bring out is the still crucial role of the teacher in scaffolding the interaction between students and digital resources so as to increase the depth of reflection on results and the quality of mathematical interpretation.

12.4 Conclusion

I chose the examples of research which have been examined here so as to bring out some of the diversity both in the types of digital resources which are being adopted for teaching school mathematics and in the ways in which these are being taken up in instructional activity. Perhaps the first important lesson that can be drawn is that any particular resource can be used in very different ways: as illustrated—in the Clark-Wilson study—by the multiplicity of usages of each of the three system functionalities, and—in the Ruthven, Hennessy and Deaney study—by the differing forms of instructional activity that teachers employed in incorporating dynamic geometry into their classroom practice; notably the contrasting degrees of direct student interaction with the software. In particular, this signals that teachers may make use of a digital resource in ways quite different from those espoused by its advocates or intended by the designer: as illustrated—in the Murphy et al. study—by the way in which teachers tended to favour those Khan Academy facilities which they saw as enhancing their established pattern of instructional activity rather than shifting either to the 'flipped classroom' approach popularly associated with the product or to the more 'personalised' instructional model influencing the system designers.

Against this background, the cautiously innovative characteristics of the digitised versions of traditional textbooks—as studied by Choppin et al.—are not surprising from established publishers that currently dominate the market for curriculum resources. Equally, the drive from insurgent enterprises to promote individualised learning designs in terms of the more 'personalised' instruction that they make possible has its own commercial rationality. Clearly, these individualised programmes could draw further on the now well established tradition of research on intelligent tutoring systems as well as capitalising on continuing technical developments to enhance the interactive and adaptive functionality of such products and reduce their cost. Already—as the Murphy et al. study illustrates—the practice exercise components of current systems are seen by teachers and students as sufficiently advantageous for them to be routinely incorporated into instructional activity.

Nevertheless, it appears that a number of barriers remain to the widespread adoption of individualised learning designs. The first is that the expository components of such systems, while they largely follow established classroom conventions of narrated written presentation, are perceived to be less well adapted to curricular and pedagogical requirements than in-class exposition by the teacher. A second barrier is that most of these systems have not yet adequately incorporated the new kinds of digital mathematical tool and dynamic software which are increasingly used in school mathematics, although—as the Choppin et al. study establishes—some are making moves in this direction. A third barrier is the limited range of types of interpersonal interaction that present programmes appear to be able to foster: in part this is inherent in the attempt to individualise instruction, but it also reflects the limited aspiration or achievement of both types of digital curriculum programme in fostering forms of interaction through which students can productively exchange and discuss mathematical ideas with their peers or with a teacher (although, of course, such interaction could be organised off-line). Finally, there is the barrier of what teachers may see as a diminution of their role: towards manager and adviser of learning, rather than as more active initiator and director: indeed, the current attractiveness to the teaching profession of the re-sourcing of their own curriculum materials (Ruthven 2016) is, at least in part, attributable to the greater opportunities that such an approach confers on teachers to originate and curate their own resources and customise them to their situation and preferences. Thus a more realistic niche for individualised learning designs may be as a complement to teacher-led forms of instruction rather than as a replacement for them.

In general—in the studies by Clark-Wilson and by Ruthven et al.—the use of digital mathematical tools and accompanying resources appeared to support relatively exploratory patterns of mathematical activity and inquiry-based approaches to learning. Underpinning this was the way in which students' interaction with digital systems could provide them with feedback. A relatively open task structure and the responsiveness of a digital task environment appeared to support teachers in adopting roles as co-enquirer with, or coach to, students, although such patterns were not uniform. Feedback from the digital system, discussion between students and metacognitive scaffolding by the teacher generated a rich base for students to engage in formative assessment.

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