ICME-13 Monographs

Lianghuo Fan
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Jana Visnovska *Editors*

Research on Mathematics Textbooks and Teachers' Resources

Advances and Issues

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Editors and Contributors

About the Editors

Lianghuo Fan obtained an M.Sc. from East China Normal University, Shanghai, and a Ph.D. from the University of Chicago, USA, and is currently Professor (Personal Chair) in Education and Director of the Mathematics, Science and Health Education Research Centre at Southampton Education School, University of Southampton. He has extensive experience in education and research in China, USA, Singapore, and now the UK, and has published widely in mathematics education with a particular focus on mathematics curriculum, classroom instruction, assessment, teacher professional development, and comparison of East and West. Among his numerous publications are *Investigating the Pedagogy of Mathematics: How Do Teachers Develop Their Knowledge* (author; Imperial College Press, 2014), *Performance Assessment in Mathematics: Concepts, Methods, and Examples from Research and Practice in Singapore Classrooms* (editor; Pearson, 2011), and *How Chinese Learn Mathematics* (co-editor, World Scientific, 2004), and *How Chinese Teach Mathematics* (co-editor, World Scientific, 2015). He has served as editor, consultant, and mostly editor-in-chief of both primary and secondary school mathematics textbooks and instructional resources in the USA, Singapore, China, and the UK, including the latest series of *Shanghai Maths Project Practice Books* published in the UK by HarperCollins and its Shanghai English edition of *Shanghai Maths One Lesson One Exercise* by East China Normal University Press. He was Chief Editor of *The Mathematics Educator* (2006–2010) and is currently Specialty Chief Editor of *Frontiers in Education* (for STEM Education). He organized, as Chair of the International Programme Committee (IPC), the inaugural International Conference on Mathematics Textbook Research and Development (ICMT-1) held in the UK, 2014. He was also Co-Chair of the IPC for ICMT-2 held in Brazil, 2017, and continues to serve as Co-Chair of the IPC for ICMT-3 to be held in Germany, 2019.

Luc Trouche is Professor of Didactics of Mathematics in the French Institute of Education, École normale supérieure de Lyon, France.

In previous years, his research was dedicated to studying ICT integration in Mathematics Education. In particular, he has studied the interplay between *instrumentation* processes and *conceptualization* processes. This work has led him to analyse the teacher's role, introducing the notion of *instrumental orchestration* for modelling the management of available artefacts (for teaching a particular mathematical topic) in the classroom. He focuses now on resource design and teacher professional development in the time of digitalization. This has led him to develop, in a joint work with Ghislaine Gueudet and Birgit Pepin, the *documentational approach to didactics*. In this perspective, the notion of teacher *resource system* appears crucial in order to understand teacher's (developing) knowledge and the coherence of his/her activity. Investigating teachers' work with resources leads to consider teachers' collective work. Indeed, digitalization favours the emergence of more or less formal collectives: networks; teaching groups; and learning communities within or out of schools. Studying the interactions between individual and collective teachers' resource systems gives means for understanding the dynamics of these collectives, and for rethinking the way of supporting teacher development at a time of the "metamorphosis" of teaching environments.

He is currently involved in collaborative research at an international level, particularly with China, Brazil, or Mexico. He is the Chair of the International Conference Re(s)sources 2018, to be held in France in 2018. Among his publications: *From Text to 'Lived' Resources: Mathematics Curriculum Materials and Teacher Development,* co-edited with Birgit Pepin, and Ghislaine Gueudet (Springer, 2012) and *Tools and Mathematics: Instruments for Learning*, co-written with John Monaghan and Jon Borwein (Springer, 2016).

Chunxia Qi is a Full Professor and Deputy Head of the Institute of Curriculum and Pedagogy, Faculty of Education, Beijing Normal University (BNU), China. She is an Associate Editor of *EURASIA Journal of Mathematics, Science and Technology Education*, a member of the Editorial Board of *the Educational Review*, USA, and Editor-in-Chief of project-based learning mathematics textbooks for grades 7–9. She is also an executive member of the Curriculum Sub-commission of Chinese Society of Education. She holds a doctorate in Comparative Mathematics Education from BNU and was a Fulbright Scholar at Teachers College, Columbia University, from 2006 to 2007. Her research interests include mathematics curriculum comparison and mathematics textbooks theory and practices. She has published eighty articles (in both Chinese and English). Since 2000, she has served as a key member on the National Research and Development Team for Mathematics Curriculum Standards at the Compulsory Education Stage, the team leader for Macao Mathematics Curriculum Standards at junior high school, and the team leader for Macao Mathematics Curriculum Standards at senior high school. She is also the Chief Expert both in Collaborative Innovation Center of Assessment toward Basic Education Quality, China and Beijing Advanced Innovation Center for Future Education, Beijing.

Sebastian Rezat is a Full Professor of Mathematics Education at the Institute of Mathematics, Paderborn University, Germany. In his research, he investigates the use of traditional and digital mathematics textbooks at primary and secondary levels. He is particularly interested in students' interactions with textbooks and the interplay between students' and teachers' use of textbooks. His doctoral thesis on students' use of their mathematics textbooks has been awarded by the German Society of Didactics of Mathematics (GDM). In collaboration with Rudolf Sträßer he has developed the socio-didactical tetrahedron that models the whole situation of textbook and resource use from an activity theoretical perspective including the two main users—teachers and students—as well as social and cultural factors. His current research also includes linguistic aspects of mathematics teaching and learning with a focus on subject-specific genres in mathematics education. Together with Sara Rezat he has developed a model for analysing the features of subject-specific genres and their implementation in mathematics textbooks and classrooms. As Chair of the International Programme Committee (IPC), he is going to host the Third International Conference on Mathematics Textbook Research and Development (ICMT-3), to be held in Germany, 2019.

Jana Visnovska is a Lecturer in Mathematics Education at the University of Queensland, Australia. Her research interests focus on the means of supporting mathematics teachers that empower them in providing all students with opportunities to learn meaningful mathematics. To this end, she explores features of teachers' resources and forms of professional development that are in particular effective in supporting the learning and instructional interactions of teachers in transition. In ongoing research collaboration with Jose Luis Cortina, she contributes to instructional design research in area of fractions as measures and has been intrigued by the role of quantity in school mathematics education. Through her research, she seeks opportunities to learn about mathematics teaching and learning in different educational contexts, including severely under-resourced schools and classrooms in Mexico, and more recently, via a collaboration with Rhodes University South African Numeracy Chair Project, South Africa. She is a Co-Editor of the last four-yearly volume on *Research in Mathematics Education in Australasia 2012–2015*, and a Contributing Researcher on Australian multi-university project Inspiring Mathematics and Science in Teacher Education (2015–2017). Alongside many Australian teachers and researchers, she participates in re(Solve) project initiatives (http://www.resolve.edu.au/), by contributing to the development of resources and building the collaborative community of mathematics educators centred in teaching mathematics by inquiry (2017–2018). She has an emerging record of over 10 contributed chapters and journal publications.

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Mathematics Textbooks and Teachers' Resources: A Broad Area of Research in Mathematics Education to be Developed

Luc Trouche and Lianghuo Fan

Abstract

This introduction aims to situate this monograph as a result of the work of the Topic Study Group (TSG) 38 at the 13th International Congress on Mathematical Education, held in Hamburg in July 2016. Beyond this goal, it aims to evidence the fact that, far from being an isolated work, the contributions received to this TSG, their number as well as their content, witness the emergence, at an international level, of a new field of research, dedicated to the teachers' resources. After having set this scene, this introduction shortly overviews the chapters of this monograph, structured in five parts.

Keywords

Mathematics teachers' resources • Learning materials • Textbooks • Mathematics education • ICME-13

The Origins of this Monograph

This monograph originates in the work of the 38th Topic Study Group (TSG 38) of the 13th International Congress on Mathematical Education (ICME-13), held in

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Hamburg, Germany in July 2016. TSG 38 was dedicated to *Research on resources* (*textbooks, learning materials, etc.*), and was intentionally broadly inclusive of school mathematics textbooks, all forms of teacher manuals, learning and assessment materials, and in particular of digital and online resources.

One of the aims of TSG 38 was to bring to the foreground and examine various theoretical and methodological approaches that were being used to study teaching and learning resources. In relation to this, the TSG 38 call for papers sought contributions that addressed broadly the areas of resources, teachers, and students, and in particular encouraged contributions that presented analyses of the evolution of interactions between resources, teachers, and students in a time of transition.

To further guide and elicit the richness of contributions and to shape the future work of TSG 38, the call included following questions about the resources:

- Among the teaching and learning materials available in mathematics classrooms in different countries, what role do textbooks and other curricular or learning resources play in mathematics teaching, learning, and assessment?
- Could we consider different types of textbooks which suit for project-based, inquiry-based, or problem-based learning?
- How does the digitalization of information and communication affect the role of textbooks?
- What are defining features of e-textbooks, and how could we characterize the differences between the traditional textbooks and e-textbooks?

Guiding questions that focused on teachers' role included:

- What are the relationships between teachers' individual and collective resources, and how could we model such relationships?
- What is the role of teachers in developing textbooks and other teaching and learning materials?
- What are the relationships between resource designers and users?
- What are the consequences of evolutions at stake for the teaching of mathematics, and for teacher's knowledge and professional development?

Finally, guiding questions that focused on students' interactions with learning resources included:

- How do students, as well as their teachers, interact through resources?
- What are the effects of modern ICT (particularly Internet) on students' use and the design of resources?
- How do these evolutions affect student behaviour, learning, and relationships to the subject of mathematics?

(See also http://www.icme13.org/files/tsg/TSG_38.pdf for the call for papers.)

The organizers of TSG 38—editors of this monograph—received about 80 answers to this call. Sixteen of them were retained as long contributions to four plenary sessions, each dedicated to one of the following topics:

(1) What role do textbooks and other curricular or learning resources play in mathematics teaching, learning, and assessment?
(2) How does the digitalization of information and communication affect the role of resources?
(3) Teachers' collective work through resources;
(4) Teachers' and students' interactions through resources.

Out of the submissions received, 45 were retained as short oral contributions, which were presented in several chaired parallel sessions. Far from bringing copies of the papers proposed to TSG 38 before ICME-13, this monograph is the result of a process that began with the TSG 38 call, continued with the rich discussions arising in each TSG session, plenary as well as parallel sessions, and concluded with the review and editing work on 15 selected papers, chosen for their quality, but also for their emblematic character in the field of research. Speaking of a single field of research may appear as a simplification, as the discussion about teachers' resources gathered researchers with various major interests, inevitably including textbooks, as well as teachers' digital resources, curriculum resources, ICT, task design, Internet usage, teachers' communities of practice, and teachers' professional development. The organization of this TSG was based on a main assumption: the evolution of textbooks (to e-textbooks), the dissemination, by various institutions, of curricular resources through the means of the Internet, and the emergence of teachers' communities designing and sharing their own resources, all evidence the need for a new field of research that this book aims to explore and pursue.

A New Field of Research

The emergence of the new field of research related to teachers' resources is noticeable in both the theme-specific spaces appearing within the existing scientific events of the mathematics education community, and the growth of new, dedicated scientific events with this focus.

In a sense, ICME-13 was the first of the major four-yearly international congresses of mathematics educators to dedicate a TSG to the theme of teachers' resources. In past, contributions related to this theme were scattered across various TSGs. For example, four years earlier, ICME-12 (http://www.icme12.org/) included teachers' resources in at least three different TSGs, specifically in TSG 18: *Analysis of uses of technology in the teaching of mathematics*, TSG 19: *Analysis of uses of technology in the learning of mathematics*, and TSG 31: *Task design and analysis*. This was the case even though the term of *resources* itself did not appear in the topic titles.

Similarly, the 2017 Congress of European Research on Mathematics Education (CERME) dedicated a thematic working group to teachers' resources for the first time (see *Curricular resources and task design in mathematics education* at http://cerme10.org/). Previously, some of the themes related to teachers' resources were integrated elsewhere, mostly within the CERME activities that addressed technology in mathematics education.

During the same time period, new scientific events are being established, dedicated to research on teachers' resources broadly, and textbooks more specifically. The First International Conference on Mathematics Textbooks Research and Development (ICMT, see http://blog.soton.ac.uk/icmtrd2014/) was held in Southampton in 2014 (Keith et al. 2014). The Second ICMT was held in Rio de Janeiro in 2017 (https://www.sbm.org.br/icmt2/) and integrated a symposium on *Teacher-resource use around the world* (Remillard, Van Steenbrugge & Trouche to be published). The series of ICMT conferences will be continued in 2019 in Paderborn, Germany, and 2021 in Beijing, China. Lastly, an international conference will be held in Lyon in 2018, focusing on the theme of teachers' resources (https://resources-2018.sciencesconf.org/).

We would like to point out that the emergence of a field of research dedicated to teachers' resources appears as the result of a convergence of three fields, the field of research on technology in mathematics education, the field of research on textbooks (Fan 2013), and the field of curriculum resources:

- The technological evolution itself, mainly via the widespread availability of the Internet, leads the researchers in the field of technology in mathematics education to look beyond a given technological tool (a calculator, or a tablet) and beyond given software, to map out and understand a hybrid set of components and how these shape mathematical teaching and learning. These components include different hardware and software tools, but also new problem scenarios, mathematical tasks, animations, simulations, and ways of checking one's understanding, to name but a few. The word most often used for naming this hybrid set of components, imported from the Internet universe, is *resources*. An interesting insight into this word taking hold in this space is offered by tracing the evolution of vocabulary between two successive editions of the *Handbook of Mathematics Education* (2003 and 2013). Each handbook includes a chapter dedicated to technology in mathematics education. In the chapter published in the first edition of the handbook (Lagrange et al. 2003), the word "resource" appears only once, while in the chapter written 10 years later (Trouche et al. 2013), it appears 150 times.
- The evolution of textbooks through the integration of digital components, from paper textbook to e-textbook (Pepin et al. 2017), leads to an evolution of the related field of research. As a result of this evolution, each textbook is to be considered as a composed entity, and each teacher is considered as a user of a composed set of resources, exceeding a single textbook.
- The evolution of curricular resources, mainly also due to the readily available Internet, leads to a rather staggering expansion of the providers of such resources. The various institutions that traditionally provided curricular support and guidance—such as governmental bodies that oversee education—now compete in this space with various companies that might (or might not) follow the institutional prescriptions. Resources developed by experts in mathematics

> education appear alongside those developed by practicing teachers, who share their own resources individually, or within frameworks established by various professional associations and online communities.

Importantly, the emergence of the new field of research is marked by the need for new conceptual and analytical tools, theoretical frameworks, and methodologies. By bringing proposals for a number of these tools, the current book witnesses the forthcoming evolutions.

The Structure of the Book

This book is organized into five parts, the final one being dedicated to concluding remarks. Parts 1 to 4 correspond to the four parts of TSG 38, which were slightly modified to reflect the focal themes of the TSG 38 discussions:

1. Trends in presentation of mathematics in textbooks and other resources;
2. Teacher interactions with curricular and other learning resources;
3. Teachers' collective work through resources;
4. Teachers' and students' interactions through resources.

The main difference between the initial structure of TSG 38 as aforementioned and the structure of the book is that we have not dedicated a section to the issues linked to digital resources. The reason is that digital resources, today, are everywhere, and they are actually present throughout the four parts of the book. The four parts are followed by a chapter of conclusions, written by the editors, and by a final discussion chapter that provides a global view on the work, by Birgit Pepin: we warmly thank her for having accepted this challenge.

Part I of the book aims to set the scene, offering a view on different kinds of mathematics teachers' resources, their variation over the time and in different cultural and institutional contexts.

In Chap. 1, Trouche, Gueudet, and Pepin propose a specific theoretical frame for analysing new phenomena arising through digitalization, in the context of France. The main phenomenon that they underline is the profusion of open educational resources (OER), which, according to the authors, "provides new opportunities for the design and use of mathematics teaching resources" (p. 3). Evidence of these opportunities is given through tracing the development of teacher associations, which increasingly design and disseminate their own teaching resources. The specific frame is the documentational approach to didactics, which orients researchers to view teachers' work through the lens of their interactions with resources. This approach proposes new concepts such as teachers' resource system, which appears critical for understanding teachers' work at a time when their resources are far from being restricted to textbooks.

In Chap. 2, Qi, Zhang, and Huang, in the context of China, explore a general statement, at an international level, about the nature of textbooks moving "from an 'authoritative course' to a kind of 'supporting material' both for teachers and students" (p. 30). Drawing on the model of Nicol and Crespo (2006), distinguishing three levels of relationships between teachers and textbooks—adhering, elaborating, and creating—they analyse the work of six mathematics teachers in teaching *transformation*, a topic recently introduced in geometry textbooks in China. They provide strong evidence of the differences between teachers' textbook usages with respect to their experience: "novice teachers mostly arrive at the adhering level while using textbooks … As for experienced teachers' behaviour, most of them reach the elaborating and creating level" (p. 47). These results plead for a more careful analysis of the evolution of textbook usages over time.

In Chap. 3, Fan, Mailizar, Alafaleq, and Wang present a comparative analysis of the textbook content, focusing on how geometric proof is treated in secondary school mathematics textbooks in China, Indonesia, and Saudi Arabia. By doing so, they open a window on national contexts in Asia, which are under-represented in the international research literature. This comparative analysis considers three points of view: the time and place of introducing proof in the curriculum, the distribution of the treatment of proof in the textbooks, and the type of proofs introduced to students (a classification of proofs is proposed for this last purpose). The authors document considerable differences between China on the one hand, and the two remaining countries on the other hand. The textbooks in China appear to provide a much bigger room to proof. Deepening this analysis would require, as the authors indicated, extending the analysis to the exercises proposed by the textbooks (in this chapter only the main texts of the textbooks are taken into account), and pursuing a longitudinal approach, in which the interrelated evolution of curricula and textbooks would be explored.

Part II analyses interactions between teachers and curricular and other learning resources.

In Chap. 4, Remillard proposes a theoretical and empirical contribution to understanding what she terms teacher–resource interactions. First, she proposes to define *curriculum resources* as "print or digital artefacts designed to support a program of instruction and student learning over time" (p. 71), distinguishing them from other types of resources and comprising three components: teacher's guides, students text (books), and documents designed by teachers. Noting that teachers read and interpret this variety of components, "in order to craft instructional episodes" (p. 73), on a daily basis, Remillard builds on the notion of pedagogical design capacity (PDC) in order to better understand the complex skills involved in doing so proficiently. Empirical data from a national project in the USA are provided to illustrate the complexity of teacher–resource interactions in a case of a fourth-grade elementary teacher. In this illustrative case, the author portrays PDC as an interaction between affordances of the resource and the interpretive capacities of the teacher.

In Chap. 5, Leshota and Adler also explore the notion of mathematics teacher's PDC. Drawing on a larger professional development study conducted with Grade 10 teachers, the authors investigate one South African teacher's use of a prescribed textbook. They document different degrees of appropriation of the textbook and bring to the fore the phenomena of omissions and injections of content by the teacher. Analysing these aspects and how they accompany implementation of a given lesson from a given textbook, the authors shed light on the elements of teachers' PDC, specifically the type of relationship that the teacher forges with the resource. Through their analysis, the authors propose that PDC is reflected in the quality of opportunities for mediation of mathematics that the teacher creates in using given curriculum resources.

In Chap. 6, Siedel and Stylianides analyse interviews with 36 secondary mathematics teachers in England, shedding light on teachers' resource selection processes in the context of proliferation of instructional resources. The authors document a wide range of selection processes and rationales that were shaped by resource constraints (resource accessibility and characteristics, cultural environment of instruction), mathematics topics, and teacher characteristics (personal interests and perceived student needs). The results outline new means for guiding the design of resources well fitted to teachers' needs, and for supporting the processes of selection—and appropriation—of new resources by teachers, perhaps adding a dimension to PDC that would be essential in contexts where resource selection is a teacher's responsibility.

In Chap. 7, Kynigos and Kolovou consider teachers as designers of digital educational resources for creative mathematical thinking. This contribution comes from a European project, MC2, which is further presented in Chap. 10. The authors documented how the integration of teachers into the design team, from the beginning of design process, enhanced the design of resources for benefiting and supporting creative mathematical thinking. Authors report that the design capacity of teachers appeared to be stimulated by two features of the working environment: a social feature (belonging to a community of interest that gathers a diversity of actors —teachers, researchers, and technicians) and a technological feature (benefiting from a flexible design environment and a reflective features of the tools). The collective aspect of teachers' work appears to be crucial in this analysis, and it constitutes the focus of the following part.

Part III is dedicated to teachers' collective work through resources.

In Chap. 8, van Steenbrugge, Larsson, Insulander, and Ryve propose *a collective perspective for supporting teachers' negotiation of meaning*. The issue is: to what extent is making sense of a given resource facilitated if it becomes the objective of a group of teachers? The study, grounded in a national large-scale professional development programme in Sweden, mobilized two theoretical frameworks: a social semiotic framework to analyse the meaning potential of curriculum resources, and the communities of practice framework to analyse a group of teachers' negotiation of meaning around these resources. The potential of the collective work seems to be real, developing teacher PDC in eliciting the central idea of a given resource and supporting teachers' reflective work. But the collective work seems to be more efficient for appropriating concrete ideas than abstract ones. Knowing how it is possible to combine teachers' individual learning processes and collective work around resources should require further longitudinal studies.

In Chap. 9, Wang analyses mathematics teacher expertise, curriculum resources, and collective work through windows to French and Chinese contexts. Situating her work in the documentational approach to didactics, presented in Chap. 1, she develops the notion of *documentation expertise* and suggests that it could broaden the notion of PDC, covering the whole process of teacher interacting with resources. She documents the effects of teachers' collective work on documentation expertise in two very different contexts: in China, where collective work has long been considered an inherent feature of teachers' professional activity; and in France, where teachers' collective work can be observed only in some settings.

In Chap. 10, Essonier, Kynigos, Trgalova, and Daskolia analyse the role of context in social creativity for the design of digital resources, within the frame of a European project (MC2, addressed also in Chap. 7). The authors document the productive effect of a community of interest for developing mathematical creativity. To do so, they investigate social interactions in a community of interest, composed of participants with different backgrounds, each bringing their personal experience and expertise of their respective community of practice to the shared task of instructional design. Participants' past personal experiences and expertise *create* the social context for the work of the design group. The analysis of the work of the French community of interest that was part of this project reveals the importance of the roles played by different actors (designers, as well as reviewers), the tools for supporting the design process, and the contextual factors, for collectively transforming personal experiences into designed mathematical problems.

In Chap. 11, Rocha examines teachers' experience resulting from their interactions with resources. She analyses, in the French context, the case of a mathematics teacher association, Sésamath (also addressed in Chap. 1), within which teachers collaboratively design resources. She proposes the concept of *documentational trajectory* for modelling teachers' work with resources over time and analyses the effect of collectives on these trajectories. Rocha draws on Fleck's (1981) notions of thought collective and thought style as she illustrates documentational trajectories in two analysed cases. In the first case, a new teacher was strongly involved in Sésamath, both as a designer and a user, providing an illustrative case of how a thought collective of a group designing a Sésamath textbook strongly influenced one participating designer's documentational experience and trajectory. In the second case, a teacher was involved in the Sésamath sphere as an occasional user, but her participation in several collectives positioned her as a broker of different thought styles across these collectives, making brokering a characteristic feature of her documentational experience and trajectory.

While the three previous parts focus on teachers' interactions with resources, Part IV opens a window on students, analysing the role of resources as mediating tools between teachers and students.

In Chap. 12, Ruthven examines the links between instructional activity and student interaction with digital resources. Drawing on recent studies, he analyses the way digital resources—in particular digital curriculum programmes in the case of the USA and dynamic geometry tools in the case of the UK—modify both instructional activities and student interactions through changing the task environments in mathematics classrooms. Documenting "particular patterns of interaction between students and digital systems, and between students using such systems and their teachers" (p. 272) associated with different resources, Ruthven highlights that digital curriculum programmes, at this point, are reasonably successful in supporting student interactions with the programme system. However, they also suffer from lack of attention to facilitating peer interactions between students and student–teacher interactions. Indeed, the role of the teacher in scaffolding the interactions between students and digital resources remains crucial, making the difference between activity of producing solutions on the one hand, and that of making mathematical interpretations grounded in reflection on the solutions on the other hand.

In Chap. 13, Visnovska and Cortina analyse how resources could help teachers to plan for interactions with students' ideas. The authors focus on a particular kind of resources—instructional sequences produced in classroom design experiments—that were designed to support teachers in establishing, in their classrooms, particular types of interactions with the aim of facilitating the emergence of specific students' mathematical ideas. Building on two professional development studies conducted with teachers in the USA and in Mexico, the authors trace both specific challenges that the teachers encountered when adapting the resources to their classrooms, and supports provided within the instructional sequences, and within the co-participation structures of professional development programmes. The authors highlight the opportunities the sequence provided for (1) targeting teachers' enactment-related decisions, (2) reframing the teacher's role in instigating progress within the sequence, and (3) experiencing early success when resources were used in the teachers' classrooms.

In Chap. 14, Naftaliev examines prospective teachers' interactions with interactive diagrams. Interactive diagrams are the mathematical activities in interactive textbooks presented by software applications. The author analyses the components of such diagrams and underlines their potential for opening students' mathematical activity. Drawing from an experimental study mobilizing 25 prospective teachers (PT), she documents teachers' difficulties related to benefiting form this potential, even in situations where the teachers were aware of it. Choosing to stay within their own security zones, the PTs preferred to follow "*well-trodden paths*" (p. 311). Naftaliev uses her analysis to argue that "developing new practices and experiences in teaching-learning with interactive textbooks seems to be a necessary condition for PTs' implementation of these sources in their future teaching" (p. 312).

In Chap. 15, Kim examines teacher decisions on lesson sequence and their impact on opportunities for students to learn. When using a given resource, teachers often make adaptations to the sequence of tasks provided by the resource, for enacting a given lesson. Kim analyses two teachers' decisions in making such adaptations, and how these decisions shaped the quality of instruction and opportunities for student learning. The cases are situated within a larger study of 11 elementary teachers in the USA, who followed three different curricular programmes. Kim's analysis supports the argument that there is higher demand on teachers who use programmes incorporating conceptual support for learners. These teachers were more likely to alter the sequence of tasks proposed by the resource, at times to the detriment of mathematical coherence of resulting learning opportunities. The author argues that teachers need to both reason *with* resources and appreciate the reasoning *in* the resources, if they were to adapt these productively to their specific classroom situations. She outlines implications of the study for curriculum designers and teacher educators.

Part V is dedicated to concluding remarks.

In Chap. 16, coordinated by Rezat, the editorial team summarizes and discusses the state of the art of research on mathematics textbooks and other resources and their use as depicted in the current volume. Based on the discussion, conclusions are drawn regarding perspectives of future research on mathematics textbooks and resources, and their use.

In Chap. 17, Birgit Pepin, as a critical friend invited by the editors, in a general commentary paper on the book, develops an argument for a complementary line of research, deserving an increased attention in mathematics education: teacher learning with educative curriculum materials.

A Broad Area of Research in Mathematics Education to be Developed

It appears clear to us that the last two decades or so have witnessed a growing interest by the international mathematics community in the study of resources, as suggested in the title of our TSG 38. Various issues related to the concept, the role, the design and the development, the use and the enactment of resources of different types such as mathematics textbooks, teaching resources, learning resources and digital resources have received increasing attention from many researchers in different parts of the world. Accordingly, new conceptualizations, new themes, and new methods have clearly emerged, providing new insights into mathematics teaching and learning, as evidenced in the chapters of this book. We hope that the book will make a meaningful contribution to the further development of this relatively new and broad area of research in mathematics education.

Finally, we wish to record our appreciation to all the contributors to the TSG 38 and the authors of this book for their contribution and all the anonymous reviewers for their generous help in reviewing all the chapters submitted for the book. We also wish to especially thank the remaining members of the organizing team, who are also co-editors of this book, for their unwavering support and commitment throughout the whole process of producing this book.

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Part I

Trends in Presentation of Mathematics in Textbooks and Other Resources

Chapter 1
Open Educational Resources: A Chance for Opening Mathematics Teachers' Resource Systems?

Luc Trouche, Ghislaine Gueudet and Birgit Pepin

Abstract This chapter proposes a theoretical frame, the documentational approach to didactics (DAD), as a tool for analyzing the changes brought about by digitalization in the design and uses of mathematics teaching resources. One of the major changes appears to be initiated by the profusion of Open Educational Resources (OER), which provide new opportunities for the design and use of teaching resources. In order to analyze the effects of such opportunities, we focus on two cases: the French Sésamath association, providing OERs at a large scale; and a French mathematics teacher using OERs as a means for accomplishing her teaching. Through the lens of DAD, we investigate the implication of this provision of 'resources-on-offer' for teachers' practices.

Keywords Mathematics teachers' resources · Open Educational Resources Documentational approach to didactics · Resource system · Teachers' design work Teachers' collective work

1.1 Introduction

In many countries teachers now have access, via the Internet, to a profusion of digital resources, and among them, *Open Educational Resources* (OERs), defined, by OECD (2007) as "digitized materials offered freely and openly for educators, students, and self-learners to use and reuse for teaching, learning, and research. OER includes learning content, software tools to develop, use, and distribute content, and implementation resources such as open licenses" (p. 10).

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The availability of such resources affords drastic changes in education (European Commission 2013; Williamson 2013), regarding both the nature of resources—see the emergence of e-textbooks (Pepin et al. 2016a) or digital curricula (Choppin et al. 2014; Pepin et al. 2017a) in mathematics education—and teachers' interactions with such resources, as *users* (Pepin et al. 2013) as well as *designers* (Pepin et al. 2017b). As *users*, teachers have new means for searching materials, selecting and adapting them, using them in class, revising and sharing them. As *designers*, teachers have new opportunities for developing resources themselves and publishing them on the web, as individuals, or as members of communities or professional associations. Whilst differentiating between *users* and *designers*, we acknowledge that in our perspective *use* and *design* are intertwined processes (ibid). Once edited, the life of a resource is not finished: each user may be involved in re-designing, or in a new design, using OERs to enhance his/her own teaching resources. A fundamental feature of OER is that they are not static; they are living resources shared and transformed by groups, such as teachers, students, teacher educators, visiting lecturers, for example.

The research question we study is the following: to what extent does the provision of such 'resources-on-offer' lead to an 'opening up' of teachers' own resource systems? As a meta objective, we want to evidence in this paper the need for a specific theoretical framework for addressing this question.

In the following (second) section we briefly review the relevant literature before introducing, in the third section, the documentational approach to didactics, grounding the methodological approach used for the purpose of our study. In the fourth section we present the collaborative design of OERs by a French mathematics teacher association; and in the fifth we focus on the use of OERs by a French mathematics teacher. Subsequently, in the last section, we discuss the two cases, include the implication of the results for policy and practice, and we suggest future research.

This chapter is anchored in a national research project in France, ReVEA,[1] which investigates the evolution/change of teachers' work in an OER context for four academic subjects: English, Science, Technology, and Mathematics. Both individual as well as collective teachers' documentation work has been investigated, and for this chapter we have chosen two mathematics education cases from this project.

1.2 Literature Review and Conceptual Framing

In this section we review the literature and explain the main concept and notions used, in particular the notion of Open Educational Resources; e-textbook/s; and teacher appropriation and (re-)design of such resources.

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[1] www.anr-revea.fr, 2013–2018, ReVEA standing for "Living Resources for Teaching and Learning".

We started our chapter by distinguishing, among digital resources, between Open Educational Resources (OERs) and implicitly ‘non open’ (restricted, access needs to be paid for) educational resources. OERs are heralded as new opportunities for both students and teachers to access knowledge that has previously been only accessible to a few who (or whose institutions) could pay for such resources. This evolution is a consequence in particular of the digital nature of educational resources, a more recent evolution in a ‘connected’ world. In that sense OERs join the most commonly known ‘opens’, such as Open access, Open Data, Open Science, and greater access seems to be running parallel to greater accountability (i.e., providing a better return on investment for producing them). The distinction between OER and ‘non open’ educational resources is not always easy to identify: most of the commercial resources have an ‘exhibit’ version; the MOOC (Massive open online courses) are free access for looking at the videos, for example, or using quizzes, but require registration, and sometimes fees, for validating one’s participation. But of course, to design and set up a MOOC, for example, and to sustain it, is not “cost-free” (McAuley et al. 2010). Hence, what we call OERs are open and freely accessible as long as the designers and distributors can pay for their design and sustainability, possibly with a particular interest. This raises the important issue of an economical model for OERs’ design and usages, which we will not address in this chapter.

OERs cover a wide range of resources: from isolated digital resources that a teacher uploads onto her website, to a whole teaching sequence for a given grade. This diversity is not specific to OERs, but could be considered as a feature of the digital era. Choppin et al. (2014) distinguish between *educational technology* and *digital curriculum programs,* which “incorporate a variety of features, including multi-media content indexed by topic, assessment systems that electronically record student data and automatically summarize the data in reports and tables, and access to a full range of grade-level content, as specified by national standards documents…” (p. 11). Furthermore, Pepin et al. (2017a) have discussed *digital curriculum resources* (DCR) in the following way:

“In terms of distinguishing research on DCR from research in digital technologies, we see the main differences as being the particular attention that the former pays to:

- The aims and content of teaching and learning mathematics;
- The teacher’s role in the instructional design process (i.e., how teachers select, revise, and appropriate curriculum materials);
- Students’ interactions with DCR in terms of how they navigate learning experiences within a digital environment;
- The impact of DCR in terms of how the scope and sequence of mathematical topics are navigated by teachers and students;
- The educative potential of DCR in terms of how teachers develop capacity to design pedagogic activities. (p. 3)”

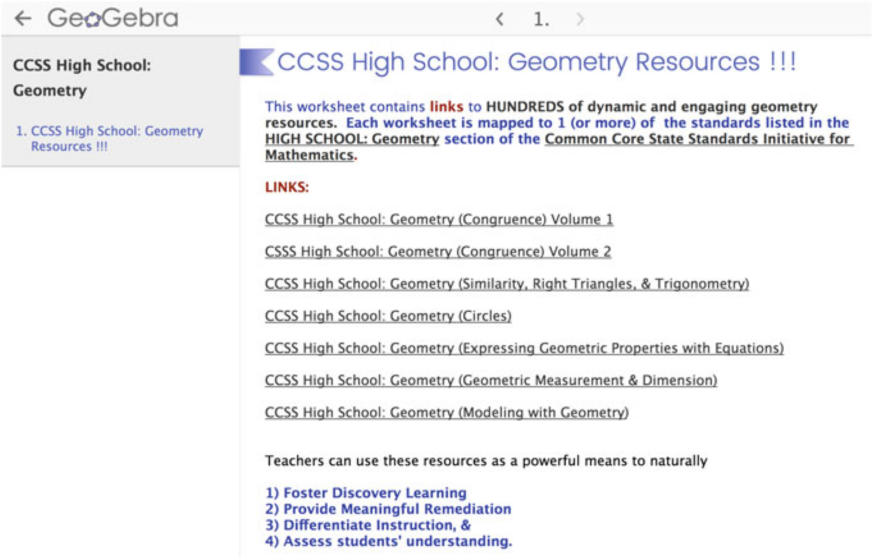
It is clear that the distinction is not always straightforward; see for example GeoGebra, which could be considered either as an educational technology, and as a wide repository of digital resources, with respect to various curricula (Fig. 1.1).

Actually, there is a large diversity of OERs, which we can classify according to:

- Their authorship (e.g., an association; individual authors; an institution; commercial organizations/designers);
- The entity sharing their usage (it could be a teacher, or a group of teachers, or a network…), the group of designers being sometimes the same as the group of users;
- Their purposes: (a) learning resources (e.g., courseware, content modules, learning objects); (b) resources to support teachers (educative materials, see Davis and Krajcik 2005), or tools for teachers and support materials to enable them to create, adapt and use OER, as well as training materials for teachers and other teaching tools.

It is likely that the development and availability of digital resources provide opportunities for teaching and learning, for the teacher as well as for the student. According to Johnstone (2005):

> By 2004, OER was defined to include: learning resources – courseware, content modules, learning objects, …; resources to support teachers – tools for teachers and support materials to enable them to create, adapt and use OER, as well as training materials for teachers and other teaching tools; resources to assure the quality of education and educational practices. (p. 16)



CCSS High School: Geometry

1. CCSS High School: Geometry Resources !!!

CCSS High School: Geometry Resources !!!

This worksheet contains links to HUNDREDS of dynamic and engaging geometry resources. Each worksheet is mapped to 1 (or more) of the standards listed in the HIGH SCHOOL: Geometry section of the Common Core State Standards Initiative for Mathematics.

LINKS:

CCSS High School: Geometry (Congruence) Volume 1

CSSS High School: Geometry (Congruence) Volume 2

CCSS High School: Geometry (Similarity, Right Triangles, & Trigonometry)

CCSS High School: Geometry (Circles)

CCSS High School: Geometry (Expressing Geometric Properties with Equations)

CCSS High School: Geometry (Geometric Measurement & Dimension)

CCSS High School: Geometry (Modeling with Geometry)

Teachers can use these resources as a powerful means to naturally

1) Foster Discovery Learning
2) Provide Meaningful Remediation
3) Differentiate Instruction, &
4) Assess students' understanding.

Fig. 1.1 A proposition for teaching geometry with GeoGebra according to given standards

This quote reflects the view that teachers can learn from 'quality' resources, perhaps by aligning their teaching according to designers' views. However, this seems to have changed, possibly due to the diversity and the dynamic aspects of an OER as a living entity. Hence, the issue of what is 'quality' has become a complex problem (Gueudet et al. 2012b, 2013).

In the case of a community of designers/users, this quality has then to be appreciated with respect to the usages in this community. Larsen and Vincent-Lancrin (2005) state that in OER communities "the innovation impact is greater when it is shared: the users are freely revealing their knowledge and thus work collectively." It can be argued that a network of OERs would be of great benefit to the community, increasing the value of individual resources and perhaps increasing the esteem of the community as a whole. However, how can the community be sustained, if the OER/s are not paid for? This raises the question of sustainability of OERs and their producing communities/networks, as well as question of intellectual property policies and useful standards (who holds the intellectual property license?).

Trgalová and Jahn (2013) claim that mathematics teachers "often find themselves unable to choose from among [the resources available] those that would be most relevant to their educational goal and to the context of their classes." (p. 973). Hence, they explore how teachers take quality criteria into account when designing and using/appropriating resources and argue that acquiring resource analysis skills must be one of the "keys to the teachers' professional development supporting the integration of dynamic geometry systems" (p. 973). This raises the issue of "design for whose learning": e-textbooks designed for student learning; or to promote teacher learning (i.e., designing educative materials, see Davis and Krajcik 2005).

For our purpose, we want in this paper to focus on the resources to support teachers. This is also the purpose of selected other chapters in this book: Siedel and Stylianides (Chap. 6) have drawn attention to the need to study teachers' selection of resources, whilst Naftaliev (Chap. 14) has explored the educational potential for prospective teachers of different types of teaching simulation around one innovative feature (i.e., interactive diagrams) of an e-textbook. Two further groups of researchers (Essonier et al., Chap. 10; Kynigos and Kolovou, Chap. 7) involved in the same project (i.e., MC Squared) have highlighted the collective design aspects related to digital educational resources and the development of "social creativity" and expertise when designing "c-books" (c for creative). The findings suggested that the interactions due to the socio-technical environment/s enhanced teachers' design creativity.

We summarize from this section that:

1. OERs include a variety of digital resources, from educational technology to digital curriculum programs (such as interactive e-textbooks);
2. OERs develop over a variety of processes (authorships, usages);
3. OERSs are expected to support teachers' work, but their abundance and heterogeneity could also constitute an added difficulty for teachers.

Our research question is the following: to what extent does the provision of such 'resources-on-offer' lead to an 'opening up' of teachers' own resource systems (i.e., teachers are more inclined to appropriate new resources, and at the same time to expose their own resources)? Answering this question gives us an opportunity to propose an overview of the state of the art of the documentational approach to didactics and related methodological aspects.

1.3 Analytical Framework and Research Design

We draw in this paper on the documentational approach to didactics (DAD; Gueudet and Trouche 2009; Gueudet et al. 2012a), which acknowledges the central role of teacher interactions with resources in their daily work. We introduce in this section its main concepts, then we highlight its holistic character, and finally we present its methodological principles.

1.3.1 The Documentational Approach to Didactics: Main Concepts

Adler (2000) proposes a wide definition of resources, as anything having the potential for *re-sourcing* teachers' work. DAD uses the notion of resource as encompassing the curriculum materials and texts, and resources that teachers select (for example OERs), use and develop in their daily practice in and for their teaching. In line with the work of Remillard (Chap. 4), DAD acknowledges "the multiple ways teachers interact with resources to design and enact mathematics instruction". Moreover, DAD emphasizes the dialectic nature of the teacher-resource interactions combining *instrumentation* and *instrumentalisation* (Rabardel 1995): the instrumentation process focusing on the effects of resources on a teacher's work; the instrumentalisation process focusing on the effects of a teacher on the resources she works on/with. These processes are also known as *mutual adaptation/s* in the literature (Berman and McLaughlin 1978).

In order to reach a given teaching goal, in a given *institution* (i.e., a school, a curriculum), a teacher interacts with a set of resources: some of them are 'old resources', already appropriated; some of them are 'new', often OERs, found on Internet, or selected or designed by colleagues, or presented in in-service training sessions. The DAD considers that this work results in a document,[2] defined as a

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[2] The name of *document* is taken from the field of information architecture, a field emerging with the rise of the Internet. A multidisciplinary French collective (Pédauque 2003) has been formed to study the conceptual needs of the digital era. It evidenced the crucial notion of document, as a *form*, a *sign* and a *medium*. Taking this further into the didactics of mathematics, Gueudet and

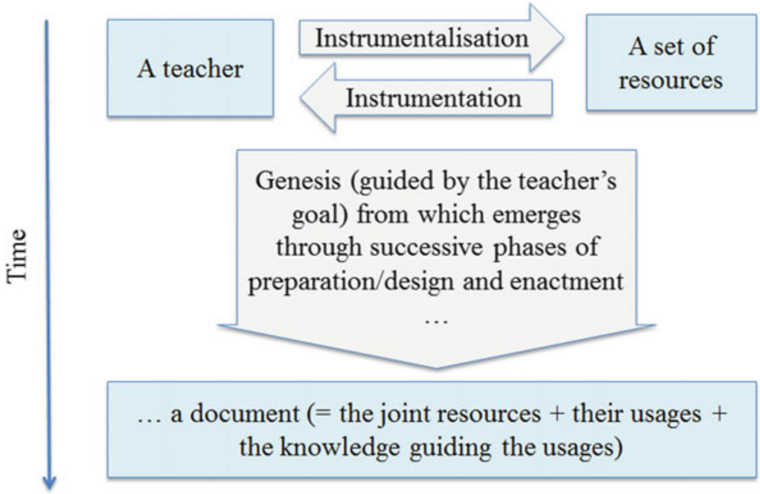

Fig. 1.2 A schema of a documentational genesis

mixed entity integrating a *material* component (the resources gathered for a given teaching objective), a *practice* component (the usages of these resources) and a *cognitive* component (knowledge guiding these usages). The DAD labels this process of developing a document *documentational genesis* (Fig. 1.2).

This process cannot be reduced to a 'one shot' activity, resulting from a single preparation of a given lesson: a document emerges through successive phases of design/enactment of teaching material, leading to a relatively stabilized organization of the activity. The essential point here is the dual nature of a document: of material nature, and at the same time of practical and cognitive nature. The way of analyzing this practical and cognitive nature could vary, according to the context of the study. In this chapter we have chosen to describe it as "usages and knowledge guiding the usages", as we did in Gueudet et al. (2012a), defining "knowledge" in the sense of "professional knowledge", as a set of pedagogical, didactical, mathematical knowledge and beliefs. In our seminal paper (Gueudet and Trouche 2009), we analyzed it based on the notion of *scheme*, defined by Vergnaud (1998) as an invariant organization of activity, made of teaching goals, rules, operational invariants, and inferences (see Chap. 9, choosing this model for applying DAD).

Trouche (2009) defined a *document* as the support of the teacher's didactical action, resulting from her interactions with resources.

1.3.2 A Holistic Approach

DAD constitutes a holistic approach to teachers' work at five levels:

- It considers teachers' work beyond the limits of the classroom, considering the time in front of students as an element of a process, their documentation work, where design and enactment are intertwined;
- It considers the different documentational geneses and their interactions. The resources a teacher works on/with do not remain as isolated entities, but constitute, over time, a *resource system*, understood as a structured entity, aligned with mathematics teachers' needs, practice, and professional knowledge;
- It considers teachers' work on the long term: Rocha (Chap. 11) proposes the notion of *documentational trajectories* for analyzing the interplay between teachers' activity and teachers' resource systems over their careers;
- It considers teachers' professional development as the joint development of knowledge and competencies that Pepin et al. (2017b) coin a *mathematics teacher design capacity*;
- It considers that a teacher's documentation work does not happen in isolation. DAD develops a "collective perspective on resources, their use and transformation" (Pepin et al. 2013). Teachers' documentation work has an essential collective component due to the social context of this work: in schools, with colleagues and students drawing on a given curriculum, using resources, such as textbooks as well as digital resources as social products. This is reinforced by the affordances of the Internet connecting teachers (see this chapter, following section).

DAD constitutes an evolving theoretical approach, due to both internal and external evolutions:

- By internal evolutions, we mean the need for or the emergence of new concepts by/when applying the documentational approach: for example, for naming specific elements structuring teachers' resource systems; for analyzing different steps of the documentational genesis, the notion of *distance* between two successive forms of a resource a teacher is working on has been introduced by Essonier et al. (Chap. 10);
- By external evolutions, we mean evolutions coming from theoretical networking (Prediger et al. 2008). For instance, for analyzing teachers' collective documentation work, Gueudet and Trouche (2012b) used the notion of *community of practice*. According to Wenger (1998), a community of practice is a group of persons sharing the same practice. It has three central features: the members of a community of practice have a mutual enterprise; a shared commitment; and a common repertoire. This repertoire can contain material objects, but also stories or signs that are shared by the members of the community. This notion is very relevant for analyzing the documentation work of a group of teachers as soon as they have a mutual enterprise, a shared commitment, and develop a common set of resources (see Gueudet et al. 2013). This theoretical appropriation leads to the

question of the relevance of new concepts, as a *community resource system*, or a *community* documentational genesis (see this Chapter, Sect. 1.4).

Following such a holistic approach calls for new methodologies. One of these is explained and substantiated in the following section.

1.3.3 Methodological Issues

Being consistent with DAD means to follow the documentation work of a teacher, over a long period, with respect to its continuity and to the diversity of places where this work takes place (e.g., in school, at home, in group sessions at university). Due to the difficulty of the researcher to follow the teacher in all these places, it is necessary to look at a teacher's documentation work through her/his own eyes. This is the principle of reflective investigation (Gueudet and Trouche 2012a), fostering the reflection of the teacher on her own documentation work. This methodology is in line with ethnographic traditions, and includes a case-based approach: the unit of analysis is the teacher, and/or a group of teachers working together for a documentation purpose (e.g., Chap. 11 studies this way collective work, also within the Sésamath association). The data collection strategy uses tools commonly associated with a case approach (e.g., video observation; interview; logbooks), in addition to selected new tools. Among these new tools is the *schematic representation of the resource system* (SRRS), where the researcher asks the teacher to draw his/her resource system (e.g., Pepin et al. 2016b). An SRRS opens up a window onto the way a teacher reconstructs, for the outsider/researcher, their main resources and how they interrelate, together with the aims and intentions of their use (Fig. 1.3). More recently SRRS has been renamed *reflective mapping of a teacher's resource system* (see Rocha, Chap. 11) for better capturing the dynamic process of describing her own resource system as designing a map for exploring a new territory, evolving as the exploration goes on. This reflective tool has been applied for exploring other concepts, such as documentational trajectories (Chap. 11), or teachers' interactions (Chap. 9).

We have used this methodology focusing on OERs for the two cases we now present.

1.4 Design of OERs by a Teacher Association: The Case of Sésamath

We present in this section the case of Sésamath, which has already been the subject of a number of analyzes due to its advanced character in terms of sharing OERs at large (Gueudet et al. 2016; Pepin et al. 2016a, 2017a; Sabra and Trouche 2011; also investigated in Rocha's contribution, Chap. 11). Sésamath is a French online

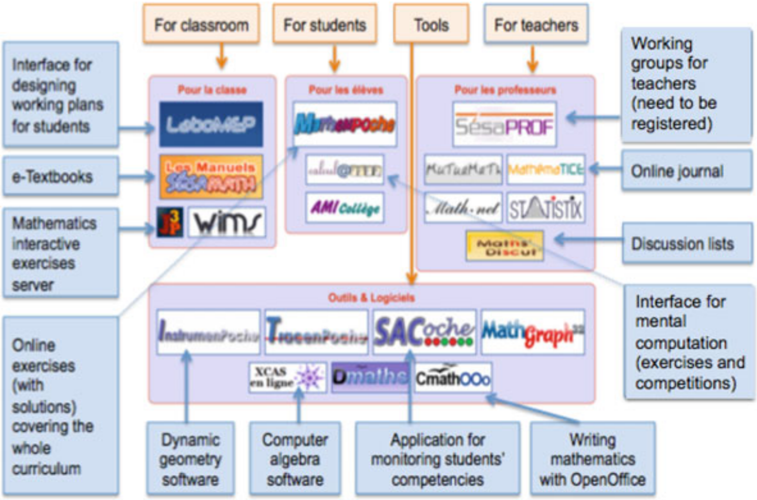

Fig. 1.3 Sésamath website front page (www.sesamath.net)

association of mathematics teachers (most of them teaching in lower secondary school, grades 6–9). The Sésamath association started in 2001 with the aim of designing OERs for mathematics teaching. We focus here on three aspects, zooming from the association itself to one of its members: the nature and structure of the Sésamath OER system; the nature of its design process in the case of a textbook chapter *Functions* (Gueudet et al. 2016); and a case study of a teacher involved in this design.

1.4.1 Sésamath: From OER to E-textbooks

The Sésamath spirit is summarized on its website (http://www.sesamath.net/) as “Mathematics for all”, which suggests (at least) two things: (a) giving teachers access to all the resources—Sésamath resources are true OERs,[3] open source, and free of charge; and (b) covering different mathematics teachers’ needs (e.g., allowing them to adapt resources for students’ individual needs; to discuss with colleagues; to use software for geometry, algebra, calculus). Figure 1.3 illustrates the large diversity of the Sésamath resources. Starting in 2001 from the design of a ‘drill-and-practice’ software program (Mathenpoche—standing for ‘Mathematics in the pocket’), Sésamath offered textbooks (with free pdf/odt version, and complements online) for the whole grade 6–10 curriculum in 2005. For most mathematics

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[3] The name of the association itself, “Sésamath”, is certainly revealing of this open-mindedness, as a wink to “Open sesame”, the famous phrase from the Tales of the Arabian Nights.

teachers, also in France, the textbook is still a central resource (Pepin et al. 2016a). Sésamath offers e-textbooks as elements of a wider digital interactive program, which is still in progress.

This program appears as a true *kaleidoscope*, that is, a colorful arrangement of interrelated resources (see Sect. 1.5: the case of Valeria). Sésamath offers generic educational technology (e.g., an application for writing mathematics with OpenOffice), as well as specific educational technology (e.g., LaboMEP standing for 'Laboratory for mathematics in the pocket'), allowing teachers to adapt learning materials to students' needs, and thus create "evolving artifacts that support interaction and collaboration" (see Kynigos and Kolovou, Chap. 7). Sésamath resources, including the textbooks, are widely used in France: since 2005, 300,000 Sésamath textbooks (a printed copy of the free online textbook) have been sold representing approximately 20% of the (very competitive) French textbook market. In addition, teachers connect to the Sésamath website for using selected resources: more than 1 million connections/hits per month.

Compared to 'ordinary' textbooks, we hypothesize that this easy access to a wide range of resources has the potential to *diversify* teachers' resource systems, if teachers use Sésamath, or if they use Sésamath in addition to other resources (e.g., other textbooks, see Rocha, Chap. 11). The issue of *enrichment* of their resource system has to be considered with respect to the didactical quality of Sésamath resources. Gueudet et al. (2013) have compared the nature of two textbooks, a Sésamath textbook designed by a number of regular teacher members of the association, and a textbook ordered by a commercial publisher, designed by a small team of experts (researchers and teachers trainers). They revealed/highlighted the following differences:

- Regarding the structure: the Sésamath textbook is an atomistic system that can be arranged differently by different users; the "expert textbook" is a single whole with an organized structure;
- Regarding the links to users: the Sésamath textbook is a proposal to be enriched by teachers' contributions; the "expert textbook" is a final product given to the teachers;
- Regarding the organization of the content: the Sésamath textbook is aligned with the national curricular requirements; the "expert textbook" is more didactically original, linked to the didactical choices of a reasonably homogeneous author team.

The president of Sésamath (Gringoz 2015) claims that these characteristics are related to the philosophy of the association: the only author guidelines are the ones provided by the curriculum, and the designers have to take care not to follow a specific didactical line (e.g., "designing for flipped classroom"). This is done to allow each teacher, regardless of their habits, to adapt Sésamath resources for their pedagogical goals and teaching. The interactions between Sésamath resources and their users are bi-directional: a user modifies and adapts the Sésamath resources, and they may also propose their changes to Sésamath, which in turn may accept and implement them. The Sésamath resources are in a flow of continuous evolution benefitting from the contributions of a wide community of users. The current quality of a given resource depends then on the number and different orientations of teachers' feedback comments, and quality becomes a dynamic process (Trgalová et al. 2009; Gueudet et al. 2013). It also depends on the initial conditions of this process, specifically the first version of a given resource.

1.4.2 Mode of Design of a Sésamath Textbook

For analyzing this mode of design, we will draw on the work of Sabra (2016; Sabra and Trouche 2011). The *mode of design* of Sésamath textbooks involves a number of teachers (approximately 20 for each textbook). They are all volunteers, having answered to a call launched by Sésamath on its mailing list. Their main motivation for joining a Sésamath project is the *exchange of experience*.[4] They could be members, or not, of the Sésamath association; having, or not, a previous experience in textbook design. But they freely decide to have a mutual enterprise and a shared commitment: designing a textbook in the context of Sésamath, which includes accepting the 'philosophy' of the association. Gringoz (2015) underlines the main rule of the design: the designers have to engage in a *collaborative* (for writing chapters) and *iterative* (alternating phases of design and experimentation/evaluation) process, in order to reach consensus about the textbook content. During this process they gradually build a shared repertoire. This is the reason why Sabra and Trouche (2011) characterize the textbook designer group as an emerging community of practice (see Sect. 1.3.3, this chapter), its participants sharing the same interest for OER design, use, and *adaptation*. These authors purposefully characterize the design process as *community documentation work*, indicating that this process of design results not only in a textbook, but also in shared knowledge. Sabra (2016) distinguishes particular moments, in this community documentation work, where a *community documentation incident* (i.e., the unexpected arrival of a new resource in the community documentation group) occurs, hypothesizing that such moments constitute an opportunity for analyzing the tensions between individual and community documentation work.

Sabra and Trouche (2011) ground their analysis in the follow-up of the design process of the Sésamath digital textbook for grade 10 (first year of French high schools), particularly the *Functions* chapter. This chapter has been chosen due to its potential to assemble resources of different nature and different representations: graphs, tables, contextualized problems, animations, to name but a few. In the following we will call DT10 the corresponding group of designers. It is the first

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[4] This common professional experience seems to constitute an important difference with the Communities of Interest (see Kynigos and Kolovou, Chap. 7), gathering practitioners from diverse disciplinary and professional domains.

digital textbook designed by the association, leading the designers to face both didactical and technological issues. DT10 members can have one or several roles: including content authors; designers of didactical scenarios; testers (for correctness of a particular chapter), or experimenters in a classroom. In order to follow the community documentation work of DT10, the tools of reflective investigation (see Sect. 1.3.3) were adapted. Three members of DT10, with different roles in the design process, were chosen as *witnesses* of the community documentation process, and asked to fill in a 'small follow-up log' (Sabra 2016). They pinpointed what for them constituted a community documentation incident, describing its nature as well as its effects, both for themselves and for DT10. The adjective 'small' is chosen to make clear that this methodological tool should not bother teachers who only have to identify and write down the main incidents happening in this process. The analysis is based both on the data collected via these logs, and on the "natural digital data" (i.e., emails exchanged in DT10, traces of the design process on the collaborative platform).

In terms of designing the *Functions* chapter, this analysis evidences two main trends for DT10 members documentation work: the DT10 members, who already participated in previous Sésamath textbook designs, contributed to this new design using *mainly* Sésamath resources; the new members contributed *mainly* by adapting their own resources to fit the textbook structure. 'Mainly' does not mean 'only': all DT10 members used a variety of OERs outside of Sésamath and outside of their own resource system (see the case of Anais, following section).

The search of OERs for designing the *Functions* chapter is all the more true when an incident happens. Sabra (2016) distinguishes different community documentation incidents: a change of mathematics curriculum; the fusion of two Sésamath OER repositories; the integration of a new software allowing to animate the online resources; and the discussion about the way for introducing the *Functions* chapter. Each incident stimulates both the opening of individual documentation work towards new resources, and the interactions within DT10, as we will evidence in the following section.

1.4.3 A Case Study of a Teacher Involved in a Sésamath OER Design

Sabra and Trouche (2011) have followed the documentation work of Anais, a member of DT10, during the group's collaborative work and discussion phase on the *Functions* chapter. Anais was one of the members witnessing the community documentation process via a Small follow-up log. She was chosen due to her profile (having already been involved in a Sésamath project, as a member of the French mathematics teachers association) and her activity in DT10 (the most active participant on its mailing list).

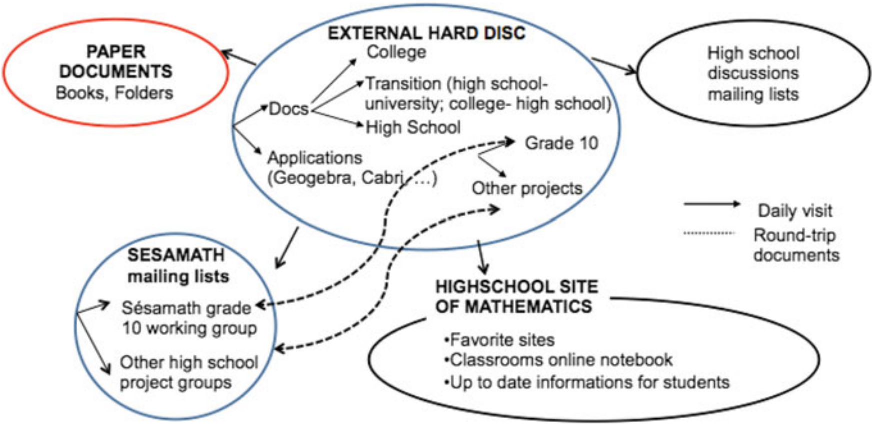


Fig. 1.4 Anais's SRRS (Sabra and Trouche 2011, p. 2361)

For Anais participating in this group constituted an opportunity for searching ideas for her own classroom teaching: "for example, for the topic *Variations and Extrema of functions*, I have not constructed the lesson yet. In fact, […] I will adapt the Sésamath book chapter for my class" (ibid, p. 2362).

Sabra and Trouche used the term *osmosis* (ibid, p. 2360) to highlight how Anais's and Sésamath's OER systems feed into each other. For example, Anais' own lesson plan introduces the concept of function with what she calls a complex 'start-up' activity, with the aim of reflecting on the semantic values of the language of functions; whereas the Sésamath textbook chapter starts with very simple exercises (reading of values from graphs and tables). These differences between each member's resource system and the textbook in progress foster the discussion in the design group, leading to a community documentation incident.

Anais' SRRS drawn by herself (Fig. 1.4), shows the central position of a digital resource: her external hard disc allowing her to 'travel with her resources', between the different places where her documentation work took place (at home, at school, or in Sésamath meetings). This central resource is connected to four sets of resources:

- Her 'traditional' paper resources;
- Sésamath resources;
- The website of her high school, opening her work to other websites providing OERs, and allowing her to interact with her students;
- Mailing lists (both in Sésamath and in her high school) for discussing questions related to mathematics teaching at the high school level.

Anais connects to Sésamath only via its mailing lists and the working groups she is involved in. She is an active participant in these forums. It appears also that Sésamath is not the only resource feeding her work; Anais uses a large range of OERs, not as a passive consumer, but also as an active discussant and producing member. Anais' resource system appears then to be a living entity; it is permanently renewed by a flow of OERs she was connected to.

Anais is not an ordinary teacher, as she is involved in the design of an e-textbook framed by Sésamath. It was noticeable that her resource system is not closed to Sésamath, quite the contrary: working with and contributing with new resources to the Sésamath resource system appears to open up, to her, a universe of other mathematics OERs. This seemed to be a common feature of Sésamath designers (see Sect. 1.4.2; and the case of Pierre, in Gueudet and Trouche 2012b).

After considering teachers' documentation work and expansion of their resource systems associated with the design of OERs, we focus in the next section on developments linked to their use.

1.5 Use of OERs by a Mathematics Teacher: The Case of Valeria

For the purpose of this paper, we selected one mathematics teacher (Valeria), followed in the frame of the ReVEA project during her teaching of functions at grade 10. Valeria was an experienced mathematics teacher, working for 29 years at upper secondary high school, when we started collecting data about her documentation work. In this section we focus on OERs within Valeria's documentation work. In France mathematics teachers (and Valeria in particular) have access to many OERs, according to their authorship or content (see Sect. 1.2). Thus our first aim was to observe which OERs Valeria uses, how and why she chooses these OERs. In terms of DAD, this means: "which documents developed by Valeria incorporate OERs, which place have these OERs in her resource system?" We firstly present the data we collected, the general organization of Valeria's documentation work and of her resource system and the place of OERs in it. Then we analyze two documents developed by Valeria, linked with two different kinds of OERs.

1.5.1 Reflexive Investigation of Valeria's Documentation Work: Data Collected and First Statements

In terms of methodology, we used the reflexive investigation method when studying Valeria: we visited her twice at home, the first time for general visit of her resources; the second time for the preparation of the *Functions* chapter teaching (both visits lasted around one hour, they were video recorded and transcribed). Regarding data collection strategies, we also video recorded her lesson in class about the introduction of the variation of functions. We met her for an interview at the end of the chapter, using extracts of the videos and asking her about her choices of resources; the adaptations she planned during her preparation, or she did on-the-spot; adaptations she envisions for future uses of these resources. We also collected all the resources she used or designed.

Concerning the data analysis, we analyzed the data firstly by identifying aims of her activity: a document is indeed characterized by this aim of the activity. There were always several possible choices for the aim, in terms of scale in particular. Since we considered here the documents developed for a given chapter, we retained aims corresponding to different moments of the chapter. Hence, these aims were not deduced from the data, but a priori chosen by the researchers; then the data were investigated to identify which resources and which beliefs were involved in the teacher's documentation work for a particular aim. If the investigation of the data evidenced new aims (declarations of the teacher formulated as "I wanted to…" or "I needed to…" etc.) we added them to this initial list. In terms of documentation work these aims can be formulated as "prepare and implement…":

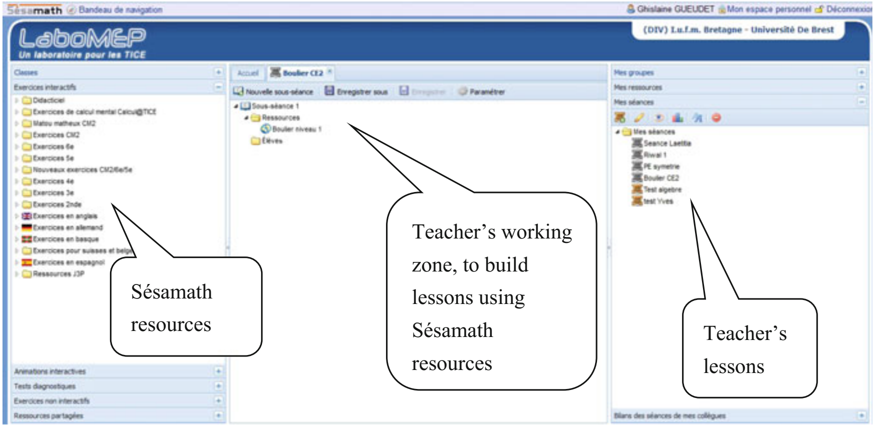
1. The stabilization of background knowledge;
2. An introductory problem;
3. The course (synthesis);
4. Exercises, in class and at home;
5. An assessment.

As Valeria is a very experienced teacher, her central resource for the course (aim 3) is her own course from previous years. For the aims 4 and 5, her central resources are different textbooks (on paper): the classroom textbook for the exercises, and other textbooks for the assessment. In fact, OERs in Valeria's resources system were not central; she started using them around five years ago, and they intervene only in documents developed for the aims (1) and (2). We propose below a detailed analysis of these documents.

1.5.2 LaboMEP in a Document Developed for the Stabilization of Background Knowledge

Valeria has a particular professional belief in terms of pupils' knowledge when they entered grade 10, coming from different lower secondary schools: she considered that her grade 10 students came with very different background knowledge concerning functions. This conviction leads her to use LaboMEP, a resource developed by the Sésamath association (see Sect. 1.4), which offers in particular the online exercises of Mathenpoche. LaboMEP provides her with opportunities to program/provide different Mathenpoche exercises for different students, and she chooses particular exercises selected on the basis of particular mathematical objectives (Fig. 1.5).

Before introducing variations of functions, she wants to be sure that the students know the basic vocabulary used with functions, and that they could read the graph



Sésamath resources

Teacher's working zone, to build lessons using Sésamath resources

Teacher's lessons

Fig. 1.5 LaboMEP, teacher's interface

of a function; for this purpose she organizes an one-hour session in a computer lab. We consider this to be an instrumentalisation process: she selects this particular (Sésamath) resource and uses it to design her own session, corresponding to her teaching objectives. Moreover, Valeria has developed a document involving LaboMEP, in order to better manage her heterogeneous group of students. LaboMEP allows her to organize different lessons (with a different content) for individual students—an instrumentation process. LaboMEP is the only Sésamath resource used by Valeria. In particular, she is very critical about the Sésamath textbook, considering that it does not offer enough exercises. She does not develop a document involving the Sésamath textbook; and this also links with her professional belief that "the paper version of a textbook must offer enough exercises". She does not want to connect to the Internet for her usual classroom sessions: exercises should be available on paper. Her use of LaboMEP is limited to the management of heterogeneity, and in particular in the case of the *Functions* chapter to the stabilization of background knowledge, with a session programmed out-of-class.

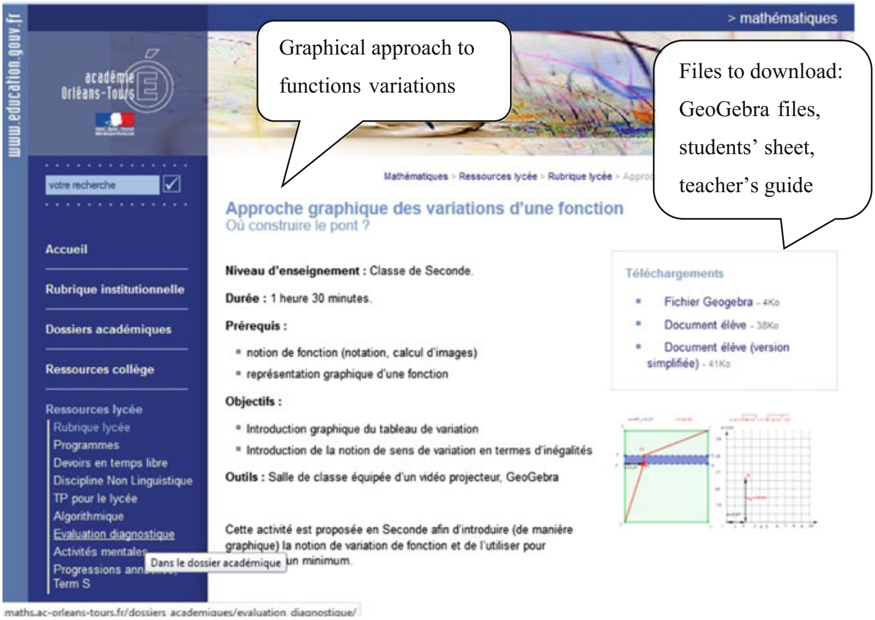
1.5.3 Institutional Repositories and the Design of Introductory Problems

Another of Valeria's professional beliefs concerns the importance of providing rich introductory problems to start her introduction of a topic area. For each grade, she wants to renew her introductory problems/activities, and uses for this purpose two kinds of resources: textbooks on paper, and institutional repositories. We observed this kind of documentation work for the Functions chapter. Below we firstly describe the steps of her documentation work and the associated beliefs, and then we present further analyses in terms of documents, instrumentation and instrumentalisation processes.

To search for a suitable problem, Valeria first types her aim into an Internet browser, “introducing variations of functions”, which provides her with a list of links. Then she browses the list of results obtained by keeping only the answers corresponding to institutional websites (in particular Eduscol, a national repository). This is linked with a belief developed along her use of such resources: “institutional websites offer resources where quality is controlled”.

Valeria knows that she wanted a problem with a dynamic figure to illustrate the mathematical concept of variation. This is linked with a belief she expressed in one of her interviews: “a dynamic figure is helpful for the students to develop an intuitive image of the variation of functions”. In the list of links sent by the ‘general’ Internet browser after her first request, she chose the Eduscol link, which led her to the Eduscol’s browser (Fig. 1.6). There she typed “grade 10”, “Variation of functions”. She obtained a list of 22 propositions; the next step was thus to make a choice within the list.

Valeria read the list, and firstly used didactical criteria linked with her precise mathematical objective in order to dismiss inadequate problems: indeed most of the problems address in fact “optimization”, and not directly “variations”. Moreover, the problem should allow for discovering variations at the very beginning of the



Approche graphique des variations d’une fonction

Où construire le pont ?

Niveau d’enseignement : Classe de Seconde.

Durée : 1 heure 30 minutes.

Prérequis :

- notion de fonction (notation, calcul d’images)
- représentation graphique d’une fonction

Objectifs :

- Introduction graphique du tableau de variation
- Introduction de la notion de sens de variation en termes d’inégalités

Outils : Salle de classe équipée d’un vidéo projecteur, GeoGebra

Cette activité est proposée en Seconde afin d’introduire (de manière graphique) la notion de variation de fonction et de l’utiliser pour un minimum.

Fig. 1.6 The introductory problem chosen by Valeria chapter. She also wanted the problem to start from a ‘concrete’ situation (this is associated with a belief also expressed in the final interview: “students are more motivated by problems corresponding to concrete situations”). Only three problems on the list corresponded to this objective. For each of these three, she followed the link giving access to details. One problem is dismissed due to a practical criterion: it uses the software called Geoplan, which is not available on the school computers. Finally she retained a problem entitled “A Graphical approach to function variation” (Fig. 1.6), in particular because it proposes a ready-made GeoGebra file (Fig. 1.7).

In this activity, the problem is formulated as follows: “A river flows in a square park ABCD. An architect wants to build a bridge P_1P_2 over the river so that the length of the path from A to C is minimal”. The GeoGebra file on the left provides a dynamic representation of the park, with the path in red; the point P_1 can be moved. On the right, the file displays the trace of the point M whose coordinates are x: the length EP_1, and y: the length of the red path. This trace coincides with the graph of the function representing the length of the path according to the position of the bridge. The students can observe on it that the point M firstly goes down, and then moves up again: this is exactly the kind of dynamic representation which, according to Valeria, supports an appropriate understanding of function variation.

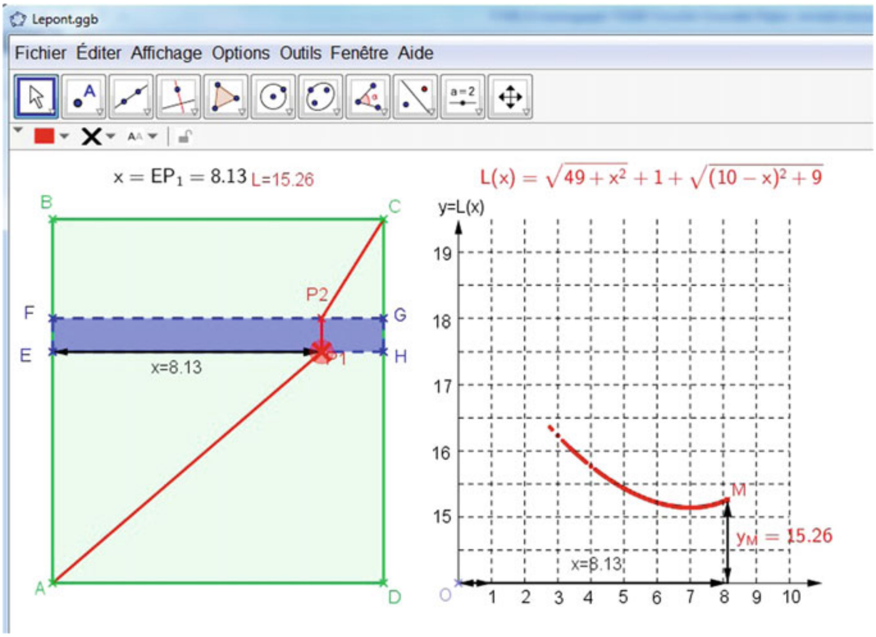


Fig. 1.7 The GeoGebra file, details

The next step is to print the text of this problem, even though at this stage Valeria has not yet decided to keep it. She compares the description with descriptions of problems in textbooks. She searches in five different textbooks; and finally decides to keep the problem found in the institutional repository, because of the usefulness of the software, which corresponds to her expectation (and because of the ready-made file).

Then she starts solving the problem herself, with the objective of modifying it for her students if needed. She decides to suppress some parts of the problem because of time limitations. She also changes the order of the questions. Indeed the original activity starts with a table of values of the function, in order to draw the graph, and then directly introduces the variation table of the function before asking the students to fill in sentences like: "On the interval $[0, \alpha]$, when x increases $L(\mathrm{x})$ … We say that the function L is … on the $[0, \alpha]$ interval." Valeria moves the table of variations to the very end of the activity, because she wants to introduce the terms "increasing" and "decreasing", drawing only on the graph of the function, to emphasize the intuitive aspects of variations. Finally, she types her own students' sheet, incorporating these modifications.

We observe here an instrumentalisation process: starting from a resource found in an institutional repository, she transforms it according to her professional knowledge and beliefs. At the same time an instrumentation process takes place: using this resource leads her to develop her use of the GeoGebra software in class. It is well known that sharing lessons with dynamic geometry software supports documentational geneses for teachers with this software (an assumption that inspired for example the Intergeo project, see Trgalová et al. 2009).

Hence, we argue that the open resources led to an enrichment of Valeria's resource system, which now encompassed GeoGebra files linked to introductory problems and online exercises. However, we also maintain that, when new kinds of resources were added, the 'traditional' resources remained present. In particular, Valeria could not consider abandoning and replacing the paper textbook by an e-textbook. This was a limitation for the evolution of her resource system, but at the same time led to new combinations between 'old' and 'new' resources.

We observed the corresponding lesson in her classroom. It took more time than initially planned by Valeria (around 2 h, instead of 1 h 30), because the students spent a lot of time on the start of the activity. This start-off was a modeling task, using geometrical knowledge that Valeria considered as basic. It was in fact more difficult than expected and lead her to declare (in the post-lesson interview) that she should have given the start-off task as homework, to avoid this "waste of time". But the students were actively involved in the mathematical activity, and they were, as planned, interested in the use of GeoGebra. Valeria considered that the dynamic figure played its intended role.

We retain that the design process is a continuous process: the resource found online already incorporated remarks on the use in class; it has been designed and tested by colleagues. Nevertheless, Valeria's experience with the students was very important, and would lead her to a new stage of design for a further use of the resource.

1.6 Discussion

Our initial question was: to what extent does the provision of "resources-on-offer" lead teachers to open their own resource system? And our meta-objective was to question the interest of the DAD for addressing this question.

We have presented two cases. Drawing on these two cases, we propose the following main arguments:

(1) The increase and large variety of available OERs open up new opportunities for discovering 'new' and/or 'missing' resources: this initial statement is straightforward. Our cases illustrate that "discovery" does not happen by chance: the documentation work of teachers is driven by their aims for the activity and by didactical beliefs. These aims and beliefs have led Valeria to search for new resources, offering her opportunities for adapting exercises to individual students' needs: LaboMEP fitted this objective. Moreover, her beliefs and the institutional expectations concerning the use of software have also guided Valeria to search for rich introductory problems integrating a software: her browser opened the way to an institutional repository. Once a given repository had been chosen, didactical criteria guided her choices. Finding "missing" resources led her to explore the OER 'universe', and each new resource discovered provided opportunities for discovering further (new) resources. The process of developing and appropriating new resources has then been a process of both instrumentation (a new resource allowing a teacher to reach her didactical objective, sometimes in a different way), and of instrumentalisation (*appropriating* a new resource always includes *adjusting* it; we observed here that Valeria modified the activity to emphasize the graphical interpretation of variations according to her beliefs). Both together, these processes composed the documentational genesis.

(2) The 'new' resources seem to complement the 'old' ones, but not to replace them. What has been claimed in the case of artifacts—the introduction of calculators did not remove 'pen and pencil' or mental calculations—seems also true in the case of resources. Valeria used OERs for a new aim (corresponding to an institutional expectation): managing the heterogeneity of her students. She also used OERs (from institutional repositories) to search for introductory problems; for this aim, she also searched in textbooks, and retained an OER mostly because it incorporated a GeoGebra file. For her other aims: prepare and implement the course, the exercises, the assessments, she did not use OERs but kept using mostly paper textbooks. Thus in terms of impact of OERs on her resource system, we observe an enrichment of her system for the aim "prepare and implement an introductory problem"; we also observe the emergence of a new part of the system, for the aim "prepare and implement the stabilization of background knowledge".

    Introducing a new resource does not necessarily lead to the replacement of the old one, but leads to a re-organization of a teacher's resource system. Hence, the increase of OERs-on-offer is likely not only an opportunity for extending teachers' resource systems, but also for their re-organization.

At a time of transitions from traditional to digital resources, this seems to be all the more true: the ReVEA study on teachers' use of resources showed an increase in the use of digital resources, and at the same time a continued use of paper resources.

(3) The question of resource usages cannot be disconnected from the question of the resource designs. Firstly, this is because in each usage, we have argued, there is a part of design, or re-design, or "design-in-use" (see Pepin et al. 2013)—what we call the instrumentalisation process. Secondly, this is because the ways the resources are designed "condition", or afford, the ways they may be used. The uploading of a large variety of OERs by teachers provides new opportunities for teachers to find new resources fitting their needs, and to adapt and combine them with their "own" resources.

The case of Sésamath has been interesting to analyze, if we consider such an association as an advanced stage of a larger process: teachers working together for designing a collective resource system; they share some of their own resources, and by working with them develop a "new" system overtaking their initial system. This process results in the enrichment of each designer's resource system, as well as of course enriching the common resource system. The term of osmosis seems in particular appropriate for describing such a mutual enrichment. In such a way Sésamath acts as a catalyst for the production of OERs: offering teachers access to a kaleidoscope of resources, and at the same time encouraging them to add to the kaleidoscope. Teachers choose resources based on their views of the suitability of the resources-on-offer, which in turn is driven by their beliefs and dispositions: the chosen resource, via its links to other Sésamath resources (as LaboMEP to Mathenpoche), opens new opportunities for interactions between teachers' and Sésamath's resource system. Is Sésamath an isolated phenomenon, or is it a significant and emerging trend in the era of digitalization in the documentation work of mathematics teachers? The different chapters of this book seem to evidence that, far from being isolated, Sésamath is the visible component of a deep evolution of teachers' work in the thread of digitalization.

The DAD provided relevant theoretical and methodological tools for structuring the analysis of these two case studies: the concepts of instrumentation, *instrumentalisation*, and resource systems as well as the *methodology of reflective investigation*, appeared critical in this perspective. Moreover, the 'resource point of view' appears efficient for analyzing teacher activity, particularly in a time of OER profusion.

At the same time we are aware of a much greater potential of this frame for analyzing long-term processes. Understanding documentational geneses over a long period, combining individual and collective aspects, inferring the schemes both guiding, and developed by, these geneses needs time often not available for case studies. This is a subject for further research projects.

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Birgit Pepin is a full professor of mathematics/STEM education at the Eindhoven University of Technology. In her research in mathematics education she has investigated (digital) curriculum materials (including e-textbooks), their design, and teachers' and students' interactions with such resources in secondary and university mathematics education contexts. She is in particular interested in the design capacity that mathematics teachers develop when working with (digital) curricula. Her current work includes educational design research studies in mathematics and engineering university departments (and their potential for teacher professional learning). Most of her recent research has been conducted with Ghislaine Gueudet and Luc Trouche.

Chapter 2
Textbook Use by Teachers in Junior High School in Relation to Their Role

Chunxia Qi, Xinyan Zhang and Danting Huang

Abstract Based on Nicol's views about the levels of use of textbooks, the present study selected six junior high school teachers in China as the subjects and conducted a qualitative analysis of their teaching of geometric transformations. It was found that the use of textbook reaches the level of elaborating and creating but most teachers still focused on elaborating level. Meanwhile, great differences existed among teachers with different years of teaching. Teachers changed differently in the five aspects of teachers' roles which affect mathematical communication, interaction with students, validation of knowledge, source of knowledge and students' autonomy. These changes improved the use of textbooks.

Keywords Mathematics teachers' resources · Textbooks
Geometric transformation · Teachers' role

2.1 Introduction

Dating back to 1658, the book called *Orbis Pictus* written by Czech educator Comenius became the first textbook in the world. Nowadays, five centuries' later, great changes have been introduced to textbooks' connotation, functions, characteristics and structures (Ding and Sun 2011). However, textbooks (at most times equal to the term *curriculum material*) could be viewed as a materialized form of

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school curriculum (Davis and Krajcik 2005; Remillard 2005). They can also be considered as the basis of instruction and the most fundamental and popular teaching medium at schools (Fan and Zhuo 2007; Plianram and Inprasitha 2012). Fan and colleagues gave an overview of textbook research, related to aspects including the role of textbooks in teaching and learning and textbook analysis and comparison (Fan et al. 2013). According to Chambliss and Calfee (1998) textbooks still determine 75–90% of instructional content and activities in American schools. Valverde et al. (2002) argued that the form and structure of textbooks advance a distinct pedagogical model and thus embody a plan for the particular succession of educational opportunities. Because of the significant functions of textbooks in teaching and learning, research on and analysis of textbooks attracted scholars' attentions continuously (e.g. Fan et al. 2013, 2016; Fan 2013). As early as in 1931, the American educator W.C. Bagley recorded the use of textbooks in class. In addition, Cronbach called for research on the use of textbooks in 1955 (Cronbach 1955). Although experts stressed the significance on studying the use of textbook in class, people still were not concerned too much about how and to what extent the textbooks can be suitably used both in and after class, especially in mathematics (Plianram and Inprasitha 2012). Previous studies on textbooks and its use, both in teaching and learning mathematics, raised two important questions: (1) How do textbooks represent the curriculum and (2) How does this instrument assist the teaching and learning of mathematics (Valverde et al. 2002). Rezat (2012) summarized the conceptualization of different ways by which teachers mediate textbook use, obligatory and voluntary (Fig. 2.1) and draw attention to the fact that all three dimensions he outlined are intertwined in a concrete way of using a textbook.

Research on textbooks regarding the use of compulsory curriculum is necessary especially in China. Meanwhile, the new ideas and perspectives proposed during the Chinese mathematics curriculum reform (i.e. year 2001 to 2011) radically shook the authority of textbooks (Huang 2011; Sun 2008). Gradually, the role of textbooks switched from an "authoritative course" to a kind of "supporting material" both for teachers and students. This subsequently led to a fundamental shift in curriculum philosophy from "teaching the textbook" to "using the textbook for teaching" (Li 2008; Guo 2016). In other words, the new role of textbooks indicated that mathematics textbooks began to "serve for teaching" instead of "controlling

Fig. 2.1 Conceptualization of ways teachers mediate textbook use

mediation		obligatory	voluntary
Direct	Specific		
	General		
Indirect			

teaching" (Guo 2001). In the meantime, this change has been challenging and influencing mathematics teachers' views on instruction so as to embrace the moment to readjust teachers' behaviors and teaching methods to face the transformation. Are teachers the only communicators of mathematics? How to interact with students in today's mathematics class? Such changes challenge teachers' role in today's class. Besides, teachers also have to re-examine textbooks' functions and re-consider the use of textbooks in school. Therefore, the following questions arise: (1) What kinds of roles do teachers play in teaching a specific concept? (2) How do teachers use textbooks in the different parts of the teaching process? (3) What are the differences in the roles of novice teachers and their experienced peers? This study aimed to address the above issues. Specifically, the present study attempted to reveal the use of textbooks in the different parts of the teaching process and analyzed the presentation of teachers at different development stages in order to inspire teachers to use textbooks more effectively.

2.2 Framework of Research

According to Herbart (1904), the process of a whole lesson can be divided into five parts (Ellerton and Clements 2005): Setting the scene, abstracting the concepts, deepening the concepts, solving the problem and summarizing. Here, a "part" of a lesson refers to teaching procedures in mathematical class specifically and Chinese mathematics teachers' structure lessons by following the five procedures (Zhang 2006). In different parts of a lesson, mathematics teachers play different roles according to the specific requirements and objectives so as to guarantee the fluency of the whole process. In other words, in order to generate best teaching effects, teachers are requested to use textbooks in different ways that adjust to different parts.

2.2.1 The Role of Teachers

As the implementers of textbooks, teachers' use of the materials should be innovative and adaptive. Ben-Peretz (1984) distinguished two basic forms of the textbook and its use: On the one hand the textbook is a sort of *proposed* text, while, on the other hand, it is *to be enacted* by teachers. She also pointed out that teachers' professional experience lies in assigning meaning to the teaching materials and enacting them suitably. Similarly, Klein et al. (1979) put forward five dimensions of curriculum to differentiate it into such types as ideal, formal, instructional, operational, and experiential. Other researchers (e.g. Lloyd 1999; Remillard 2005) resonated with this view and argued that curriculum implementation should consider the dynamic interaction between teachers and teaching materials. Furthermore, Gueudet and Trouche (2009) suggested that documentation work should be taken into account when using the proposed text so as to search for valuable perspectives and design meaningful cognitive tasks and learning paths. Actually, the study by Hiebert et al. (2003) based on TIMSS data has shown that students who come from high achieving countries usually engaged in more mathematical activities than their counterparts who come from low achieving countries, which mirrored that teachers from the former countries may have implemented classroom teaching more dynamically and adaptively. Apart from the innovative re-developing of textbooks, technology has been playing an increasingly important role in curriculum implementation. Previous studies have confirmed the necessity and impact of using technology both in creating the collaborative environment (Chazan and Yerushalmy 1998) and stimulating students' motivation to explore (Olive 2002). In today's Chinese mathematical class, teachers are used to applying some fancy technologies to their teaching in order to increase class interactions. Such mathematical classes increasingly focus on students instead of mainly focusing on knowledge and teachers' lecturing as in traditional Chinese class. Being an important media and supplement to textbooks, technology, especially dynamic software is one of the classroom resources which students could use to explore problem situations such as 3-D solid geometry problem like scientists (Ruthven 2012; Olive 2002). Furthermore, teachers reported that dynamic geometry is their most valued software and that skill-development is their main objective for computer use (Becker et al. 1999).

In today's mathematical class, technology is widely used and has a great influence on class teaching. Trigueros et al. (2014) proposed a framework to analyze the role of the teacher when using digital resources. In our research, we used this framework in order to analyze the changes of teachers' role during their implementation of the curriculum reform in teaching geometric transformation and compared the differences of using textbooks from the perspective of different developmental stages. Trigueros et al. (2014) five different aspects of teachers' role being aware that the different aspects overlap and cannot be clearly separated. Our research is based on Trigueros et al.'s classification. Each role is explained below:

1. Role in terms of communication of mathematics. In conveying mathematical knowledge teachers play the role as an assistant in helping students understand mathematical concepts, regulations and propositions. Teachers' performance could be divided into three levels in this process. The first level is that the teacher is the only source of mathematical information for students and that communication only occurs between the students and the teacher. The second level is that communication occurs among students, teachers and textbooks partly. The third level is that textbooks also facilitate mathematical communication besides teachers.
2. Role in terms of interaction with students. This role of teachers in the context of the original study refers to the way by which teachers interact with students and how they manage and regulate what happens in the classroom (Trigueros et al. 2014). While playing this role, teachers' performance still can be classified into three different levels. The first level is that teachers follow their predetermined

plan strictly and make few responses to students' questions and answers. The second level is that teachers listen to students' answers attentively and respond to them immediately. The third level of teachers' presentation is that teachers can modify their original teaching plan corresponding to students' participation.

3. Roles in terms of validation of mathematical knowledge. In our research, we only consider the role the teacher has in validating the correctness of knowledge. Teachers show two levels of validating the correctness of mathematical procedures and answers: The basic level is that teachers are the only source of validation. On the second level, teachers, technology and other sources can support the validation together.
4. Roles in terms of the source of mathematical problems. In our research, we discuss the role of the teacher in selecting problems in class. The basic level of this role is that teachers are the unique source of mathematical problems for students to solve. Even if problems from textbooks are solved, it is the teacher who decides which problems or exercises are to be worked on (Trigueros et al. 2014). The second level is that teachers and textbooks can both provide problems for students to practice. The third level is that valid problems could emerge from some other textbooks or resources and students themselves.
5. Role in terms of actions and autonomy of students. In our research, we discuss the roles of teachers in terms of nurturing students' autonomy. This last role teachers play in class is taking charge of students' self-motivated learning. The first level is that students only obey, repeat and answer the questions provided by teachers. The higher level is that students have the autonomy to decide what to do and how to do it.

2.2.2 Use of Textbooks

Trigueros et al.'s (2014) study us the different roles teachers play in classroom teaching. Even playing the same role, different teachers may perform at different levels. However, in each role and on every level teachers can still utilize textbooks to facilitate their teaching, but in distinct ways.

Concerning the use of textbooks, several studies describe typical types of teacher-textbook relationship and classify hierarchical levels of teachers' interaction with textbooks. For instance, Brown (2009) stated that teachers' interactions with textbooks can be differentiated into the three levels of *offloading*, *adapting*, and *improvising*. In his definition, the offloading way of using textbooks usually refers to teachers' use of textbooks in a literal manner following the established contents as closely as possible; the adapting way of using textbooks usually refers to teachers' use of textbooks in a more flexible manner by following the established contents partially, but also adjusting the contents if necessary; The improvising way of using textbooks usually refers to teachers' use of textbooks in a highly flexible and innovative way by effectively integrating teachers' spontaneous adjustments into the teaching process. Similarly, Nicol and Crespo (2006) provided a hierarchical model of using textbooks, in which they classified the use of textbooks by teachers in mathematics class into the three levels *adhering*, *elaborating* and *creating*. Literally, *adhering* refers to considering textbooks as "an authority" deciding what to teach and how to teach it. Teachers always make no or few adjustments and modifications to textbooks, or only make some superficial changes in teaching. *Elaborating* refers to considering textbooks as "a guidance" to tell teachers what to teach and how to teach it. At this level, teachers can take advantage of other sources to amplify the questions, tasks and exercises in textbooks. *Creating* refers to utilizing the textbook critically and innovatively in order to find out the intention and limitations of textbooks. At this level, teachers can optimize teaching structures, sequence, etc. through setting up appropriate problems.

In the present study, we combine the five roles proposed by Trigueros et al. (2014) with Nicol and Crespo's (2006) levels of the use of textbooks. We use two dimensions—the role of teacher and the level of using textbooks—to describe teachers' behaviors in class. Even if teachers are at the same level of the teachers' role, they still adopt various methods to use the textbooks in class. In this study, we observe and analyze the transformation of different teachers both on the dimension of teachers' role and the use of textbooks. It is worth mentioning that although both Brown's and Nicol and Crespo's three-level models have classified different levels of teachers using textbooks, the two models share some common ideas on defining the extent of the interaction between teacher and textbook. We consider Nicol and Crespo's classification as a more suitable model for the present study since the term *creating* is more appropriate than *improvising* in describing the highest level of teachers' use of textbooks in Chinese classroom teaching. In other words, we encourage teachers to perform innovatively and adaptively, but all the related novel adjustments and activities should be well-designed. Even though improvising can be considered as a highly flexible interaction between teacher and textbook, and actually many teachers perform classroom teaching in this way well, at most times Chinese excellent teachers teach mathematics in a well-designed but not a in just-in-time innovative way.

2.3 Research Design

Both international and Chinese researchers consider geometric transformation as a foundational content to construct the system of geometry in junior high school (Daniela et al. 2014; Hollebrands 2004; Tao 2013). In China, this field has only recently been introduced to textbooks. Therefore, related research on this topic is rare. Although some researchers compared the related contents in different versions of textbooks (Lu 2006; Zhang 2007; He 2011; Ni 2012), the application of these textbooks in teaching is still to be discussed. Therefore, we selected geometric transformation as the topic and used the video analysis method to analyze teaching practice of six mathematics teachers by the software NVivo 8.

2.3.1 Samples

Six mathematics teachers from different regions were selected. Three of them have taught mathematics less than five years and are called novice teachers in this study. The other three are experienced teachers who have taught for over 10 years and are called experienced teachers in this study. Detailed information about the six teachers can be seen in Table 2.1.

2.3.2 Data

Six videos were recorded—one of each teacher teaching geometric transformation in class. Before videotaping the teachers' classes, we explained the purposes of the observation and asked them to present an usual lesson. We adopted both participatory and non-participatory observation in order to guarantee the accuracy and integrity of the observation materials. We also conducted personal interviews with each teacher for about 40 min in order to acquire further information on the intentions of the design and the activities in particular parts of the teaching process. In order to analyze the differences among the teachers at different developmental stages, we divided the six teachers into three groups; each group had one novice teacher and one experienced teacher. Teachers who were in the same group chose the same contents to teach, for example, both teachers in group 1 taught translation, group 2 taught axial symmetry and group 3 taught rotation. After recording the videos, the conversations in class were transcribed and transcripts were used as the basic data to subsequently analyze the roles of teachers and theuse of textbooks in teaching geometric transformation.

Table 2.1 Basic information about the selected six mathematics teachers

Code	Sex	Years of teaching	Diploma	Relative information	Subject
T1	F	2	Master	Basic teaching experience	Translation
T2	M	2	Bachelor	Basic teaching experience	Axial symmetry
T3	F	4	Bachelor	Has experience in teaching senior high school	Rotation
T4	F	11	Bachelor	Responsible for the research on mathematics in 8th grade	Translation
T5	F	13	Bachelor	Grade 9 teaching experience for many years	Axial symmetry
T6	M	12	Bachelor	Has experience in teaching senior high school	Rotation

2.3.3 Coding

Based on Nicol and Crespo's views about the combination of teaching activities and the use of textbooks, the six videos-recorded lessons have been split into five parts according to Hebart. As we have mentioned before, teachers used textbooks in each part except for the summarizing part (i.e. the last part) of the lesson. Thus, the last part was excluded from the analysis. The first four parts were analyzed with respect to the teachers' role and the use of textbooks. In the next section, we present the analysis of teacher T4 as an example.

We coded the collected data. First of all, we selected the teaching parts where textbooks contents used and handed the related videos to three experts. One expert a professor in mathematics education, another one an editor of a mathematics textbook, and the third one a mathematics teacher in junior high school. Secondly, the three experts were asked to individually determine the level of use of textbooks according to Nicol and Crespo's framework. Finally, they discussed the different decisions they had made, determined individually again and discussed once more until they got consistent coding.

2.4 Results

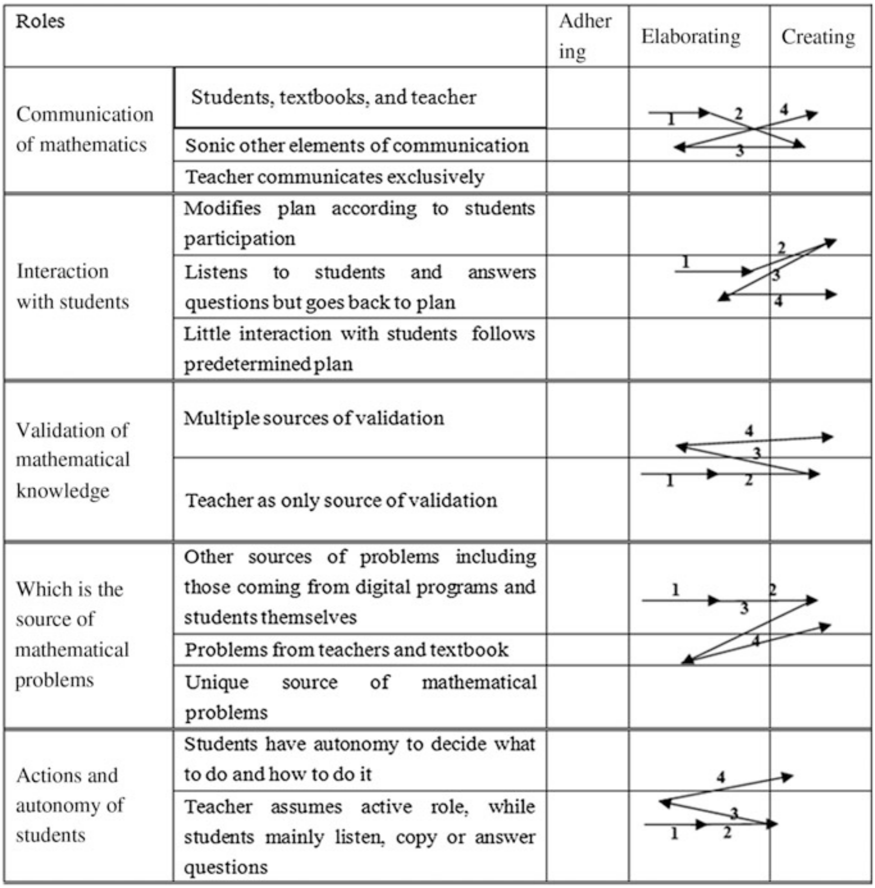
Teacher T4 taught translation and the textbook she used was edited by Beijing Normal University (hereinafter referred to as *BNU version textbook*). We selected the video materials which contained the records of teacher's class teaching from the first part (i.e. setting the scene) as an example of part 1 and analyzed her behavior in order to explain how we have arrived at the corresponding arrow in Table 2.2.

Part 1

Teacher T4 introduced translation by asking students to watch a video about two spaceships.

T4 What kind of movements did the two aircrafts make?
S (In chorus) Translation.
T4 Yes, translation. You have learnt it in primary school. Can you find translation in your life?
S (Points to the curtain in the classroom) The Curtains.
T4 Excellent! You found the translation in our life. How did the curtains do the translation?
S From left to right, or from right to left.
S The first layer of blackboards in our classroom can moving from right to left.
T4 Right, our blackboards do the translation every class.

Table 2.2 T4's use of textbook in different roles



Roles		Adhering	Elaborating	Creating
Communication of mathematics	Students, textbooks, and teacher			
	Sonic other elements of communication			
	Teacher communicates exclusively			
Interaction with students	Modifies plan according to students participation			
	Listens to students and answers questions but goes back to plan			
	Little interaction with students follows predetermined plan			
Validation of mathematical knowledge	Multiple sources of validation			
	Teacher as only source of validation			
Which is the source of mathematical problems	Other sources of problems including those coming from digital programs and students themselves			
	Problems from teachers and textbook			
	Unique source of mathematical problems			
Actions and autonomy of students	Students have autonomy to decide what to do and how to do it			
	Teacher assumes active role, while students mainly listen, copy or answer questions			

In this part, students evaluated the translation movement first and further examples of translation they remembered or had seen before. In this activity, students actually to perform the movement of translation in mind many times and built the connections between the movement and the definition of translation. From the perspective of teacher's role, communication occurred between students and the teacher via using technology to imitate translation and by observing the movement of curtains and blackboards. However, their communication only related to some simple problems while the teacher wasted the opportunities to deepen the concept of translation. In this activity, the teachers' role of communication only occurred at the first level. The teacher also offered enough time and opportunities for students to describe their viewpoints. Referring to the students' answers, the teacher T4 posed two further questions naturally: "How did the curtain translate?" and "How did the blackboard translate?" Teacher T4 also selected the problems related to the latest news. Questions raised in this part did not emerge from the textbook, but satisfied the original intention of the textbook. Consequently, in this part the teacher *elaborating* level in interacting with students while she *creating* level in the dimension of source of mathematical problems. It is obvious to see that the teacher was the only judge to evaluate the students' answers, other students in that class not express their opinions. Therefore, referring to the role of validating knowledge, the teacher only the first level. As for the roles in students' autonomy, although students found examples of the movement of translation by themselves, they still gave the corresponding answer under the teachers' guide. The space for students to act autonomously was limited, so teacher T4 reached the first level in taking charge of students' self-motivated learning. As for the whole situation of the use of the textbook, teacher T4 did not use the reference problem from the textbook, but she problems with contexts that were likely to arise students' interests and generate new problems depending on students' answers. Though the videos and questions did not came from the textbook, they elaborated on the textbook because the intention of this part and the purpose of the textbook reached consensus. Resources like technology and real objects only supported the purpose of the textbook.

Part 2 is selected from an "abstracting the concepts" part. The following transcript can be seen as an example. The teacher in this part to abstract the concept from the scene in part 1.

Part 2

T4 How to determine whether the two spaceships connect successfully?
S The connected part of the two spaceships should be in the same position or overlapped.
T4 What if the spaceships turned to this situation (teacher imitates the two spaceships with a certain angle with two pens), what's wrong with the situation?
S Direction. Something wrong happened to the orbit's angle.
T4 The two spaceships failed to connect because of their wrong direction. Now let's adjust their directions in a same line (represents the two pens in a collinear line but no connection), does it mean successfully connected?
S No (in choir)
T4 Why not? What else should we adjust?
S Distance.
T4 Now could you tell me the meaning of translation specifically?
(Then, the teacher the two key elements of translation in Translating Triangle Activity and the abstract concept of translation)

In this part, the teacher communicated with the students by asking questions trying to lead the students close to the essential characteristics of translation emphasized in the textbook. However, the textbook not support the communication between the teacher and the students directly, so the role of communication reached the second level, which means that the communication occurred among the students, the teacher, and the textbook partly. Teacher T4 did not plan to simulate the movement of the spaceship by pens, while in the group discussion, the teacher recognized that students had difficulties to understand this. After the students answered "The two spaceships should be overlapped," teacher T4 readjusted her teaching contents by using pens to demonstrate a counter-example. Therefore, we can claim that the teachers' behavior in the interaction reached the third level. Meanwhile, the teacher also proposed questions like "What can we adjust in order to connect the two aircrafts?" These questions originated from the students' confusion. Teacher T4 solved the difficulty of understanding the two characteristics—distance and direction—by solving the problem. Consequently, in this part, the role of the teacher related to mathematical problems was on the third level. While thinking about the conditions of connecting successfully, the students and the teacher determined the correctness and rationality of the problem together. So the role of validation the mathematics teacher T4 was on the 2nd level. However, in this teaching part, the teacher occupied a dominant position and students answered the teacher's questions in a choir, paying less attention to students' autonomy. Thus, the role teacher T4 plaid in students' actions and autonomy only reached the 1st level. In addition, from the perspective of the use of textbook, teacher T4 encouraged students' participation by using a counter-example and object simulating. Hand operation and brainstorm, both optimized the teaching structure and promoted students' understanding of mathematics. Therefore, the use of textbooks in this part achieved *creating* level.

Part 3 is a section of the third teaching part—deepening the concept. Teacher T4 planned to deepen students' understanding of translation by expanding the translation from lines to two-dimensional figure.

Part 3

After summarizing the definition of translation, teacher T4 reused the moving triangle activity to strengthen students' understanding of the concept. Students were able to describe the translation triangles with mathematical language. They also understood that the direction and distance that every point on the triangle moved, was identical to the translation of the whole triangle. Then students got the two properties of translation fluently.

T4 Every point moved the same distance, so the line segments all have the same measures. According to the translation of triangles, we know that the line segments connecting corresponding points have the same length and are parallel to each other. We must validate the correctness of the conclusion by Sketchpad. Do you remember the way of validating the equal relationship of two line segments?

S We should measure the two line segments respectively. (Teacher operated on Sketchpad—a software for science teachers to sketch graph—following the student's description)

T4 How to validate whether the two line segments are parallel or not?

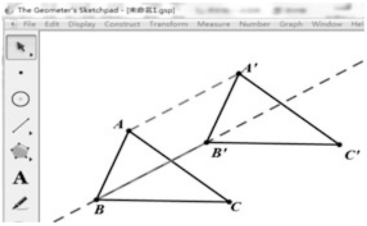
S We can validate it by drawing parallel lines.

T4 We have made a parallelogram on Sketchpad, have you recalled the method of sketching parallel lines? Any volunteers can make an example here?

S First of all, we should select a line segment AA′, and choose any point B or C outside the line segment. Then we can make a parallel line through point B or C, if they can coincide with line segment AA′, we can validate that the line segment connecting corresponding points are parallel. (Student operates the Sketchpad with description)

T4 Great, we can draw parallel lines accurately and validate the position of the other connected lines.

In this part, the teacher and the students interacted by discussing mathematical problems, but ignored the communication of mathematical knowledge and mathematical ideas. Moreover, the whole teaching process followed the original lesson plan directly, so the role teacher T4 in both communication and interaction stayed on the 2nd level. The use of Sketchpad enriched the method of knowledge validation and thus the teacher was not the unique source of validation. Then, teacher T4 reached the 2nd level related to her role of validation of mathematical knowledge. Although the students in class thought intensely about the problems raised by the teacher and verified the conclusions through Sketchpad, problems still emerged from the teacher and the textbook. The teacher designed the problems according to the textbook and determined students' learning procedure, which means that students participated in class activities in authentically. Therefore, teacher T4 only achieved the 1st level both in the role of students' autonomy and the source of problems. In addition, as for the use of the textbook, the process of exploring the nature of translation was based on the textbook arrangement, but teacher T4 provided Sketchpad for her students to validate in order to let students experience the process of acquiring mathematical knowledge from conjecture to validation. Students also experienced the convenience brought by the mathematical app, so in this part, the use of textbook reached the *elaborating* level.



Part 4 is selected from the "solving problem" part. This part aims to apply the knowledge learned in this class to solve problems.

Part 4

In order to help students recognize the random direction of translation, the teacher broke the limits of horizontal or vertical direction and oriented/encouraged students to realize the translation by marking vectors. Moreover, she asked students to design pictures with the help of resources, which were provided by the teacher, and to present their accomplishment via local area network.

T4 Now I'll introduce a new button to you. In Geometer's Sketchpad, there is a button controlling the direction and distance of a translation, it is called vector. First of all, let's draw the line segment *AB*, then choose a point *E* outside *AB*. The order to select is from *A* to *E*. Then press the button mark vector and choose the line segment. The button translation will remind you whether to translate from *A* to *E*. When you press OK, connect *AE* and *BB′*. Now let's draw a parallelogram with marked vector first and construct a Christmas tree by translation.

(Students operate Sketchpad and design the pictures independently.)

T4 You guys have designed all kinds of Christmas trees, let's appreciate others' work.

(Students present their work to others and the teacher picks some students' pictures to show them to the other students.)

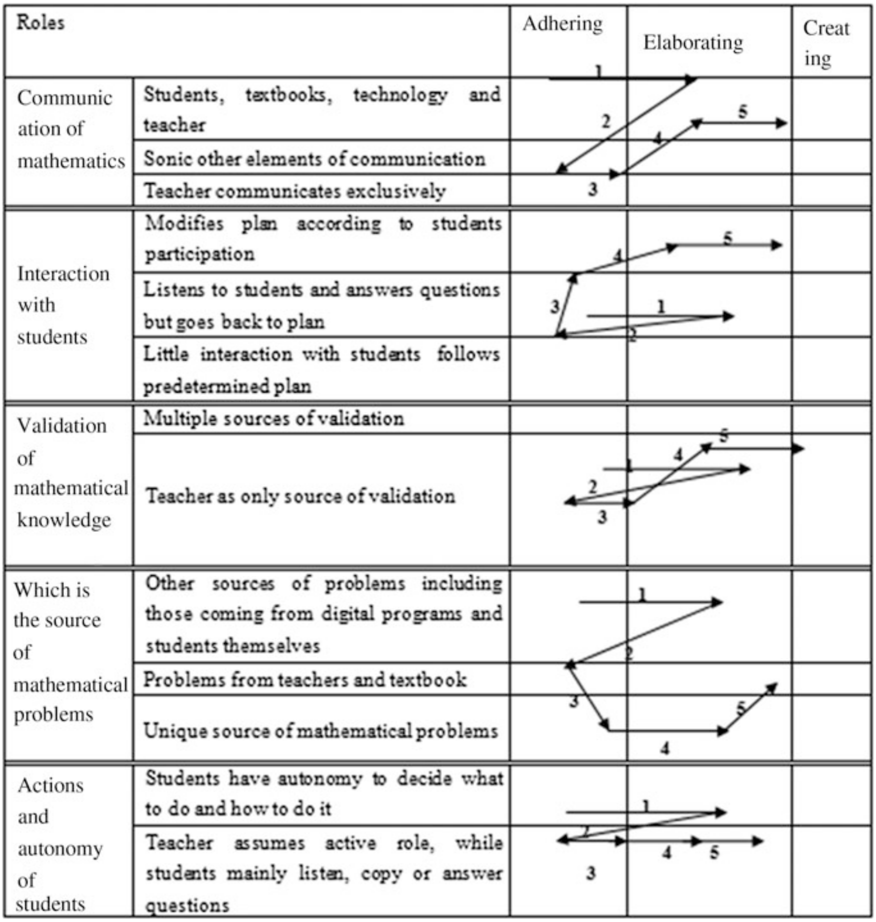
In this part, students designed beautiful patterns by applying Sketchpad operation skills and their knowledge about translation. This realized sufficient interaction among students, teachers, and other resources, so that the role of mathematical communication achieved the 3rd level. Even so, teaching in this part was carried out in accordance with the lesson plan, but the feedback on the students' works was not sufficient, so that the interaction with students achieved the 2nd level. In terms of the assessment of students' final work, teacher T4 did not evaluate directly but students' mutual assessments. This evaluation method turned students into judges and reflected the evaluation diversity, so the role of validation reached the 2nd level. In this part, students had high autonomy to design patterns, so the teacher reached the highest level (i.e., the 2nd level) in playing the role of nurturing students' autonomy. Besides, in this part, the teacher created the use of textbooks completely. The teacher promoted students' feeling for translation and stimulated the interaction among students and teacher by making use of computers. Therefore, the use of the textbook in this part achieved *creating* level.

On the basis of the analysis above, we used arrows to describe the use of the textbook in Table 2.2.

Table 2.2 indicates the changes in teacher's role and the use of the textbook. The vertical components of the arrows express the transformation of the teachers' role, while the horizontal components indicate the alteration of the use of the textbook. The starting point and the end point of the arrows represent the specific behavior in the teaching part. If the arrow closes to the border, it signifies that the teacher has taken two different roles. The position where the arrows stay stands for the original meaning primarily. The number on the arrow refers to the number of the part in the sequence of teaching parts.

We will describe the meaning of the arrows using the example of the role of interaction with students. In part 1, the use of the textbook gets to/reaches the *elaborating* level, and the teacher mainly communicates with students by questions

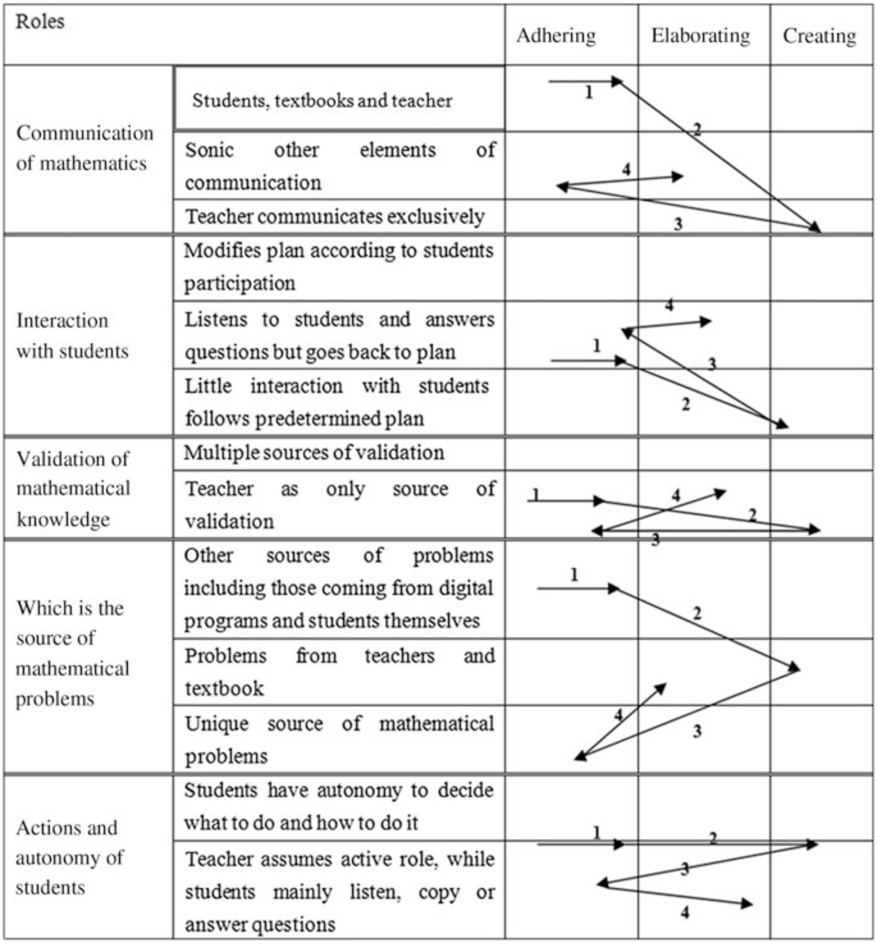
Table 2.3 T1's use of textbook in different roles



Roles		Adhering	Elaborating	Creating
Communication of mathematics	Students, textbooks, technology and teacher			
	Sonic other elements of communication			
	Teacher communicates exclusively			
Interaction with students	Modifies plan according to students participation			
	Listens to students and answers questions but goes back to plan			
	Little interaction with students follows predetermined plan			
Validation of mathematical knowledge	Multiple sources of validation			
	Teacher as only source of validation			
Which is the source of mathematical problems	Other sources of problems including those coming from digital programs and students themselves			
	Problems from teachers and textbook			
	Unique source of mathematical problems			
Actions and autonomy of students	Students have autonomy to decide what to do and how to do it			
	Teacher assumes active role, while students mainly listen, copy or answer questions			

and answers. Thus, the arrow lies under the second role (i.e. elaborating). In part 2, the use of the textbook reaches *creating* level. In part 3, the teacher the textbook again. She students to answer her questions, so the arrow lies under the second role and the elaborating status. In the last part, she the textbook creatively for she students' autonomy but still the original teaching plan. Consequently, the arrow in this part stays the same as the arrow in part 3. Tables 2.3, 2.4, 2.5, 2.6 and 2.7 indicate the use of textbooks of other five teachers.

Table 2.4 T2's use of textbook in different roles



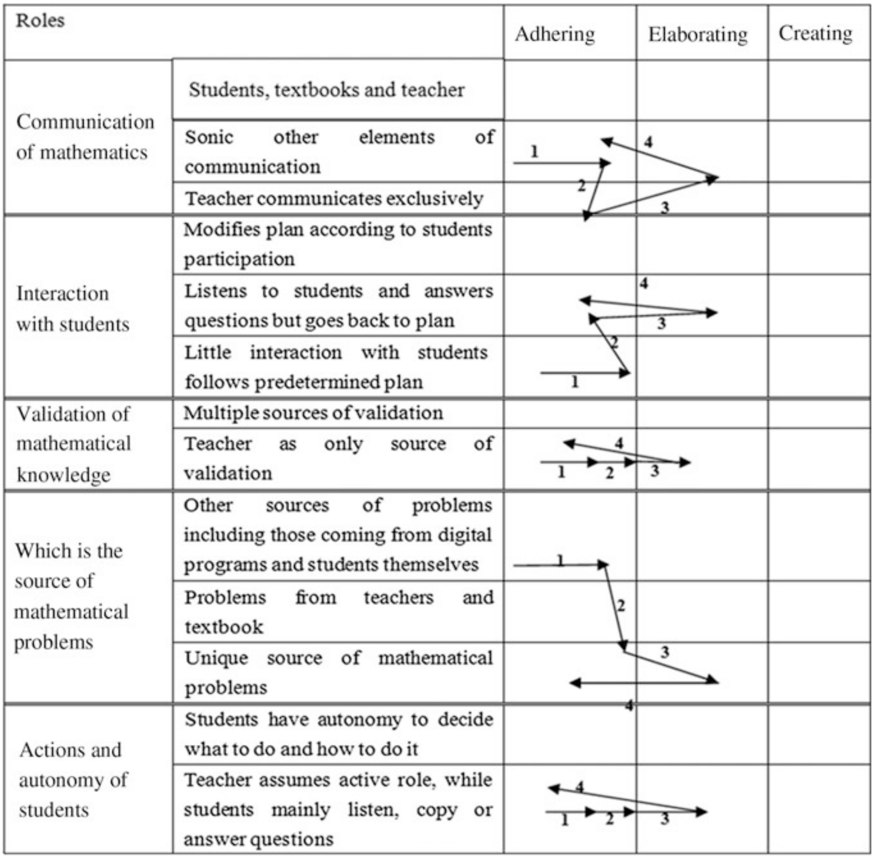
Roles		Adhering	Elaborating	Creating
Communication of mathematics	Students, textbooks and teacher			
	Sonic other elements of communication			
	Teacher communicates exclusively			
Interaction with students	Modifies plan according to students participation			
	Listens to students and answers questions but goes back to plan			
	Little interaction with students follows predetermined plan			
Validation of mathematical knowledge	Multiple sources of validation			
	Teacher as only source of validation			
Which is the source of mathematical problems	Other sources of problems including those coming from digital programs and students themselves			
	Problems from teachers and textbook			
	Unique source of mathematical problems			
Actions and autonomy of students	Students have autonomy to decide what to do and how to do it			
	Teacher assumes active role, while students mainly listen, copy or answer questions			

2.5 Conclusions and Analysis

2.5.1 *Mathematics Teachers' Reliance on Textbooks Is Decreasing*

From Tables 2.2, 2.3, 2.4, 2.5, 2.6 and 2.7, it is clear to see that less than 1/3 of the teachers adhered to the textbook while teaching geometric transformation. The use of the textbook reaches the level of elaborating and creating and most teachers elaborate the textbook. Specifically, the application of technology is the main method to use textbooks creatively. For example, teachers T4 and T6 their

Table 2.5 T3’s use of textbook in different roles

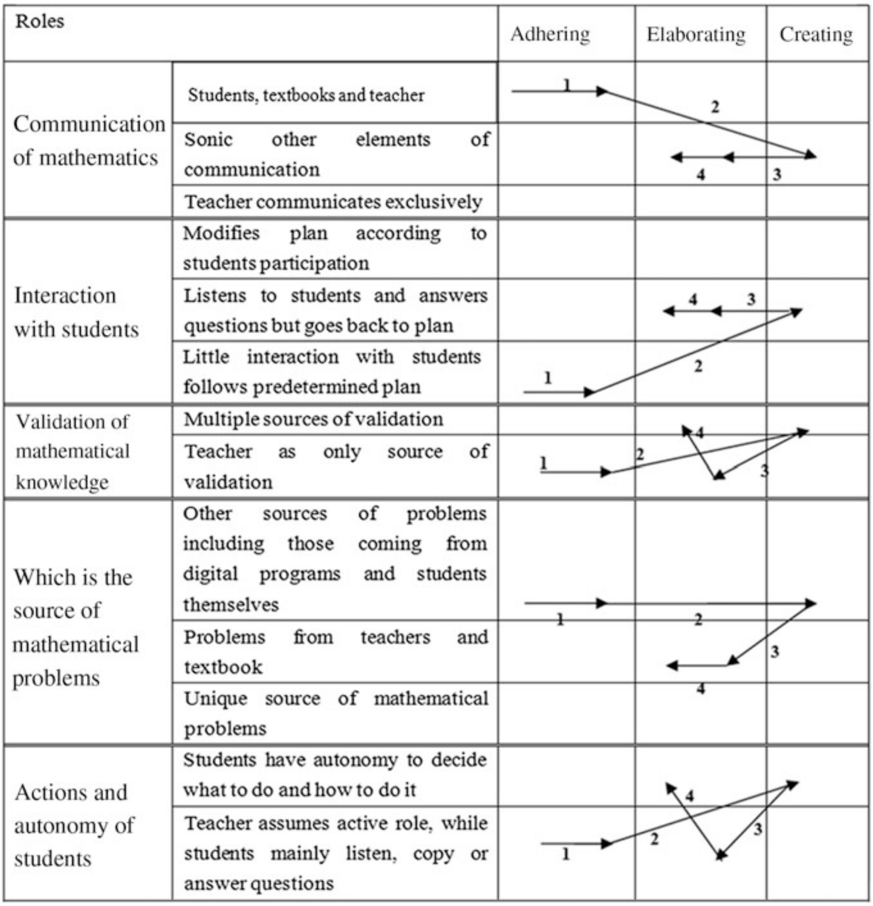


Roles		Adhering	Elaborating	Creating
Communication of mathematics	Students, textbooks and teacher			
	Sonic other elements of communication			
	Teacher communicates exclusively			
Interaction with students	Modifies plan according to students participation			
	Listens to students and answers questions but goes back to plan			
	Little interaction with students follows predetermined plan			
Validation of mathematical knowledge	Multiple sources of validation			
	Teacher as only source of validation			
Which is the source of mathematical problems	Other sources of problems including those coming from digital programs and students themselves			
	Problems from teachers and textbook			
	Unique source of mathematical problems			
Actions and autonomy of students	Students have autonomy to decide what to do and how to do it			
	Teacher assumes active role, while students mainly listen, copy or answer questions			

hypothesis by Geometer’s Sketchpad, and teachers T1 and T4 students to design their own patterns on Geometer’s Sketchpad. Technology helps students to strengthen their understanding and facilitates the advancement of their innovation. Furthermore, the elaboration of the textbook is eminently embedded in the supplement to the exercise in the textbook and the complement in the introduction scene, which manifests the contradiction of the uniqueness teaching situation in mathematics and the universality of textbook pursuit, as well. In addition, the exploration of the transformation concepts always follows the procedures listed in the textbook.

Additionally, Tables 2.2, 2.3, 2.4, 2.5, 2.6 and 2.7 manifest multiple sources of teaching problems. For example, problems raised in introduction part come from Internet, real life, campus activity, etc. Teacher T6 the problem “Do you know the way in which objects move?” by reviewing previous lessons. However,

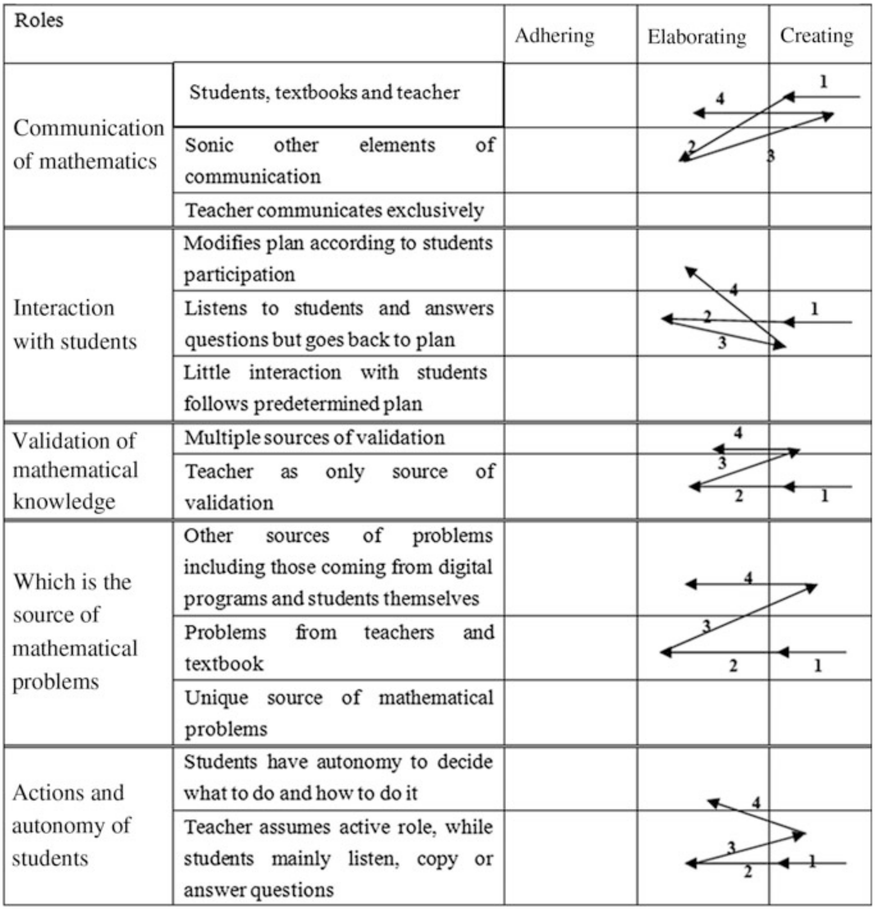
Table 2.6 T5's use of textbook in different roles



Roles		Adhering	Elaborating	Creating
Communication of mathematics	Students, textbooks and teacher	1	2	
	Sonic other elements of communication		4	3
	Teacher communicates exclusively			
Interaction with students	Modifies plan according to students participation			
	Listens to students and answers questions but goes back to plan		4 3	
	Little interaction with students follows predetermined plan	1	2	
Validation of mathematical knowledge	Multiple sources of validation		4	
	Teacher as only source of validation	1	2 3	
Which is the source of mathematical problems	Other sources of problems including those coming from digital programs and students themselves	1	2	
	Problems from teachers and textbook			3
	Unique source of mathematical problems		4	
Actions and autonomy of students	Students have autonomy to decide what to do and how to do it		4	
	Teacher assumes active role, while students mainly listen, copy or answer questions	1	2 3	

in exploring new knowledge, teachers questions suggested in the textbook, only sometimes the questions the teachers from the interaction between teachers and students. For example, T3 a range of questions while discussing rotation properties via operating with the parallelogram ABCD. "(1) What's the center of rotation? (2) After rotating, where did point A and B move? (3) Compare the measure of OA and OD. What about OB and OE, OC and OF?" These questions are identical with the problems of the textbooks by BNU (Beijing Normal University Press). Especially in the practicing part, more questions were chosen from textbooks. Only T4 and T6 adjusted the practice problems according to students' presentation. Otherwise, although all six teachers used PowerPoint, they only presented materials collected from the internet, the textbook or other reference books in a static order without guiding mathematical thinking and mathematical methods.

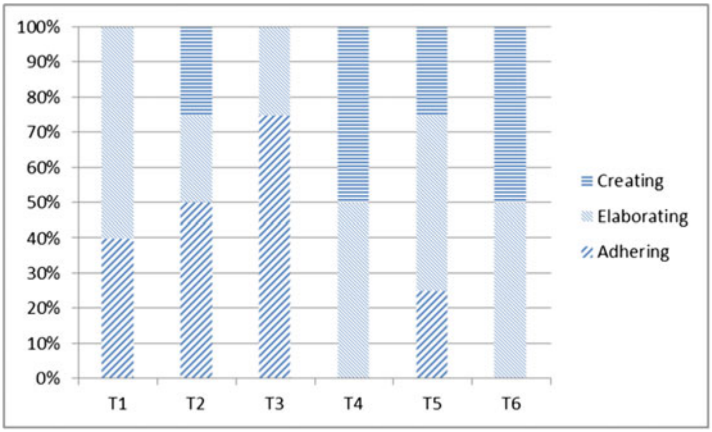
Table 2.7 T6's use of textbook in different roles



Roles		Adhering	Elaborating	Creating
Communication of mathematics	Students, textbooks and teacher		4	1
	Sonic other elements of communication		2	3
	Teacher communicates exclusively			
Interaction with students	Modifies plan according to students participation		4	
	Listens to students and answers questions but goes back to plan		2 3	1
	Little interaction with students follows predetermined plan			
Validation of mathematical knowledge	Multiple sources of validation		4	
	Teacher as only source of validation		3 2	1
Which is the source of mathematical problems	Other sources of problems including those coming from digital programs and students themselves		4	
	Problems from teachers and textbook		3 2	1
	Unique source of mathematical problems			
Actions and autonomy of students	Students have autonomy to decide what to do and how to do it		4	
	Teacher assumes active role, while students mainly listen, copy or answer questions		3 2	1

According to video analysis, some mathematical teachers could use the textbooks innovatively. Teachers turn their attitudes from "teaching what is in textbooks" to "making better use of textbooks". Compared with the results in previous studies (Yang 2005), teachers' reliance on textbooks has been decreased. However, as far as the whole situation of the usage of textbooks, teachers still tend to rely on textbooks. When compiling textbooks, though mathematics educators and mathematicians take students' cognitive characteristics into account, the logical order of the textbooks is not same as students' cognitive order. Therefore, teaching cannot completely duplicate textbooks step by step. Teachers should use teaching materials innovatively and flexibly and make the appropriate adjustments at some suitable occasions, according to the specific requirements, otherwise it will affect the effectiveness of teaching (Li 2008; Guo 2016). We agree with Zhong (2010)

Table 2.8 Six teachers' usage of textbooks



who says, "Although textbooks lie everywhere, teachers must experience and grasp them on their own. Current textbooks may not be useful, we must continue to develop new materials and make them become my textbook".

2.5.2 Teachers at Different Development Stages Differ in Using Textbooks

Differences existed among teachers at different developmental stages. The Table 2.8 showed the proportion of "*creating*", "*elaborating*", *and* "*adhering*" levels for the 6 teachers' data on use of textbooks. The table showed that novice teachers (T1, T2, T3) mostly arrived at the "adhering" level while using textbooks. Only sometimes they reached the "creating" level. As for experienced teachers' behavior, most of them reached the "elaborating" and "creating" levels. From this table, we can see that experienced teachers were able to use textbooks at higher levels than novice teachers.

Moreover, experienced teachers and novice teachers differed in specific teaching parts. First of all, novice teachers utilized textbooks predominantly at *adhering* level. That is to say, they mainly designed their lesson plans according to the contents in textbooks. For example, teacher T3 only compared the similarities and differences of rotation and translation after exploring the nature of rotation. In other parts, she also carried out her teaching in accordance with the arrangement in the textbook. In comparison, teacher T6 used the textbook creatively. He changed the exploration of rotation's nature into validating the hypothesis by Geometer's Sketchpad. Secondly, senior and novice teachers showed differences in using textbooks in different parts of the teaching process, especially in validating mathematical knowledge and in charge of inspiring students' study motivation. Novice teachers planed their teaching mainly according to textbooks. They usually followed the order in their textbook and only focused on the knowledge instead of putting themselves in students' shoes to figure out a way that students can accept and understand much more easily. Experienced teachers always considered students' previous understanding and rearranged textbooks; they transferred the contents into a way students can understand. For example, teacher T6 introduced the three key elements after operating with triangle rotation. The teacher raised the question "How to get ΔABC by rotating $\Delta A'B'C$?" This question helped students understand the reversibility of this transformation. This complementary part indeed improved students' analysis and understanding of rotation. It also nurtured students' reverse thinking. This process made the acquisition of knowledge a process of self-learning and developing thinking. Thus, experienced teachers performed better at using textbooks and grasping the materials, which is consistent with the results from previous research (Borko and Livingston 1989; Westerman 1991).

2.5.3 Changes of Teachers' Role Improved the Use of Textbooks

In mathematics teaching, teachers transformed their roles in mathematics communication, interaction with students, validation of knowledge, source of knowledge and students' autonomy, which improved the use of textbooks.

First, the communication of mathematics diversified. Teachers and textbooks were not the only way to acquire mathematical knowledge; technology became an external resource for students and teachers to get mathematical information. The use of technology helped students to communicate with mathematical knowledge directly and provoked teachers to acquire, represent, broadcast and communicate knowledge beyond textbooks. Technology's unique function realized the creativity of using textbooks.

Second, the validation of mathematical knowledge produced multiple ways to verify mathematics instead of depending on mathematics teachers as the only source. When information technology began to support the validation, it affected the validation that depends on deductive reasoning in the textbook. Current textbooks usually do not suggest to use information technology, especially professional mathematics software, which do not adhere to the basic philosophy of the integration of technology and curriculum. During the compilation and revision of the textbook, the application of technology needs attention. Experts who prepare the textbooks should think about how to present the basic philosophy of technology in textbooks and how to develop students' literacy in information via textbooks.

Last, students' autonomy improved and consequently they broadened their scope of study resources instead of only relying on textbooks. When teacher T5 asked students to explore the concepts of rotation, she allowed them to use the internet. Hence, technology provided a resource for students to study mathematics, which urged teachers to go beyond textbooks and expand their horizons. This also suggests that teachers are already aware of the significance of resources and are overcoming the limitation of textbooks by using technology to provide more resources to facilitate students' mathematical thinking. Because the other resources have deepened the understanding of some issues, now textbooks truly become dynamic, generative curriculum resources. According to the analysis above, there exists a close correlation between teachers' roles and textbook use in teaching mathematics. The transformation of teachers' roles, especially the use of technology, supports teachers in creating favorable mathematics learning environments, facilitating students' communication and activating students' effective learning behavior. Meanwhile, the transformation also urges teachers to consider the applicability of textbooks and allows them to avoid the problem of "deskilling" (Apple 1986) through appropriate adjustment in content and order.

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Chapter 3

A Comparative Study on the Presentation of Geometric Proof in Secondary Mathematics Textbooks in China, Indonesia, and Saudi Arabia

Lianghuo Fan, Mailizar Mailizar, Manahel Alafaleq and Yi Wang

Abstract This chapter presents a comparative study aiming to examine how geometric proof is treated in secondary school mathematics textbooks in China, Indonesia, and Saudi Arabia, and explore the similarities and differences revealed in these three countries' textbooks. The results show that, although all the selected textbooks from these countries introduced mathematics topics related to geometric proof, they differed considerably in three aspects: the number of examples, the distribution of contents and, to a lesser degree, the types of proof. The textbooks in China contain the highest percentage of geometric contents and pay the most attention to the topic of geometric proof itself. The national mathematics curricula are clearly a main factor for the differences revealed.

Keywords Mathematics teachers' resources · Mathematics textbook research
Geometric proof · Chinese mathematics education · Indonesian mathematics education · Mathematics education in Saudi Arabia

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3.1 Introduction and Background

School textbooks have been the subject of research internationally for quite a long time. In the mathematic subject, school textbooks have also received increasing attention in the international research community over the last few decades (Fan et al. 2013). More recently, the inaugural International Conference on Mathematics Textbook Research and Development (ICMT) held in 2014 in Southampton, UK, and the second ICMT held in 2017 in Rio de Janeiro, Brazil attracted many researchers around the world (Fan and Wu 2015; also see www.im.ufrj.br/~ictm2).

In this paper, we report a part of a larger scale study that we have conducted recently comparing China, Indonesia, and Saudi Arabia mathematics textbooks. In a previous study, we examined the mathematics textbooks in those three countries at the primary schools level, aiming to document and analyse how the three countries introduce the equality and inequality of whole numbers. The results revealed a high level of consistency in the way of introducing the comparison of whole numbers in the textbooks across the three countries (Alafaleq et al. 2015). In a more recent study, we compared how Pythagoras' theorem was treated in secondary mathematics textbooks in these three countries. We found that it appeared overall highly similar across the three countries in terms of the total number of problems provided, when and how the theorem was introduced (i.e., all the countries introduced it in the first semester of grade 8 and all provided students' own exploration activities), though a higher percentage of real life problems were designed in the Chinese and Saudi Arabian textbooks than the Indonesian textbook. In addition, both Chinese and Indonesian mathematics textbooks introduced some historical knowledge about Pythagoras' Theorem, while this was not the case in Saudi Arabian textbooks (see more details in Fan et al. 2016).

In the present study, we focus on how proof, as a special kind of argumentation, is presented in the secondary mathematics textbooks in these three countries.

As is well known, geometry has traditionally played an important role in school mathematics curriculum in many countries. However, it has also proven to be one of the most difficult and challenging areas for both mathematics teaching and learning, and attracted considerable attention and controversial debate over the last century (especially during the new math movement in the 1960s), among mathematics education researchers, curriculum reformers, textbook developers, and school practitioners (e.g., see Burger and Shaughnessy 1986; Jones 2002; Kapadia 1980; Kilpatrick 1992; Usiskin 2014). In particular, it has been widely recognized that geometric proof is one of the most difficult parts in students' learning of mathematics and due to its difficulty, researchers have advocated different approaches to the teaching and learning of geometry and geometric proof in school mathematics (Boero et al. 2010; Fan et al. 2017; McCrone and Martin 2004; Kollar et al. 2014; Senk 1985; Usiskin 1972; Usiskin and Coxford 1972; Weber 2001). Given the history and debate of geometry in school mathematics curriculum reform and development, we are interested to know how geometry and geometric proof are presented in current school mathematics curriculum and textbooks in different countries.

We selected the textbooks from China, Indonesia and Saudi Arabia for comparison for several reasons: On the one hand, as a team, we have different members who have received education and worked in and hence have reasonable knowledge about all the three countries including their education systems; it is also practically feasible for us to access textbooks from these countries. Moreover, all these countries have a centralized education system, and they have the same system of grade levels in terms of student ages, namely, 6 years of primary education (starting at the age of 6), 3 years of junior high school (secondary) education, and 3 years of senior high school education.

On the other hand, these countries have different cultural traditions and social contexts. In terms of international student assessments, these countries also differ widely from each other. For instance, in the Programme for International Student Assessment (PISA) 2012, China (Shanghai) placed top while Indonesia placed second lowest out of the 65 participating countries or economies on the mean score table (OECD 2014). Saudi Arabia did not participate in PISA 2012, but according to TIMSS 2011 results, Saudi Arabia is regarded as a lower middle achiever in the assessment (Mullis et al. 2012). We are interested to know if curriculum and textbooks could be a factor contributing to the difference of student achievement as reflected in these international assessments. Having realised the challenge in establishing such a connection between textbooks and student achievement (e.g., see Fan et al. 2013), and as a first step in a sense, we wish to know how the textbooks in these countries represent mathematics topics, geometric proof specifically.

3.2 Literature Review

Researchers have offered a variety of definitions about proof and many linked it closely with argument, which appears more from the perspective of local organisation (for mathematics as an activity) instead of global organisation as Freudenthal (1971) once distinguished. For example, Conner (2008) defined proof "as logically correct deductive argument built up from given conditions, definitions, and theorems within an axiom system". Similarly, Clapham and Nicholson (2009) defined proof as "a chain of reasoning, starting from axioms, usually also with assumptions on which the conclusion then depends, that leads to a conclusion and which satisfies the logical rules of inference" (p. 638).

In recent years, many researchers have indicated the central importance of proof and proving to students' school mathematical learning in all content areas (not just in geometry) and at all grade levels (Knuth 2002; Stylianides 2007). This is so because proof and proving are fundamental to doing and knowing mathematics. Moreover, proof and proving are essential in developing, establishing, and communicating mathematical knowledge and understanding (Bartlo 2013; Hanna and Jahnke 1996; Hanna 2000).

Some researchers have investigated the nature of proof, justification and explanation presented in school mathematics textbooks. For example, Stylianides (2008) examined how proof is promoted in a popular US standards-based curriculum for middle grades. He found that about 5% of student tasks involved proof. Cabassut (2006) compared the reasoning presented in proofs in French and German school mathematics textbooks and found that deductive arguments often occur in conjunction with empirical arguments.

Nordstrom and Lofwall (2006) investigated proof in Swedish secondary mathematics textbooks. They revealed a very low occurrence of proof, and that proof was seldom made explicit in explanatory texts. Focusing on explanatory texts that introduced new mathematical rules or relationships, Stacey and Vincent (2009) examined the reasoning presented in seven topics in nine Australian eighth-grade textbooks. They classified explanations and found that textbooks generally did not distinguish between the legitimacies of deductive and other modes of reasoning.

Overall, it can be seen that the available research literature in this area has been scattered and focused more on western countries, and it merits a more systematic look at a wider international level.

3.3 Research Questions

With a focus on geometric proof, this study aims to investigate the presentation of proof in the school mathematics textbooks at the secondary school level, that is, from grade 7 to grade 9 in China, Indonesia, and Saudi Arabia.

More specifically, we are interested in the following three research questions:

1. When and where proof is first introduced in the textbooks concerned?
2. How the treatment of proof is distributed in the textbooks?
3. What types of proof are introduced in the textbooks?

Through addressing these questions, we hope to reveal and document the similarities and differences in the secondary school textbooks from these three countries in terms of grade levels, topics, and types of proof in relation to the treatment of proof, and explore possible reasons for the similarities and differences, and their implications for mathematics textbook research and development.

3.4 Methodological Matters

3.4.1 Textbook Selection

It is important to note that all the three countries selected have 6–3–3 school system, as mentioned earlier, 6 years of primary school, 3 years of junior high school, and 3 years of senior high school. Students are at about the same age for the same grade level, which makes it more comparable across the three countries.

In total, nine mathematics textbooks, that is, three textbooks from each country, were selected. For Chinese and Indonesian textbooks, there are a variety of mathematics textbook series being used in the secondary schools (Malizar et al. 2014; Xu 2013), while there is only one series being used in Saudi Arabia's schools.

For the Chinese textbooks, the latest series published by Beijing Normal University Press in 2014 were selected. As many authors and editors of this series of mathematics textbooks were also key members of the team who developed the national mathematics curriculum, this series was believed to largely reflect the ideas and purposes of the new national curriculum (Ma 2014).

In Indonesia, all the textbooks are published and distributed for free by the government on http://bse.kemdikbud.go.id/. There are five series of mathematics textbooks for each grade at the secondary level. It should be pointed out that for the same grade, all school mathematics textbooks in Indonesia contain the same mathematics topics and contents, as they must follow the national syllabus in which the progression of learning including learning objectives, topics and contents for each grade are clearly stipulated.[1] For this reason, we decided to choose the most popular textbook in terms of the number of users for each grade in the case of Indonesia, even though it could mean that they could be from the different series. As a result, the three textbooks we selected from grade 7 to grade 9 were indeed from three different series. Each was the most widely used textbook for the grade as indicated by numbers of downloader's revealed in the above website.

For Saudi Arabia textbooks, there is only one textbook series for the secondary schools and this series was developed and published by the Ministry of Education of Saudi Arabia (see Ministry of Education of Saudi Arabia 2008). Therefore we naturally chose the latest series developed by the government (Ministry of Education of Saudi Arabia 2014).

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[1] Note this is not necessarily the case in other countries. For example, in China and England, the learning progression stipulated in the national syllabus (standards) is classified into different learning stages with each stage consisting of a few grades or years, and grade 7 to grade 9 (or year 7 to year 9 in England) are in the same learning stage (the third stage). Hence, textbook developers and authors might introduce different mathematics topics and contents for the same grade level, resulting in different sequencing of mathematics topics from grade 7 to grade 9.

3.4.2 Coding Scheme

Different researchers have established and used a variety of frameworks of proof levels or classifications for different purposes. For instance, Blum and Kirsch (1991) classified levels of proof into experimental, pre-formal, and formal proof. Harel and Sowder (1998, 2007) proposed a proof scheme which consists of three levels, namely external conviction, empirical proof, and deductive proof.

For this study, we used the following criteria to classify types of proof presented in the textbooks.

1. Direct proof (P → Q: assume that P is true. Use P to show that Q must be true)
2. Proof by contradiction/indirect proof (P → Q: assume P is true, and assume that $\neg$Q is true. Use P and $\neg$Q to demonstrate contradiction)
3. Counter examples (to prove that a property is not true by providing a counter example where it does not hold).

It should be noted that the above classification is based on the fact that this study focused on proof at the secondary school level, and hence from the perspective of school curriculum, it is more aligned with Freudenthal's conceptualization viewing proof being part of mathematics as an activity in the sense of local organization, as mentioned earlier (Freudenthal 1971).

In addition, in case that the above three categories do not cover all the types of proof introduced in the textbooks to be examined, we added a fourth type, that is, others, to cover all the other types of proof introduced. The result shows that it was in fact not necessary as later we found no proof introduced in the textbooks falling into this category (see Table 3.3).

Using the above classification, we examined all the main texts of the textbooks which contain proof with focus on geometric topics. Then the researchers first coded the results according to the grade levels, then topics of geometry, and finally the types of proof.

Furthermore, to ensure the reliability of coding, an external coder from each of the three countries was invited to code all of the examples independently and overall, the results of coding were highly consistent with an average agreement of 94.6%.

To end this section, we wish to point out that this study did not take into account the proof problems in the exercises in these textbooks. The first reason is pedagogical, that is, how teachers and students approach these proof problems is unclear to us given this is a textbook analysis study, and the second reason is mathematical, that is, there is often more than one way in mathematics to solve proof problems. Therefore, readers are reminded that the result of the study only reflects the intended experience of students' learning of geometric proof in these countries. Different research methods are needed to reveal the actual experience of students' learning in this area using these textbooks, which is a challenge for textbook research, as Fan (2013) argued.

3.5 Findings and Discussion

Before we report the results about the specific research questions, we first point out differences in the weight (or sheer exposure) of geometric contents in the textbooks we selected. For this purpose, the ratio of the number of pages which contain geometry contents/topics to the number of pages in the whole textbook was employed as an indicator.

The results showed that Chinese mathematics textbooks had the highest frequency. About 45% of the content pages are on the topics of geometry, while the figure is about 35% in Indonesia and 24% in Saudi Arabia textbooks.

3.5.1 *The First Introduction of Proof in the Secondary Mathematics*

In Indonesia secondary mathematics textbooks, proof is introduced for the first time in the first semester of grade seven in the topic of algebra about the properties of exponents. One of the examples was presented in the grade 7 textbook, as follows:

> Given m, n are positive integers, and p is a positive integer,
>
> prove that $(p^m)^n = p^{m \times n}$. (Nuharini and Wahyuni 2012, p. 29)

The textbook used the definition of p^m as the product of p multiplying itself for m times to deduce the property (which we consider as direct proof). However, all the three textbooks across the three grades do not mention any definition of proof in mathematics. Moreover, all the textbooks do not provide any explanation or a brief introduction of proof.

The same situation was also found in Saudi Arabian textbooks in which proof is introduced for the first time in an algebra chapter in grade seven, and the textbooks in all the three grade levels do not offer any explicit introduction about proof in mathematics (Ministry of Education of Saudi Arabia 2014).

On the other hand, Chinese mathematics textbooks have a separate chapter that introduces definition, statement and proof, which is not the case in the textbooks of the other two countries. The Chinese textbook defines formal proof as “the process of deduction”. Considering the sequences of all mathematics topics, the author arranged the chapter of proof in the first semester in grade eight, connected to the Pythagoras’ theorem (called “Gou Gu Theorem” in Chinese) and followed by parallel lines. The textbook firstly introduces that when proving a statement is a false statement, we usually use counterexamples. Then in order to prove that a statement is true, the textbook introduces nine basic facts as the foundation of proofs, which are based on the curriculum, and all of them are about geometry:

1. Two points determine a line.
2. The shortest distance between two points is a segment.
3. There is one and only one line that is vertical to a given line in a plane.
4. If two lines are intersected by a third line in a plane and the corresponding angles are equal, then the two lines are parallel to each other.
5. Given a point outside a given line, there is one and only one line passing through the point that is parallel to the given line.
6. If two sides and their included angle of one triangle are equal to the corresponding sides and angle of another triangle, the two triangles are congruent.
7. If two angles and their included side of one triangle are equal to the corresponding angles and side of another triangle, the two triangles are congruent.
8. If all three sides of one triangle are equal to the three sides of another triangle, the two triangles are congruent.
9. If two lines are intersected by a set of parallel lines, the segments obtained are in proportion.

All the proofs in this Chinese mathematics series take the nine facts as the basis and according to the facts, all the theorems introduced in the textbooks can be proved. Beyond the geometric curriculum, the textbooks mentioned one foundational concept related to algebra, which is the basis of proving equality and inequality. For example, if $a = b$ and $b = c$, then there must be $a = c$, which is known as "substitution of equal quantities". In the second semester of grade 8 and in the topic of triangles, the last proof technique is introduced: reduction to absurdity. The textbook offers only a simple example using that method and guides students to understand the process of its deduction.

From the above, it appears clear that the Chinese mathematics textbooks paid the most attention to the introduction of proof.

3.5.2 *Proof in Geometry of Secondary Mathematics*

Table 3.1 shows a summary of the distribution of texts (examples) introducing proof in the selected mathematics textbooks in these three countries. Overall, proof in geometry is presented in all the selected textbooks in the three countries.

Table 3.1 Distribution of texts (examples) introducing proof in the selected textbooks

Grade level	China		Indonesia		Saudi Arabia	
	Semester 1	Semester 2	Semester 1	Semester 2	Semester 1	Semester 2
7	0	0	0	9	0	1
8	9	29	0	1	4	3
9	11	5	2	0	0	8
Total	20	34	2	10	4	12

From Table 3.1, we can find that there exist two large differences among the three countries.

First, in terms of the total numbers of examples, China's textbooks present many more examples (54) than Indonesia (12) and Saudi Arabia textbooks (16).

Second, in terms of grade level, while Indonesia's textbooks present examples of proof at all the grade levels in the secondary school, most examples of proof were found in Semester 2 of grade 7.

In contrast, the Chinese textbooks do not introduce proof until grade 8, which also has the most intensive presentation of proof in all the three grade levels, while Saudi Arabia's textbooks almost introduce proof equally in grade eight and grade nine. Overall, we can see that the Chinese textbooks emphasize the proof the most.

The three countries also show large differences in the distributions of proof across the different topics in geometry. Table 3.2 summarizes the results.

In the Chinese textbooks, formal proof is presented in the topics of parallel lines, triangle, circles, and parallelogram including rhombuses, rectangles and squares, which appear to follow closely the Chinese National Mathematics Curriculum Standards for Compulsory Education. Furthermore, there are substantial examples in the topics of triangles and parallelograms since the textbooks take them as the focus of teaching on deduction and proof.

In Indonesia and Saudi Arabia's textbook series, no proof examples were found in parallel lines lessons. Moreover, it should be pointed out that proof examples are mostly concentrated in the topic of triangles in the three countries textbooks, particularly in the Indonesia and Saudi Arabia textbooks.

Regarding different types of proof, it was found that only the Chinese textbooks have more than one type of proof, that is, direct proof and proof by contradiction, while the textbooks in the other two countries only have introduced direct proof. In addition, no textbooks in all the three countries provided any example of proof by counterexample.

Table 3.3 summarizes the distribution of the numbers of different proof types in these textbooks. It seems apparent again that the Chinese mathematics textbooks set the highest requirement in this area of proof.

Table 3.2 Numbers of examples of proof across different topics of geometry

Topic	China			Indonesia			Saudi Arabia		
	G7	G8	G9	G7	G8	G9	G7	G8	G9
Parallel lines	0	9	0	0	0	0	0	0	0
Triangle	0	17	3	2	0	2	0	7	5
Parallelogram	0	12	0	1	0	0	0	0	3
Rhombus, rectangle and square	0	0	8	1	0	0	0	0	0
Circle	0	0	5	0	1	0	0	0	0
Angle	0	0	0	4	0	0	1	0	0

Table 3.3 Numbers of examples of different types of proof in the textbooks

Proof	China			Indonesia			Saudi Arabia		
	G7	G8	G9	G7	G8	G9	G7	G8	G9
Direct	0	36	16	9	1	2	1	7	8
Proof by contradiction	0	2	0	0	0	0	0	0	0
Proof using counterexample	0	0	0	0	0	0	0	0	0
Others	0	0	0	0	0	0	0	0	0

3.6 Concluding Remarks

In this comparative study, we aim to examine how geometric proof is presented in the secondary school mathematics textbooks from grade 7 to grade 9 in China, Indonesia and Saudi Arabia. Our comparison of the selected textbooks from the three countries revealed that the Indonesian and Saudi Arabian mathematics textbooks introduced proof earlier than the Chinese textbooks in the sense that the former introduced it in algebra in grade 7 while the latter in grade 8.

However, regarding the number, type, and distribution of proof in geometric contents, Indonesian and Saudi Arabian mathematics textbooks gave much less emphasis on proof compared with the Chinese mathematics textbooks. The textbooks in China presented many more proof examples at grade 8 and grade 9.

It is also important to notice that Indonesian and Saudi Arabian secondary mathematics syllabi do not explicitly mention the concept of proof, which is not the case in Chinese national curriculum, which might explain why the Chinese textbooks devoted a whole chapter to the topic of mathematical proof, while while Indonesianand Saudi Arabian textbooks did not pay significant attention to proof. As pointed out earlier, all the three countries have adopted a centralized education system, and all the textbooks are required to follow the national curriculum standards (or syllabus).

As mentioned at the beginning of this article, our earlier studies on the introduction of the comparison of whole numbers at the primary school level and on the presentation of the Pythagoras' theorem at the secondary school level in the mathematics textbooks across the three countries revealed a high level of consistency (Alafaleq et al. 2015; Fan et al. 2016). In contrast, this study reveals more inconsistencies or differences than similarities about the treatment of proof in the three countries, which are particularly evident in the three aspects: the number of examples, the distribution of contents and, to a lesser degree, the types of proof. We think these differences clearly reflect a lack of consensus in the international research community about the role and importance of proof in the mathematics curriculum and the way of teaching proof in school mathematics. It seems clear to us that more sound research is much needed in this area before any consensus can be truly reached.

Finally, we wish to emphasize that the development of textbooks is affected by a variety of factors including, for example, curricular, educational, social, and cultural factors, as researchers have argued (Fan 2013; Rezat 2006; Rezat and Sträßer 2012). Nevertheless, given the purpose, design, and scope of this study, it is less clear that, in addition to the national curricula and educational systems, how other factors have played a role in the development of the textbooks in these three countries in relation to the presentation of proof. To address these issues in the development of textbooks would require a different methodology, for instance, interview with the textbook developers, historical and longitudinal approach, which we believe in a large sense will be more challenging as well as interesting.

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Part II

Teacher Interactions with Curricular and Other Learning Resources

Chapter 4
Examining Teachers' Interactions with Curriculum Resource to Uncover Pedagogical Design Capacity

Janine T. Remillard

Abstract This chapter considers how teachers interact with curriculum resources to design and enact mathematics instruction and the capacities involved in doing this work. It begins with a discussion of conceptual and empirical issues related to curriculum, resources as a genre of tools, and pedagogical design capacity (PDC). These concepts are then illustrated, using one elementary teacher's interactions with an unfamiliar curriculum resource. Analysis of the teacher's reading of the guide, an enacted lesson, and pre- and post-observation interviews, identified robust and underdeveloped aspects of the teacher's PDC. Analysis of the teacher's guide indicates a lack of transparency about key mathematical and pedagogical concepts, which shed light on these findings.

Keywords Mathematics teachers' resources · Pedagogical design capacity
Teacher's guides · Elementary teachers

4.1 Introduction

A growing body of research around the world, represented in part by the work shared in Topic Study Group 38, is seeking to uncover and conceptualize the multiple ways teachers interact with resources to design and enact mathematics instruction and to identify the capacities involved in doing this work. In this chapter, I explore these issues theoretically and empirically. First, I discuss some conceptual and theoretical issues related to curriculum resources, including the meaning of curriculum as an adjective, how resources can be understood as a genre of tools used by teachers, and what is involved in reading and using them. I then discuss what I mean by teacher-resource interactions and the capacity involved in this work. Finally, I offer an illustrative example of one elementary teacher's interactions with a mathematics curriculum resource to explore how research on

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teachers' interactions with particular curriculum resources can inform our understanding of this capacity.

4.2 What Are Curriculum Resources?

In this TSG, mathematics teaching and learning resources refers to a genre of materials and tools, including, but not limited to textbooks, designed to guide, support, and enhance mathematics teaching and learning in schools. For many years, the textbook, accompanied in some countries by a teacher's guide, served as the primary instructional resource found in mathematics classrooms. In 2017, the list of types of resources available is long and diverse, including, but not limited to print, digital, and online materials and tools used either periodically or over an extended period of time. Some resources are designed to be used to support and guide instruction; others are resources taken up by teachers and deployed as instructional tools.

4.2.1 Distinguishing Different Types of Resrouces

There is an equally long list of terms used to refer to teaching and learning resources. I begin by clarifying some terms that may assist in discussions of different types of resources.

I use the term *instructional resource* to refer to tools provided to, appropriated by, or generated by teachers to guide or support instruction. Instructional resources represent a broad category of artifacts, as shown in the largest region in Fig. 4.1. These resources include curriculum resources and others that, alone, are not curricular in nature. I discuss the meaning of curriculum below.

Five or ten years ago, I might have used the term *materials* instead of resources. This term may be a holdover from print materials; it may also be unique to the United States. In using the term *resources,* I am following the lead of many of my European and South African colleagues who use the term resources to include the wide range of possible types of tools, including print or digital instructional materials, simulations, videos, interactive tools, and the like. I also find Jill Adler's (2000) ideas about resource use as "re-sourcing" constructive. Describing teachers' work as re-sourcing hints at the active nature of the work. By using resource as "both noun and verb, as both object and action," Adler focuses attention on teachers working with resources in practice (p. 207).

The term resource also draws attention to a critical aspect of teachers' work—interpreting and transforming available artifacts for one's own instructional purposes. Gueudet and Trouche (2009) use the term documentational genesis to describe this process. Informed by Rabardel's (1995) articulation of how artifacts, when used for specific purposes, become instruments, endowed with human

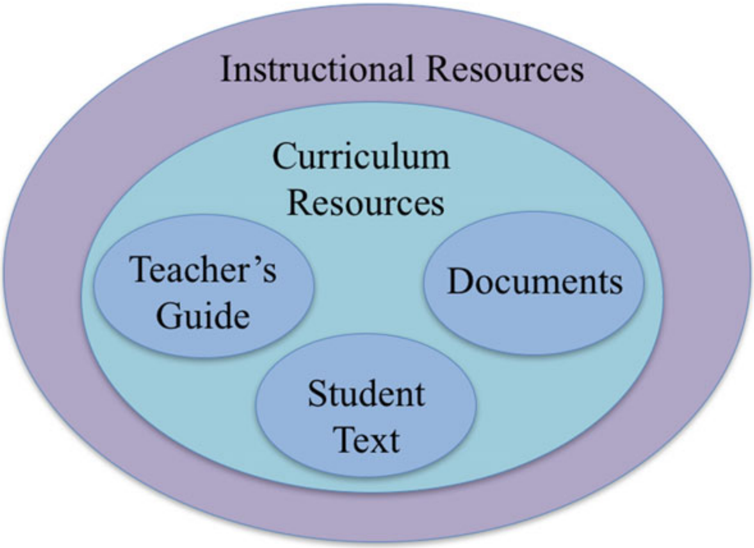

Fig. 4.1 Relationships among different types of instructional resources

purpose, and the participatory perspective on teachers' use of curriculum resources (Remillard 2005), Gueudet and Trouche argue that teachers appropriate and transform available resources as they design instructional plans. The documents, which are the products of this work, are then used by teachers. In the next round of usage, the documents are transformed further, informed by the experience of the first round. In this way, the documentational genesis process is a dialectic and dynamic one.

I use the term *curriculum resources* to refer to print or digital artifacts designed to support a program of instruction and student learning over time. The term *curriculum* refers to the course or pathway on which learners are guided. Resources that attend to sequencing or mapping students' learning over a period of time, such as a lesson sequence, a set of lessons, a year of instruction, or more, I argue, are curricular in nature. From this perspective, as shown in Fig. 4.1, all curriculum resources are examples of instructional resources, but not vice versa. This aspect of sequencing, appears to be an important component of curriculum resource design, as it proposes an intended learning progression for particular mathematical domains. Choppin (2011) has identified these learning sequences as a critical element of many curriculum programs that is not always made visible to the teacher. Moreover, Sleep (2009) identifies identifying learning sequences as an important feature of content-specific curriculum knowledge. I argue that sequencing or curriculum mapping (Remillard 2016) is an under-appreciated aspect of curriculum design and curriculum knowledge.

The curriculum resource domain in Fig. 4.1 includes three examples of such resources. *Student texts* or textbooks and teacher's guides are curriculum resources designed for teachers' use. The term textbook is most frequently used in the field to refer to the resource designed for the students' consumption (Fan et al. 2013). Textbooks include exercises, problems, and other tasks for students, along with worked examples, and definitions. Many curriculum resources also include guidance prepared specifically for the teacher, often referred to as the *teacher's guide*. In the U.S. and a number of other countries, guides are written to communicate to the teacher and support them in shaping instruction (Remillard et al. 2016). In cases where there are no written teacher's guides, teachers draw on their own experience and the student text to make inferences about the intent of instruction plans. Documents, drawing on Gueudet and Trouche (2009), refer to the products of teachers' curriculum design work, which approximate what Remillard and Heck (2014) refer to as the teacher intended curriculum.

In the analysis presented in this chapter, I focus on the relationship between these three types of curriculum resources. I look closely at one teacher's guide, to understand what is involved in interpreting it, and how one teacher appropriates the resources it contains. First, I conceptualize Teacher's guides as a genre of communication and the craft involved in using them.

4.2.2 Teacher's Guides as a Genre of Communicaton

Curriculum resources are cultural artifacts (Vygotsky 1978). They are the products of cultural activity and reflect norms, values, and practices specific to the local cultural context (Pepin and Haggarty 2001; Pepin et al. 2013). In this sense, they hold cultural knowledge. Curriculum resources designed for teachers are intended as a resource for teachers, but making sense of them requires a process of interpretation.

I think of teacher's guides as a genre of communication within a larger class of written and visual communication. They are designed to offer information, instructions, and suggestions that will aid in the construction of curriculum in the classroom. In essence, they are meant to guide action and decision making. As a genre, they communicate with readers with action in mind. In the case of mathematics, these actions are contingent on the development of mathematical understanding in others.

The notion of genre is elaborated by Ongstad (2006) in his semiotic analysis of communication in mathematics and mathematics education. "Genre precisely presupposes much of what can be expected in the kind of communication in question" (p. 262). Its familiarity conjures a "zone of expectation" and aids in how one makes sense of any form of communication, textual, or discursive. For teachers, the familiarity they might have with curriculum resources as a genre influences how they read, interpret, and use them.

As I discuss below, not all reading and interpreting of teacher's guides, despite their familiarity, is straightforward. Furthermore, teacher's guides are likely to contain unfamiliar elements as well. Ongstad uses the term "rheme" to identify the unfamiliar or new ideas in a familiar form, those that fit outside of the zone of expectation. Reading teacher's guides involves an interaction between the familiar (the theme) and the new (the rheme) in which the theme contextualizes and aids in the interpretation of the rheme (p. 263).

To further examine the genre of teacher's guides, I draw on Brown's (2009) sheet music metaphor for curriculum resources. They are both "static representations" of intended activity and the "means of transmitting and producing" it, but not the activity itself. In other words, they are not the music or the curriculum, but offer guidance to the agent responsible for enactment. As such, they serve as "an interface between the knowledge, goals, and values of the author and the user" (p. 21).

Curriculum resources and sheet music are also similar in that "they are intended to convey rich ideas and dynamic practices," but they do so "through succinct shorthand that relies heavily on interpretation" (Brown 2009, p. 21). They often rely upon "culturally shared notational rules, norms, and conventions" and "reflect common or existing practices." At the same time, curriculum materials are often designed to "influence common practice by introducing innovative approaches and ideas." Most critically, and often overlooked, curriculum materials "require craft in their use; they are inert objects that come alive only through interpretation and use by a practitioner" (p. 22).

4.2.3 The Craft of Resource Use: Pedagogical Design Capacity

Seeing curriculum resources as a genre highlights the complexity of reading and using curriculum resources. As Brown (2009) points out, curriculum resources tend to represent complex and multifaceted ideas in succinct shorthand. Teachers must read and interpret a variety of components of curriculum resources and determine their meanings and implications relevant to their teaching context. Engaging in this type of analysis involves substantial ability on the part of teachers.

Brown (2009) introduced the term pedagogical design capacity (PDC) to refer to a teacher's ability to perceive and mobilize curriculum resources in order to "craft instructional episodes" (p. 29). He points out that, together, perceiving and mobilizing include knowing and doing, making determinations and then acting on them. The concept of PDC is important because it signals that the task of using resources is not straightforward and involves skills that teachers need to develop. PDC is also mediated by the particular resource a teacher is using. Some resources are more demanding for teachers to read and mobilize because of their complexity or unfamiliarity; others may include additional, "educative," supports that increase the transparency or explicitness of guidance included (Davis and Krajcik 2005; Stein and Kim 2009). Unfortunately, because the work of using resources is not well understood, insufficient attention has been paid to how to delineate these capacities and help teachers develop them. Moreover, at least in the U.S., the myth that the best teachers do not rely on resources, but develop lessons on their own, works against nurturing these capacities (Remillard and Taton 2015).

Over the last several years, researchers in the ICUBiT[1] project have studied elementary teachers' interactions with curriculum resources in the U.S., with the goal of elaborating PDC. In order to understand what is involved in perceiving resources, we have analyzed mathematics teacher's guides to uncover and elaborate the succinct content of these guides that teachers read, interpret, and reason about in order to use. We then observed and interviewed teachers using these resources to understand the process of mobilizing them to design and enact instruction.

In the analysis that follows, I examine one teacher's interactions and use of a single lesson from the teacher's guide, drawn from the ICUBiT data set. My aim is to illustrate how analyzing curriculum resources, both teacher's guides and related documents associated with use, can shed light on aspects of PDC.

4.3 A Case of Teacher-Resource Interactions

Elsewhere, I have argued that in the process of using them, teachers interact with their curriculum resources (Remillard 2005). This perspective challenges the assumption shared by some educational researchers and decision makers that professionally designed curriculum resources decrease the demands on the teacher using them; teachers simply pick them up and use them. As discussed earlier, evidence from research suggests that using curriculum resources is a dynamic process involving reading, interpretation, appropriation, and design (Brown 2009; Gueudet and Trouche 2009; Remillard 2005). I think of it as a participatory process, through which teachers actively partner with resource designers, the resources they designed, and, potentially, other teachers. In our research on teacher-resource interactions, the ICUBiT team adopted a participatory perspective. After a brief description of our methods, I provide an analysis of the multifaceted and layered nature of one curriculum resource, focusing on a single lesson. I then describe and analyze one teacher's interactions with and use of this lesson.

4.3.1 Methods

The team analyzed five elementary mathematics curriculum programs used in the U.S., focusing on the mathematical and pedagogical demands placed on teachers and how authors communicate with teachers about different aspects of the intended

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[1] ICUBiT stands for Improving Curriculum Use for Better Teaching. It is a project funded by the National Science Foundation in the U.S., directed by Janine Remillard and Ok-Kyeong Kim.

curriculum. Using the lesson as a unit of analysis, we identified the different components of the resource and the ways mathematical ideas were represented. We also coded all written and visual aspects of the teacher's guide for what the authors communicate to the teacher about. Looking across teacher's guides for the five different programs, we compared the different amounts and types of information provided to teachers. We also identified a set of common forms of communication used across the guides. In the analysis presented in this chapter, I use a single curriculum program, called *Mathematics in Focus* (MiF) (Kheong et al. 2010). MiF was a modified version of one of the mathematics programs used in Singapore, developed by Marshall Cavenish, for sale in the U.S.

Data collection of teachers using the curriculum resources relied on a *teaching set* methodology (Cobb et al. 2009; Simon and Tzur 1999), which involves collecting video records of multiple lessons along with associated artifacts and then using specific events or practices observed in the data as a basis for teacher interviews. The ICUBiT study collected teaching sets for 25 teachers from four states in the U.S. The teacher discussed in this chapter, Maya Fiero,[2] was a 4th grade teacher in an elementary school in the eastern United States. She was in her tenth year of teaching and was using MiF for the second year.

Two teaching sets were collected for each teacher, one in the fall and one in the spring. The teaching set included 3 video recorded lesson observations, a completed curriculum reading log (CRL) for the lessons taught during the week of observation, and a follow-up interview. Prior to the fall teaching set, each teacher completed an introductory interview, during which the teachers provided information about professional background and curriculum use. The CRLs consisted of a copy of the relevant lesson in the teacher's guide on which teachers used coloured highlighters to indicate which parts of the guide they read and how they planned to use them: General reading (yellow); plan to use (green); other portions that were helpful when planning, include those to be adapted (orange). During the follow-up interview, the interviewers asked teachers to respond to questions about the observed lessons and the CRL.

4.3.2 Findings from Curriculum Analysis

Brown (2009) described curriculum resources as conveying "rich ideas and dynamic practices through succinct shorthand" (Brown 2009, p. 21). Further, as Brown points out, the representations of these forms are static, although they intend to communicate dynamic practices. Our analysis of all five curriculum guides was aimed at uncovering how they communicate with teachers and what is involved in perceiving their content and messages.

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[2] Pseudonyms.

Our analysis surfaced three layers of communication that teachers routinely encounter and must interpret when designing instruction: mathematical-instructional objects, pedagogical guidance and insights, curricular sequences. I use the term layered to refer to the way concepts and ideas are packaged in instructional activities, which are sequenced to develop over time.

Figure 4.2 shows the first page of a Grade 4 lesson in a teacher's guide taken from *Mathematics in Focus* (Lesson 3.2, p. 86). This lesson introduces an approach to multiplying two-digit numbers by two-digit multiples of 10. The column on the left-hand side of the page provides an overview of the lesson, detailing the objectives and the various resources the teacher might draw on from elsewhere. At the bottom of the left-hand margin there is a description of a short activity, entitled "5-minute Warm up." An image from the students' book is captured on the upper right-hand side, showing three ways to represent and solve, first, 4 × 10 and then 3 × 20: (a) a story context, (b) a place value chart, and (c) an equation-based approach to multiplying by multiples of 10. Beneath the student page are suggestions for the teacher to use when teaching the lesson.

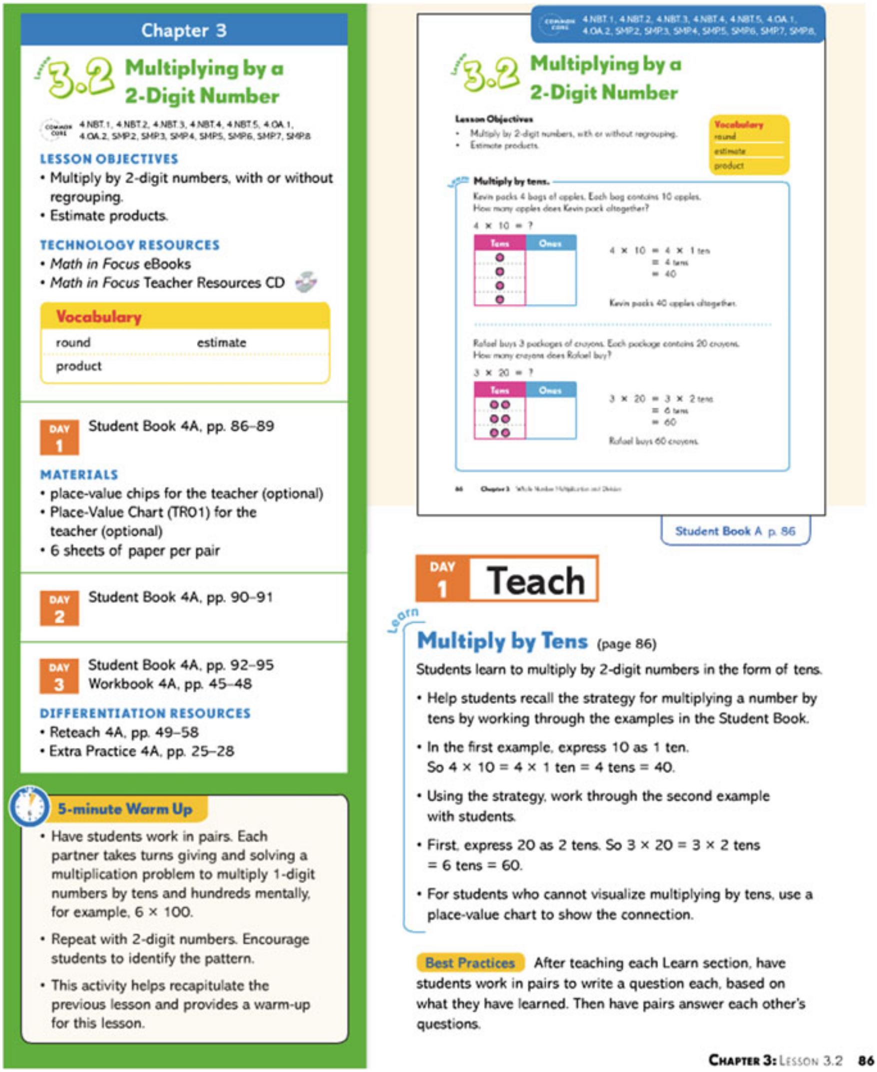
4.3.2.1 Mathematical-Instructional Objects

The first layer, *mathematical-instructional objects*, are mathematical ideas packaged for the purpose of instruction. These objects include tasks, activities, strategies, models, and representations, are designed for direct engagement with students and are intended to facilitate their learning of mathematics. Lessons in curriculum guides typically consist of a variety of tasks and activities designed for students to do, which are seen as the primary vehicle through which the learning takes place and by which teachers assess students' understanding (Doyle 1983; Stein et al. 1996). The story problems in Fig. 4.2 (upper right-hand side) provide one example of tasks. Others include the "5-Minute Warm Up" at the beginning of the lesson, during which students work with partners to practice multiplying "1-digit numbers by tens and hundreds mentally" and the set of "Guided Practice" problems on the following page (not pictured), which include 14 × 10, 9 × 40, and 47 × 80.

Mathematical-instructional objects are the primary vehicles through which students encounter and engage with mathematical ideas, but as Sleep (2009) emphasizes, the mathematical point is not always clearly articulated. In other words, there is a distinction between instructional activities and the key mathematical points underlying them. Later, I discuss the mathematical complexity embedded in the mathematical-instructional objects in Fig. 4.2.

4.3.2.2 Pedagogical Guidance and Insights

The second layer of communication is *pedagogical guidance and insights*. To differing extents, teacher's guides provide information teachers might use to plan lessons and follow during the lesson. This layer is intended for the teacher, not the



Chapter 3

3.2 Multiplying by a 2-Digit Number

4.NBT.1, 4.NBT.2, 4.NBT.3, 4.NBT.4, 4.NBT.5, 4.OA.1, 4.OA.2, SMP2, SMP3, SMP4, SMP5, SMP6, SMP7, SMP8

LESSON OBJECTIVES

- Multiply by 2-digit numbers, with or without regrouping.
- Estimate products.

TECHNOLOGY RESOURCES

- *Math in Focus* eBooks
- *Math in Focus* Teacher Resources CD

Vocabulary

round estimate

product

DAY 1 Student Book 4A, pp. 86–89

MATERIALS

- place-value chips for the teacher (optional)
- Place-Value Chart (TR01) for the teacher (optional)
- 6 sheets of paper per pair

DAY 2 Student Book 4A, pp. 90–91

DAY 3 Student Book 4A, pp. 92–95
Workbook 4A, pp. 45–48

DIFFERENTIATION RESOURCES

- Reteach 4A, pp. 49–58
- Extra Practice 4A, pp. 25–28

5-minute Warm Up

- Have students work in pairs. Each partner takes turns giving and solving a multiplication problem to multiply 1-digit numbers by tens and hundreds mentally, for example, 6 × 100.
- Repeat with 2-digit numbers. Encourage students to identify the pattern.
- This activity helps recapitulate the previous lesson and provides a warm-up for this lesson.

4.NBT.1, 4.NBT.2, 4.NBT.3, 4.NBT.4, 4.NBT.5, 4.OA.1, 4.OA.2, SMP2, SMP3, SMP4, SMP5, SMP6, SMP7, SMP8.

3.2 Multiplying by a 2-Digit Number

Lesson Objectives

- Multiply by 2-digit numbers, with or without regrouping.
- Estimate products.

Vocabulary
round
estimate
product

Multiply by tens.

Kevin packs 4 bags of apples. Each bag contains 10 apples.
How many apples does Kevin pack altogether?

4 × 10 = ?

4 × 10 = 4 × 1 ten
= 4 tens
= 40

Kevin packs 40 apples altogether.

Rafael buys 3 packages of crayons. Each package contains 20 crayons.
How many crayons does Rafael buy?

3 × 20 = ?

3 × 20 = 3 × 2 tens
= 6 tens
= 60

Rafael buys 60 crayons.

Student Book A p. 86

DAY 1 **Teach**

Multiply by Tens (page 86)

Students learn to multiply by 2-digit numbers in the form of tens.

- Help students recall the strategy for multiplying a number by tens by working through the examples in the Student Book.
- In the first example, express 10 as 1 ten. So 4 × 10 = 4 × 1 ten = 4 tens = 40.
- Using the strategy, work through the second example with students.
- First, express 20 as 2 tens. So 3 × 20 = 3 × 2 tens = 6 tens = 60.
- For students who cannot visualize multiplying by tens, use a place-value chart to show the connection.

Best Practices After teaching each Learn section, have students work in pairs to write a question each, based on what they have learned. Then have pairs answer each other's questions.

CHAPTER 3: LESSON 3.2 86

Fig. 4.2 Excerpt from *MATH IN FOCUS: The Singapore Approach* (Kheong et al. 2009, Grade 4, Lesson 3.2, p. 86). Copyright © 2009 by Houghton Mifflin Harcourt Publishing Company. All rights reserved. Reprinted by permission of the publisher

student. All lessons include statement of the mathematical learning goal or objective. Teacher's guides also include instructional suggestions on what to do or say during the lesson. For example, guidance that accompanies the page in Fig. 4.2 includes the following recommendation: "Help students recall the strategy for multiplying a number by tens by working through the examples in the Student

Book" (p. 86). I think of this type of guidance as directing teachers' actions or moves (Remillard 2013).

Other guidance provides information about the rationale behind particular design decisions, how students might or should respond to various tasks, or the underlying mathematical ideas (Davis and Krajcik 2005; Stein and Kim 2009). The guidance that accompanies the 5 Minute Warm Up that begins the lesson, includes the following rationale: "This activity helps recapitulate the previous lesson and provides a warm-up for this lesson" (p. 86). Several of the teacher's guides we examined include examples of common student errors or anticipate difficulties they might have along with ways teachers might address these. The guidance with Fig. 4.2 includes: "For students who cannot visualize multiplying by tens, use a place value chart to show the connection" (p. 86).

4.3.2.3 Curricular Sequences

The third layer of communication, *curricular sequences*, is structural in nature and refers to how the mathematical content is organized for learning over a variety of different timeframes. Curriculum materials offer a sequenced learning pathway for the development of identified mathematical goals (Choppin 2011). I use the term *learning pathway* to refer to the planned development of mathematical ideas and related skills over time, within a lesson, within a year, or over multiple years (Sleep 2009). The sequences built into these pathways are critical to the designed curriculum.

One marker of the sequence built into Lesson 3.2 includes the warm-up described above. This short activity is designed to activate students' fluency with multiplying by 10 or 100, which will be drawn on in this lesson. The sequencing is also evident in the way the lesson introduces the equation-based structure with 4×10 and 3×20 before using the same structure to multiply larger numbers, including multiples of 100. Over the next several lessons, the curriculum uses the structure for multiplying by multiples of ten to introduce the steps of multiplying any two 2-digit numbers using the standard, multiplication algorithm. Common pathways built into all of the curriculum programs we analyzed included movement from use of visual or concrete models to abstract representations of procedures and relationships and gradual increase in the size or complexity of number students are expected to deal with. Other learning pathways we identified included connections across topics, such as the fractions and decimals or different operations.

Understanding curriculum resources as containing multiple layers of communication has roots in didactic transposition theory (Chevallard 1988), the notion that in order to be packaged for the purpose of teaching, ideas, such as mathematical concepts, are structured into pedagogical forms. It is representations of these forms that are encountered by teachers and from which the mathematical and pedagogical ideas must be perceived. Perceiving curriculum resources involves reading on each of these three levels, uncovering the embedded mathematical and pedagogical meanings, and assessing their relevance to the goals at hand.

4.3.2.4 Unfamiliar Mathematical and Pedagogical Approaches

Because it was based on curriculum resources used in Singapore, MiF tended to present complex mathematical concepts using approaches not typical of instructional approaches used in the U.S. Lesson 3.2, excerpted in Fig. 4.2, provides an example of one such strategy presented in MiF and is typical of the way the authors build on mathematical structures throughout the program. The symbolic approach shown to multiply any number by a multiple of 10 is not commonly taught in the U. S. It uses an equivalent relationship to guide students through steps of rewriting the original expression in equivalent forms in order to solve: 4 × 10 can be written as 4 × 1 ten; and then 4 and 1 can be multiplied, resulting in 4 tens, or 40, also shown in the place value chart. Although it is likely that students will know that 4 × 10 is 40, the authors appear to be building a structure that will be applicable to all cases involving multiplying by multiples of 10, such as the second problem, 3 × 20, or 24 × 300, a problem that appears on the following page in the lesson.

This structure relies on several foundational mathematical ideas that underlie the structure of numbers and multiplication and will be critical to algebraic thinking. One idea is that numbers can be decomposed into workable components that can facilitate operating on them (e.g., 20 = 2 × 10); further, any number with a zero in the ones place can be considered as a certain number of tens (e.g., 20 = 2 tens); a third, related idea is that "tens" refers to units (e.g., *2 tens* means *2 units of ten*), which then can be counted as a group (e.g., 3 × 2 tens means 3 groups of 2 tens). By rewriting 10 as 1 ten, the curriculum implicitly emphasizes ten as a unit; rewriting 3 × 20 as 3 × 2 tens emphasizes that the problem involves 3 sets of 2 tens, also illustrated by the model on the left, showing tens as circles and each package of 20 crayons as 2 tens. This model further represents the equivalent relationship between 3 × (2 tens) and 6 tens, as well as illustrates the associative property of multiplication: 3 × (2 tens) = (3 × 2) tens, i.e., 3 × (2 × 10) = (3 2) × 10. The final step in the approach, using the understanding of sets of ten (or 100s) to multiplying a number by 10 (or 100) by adding zeros, assumes an understanding of iterating composite units, a concept foundational to multiplicative reasoning (Ulrich 2015).

4.3.2.5 Minimal Transparency

Our analysis of how the authors of MiF communicated with teachers revealed a tendency to provide directive guidance, offering suggested actions or moves teachers might make. At the same time, the authors provided minimal explanation or descriptions of the rationale behind the instructional approach of the intended sequencing of the tasks. For this reason, we describe this approach to communicating with teachers as lacking transparency about the mathematical or pedagogical designs. Few of the mathematical ideas detailed above are discussed explicitly in the teacher's guide, although the role of the associative property of multiplication is mentioned later in the lesson.

4.3.3 The Work of Perceiving Curriculum Resources

My aim in analyzing the layered content of a curriculum resource and its approach to communicating with teachers was to illustrate the complexity of curriculum design work from both the curriculum designer's perspectives and that of the teacher using it. Perceiving the affordances of designed curriculum resources, a key component of PDC, involves identifying the mathematical purpose or point underlying mathematical-instructional objects, considering the rationale behind recommended pedagogical actions, and mapping the learning pathways underlying curricular sequences. When teachers perceive these affordances, even when they are not explicitly stated in the resource, they are better situated to use them to design instruction. The following section provides an illustration of one teacher's perception and mobilization of some of the resources in this lesson.

4.3.4 One Teacher's Interaction with the Teacher's Guide

When we observed and interviewed Ms. Fiero during the 2011–12 school year, her school had just adopted MiF. She was one of the teachers who had piloted the program the previous year. She told us, "Math is my subject. I like math. I like to teach it."

4.3.4.1 Ms. Fiero's View of the Resource

Ms. Fiero explained that, after using MiF during the first year, she "really liked it," although many of her colleagues did not. When asked what she liked, she provided two reasons. First, she liked that the approach to teaching math was somewhat different than what she was familiar with. She especially liked that the guide almost always offered more than one way to approach a problem. "I want to give them other strategies," she explained. "Some kids can't solve one [problem/task] one way, but they can solve it another way." Her second reason was that the guide provided a lot of details on how to introduce the strategies to students. These details helped her because many of the strategies introduced in MiF were unfamiliar to her. For this reason, she said that she read every detail in the teacher's guide and tried to use most of it. This approach is supported by her detailed highlights and annotations in the CRL. She found that it was helping her develop "a new mindset. . . I'm a lot older; my mind doesn't think this way either," she said, "but I find it interesting and I get excited about it."

4.3.4.2 Ms. Fiero's Reading of Lesson 3.2

As is mentioned above, Ms. Fiero appeared to read the teacher's guide thoroughly. An excerpt from her CRL is shown in Fig. 4.3. We asked teachers to mark in yellow all parts of the guide that they read when planning their lesson. We asked them to mark in green parts that they planned to use during the lesson and we asked them to highlight the parts in orange that influenced their planning or that they found helpful. This might include parts that they planned to modify. Ms. Fiero not only marked the parts she planned to use, but added additional comments.

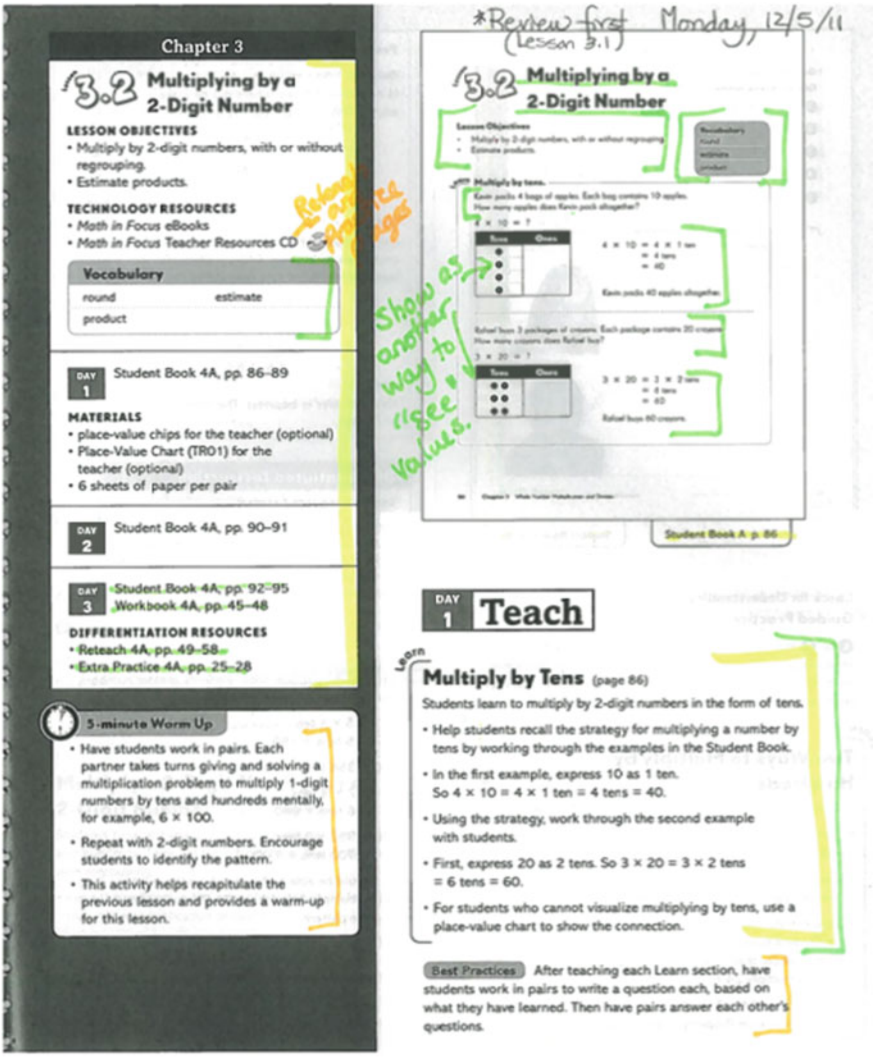
It is worth nothing that the 5-minute warm-up is marked in orange. When asked about it during the interview, she said that they had spent some time multiplying by tens and hundreds the previous day. "We just built on the multiplying by tens and just you know, we have hundreds with two zeros instead of one zero. And they seemed to get that pretty well. . . I didn't want to go too in-depth because like we spent a little bit of time on this and I walked them through. . . I felt that was sufficient." As I describe in the brief synopsis of Ms. Fiero's lesson below, she replaced this 5-minute warm-up with a multiplication practice sheet, which included several 2- by 1-digit multiplication exercises.

4.3.4.3 Synopsis of Lesson 3.2

The lesson began, as is typical for Ms. Fiero, with the 2- by 1-digit multiplication exercises, presented in a vertical format. As they practiced multiplying by the number in the ones place and then the tens place Ms. Fiero emphasized the use of "basic facts," such as 3 times 6 = 18, in 62 × 3. The students spent about 12 min on the exercises and then Ms. Fiero led the class in reviewing them on the board. She then guided students through a review of the previous day's multiplication work. Twenty-five minutes into the lesson, Ms. Fiero began the introduction shown in Fig. 4.2. They began with a short discussion of title, objective, and key vocabulary listed on the first page of the student workbook, all items marked in the CRL. She then moved the class to considering how to multiply numbers by 10. The guidance in the teacher's guide states: "Help students recall the strategy for multiplying a number by tens by working through the examples in the Student Book." Rather than using the examples in the student book, Ms. Fiero, reminded the students that they had multiplied numbers by 10 the previous day. She wrote 81 × 10 on the white board to illustrate. For the next several minutes, the students struggled to provide an answer. One students said, "We haven't learned that yet."

Ms. Fiero responded: "We don't know how to do 2-digit multiplication. No we don't, yet. But do we know how to multiply things by 10? She called on several students who offered incorrect answers. Then, she called on a student who said: "You multiply the first number by the one, the 10, and then you just add the zero at the end."

Ms. Fiero then moved the class onto the two examples on the page 86 (Fig. 4.2). She copied the following from the student book, guiding them through each step:



Chapter 3

3.2 Multiplying by a 2-Digit Number

LESSON OBJECTIVES
- Multiply by 2-digit numbers, with or without regrouping.
- Estimate products.

TECHNOLOGY RESOURCES
- *Math in Focus* eBooks
- *Math in Focus* Teacher Resources CD

Related [illegible] Practice pages

Vocabulary

round	estimate
product	

DAY 1 Student Book 4A, pp. 86–89

MATERIALS
- place-value chips for the teacher (optional)
- Place-Value Chart (TR01) for the teacher (optional)
- 6 sheets of paper per pair

DAY 2 Student Book 4A, pp. 90–91

DAY 3 Student Book 4A, pp. 92–95
Workbook 4A, pp. 45–48

DIFFERENTIATION RESOURCES
- Reteach 4A, pp. 49–58
- Extra Practice 4A, pp. 25–28

5-minute Warm Up
- Have students work in pairs. Each partner takes turns giving and solving a multiplication problem to multiply 1-digit numbers by tens and hundreds mentally, for example, 6 × 100.
- Repeat with 2-digit numbers. Encourage students to identify the pattern.
- This activity helps recapitulate the previous lesson and provides a warm-up for this lesson.

*Review first Monday, 12/5/11
(Lesson 3.1)

3.2 Multiplying by a 2-Digit Number

Show as another way to "see" values.

4 × 10 = 4 × 1 ten
= 4 tens
= 40

3 × 20 = 3 × 2 tens
= 6 tens
= 60

Student Book A p. 86

DAY 1 Teach

Learn

Multiply by Tens (page 86)

Students learn to multiply by 2-digit numbers in the form of tens.
- Help students recall the strategy for multiplying a number by tens by working through the examples in the Student Book.
- In the first example, express 10 as 1 ten. So 4 × 10 = 4 × 1 ten = 4 tens = 40.
- Using the strategy, work through the second example with students.
- First, express 20 as 2 tens. So 3 × 20 = 3 × 2 tens = 6 tens = 60.
- For students who cannot visualize multiplying by tens, use a place-value chart to show the connection.

Best Practices After teaching each Learn section, have students work in pairs to write a question each, based on what they have learned. Then have pairs answer each other's questions.

Fig. 4.3 Maya Fiero's Curriculum Reading Log for Lesson 3.2, p. 86

$$\begin{aligned} 4 \times 10 &= 4 \times 1 \text{ ten} \\ &= 4 \text{ tens} \\ &= 40 \end{aligned}$$

Students had difficulty with several aspects of this approach. They appeared to be confused by representing the two equivalent expressions: 4 × 10 = 4 × 1 ten or 3 × 20 = 3 × 2 tens. Rather than represent 4 × 10 in an equivalent form, they wanted to provide an "answer" to the right of the equal sign. In response, Ms. Fiero focused on the meaning of the equal sign in the number sentence. She asked questions like, "What does the equals sign mean?" and emphasized that it indicated that "both sides are equal or the same value." She also told the students, "When you get into algebra and things when you're bigger, you're gonna have tons of stuff on this side equal to tons of stuff on this side."

Another difficulty students had during this part of the lesson involved recognizing that 4 × 10 was indeed equivalent to 4 tens. A related difficulty students had, especially when the numbers in the problems increased in size, was converting a number of tens, such as 6 tens (or 18 tens, on the following page) to 60 (or 180). Ms. Fiero's approach was to encourage them to recall "basic fact" that when multiplying by 10, they could simply add a zero.

The teacher's guide included the following suggestion: "For students who cannot visualize multiplying by tens, use a place-value chart to show the connection." Even though the place value charts were on the student page, Ms. Fiero did not refer students to these models once. Instead, she went through several examples, emphasizing the process of rewriting the values in equivalent forms and reiterating the meaning of the equal sign. As they practiced similar exercises on the following page, she announced, "Remember the equals sign is very, very important. Gets a bad wrap, it just hangs out there. But it tells you what you need to equal."

4.3.4.4 Analysis of Lesson 3.2

In the analysis, I focus on how Ms. Fiero perceived and leveraged elements in the teacher's guide and what she appeared to have missed. It is evident that she grasped the structural approach introduced in this lesson, seeing it as an opportunity to emphasize the meaning of the equivalence and as a precursor to algebra. When asked about this approach in the follow-up interview, she emphasized:

> I think that's something they should have further on down and I know they've learned about the equal sign, but they don't seem to understand the importance. And then you know when you go to algebra and you have a huge thing over on this side you know, I think that's going to be important.

Ms. Fiero expressed some surprise that her students had difficulty with this approach. She attributed their struggles to the use of the unfamiliar notation used by MiF. "The way they reword and set up problems differently really does throw them off." She did not appear to appreciate that students may have been struggling with the multiplicative meaning of 4 tens or the relationship between 4 × 10 and 4 tens. During the interview, Ms. Fiero was puzzled by this particular confusion. She indicated that students did work on place value in other grades and for several lessons at the beginning of fourth grade. She wondered if they were "still caught up and confused on how *Math in Focus* presents itself."

Ms. Fiero appeared to have missed an important element of the curricular sequencing built into the designed lesson. The 5-minute warm-up activity was intended to provide students with review and practice multiplying numbers by ten. This was one of the steps that students struggled with during the lesson. It is possible that, had she assigned students the warm-up in the guide, Ms. Fiero may have been alerted to their difficulty and made modifications to build the foundation for this task.

She also opted to not use or refer to the visual models (place value charts) on the student page to emphasize the relationship between 4 × 10 and 4 tens, even though she had marked it as something she would use in the CRL. When asked about this during the interview, she explained that she thought the representation used on the page was somewhat confusing.

4.4 Discussion

The analysis of lesson 3.2 in the MiF teacher's guide, alongside the analysis of how Maya Fiero read, interpreted, and used it to design and enact a lesson, illustrates and provides insight into several important aspects of teacher-resource interactions, the complexity of reading curriculum resources as a genre, and the craft involved in using them to design instruction. Using Ms. Fiero's interaction with the MiF excerpt, I discuss these aspects and consider the meaning of pedagogical design capacity (PDC).

The analysis of the lesson excerpt from the MiF teachers' guide illustrates several aspects of the curriculum resource genre of communication. First, it offers different types of information, which teachers are to read and interpret. The example illustrates how a complex set of mathematical concepts, tasks, and representations are intertwined with pedagogical suggestions and explanation. Further, these concepts and tasks are intentionally sequenced. Second, these forms are represented succinctly and often with limited transparency about their intent. Third, in this genre of communication, complex mathematical and pedagogical ideas are often communicated through straightforward directives, such as "Help students recall the strategy for multiplying a number by tens by working through the examples in the student book." These directives do not necessarily offer insight into their intent or the rationale behind them.

Because of the succinctness and general lack of transparency of this genre, teachers are expected to do significant interpretive work (Remillard and Kim 2017). The analysis of MiF suggests that these characteristics were especially problematic, placing greater demands on the teacher. The brief explication of Maya Fiero's interpretations of the resource revealed that some aspects of her PDC were more robust than others. Her grasp of the foundational mathematical ideas underlying the unfamiliar approach to multiplying by factors of ten appeared to be sufficiently strong. She saw the mathematical importance of restating the expressions in equivalent forms, the value of understanding the meaning of the equal sign, and the ways this approach was a precursor to algebraic manipulation. At the same time, she did not seem to appreciate the meaning of multiplication as iterating composite units; nor did she fully understand the developmental progression needed for children to work flexibly within multiplicative structures (Ulrich 2015). As a result, Ms. Fiero did not recognize or find value in other components of the lesson aimed at helping students understand the meaning of multiplying composite units of ten.

The areas where Ms. Fiero's understandings appeared to be more fragile are characteristic of what Ball et al. (2008) identify as *specialized content knowledge* and *knowledge of content and students,* because they involve forms of content knowledge deeply connected to how it is learned. These possible gaps in Ms. Fiero's understanding of the development of multiplicative reasoning may help to explain why she underestimated the challenge it would present to students and did not interpret the warm-up activity as a useful precursor to the focal tasks of the lesson.

It is important to emphasize that PDC is not a static capacity residing in individual teachers. It is mediated by characteristics of the resource. As I have argued, curriculum resources are layered and complex to use. Nevertheless, some curriculum authors provide more transparency and explication than others to guide teachers' interpretations and assist them in anticipating student difficulties (Davis and Krajcik 2005; Stein and Kim 2009). My analysis of the excerpt of the MiF teacher's guide raises questions about the extent to which the curriculum resource was designed to support teachers to activate their PDC. As Ms. Fiero pointed out in her interview, the teacher's guide provides extensive guidance on what the teacher might say or demonstrate during a lesson. At the same time, it communicates with teachers primarily through directing their pedagogical actions. It provides little in the way of transparency or other educative supports that might provide insights into the underlying mathematical ideas or design rationale.

4.5 Concluding Thoughts

The analysis of Maya Fiero's interpretation and use of the MiF excerpt underlines the complex characteristics of curriculum resources and illustrates that PDC is an interaction between affordances of the resource and the interpretive capacities of the teacher. These findings have implications for research on resource use and design. Studying or assessing PDC involves examining the teacher's interpretive interactions with the resource and accounting for characteristics that both the teacher and the resource bring to and leverage in the interaction. Here, I return to Ongstad's (2006) insights about the genre of curriculum resources to elaborate this process and its challenges. He argues that teachers' interpretive work is often mediated by familiar aspects of the genre, such as the common forms and structures used to communicate. These familiar forms conjure a "zone of expectation" that aid the interpretive process. Ongstad also posited that these familiar forms also support teachers' sense making when curriculum resources present novel components or approaches. In this light, recall that Ms. Fiero was in her second year of using MiF, and she described it as using a number of unfamiliar approaches and representations. The novel approaches, together with the lack of transparent supports in MiF may have constrained, rather than supported, her interpretive work. Her case raises an important question about resource design: Can curriculum resources be designed to support teachers' interpretive work with resources that use unfamiliar approaches and representations?

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Chapter 5
Disaggregating a Mathematics Teacher's Pedagogical Design Capacity

Moneoang Leshota and Jill Adler

Abstract The analytical approach for describing teachers' Pedagogical Design Capacity (PDC) adopted in this chapter is part of a larger study investigating mathematics teachers' use of a prescribed textbook. In this chapter, we describe teachers' PDC through (i) the type of use of the curricular resource, that is, whether use is deliberate or tacit; as well as, (ii) the type of a relationship the teacher forges with this resource. We find that a deliberate use of the textbook and an intimate relationship with the textbook reflects a high PDC. We argue that PDC is more than the degree of appropriation of the affordances of curricular resources by the teacher: it is also about the quality of opportunities for mediation of mathematics that the teacher creates.

Keywords Mathematics teachers' resources · Pedagogical design capacity
Omissions · Injections · Teacher-textbook relationship

5.1 Introduction

The present chapter derives from a study which investigated teachers' use of a prescribed mathematics textbook in South Africa. The motivation for the study arose while conducting preliminary classroom observations of teachers who were

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going to participate in a professional development programme.[1] While the programme was not on teachers' use of textbooks, the observations of how two teachers in two different schools interacted with their textbooks piqued our interest in teachers' relationship with their textbooks. For example, in one teacher's classroom, Grade 9 learners were instructed to do exercises from the textbook which required generating tables of values for the expressions, $H = (t-3)(t-3)$ and $K = t^2 - 6t + 9$, for different values of t. A question which followed the generation of tables of values compared the values of H and K for the same value of t to show learners that the two expressions were equivalent. However, after learners had completed the tables of values, the teacher told them to skip that particular question on comparing H and K, and moved on to the next set of exercises which was completely different from the ones learners had been doing. Our question was why the teacher omitted that particular question which in our opinion was critical to the understanding that the two expressions were equivalent. Ultimately, the exercise which was intended as an introduction to transformational algebra by the textbook authors ended up being about multiplying factors of H and a substitution exercise in the case of K.

In another lesson, on Variance at Grade 11, learners were given a set of data from which to calculate Variance whose formula the teacher wrote out as, $\text{Variance} = \sum \frac{(x_i - \bar{x})^2}{n}$. The set of data was written on the chalkboard by the teacher. We would later on observe that it was an exercise from a prescribed learner textbook, as learners started paging through the textbook for guidance on how to calculate Variance. The teacher complained that learners were taking too long and promised them a shorter method which he obtained from a workbook[2] in the cupboard. The formula the workbook used was given as, $\text{Variance} = \sum \frac{x_i^2}{n} - \bar{x}^2$. Some learners continued with the first formula while others utilised the latter formula. When learners called out their answers, the teacher observed that four different answers were provided by the learners, at which stage he asked the researchers which of those we thought were correct. We wish to point out that the two formulas are in fact correct and are both used to calculate the Variance. However our attention was once more called to the teacher's use of the textbooks: two seemingly different formulae for Variance whose difference was not explained by the teacher. Furthermore, the teacher had not done the exercise himself before assigning it as a class activity.

The two examples narrated above prompted an interest and consequently an investigation of the teacher-textbook relationships and the processes by which

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[1] The Wits Maths Connect Secondary Project (WMCS) is a research and development programme funded by the FirstRand Foundation (FRF), and the Department of Science and Technology offering a school-based professional development and research programme for mathematics teachers.

[2] We adhere to terminology of Taylor and Vinjevold (1999) who distinguish between textbooks and workbooks by defining textbooks as providing a systematic learning programme while workbooks providing supplemental and revision materials in support of the textbooks.

seven teachers from three different schools mobilised the affordances of the textbooks they were using. A multilayered analytical process was used to determine how teachers incorporated their textbooks and other resources in designing effective classroom episodes. Brown (2002, 2009) named this capability, *pedagogical design capacity* (PDC), which shall be described in detail in latter sections. The present chapter however focuses on only one of these seven teachers, whom we gave a pseudonym, Mpho. Mpho's case is used as an illustration of the analytical process of describing the teachers' PDC in the study in an effort to understand better the relationship between the teachers and their curricular resource.

We suggest that through our analysis of Mpho's lessons, the process of describing her PDC illuminates our understanding of the teacher resource relationship, with possibilities to inform policy and professional development programmes on teachers' use of curricular resources. We are however cognizant of the complex nature of this undertaking as evidenced in research on teacher–resources interactions (Choppin 2011; Remillard 2005).

5.2 Teacher–Resource Relationships

Investigations of teachers' PDC fall within the larger field of teachers' use of curriculum resources and the impact of these resources on teaching. Within this field, there are studies that investigate the teacher–resources interrelationships looking into the factors that influence the teacher resources interactions. Among them is Remillard's (2005) synthesis of over 25 years of research on curriculum use in mathematics. In this synthesis, Remillard suggested four different conceptualisations of curriculum 'Use' as: 'Use' as following or subverting the text; 'Use' as drawing on the text; 'Use' as interpretation of text; and 'Use' as participation with the text. Studies on teachers' PDC align with the conceptualisations of use as participation with text and that of use as interpretation of text. These two conceptualisations of use emphasise the participatory interrelationship between the teacher and the resource, as well as the interpretative nature of teachers' interactions with the resource. Teachers and resources are viewed as engaging in a dynamic interrelationship in which the teacher shapes the resource, and the resource in turn shapes the teacher while they both shape the outcome of instruction (Stein and Kim 2009). On the other hand, research recognises that teachers interpret the intentions of the authors of the resource to suit their classroom goals (Ben-Peretz 1990; Chavez 2003).

Other than the 'what' of the teacher resource interactions, there is also research on the 'how' of the interactions. On one hand, Brown (2002) studied the degree of use of a given science resource by teachers and suggested a continuum of three differential ways through which teachers engaged with resources and called them: *offloading, adapting*, and *improvising*. At the far opposite extremes of the continuum are *offloading* and *improvising* with *adapting* nested in the middle. When offloading, the agency for the delivery of content lies with the resource, whereas with improvising the agency lies with the teacher. Adapting reflects both teacher and resource agency equally in a lesson. Thus, the three processes illuminate the degree of appropriation (Wertsch 1998) of the affordances of the curriculum resource. However, a cautionary observation from Brown (2002) is that the categories do not necessarily correlate to teacher expertise or the quality of educational designs and therefore their occurrence may not be used as an indication of such.

In our study we have adopted these categories as we found them useful in the initial stages of analysis for providing information on how Mpho mobilised the resources available to her. Similar studies include those of Sherin and Drake (2009) in which the authors showed that teachers have three general approaches lying on a continuum too, when adapting the resource. They either *omit* components of a lesson, or *replace* one component with another, or completely *create* new components. Kim and Atanga (2014) studied teachers' decisions on whether to use, modify, omit, or make additions to the curriculum lesson, and concluded that these decisions have a bearing on opportunities for learning that teachers make available. The wider study from which this chapter has been drawn, examined the omissions from the textbook as well as the injections of content not available in the textbook (Leshota 2015; Leshota and Adler 2014), and concluded too that the types of omissions and injections teachers make have a bearing on the opportunities for mediation being opened up in the classroom.

The studies discussed above begin to illuminate teachers' pedagogic actions that are part of their PDC. Analysing these actions thus highlights important elements of the teachers' capacity to utilise their resources in ways that open up opportunities for mediation in the classroom. The next section provides a detailed description of the notion of teachers' PDC.

5.2.1 The Notion of Teachers' Pedagogical Design Capacity

Teachers' pedagogical design capacity (PDC) coined by Brown (2002, 2009) is a theoretical construct. It describes teachers' unique skill of perceiving the affordances of a resource, and reflects teachers' ability of creating "deliberate, productive designs" (Brown 2009, p. 29). In the teacher-curriculum interaction framework (Remillard 2005) illustrated in Fig. 5.1, PDC falls among the features which the teacher brings to the interaction.

It is worth noting from Fig. 5.1 that PDC is different from teacher knowledge, that is, from pedagogical content knowledge (Shulman 1986), or subject matter knowledge. PDC is not what a teacher "has", like knowledge, but characterises *a process* by which the teacher utilises their knowledge and other features together with features of the resource to design instruction for students. It is therefore more than teacher knowledge; it is about what teachers are capable of doing with that knowledge to 'craft' (Brown 2002, 2009) classroom episodes. Thus, teachers need to be able to recognise and understand the affordances and constraints of available

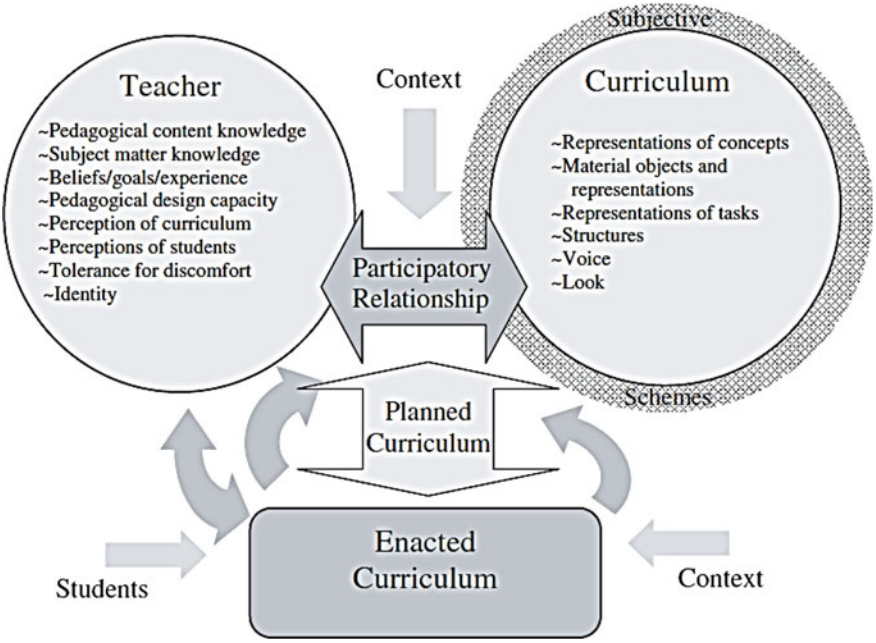


Fig. 5.1 Framework of components of teacher-curriculum relationship. Taken from Remillard (2005, p. 235)

resources, and weave these with their personal capabilities to generate episodes that create opportunities for mediation in the classroom.

Teachers' PDC hence, depends on two major aspects. Firstly, on the teacher's capacity to perceive needs and opportunities in their classrooms; and secondly, on the teachers' capacity for opening up opportunities for mediation with the available personal and external resources. Thus we expect each teacher's PDC to have specificity; reflecting her preferences, her context, and her understanding of different features of the resources. In describing Mpho's PDC in this chapter, our aim is to understand her pedagogic design capacity as reflected in her actions. We achieve this aim through identifying the patterns by which Mpho mobilises the affordances and constraints of available resources together with her personal capabilities to open up opportunities for mediation in her classroom. We operationalise Mpho's PDC by considering the content she omits from the textbook (or other curricular resources she utilises), the content she injects into the lesson that is not available from these curricular resources, and how these omissions and injections open or close the opportunities for mediation.

Before we move on to elaborating the overarching theoretical framing in the chapter, we have found it prudent to provide a background of textbook use and textbook availability in the South African mathematics education context, and thus some of the contextual conditions within which Mpho and other teachers in our study worked.

5.2.2 Mathematics Textbooks in South Africa

In South Africa, textbooks undergo a process of approval by the Department of Basic Education, with the major criterion for successful approval being alignment to the curriculum statement. This implies hence that a textbook approved for a particular grade represents the approved official curriculum in terms of the grade specific curriculum and the expected sequencing of topics. The context is such that teachers are under considerable pressure for curriculum coverage, and so usually align their goals with those in the prescribed textbook.

For the majority of teachers in South Africa, the print textbook is still the most accessible teacher resource, and in some cases, the only teacher resource. The state provides textbooks for learners and teachers in government schools even though in schools with large numbers of learners, it is not uncommon to find up to six learners sharing one textbook. A timetable would be drawn that showed who got the textbook, and when. Approved textbooks are highly regarded as a useful learning resource for school in South Africa. To illustrate this assertion, we recall an incident in 2012 where government was taken to court by nongovernmental organisations for failing to deliver textbooks to schools on time, in what was to become known as the "Limpopo Saga". In the Limpopo province, it was discovered that textbooks had not been delivered to schools six months into the beginning of the school year. An uproar by parents, teachers and NGO's prompted a parliamentary commission of inquiry with human rights organisations conducting their own investigations into the matter. The Minister of Basic Education had to make presentations to a parliamentary committee, and the matter ended up in the High Court where government was directed to make arrangements for a six-month 'catch-up' plan for the affected learners. The assumption here seemed to be that if there were no textbooks, then there was no learning. Textbooks delivery has since become a burning national issue and government is called upon to account for delivery of textbooks on a yearly basis.

The textbook package usually consists of a learner textbook and a teacher manual, and sometimes additional exercises on CD. The teacher manual typically only includes answers to questions in the learner book. This means that for purposes of teaching, teachers use the learner book as it includes mathematics explanations, tasks and exercises. In the schools where this particular study took place, the project provided some two hundred (200) textbooks to all learners and teachers in participating classrooms to curb the issue of unavailability. Furthermore, the present study took place during the period when a new curriculum was being implemented in South Africa, with new textbooks being developed. It was possible to obtain access to these new materials for teachers in the project. More details will be provided in later sections. At the same time, WMCS had begun the professional development programme for teachers and provided additional handouts for teachers through the programme's workshops. Some teachers also utilised other textbooks besides the prescribed textbook for teaching. Therefore while the study was about the teachers' relationship with the prescribed textbook, with respect to the teachers'

PDC it also considered the other resources which teachers mobilised for teaching. We shall see later in the chapter, that Mpho actually utilised the workshop materials in one of her lessons and not the prescribed textbook.

5.2.3 Theoretical Considerations

Vygotsky's (1978) Sociocultural theory wherein all humans are inherently social beings and grow from and through the use of tools, provides the overarching theoretical framework of this study. In this framework, cultural artefacts and tools mediate the relationship between subject and object, and as Wertsch (1991) argues, the cultural tools that are employed in mediated action are the key to understanding the relationship between sociocultural settings and human action. In appreciating this "agent-acting-with-mediational-means" (Wertsch 1998, p. 24) theory, we are forced to "go beyond the individual agent when trying to understand the forces that shape human action" (ibid) and to focus on the agent-instrument dialectic. In the teacher–resource interactions studies, whether following the theory of *documentational genesis* (Gueudet and Trouche 2009) or the *interpretation of and participation with resources* (Remillard 2005), analysis emphasises on one hand the affordances (Gibson 1977) of the resource, and on the other hand how the teacher appropriates (Wertsch 1998) these affordances and constraints (Norman 1999) of the resources.

We have already mentioned that our study aligns with the *interpretation of and participation with resources theory* which conceptualizes the teacher-curriculum relationship as a "dynamic interrelationship" involving participation of both the teacher and the curriculum, and where the teacher and resource shape each other and both shape the instruction (Remillard 2005). Remillard, in reviewing studies on teacher-curriculum relationships highlights the participatory relationship between the teacher and curriculum as a significant construct in greater understanding of these relationships. The participatory relationship views teachers as 'active' designers of the curriculum (Brown 2009; Pepin et al. 2013; Remillard 2005) and not just mere conduits of the authors. Curriculum materials are regarded as artefacts (Brown 2009; Vygotsky 1978; Wertsch 1998) with affordances and constraints on teachers' activities, while the activities are conceptualized as design processes (Brown 2002, 2009) where teachers use these resources in specific ways. This 'specificity' of use illuminates each teacher's design capacity in selecting the resource; in adapting it to suit the teacher's classroom needs, and in utilizing it 'as is' depending on her classroom needs. However, it does highlight the complex nature of studies such as this where there are many elements that influence each teacher's decision making process including the teacher's own beliefs.

How then is the teacher-curriculum materials relationship to be observed and described? In the next section we outline our methodology with a focus on what is pertinent for this chapter. Inevitably, we touch on how this methodology informed the larger study.

5.3 Methodology

5.3.1 Data Collection

The larger study involved seven Grade 10 teachers participating in a professional development programme. All teachers used the same prescribed textbook and were observed teaching a topic on functions. The textbook is one of the most popular in South Africa and a textbook of choice according to the teachers. The curriculum requirements as reflected in the prescribed textbook included three main components: (i) introducing graphs of the quadratic function, the hyperbola, the exponential function, and the three basic trigonometric functions, sine, cosine, and tangent; (ii) determining the properties of these functions; (iii) performing horizontal transformations and compressions $g(x) = af(x) + q$, and observing the effect of the parameters a and q on the parent graphs; and then (iv) interpreting functions, including determining equations of given graphs and sketching the graphs.

The study took place during an unstable phase of curriculum policy in South Africa. Specifically, as data collection was about to begin, a curriculum review process was concluded leading to the reformation of the existing NCS curriculum (National Curriculum Statement) into the present curriculum that came to be known as the CAPS curriculum (Curriculum and Assessment Policy Statement). The review and reformation was officially described as a refinement and not radical change, and in mathematics, textbooks were simultaneously being revised so as to be available for evaluation for approval by the department of basic education. The NCS aligned prescribed textbook which Mpho and the other teachers were using was still available in their class. The author who wrote the existing chapters on functions was also the author of the ‘same’ chapters in the new edition. Before commencing on the classroom observation we got hold of the author who organized a two hour workshop for the teachers, mainly to make visible her design rationale (Ball and Cohen 1996; Davis and Krajcik 2005; Stein and Kim 2009) in the new edition. Mpho and the other teachers were also provided with the draft copies of the new edition to use as they saw fit. Due to the high regard for the textbook series in the country, it was expected by teachers that the new edition would also pass the evaluation by the department of basic education, which it eventually did.

Data collection for the study also coincided with a workshop organized by the WMCS project on teaching functions for the teachers in the project schools. The teachers were furnished with handouts from this workshop to utilize as they saw fit. All these materials were collected for documentation and analysis in the study.

Three lessons on functions were observed and video-recorded in Mpho’s class where all learners had been provided with the prescribed textbook by the WMCS project. As noted, Mpho had access to the latest edition of the textbook as well to use as she saw fit.

In addition to lesson recordings, a post interview with each teacher took place some months after the lessons were taught. The post interview was designed to probe teachers’ interpretations of the affordances of their textbooks, with specific questions related to the lessons they had taught on functions, but in an atmosphere that was distanced from the observations. Unfortunately, despite coaxing teachers' reflections on their use of the curricular resources, the post interviews did not move beyond general information. For example, the teachers said that they used the textbook for preparing lessons and assigning homework or class activity. They did not talk specifically about how they did this for specific lessons. The interviews thus did not provide post hoc insight into teachers' goals and intentions, and did not function as intended for further illuminating teachers' PDC in relation to their lessons. In the wider study, we only drew on the interviews to support our interpretations across teachers of their awareness of their textbook affordances. We return to this point when we discuss limitations of our study. We note this here to flag up for readers that the descriptions we build of teachers' PDC relied on their observable actions in their classroom lessons.

The data set for this chapter thus included the two editions of the prescribed textbook, the workshop handouts, and the video recordings of the three lessons observed. The lesson observations were all transcribed before the commencement of data analysis.

5.3.2 Data Analysis

The process of data analysis of the larger study entailed two main phases. In the first phase, the prescribed textbook was analysed for its affordances to the teachers' classroom. This process while it does not form part of the present chapter, became instrumental in the second phase of analysis when we were determining how the teachers mobilized these affordances. Each lesson was chunked into analysable episodes and the episodes were subjected to a process of coding for determining appropriate themes to determine how teachers mobilized their curricular resources.

5.3.2.1 Determining Textbook Affordances

The process of analysis for the larger study began with a thorough and detailed analysis of the affordances of the prescribed textbook in line with the *interpretation of and participation with resources theory* (Remillard 2005). In the analysis we looked for three main aspects: (i) content areas or subthemes covered under the topic of grade 10 functions; (ii) how the content was presented and sequenced; (iii) the embedded instructional approach of the textbook, and (iv) the conception of function that the textbook conveyed, that is, whether pointwise or global or a progression from pointwise to global strategies (Even 1998). This analysis does not form part of this chapter, and therefore we shall not elaborate deeply on it. However, it was important to note the four main subthemes expected to be taught and their sequencing as follows: *Introduction to function notation and terminology; determining properties of functions; interpreting functional properties;* and, *transformation of functions*. These results made it possible for us to compare the content and sequencing of Mpho's lessons to those of the textbook to be able to determine how much of the textbook had been utilised in Mpho's lessons. The results would also enable us to determine what the teacher omitted from, or injected to the textbook content. We wish to reiterate the context for this study where an approved textbook is regarded as being representative of the official curriculum and therefore the goals of the textbooks can be expected to align with goals in the classroom.

5.3.2.2 Determining Degree of Appropriation

The first step in the analysis of the lessons was to utilise Brown's (2002, 2009) scale of artefact appropriation. By comparing Mpho's lesson content and its sequencing with that of the textbook we were able to determine whether the teacher was *offloading* or *adapting* or *improvising* the textbook content in each lesson. This provided an indication of whether or not the teacher utilised the prescribed textbook; and if they did, how much had been utilised. Where Mpho utilised curricular resources other than the prescribed textbook, we were also able to determine which those were.

5.3.2.3 Identifying Omissions and Injections

It is important to mention at this point that the main aim of the analysis of the lessons was to investigate how teachers used the prescribed textbook, especially in the classroom. However, after determining the degree of use as outlined above, we could not fully describe Mpho's PDC with the information at hand. At this stage we could only say how much or how little of the textbook Mpho had used, but the analysis could not help us say what it meant for the opportunities for mediation opened up in the classroom.

While we were determining the degree of textbook use by Mpho in the lessons, we were at the same time noting elements of content which were omitted by Mpho, and those which were not in the textbook but which Mpho inserted into her lessons. For example, the table for analysis of her first lesson was as follows:

The third column of Table 5.1 was used for any observations which we deemed important, and what started emerging was that there was textbook content which Mpho would omit, while at the same she would insert some content which was not available in the textbook. This led to the emergence of new analytical constructs. We named the content that Mpho omitted from her lessons as "Omissions". It was at this stage of analysis that we found we needed to make a distinction between content that was improvised and content that was newly inserted.

We described "Improvisations" as the content that was required at the grade level by the curriculum, and which therefore was available in the prescribed textbook, but which the teacher decided to bring from a different resource other than the

Table 5.1 An example of the process of analysis for lesson one

Episode	Degree of appropriation	Comments
Function notation	Offloaded	Mpho uses the CAPS textbook for definition
Worked examples	Offloaded	An omission: two other worked examples from textbook omitted in the lesson
The vertical line test for functions	Not in textbooks	An injection: the vertical line test for distinguishing between functions and non-functions inserted into the lesson
Practice exercises	Offloaded	Practice exercises do not include questions on the vertical line test

prescribed textbook. The "Injections" (or insertions) we then described as all content which the teacher introduced into the lessons but which was not specifically required by the curriculum at the particular grade level. This content would not be available in the prescribed textbook. In our analysis, the *omissions* and *injections* became the first indication of the teachers' ability to mobilise curricular resources and a demonstration of the "creative and constructive dimensions of teachers' instructional capacities" (Brown 2009, p. 29). We took the analysis to a new level where we further categorised the *omissions* and *injections*.

5.3.2.4 Categorising Omissions and Injections

Leshota (2015) building on our earlier work (Leshota and Adler 2014) categorizes injections into *robust* and *distractive*. *Robust injections* referred to injections which were enhancing the content while *distractive injections* led to erroneous mediation. Omissions were categorized into *productive*, for those that did not detract from the opportunities for mediation, versus *critical omissions*, that is, "those aspects of the object of learning that are critical to its mediation that teachers omit from the lessons" (Leshota 2015, p. 96). The question at this point would be how the researchers decided what was critical and what was not critical. We have already mentioned that there is a process of selection and approval of prescribed textbooks in South Africa by the Department of Basic Education. The textbooks have to align themselves with the curriculum statement, but more than that, they align with the expected teaching sequence in each grade as well. If a particular textbook is approved as a prescribed textbook in a certain grade, then it means the textbook has satisfactorily sequenced the topics in a way teachers are expected to teach in that grade, but most importantly, the textbook prescribes the minimum requirement on content expected in that classroom.

In the analysis hence, if the teacher omitted content that was outlined as a minimum requirement in the textbook, and coupled with our own knowledge of mathematics, we were able to determine whether such an omission would be critical to the opportunities for learning the particular aspect of the topic or not. We also thought about what if the teacher omitted the particular aspect at that time but could still pick it up at a later date? We resolved that while this was a possible scenario, our focus was on the actions at the time in the classroom. We wanted to find out what was the space for learning the teacher was opening.

The constructs of *omissions* and *injections* thus emerged as the result of the interaction between theoretical resources and the empirical texts, that is, the lessons themselves, in this study. We used these constructs to develop an analytical framework that illuminates types of teacher–resource relationships at play. The relationships were then used to evaluate Mpho's PDC, and PDC for the rest of the teachers in the study.

We now elaborate on the teacher–resource relationships which we deduced from analysing the *omissions* and *injections* in Mpho's lessons.

5.3.2.5 Classifying Teacher–Resource Relationships

Our definition of *robust injections*, that is, injections that are enhancing, suggests an existence of a relationship between the teacher and the resource. This is a teacher who is able to mobilize the resource productively. Similarly, we infer that a teacher who omits content that does not detract from what is being learned has some prior transaction with the textbook. We argue that the presence of *robust injections* and *productive omissions* (omissions that are not distractive) indicate deliberateness in the teacher's use of the resource. We further argue that *deliberateness* reflects a *participatory* resource use, and we describe the relationship that is forged between the teacher and resource in this case as an *intimate relationship*. If the teacher–resource relationship is *intimate*, then the teacher's capacity for pedagogic design, her *PDC is high*.

On the other hand, the teacher who *omits critical* content that is available in the resource raises questions about her interaction with the resource. We argue that this is a reflection of a non-participatory and therefore non-deliberate interaction with the resource. We have referred to this kind of resource utilisation as *tacit* (Polanyi 1967), and argue that relationships of this kind lack intimacy and imply *low* levels of PDC.

We draw similar conclusions for *distractive injections*: injecting content that is not enhancing and which may lead to erroneous mediation, while it might say something about the teacher's subject matter knowledge (SMK) (Shulman 1986), is indicative of a *tacit* use of the resource. We consequently conclude that the presence of *critical omissions* or *distractive injections* in a lesson suggest low PDC.

The categorizations of omissions and injections, their implications for resource use, and subsequent teacher–resource relationships forged are summarized in Table 5.2.

Table 5.2 shows that an *intimate* teacher–resource relationship, and consequently high PDC is produced from the combination of *robust injections* and *productive omissions* only. The rest of the combinations result in tacit textbook use, teacher–resource relationships which are not intimate, and PDC levels that are not

Table 5.2 Relating omissions and injections with teacher-textbook relationships

	Robust injections	Distractive injections
Productive omissions	Deliberate and participatory textbook use	Tacit textbook use
	Intimate teacher–resource relationship	Teacher–resource relationship not intimate
	High PDC	PDC not high
Critical omissions	Tacit textbook use	Tacit textbook use
	Teacher-textbook relationship not intimate	Teacher-textbook relationship not intimate
	PDC not high	PDC low

high. The combination of *critical omissions* and *distractive injections* indicate low levels of teachers' PDC.

Thus, the constructs of *omissions* and *injections* as these emerged through the analytic process in the wider study provide this chapter with indicators for *deliberate* versus *tacit* resource use; and *intimate* teacher–resource relationships versus relationships that lack intimacy. In the next section, we outline the results from the analysis of Mpho's three lessons.

5.4 Results

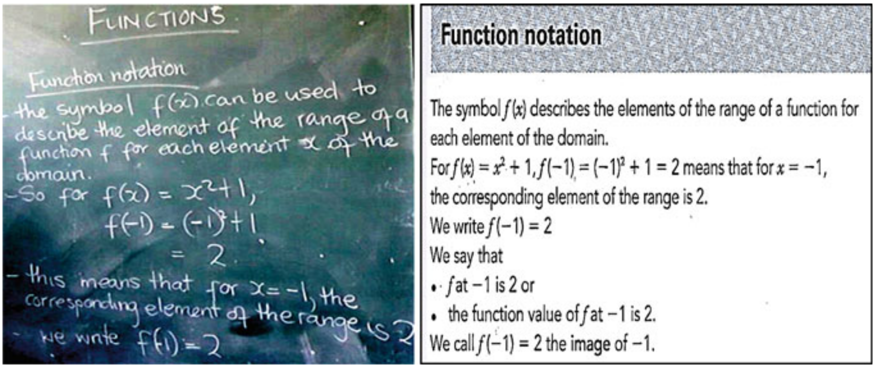
5.4.1 Mobilization in Lesson One

5.4.1.1 Degree of Appropriation

In the first lesson, Mpho introduced the functional notation, $f(x)$, and how to evaluate function values if given a function, $f(x)$. The lesson began with Mpho copying notes from the textbook page to the chalkboard before she explained them. Figure 5.2 shows the notes Mpho had copied alongside the textbook page she was copying from.

Figure 5.2 shows that Mpho copied the notes from the textbook almost word for word. After explaining how to evaluate the function value, $f(-1)$ given that $f(x) = x^2 + 1$, Mpho gave learners an example which she worked on the chalkboard guiding the learners on the procedure for evaluating $f(3), f(-3)$, and $f\left(-\frac{3}{2}\right)$ given the function $f(x) = 2x - 3$. This example is shown in Fig. 5.3 alongside a textbook page of exercises for evaluating function values.

Figure 5.3 shows that Mpho has used the same example from the textbook as well. So far, the two examples from the textbooks have been used by Mpho in her lesson. As learners continued to write in their books, Mpho wrote the following notes on the chalkboard, and this time she was not copying from a book or any other resource.



FUNCTIONS

Function notation

- the symbol $f(x)$ can be used to describe the element of the range of a function f for each element x of the domain.
- So for $f(x) = x^2+1$,

$f(-1) = (-1)^2+1$

$= 2$

- this means that for $x=-1$, the corresponding element of the range is 2
- we write $f(-1) = 2$

Function notation

The symbol $f(x)$ describes the elements of the range of a function for each element of the domain.

For $f(x) = x^2 + 1$, $f(-1) = (-1)^2 + 1 = 2$ means that for $x = -1$, the corresponding element of the range is 2.

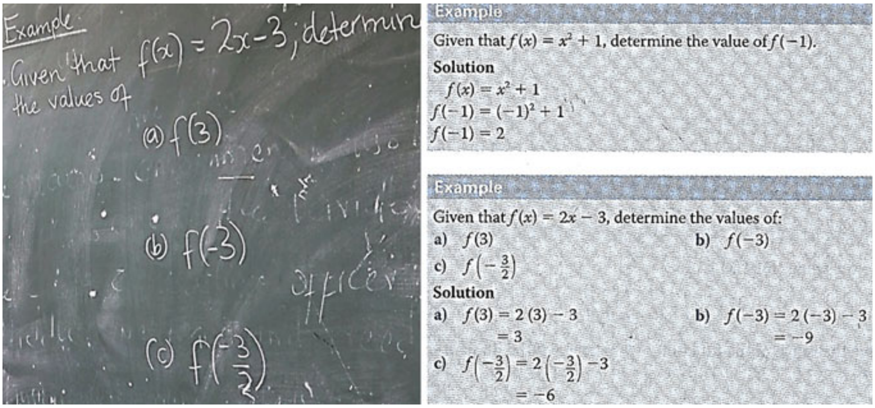
We write $f(-1) = 2$

We say that

- f at -1 is 2 or
- the function value of f at -1 is 2.

We call $f(-1) = 2$ the image of -1.

Fig. 5.2 Mpho's notes in lesson one alongside a textbook page



Example:

Given that $f(x) = 2x-3$, determin

the values of

(a) $f(3)$

(b) $f(-3)$

(c) $f(-\frac{3}{2})$

Example

Given that $f(x) = x^2 + 1$, determine the value of $f(-1)$.

Solution

$f(x) = x^2 + 1$

$f(-1) = (-1)^2 + 1$

$f(-1) = 2$

Example

Given that $f(x) = 2x - 3$, determine the values of:

a) $f(3)$ b) $f(-3)$

c) $f\left(-\frac{3}{2}\right)$

Solution

a) $f(3) = 2(3) - 3$
$= 3$

b) $f(-3) = 2(-3) - 3$
$= -9$

c) $f\left(-\frac{3}{2}\right) = 2\left(-\frac{3}{2}\right) - 3$
$= -6$

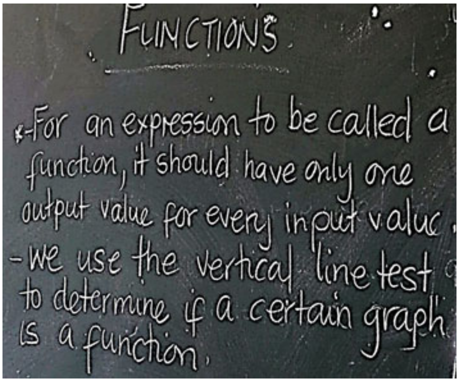
Fig. 5.3 An example in lesson one alongside examples from the textbook page

The information in Fig. 5.4 about the *Vertical line test* which is used for determining whether graphs in the Cartesian plane represent functions or not, is not available in the textbook. Mpho spent some few minutes demonstrating on the chalkboard to learners how the vertical line test was used, after which she assigned a class activity, shown in Fig. 5.5a. We noted that up to this point, the vertical line test had been the only aspect which Mpho had not extracted from the textbook.

Mpho wrote the class activity on the chalkboard for learners to work on individually as she moved around helping them.

The exercises from Fig. 5.5a above were extracted from two different exercises from the textbook, shown in Fig. 5.5b.

Figure 5.5b shows that Mpho took questions 1 a), b), and c) from Exercise 7.1 of the textbook and question 1 of Exercise 7.2 to design a class activity.



FUNCTIONS.

-For an expression to be called a function, it should have only one output value for every input value.
-We use the vertical line test to determine if a certain graph is a function.

Fig. 5.4 Notes on the vertical line test

All in all, in lesson one, all content which Mpho used except for the vertical line test, she had extracted from the textbook. We concluded therefore that lesson one was an *offloaded lesson* because only a small part of that lesson did not come from the textbook.

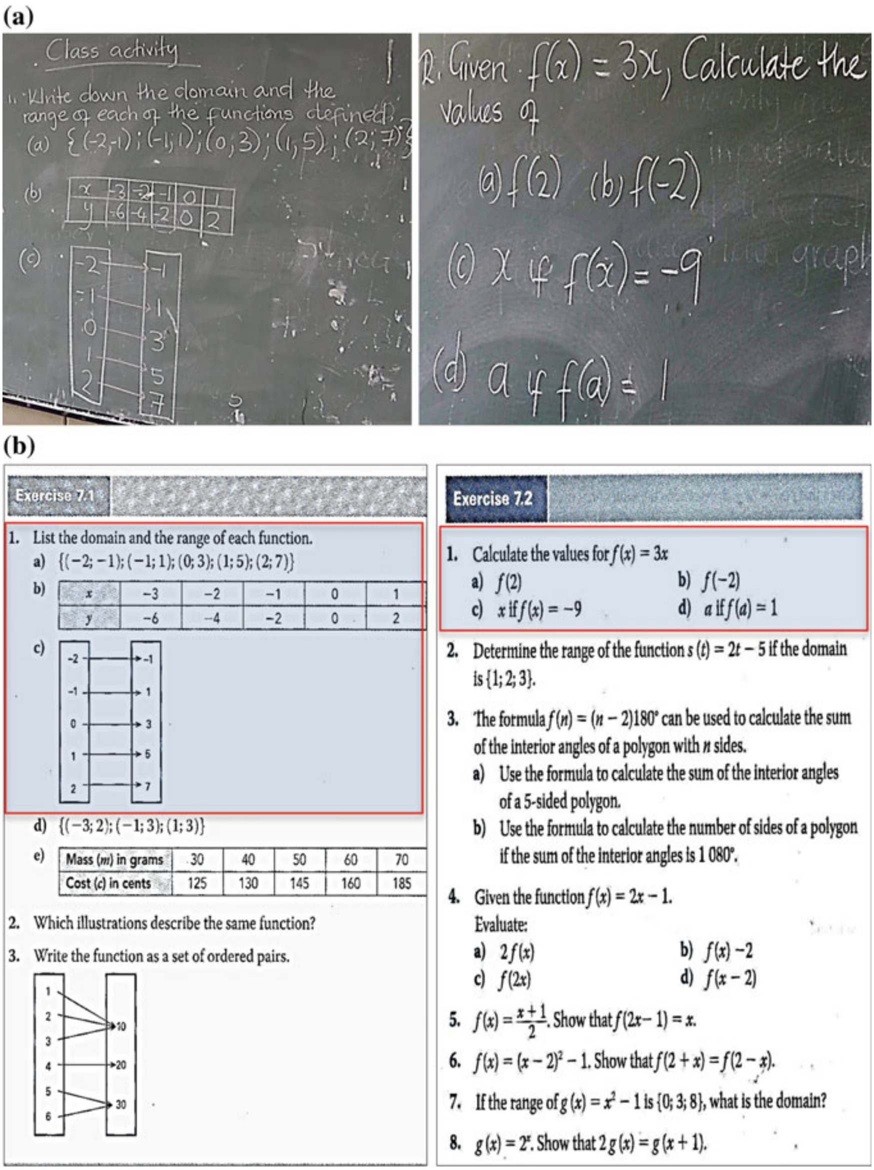
5.4.1.2 Omissions and Injections

Injections

We begin with injections in this lesson because there were very few of them. The only injection in the lesson is that of the *vertical line test*, as we have already shown. It is considered as an injection instead of an improvisation because it is not stipulated in the textbook, and therefore its absence from the prescribed textbook indicate that it is not regarded as part of minimum requirements of the curriculum. However, the vertical line test is regarded as an important aspect of the teaching and learning of functions which is featured in many textbooks on functions. It is used as a visual means to distinguish between graphs in the Cartesian plane which represent functions and those which do not. In fact, it was featured in the workshop on functions organised and conducted by the WMCS for teachers in the project schools which Mpho also attended. We therefore regard Mpho's inclusion of the vertical line test as enhancing to the learning of functions, and categorise it as a *robust injection* hence.

Omissions

Even though this lesson was largely composed of content from the textbook, we observed that Mpho did not utilize all the content which the textbook had provided. As Fig. 5.5b shows, in Exercise 7.1 Mpho omitted questions 1 d), e), question 2 and question 3. In Exercise 7.2, she only used question 2 and omitted all the other seven



Class activity

1. Write down the domain and the range of each of the functions defined.

(a) $\{(-2,-1);(-1;1);(0,3);(1,5);(2;7)\}$

(b)

x	-3	-2	-1	0	1
y	-6	-4	-2	0	2

(c)

2. Given $f(x) = 3x$, Calculate the values of

(a) $f(2)$ (b) $f(-2)$

(c) x if $f(x) = -9$

(d) a if $f(a) = 1$

Exercise 7.1

1. List the domain and the range of each function.
   a) $\{(-2; -1); (-1; 1); (0; 3); (1; 5); (2; 7)\}$
   b)

x	−3	−2	−1	0	1
y	−6	−4	−2	0	2

   c)

   d) $\{(-3; 2); (-1; 3); (1; 3)\}$
   e)

Mass (m) in grams	30	40	50	60	70
Cost (c) in cents	125	130	145	160	185

2. Which illustrations describe the same function?
3. Write the function as a set of ordered pairs.

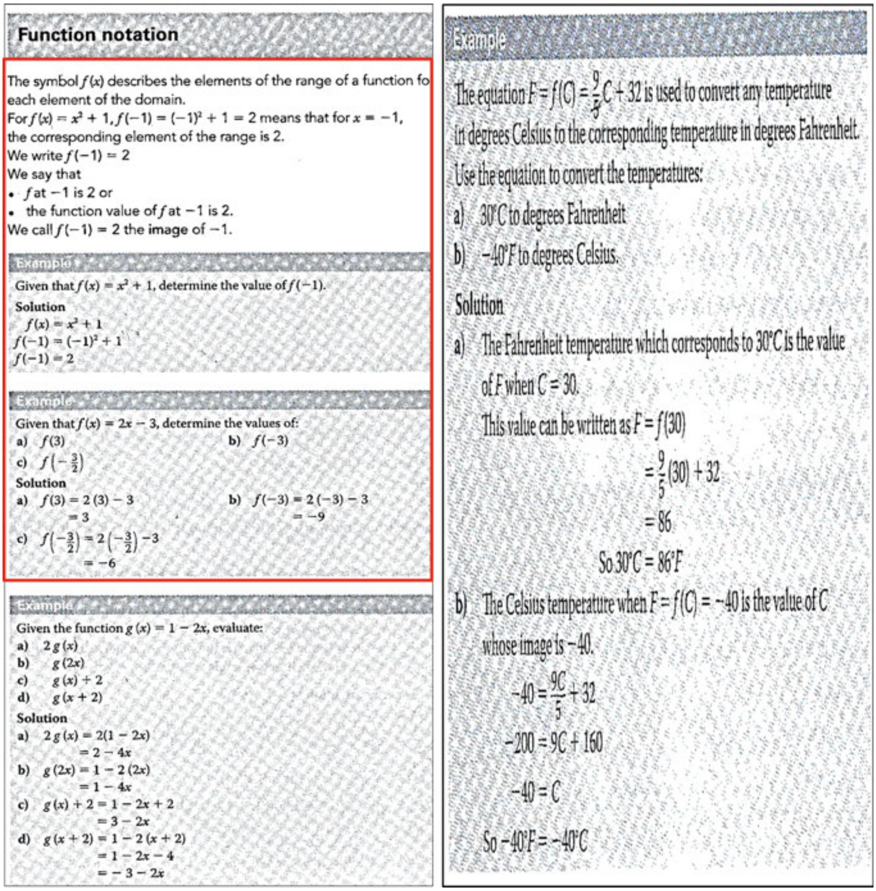
Exercise 7.2

1. Calculate the values for $f(x) = 3x$
   a) $f(2)$ b) $f(-2)$
   c) x if $f(x) = -9$ d) a if $f(a) = 1$
2. Determine the range of the function $s(t) = 2t - 5$ if the domain is $\{1; 2; 3\}$.
3. The formula $f(n) = (n - 2)180°$ can be used to calculate the sum of the interior angles of a polygon with n sides.
   a) Use the formula to calculate the sum of the interior angles of a 5-sided polygon.
   b) Use the formula to calculate the number of sides of a polygon if the sum of the interior angles is 1 080°.
4. Given the function $f(x) = 2x - 1$.
   Evaluate:
   a) $2f(x)$ b) $f(x) - 2$
   c) $f(2x)$ d) $f(x - 2)$
5. $f(x) = \frac{x+1}{2}$. Show that $f(2x - 1) = x$.
6. $f(x) = (x - 2)^2 - 1$. Show that $f(2 + x) = f(2 - x)$.
7. If the range of $g(x) = x^2 - 1$ is $\{0; 3; 8\}$, what is the domain?
8. $g(x) = 2^x$. Show that $2g(x) = g(x + 1)$.

Fig. 5.5 **a** Class activity in lesson one. **b** Textbook exercises from where Mpho developed the class activity

questions. In fact, these were not the only omissions in the lessons as indicated in Fig. 5.6.

In the lesson, Mpho omitted the last example indicated in Fig. 5.6 on the left diagram on evaluating functions $2g(x); g(2x); g(x) + 2;$ and $g(x + 2)$ given that



Function notation

The symbol $f(x)$ describes the elements of the range of a function fo each element of the domain.
For $f(x) = x^2 + 1, f(-1) = (-1)^2 + 1 = 2$ means that for $x = -1$, the corresponding element of the range is 2.
We write $f(-1) = 2$
We say that

- f at -1 is 2 or
- the function value of f at -1 is 2.

We call $f(-1) = 2$ the image of -1.

Example

Given that $f(x) = x^2 + 1$, determine the value of $f(-1)$.

Solution

$$f(x) = x^2 + 1$$
$$f(-1) = (-1)^2 + 1$$
$$f(-1) = 2$$

Example

Given that $f(x) = 2x - 3$, determine the values of:

a) $f(3)$ b) $f(-3)$
c) $f\left(-\frac{3}{2}\right)$

Solution

a) $f(3) = 2(3) - 3 = 3$
b) $f(-3) = 2(-3) - 3 = -9$
c) $f\left(-\frac{3}{2}\right) = 2\left(-\frac{3}{2}\right) - 3 = -6$

Example

Given the function $g(x) = 1 - 2x$, evaluate:

a) $2g(x)$
b) $g(2x)$
c) $g(x) + 2$
d) $g(x + 2)$

Solution

a) $2g(x) = 2(1 - 2x) = 2 - 4x$
b) $g(2x) = 1 - 2(2x) = 1 - 4x$
c) $g(x) + 2 = 1 - 2x + 2 = 3 - 2x$
d) $g(x + 2) = 1 - 2(x + 2) = 1 - 2x - 4 = -3 - 2x$

Example

The equation $F = f(C) = \frac{9}{5}C + 32$ is used to convert any temperature in degrees Celsius to the corresponding temperature in degrees Fahrenheit.
Use the equation to convert the temperatures:

a) 30°C to degrees Fahrenheit
b) −40°F to degrees Celsius.

Solution

a) The Fahrenheit temperature which corresponds to 30°C is the value of F when $C = 30$.
This value can be written as $F = f(30)$

$$= \frac{9}{5}(30) + 32$$
$$= 86$$

So 30°C = 86°F

b) The Celsius temperature when $F = f(C) = -40$ is the value of C whose image is -40.

$$-40 = \frac{9C}{5} + 32$$
$$-200 = 9C + 160$$
$$-40 = C$$

So −40°F = −40°C

Fig. 5.6 Worked examples from the textbook

$g(x) = 1 - 2x$. She omitted the question on evaluating functions based on temperatures on the right diagram of Fig. 5.6 as well.

We begin the categorization of the omissions with these worked examples.

Omission of $f(x)$ as an entity in its own right

In the worked example on the left diagram in Fig. 5.6 which Mpho omitted, the function $g(x)$ is regarded as an object that can be operated on in its own right, for example, as in adding a number 2 to the object, or multiplying the object by 2. This example is different from the other examples and exercises Mpho was doing with learners in the classroom. In the classroom, the input value was given as an integer, for example, -1, and through a process of substituting x for the given integer, a function value, which was another integer was obtained. With the omitted example came a new concept where there was no substitution of integers, and the outputs were no longer integers but other functions.

We observed further that Mpho actually omitted all similar questions in the practice exercises of Exercise 7.2 (see Fig. 5.5b).

Looking at what Mpho was doing in the lesson, that is, getting learners to substitute a value for the input given by x in the equation, and then obtaining another integer as the output, the omission of these examples did not detract from what was being learned in the lesson. In order to introduce this new aspect, for example, $2g(x)$, Mpho would need to bring in the aspect of transformations of functions, which actually, came later in the curriculum after determining the functional properties. We therefore regarded not using these examples in the present lesson as a productive omission. It is however worth noting that Mpho did not completely ignore these kinds of examples as they were part of her second lesson.

Omission of 'applied' functions

The worked example on the right diagram in Fig. 5.6 used a real life application of functions involving temperature. We termed functions of this type as 'applied' functions in the analysis. This worked example and a similar question (question 3, Exercise 7.2) in Fig. 5.5b were omitted from the lesson by Mpho. Question 3 in Exercise 7.2 expressed the sum of interior angles of a polygon, as a function of the number of its sides, n, that is, $f(n)$. We considered the 'applied' functions as particular types of questions which served a particular purpose of using mathematics to understand phenomena in the real world. In another 'applied' example (see Fig. 5.5b) the input of a function was expressed in terms of the Mass in grams, and the output as the Cost in cents. We argue that while the application of mathematics to real life is important, as is mostly desired and encouraged in the teaching and learning of mathematics, in this particular lesson of introducing function terminology and notation, it was not critical in that its omission did not detract from the opportunities for mediation. As in the case of function as an entity which Mpho omitted, we regarded this type of omission as well, as being *productive*.

Omission of questions with similar structure

At the same time, we observed that question 1 e) in Exercise 7.1 was similar in structure to 1 b) which was assigned in the class activity, the difference being that in 1 b), the inputs were represented by the variable, x and the outputs by the variable, y. Similarly, question 1 a) and 1 d) had the same structure, and therefore omitting 1 d) did not detract from the opportunities for mediation. Learners had done similar things in question 1 a). These omissions were once more, *productive* omissions as opposed to being distractive.

In summary, lesson one was an *offloaded lesson* in which *robust injections* occurred and the *omissions* made were all *productive*.

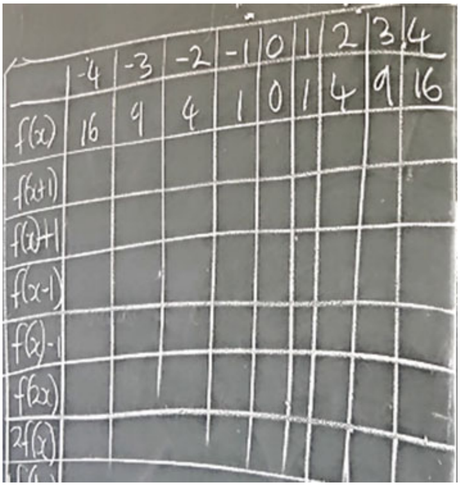
5.4.2 *Mobilization in Lesson Two*

5.4.2.1 Degree of Appropriation

In the second lesson, Mpho did not use the content from the prescribed textbook, but drew content from a workshop on grade 10 functions organized by the WMCS project for all teachers in the project schools. This particular workshop done once a week over a period of four weeks looked at the following key features of school functions:

- Key aspects of functions: *definition; domain and range; static points (intercepts and turning points); function behaviour (symmetry, gradient, concavity, end behaviour, continuity)*
- Transformation of functions: *vertical and horizontal shifts; vertical and horizontal stretches; reflections*
- Multiple representations of functions: *algebraic; verbal; graphical; and numeric*
- Ways of approaching functions: *pointwise; global; function as object*
- Applications of functions in the real world.

The content aligned with the curriculum specifications but was organized and packaged differently from the textbook. For example, lesson two began with Mpho drawing a table of values shown in Fig. 5.7 and asking learners to provide answers to the missing cells on the table. The given function for the activity is, $f(x) = x^2$. This activity had been part of the workshop activities for the teachers with two main foci: to determine the effect on the graph of $f(x) = x^2$ of the transformations of the form: $f(x) + a$, $f(x + a)$, $af(x)$ and $f(ax)$; and to contrast the forms $f(x + a)$ and $f(x) + a$, as well as the forms $f(ax)$ and $af(x)$. For Mpho's second lesson, learners



	-4	-3	-2	-1	0	1	2	3	4
f(x)	16	9	4	1	0	1	4	9	16
f(x+1)									
f(x)+1									
f(x-1)									
f(x)-1									
f(2x)									
2f(x)									

Fig. 5.7 Transformations of $f(x) = x^2$ in lesson 2

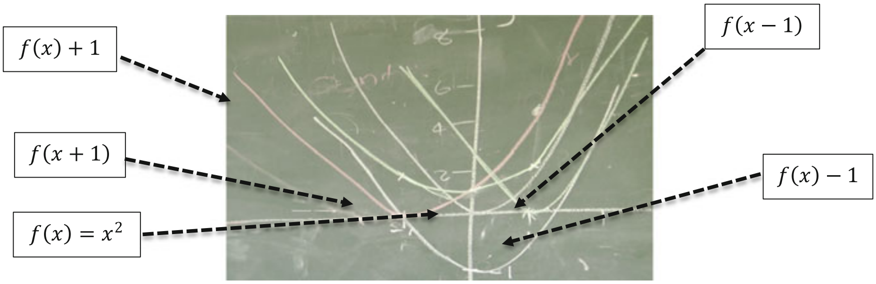


Fig. 5.8 Graphs of transformations of $f(x) = x^2$

had been assigned this table to fill out as homework and the process of completing it in the lesson went very quickly with learners calling out the answers and Mpho filling in the cells.

When the table of values had been completed, Mpho drew a sketch of the graph of $f(x) = x^2$ on the chalkboard and invited a few learners to come to the chalkboard to draw the rest of the functions on the same set of axes, using different colours. Up to the function $f(x) - 1$ the graphs looked as illustrated in Fig. 5.8.

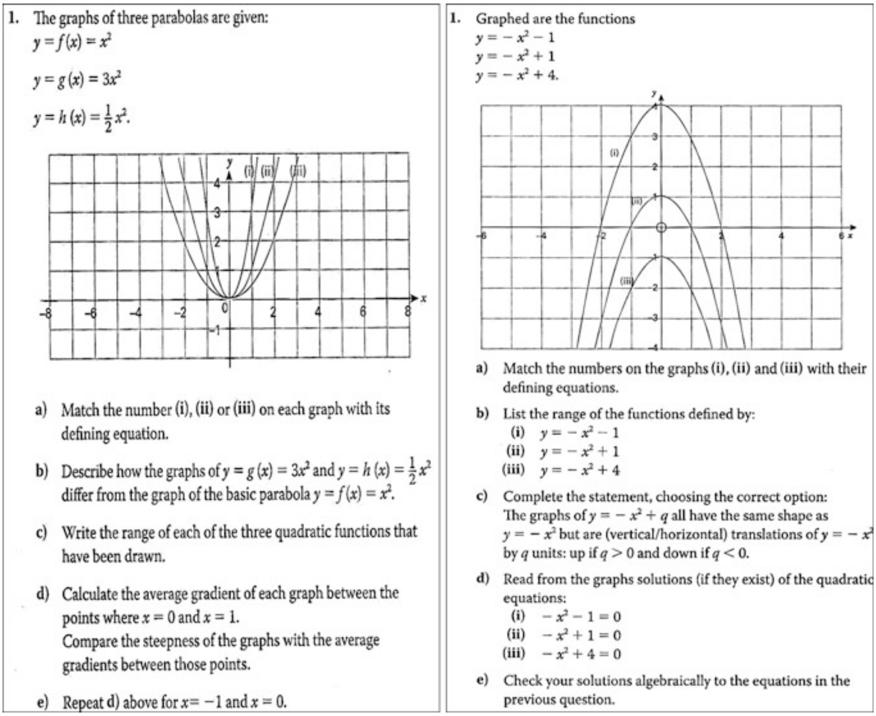
When all graphs had been drawn, Mpho led a discussion with learners that concluded that functions of the form $f(x) + a$, resulted in vertical shifts of $f(x) = x^2$ while the functions of the form $f(x + a)$, resulted in horizontal shifts of $f(x) = x^2$.

The vertical shifts are a grade 10 curriculum requirement and would therefore be available in the textbook. In the textbook, however, the presentation is different as it does not include the contrasting of $f(x + a)$ with $f(x) + a$. In the textbook, the activities on transformations of the quadratic function are as shown in Fig. 5.9.

In the textbook, the forms $af(x)$ are dealt with separately from $af(x) + q$ as Fig. 5.9 shows. Mpho used the materials from the workshop instead of the textbook. Since the material Mpho used was available in the textbook, this was an *improvised* lesson.

5.4.2.2 Omissions and Injections

We noted that functions of the type $f(x + a)$ and $f(ax)$, that is, horizontal shifts and stretches respectively, have been included in the table of values in Fig. 5.7. These functions are not required at grade 10, and therefore their inclusion is considered an *injection* of content in this lesson. The question to ask is what purposes did they serve at this grade? We argue that the inclusion of these functions was significant in showing the differences in the shifts of the graph of $f(x) = x^2$. During the lesson, Mpho focused learners' attention on these differences, suggesting it was her intention that these shifts be observed. The transcript below shows how Mpho contrasted the graphs of $f(x + 1)$ and $f(x) + 1$.



1. The graphs of three parabolas are given:
$y = f(x) = x^2$
$y = g(x) = 3x^2$
$y = h(x) = \frac{1}{2}x^2$.

a) Match the number (i), (ii) or (iii) on each graph with its defining equation.

b) Describe how the graphs of $y = g(x) = 3x^2$ and $y = h(x) = \frac{1}{2}x^2$ differ from the graph of the basic parabola $y = f(x) = x^2$.

c) Write the range of each of the three quadratic functions that have been drawn.

d) Calculate the average gradient of each graph between the points where $x = 0$ and $x = 1$.
Compare the steepness of the graphs with the average gradients between those points.

e) Repeat d) above for $x = -1$ and $x = 0$.

1. Graphed are the functions
$y = -x^2 - 1$
$y = -x^2 + 1$
$y = -x^2 + 4$.

a) Match the numbers on the graphs (i), (ii) and (iii) with their defining equations.

b) List the range of the functions defined by:
(i) $y = -x^2 - 1$
(ii) $y = -x^2 + 1$
(iii) $y = -x^2 + 4$

c) Complete the statement, choosing the correct option:
The graphs of $y = -x^2 + q$ all have the same shape as $y = -x^2$ but are (vertical/horizontal) translations of $y = -x^2$ by q units: up if $q > 0$ and down if $q < 0$.

d) Read from the graphs solutions (if they exist) of the quadratic equations:
(i) $-x^2 - 1 = 0$
(ii) $-x^2 + 1 = 0$
(iii) $-x^2 + 4 = 0$

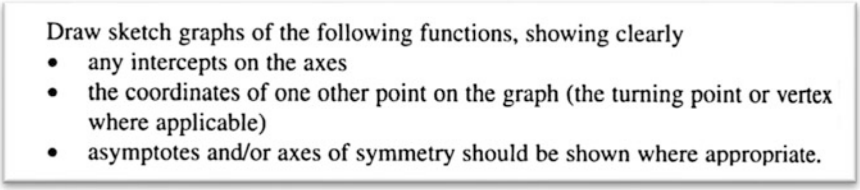
e) Check your solutions algebraically to the equations in the previous question.

Fig. 5.9 An example of activities on transformations of $f(x) = x^2$ in the textbook

Mpho	Okay, so this is a shift in the y-axis and remember here we added one inside and now this one is being added to the function (*points to* $f(x) + 1$). Okay?
Students	Yes.
Mpho	So, when you add one inside of the bracket, the movement is being affected in which axis? (*Pointing to* $f(x + 1)$). In the x-axis. Now we added outside the function, in the function plus one the movement is going to be affecting the y-axis. Okay?
Students	Yes.

Thus, the inclusion of the horizontal shifts and stretches enhanced the lesson. Instead of waiting to do these shifts and stretches at grade 11, the learners were given an opportunity to 'see' the difference between a parameter forming part of an argument of a function and when it was not part of the argument. We thus considered these to be *robust injections*. With regard to transformations of the function, $f(x) = x^2$, we could not identify any omissions in this lesson.

In summary, the second lesson was an improvised lesson from the workshop handouts. There were no apparent *omissions* when we compared the lesson content to the textbook content, but Mpho made *robust injections* to the content nevertheless.



Draw sketch graphs of the following functions, showing clearly

- any intercepts on the axes
- the coordinates of one other point on the graph (the turning point or vertex where applicable)
- asymptotes and/or axes of symmetry should be shown where appropriate.

Fig. 5.10 An extract of the procedure for sketching graph

5.4.3 *Mobilisation in Lesson Three*

5.4.3.1 Degree of Appropriation

The third lesson began with the correction of homework questions where Mpho worked out answers to the questions on the chalkboard. In the homework learners had to find equations of given graphs using the information provided. An inspection of the textbook showed that all the questions came from an end of chapter exercise in the textbook. After the correction of homework, Mpho introduced the topic on sketching functions, and began this part of the lesson by reading out and explaining the procedure (presented in Fig. 5.10) to be followed when sketching functions as outlined in the textbook.

The procedure in Fig. 5.10 outlines features that are key in sketching graphs of functions, such as: *the intercepts, turning points, asymptotes, axes of symmetry.* In order to demonstrate the procedure to the learners, Mpho used one of the functions, $f(x) = \left(-\frac{3}{2}\right)x^2$, from the textbook, as an example. After the demonstration there was very little time left and Mpho used the few minutes left to assign homework questions to be discussed in the next lesson. The homework questions too were selected from the textbook.

This third lesson hence was an *offloaded* lesson as all content which was dealt with in the lesson came from the textbook.

5.4.3.2 Omissions and Injections

We did not observe any injections of content in this lesson and therefore the discussion in this section shall be about the omissions only.

We firstly checked the Exercise in the textbook from where the homework which Mpho and learners were correcting at the beginning of the lesson came. There were two questions in this Exercise. The first question contained six graphs and a general equation of each graph was given. For example, for the straight line graph, the equation was given as $y = a.x$, for the hyperbola as $y = \frac{a}{x}$, for the quadratic function graph as $y = a.x^2$, and so forth. There was additional information provided on the graph, for example coordinates of one or two points. Learners were to use the information to find the value of a for each graph. To answer the question, learners needed to substitute a value of x and y in the given general equation and solve for a. Mpho had selected two graphs out of the six given for homework from this question. These were an upward facing parabola, and an increasing exponential graph.

The second question consisted of eighteen (18) graphs. Learners had to first decide to which of the given general equations the graph belonged. For example, the general equations provided included: $y = \frac{a}{x}$, $y = a.b^x$, $y = ax$, $y = ax^2$, $y = a\sin x$, $y = a\cos x$, and $y = a\tan x$. These general equations involve all the seven different functions learners are expected to learn at grade 10. Using the information provided on each graph, learners had to determine the equation of each graph. Mpho assigned four graphs in this question which included: a downward facing parabola, a decreasing exponential graph, a tangent function graph, and a hyperbola.

In the first place, there were numerous graphs, and thus too many for all to be assigned for homework. A selection was required. We observed that in the first question, an upward facing parabola was chosen and in the second question, a downward facing parabola. Similarly for the exponential function, in the first question, the chosen graph was increasing and a decreasing one chosen in the second question. This showed Mpho's deliberateness in the choice of questions for homework. The selection does not look haphazard but seems to have been deliberately worked out: The choices would illuminate the differences in the values of a in $y = a.x^2$ which depended on the direction the parabola was facing. Similarly for the exponential function, $y = a.b^x$. A trigonometric graph and a hyperbola were also chosen ensuring that the homework included all different functions studied and the homework was thus representative of key issues to be learned in determining equations of graphs at this grade. The selection of the homework exercises reflected a deliberate choice, where the omissions made were not distractive. We made similar observations regarding the homework which was assigned at the end of lesson three. There was no time left in the lesson for learners to practice the sketching of graphs. The homework was assigned quite hurriedly and Mpho called out the first four graphs out of the seven for homework. We regarded these omissions as a result of time constraints.

We made a further consideration of omissions in this lesson by looking at the demonstration and subsequent discussions of the procedure for sketching graphs of functions. We saw the four key features as outlined by the textbook when sketching graphs of functions as: *the intercepts, turning points, asymptotes,* and *axes of symmetry* (see Fig. 5.10). As Mpho illustrated the procedure to the learners, we observed that she guided them to determine the x- and y-intercepts of the graph by equating the function to zero and finding the x values which caused this, and substituting $x = 0$ in the equation, respectively. Mpho guided learners to determine that the graph would face downwards because the coefficient of x^2 was negative. However, we noted that Mpho never mentioned the axes of symmetry for the graph. This feature was explicitly mentioned in the procedure in the textbook as one of the

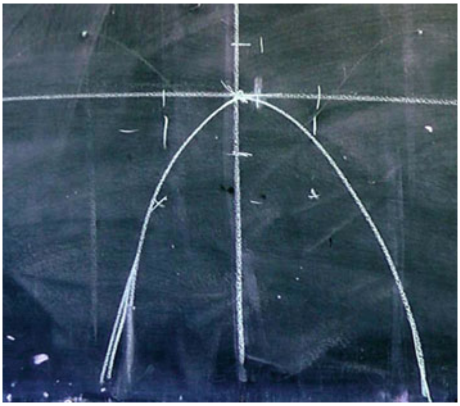


Fig. 5.11 Mpho's sketch of the graph of $y = -\frac{3}{2}x^2$

key characteristics in sketching graphs. What Mpho did was to guide learners to determine that the graph would be passing through the points $\left(-1, -\frac{3}{2}\right)$ and $\left(1, -\frac{3}{2}\right)$. This in our opinion had also provided an opportunity for learners to see that the one point was a reflection of the other on the y-axis, and therefore an opportunity to 'say' something about the axes of symmetry, but this did not happen. Mpho drew the sketch of the graph on the chalkboard as shown in Fig. 5.11 but still made no mention of the axes of symmetry.

The sketch of the graph in Fig. 5.11 is not quite symmetric. On the one hand, this is understandable considering that Mpho was using a free hand to sketch the graph. On the other hand, the fact that Mpho did not mention the property of the axes of symmetry for this graph made us wonder if she had paid attention to the symmetry in her sketch. This was a problem for us: If a learner were to sketch the graph of a hyperbola and did not mention an asymptote, we would have the same problem. We considered the omission of the feature of axes of symmetry as a *critical omission*.

One of the content areas for grade 10 functions has been mentioned as determining the properties of the functions being studied which include the quadratic function. The feature of symmetry is fundamental to quadratic functions and is one distinguishing feature of quadratic functions. We consider it distractive if the feature of symmetry is not mentioned in the procedure to illustrate how to sketch the graphs of functions. This is an activity that demands one to be conversant with the critical features of the different functions, as the opportunity to graph the function using point by point methods and be accurate in the graphing has been taken away. For these reasons, we regard the omission of determining the axes of symmetry in sketching the function $f(x) = -\frac{3}{2}x^2$ as a *critical omission*.

The analysis of Mpho's three lessons has been summarized in Table 5.3 and indicates the degree of appropriation, and the different types of omissions and injections made.

The summary in Table 5.3 shows that Mpho offloaded content in two out of the three lessons, and improvised in one lesson, meaning that for the three lessons, the

Table 5.3 A summary of Mpho's mobilization of curricular resources

Lesson	Mobilizing of the textbook	Omissions and/or injections
1	Offloaded	Robust injections Productive omissions
2	Improvised	Productive omissions Robust injections
3	Offloaded	Productive omissions Critical omissions

textbook remained Mpho's main resource. With respect to *omissions* and *injections,* the results show that in lessons one and two, Mpho produced a combination of *robust injections* and *productive omissions*, whereas in lesson three both types of *omissions*, *productive* and *critical* occurred. We further note that there were no *distractive injections* in all the three lessons.

In the next section we describe the implications for Mpho's PDC as determined through the analysis of the *omissions* and *injections* she made in the three lessons.

5.4.4 Determining Mpho's Pedagogical Design Capacity

What do we know about Mpho's mobilisation of the available resources from her lessons? We know that Mpho utilised available resources in the form of the textbook and the workshop handouts in her lessons, but mostly the textbook, from our observation that she offloaded in two lessons and improvised in one. We also know that in all the three lessons, there were no *distractive injections*, meaning that Mpho, while she was able to improvise content from other resources than the textbook, did not introduce in her lessons content that would reduce the opportunities for mediation. Instead, the injections in her lessons were *robust,* meaning that when she brought in content which was not yet required at grade 10 level, Mpho only brought in content that enhanced her lessons. As our analytical framework (Table 5.2) suggests, this is indicative of deliberateness in Mpho's mobilisation of the available resources, and therefore suggests an *intimate* teacher–resource relationship between Mpho and her available curricular resources.

Furthermore, when it came to omissions in the lessons, in all the three lessons, Mpho made *productive omissions* of content, meaning that in all her lessons, she had omitted content from the textbook in a way that did not detract from the goals of the lesson. We appreciate that the content made available in the textbook is important, the analysis process has also revealed that there are many valid reasons why any teacher would find themselves in a situation where they may need to cut on some content, including time constraints, for example. More indepth analysis may be needed including asking the teachers about their process of selection to understand why they omit content that is available in the textbook and therefore regarded as important. However, what we are saying about the *productive* *omissions* in the lessons is that Mpho's selection of content to include or to omit did not seem haphazard as it did not detract from the goals of the lesson. This serves as a confirmation of a participatory way of interacting with the resources, and again, a suggestion of an intimate teacher–resource relationship. To reiterate, PDC reflects the teacher's ability to create "deliberate, productive designs" (Brown 2009, p. 29) for her classroom.

Up to this point, all the signs of a high level of capacity for pedagogic design are showing for Mpho: the *intimate* relationship with the resources forged through Mpho's ability to weave together affordances from different resources; deliberate and participatory resource use that ensures only *robust injections* and no *distractive injections;* and *productive omissions* which do not harm the opportunities created in the classroom.

However, in continuing about what we know about Mpho's mobilisation of the resources, we know that Mpho made a *critical omission* of content as well in lesson three. That is, Mpho omitted content that was considered to be critical in the learning of the particular object, but most importantly, this content was available in the textbook/resource she was utilising at that time in the classroom. We regard this *critical omission* as being indicative of a "break" in communication between Mpho and the resource, and in terms of our analysis, indicative of *tacit use*. However this shows some inconsistency in how Mpho transacted with the resources. The analysis shows that overall, Mpho's pattern of use across the lessons is one of participation, and *intimate* relationships with her resources, and therefore with respect to PDC, we conclude there are relatively high levels of PDC for Mpho. The occurrence of this unevenness suggests how important the interview could have been in probing this inconsistency.

5.5 Discussion and Conclusion

We set out in this chapter to disaggregate or 'unpack' Mpho's capacity to mobilise the resources available to her effectively in her lessons, and we have done so, even though not without challenges. The analysis of Mpho's lessons has shown that a teacher's PDC manifests itself in the kinds of teacher–resource relationships that the teacher forges with her curricular resources. We have demonstrated that *intimate* teacher–resource relationships occur as a result of deliberate and participatory use of the resources thus demonstrating high levels of PDC. We have presented the notions of *omissions* and *injections* as indicators for determining deliberateness in use and argue that these analytical constructs provide a lens for describing teachers' capacity and competence to mobilise their curriculum resources in productive ways.

However, we recognise that although the final product appears to be smooth, we have seen that the analytical process is not smooth and clean, but involves a fair amount of complexity. This implies that the theoretical base for these notions still needs tightening, but we reiterate that they have a potential in disaggregating teachers' PDC.

Teachers' PDC is defined as the teachers' capacity to perceive the affordances of the resources and to then mobilise them to create instructional designs that respond favourably to teachers' classroom needs. Due to challenges which we encountered when trying to determine the teachers' processes of perception of affordances through interviews, we missed out on this critical aspect of teachers' PDC. The implication is that our story in this chapter is not complete and suggests that there is more work that still needs to be done to complete the description of teachers' PDC. We have mentioned that we could not obtain the information which we needed from the interviews which we conducted with the participating teachers, and we suggest that one of the challenges with teachers in the South Africa context could well be the high level of prescription in the textbooks which has a potential to mask the individual teacher's goals and intentions.

Secondly, research has determined that curricular resources do not only provide the teacher's practice with content, but with instructional approach as well (Ensor et al. 2002; Leshota 2015). The *omissions* and *injections* as used in the chapter take the instructional approach of resources into the background. This is another limitation of the study which needs to be explored further.

In concluding this chapter, we wish to comment that while the world at large seems to be moving towards digital resources, the situation in countries such as ours remains that of a print textbook still being the most accessible resource for teachers and learners alike, and therefore a very precious resource. The need to re-source (Adler 2000, 2012) teachers and to help the teachers, teacher educators, policy makers, and textbook authors understand that availability of textbooks does not imply use (Adler 2000) through concerted efforts to research the teacher–resource relationships and conduct professional development on resource use remains as intense as it has ever been.

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Moneoang Leshota currently holds a Postdoctoral Fellowship in Mathematics Education at the University of the Witwatersrand (WITS) in South Africa (2017–2019), working with Professor Jill Adler on teacher education and resources. Her research focused on teacher–textbook relationships especially the teachers' capacities to mobilise the affordances of their mathematics textbooks for effective classroom mediation. She is currently extending this focus to include other instructional resources that are being purposefully introduced by government into the mathematics classroom as teacher guides for instruction. Prior to pursuing her doctoral studies she was a member of the teaching staff in Mathematics at the National University of Lesotho (NUL) for over two decades, with her main teaching assignments being at first year for science-related fields. It was her experiences of teaching first year mathematics which made her want to learn more about what it took to teach and learn mathematics better and subsequently led her to changing her research interests from mathematics to mathematics education.

Jill Adler holds the SARChI Mathematics Education Chair at the University of the Witwatersrand, which focuses on research and development in secondary mathematics education, and is the President of the International Commission on Mathematical Instruction (ICMI). Jill has spearheaded several large-scale teacher development projects. The most recent one, within the Chair ambit begun in 2009, is called the Wits Maths Connect Secondary project. This work builds on her research on teaching in multilingual classrooms, and teacher professional development. Jill is a Visiting Professor of Mathematics Education at King's College London, UK. She is the recipient of numerous awards, the most significant of which are the 2012 Academy of Science of South Africa (ASSAf) Gold Medal for Science in the Service of Society, and the 2015 Freudenthal Award.

Chapter 6
Teachers' Selection of Resources in an Era of Plenty: An Interview Study with Secondary Mathematics Teachers in England

Helen Siedel and Andreas J. Stylianides

Abstract The proliferation of instructional resources and the potential impact of teachers' resource selection on students' learning opportunities create a need for research on teachers' selection of resources. We report results from an interview study with 36 secondary mathematics teachers in England, designed to find out (1) what instructional resources teachers choose for their everyday practice, thus beginning to document what we call a resource "pool of possibilities" to represent teachers' resource options, and (2) the reasons for teachers' choices, thus beginning to construct a taxonomy of what we call teachers' "resource pre-disposition" to schematize teachers' selection decisions. Our results show a large pool of possibilities and a complex taxonomy of teachers' resource pre-disposition.

Keywords Mathematics teachers' resources · Resource selection
Instructional resources · Curriculum · Textbooks

6.1 Introduction

One result of several decades of fast-paced technological change affecting the way information is produced and disseminated is that mathematics teachers in much of the world must increasingly contend with a cornucopia of resources they might use to support instruction. In mathematics education, where teachers' interactions with resources include finding, selecting, adapting, using, and developing resources (e.g., Gueudet and Trouche 2009b), little is known, as yet, about what resources

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mathematics teachers actually choose to use, and why, when choice is an option. Weller (2011) describes the shift within education that is necessitated by the technological changes as moving from a *pedagogy of scarcity* to a *pedagogy of abundance,* and suggests that two key issues in the shift are how educators are "coping with abundance" and how they can "best take advantage of abundance in their own teaching practice" (p. 232).

England is a fertile site for asking emergent questions about how mathematics teachers cope with an abundance of resources, and thereby illuminating issues about resource selection that may exist but be less visible elsewhere. This is because mathematics teachers in England typically have both the luxury and the responsibility of deciding what instructional resources to select, adapt, and use (Ruthven 2013b). Mathematics teachers' interactions with resources in England typically represent what Remillard (2005) identified as a "drawing on" conceptualization of teacher-resource interactions. Unlike what happens in countries such as Japan and the United States, where mathematics teachers tend to use textbooks quite heavily in planning and delivering their lessons (Mullis et al. 2012), utilizing adopted or mandated textbooks (e.g., Alajmi 2012; Fujita and Jones 2014; Leshota and Adler, Chap. 5), in England textbooks are not the central resource for mathematics teachers, supplemented by other resources a teacher might choose to use (Mullis et al. 2012). In England textbooks may have equal status with other available resources, or even lesser status, thus creating a natural context for the phenomenon of resource proliferation, and teachers' interaction with these resources, to materialize and be studied.

In this chapter, we report results from an interview study designed to address the following research questions (RQs):

RQ1: What instructional resources do English secondary mathematics teachers choose as support for their everyday teaching practice?

RQ2: What are teachers' reasons for their choices?

By examining what instructional resources English secondary mathematics teachers choose to use and why, we begin to construct a taxonomy of what we call teachers' "resource pre-disposition" in an era of plenty. The term "pre-disposition" derives from Bergmann's (1998) discussion of "acquired behavioral pre-disposition," extending Campbell's (1963) use of the term "acquired behavioral disposition." As expressed by Bergmann, a person has a tendency to act in ways shaped by their personal experience in various environments, past or present. Applied to teachers' selection of resources, we propose that a teacher's resource pre-disposition, formed by personal experience over time, may shape which resources they select from an array of choices and for what reasons. A resource "pre-disposition taxonomy" could serve as a generic template for categorizing teacher choices, helping the mathematics education community to examine the selection process and determine, for example, whether teachers' pre-dispositions influence their choices in significant ways. Thus deepening our understanding about teachers' choices and rationales, the pre-disposition taxonomy may be useful for designing professional development that supports teachers' interactions with resources in ways that can serve student learning.

Alongside teachers' resource pre-disposition, we begin to document a resource "pool of possibilities" for teachers in England, an inventory of the resource choices available to teachers in that country. The pool of possibilities, which could be annotated in various ways, might enhance teachers' resource selection by making it easier for teachers to discover what's possible, effectively increasing resource accessibility. The documentation of the pool of possibilities can also allow the mapping of teachers' selections against the wide range of options, thus illuminating teachers' response to an abundance of resources. Items in a pool of possibilities represent what Trouche (2004) described as the "tool before considering its users and its uses" (p. 282), that is, resources that teachers might, or might not, draw on. A comprehensive pool of possibilities, which to date has not been an explicit focus of study, resembles what Folcher (2003) referred to as a "database of propositions" (p. 638) used by a group of people.

As background for the interview study, we further discuss the conceptual and empirical setting by addressing the meaning of "resource," findings of existing studies about resource selection, and the cultural context of England with regard to mathematics education resources.

6.2 Background

6.2.1 The Meaning of "Resource"

The term "resource" has a multitude of everyday and educational meanings (Ruthven 2013a). In this chapter, we use both "resource" and "instructional resource" to refer to any assets teachers might draw on to support any stage of their everyday teaching practice. We base our interpretation primarily on Adler (1998, 2000) and Cohen et al. (2003). These researchers not only expanded the notion of resource at the time they wrote, they contributed to a new focus on the study of resources-in-use by teachers.

Demonstrating that "resources are not self-explanatory objects with mathematics clearly shining through them," Adler (2000, p. 207) introduced the influential term "re-sourcing" to reflect that what teachers do with resources, in context, matters as much or more than the possibilities embedded in inert material resources such as textbooks. Importantly, in categorizing resources for school mathematics she included human, social, and cultural resources as well as material resources. Cohen et al. (2003), wishing to influence research about effective instructional programs, similarly considered "resources" or "sets of resources" broadly, as composed of "conventional, personal, environmental, and social resources" (p. 123). They emphasized that resources must always be associated with instructional goals, and that "the value of resources is likely to depend on the ways they are used" (p. 138).

They defined instruction as “interactions among teachers and students around content, in environments” (p. 122).

To indicate our interest in resources for a teaching-learning context, we use the expression “instructional resource.” Adjectives such as educational, teaching, learning, curricular, digital, online, physical, intellectual, or instructional, all modify the scope of the term resource to some degree. After reviewing how adjectives were used with “resource” in the literature (e.g., Pepin et al. 2013, p. 969; Recker et al. 2004, p. 93; Remillard 2005, p. 213; Stylianides 2016, pp. 1–2), we chose “instructional” as having perhaps the broadest application, reflecting our interest in the entire range of teachers’ work with resources. This not only aligns with the broad interpretations of resources introduced by Adler (1998, 2000) and Cohen et al. (2003), it corresponds with the definition of instruction used by Cohen et al. (2003).

Investigations of teachers’ interactions with resources in general and investigations of technology integration (e.g., Windschitl 1998) were not affiliated when Adler and Cohen et al. defined resources. As new types of resources appeared and the resource pool of possibilities expanded, teachers’ interactions with resources remained important to the research community. New perceptions of teachers’ resource use included the recognition that teachers work with multiple resources, including both technological and general resources (e.g., Ruthven 2007); the introduction of the documentation approach as a theoretical attempt to link “studies about the use of teaching resources, and studies about the way in which teachers work in a technology-rich environment” (Gueudet and Trouche 2009a, p. 1360); and an emphasis on teachers as resource designers (e.g., Pepin et al. 2013). The meaning of resource as developed by Adler and by Cohen et al. remains applicable, able to encompass all of the new types of resources and evolving perceptions of teacher interactions with resources.

The term “curriculum” relates, or can relate, to instructional resources, and so a discussion of the meaning of this term is relevant here. In our definition of curriculum we draw primarily on Remillard (2005), whose research on curriculum materials, as one type of instructional resource, has contributed one of the important theoretical perspectives on teachers’ work with resources (Jones and Pepin 2016). Also, similar to Adler (1998, 2000) and Cohen et al. (2003), Remillard (2005) attended to the importance of what teachers do with resources, focusing on curriculum as it pertains to classroom work, rather than a larger policy document.

Remillard’s work developed within the context of the United States, where textbooks are the norm, there is no national curriculum, and adopted textbooks may define the curriculum (e.g., Apple 1990). Given the different circumstances in England, with a national curriculum, where teachers might not be working with published print or digital curriculum materials (see Sect. 6.2.3), we employed a broader definition of curriculum as a nested set of related plans. This definition derives from Voogt et al. (2011), who described curriculum as “a plan for learning,” attributing this description to Taba (1962). Curriculum materials, according to Voogt et al. (2011, p. 1236), are “concrete products of the plan,” and are “designed for several curriculum levels,” as small as a lesson plan or as large as a national plan.

6.2.2 Existing Studies of Resource Selection

Few research studies have addressed the decisions teachers make when finding and selecting instructional resources and we found no studies that surveyed the full range of resources from which teachers select. In this section, we review some studies that feature teachers' resource selection, highlighting their relationship to the notion of teachers' resource pre-disposition.

In a study of technology integration preceding the development of online resources, Kerr (1991) observed that teachers may be disposed to select a new resource based on its ease of integration with classroom norms, and be less inclined to select resources that disrupt the established teaching-learning context. This suggests that teachers have a pre-disposition with respect to resources, favouring some while not selecting others. "Ease of integration with classroom norms" could be included in a pre-disposition taxonomy as one teacher criterion for assessing whether to select a resource.

Also investigating teachers' use of computer-based tools and resources, Ruthven and Hennessey (2002) found that some teachers in England were choosing resources apart from any identifiable pedagogical alignment, such as "transmission" or "constructivist," in order to carry out a variety of classroom activities and purposes. They also found that teachers in England were intrigued by what technology afforded, but slow to change their traditional ways of teaching, which affected their choice of resources. The study suggests that items for a pre-disposition taxonomy might include characteristics of teachers, such as their "interest in innovation," "reluctance to change," "rigid pedagogical approach," or their pre-disposition for "activity-oriented selection."

Recker et al. (2005) investigated large-scale online repositories that were developed to collect a large number and variety of resources in one place. Working with a group of mathematics and science teachers who were learning to find and select resources from such a large collection, the authors highlighted resource selection by asking participants about their search strategies, their motivations for using online resources, barriers to and benefits of use, and selection criteria. Participant teachers expressed a preference for resources that were small and less complicated, reiterating that "ease of integration with classroom norms" is important for a pre-disposition taxonomy. Participants also reported that abundance and variable quality were barriers to selection, suggesting that items for a pre-disposition taxonomy include the notion of selection constraints. Teachers may be "challenged by abundance," "have difficulty finding resources," or "have difficulty assessing resources."

Gueudet and Trouche (2009a, b) interviewed nine secondary mathematics teachers in France to explore teachers' decisions about resource selection during the planning stage of their work. These case studies contribute characterizations of the teachers' pre-dispositions, influenced by personal history and environment. Concerned with showing resources in use, rather than with zooming out to sketch the resource pool from which teachers in France are making selections, the authors (2009b) nevertheless contributed an example of one teacher's rationale for rejecting Mathenligne, a resource in her pool of possibilities. Her rationale included that "its mathematical content is unsatisfying," and the files "cannot be easily modified" (2009b, p. 213). Such appraisal represents important information about a non-selected resource in the pool of possibilities. This case study also suggests that "quality of mathematical content" and "adaptability" be included in a pre-disposition taxonomy.

More recently, Quentin and Bruillard (2013) investigated online teacher networks in France where teachers can work with others to design, share or discuss resources. The authors showed that these networks are cultures with differing possibilities for teacher participation. Teachers might be pre-disposed to selecting a network based on their preference for flexible or rigid participation, for collaborative or individual resource design, or for copying or constructing resources. This suggests that "interest in resource design" and "interest in working with others" be included in a pre-disposition taxonomy.

A US study of teachers' resource selection was based on theories related to information science, in particular the theoretical framework of relevance. Diekema and Olsen (2012), investigating "how teachers manage their personal information spaces" (p. 1), found that "relevance," which they describe as "usefulness" played the biggest role when teachers sought new information. Relevance refers to a judgment teachers make when they need information and perceive a relationship between information they need and information on offer from a source they find. Teachers "make inferences about the resource and make a selection based on the relationship between the resource, the topic at hand, and properties of their students" (p. 2). This study suggests that teachers have established "criteria for choosing resources," an item for a pre-disposition taxonomy.

Two US studies by Abramovich and colleagues looked at sites where teachers exchange resources and explored teachers' selection criteria using data available from teachers' ways of interacting with the site. One study analyzed download decisions (Abramovich et al. 2013); the other (Abramovich and Schunn 2012) analyzed payment patterns from a site where teachers offer self-designed resources for sale to other teachers. According to the authors, the importance of knowing more about teachers' resource selection is that it can help improve these large sites, which are resources where teachers search for resources. Complex sites are similar to complex resources. If they are difficult for teachers to use, they are likely to be non-selected resources. These studies began to look at teachers' selection from a particular pool of possibilities while suggesting that teachers have search criteria representing their preferences, or pre-dispositions. The researchers noted a lack of consensus about what constitutes a high quality resource and the need for defining "high quality."

All of these studies provide useful information about teachers' selection of resources. Although this was not their main focus, they also suggest a range of possible items for a pre-disposition taxonomy.

6.2.3 The Cultural Context of England

In some countries, mathematics textbooks play an important role in everyday teaching practice. In these countries, there seems to be little doubt about whether textbooks have an important role to play, with significant efforts being placed on examining what is involved in designing textbooks that can better support classroom work and promote teacher understanding alongside student learning (e.g., Davis et al. 2014; Stylianides 2016). In England, the textbook situation differs.

Although English teachers participating in TIMSS (Mullis et al. 2012) approximated the international norm in reporting that they *use textbooks*, their reported use of textbooks as *the basis for instruction* put England among the lowest internationally (Mullis et al. 2012). In England, textbooks are not highly regarded; the general sentiment "seems to be that the knowledge in textbooks is in some way second-rate knowledge and that the teachers, the writers and learners who engage with them in their different ways are somehow doing something second-rate" (Issitt 2004, p. 690). Even clearer was a statement in a speech by Education minister Elizabeth Truss (2014) who talked about "an 'anti-textbook' orthodoxy" in the country.

There are few research studies of English textbooks or their use in the English context. A recent study by Edwards et al. (2014) investigated textbook use by 42 secondary mathematics trainee teachers and considered the significant impact of the cultural and political climate of England. The government determines the curriculum, resulting in frequent curriculum revisions as the government changes. The researchers' findings agree with the TIMSS findings (Mullis et al. 2012) on the low use of textbooks as the basis for instruction in England. Moreover, the degree of use ranged widely, determined by teachers individually rather than by their school.

Hodgen et al. (2010) analyzed algebra textbooks for lower secondary mathematics. They suggested that a collection of factors in England have contributed to lower textbook use: increased and early use of Internet resources; government promotion of textbook alternatives; the "view amongst the educational establishment that schools over-rely on textbooks rather than undertaking their own detailed planning" (p. 190); and frequent changes to the national curriculum that limit time for developing, let alone "vetting," textbooks. Hodgen et al. also found, as did Haggarty and Pepin (2002), that "English textbooks are simple and routine with little requirement for deeper thinking" (p. 197) and tend to lack any foundation in research, suggesting these textbook characteristics might also have contributed to a decline in textbook use.

A significant influence on the school culture in England is a government-mandated document called the "scheme of work," constructed by individual schools. Each scheme of work represents a localized, school-level interpretation of the national curriculum. According to Ruthven (2014), the tradition of localized efforts in England represents the aims of the re-sourcing movement in that teachers are encouraged to use and develop a range of resources beyond conventional curriculum materials such as textbooks. However, although the perceived poor quality of textbooks in England made the scheme of work an attractive alternative, the quality of the locally developed schemes of work is highly variable (Ruthven 2013b).

England, extreme in its low reliance on textbooks and high degree of teacher choice, can thus serve as a very good case study to better understand the phenomenon of teachers' pre-disposition toward the pool of possibilities for resource selection in the current era of plenty. However, beyond anecdotal reports, little is known about the textbook alternatives available to teachers in England and about teachers' reasons for their resource selections.

6.3 Methods

6.3.1 Research Participants and Context

Participants for this interview study were thirty-six mathematics teachers at six secondary schools, from two counties in England; this included five state schools and one independent (private) school. Eleven had been teaching less than 5 years; fifteen, 5–9 years; ten, more than 10 years. Thirty-four were teaching 11–16 year olds (key stages 3 and 4) and ten of these were also teaching 16–18 year olds (key stage 5, known as A Level and not offered at all schools in the study); for two teachers, information about levels taught was missing.

None of the schools had a policy about the use of textbooks (although this was not a factor for a school's inclusion in the study). Teachers at these schools could largely decide for themselves whether and how much to use textbooks or other resources. At all the schools, textbooks were available for every student year, but these were not always still in print and might vary by year.

Each school had developed its own scheme of work. For five schools, the scheme of work represented a local interpretation of the national curriculum; the independent school did not use the national curriculum. Although often encouraged or expected to use resources identified in the scheme of work, teachers were not required to use those resources at any school. The scheme of work and the available textbooks were aligned at two schools, at least; one of these had adopted a current textbook to cover more than one year.

The participating teachers and schools constituted a rather diverse sample that allowed us to begin to explore the mostly uncharted territory of teachers' resource pre-disposition in a cultural context where the pool of possibilities for resource selection is very large. Generalization was not a concern in this research, and so we did not impose any restrictions on the participation of any teachers or schools that were willing to participate in the interview study.

6.3.2 Procedure

To allow space for the emergence of teachers' own views about what constitutes a resource for them, we explicitly invited participating teachers to interpret the term "resources" broadly, as any form of educational apparatus, including, but not limited to, textbooks and other print material, digital and online resources, and people. They were aware the study was partly motivated by the perception that teachers in England do not rely on textbooks in their everyday planning and teaching.

Interviews were audiotaped and conducted using a four-part questionnaire employing survey and semi-structured interview techniques. In Part I (survey interview), participants were asked to list what resources they use. In Part II (semi-structured), participants discussed their selection of resources. To support this discussion, we asked questions like, "What makes you look for a resource?" and "What makes you choose one resource over another?" In Part III (semi-structured), not discussed in this paper, participants were invited to discuss their approach to a focal mathematical topic (negative number operations). Part IV (survey interview) consisted of a checklist of 12 resource types, intended to prompt participants to identify any resources they were using but had omitted to mention in Part I. Part IV was omitted in three interviews due to time constraints. Four teachers were contacted for follow-up interviews, for clarification.

Data used to address RQ1 (identification of the resource pool of possibilities) came primarily from Parts I and IV, while data addressing RQ2 (reasons for resource selection) came primarily from Part II. Due to the non-rigid nature of the interviews, during some parts of the interview teachers could make comments that we had aimed to elicit in different parts, so there was not a strict correspondence between interview parts and research questions.

6.3.3 Analysis

Regarding RQ1, resources mentioned by teachers were identified as either *specific* if they have a brand name (e.g., the NRICH website) or *generic* (e.g., colleagues) if they represent a broader class of resources. For example, if in the course of an interview a teacher mentioned the name of a textbook they use, the particular textbook (brand name) would be recorded on the list of specific resources. If, however, the teacher referred to a textbook without mentioning the textbook's name (as happened in most of the cases), "textbook" would appear as a generic resource. We recorded the resource according to the language of the teacher.

If a teacher mentioned the same resource more than once, it was only counted once. We also distinguished between "prompted" and "unprompted" teacher references to a resource: a reference to a resource was considered to be *unprompted* if mentioned during Part I of the interview, and *prompted* if mentioned during Part IV.

If a resource was mentioned in discussion but not as a resource, the reference to it was coded as *other*.

Regarding RQ2, we used qualitative content analysis to identify themes from teachers' Part II discussions of their reasons for resource choice. We also included in our analysis any comments from other parts of an interview if the comments pertained to resource selection. Six themes emerged about what drives teachers' choice of resource, as discussed in the Results section, including student-driven resource selection and teacher-driven resource selection. To categorize comments we established criteria for coding the data. For example, we considered data as relating to student-driven resource selection when a teacher talked about what students would be doing, focusing on the nature of their activity; we considered data as relating to teacher-driven resource selection if a teacher talked about what students would be doing, but provided a rationale for student activity, opening a portal on their teaching philosophy with respect to resources. All six themes, as drivers for resource selection, could arise in a single interview. Within each theme, we identified sub-themes, essentially items in a pre-disposition taxonomy. Some items coincided with items we had derived or inferred from the literature (see Sect. 6.2.2), while others emerged uniquely from the data. A comment could fit under multiple themes or sub-themes.

Although some themes and sub-themes were more prominent than others, we did not rank these by counting frequencies. This decision was consistent with our objective to identify themes and sub-themes, treating them as tentative propositions to be explored further in future research, rather than try to determine their relative importance. For the same reason a precise demarcation between different themes or sub-themes was not a goal (besides, multiple coding of comments was possible), thus downplaying the necessity of both of us coding all comments and determining inter-rater agreement.

6.4 Results

6.4.1 Identification of Resources (RQ1)

Table 6.1 is the frequency table for teachers' mention of *specific* resources. The table offers information about whether a teacher's mention of a resource was *unprompted* (during Part I of the interview), *prompted* (during Part IV), or *other*. Recognizing that readers may be unfamiliar with some of the resources, we have included the relevant web addresses where available.

A documentation of the complete pool of possibilities of specific resources emerging from the interviews would require the inclusion of all specific resources mentioned by teachers. For practical reasons, we restricted the list of specific resources in the table to those that were mentioned by at least five teachers, which also suggests which specific resources may be most influential in England.

Table 6.1 Specific resources mentioned by 5 or more teachers (N = 36)

Resource	Web address	Unprompted	Prompted	Other	Total
NRICH	nrich.maths.org	17	10		27
TES[a]	tes.com	16	6		22
My Maths	mymaths.co.uk	12	8	2	22
YouTube	youtube.com	2	18		20
IWB[a]		10	3	1	14
NCETM[a]	ncetm.org.uk	5	7		12
ATM[a]	atm.org.uk	5	7		12
Mr. Barton Maths	mrbartonmaths.com	6	4	1	11
Tarsia Jigsaw	mmlsoft.com/index.php/products/tarsia	6	3		9
Standards unit[a]	stem.org.uk/elibrary/collection/2933	5	3		8
Dan Meyer	mrmeyer.com	3	4		7
Maths Watch	mathswatch.co.uk	2	5		7
MEI[a]	mei.org.uk	3	4		7
Twitter	twitter.com	3	2	1	6
Suffolk Maths	suffolkmaths.co.uk	5	1		6
STEM website	stem.org.uk	2	3		5

[a] *TES* Times Educational Supplement; *NCETM* National Centre for Excellence in the Teaching of Mathematics; *ATM* Association of Teachers of Mathematics; *IWB* Interactive White Board; *Standards Unit* is also known as Improving Learning in Mathematics; *MEI* Mathematics in Education and Industry (teachers could have mentioned MEI or one of the services offered by MEI, such as the Further Maths Support Programme)

Altogether, teachers mentioned more than 70 specific resources, with most of these mentioned by fewer than five teachers (and thus not included in Table 6.1).

The number of specific resources per teacher varied from zero to 16: 14% of the teachers mentioned fewer than five; 50% mentioned five to nine; and 36% mentioned 10 or more. Of 16 specific resources mentioned by five or more teachers, the four most popular were NRICH, the Times Educational Supplement (TES), MyMaths, and YouTube. YouTube differed from the other three in that the vast majority of teachers who mentioned it (18 of 20) did not do so until prompted by the question, "Do you use YouTube?" during Part IV of the interview. The remaining three are mega-sites, referred to by one teacher as "sources of resources."

We acknowledge that Table 6.1 does not reflect any resource overlapping. For example, NRICH and Mr. Barton Maths partner with TES, as do others; many Tarsia Jigsaw puzzles are available at TES; and the National STEM Centre is archiving and making available Standards Unit materials at the STEM Website. However, we are reporting the resources as mentioned by the teachers, who may have not recognized, been aware of, or considered important any such overlaps.

Table 6.2 is the frequency table for teachers' mention of *generic* resources, restricting the list of generic resources to those that were mentioned by more than

Table 6.2 Generic resources mentioned by more than half of the teachers (N = 36)

Resource	Unprompted	Prompted	Other	Total
Student textbooks	32	2		34
Colleagues	20	13		33
Materials at school[a]	20	11		31

[a] Materials at school do not include student textbooks, but they do include any other physical materials, materials in shared digital folders, and a school's scheme of work

half of the teachers. The cut-off was chosen to depict the contrast between these three generic resources and others, in that all others were mentioned by much fewer than half. This, then, also suggests the likely significant influence of these three. Like Table 6.1, this table also offers information about whether a teacher's mention of a resource was *unprompted*, *prompted*, or *other*.

Table 6.2 shows that three generic resources were mentioned by nearly all teachers: student textbooks, colleagues, and materials at school (other than student textbooks). A few teachers said they use the teachers' editions of student textbooks (teacher guides), professional development materials, professional organizations, journal articles, or books other than textbooks. The teachers who mentioned professional organizations meant mostly materials or resources produced by the organizations (e.g., resources found on their websites) rather than benefits of membership. As noted earlier, most teachers did not mention the name of any student textbooks they use, and therefore the textbooks were not included as specific resources.

From the RQ1 analysis, a portrait emerged of thirty-six highly individualized packages of resources, with little commonality except for the top three generic resources and the fact that some specific resources appeared in more packages than others. Teachers freely drew on multiple resources; their individual resource packages consisted of resources external to the school context, resources at school, and their personal resource banks. We refer to the phenomenon of multiple resources and the pronounced variation of resources as "plurality and variation."

6.4.2 Selection of Resources (RQ2)

6.4.2.1 Overview

Our analysis of teachers' reasons for their selection of resources identified six key themes: *student-driven* selection, *teacher-driven, mathematics-driven, constraints-driven, resource-driven,* and *culture-driven.* The first three pertain to classroom interactions; the last three to influences other than classroom context. We report results for these selection themes in Sects. 6.4.2.2–6.4.2.7, where we introduce the themes and sub-themes and use teacher remarks to illustrate them.

In what follows, we present longer excerpts from the interview with Teacher 35 to exemplify that (1) a teacher's selection of resources could have multiple drivers and (2) data from which the six themes were derived were not exclusive to Part II of an interview. With regard to the second point, as we noted in Sect. 6.3.3, for our analysis of RQ2 we incorporated teacher comments from other parts of an interview if they provided insights about selection rationale. For example, when asked in Part I what resources they are using given our broad definition of resources, teachers often responded with their planning sequence, which wrapped their contribution of a list of resources in sentences that also illuminated their selection process. Teacher 35 was one of these teachers.

When asked in Part I what resources he uses, given our definition of resources, Teacher 35 took ten minutes to respond, mentioning a large number of resources (both specific and generic) alongside reasons for their selection. He said: "I use all of them. First I find myself looking at the syllabus, the *scheme of work* [italics are used to highlight a resource]. I also want to look at the *textbooks* I have." When asked what textbooks, he said EdExcel but that "depends on the school and the examining board," clarifying that he also looks at some other textbooks. Next he said, "I might want to reflect on my personal experience" and "I do collaborate with *colleagues*." Asked to distinguish between sharing and collaboration he replied that sharing is asking colleagues what resources they have; collaboration is "this is what I've done before and it worked well… and we try to blend." Thus far in the interview, Teacher 35 mentioned all three common generic resources reported in Table 6.2: the school's scheme of work (an example of the broader category "materials at school"), student textbooks, and colleagues.

Then Teacher 35 went on to mention other (specific) resources and his rationale for their selection. First was the Maths Enhancement Program (MEP), an online textbook available at the Centre for Innovation in Mathematics (CIMT):

> Then I have my online resources and the online resource I use a lot is the MEP… yes, CIMT. I use that extensively… They have done extensive research on how maths is taught… it makes sense with my thinking about how maths could be taught and the way of organizing structure and my approach to teaching maths as well… students can go (online) and read about… things we've done in class.

The rationale offered by Teacher 35 for using the MEP in his work is an example of a *teacher-driven* resource selection: the particular program fits the teacher's personal teaching style and his beliefs about how mathematics should be taught. Later during the interview, the teacher noted that the MEP is charitable, with no charge for using it, an example of a *constraints-driven* resource selection. The teacher noted further that for several years he did one-to-one tutoring with students from a variety of backgrounds and felt that the MEP worked well with students of different backgrounds and from all over. This suggests that the teacher's selection of the MEP considered students' needs and thus was also *student-driven*.

6.4.2.2 Student-Driven Selection

Many teachers reported their resource selection as driven by consideration of their students' needs. Table 6.3 offers some interview excerpts (ordered by participant number) illustrating student-driven resource selection and two sub-themes—"resource variety" and "mathematical value"—that contribute to a pre-disposition taxonomy.

Regarding the sub-theme *resource variety*, teachers emphasized the need for variety as a way to promote student engagement (Teachers 6, 8), accommodate the personality of the student group (Teachers 25, 27), differentiate by attainment (Teachers 10, 19), and enable particular and variable types of activity, including varied lesson types (Teachers 25, 30). Regarding the sub-theme *mathematical value*, teachers selected resources they thought would add value for students' mathematical learning (Teachers 6, 8).

It was worth observing that, in response to student needs, teachers would not necessarily look for new resources. Teacher 6 mentions resources she already has that have proved reliable; Teacher 10 would design a resource; Teacher 30, needing a new activity type, had a resource in mind, one with which he was already familiar.

Table 6.3 Interview excerpts illustrating student-driven resource selection

Teacher	Interview excerpt
Teacher 6	"The resources I keep coming back to are ones that have engaged the class and showed me that they have understood it better as a result of my using that."
Teacher 8	"[A resource works when] the pupils are engaged in the activity, they enjoy doing it, and they're actually learning through the process."
Teacher 10	[Discussing how to reach lower end students by varying the approach to a topic:] "They basically repeat and repeat and repeat." "If students aren't getting it, I know I need something different to what's already out there because what's out there tends to be the same thing, so I like to redesign the wheel."
Teacher 19	"We teach mixed ability… so it's usually either… you're trying to think of an extension activity [for the higher ability students], or an activity that will support those students that are going to find it difficult."
Teacher 25	"It's choosing things that are appropriate for particular groups and you may well be following the same scheme of work with two or three different groups… One group [I worked with] was able to work with 'less structure' and could do more 'investigative' work [than another]."
Teacher 27	"I always think of a particular group of students when choosing a resource. Different classes react to different things."
Teacher 30	"Let's say you feel they need more practice. Giving them another worksheet isn't going to keep them motivated but something like this [Tarsia puzzle] might."

6.4.2.3 Teacher-Driven Selection

Many teachers reported their resource selection as driven by the way they prefer to teach. Table 6.4 offers some interview excerpts illustrating teacher-driven resource selection and two sub-themes—"teaching approach" and "planning strategies"—that contribute to a pre-disposition taxonomy.

Regarding the sub-theme *teaching approach,* individual teaching approaches and preferences varied a great deal, highlighting teachers' autonomy even when teachers valued collegiality. Teachers selected resources to fit their preferred teaching style (Teacher 28) or to keep themselves motivated in their teaching (Teacher 12), but also according to their convictions about the nature of mathematics (Teacher 2) or about student learning (Teacher 9). Some labeled their personal teaching approach (Teacher 22).

Regarding the sub-theme *planning strategies,* the teaching point at which resources were selected was often mentioned. For some teachers a reason for looking for resources is a need arising as they try to implement a plan they have already developed (Teacher 9). For others finding resources might begin the planning process (Teacher 16). Altogether, selection might occur in a particular sequence or at various points in a planning phase.

Table 6.4 Interview excerpts illustrating teacher-driven resource selection

Teacher	Interview excerpt
Teacher 2	"I think of maths as questions."
Teacher 9	"I know exactly how I want to teach this." "I look for resources that match what's in my head." "I like to write on the board in front of the students… while still talking to them … it's nice to show them… how to approach the method… show them exactly how they should be thinking and setting out." "The most important resource is probably what's in their heads." "I like to bring out their misconceptions."
Teacher 12	[Why would you choose one resource over another?] "Probably for my own sanity… I might change a resource if I've been doing it for years, just to do something different."
Teacher 16	[What makes you look for a resource?] "A new topic I haven't taught before."
Teacher 22	[He described his approach as structured. His pupils complained about his structured approach to ratio, so he tried something another teacher did.] "It went disastrous." "I wasn't happy with the way it went." "It works very well for [the other teacher]."
Teacher 28	[Discussing a set of old textbooks he uses a lot:] "They weren't for everybody but they fit my style of teaching." "I can't step into other people's shoes."

Table 6.5 Interview excerpts illustrating mathematics-driven resource selection

Teacher	Interview excerpt
Teacher 6	"Bitesize is good for [teaching] loci."
Teacher 21	"Well, if I just talk about today, I used electronic resources, Geogebra, because we've been doing some work on graphs, straight line graphs, and I've used pre-printed coordinate grids, and I've used questions that I've prepared on an interactive white board. And then later on I'm going to be teaching probability so I'm going to be using some interactive resources called Interactive Teaching Programs…"
Teacher 30	(a) "We don't have quality resources for units of measure." (b) [Discussing a specific resource] "It's not wonderful but it gets them something."

6.4.2.4 Mathematics-Driven Selection

Few teachers reported their resource selection as driven by mathematics. Table 6.5 offers some interview excerpts illustrating mathematics-driven resource selection, and two sub-themes—"topical selection" and "content appraisal"—representing items for a pre-disposition taxonomy.

Regarding the sub-theme *topical selection*, some teachers associated the selection of resources with particular mathematical topics (Teachers 6, 30a). Comments about topical selection sometimes indicated why teachers might select multiple resources (Teacher 21); a resource might be deemed especially good for a particular topic but possibly not used with any other topic, and several resources might be needed per topic. Regarding the sub-theme *content appraisal*, mathematics-driven comments sometimes occurred when teachers talked about why a resource did or did not work or that a resource might work in some ways but not in others (Teacher 30b) with respect to student learning.

6.4.2.5 Constraints-Driven Selection

Many teachers reported their resource selection as influenced by constraints on accessibility. Table 6.6 offers some interview excerpts illustrating constraints-driven resource selection and five sub-themes—"time," "money," "technology," "awareness," and "other." Each sub-theme is characterized by a "lack of" something that restricts resource accessibility and represents an item for a pre-disposition taxonomy.

Familiar constraints such as *time* (Teacher 24), *money* (Teacher 30) and *technology* (Teacher 33) reduced teachers' access to the full range of resources. Time was most prominent, notably the lack of time for finding resources (Teacher 18), or for getting familiar with resources that are not classroom-ready (Teacher 8). Most teachers preferred to use what they already have and reduce the time spent searching for resources (Teacher 1).

Table 6.6 Interview excerpts illustrating constraints-driven resource selection

Teacher	Interview excerpt
Teacher 1	[Using resources in the scheme of work is efficient because these are provided by colleagues who know the student population.] "Working within the school … they know where we're going with it."
Teacher 8	"You find these things and you're not really sure how to use them… [you] can't use the resource without training."
Teacher 12	"You have to know it exists." "There are so many resources."
Teacher 17	"I don't have a room allocated to myself … carrying piles of textbooks was becoming more and more problematic."
Teacher 18	"There's so much out there."
Teacher 23	"Things disappear." [She mentioned her personal experience with a website that disappeared, government materials now archived, and a folder at school that had been emptied.]
Teacher 24	"Time is a resource and a filter."
Teacher 27	"A lot of the American resources use a different language to what we use."
Teacher 29	[Discussing resources for teacher learning] "I wouldn't think to look on the Internet for something like that." [When he has questions he asks colleagues or looks at materials at school.]
Teacher 30	"You can buy the… manuals that go with different courses but they're quite expensive." "Because people dip in and out of textbooks… we don't end up buying them." "[If teachers used textbooks more,] it would be worth the money."
Teacher 33	[This teacher writes lessons for the department.] "I would like to use Mr. Bowland more but quite a lot of their activities require access to PCs and we don't have them as a department."

The sub-theme *awareness* related to teachers' lack of awareness of existing specific resources (Teacher 12) or lack of information about how newer types of resources might benefit their work, such as sites about mathematics or the published results of research aimed at practice and having unrestricted access (Teacher 29).

The sub-theme *other* includes constraints such as available resources having different terminology from that used in school (Teacher 27), resources being no longer available (Teacher 23), and limitations of the physical environment (Teacher 17).

6.4.2.6 Resource-Driven Selection

Many teachers reported their resource selection as influenced by characteristics of the resources themselves. Table 6.7 offers some interview excerpts illustrating resource-driven selection and seven sub-themes—"ease of use," "adaptability," "flexibility," "aesthetics," "accessibility," "reliability," and "fit for purpose"—that contribute to a pre-disposition taxonomy.

Table 6.7 Interview excerpts illustrating resource-driven selection

Teacher	Interview excerpt
Teacher 1	(a) "The content… it's easy to use… what I think the students would think of it… visually, what does it look like, is it aesthetically pleasing… the key thing is does it cover what I want it to cover?" (b) "I would not rely on the textbook to do any of the delivery, obviously." "When I plan my lessons, I would look to see what sort of questions they are using, but I often find them quite limited in how many questions they ask, or if I taught a lower set there aren't enough questions or they get too hard too quickly, or if I have higher set they don't extend them as much as one would want."
Teacher 6	"Once they've got the concept, student texts are good to give them practice."
Teacher 7	[Discussing why he uses students as a resource] "The student sitting next to them then pipes up and they give a much better explanation than you."
Teacher 11	(a) "Some resources are not well-organized." (b) "Textbooks here are not known for teaching negative numbers in any kind of meaningful way."
Teacher 12	"Some sites understand teachers." "They offer more."
Teacher 13	"No textbook promotes conversation [classroom dialogue]."
Teacher 17	"If it's recommended by someone in the department, I'll certainly look at it."
Teacher 18	"It's user-friendly. I don't have to work too hard."
Teacher 19	"[Textbooks are rigid.] There is a cap on the learning."
Teacher 24	"[Websites that have worksheets or games or puzzles that can be used across the levels offer] an endless supply for every topic we have."
Teacher 25	"[I use an online maths program] quite a lot because it is colorful, interactive, and can be projected in front of the class… I never use it for homework because it encourages students to just get the answer. [It] doesn't convey the importance of the structure of getting an answer."
Teacher 27	"Don't have to adapt too much." "Ease of access for my students."
Teacher 29	"Adaptability." "A pdf cannot be changed."
Teacher 33	(a) [A resource must be] "fit for purpose." (b) [Commenting on a mega-site] "You have to know what you're looking for."

The illustrative comments for the resource-driven theme reflect teachers' criteria for selecting material or digital resources, including textbooks, as well as human resources. The comments from Teacher 1 in this table and in Table 6.6 indicate that many teachers had well-developed criteria, including criteria for different types of resources.

Regarding the sub-themes for resource-driven, selection based on characteristics of the resources themselves included *ease of use* (Teachers 1a, 11a, 12, 18), *adaptability* (Teachers 27, 29), *flexibility* (Teachers 1b, 19, 24), *aesthetics*

(Teachers 1a, 25), or *accessibility* (Teachers 7, 18, 33b). Some teachers spoke about whether a resource "works." This notion is captured in the sub-theme *reliability* (Teachers 11b, 17). Related to reliability, the sub-theme *fit for purpose* reflects that resource features might be assessed together with objectives for teaching and learning (Teachers 1, 6, 7, 25, 33a); the same resource might have both advantages and disadvantages (Teachers 25, 30).

6.4.2.7 Culture-Driven Selection

In addition to teacher comments reflecting the national characteristics of England (see Sect. 6.2.3), some teacher commentary indicated how department-level characteristics at a school can impact individual resource selection. Table 6.8 offers some interview excerpts illustrating department-level culture-driven resource selection and four sub-themes—"teacher autonomy," "sharing," "collaboration," and "structured"—to contribute to a pre-disposition taxonomy.

Regarding the sub-themes for culture-driven selection, although collegiality was important to teachers at all the schools in the study, departmental cultures varied in

Table 6.8 Interview excerpts illustrating culture-driven selection

Teacher	Interview excerpt
Teacher 8	[At a school that wanted to increase resource sharing, and recognized that individual conversations limited departmental sharing] "We're good at talking to each other whereas we're not as good at uploading a bank of resources."
Teacher 16	[In a department that is increasingly collaborative, this teacher noted that this department is more collaborative than at his previous school]. "I use collaboration and our shared resource bank" [with resources written by teachers in the department] "and adapt for my style. Collaboration makes life easier."
Teacher 21	"I do try to use lots of very different resources and I don't use textbooks really at all."
Teacher 23	[Does not often look beyond what's available at the school] "I make a lot of them myself; that's the way I prefer to teach."
Teacher 28	[Did not mention any specific resources but relied on four sets of out-of-print textbooks] "I like the constraints of following their [the textbooks'] path."
Teacher 30	[At a school where there were some resources all mathematics teachers were expected to use, including resources to uniformly structure student work] "Structuring students' time outside of school, to extend students' working hours" had "really paid dividends" for them.
Teacher 35	[Struggling to teach a topic using a new approach recommended by his department, which is moving towards uniform lesson plans that are collaboratively developed but written by one teacher]. "It's like teaching an old dog a new trick." "I came more through the traditional way of teaching and learning" and "teach it quite successfully." "I have not made" the new approach "my own way of thinking about it, using it, doing it."

the degree to which teachers autonomously selected resources. At a school that discouraged textbook use and emphasized *teacher autonomy,* teachers demonstrated an accompanying variation in resource selection (Teachers 21, 23, 28). At a school that recognized the way in which their sharing by way of conversations limited overall departmental sharing, there were attempts to increase *resource sharing* using a digital tool (Teacher 8). At a school that attributed its increase in collaboratively designing and using resources to successful digital sharing and the absence of a textbook, several teachers spoke of the benefits of the more *collaborative* approach (Teacher 16). At a school that was moving towards a more *structured* approach, positive results for students were attributed to a higher degree of uniformity with respect to resource use (Teacher 30). Using particular resources to structure students' time outside of school was just one example of their expectations for teachers. Finally, at a school that was moving towards greater uniformity and had expectations for teachers' resource use, teachers who were used to more autonomy were resistant (Teacher 35), highlighting the possible tension between a department culture and an individual teacher's preferences.

6.5 Discussion

In this chapter we have introduced the constructs of *pool of possibilities* to represent teachers' resource options and *pre-disposition taxonomy* to schematize teachers' selection decisions. These constructs have also served as organizers for the results we obtained from the interview study.

The results have enabled us to begin construction of the pool of possibilities for secondary mathematics teachers in England and to specify resources in teachers' individual packages. Resources in these packages are those teachers will likely draw on when planning, and include specific and generic resources and resource banks, but these resources are not necessarily used together and may not even be compatible.

The multiplicity of resources selected by individual teachers, coupled with the large variation across teachers in the resources they select, suggest the existence of a similarly large variation in the nature of learning opportunities offered to students across English classrooms. According to Morris and Hiebert (2011), the large variation in learning opportunities for students across classrooms is an important problem in education, primarily due to the quality of these learning opportunities also being highly variable. We argue that this "variation" problem is amplified in England by: (1) the tradition of localized interpretations of the national curriculum (Ruthven 2013b); and (2) the lack of research on the mathematical and pedagogical affordances and constraints of even popular resources such as MyMaths selected by English teachers. Another issue with the plurality of resource use, possibly a growing issue elsewhere as textbooks face competition, is the obstacle that this may create for systemic improvement of classroom instruction. We do not argue for more textbook-based teaching in England as a solution to the variation problem, but we think one cannot underestimate the idea that, when well-designed textbooks play a key role in everyday classroom instruction, accompanied by teacher guides, instructional reform at scale receives support (Ball and Cohen 1996).

The results have also enabled us to begin construction of a pre-disposition taxonomy to characterize the complex set of drivers influencing teachers' choice of instructional resources, when choice is an option. The six drivers of resource selection that emerged from our analysis are in line with ideas we discussed in the Background (see Sect. 6.2). The correspondence with Cohen et al.'s (2003) definition of instruction, as "interactions among teachers and students around content, in environments" (p. 122), is notable: teacher-driven, student-driven, and mathematics-driven resource selections correspond directly to the three main components of instruction, while the remaining three drivers—constraints-driven, resource-driven, and culture-driven resource selection—correspond to the notion of environments.

Our analysis delved deeper into each driver of resource selection, identifying sub-themes that may represent items for a pre-disposition taxonomy. Our results overall suggest that teachers in England may be choosing resources for lesson-level work but that their departments may be promoting sequences of lessons or promoting pedagogical approaches that may differ from teachers' individual approaches. An important cross-cutting theme in England, then, is the relationship between individual teachers, including the notion of teacher autonomy that appears to be a characteristic of the cultural environment, and their departments in designing and implementing the school's scheme of work. Another important cross-cutting theme that affects teachers' ability to select resources is change that occurs over time. Teachers, their departments, and resources in the pool of possibilities may be out of sync in many ways. Teacher autonomy in relation to department-level decisions, change over time, and the effect of these on children's opportunities to learn are topics about resource selection for further research.

Items for a pre-disposition taxonomy that were suggested by existing studies (see Sect. 6.2.2) could enhance further development of the pre-disposition taxonomy that developed from the interview study. Some items proposed from existing studies align with items on the pre-disposition taxonomy. For example, "quality of mathematical content" from Gueudet and Trouche (2009a, b) corresponds to the sub-theme "content appraisal" for mathematics-driven selection. Other items from the existing studies could help delineate sub-themes. Examples include "interest in innovation" from the Ruthven and Hennessey (2002) study and "interest in resource design" from the Quentin and Bruillard (2013) study, both fitting within the sub-theme "teaching approach" and helping articulate what teaching approach might encompass. Existing studies also raise questions about the pre-disposition taxonomy from the interview study. Using Recker et al. (2005) as an example, if a teacher has "difficulty finding resources," is that a teacher characteristic or a constraint reflecting some lack, or should it be recognized as cutting across themes, relating them? In our view, such questions show how the notion of a pre-disposition taxonomy may be used to promote discussion about resource selection that could ultimately provide support for teachers.

Another interesting result from our analysis of teachers' choice of resources is that teachers in England do not appear to choose resources that enhance their knowledge of teaching or mathematics, the kind of professional learning supported by educative curriculum materials (Davis and Krajcik 2005; Davis et al. 2014). When asked how she learned about the errors students might make when learning mathematics, a teacher responded, "My experience." It takes time to learn from experience, and this can affect the quality of students' opportunities to learn. Knowledge of probable student responses when learning a topic, including their misconceptions and errors, are part of what is required for effective teaching (e.g., Ball et al. 2008), and teachers should be able to easily access this information, if available.

A leitmotiv in the literature on teachers' interactions with resources, common to various theoretical perspectives, is that resources and teachers influence one another (e.g., Remillard 2005; Stylianides 2016). However, teachers cannot be influenced by a resource unless they know of its existence and it is accessible to them. This is one reason for the mathematics education community to pursue the *curation* of resources on behalf of teachers (Albion 2014) so that teachers can more effectively take advantage of resource abundance. A pool of possibilities as we envision it would identify, classify and annotate resources, providing information about use, and about the affordances and constraints of available specific resources or resource types. This would be especially important for resources, such as those in Table 6.1, that appear to be having a major influence on children's opportunities to learn mathematics in England. Of course, supporting teachers in selecting resources is not merely a matter of curation. If teachers are to access resource quality, they need to develop a reflective stance towards not only what constitutes a quality or reliable resource for a given purpose (e.g., Rolando et al. 2013), but also what constitutes quality mathematics instruction for offering students sound learning opportunities. Teacher education has an important role to play in helping teachers develop this reflective stance.

Our research indicates that resource selection, including the issue of finding resources in an era of abundance, is complex and deserves more attention in studies of instructional resources. This is particularly applicable for studies conducted in countries where there is no single dominant resource (notably textbooks), but it can also be relevant to studies conducted elsewhere, for all teachers may be influenced by the current abundance of resources. Future research on resource selection and use might examine issues such as the following: relationships between specific and generic resources in a pool of possibilities, including re-designed resources resulting from the combination of other resources; the relationship between the popularity of a selected resource and resource quality (teachers may be relying on filters such as 'hits' and 'retweets' when selecting resources); factors influencing resource selection from among competing options, mapped against the pre-disposition taxonomy; and support for informed choices. Although some of these issues have a footprint in the existing literature on resources, research should, we believe, consider these issues with a view to the full spectrum of instructional resources teachers might be exploring from the pool of possibilities, thus enhancing the field's understanding of the nature of teachers' resource pre-disposition and the role that this plays in their everyday planning and teaching.

The study of these issues can draw upon, or further clarify, the constructs of pool of possibilities and pre-disposition taxonomy that we have introduced in this chapter. At a practical level, these constructs, used separately or together, can provide support for teachers' interactions with resources, called for in the literature (e.g., Rolando et al. 2013). A better understanding of the pool of possibilities can enhance resource accessibility while a better understanding of teachers' resource pre-disposition (as reflected in the pre-disposition taxonomy) can support more deliberate and advantageous selection from the pool of possibilities. Although inaugurated in the English context, these constructs are context-neutral, potentially suitable for development in other national or local contexts, for use by individual teachers, groups of teachers, teacher educators, or researchers, and for comparative studies.

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Author Biographies

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Chapter 7
Teachers as Designers of Digital Educational Resources for Creative Mathematical Thinking

Chronis Kynigos and Angeliki Kolovou

Abstract This chapter focuses on the design of digital C-books, c for creativity, incorporating dynamic constructionist artefacts that aim to induce creative mathematical thinking (CMT). We studied the design of C-books by mathematics teachers in collectives of educational professionals with a diversity of expertise. The analysis of the design process of a C-book on Curvature shows that the interactions fostered allowed mathematics teachers to learn from diverse practitioners. The C-book was developed as a collective document combining a variety of shared resources. The C-book technology allowing for the design of diverse malleable and improvable resources supported the infusion of constructionist and creative elements in the C-book resulting in more innovative approaches to teaching curvature.

Keywords Mathematics teachers' resources · Constructionist design
Social creativity · Creative mathematical thinking · Curvature

7.1 Introduction

Technology was first introduced into mathematics education as holding the potential to bring about fundamental changes in the experience of learning mathematics or even the power to transform mathematics teaching and learning. However, research has shown that the impact of technology on students' learning is less significant than initially anticipated (Clark-Wilson et al. 2014) and technology still plays a marginal role in mathematics classrooms. Since teachers have a key role in the uptake of digital technologies in classrooms, several researchers have focused

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on investigating teachers' interactions with digital resources (Pepin et al. 2013) or teacher expertise related to the integration of digital technologies into everyday teaching practice (Mishra and Koehler 2006; Ruthven 2014). Others (Fuglestad et al. 2010; Laurillard 2012) have stressed the importance of involving teachers in all stages of the design process of learning activities involving the use and design of digital tools. In this way, teachers can become aware of the complexities involved in the use of digital tools in the classroom and become "active agents in the process of creating new cultures of practices capitalizing on the possibilities of digital tools" (Fuglestad et al. 2010, p. 309).

Acknowledging the complexity of technology integration into mathematics teaching thus induced a shift towards supporting teachers not only by providing them with resources, but by helping them to design their own resources individually and collaboratively. The main avenue for providing such support is through teacher professional development courses placing teachers in communities of practice (CoP, Lave and Wenger 1991) where resource production becomes an experience over which technological, pedagogical, and content knowledge (TPaCK, Mishra and Koehler 2006) is shared and cultivated. This support entails, among other things, conceiving of new media for ongoing exchanges (e.g., via platforms) and taking into account student and classroom interactions, as well as teachers' interactions (Trouche et al. 2013). Trouche et al. (2013) show how resource production and perpetual reformulation (they aptly use the term 'a living document' for a resource under continuous improvement) is an integral part of the teacher's professional life and an important vehicle for manifesting reflexive teaching practice. In this wake, some in-service teacher education programs have placed the teacher at the centre of the design process (Kynigos and Kalogeria 2012; Trouche et al. 2013), and have supported their ensuing exchanges in CoP to focus on the sharing of resources and even their collaborations in joint resource productions. So, individual and joint production of digital resources becomes a natural professional development activity involving the sharing and discussing over this kind of resource and its use in the classroom.

In this chapter we take the idea of joint construction of digital resources for teacher professional development further by means of placing the mathematics teacher in hybrid communities where she/he collaboratively designs with colleagues of diverse expertise. We ask the question 'how can this experience inspire creativity in the design process and product'?

We do acknowledge that much more research is needed in order to understand the complex interplay of several mechanisms that are being put into action during a collaborative design processes (Kynigos and Kalogeria 2012; Pepin et al. 2013) and, in particular, to clarify the role of the mathematics teacher as designer of digital resources in collectives.

The design of digital resources is not a neutral process, in the sense that it reflects certain views on teaching and learning, which may be affected by the new opportunities technology offers (Trouche et al. 2013). Take for example the case of expressive digital media for mathematical meaning-making. Hoyles and Noss (2003) use this term to signify customizable tools that offer learners various semiotic registers with which mathematical knowledge can be explored, such as dynamic geometry systems. Expressive media for mathematics education afford dynamic manipulation, interlinked representations, simulations of phenomena and situations embedding mathematical rules. They are thus designed to enable knowledge construction through creative engagements with (advanced) mathematical concepts. In fact, meaning-making through the process of tinkering with digital media has been perceived as a creative activity (Kafai et al. 2009). Likewise, interactions with such media challenge teachers' epistemology and perceptions of teaching and learning mathematics (Kynigos 2007) and might induce novel pedagogical approaches and teaching strategies. Designing for students to engage in creative activities with digital media is, thus, a challenging endeavour, which also brings the issue and study of creativity in the design to the fore. In collaborative design activity, process and product are inextricably linked, in the sense that the former draws its very existence on the pursuit of the creation of some novel product, and that a creative product acquires its substance as the end-result of some design processes. However, the role of creativity in instructional design (or specifically in educational resource design) has been only recently acknowledged (Clinton and Hokanson 2012).

In this chapter, we look for ways to enhance this kind of creativity by means of addressing it as a social phenomenon in collaborative design (Daskolia et al. 2016). In particular we are interested in the question of how can the mathematics teacher be inspired to think creatively when working in a group of colleagues with diverse expertise. How can this diversity generate creativity in the design process and product? We thus look at mathematics teachers working with researchers, designers of digital tools, teachers educators, teachers of other domains and creative writing specialists.

To make this activity meaningful and challenging, we provided our diverse groups with a new genre of resource, a particular type of authorable e-book which we called the 'C-book' which was used to encourage the re-considering of the structure of mathematics curricula in the quest for innovative conceptual fields (Vergnaud 2009) enabled by the new medium and aiming to generate creative mathematical thinking in students. From a technical point of view, a 'C-book' allows for text to intertwine with any kind of expressive media. A mathematical story or problem may easily involve different digital widget instances for students to dynamically manipulate, construct and tinker with (Kynigos 2015). In this way, a medium meshing narrative with dynamic constructionist widget instances may enhance student creativity in mathematical thinking and meaning-making. Before moving on to how we viewed collaborative design of C-books, let us focus for a moment on how we view creative mathematical thinking as it constitutes a main design target for the C-book medium.

Our perception of creativity is aligned with the view of Silver (1997) who sees creativity as a disposition toward mathematical activity that can be broadly fostered in the general school population through engaging students with ill-structured, open-ended problems. Mathematical creativity is susceptible to instruction and can be supported with tools that integrate open, real-life problems amenable to multiple solutions, offer students a multiplicity of representations, promote inter/intra-disciplinarity and foster constructionist activity (Papadopoulos et al. 2016). So, narrative and stories embedding opportunities for mathematization through the use of digital widget instances seemed an appropriate medium to encourage mathematical creativity in students.

We began from the premise that collaborative design by educational professionals with diverse expertise would generate a socio-technical environment likely to produce creative ways to enhance creative mathematical thinking (CMT) for students. In addition, by involving mathematics teachers in all stages of the design process, we aimed to induce their reflection on the affordances and pedagogies incorporated in the tools, the tasks and narratives under development, as well as the changes to the mathematical content and the classroom practices that the presence of technology brings about.

Addressing this problem is a complex venture. Not surprisingly, we did not find a single theory or framework which would enable us to make sense of creativity emerging in collaborative design interactions and production versioning. Based on our experience with forging connections between frames for mathematical learning with digital media (Artigue and Mariotti 2014), we engaged the employment of four theoretical constructs which we felt would critically inform our study of collaborative design process and production: Social Creativity, Constructionism, Boundary Crossing (BC), and the Documentational Approach (DA). Instead of adopting the metaphor of a theoretical landscape where priority is given to ad hoc connections between fragmented theories (Artigue and Mariotti 2014), we found it more useful to think of these four theories in a nested structure. We thought of social creativity as being our focal point and constructionism enhanced by the other two constructs, BC and DA as being the tools to think of the former. Our connections thus had a sense of directionality and complementarity, as shown in the subsequent sections (Kynigos et al. 2016). BC and DA fed our understanding of constructionism in collaborative design which in turn helped us make sense of emergent social creativity in the design groups. In the following sections, we explicate how these frameworks were employed in an integrative approach to teachers' collaborative design activity. In particular, the first, social creativity, is delineated in Sect. 7.2 of this chapter, while the others are explained in Sect. 7.3.

7.2 Social Creativity in the Collaborative Design of Digital Educational Resources

In order to form diverse communities in which teachers would be prone to be creative and produce out-of-the box but relevant ideas, we found Fischer's "community of interest" (CoI) to be directly relevant and useful (Fischer 2001; Kynigos and Daskolia 2014). A CoI is a collective of practitioners from diverse disciplinary and professional domains and differs from a Community of Practice (CoP, Lave and

Wenger 1991)—defined as a community of practitioners in a particular domain who undertake a similar work—in that CoI members are representatives of more than one CoP who were brought together to solve a particular problem of common concern. A CoI collaboratively designing a C-book should comprise professionals representing diverse CoP, each with a distinct disciplinary and knowledge background, field of professional practice and area of expertise. At the same time, many CoI members bring overlapping fields of experience by simultaneously participating in more than one CoP (e.g., a teacher with design expertise). Thus, each CoI member carries one main CoP membership but also potentially a subsidiary working knowledge and practical experience background from participating in more than one CoP in parallel. Inside the CoI, these members participate as representatives of the CoP they choose as most appropriate, according to their personal statements for their everyday professional activity, their personal interests and the task at hand. In the current study, the CoI consisted of teachers from different/ diverse educational domains with a shared interest in designing digital resources for CMT.

The social dimension in collectives of educational designers and its role in enhancing both individuals' and the group's creative capacity has gained, so far, little attention in mathematics education research. Social Creativity (e.g., Fischer 1999) offers an appropriate theoretical frame for understanding and fostering the creativity of teachers working in collectives operating within particular technological environments. It addresses creative performance both as a process and through its outputs, as they grow out of the interaction taking place among the individual members of a specific group or community and between them and computational media, technologies and artefacts, all of them constituting and acting as a context to this end.

Social creativity can be fostered by innovative digital environments that enable users to frame and solve problems collaboratively. Meta-design is a methodology that addresses the design of socio-technical environments that help users "to be creative by allowing them to go beyond the explicitly described functionality of any artefact, to use it in new ways, to evolve it, and to explore its potential for new processes" (Fischer 2002). This is a perspective allowing socio-technical environments to stay open to modification and customization to new needs and opportunities that emerge during use. As such, meta-design addresses critical design challenges, including coping with ill-defined problems, supporting reflective practitioners and design as a collaborative process (Giaccardi and Fischer 2008) thus, eventually, design as a constructionist activity.

In the context of the research presented in this chapter, social creativity in the design of digital educational resources is addressed, on the one hand, through specially formed Communities of Interest. In addition, a constructionist digital environment (the C-book environment) was specially developed to allow CoI members to coordinate their efforts in designing CMT resources. To analyse the creative interactions taking place during collaborative design of digital media fostering CMT, three other relevant constructs were used, namely constructionism, boundary crossing and community documentational genesis. In the following section, we elaborate on these constructs and attempt to draw connections between them.

7.3 Constructionism and Its Intersection with Boundary Crossing and Documentational Appoach Frameworks

Constructionism (Harel and Papert 1991) emerged through the study of ways in which technology can be designed and used to enhance mathematical learning. It addresses the design for and the study of constructivist learning, in particular, the process of generating meanings and expressing them through interacting with the structural affordances of artefacts, mainly digital artefacts (Kynigos 2015). A key notion here is that of 'half-baked microworlds' coined by Kynigos (2007). Half-baked microworlds are digital artefacts intentionally designed and given to students as malleable and improvable asking of them to engage in discovering faults and shortcomings and changing them. It is through this process of exploration, bricolage and construction with expressive digital media that students—individually and collectively—are enabled to construct sound mathematical meanings.

Apart from providing analyses of designs affording constructionist learning, constructionism also provides analyses of design processes with an emphasis on understanding the meanings that designers generate through and with the changes they implement to the artefact under construction.

7.3.1 Constructionist Artefacts as Boundary Objects Facilitating Boundary Crossing Between Designers

In the process of communal design of digital resources, the diversity in collectives of professionals causes difficulties in communication and collaboration, but it also provides unique opportunities for the creation of new shared knowledge. According to Akkerman and Bakker (2011) boundaries are defined as the "socio-cultural differences that give rise to discontinuities in action and interaction" (p. 139), which can be overcome through *boundary crossing processes*, i.e., efforts made by individuals or groups 'at boundaries' to establish or restore continuity in action or interaction across practices. Hence, social creativity can be viewed as located in and nurtured by the boundary crossing encounters among the CoI members, in the mechanisms and strategies employed and as an outcome of them. In fact, Akkerman and Bakker (2011) distinguished between four mechanisms of learning at the boundary: (a) *identification* processes through which boundaries are reconstructed without necessarily overcoming discontinuities, leading to a renewed sense-making of different practices, (b) *coordination* which entails processes such as communicative connection between diverse practices, leading to the overcoming of boundaries, facilitating effortless movement between different sites, (c) *reflection* on the differences between practices leading to an enrichment and new construction of identity, and (d) *transformation* leading to profound changes in practices and the emergence of new in-between practices.

Boundary crossing efforts are facilitated by *boundary objects* that fulfil a bridging function between 'intersecting social worlds' (Star and Griesemer 1989). They come in the form of artefacts (such as specially designed computer tools), discourses (as a common language), or processes that allow the coordination of actions. In the course of designing a C-book, designers negotiate and collectively implement changes to the artefacts under construction. The malleability of constructionist artefacts permits their fine-tuning so they are appropriate for the practices of specific communities of designers working together in a CoI as well as across these communities (Healy and Kynigos 2010). In this way, these artefacts play the role of improvable boundary objects that help to establish and maintain a common ground supporting communication and shared understanding between different communities of practice involved in a CoI.

7.3.2 Constructionist Design for Collective Documentational Genesis

For analysing teachers' interactions with constructionist media, the *instrumental approach* (Guin et al. 2005) is highly relevant. A key notion of this approach is *instrumental genesis*, a process through which an artefact is associated with a set of schemes resulting in the transformation of artefacts into instruments (Gueudet and Trouche 2009). This is a reciprocal process, since the tool shapes the user activity (a process called *instrumentation*) and, vice versa, the tool is shaped by the user activity (a process called *instrumentalization*). The latter process is particularly enabled when interacting with incomplete, malleable, improvable constructionist tools (Kynigos and Psycharis 2013).

Stemming from the instrumental approach, the *documentational approach of didactics* (Gueudet and Trouche 2009) focuses on the interactions between mathematics teachers and resources and on the implications for teachers' professional development. Gueudet and Trouche (2009) coined the term *documentational genesis* to describe the ongoing process through which a new resource and a scheme of utilisation of this resource are jointly generated. In relation to collectives instead of individual teachers, *community documentational genesis* describes the process of gathering, creating and sharing resources to achieve the teaching goals of the community. The result of this process, the community documentation, is composed of the shared repertoire of resources and shared associated knowledge (Gueudet and Trouche 2012). A collaborative design activity involving teachers as designers of creative educational resources, is thus a process that is expected to trigger collective documentational genesis.

7.4 Design of a C-book Around Curvature

Curvature has hardly been addressed as a conceptual field (Vergnaud 2009) with respect to its potential to generate environments rich in opportunities for mathematical meaning-making by students. In traditional curricula, it lies disparately in Euclidean Geometry sections, in Algebra and Calculus depicting systematic co-variation, in 3D Geometry but in simple applications like conic sections. The dynamic and diverse representational repertoire provided by digital media allows us to approach curvature anew with a disposition to re-structure (to use Wilensky and Papert's term, 2010) the ways in which mathematics is conceptualized in education in the quest to make it more attractive to students, affording meaning-making, creative mathematical thinking and engagement. Previous research has indeed shown that dynamic digital environments and especially 3D spatial environments support students in constructing meanings about challenging conceptual fields (Kynigos and Psycharis 2003; Latsi and Kynigos 2011; Zantzos and Kynigos 2012). The study presented here focuses on the collective design of the 'Curves in space' C-book which eventually revolved around a story involving comparisons between Archimedes' and exponential spirals.

7.4.1 The CoI

The recruitment of CoI members was based on former collaborations developed as part of educational innovations related to the design of digital resources initiated by the Educational Technology Lab, the coordinating institute of this research study. In addition, CoI members had to exhibit particular traits, such as professional responsibility and sense of commitment, 'playful' attitude to work and sense of satisfaction out of professional collaboration with others, being open to accepting or acknowledging the perspectives projected by other communities, and willingness and readiness to cross own 'boundaries', critical, argumentative, and creative stance. The seven CoI members that were invited to participate in the joint design of the C-book 'Curves in Space' were graduate or in-service teachers in different levels of education (from primary to tertiary education) specialized in mathematics, mathematics education, creative writing, computer mediated communication and the design of digital tools for mathematics education. Among them, four had a strong mathematical background. This diversity in knowledge domains, perspectives and cultures as well as complementarity of expertise, were important factors for boosting social creativity in the design of the C-book.

7.4.2 The Computational Environment

The C-book environment provides the 'CoICode workspace', a tool for asynchronous online discussions allowing designers to choose between a threaded forum discussion organised in a tree-like structure (see Fig. 7.1) and a mind map view. When posting a contribution, CoI members have to state its nature (i.e., alternative, contributory, objecting, off task, or management) by using a specific icon, and can attach and refer to objects like online resources, texts or widget instances that reside in the C-book under construction. Furthermore, the CoICode tool provides the designers with the possibility to rate any idea or digital artefact in the form of CoICode contribution against three criteria[1]: (a) novelty, (b) appropriateness, and (c) usability of the contribution, on a yes/no basis. A creativity score per contribution is then automatically calculated as the aggregate score from the votes received on each criterion and across raters. Based on this score all generated ideas per C-book can be classified in terms of creativity, as well as in terms of their degree of perceived novelty, appropriateness, and usability.

In addition, the environment contains a platform which is the space for authoring (the C-book authoring tool) and the space where students interact with the C-book (the C-book player). The platform is designed to incorporate pages with dynamic and configurable widget instances accompanied by corresponding narratives (see Fig. 7.2). The authors can write text, attach links, files, or widget instances out of a set of widget factories (e.g., Geogebra is a 'widget factory', and a microworld of this factory incorporated in a C-book page is a 'widget instance'). In the C-book page depicted in Fig. 7.2, MaLT Turtlesphere, a 3D Logo-Based Turtle Geometry tool affording dynamic manipulation of variable values was used (http://etl.ppp.uoa.gr/malt2). With this tool, spirals can be generated by either constant or incremental curve and torsion changes to a turtle respectively repeating very small displacements.

7.4.3 Analysis of the C-book Design Process

Our data were: (a) the 124 contributions uploaded in the 'Curves in Space' workspace from the outset of the design process (6/4/2015) until the final version of the C-book was released (23/7/2015), (b) the actual C-book produced in terms of structure (pages) and contents (the 'script', the widget instances and the respective narratives per page), and (c) the data automatically collected and analyzed by CoICode, such as time series interaction data and/or creativity scores extracted per idea posted as contribution in CoICode workspace. The level of analysis employed

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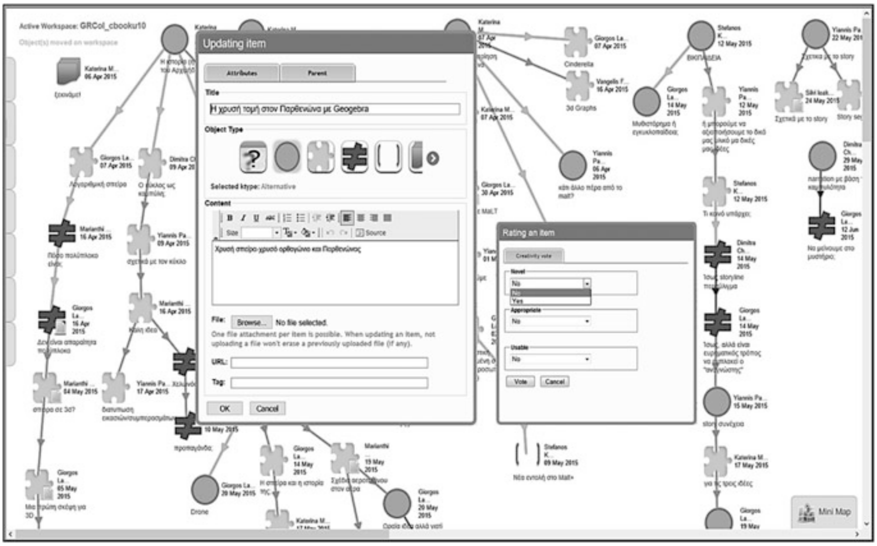
[1] These three criteria were developed throughout several cycles of work of a larger project focusing on the communal design of digital resources for fostering mathematical creativity (www.mc2-project.eu), of which the current study constitutes a part.



Updating item

Title: Η χρυσή τομή στον Παρθενώνα με Geogebra

Object Type

Selected ktype: Alternative

Content: Χρυσή σπείρα-χρυσό ορθογώνιο και Παρθενώνας

File: Browse... No file selected.

One file attachment per item is possible. When updating an item, not uploading a file won't erase a previously uploaded file (if any).

URL:

Tag:

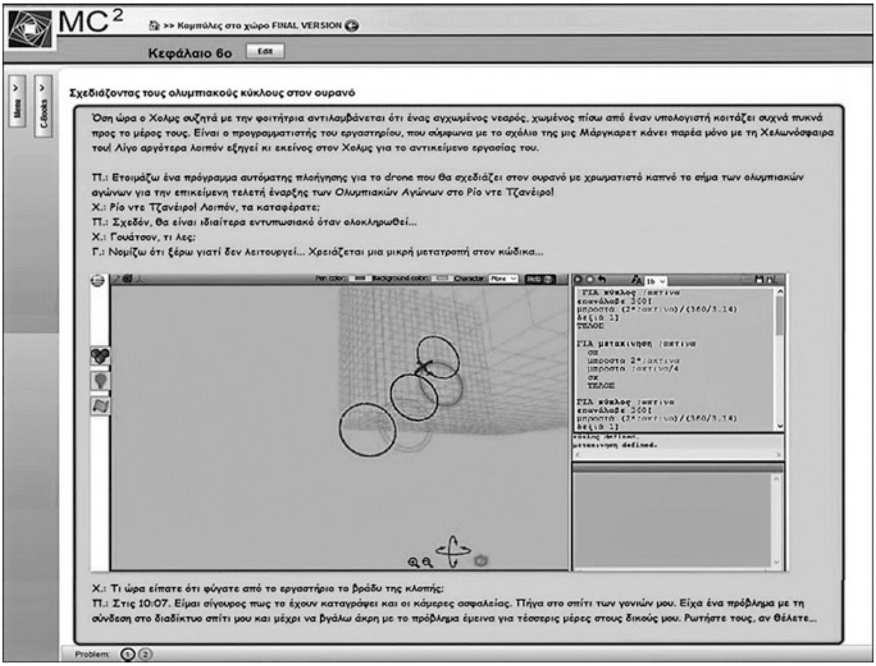
Rating an item

Novel: No

Appropriate: No

Usable: No

Fig. 7.1 The CoICode collaborative workspace of the ‘Curves in space’ C-book



Σχεδιάζοντας τους ολυμπιακούς κύκλους στον ουρανό

Όση ώρα ο Χολμς συζητά με την φοιτήτρια αντιλαμβάνεται ότι ένας αγχωμένος νεαρός, χωμένος πίσω από έναν υπολογιστή κοιτάζει συχνά πυκνά προς το μέρος τους. Είναι ο προγραμματιστής του εργαστηρίου, που σύμφωνα με το σχόλιο της μις Μάργκαρετ κάνει παρέα μόνο με τη Χελωνόσφαιρα του! Λίγο αργότερα λοιπόν εξηγεί κι εκείνος στον Χολμς για το αντικείμενο εργασίας του.

Π.: Ετοιμάζω ένα πρόγραμμα αυτόματης πλοήγησης για το drone που θα σχεδιάζει στον ουρανό με χρωματιστό καπνό το σήμα των ολυμπιακών αγώνων για την επικείμενη τελετή έναρξης των *Ολυμπιακών Αγώνων* στο Ρίο ντε Τζανέιρο!
Χ.: Ρίο ντε Τζανέιρο! Λοιπόν, τα καταφέρατε;
Π.: Σχεδόν, θα είναι ιδιαίτερα εντυπωσιακό όταν ολοκληρωθεί...
Χ.: Γουάτσον, τι λες;
Γ.: Νομίζω ότι ξέρω γιατί δεν λειτουργεί... Χρειάζεται μια μικρή μετατροπή στον κώδικα...

```
 ΓΙΑ κύκλος :ακτίνα
επανάλαβε 360[
μπροστά (2*:ακτίνα)/(360/3.14)
δεξιά 1]
ΤΕΛΟΣ

ΓΙΑ μετακίνηση :ακτίνα
  σα
  μπροστα 2*:ακτίνα
  μπροστα :ακτίνα/4
  σκ
  ΤΕΛΟΣ

ΓΙΑ κύκλος :ακτίνα
επανάλαβε 360[
μπροστά (2*:ακτίνα)/(360/3.14)
δεξιά 1]
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κύκλος defined.
μετακίνηση defined.
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Χ.: Τι ώρα είπατε ότι φύγατε από το εργαστήριο το βράδυ της κλοπής;
Π.: Στις 10:07. Είμαι σίγουρος πως το έχουν καταγράψει και οι κάμερες ασφαλείας. Πήγα στο σπίτι των γονιών μου. Είχα ένα πρόβλημα με τη σύνδεση στο διαδίκτυο σπίτι μου και μέχρι να βγάλω άκρη με το πρόβλημα έμεινα για τέσσερις μέρες στους δικούς μου. Ρωτήστε τους, αν θέλετε...

Fig. 7.2 A ‘Curves in Space’ C-book page asking students to fix the code for programming a drone to imprint circular traces in this study involved the selection and analysis of critical episodes, i.e., relatively brief and uninterrupted periods in CoICode discussion, shedding light on some important aspect of the social creativity processes and/or products developed, by focusing on the interactions among the CoI members and with the C-book technology. The identification of each critical episode was guided and supported by the data automatically collected and analysed by CoICode. Furthermore, we traced paths of socially creative ideas, which stretch over longer periods of time and include several critical episodes, in terms of the critical moments in their evolution from the initial to the final idea (i.e., an idea implemented and incorporated into some part of the C-book). Again, the identification of creative ideas was guided and supported by the CoICode data. The analysis of the path of creative ideas enabled us to highlight conditions of the emergence of creative interactions among the CoI members, such as the role of specific resources functioning as boundary objects, or the complementarity of background of the C-book unit designers. This qualitative approach to understanding social creativity in the design of C-books was chosen to shed light on the social nature of the processes involved in the development of ideas and in the examination of the C-book environment features which added to the formulation, elaboration, and cross-fertilisation of the CoI members' ideas.

7.4.3.1 Critical Episode: The Design of a Widget Instance as a Result of the Creative Interaction Between CoI Members with Diverse Expertise

The episode selected (see Fig. 7.3) started one month after the outset of the design process, it lasted 8 days (14/5–20/5/15), and the participants in it were three CoI members: George, Mathematics teacher and graduate student in Mathematics education, Dimitra, Literature teacher and graduate student in ICT in Education, specialised in creative writing, and Marianthi, MA ICT in Education graduate and developer. At that time an exchange of resources was taking place on the mathematical idea of Helix-Spiral between Stefanos, a senior figure in the CoI from the Mathematics CoP and teacher educator) and George. The resources shared in the CoICode workspace triggered a discussion among all CoI members, which revolved around the history of curves going back to Ancient times. The episode is initiated by Dimitra (14/5) who, inspired by the airplane functionality in MaLT, suggests that students calculate the distance covered by airplanes performing spiral movement during air shows. She points out that this could be an engaging activity, though it does not relate to the historical perspective on curves, discussed so far. George (14/5) responds enthusiastically, stating that the 'airplane idea' is a powerful one, elaborates on it and provides a Wikipedia link on jets streams. He also wonders about how Dimitra's idea could be joined together with the history of curves, suggesting, thus, the coordination of the different ideas discussed so far. Marianthi then puts forth a suggestion on a half-baked widget instance:

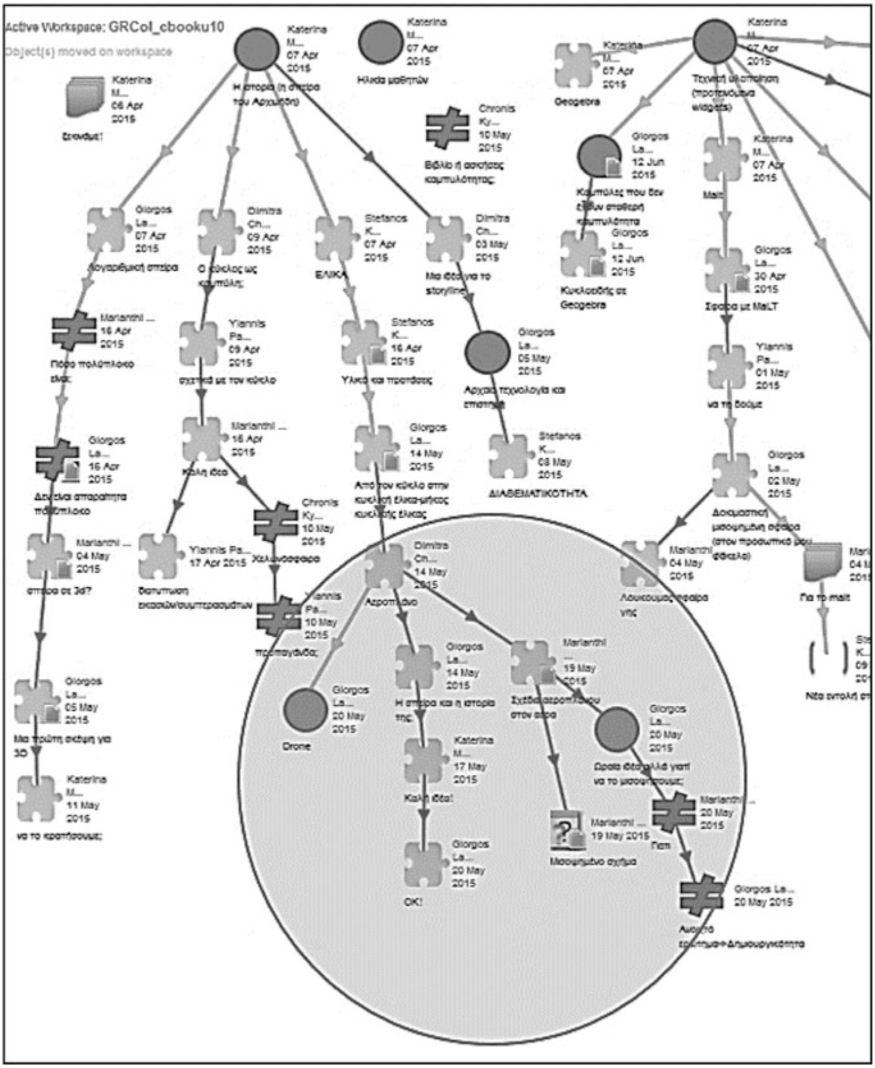

Fig. 7.3 Excerpt from the CoICode workspace depicting the critical episode

Marianthi (19/5):	[…] in MaLT + I created a procedure where the airplane movement forms the Olympic rings. I am sending you the complete code so that we can half-bake it, e.g. it can turn by a 45-degree angle in the last two turns so that the rings do not come out straight (attaches ‘Olympic_correct.txt’)
Marianthi (19/5):	I am sending you the one I wrote with the wrong angles (attaches ‘Olympic_wrong.txt’)

George (20/5):	I like it a lot! I suggest not to half-bake it, but ask students to create it by themselves by looking at an image of the Olympic rings […]
Marianthi (20/5):	[…] I think if it's half-baked it will be more challenging for students to correct it than create it from scratch. Also, we can focus on specific mathematical topics like the turn angle or the distance of cycles.
George (20/5):	[…] Since the unit addresses senior students it would be more creative to allow them work without such restrictions. If we half-bake it though, wouldn't it be better to use variables for the angles?

George (20/5) refers to drones as a more innovative alternative to airplanes and designs two alternative versions of the widget instance in which he adopts Marianthi's proposal. Finally, one of his versions was incorporated in the C-book including his suggestion of imprinting the traces of a drone instead of a plane (see Fig. 7.2).

This episode shows how the collective resource system is enriched through the sharing, reflection and transformation of individual resources to boundary resources. What is more, boundary crossing interactions between CoI members allowed the cross-fertilization of diverse perspectives: mathematics, digital tools development, and creative writing. Dimitra, having studied existing resources, is inspired to articulate the airplane idea stating in what ways it deviates from what has been heard before. Marianthi turns Dimitra's idea into a 'tangible' object, i.e., a widget instance, while George expresses considerations initiating an interesting exchange on the pedagogical affordances of different types of activities. He brings CMT to the fore and poses the challenge to other members to directly argue on specific pedagogical and technical affordances of the proposed activities. The final version of the instance appears in the C-book as a result of the coordination of George's and Marianthi's ideas. Social creativity is thus enhanced by exchanging, discussing on and modifying half-baked curve designs acting as boundary objects, allowing the communication and coordination of diverse perspectives. Mathematical resources thus take a mediational role between diverse perspectives undergoing several transformations and revisions until they are reified as widget instances in the C-book. As mathematics teachers negotiated over an emergent mathematical construction, they were challenged to reflect on and reconsider their beliefs and practices related to what constitutes a sound, challenging and creative mathematical activity, thus expand their learning.

7.4.3.2 The Evolutionary Path of the Narrative

The path presented below is related to the evolution of the narrative of the C-book. The respective path includes in total 52 contributions and stretches along the entire workspace. Early on in the design process, CoI members were concerned with devising a narrative that, together with appropriate widget instances, would provide opportunities for mathematization and meaning-making around curvature. The mathematical affordances of various digital tools also became an early topic of discussion so that tools, narrative and mathematical concepts were interrelated in the design of the C-book. Below we provide an excerpt of the path (see Fig. 7.4) including decisive contributions from individual CoI members and stress the social nature of the processes involved in the development of the scenario from its first appearance to its incorporation in the C-book. At that time a number of widget instances designed to afford creativity and meaning-making in curvature took the role of boundary objects by evolving through multiple cycles. However, a cohesive narrative that would incorporate and join together these elements was pending, despite the fact that some interesting ideas had been already suggested. The path sheds light into how the CoI members' conceptions about their productions in terms of didactical design (widget instances and corresponding learning activities) intertwined with their ideas about the narrative of the C-book.

Stefanos (22/6) presents a -rather loose- synthesis of his own and other members' ideas on the C-book narrative integrated in a new version of the C-book: the history of curves, two detectives working to solve a crime, a 3D printer laboratory, and solving riddles related to spirals. Stefanos had worked extensively in the past on curvature and had initially disposed to the CoI his own resources related to the history of curves, which directly affected the first two story versions.

George (24/6) reacts enthusiastically and attaches an elaborated version of Stefanos' story incorporating Sylvie's comments on enhancing the story: two renown detectives (Hercule Poirot and Sherlock Holmes) try to solve a mysterious robbery in a laboratory, which is connected to constructions related to curvature. Sylvie (25/6), a teacher and creative writing specialist who joins the discussion at that time, posts a new, radical version of the narrative relying on contemporary characters, which fuels an intense debate. Stefanos (25/6) objects to Sylvie's suggestion on the grounds that the storyline should blend with the widget instances so that students follow a learning trajectory working with tools of gradual increase in complexity. He also posts a document in which he justifies his rationale for building his own version in which mathematical concepts are presented in a coherent and meaningful way. George and Katerina (computer mediated communication specialist) react to Stefanos' post:

George (26/6): Very insightful comments, especially in relation to the way the current narrative supports the smooth integration of the learning sequence on curvature. […] Wouldn't it be better to make some corrections without discarding what we've done until now? […] (He attaches 'What a strange morning in the laboratory.doc' where he expands his previous version to include logarithmic spirals).

Katerina (27/7): I understand your argument related to mathematical coherence […] but I don't think that the new version rejects previous constructs and ideas […] but rather promotes them by organically binding them with a fresh, creative story.

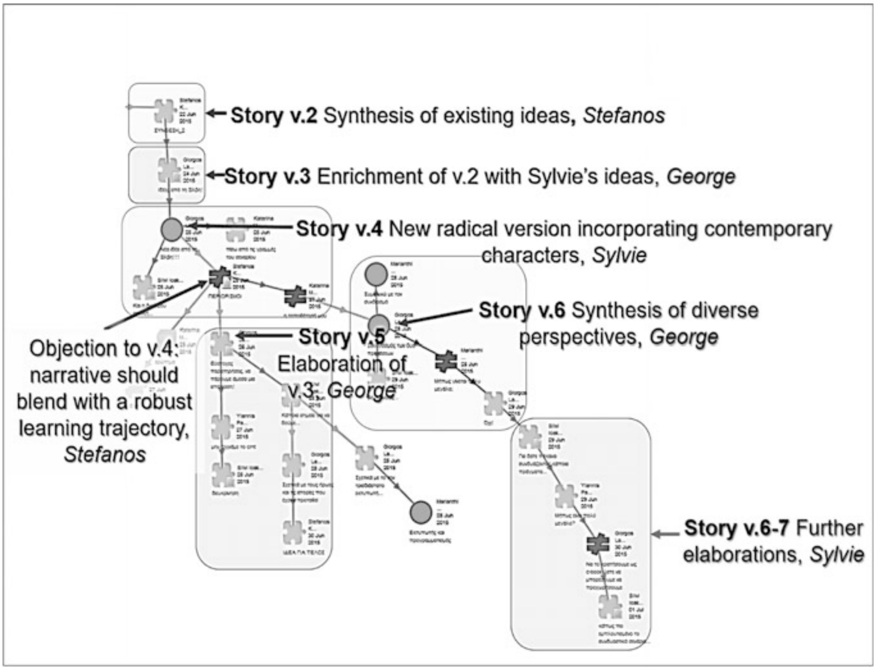

Fig. 7.4 Excerpt of the narrative path

At that moment, Ioannis, a senior researcher in mathematics education initiates a discussion on the C-book's CMT affordances with regards to students' involvement, both at the level of the narrative and at the level of activities (dealing with open problems, conjecturing, generalising, constructing, etc.).

Up to this point there are two opposing views on the scenario; a mathematics oriented strict, structured and robust learning scenario mainly supported by a senior member who is an experienced Mathematician, teacher and researcher (Stefanos) and a more innovative one which embodies a set of characters and situations of contemporary culture, aiming at increasing the readers' motivation and the narrative power of the story. This tension is released when George replying to Katerina, posts a new synthetic version of the scenario attempting also to address Ioannis's preoccupations regarding students' active involvement:

George (28/7): Here is a proposal for synthesizing the two alternative suggestions keeping the excellent work we've done till now […]

In the next two versions of the scenario Sylvie and George collaborate so that Sylvie presents a more robust synthetic version that organically integrates the designed widget instances.

The coordination of the two prevalent perspectives in the design of the C-book, i.e., the mathematics and the creative writing perspective, made possible the infusion of creative elements in the narrative, while not losing sight of its mathematical focus on curvature. It is noteworthy that these two perspectives are not only gradually reconciled after the catalytic intervention of George, they also enrich each other; on the one hand, the senior mathematician deviates from his original strictly structured storyline and proposes two additional ideas much more innovative than his initial ones. On the other hand, the creative writing specialist, after closely collaborating with mathematicians, comes to adjust her story in a synthetic version. In addition, both Sylvie and George reconsider their previous practice in the light of the rich exchange on CMT affordances. These reflective processes are essential for the interweavement of widgets instances with the narrative into a concise whole. Social creativity is thus facilitated by the meshing of the Sherlock Holmes narrative with curvature, which can only have emerged because of the diversity in the CoI. Furthermore, the process of story versioning boosts social creativity as it allows for the generation of new ideas which capitalize on, object to, and finally synthesize previous ones. It is an ongoing process where ideas are adjusted, adapted and combined to produce new documents.

7.5 Conclusions

In this study we tried to show how creativity might emerge in mathematics teachers' design processes and products under particular circumstances, that is, as they acted as members of a distinct socio-technical environment. The characteristics of the environment were based on the principle of integration. The technology afforded the meshing of narrative text and diverse constructionist widget instances. The collaboration was based on integrating expertise in a designer CoI, where our mathematics teachers worked with Dimitra, a language teacher, Sylvie, specialised in creative writing and story-telling, and Marianthi, a developer of constructionist widgets. Our analysis of social creativity in the design process of a C-book on curvature focused on the boundary crossing interactions between the CoI members and the role of the narrative in conjunction to widget instances as a key resource for the development of social creativity. Furthermore, to understand the interactions amongst the CoI members, we drew from diverse theoretical constructs. We employed BC and DA to elaborate on constructionist collaborative design processes so that we could illuminate the potential for generating social creativity in our mathematics teachers. We saw creativity as connected to the growth of their TPaCK within a teacher professional development capacity but also as a strategy to generate meaningful uses of digital media in the classroom based on the encouragement of CMT in students.

Two important boundary crossing processes, those of coordination and reflection, enhanced social creativity establishing communication between our mathematics teachers and colleagues from different communities of practice, i.e., from language teaching and computer science. Our mathematics teachers thus came to appreciate and appropriate the ideas of connecting a helix to ancient monuments and artefacts, of embedding mathematical concepts in a theft mystery storyline, and of affording digital modifiable representations of spirals for students to generate meanings of constant and systematic turn and torsion change. Reflection was a process which gave ground to the fertile synthesis of different views. Moreover, the story and its versions as a key resource was paramount to social creativity within the CoI. The story versioning process allowed for heated debate and idea exchange to take place. It created common ground for all CoI members to unfold their expertise, as well as the meshing of narrative with constructionist artefacts-widgets on curvature. As a result, a collective document, that is the C-book, was developed, associating various shared resources, i.e., activities, widget instances, text, and CMT representations and a scheme for interweaving all these elements in a coherent whole.

Along this process the CoI members were engaged in instrumentalization which in a constructionist frame can be seen to emphasise the collective modifying of the widget instances and the narrative. The issues that emerged during the construction of successive C-book versions challenged the mathematics teachers' perceptions with respect to the teaching and learning of curvature resulting in innovative approaches fostering creativity and meaning-making. Embedding the comparison of constant to incremental turn and torsion changes to generate Archimedes' and exponential spirals in space within a Sherlock Holmes 'who dunn it' story was not in any traditional curriculum section or chapter. The process challenged the mathematics teachers to step out of curriculum structures for curvature and to make a new conceptual field available to students connecting curvature with functions and 3D geometry.

By involving mathematics teachers in the collective design of digital tools we recognized them as active agents in the process of integrating technology in the mathematics classroom and not only practitioners involved in top–down integrations of technology in the classroom as suggested by other frameworks, such as TPaCK (Mishra and Koehler 2006). Socio-technical environments, like the one employed in this study, can serve as settings to study teachers' collaborative resource design and as a driving force for classroom innovations.

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Part III

Teachers’ Collective Work Through Resources

Chapter 8
Curriculum Support for Teachers' Negotiation of Meaning: A Collective Perspective

Hendrik Van Steenbrugge, Maria Larsson, Eva Insulander and Andreas Ryve

Abstract Acknowledging the central role of teachers in curriculum implementation, a growing body of research has analyzed how curriculum resources can support teachers to learn using these resources well. This has been done mainly with a focus on individual teachers' learning. Teachers' daily work encompasses many collaborative aspects around curriculum resources as well, an area too often overlooked. We address this issue by describing and discussing our initial steps in developing an approach to analyze curriculum resources from this collective perspective. The approach combines a social semiotic framework to analyze the meaning potential of curriculum resources and the communities of practice framework to analyze a group of teachers' negotiation of meaning around these resources.

Keywords Mathematics curriculum resources · Teacher support
Collective learning

8.1 Introduction

Curriculum resources, such as student textbooks, teacher guides, and professional development materials used by teachers and students, typically constitute central components within research-based initiatives to improve mathematics classroom

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practices (Cobb and Jackson 2012). The research focus on such materials is part of a larger movement in which theoretical elaborations (Pea 1993; Wertsch 2007) and recent empirical studies (e.g., Gueudet et al. 2013) emphasize that cultural tools guide teachers in establishing collegial discussions and classroom practices. To be more precisely, tools, such as student textbooks, teacher guides, and professional development materials, function as resources for teachers' practices of planning and enacting teaching and as such both afford and constrain teachers' actions (Stein et al. 2007).

An important strand of research on curriculum resources, initiated about two decades ago, launched the idea that curriculum resources should explicitly target teacher learning in order to be more supportive in relation to curriculum enactment (Ball and Cohen 1996; Remillard 2000). Along this line, Davis and Krajcik (2005) suggested that curriculum resources could be designed to support teachers' understanding of mathematics, their students' thinking of mathematics, how particular units of teaching fit within a larger curriculum context, the philosophy of the curriculum, and how to use curriculum resources more effectively. In the field of mathematics and science education, several studies have focused on such *educative* curriculum resources, for instance by analyzing how these resources communicate with the teachers and how to design educative curriculum resources (Davis et al. 2014; Remillard 2013).

Several studies on how individual teachers make use of and learn from educative curriculum resources applied a case-study approach (e.g., Davis et al. 2011; Remillard 2000). Other studies have applied other methodologies to capture, for instance, how educative curriculum resources function in large-scale settings such as schools and districts (e.g., Stein and Kaufman 2010). Interestingly, in studying the implementation of educative curriculum resources in two districts, Stein and Kaufman found that proficiency in teaching mathematics was not mainly explained by cognitive capacities such as knowing mathematics, experiences, and pedagogical content knowledge, but suggested that the capacity to effectively use curriculum resources was crucial. They stated, "*Our findings suggest that how a teacher uses a curriculum may be more important than the education, experience, and knowledge that he or she brings to the table. Perhaps another way of conceptualizing teacher capacity—as a teacher who has the capacity to seek out and productively use resources—may be in order* (*J. Greeno, personal communication, January 19, 2008*)" (p. 688). Indeed, recent studies on teacher knowledge and teachers' use of curriculum resources employ the term *capacity,* conceptualizing teaching as a design activity (e.g., Brown 2009; Pepin et al. 2017). These findings suggest the importance to study how teachers productively use curriculum resources, how the aspects of resources facilitate productive use, and how to design resources to be educative in developing teachers' *pedagogical design capacity* (Brown 2009).

As indicated above, there is a growing body of research on educative features of curriculum resources and how individual teachers approach, use and learn from such resources. Further, there exists a rather encompassing body of research on how teams of teachers learn and collaborate and develop instructional practices (e.g., Kennedy 2016; Robutti et al. 2016). However, still a lot has to be done to combine these two perspectives. That is, our field needs to put more focus on and develop analytical approaches to better understand how teams of teachers use curriculum resources to plan, enact and reflect upon mathematics instruction. This is a crucial aspect to address, as teachers' daily work as well as many professional development initiatives comprise of many collaborative aspects, often related to and through the use of curriculum resources (Gueudet et al. 2013).

Addressing this issue, this chapter reports about an initial approach to analyze how curriculum resources can support the learning of a group of teachers. The Communities of Practice framework (Wenger 1998) is applied to characterize group learning. We also borrow from Hallidays' (1978, 1985) description of functions of a text to analyze the meaning potential of curriculum resources. In his social semiotic conception, Halliday is concerned with the social aspects of language and regards the grammar of language as a resource for making meaning. According to van Leeuwen (2005), social semiotics is not considered a "pure" theory, but may be applied to specific problems and may contribute with ideas for formulating questions. Social semiotics is concerned with meaning but it is not a theory for learning. Bringing together social semiotics with the theory of situated learning (Wenger 1998) offers us a way of addressing the relations between communication, the social context and group learning.

The context of the study is a national large-scale Swedish professional development program called the "Boost for Mathematics", initiated by the Swedish National Agency for Education and developed together with many universities in Sweden. The professional development initiative is built around a total of 36 modules of curriculum resources, developed to support groups of teachers, ranging from preschool to high school and adult education to weekly plan, establish and reflect upon mathematical classroom practices throughout one school year. According to the Swedish National Agency for Education (2017), about 35,000 mathematics teachers have participated in the project up until now, which corresponds to 76% of all the mathematics teachers in Sweden. The collected data focuses on grade 1–9 teachers, for whom there are 24 modules in total. In this study, we focus on a subset of six texts in one module for grades 1–3.

8.2 Theoretical Framework: Meaning Potential of a Text and Negotiation of Meaning

Although we ultimately aim to understand how curriculum resources can support the learning of a team of teachers, in this chapter we report about the current state of our analysis. Doing so, we center on the characterization of the curriculum resources in terms of meaning potential, the identification of negotiation of meaning based on the group members' participation and reification, and the relationship between the resources and the group's negotiation of meaning.

Below, we first describe the theoretical underpinnings in relation to our analysis of the curriculum resources. We then also describe the theoretical base as to our analysis of the group processes, or how the group approaches the resources.

8.2.1 Metafunctions and Meaning Potential of a Text

Social semiotics has its origins in systemic-functional linguistics, where Halliday (1978, 1985) formed his theory of language from a sociological perspective. In a semiotic, rather than a linguistic approach, communication encompasses several modes of communication and meaning making practices. The very term multimodality marks an interest in all the culturally shaped resources that are available for making meaning; whether it be writing, speech, image or sound (Kress et al. 2005). We also regard the term "text" in an extended semiotic sense, as complex signs and as structure of messages (Hodge and Kress 1988) expressed in for instance speech, writing, image or sound. According to Halliday (1978), the meaning potential of a text depends on the interplay of three forms of meaning—or metafunctions of a text. The *ideational metafunction* relates to how semiotic resources produce representations of the world. In our study, we focus on the presence of central ideas in the curriculum resources. These central ideas can be mathematical, but also didactical and socio-mathematical. The *interpersonal metafunction* relates to how language and other semiotic resources contribute to making social and emotional interactions and relations between participants in communication. In the present study, this function addresses the relation between the text and the reader. The *textual metafunction* is about the construction and coherence of the text, and what makes it recognizable as a particular type of text. When discussing within the team about the type of text, it occurred that the methods often used in professional development programs to facilitate enactment of new ideas, as identified by Kennedy (2016), were helpful to capture differences between the texts that the teachers read.

Based on a review of 28 studies about professional development programs, Kennedy (2016) differentiates between four approaches by which the programs aim to facilitate enactment of new ideas: through (1) prescription, (2) strategies, (3) insight, or (4) presenting a body of knowledge. The approach of prescription favors literal enactment of certain ideas. A strategies approach can contain detailed prescriptions as well, but provides the rationales and goals related to the procedures as well. This approach acknowledges that teachers have to tailor instruction to their classroom and that teachers are best equipped for doing so when they understand the underlying motives for certain approaches. The insight approach capitalizes on an "aha!" experience on the side of the teachers, triggered by challenging questions that make the teachers look on well-known phenomena in new ways. Finally, the body of knowledge approach presents knowledge sources in the form of books, articles, lectures etc. to the teachers, assuming that teachers will work their own way through these materials. We applied these differing approaches to distinguish between the different types of text.

The premise of metafunctions (Halliday 1985) resonates with our view that a reader's relationship is with the text rather than with the author (Remillard 2012; Rosenblatt 1980). Halliday relates these forms of meaning to the use of language in social contexts. In this sense, we will relate the meaning potential of the curriculum resources to the negotiation of meaning within the community of practice that the teachers are involved in during the professional development program. Hence, the central unifying features of Halliday's and Wenger's analytical frameworks of relevance for our study is their focus on ways of constructing and negotiating meaning.

8.2.2 Communities of Practice and Negotiation of Meaning

We look at the group of teachers in our data from the perspective of a community of practice (CoP). A CoP is characterized as a collection of participants that build mutual and collaborative engagement, share a joint enterprise, and produce and use a shared repertoire of resources (Wenger 1998). Although at the very beginning in their process of becoming a collaborative group, which eventually might evolve into a CoP, the observed group of teachers engages collaboratively and mutually toward the improvement of their students' mathematical skills and knowledge through applying new classroom practices—their joint enterprise. In doing so, they build on the curriculum resources to "resource" their practices (Adler 2000; Gueudet and Trouche 2009) and to eventually create their own repertoire of resources. Wenger et al. (2002) and Gueudet and Trouche (2012) describe similar characteristic stages in the process toward community development. The *early stages of community development* (Wenger et al. 2002), which trigger our interest since the observations relate to the first half of a one-year lasting professional development project, are referred to as the stages of *potential* and *coalescing*. A key community issue in the potential stage is to find out which people within the organization are already networking on the topic of interest, and to reflect on how the network could be extended. A key community issue in the coalescing stage is to develop relationships within the group, and to come to trust one another so that group members feel comfortable to discuss sticky practice problems. As can be inferred from our descriptions of the group processes in the results section, we locate the observed group in the coalescing stage. Contrary to groups under study in Wenger et al. (2002) and Gueudet and Trouche (2012), membership to this professional development project was to a high extent mandatory and teachers were supposed to collaborate within given groups. This strengthens us to locate the observed teacher group in the coalescing stage. We hypothesize that as this group further develops, key issues typically for the potential stage, such as defining common knowledge needs and trying to increase the network might still be brought up, hence taking up the potential stage at a later point. Indeed, also Wenger and colleagues describe that there can be wide varieties in how communities experience the different stages, and that groups can skip stages and deal with related issues at a later time.

In following Wenger (1998), we understand teacher engagement in the professional development program as a process of negotiating meaning around the central ideas in the curriculum resources. Negotiation of meaning, in turn, draws on the processes of participation and reification. As Wenger describes, participation relates to "a process of taking part and also to the relations with others that reflect this process. It suggests both action and connection." (Wenger 1998, p. 55). Through participation, individuals contribute to the negotiation of meaning and become part of a collective process of constructing reality and directing attention. Reification, on the other hand, is the process of giving form to the group's experience by turning processes or objects into their objects. The reified objects serve as resources for teachers to negotiate meaning as they become parts of a shared repertoire. Together, participation and reification form a duality fundamental to the negotiation of meaning. Learning in this sense does not relate to a static cognitive state of mind, but rather to the process of being engaged in negotiating meaning, affecting both participation and reification (Wenger 1998).

Our approach to characterize the learning of a group of teachers relates to focusing on changes in the negotiation of meaning during the time span of the professional development program. Toward that end, we characterize learning as evolutions in participation and reification in relation to the central ideas in the curriculum resources. This chapter relates to a necessary first step in the process of analyzing changes in negotiation of meaning, the identification of participation and reification around the central ideas in the curriculum resources.

8.3 Social Semiotics and Communities of Practice in Recent Mathematics Education Research

From Halliday's seminal work, social semiotic theory has been further developed by Gunther Kress and others (see for instance Hodge and Kress 1988; van Leeuwen 2005). Critical linguistics is the basis from which Hodge and Kress (1988) developed the theory beyond "mainstream semiotics", toward inclusion of other systems of meaning. van Leeuwen (2005) extends the idea of language as resource to apply to other semiotic modes and works with the notion of meaning potential and affordance. Kress and van Leeuwen (1996) also demonstrate how visual semiotic resources have meaning potential, and further elaborated the notion of metafunctions in analyses of images.

Connecting social semiotics to mathematics education research, Morgan (2006) argues that social semiotics allows to understand different functions that features of texts fulfill for the participants in mathematical practices around these texts, and thus, also supports a better understanding of the practices themselves. Potential questions to be posed from this perspective relate to the nature of mathematics and mathematical activity as constructed in the text (ideational); the extent to which participants are characterized as specialists, whether the authors make claims about authority, and the envisioned roles for the reader (interpersonal); and what role the text plays within a context (textual) (Morgan 2006).

Gueudet and Trouche (2012) use the CoP theory as a basis for developing an analytical approach for studying an online mathematics teacher association in France. In particular, Gueudet and Trouche (2012) describe how CoP and amendment of it enabled the researchers to identify and characterize how teachers become participants of a number of CoPs in gathering, sharing and constructing resources for mathematical teaching.

Building on Gueudet and Trouche (2012), Visnovska and Cobb (2013) applied the construct of CoP to study a professional development program in which teachers collectively watched and discussed classroom video recordings. The analytical framework helped to understand differences in teachers' approaches to the same video fragments two years apart. Interestingly, Dean (2005) showed that it was not until 19 months of collective professional development that the group became what Dean (2005) and Visnovska et al. (2012) called a genuine professional teaching community (cf. Wenger 1998).

In sum, these studies from the field of mathematics education suggest (a) the usefulness of social semiotics in its focus on different functions of texts closely tied to their actual use and (b) that the CoP framework allows to track group learning while approaching curriculum resources. We aim to add to this body of research by combining these frameworks in analyzing how curriculum resources mediate a group of teachers' meaning making of these resources. More precisely, our analytical approach aims to study how resources and groups of teachers interact by conceptualizing professional development curriculum materials in terms of meta-functions (Halliday 1985) and the communication among teachers as the negotiation of meaning (Wenger 1998).

8.4 Data Sources and Method

8.4.1 Data Sources

The grade 1–9 professional development program consists of 24 modules of curriculum resources, eight per grade level 1–3, 4–6, and 7–9 (see https://matematiklyftet.skolverket.se). Each of the modules has a particular focus on a mathematical content such as number sense or geometry, or on other aspects of mathematics education such as problem solving, language, and the use of ICT in mathematics. Each module is designed to support groups of teachers during one semester to engage in one module that consists of eight module parts. Each module part further consists of four sessions: (A) individual reading of the resources,

(B) collective lesson planning with colleagues based on the reading of the resources, (C) individual classroom teaching, and (D) collective reflections with colleagues upon the classroom instruction. All schools and teachers in Sweden are expected to participate in the professional development program during one complete school year by engaging in two modules of the curriculum resources.

Table 8.1 lists the data analyzed for the purpose of this chapter. We report on the engagement of one group of teachers in relation to the module part "The didactical contract" in the module "Understanding and use of numbers" in grade level 1–3. Five teachers, all teaching at the same school, and a trained coach engaged in this module during the first half of the school year 2015–2016. We focused on this group since the group membership remained constant during the whole year and since three teachers were actually observed in their classrooms. As can be seen in Table 8.1, analyzed data consist of the curriculum resources (the six texts to be read by teachers individually); two transcribed collegial meetings; observations of three teachers' actual teaching; and pre and post lesson interviews. As described in Sect. 2.1 we understand a text in an extended semiotic sense implying that a text can refer to different modes, such as speech, writing, image and sound. Because the available texts in this stage of the professional development project include only writing, the reported analyses do not relate to other modes. As we are aware that other modules and module parts include video recordings to be watched by the teachers and texts including several pictures, we anticipate that subsequent analyses will also include pictures and videos in addition to written text.

Ethical considerations have been made throughout the data collection process, including carefully following the four ethical principles on information, consent, confidentiality and utilization (Bryman 2011).

8.4.2 Method

Our method consists of three major phases: (1) characterization of the meaning potential of the curriculum resources, (2) characterization of the negotiation of meaning in the teacher group around central ideas, and (3) discussion of possible explanations for differences in negotiation of meaning around the central ideas based on our analysis of the meaning potentials of the analyzed texts. Below, we describe in more detail our approach in relation to the three phases.

Phase 1: Characterization of the Meaning Potential of the Curriculum Resources

We characterized the six texts to be read individually by the teachers: *Didactical contract* (Helenius 2013), *Equal sign 1* (Sterner 2013), *Equal sign 2* (Olsson 2013), *Lesson suggestion 1* (Sterner and Helenius 2013), *Lesson suggestion 2* (Sterner and Helenius 2013), and *Lesson suggestion 3* (Sterner and Helenius 2013). To do so, we created a matrix aggregating the central ideas in the texts and relating them to the texts' textual and interpersonal metafunction (see Table 8.2).

Table 8.1 Overview of analyzed data per session for the module part "The didactical contract"

Module part "The didactical contract"			
Session A: Individual reading of curriculum resources	Session B: Collective planning	Session C: Individual classroom teaching	Session D: Collective reflection
– Text: didactical contract (Helenius 2013)	– Transcribed conversations	– Videotaped lesson observations of three teachers	– Transcribed conversations
– Text: equal sign 1 (Sterner 2013)		– Pre and post lesson interviews	
– Text: equal sign 2 (Olsson 2013) – Text: Lesson suggestion 1 (Sterner and Helenius 2013) – Text: lesson suggestion 2 (Sterner and Helenius 2013) – Text: Lesson suggestion 3 (Sterner and Helenius 2013)			

First, we identified the central ideas in the texts based on our analysis of the texts' ideational metafunction. Our team members[1] discussed the central ideas in each text and found out that each text focused on one or two central ideas. As indicated by the headings in Table 8.2, the three observed central ideas define the columns of the matrix. The rows of the matrix are defined by how the texts' textual and interpersonal metafunctions relate to the three central ideas.

As to the textual metafunction of the resources, we focused on how the texts were constructed, or structured. During related discussions in the team, it appeared that one feature stood out in most of the texts. In all texts, except for the concrete lesson suggestions, the introduction of the central idea tended to be followed by contrasting ideas, to illustrate how the advocated idea differed from previous or common approaches. Throughout the discussion, we also came to notice differences between the texts that we traced back to the type of text, an aspect of the textual metafunction as well. We found Kennedy's (2016) description of four approaches to facilitate enactment of ideas in professional development programs useful to capture the differences between how the central ideas were put forward in the analyzed texts: through (1) prescription, (2) strategies, (3) insight, or (4) presenting a body of knowledge (see Sect. 2.1). It appeared that each text favored enactment of the central idea in a way that resembled one or two of the approaches as described by Kennedy (2016). That way, we characterized each text by one or two of these approaches (see Table 8.2).

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[1] Collectively, the team has experience with multimodal analysis, analysis of mathematics curriculum resources, the Swedish educational context, teaching mathematics education in teacher education programs, and teaching mathematics in elementary and secondary school.

As to the interpersonal metafunction, we concentrated on the relationship between the text and the reader. It appeared during our discussions that all texts, except from the concrete lesson suggestions included references to research or national curriculum documents, which we interpreted as stressing a text's authority in relation to the advocated central idea. Such instances were typically alternated by moments in which teachers were invited to reflect on how the central idea connected to their own practices, or by concrete descriptions of a situation in a classroom to help readers to connect the central idea to their teaching practice. We understood such sections as instances to solicit for teacher engagement in relation to the central ideas. In the next section, we exemplify our approach by describing how we characterized one text's ideational, textual, and interpersonal metafunction.

Analysis of Meaning Potential of One Text: Didactical Contract (Helenius 2013)

In the text "Didactical contract", there is a specific section describing an implicit agreement between students and the teacher that determines how participants in a lesson are expected to behave. It also describes that such a didactical contract is a social agreement formed through previous experiences. The section further describes that the didactical contract helps to understand why changes in classroom practices often turn out to be difficult, and that knowledge of the didactical contract at stake can help to smoothen anticipated changes. Based on this section, we determined the central idea of the text to be the didactical contract as a means to understand and anticipate difficulties in changing classroom practices.

A subsequent section of the text stresses the relevance of the didactical contract in anticipating difficulties when changing classroom practice by making use of a contrast between a common way of teaching in Sweden, referred to as task discourse, and a more recent view of teaching that builds on conceptual understanding and reasoning advocated in the professional development program and in the Swedish national curriculum. Applying Kennedy's (2016) typology of professional development initiatives to characterize the texts, we observed that the idea about the didactical contract as described in the text aims to facilitate teachers to gain insight into the power of the didactical contracts at play in the classroom. It does so by (a) describing the concept of didactical contract and raising challenging questions in order to change the way that familiar classroom events are interpreted by teachers. It also does so by (b) providing a body of knowledge in the form of suggestions for further reading. In contrast, the two central ideas of the equal sign and classroom discussion build more directly on prescribed teaching actions, which are sometimes substantiated by explicitly stated rationales (see Table 8.2), that is, the ideas of the equal sign and classroom discussion are put forward in the texts through prescriptions and strategies.

As to the interpersonal metafunction, the text includes several references to the Swedish national center for mathematics education (NCM), to Swedish as well as international scholars, and to the Swedish national curriculum for mathematics. We understood these instances as stressing the text's authority in relation to the central idea. Furthermore, the text begins with a description of a classroom situation in which an anticipated change turned out to be less well conceived by the students than hoped for by the teacher. The reader is then asked whether such a situation sounds familiar. We understood such instances as opportunities to strengthen teachers' engagement in relation to the central idea.

Phase 2: Characterization of the Negotiation of Meaning in the Teacher Group Around Central Ideas

In a second phase, we identified the moments when the central ideas were brought up during the two group meetings (sessions B and D) and during the actual teaching of lessons (session C). At this stage in the professional development project,[2] we did not expect the group to create yet their own repertoire of resources, or to make the advocated ideas in the curriculum resources their ideas. Rather, we characterized the moments when group members referred to the central ideas akin to the descriptions in the curriculum resources as *potential starting points of reification*. See for instance in the results section the excerpt illustrating that teachers Moa, Julia, Camilla and Karin start to talk about the two advocated conceptions of the equal sign.

We described participation in relation to the key issue in the coalescing stage of community development: the development of relationships, and trust to discuss sticky practice problems (Wenger et al. 2002). Therefore, during moments that the group referred to the central ideas, we kept track of the teachers who were participating in the discussion, and whether the teachers came to bring in situations from their own classroom practice and discussed classroom-related difficulties in relation to the central ideas. For instance, in the abovementioned excerpt, four out of the five teachers are actively participating in the discussion. Also, the teachers, Moa and Camilla in particular, relate the discussion to difficulties they experience in their classroom teaching because their students seem to be more apt to think about the equality sign as "gets" rather than "is".

In sum, we identified the moments in which the central ideas were brought up in the group and characterized these moments in terms of participation and reification. Given the group's initial stage in the development toward an eventual CoP, we coined the term *potential starting point of reification* to refer to those moments when the group related to the central ideas as described in the curriculum resources. We also described participation in terms of the development of relationships and the appearance of moments when group members discussed difficulties from their own practice—a key issue of a group's coalescing stage (Wenger et al. 2002). This helped us to characterize participation and reification around the three central ideas —or the group's negotiation of meaning. Doing so enabled us to see differences as to the group's negotiation of meaning around the three central ideas, an issue we address in the findings and discussion sections.

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[2] Our observations relate to the beginning of a one-year lasting professional development program.

Phase 3: Differences in Negotiation of Meaning Related to Differences in Meaning Potential

After having characterized the meaning potential of the texts and the negotiation of meaning by the group of teachers around the central ideas, we searched for explanations for differences in participation and reification in relation to the three central ideas. We did so by tracing differences in the negotiation of meaning around the central ideas to specific aspects of the meaning potential of the texts, i.e., to the texts' ideational, textual and interpersonal metafunction. For instance, as we will describe in the results section, the negotiation of meaning around one central idea (the didactical contract) was lower compared to the other two central ideas. Looking back at the texts' three metafunctions helped to hypothesize about causes for these differences.

8.5 Findings

Following the three steps as described in the method section, we will first describe the meaning potential of the texts. We will then report about how the group negotiated meaning around the central ideas in the texts, tracing eventual differences back to the texts' meaning potential. As we found consistency both in the texts' meaning potential as well as in the group's negotiation of meaning in relation to the central idea "didactical contract" compared to the ideas "meaning of equal sign" and "classroom discussion", we opted to hold this consistency as a structure of the two subsections below.

8.5.1 Meaning Potential of Texts

Table 8.2 summarizes the analysis of the meaning potential in terms of metafunctions (Halliday 1985) of the six texts that teachers read individually before jointly planning their respective lessons. Each row in Table 8.2 is defined by the three metafunctions: (1) ideational, (2) textual, and (3) interpersonal metafunction. The columns in Table 8.2 are defined by the three central ideas as derived from the ideational metafunction in the six texts: (1) the didactical contract, (2) the meaning of the equal sign, and (3) the use of classroom discussion to challenge students' understanding. As described in Sect. 4.2, in relation to the textual metafunction, we list how the texts make use of contrasts to highlight the relevance of the central idea and how each text has a particular style to communicate about the central ideas. In the bottom row we list how the texts include external references and concrete descriptions of classroom situations in relation to the central ideas, which we interpreted to be significant regarding the relationship between the text and the reader concerning the specific central idea.

Table 8.2 Analysis of the meaning potential of the six texts[a]

	Central idea 1	Central idea 2	Central idea 3
Ideational metafunction	**Didactical contract** as a means to understand difficulties in changing teaching and to help teachers to act purposefully in the classroom (*Didactical contract*)	**Meaning of equal sign** as a relation between two expressions or numbers ("is") (*Equal sign 1&2*)	**Classroom discussion** as a means to facilitate exchange of student ideas, challenge students' conceptions, and help students develop correct, sustainable understandings (*Didactical contract, Equal sign 1&2, Lesson suggestion 1, 2, 3*)
Textual metafunction	Contrast between: A common way of teaching in Sweden (*task discourse*) and a less common way of teaching that focuses on conceptual understanding and reasoning (*Didactical contract*)	Contrasts between: (1) The equal sign conceptualized as "is" is contrasted with and stressed over the frequently understood meaning of the equal sign as "gets" (*Equal sign 1&2*) (2) Different types of tasks that give different affordances to conceptualize the equal sign (*Equal sign 2*)	Contrasts between: (1) Teaching that focuses on conceptual understanding through classroom discussions and reasoning instead of teaching that stresses procedural fluency (*Didactical contract*) (2) Challenging students' misconceptions by discussing them in class, instead of students carrying on with their misconceptions (*Equal sign 1&2*)
	Type of text: "insight" and "body of knowledge": Text builds on facilitating teachers to gain insight into the power of the didactical contract at play in the classroom and on providing a body of knowledge by suggested further readings (*Didactical contract*)	Type of text: "prescription" and "strategies": Texts build on prescribed teaching actions regarding the equal sign, often substantiated with explicitly stated rationales (*Equal sign 1 & 2, Lesson suggestion 1, 2, 3*)	Type of text: "prescription" and "strategies": Texts build on providing rationales for discussion-based lessons, with some prescribed teaching actions for having classroom discussions (*Equal sign 1 & 2, Lesson suggestion 1, 2, 3*)
Interpersonal metafunction	At the beginning of the text, teachers are prompted to reflect on a concrete description of	The text starts by providing examples of common misconceptions about	References to research and the Swedish official curriculum emphasize the importance of

(continued)

Table 8.2 (continued)

	Central idea 1	Central idea 2	Central idea 3
	an attempt to change classroom practice. This invites teachers to bring in their own experiences in relation to the idea of didactical contract. At the same time, the use of several references throughout the text to international scholars, the Swedish official curriculum, and other Swedish governmental bodies stresses the authority of the text in relation to this particular central idea (*Didactical contract*)	the equal sign, followed by directive guidelines to support students' understanding, intertwining references to research with vignettes of concrete classroom situations. This gives text authority in relation to the central idea, and at the same time allows teachers to connect the central idea to concrete classroom situations. Reflective questions at the end of text support teacher engagement in relation to the central idea (*Equal sign 1*) The beginning of the text prompts teachers to reflect on conceptual understanding of the equal sign. This may support teacher engagement in relation to the central idea. Directive guidelines and references to national tests stress the text's authority in relation to the central idea. Reflective questions at the end of the text support teacher engagement (*Equal sign 2*)	classroom discussions and stress the authority of the text in relation to this central idea (*Didactical contract*) Concrete tasks that stress classroom discussion are suggested to be taught in a specific order. Concrete questions to be posed by the teacher are listed as well. These directive guidelines put the teacher in a rather following mode (*Lesson suggestion 1, 2, 3, Equal sign 1 & 2*) Several suggestions are described to strengthen students' engagement in classroom discussions. Often, these suggestions are complemented with rationales, supporting the teachers' decision-making in the classroom (*Equal sign 1*)

[a] Texts are listed in italics in parentheses

Before delving into the findings in relation to the central idea "didactical contract" compared to the ideas "meaning of equal sign" and "classroom discussion", we will point out two issues that stand out in relation to all three central ideas. As can be seen in the row "textual metafunction" in Table 8.2, the relevance of each of the central ideas is stressed through the use of contrasts. See for instance, in relation to the central idea "meaning of equal sign", that the equal sign conceptualized as "is" gets contrasted with and stressed over the conceptualization as "gets". Further, the row "interpersonal metafunction" reveals a pattern across the central ideas.

Stressing their authority in relation to the advocated central idea, the texts make use of references to national and international studies and scholars and by means of referring to official Swedish instances or documents. At the same time, the texts open up for teacher engagement and involvement in relation to the central idea by providing concrete descriptions of classroom situations or student thinking, and by providing prompts for teacher reflection.

As mentioned, we identified patterns in the texts' meaning potential in relation to the central idea of "didactical contract" compared to the central ideas "meaning of equal sign" and "classroom discussion". These patterns became visible in studying the texts' ideational and textual metafunctions. The idea "didactical contract" relates to a new scientific construct, which has to be inferred from teaching. To help teachers understand the relevance of the didactical contract to anticipate challenges when changing teaching practices, the text builds on facilitating teachers to gain insight in the construct and on providing reading suggestions to further expand current knowledge of the construct.

In contrast to the idea "didactical contract", the ideas "meaning of equal sign" and "classroom discussion" relate to more everyday concepts and ideas that can be applied to teaching more directly. Furthermore, the related texts contain detailed descriptions for teacher actions, often supplemented with information about students' thinking and rationales behind activities or tasks. Whereas these kinds of texts are more common for the teachers than the text about the idea "didactical contract", they also acknowledge that teachers can benefit from such educative support.

8.5.2 Negotiation of Meaning Around Central Ideas

A group's negotiation of meaning draws on the interdependent processes of participation and reification. Recall that, given the initial steps of the group's development toward an eventual CoP, we referred to moments when the group related to the central ideas as potential starting points of reification. The development of relationships and appearance of moments when group members discussed difficulties from their own practice, a key issue of a group's coalescing stage, is used to characterize the group's participation around the three central ideas.

As to the group's participation, all five teachers in the group are involved in discussing the two ideas "meaning of equal sign" and "classroom discussion", relating it to their classroom practices, which is not the case for the idea "didactical contract". Concerning potential starting points for reification, teachers refer to the central ideas "meaning of equal sign" and "classroom discussion" in ways that align with how these ideas are covered in the curriculum resources. As for the idea of classroom discussion, the phases in a discussion-based lesson are repeatedly referred to as "t-p-s",[3] which stands for "think-pair-share". Further, the two

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[3] Translated from Swedish "enskilt-par-alla", usually referred to as "e-p-a" by the teachers.

meanings of the equal sign are repeatedly referred to as “is” and “gets” in the conversations, as illustrated by the following excerpt[4] from the collective planning (session B), in which four out of the five teachers are actively engaged in the discussion:

Coach: Eh Moa, have you found something in the text that you would like to raise?

Moa: What I found, eh what caught my attention ’cause it was some question here, eh, when you have three plus eight, or three plus five. Eh **is** or **gets**, (Julia: Ah I’ve written that too.) and I think I say **is**.

Coach: Yeah, that depends on how you look at the equal sign (Moa agrees), as dynamic or static.

Moa: But that, I think I’ve changed (several teachers agree) during (Karin: So have I.) during the years so to speak. (several teachers agree)

Julia: So now you say **is** (Moa: yeah) (several teachers agree). […].

Camilla: But that caught my attention too ’cause the students come already in the first grade and they use the word **gets**, (several teachers are nodding), they do, (several teachers are nodding) and then you sort of have to try to change that (several teachers are nodding). But they bring that with them (several teachers are nodding) when they start the first grade (several teachers are nodding).

Moa: They can bring that from home as well (several teachers agree) I think or they, or maybe it’s just natural (several teachers are nodding) that you say that it **gets,** not that it **is** (several teachers are nodding).

We considered this to be an instance of a potential starting point for reification, as the teachers referred to the equal sign as “gets” and “is”, akin to the description in the curriculum resources. This might create “points of focus around which the negotiation of meaning becomes organized” (Wenger 1998, p. 58) and might support further participation around the meaning of the equal sign.

Compared to the group’s participation and reification in relation to the central ideas of meaning of equal sign and classroom discussion, we observed that the participation and reification is lower in relation to the idea “didactical contract”. Although the notion of didactical contract is raised several times by the coach during the collective planning (session B), the group does not engage explicitly in discussing the notion. That is, although the coach tries to create mutual engagement in the group around the notion of didactical contract, the negotiation of meaning of what the notion entails is still not high among the teachers in the group. The participation in the discussion is low, and teachers do not explicitly relate the idea of didactical contract to their teaching practices. An illustration of how the focus of the teacher group slides over to the notion of “t-p-s” (“think-pair-share”) as a format

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[4] The original Swedish conversation is translated into English by the second author whose mother tongue is Swedish.

for discussion-based lessons instead of focusing on the notion of didactical contract is provided by the following excerpt from the collective planning (session B):

Coach: It's a lot around this didactical contract (Ann: Mm) eh between the teacher and the pupil sort of (Ann: Mm) 'cause the equal sign it's a very central (Ann: Yeah, it certainly is) concept in mathematics so that you might think around this that you can create a lot from the discourse.
Ann: 'Cause then this becomes "s" [refers to "share" in "t-p-s"] and here it becomes "p" [refers to "pair" in "t-p-s"].
Coach: Ah exactly.
Ann: And then we think that the first becomes "t" [refers to "think" in "t-p-s"] although it's "s" when you start.
Moa: But they think but they think.
Coach: Sort of by themselves.
Karin: They think individually (Moa: Individually) so it still becomes "t" (Ann: Ah).

We observed that even during pre and post interviews with the observed teachers, reference to the idea of didactical contract was low. In fact, one (Ann) out of the three observed teachers referred only once to this notion: "The idea is to try to start from this didactical contract, that it requires so much more than just the correct answer to the question, it requires that you discuss why, to start that thought. So we'll focus a lot on that I think." This might be an instance in which the teacher considers knowledge of the didactical contract at stake to be helpful in changing the existing classroom practice, to come up with the correct answer, into openness to discuss and argue for answers in the group.

During the collective reflection (session D), after having taught a lesson (session C), one teacher (Julia) brings up the power of using fictitious student solutions to facilitate students' discussions of erroneous solutions. The coach tries to relate the notion of didactical contract to Julia's reflections on her classroom practice on two different occasions, as can be seen by the bolded text in the following excerpt:

Coach: What about you others, did you feel as if this was a little different?
Julia: I felt, I did lesson suggestion eh two (coach nods) where they got to eh sort of correct children who aren't in the class but have fictitious (coach: Yeah right, mm) names and then I felt we usually sort of, here, one usually gets to openly say sort of what you (a teacher nods) think and respond. Come over and write on the board and then it's like this no but then it's no one who, not so many who see, they don't seem to want to sort of look at each other and say like this, but now when it was, it were the fictitious students

Ville and Sigrid and Olle, well then it was sort of that person has done it and they could say straight: yes he, he’s done like that and like that and like that sort of thought in that way. So it kind of became a little more open to be able to correct.

Coach: Mm, they were, they sort of dared a little more.

Julia: Yes they dared to correct (coach: Yeah) a little more.

Coach: So then you got at the mathematics itself (Julia agrees) because it were (Julia agrees) fictitious students who had done the task.

Julia: That they became a little more it wasn’t like this that. They didn’t have to care that it (coach agrees) was someone in the class who had made an error (several teachers and the coach agree) but that it was sort of someone else who had thought completely wrong (several teachers agree).

Coach: But then you sort of get on a bit with the **didactical contract** ’cause (Julia: mm) then then if you’ve worked with this for a while now with Sigrid and Olle and Kalle and (Julia: mm) who don’t exist (Julia: mm) then maybe you can later sneak in (Julia: Yeah but I’ll continue with) with their own (Julia: yeah).

Julia: ’Cause I will I thought th- th- as soon as I kind of familiarized myself into the whole lesson I thought but God and later when I had done the lesson I thought oh God this is a way of doing it (coach: ah) it doesn’t really matter whose it is.

Coach: No no ’cause ’cause it.

Julia: Whose the solution is.

Coach: ’Cause it’s it’s also kind of a eh kind of a **contract** sort of (Julia: Ah) that in this classroom it’s okay to make mistakes (Julia: Mm) and we view kind of not the errors as the worst errors but we see it as learning.

As can be seen above, only one teacher (Julia) and the coach participate actively in the discussion. Although the coach tries to weave in the idea of didactical contract, this was not actively taken up by the group. It does not seem to function as a potential starting point of reification as none discusses how the didactical contract at stake in the classes might have influenced the described classroom situation.

Although this study is based on data collected during the beginning of the first half year in the professional development project, we find it worth reporting about teachers’ collective planning at the end of the first half year. During that instance, the teachers return to the idea of discussing students’ incorrect solutions (as in the excerpt above), which is closely related to what kind of didactical contract that prevails in the classroom. This time, several teachers do participate actively in the discussion although they do not relate this to the notion of didactical contract. Although it is mentioned in the teachers’ discussion that this is a new way of teaching, they do not reflect on the process to come to this new way of teaching, and how to eventually encompass difficulties toward this new way of teaching. Thus, although the participation is higher than at the beginning of the professional development project, they still do not actively relate their experiences to the idea of the didactical contract hence not making it their own reified idea.

8.5.3 Differences in Negotiation of Meaning Related to Differences in Meaning Potential

In the previous section, we illustrated that in this teacher group, the negotiation of meaning around the three central ideas differed. Compared to the central idea "didactical contract", participation and reification in relation to the central ideas "meaning of equal sign" and "classroom discussion" were higher. We hypothesize that differences in meaning potential of the texts might relate to the observed differences in negotiation of meaning.

In relation to the texts' ideational metafunction, it might be that the ideas related to "meaning of equal sign" and "classroom discussion" are easier to negotiate meaning about by the teachers than a new scientific construct as "didactical contract". The support that these teachers get in the teaching materials they use on a daily base usually addresses concrete mathematical topics, didactical strategies, and classroom settings. Teachers' experience with concepts, such as the didactical contract, that have to be inferred from their teaching is much lower.

Further, participation and reification in relation to "meaning of equal sign" and "classroom discussion" are facilitated by different kinds of texts than is the case for the idea of "didactical contract". As to the former, we characterized the types of texts describing these two ideas as "prescription" and "strategies". These types of texts, describing teacher actions, sometimes complemented with additional declarative information correspond to some extent to the type of support teachers usually get in their teaching materials. The text about the didactical contract, which we characterized as an "insight" and "body of knowledge" text corresponds to a much lesser extent to typical support for Swedish teachers in teaching materials.

8.6 Discussion

Within mathematics education, analytical approaches have been developed for conceptualizing use of resources (e.g., Gueudet and Trouche 2009; Remillard et al. 2009) and collegial interactions (e.g., Visnovska and Cobb 2013). Moreover, some scholars have started to explicitly combine these strands of research to understand how teams of teachers use, draw upon and redesign resources in different social practices (e.g., Gueudet et al. 2013; Gueudet and Trouche 2012). We used the two analytical perspectives of Halliday and Wenger as a starting point for conceptualizing the meaning potential of curriculum resources and teachers' negotiation of meaning around these resources. In the following paragraphs, we will discuss the pros and cons of our analytical approach, with an orientation toward further refinement of the approach and a particular focus on the characterization of the resources, the negotiation of meaning around these resources, and potential influential characteristics of the teacher group and school context.

Applying Halliday's metafunctions to the analysis of the curriculum resources enabled us to focus on the central ideas advocated by the resources, how these ideas are accomplished through the resources' construction and coherence, as well as how the resources and the readers are positioned in relation to the central ideas. Such a conceptualization of the curriculum resources facilitated our analysis of how resources might mediate the teachers' negotiation of meaning. More in particular, our approach to describe the texts' ideational and textual metafunctions by means of identifying the central ideas and characterizing the type of text respectively was helpful in this regard. We could, however, not trace back the observed differences in negotiation of meaning to our approach of describing the texts' interpersonal metafunction. Therefore, we hypothesize that, in addition to describing how the text and the reader are positioned in relation to the central ideas, it might be interesting in subsequent analyses to describe how the texts position the readers in relation to other readers. In other words, it might be helpful to analyze how a text, through its interpersonal metafunction, provides opportunities to build and strengthen the relationships between teachers in the group.

Our approach to capture the group's negotiation of meaning around the three central ideas centered on describing the group's participation around and reification of the central ideas. Characterizing participation by means of the key community issue for where we situated the group of teachers regarding the stages of community development (Wenger et al. 2002) was helpful. Describing both the quantity of teachers involved in the discussion and whether or not teachers actively connected the discussion to their own classroom experiences helped us to understand differences concerning the group's participation around the three central ideas. We hypothesize that, as the group proceeds further in this professional development project, we will be able to describe the group's participation not only quantitatively but also qualitatively, characterizing participation in more detail. We characterized the group's reification in relation to the central ideas by describing when the group referred to the central ideas. Although doing so enabled us to observe differences related to the three ideas, this aspect has to be developed in more detail. Subsequent analyses of how teachers relate to central ideas (e.g., as expressed by gestures) and make them their own ideas are needed. Along this line, analytical approaches as described by Gueudet and Trouche (2012) to capture how teachers actually refer to and draw upon resources might be helpful in characterizing a group's reification of a central idea.

The analytical approach as such did not include any constructs from mathematics education research. Our idea was that we did not a priori decide about theories from mathematics education that might be essential to use. Instead, we anticipated that the first step in the analytical process, in which we characterized the ideational meaning of the texts, might give direction toward deciding about which theories and research results from mathematics education research are important to consider. For instance, in the findings section we see that the curriculum resources include information about the didactical contract. To better understand this text as well as how teachers negotiate meaning around this theme one could study Brousseau's (2002) work, the conceptualization of classroom norms (Yackel and

Cobb 1996) and meta-discursive rules (Sfard 2000). Doing so might allow for a more fine-grained analysis concerning the group's participation and reification, and to characterize evolutions in the group's negotiation of meaning, something we framed in the introduction as our characterization of group learning.

On a related note, in order to track appropriate and qualified changes in the processes of teachers' reification and participation, we argue that changes should be neither too general nor too specific. For instance, taking the central idea of the equal sign as an example, it is unlikely that the team of teachers under study in this chapter will return to this central idea throughout the year since this central idea is not at the focus in subsequent stages in the professional development project. Therefore, it is unlikely that we will be able to track changes in negotiation of meaning around that specific idea later in the professional development project. We propose that appropriate and qualified changes in negotiation of meaning should be tracked at some meso-level of negotiation of meaning, such as how teachers consider how to engage students in conceptually and cognitively demanding activities, how teachers decide about the key mathematical ideas they want to build lessons around, how teachers can build upon and extend students' mathematical thinking or how teachers can prepare and act to orchestrate productive whole-class discussions in the mathematics classroom. From such a perspective, we work on developing an analytical approach that might include three levels of analysis—a level using Halliday and Wenger, a level specifying key mathematical classroom practices of relevance for a majority of collegial interactions throughout the professional development project, and a level that digs deeper into aspects that are specific to particular sections of curriculum resources or collegial discussions, such as aspects specific to the idea about the equal sign.

Building on research that stresses that cultural tools—such as student textbooks and teacher guides—guide teachers in establishing collegial discussions and classroom practices (e.g., Wertsch 2007; Gueudet et al. 2013), we focused on how the curriculum resources in this study possibly afforded and constrained teachers' negotiation of meaning around the central ideas in the texts. An unexplored aspect as to our analysis is the characterization of the teacher group and how that might influence the group's negotiation of meaning. For instance, it is likely that teachers' mathematical knowledge for teaching, their beliefs, pedagogical design capacity and perceptions of the students and curriculum materials influence the negotiation of meaning around the central ideas (Remillard 2005). A related issue to be explored further as well is the role of the coach in the teacher group—as Wenger et al. (2002) allocate this role an important factor in a community's success. Likewise, particular aspects of the school context might influence the group's negotiation of meaning. We hypothesize that characterization of the teacher group and the school context will help to understand how different groups of teachers negotiate meaning around the same curriculum resources.

8.7 Conclusion

This chapter described an initial approach to combine two frameworks to better understand how curriculum resources might be supportive for a group of teachers' learning. The experiences with our approach—characterizing the curriculum resources' metafunctions and relating them to the group's negotiation of meaning—have been educative to us as a research team. A next step, as we see it, is to explore further the possibilities of this approach. This would involve a longitudinal study design, following different teacher groups over a longer period, as well as including curriculum resources that have other modes than writings. Doing so might allow us to refine our approach as to how teachers approach the resources and possibly help us better understand how curriculum resources might constrain and support the learning of a group of teachers.

Our preliminary findings suggest that curriculum resources can support the learning of a team of teachers in at least two ways. First, by supporting teachers' understanding of the central idea(s). This can be done by describing the central ideas explicitly and by contrasting them with other ideas. Second, by including prompts to reflect collectively, not merely only individually, on described ideas and the practical implications of these ideas.

Finally, our tentative findings indicate that teachers' negotiation of meaning is higher around concrete central ideas described in a type of curriculum resource that they are more familiar with. This might suggest that the teachers have to go through a learning process before they take up more abstract central ideas described in a type of curriculum resource other than the ones they work with on a daily base.

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Chapter 9
Mathematics Teachers' Expertise in Resources Work and Its Development in Collectives: A French and a Chinese Cases

Chongyang Wang

Abstract Designed as a case study, two mathematics teachers from two contrasting contexts, China and France, were selected and investigated. Looking through both the teachers' resource system and their resources work in collectives, it is hoped to develop a deeper understandings on teacher's expertise in working with resources, and the factors from collectives to develop it.

Keywords Mathematics teachers' resources · Documentational approach to didactics · Teacher expertise · Teachers' collective work

9.1 Introduction

Teachers always interact with resources in their work, which provides a lens to see their work and professional development in a resource dimension (Gueudet et al. 2012). What is more, the relationship between teachers and resources has changed with the evolution of technology: teachers are no longer only users anymore, but designers for developing resources, such as Open Educational Resources (discussed in Chap. 1) or online e-textbooks (discussed in Chaps. 12 and 15).

In the pilot study (Pepin et al. 2016) of our Ph.D.[1] thesis, concerning three Chinese "expert" mathematics teachers, we explored their expertise combining the

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[1] This Ph.D. project, entitled "An investigation of mathematics documentation expertise and its development in collective work: two contrasting contexts from China and France", started in September 2014, co-supervised by Luc Trouche in ENS de Lyon and Binyan Xu in East China Normal University.

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analysis of their *resource systems* (i.e. the structured set of resources supporting their teaching) with deep-interviews and observation. In this study, I paid particular attention to teachers' expertise in their work with resources. It is assumed this expertise contains a way of integrating resources: when the teachers are interacting with resources, they are in fact experiencing a process of integrating the resources from outside into their own resource systems. Since cooperation among teachers is considered important for their professional development (Hargreaves 1995; Cui and Zheng 2008), this expertise will be explored from a collective dimension: two cases from two different collective contexts, one in China, and the other one in France, will be chosen and studied.

Contributing to the exploration of the expertise in teacher's working with resources, and how it is developed through teachers' collective work, this research is hoped to develop a refined definition of such expertise and the factors that are due to collective work. The research questions are:

(1) What kinds of expertise are there in teachers' work with resources?
(2) How do teachers develop their expertise working with resources, particularly in collectives?

In the following parts, firstly the theoretical framework and methodological discussion will be presented, subsequently two cases in China and France, and a deep analysis on the French case will be expanded.

9.2 Theoretical Framework and Definition of Concepts

This study is based on two frameworks: *Documentational Approach to Didactics* (Gueudet and Trouche 2009, see also in Chap. 1) to analyse the features of teachers' resource work, and elements of the expertise in resource work; and *Activity Theory* (Engeström 2001) for tracing the influences from collective work. This section will start from the definition of resource, then the two theoretical frameworks, and a preliminary definition of expertise in teachers' work with resources will be given based on the literature review.

Regarding the notion of resource, Adler (2000) described the resource as the verb "re-source"; a resource could be anything with the potential to *re-source* a teacher's activity. This study kept the idea of Adler's definition, but in this chapter, the resource refers mainly to elements developed or used by teachers in their work about the curriculum, like textbooks, reference books, no matter digital ones, material ones, and even personal ones, etc.

9.2.1 Documentational Approach to Didactics (DAD)

According to Documentational Approach to Didactics (DAD), the interactions between teachers and resources, including restricting, selecting, implementing, modifying, adapting, saving and sharing, were defined as *documentation work*: a document is composed of a resource and schemes of utilization. The documents of a teacher are articulated in a structured documentation system, correspondingly, the resource system constitutes the "resource" part of the documentation system without the scheme part of the documents. A scheme in Vergnaud's study (2009) was defined as the invariant organization of activity for a given class of situations, comprising the goal(s) of the activity, the rules to generate activity, the operational invariants for picking up and selecting the relevant information, and the possibilities of inference. The frameworks of DAD provided a view to see teachers' expertise in resource work by analyzing the elements of scheme in using resources: goal(s), rules of generating activity (how to do), the operational invariants (why to do), and the possibilities of inference.

9.2.2 Activity Theory

While taking DAD as a framework to analyze the resource perspective on teacher work, there also comes the question of the nature of teachers' work: teachers belong to institutions (Chevallard 2006), their work is neither isolated nor individual, but part of society, their documentation work is connected to others, and culturally and socially situated (Gueudet et al. 2013). Hence, the *Activity Theory* (Engeström 1987, 2001) is adapted as the second framework. Engeström expanded a mediational triangle as "activity system" from Vygotsky (1978) and Leont'ev (1978), with six elements: subject, mediating artifacts, object, rules, community and division of labor, emphasizing that activity should be situated into a cultural and historical background with five principles: (1) the activity system as a whole as the unit of analysis; (2) multi-voicedness, "multiple points of view, traditions and interests"; (3) historicity, "activity systems take shape and get transformed over lengthy periods of time"; (4) contradictions, "as sources of change and development"; (5) the possibility of expansive transformations (ibid., pp. 136–137). Inspired from Activity Theory, teachers' resource work could be situated into an activity system with the corresponding elements: teacher (subject), resource (mediating artifacts), object, rules, collective (community), and division of labor.

While the activity system is taken as the unit of analyzing teacher's activity, the resource system of the teacher also gets developed (re-organized or enriched) along with the teacher's resource mobilization from her resource system to achieve the goal of the activity. It is believed in this study that the resource which works as the mediating artifact in the activity system comes from the teacher's resource system. This way, teachers' resource system could be understood by observing how the teacher adapts it in specific activities, while teachers' activities could be analyzed through a lens of her resource system.

As stated before, Activity Theory provided a framework to trace the influences from the collective, (1) the history and culture of the collective where the teachers work in will be considered when analyzing their collective work, (2) the points of view from other colleagues in this collective will also be paid attention to when following the targeted teachers, (3) the contradictions between the targeted teachers and other colleagues (like the conflicts in using resources), as well as the collective or the environment (such as new tasks or challenges) will be studied carefully to see the "source of change and development". The specific tools inspired from DAD and Activity Theory will be introduced at length in the methodology section.

9.2.3 Definition of Documentation Expertise

The expertise in teachers' documentation work is defined as *Documentation Expertise (DE)* in this study. According to Berliner (1988), expertise "is *specific to a domain*, and to particular contexts in domains, which is developed over hundreds and thousands of hours". Key elements of expertise are linked with teaching problems solving efficiently and creatively with a wide range of knowledge and experiences (Sternberg and Horvath 1995), or to be more precise, teachers with "adaptive expertise" were proposed as "specialists in retrieving, organizing, utilizing, and reconsidering their professional knowledge and beliefs" (Avalos 2011). Drawing from the definitions of expertise and documentation work, DE is defined preliminarily as the schemes while interacting (retrieving, selecting, organizing, modifying, adapting, creating and sharing off) with resources. DE is considered as a developing state of teacher expertise, offering a resource aspect to explore the way of teacher's professional development (Pepin et al. 2016).

In this study, we assumed that the elements of DE could be summarized from schemes. Comparing with *Pedagogical Design Capacity* (PDC) (Brown 2009; see Remillard's study in Chap. 5), DE emerges with the property of documentation work, which covers the whole process of interacting with resources. It means that DE is more than "teacher design" or "teacher design capacity" (Pepin et al. 2017), which appears not only in the design phase and the implementation phase, but also the phases of reflection, modification, saving or organizing, and sharing off etc.

To summarise, DE could be defined as a series of structured schemes in resources retrieving, selecting, organising, modifying, creating and sharing off, with an aim of achieving some teaching goals or solving some teaching problems.

9.3 Methodology

As stated in Sect. 9.2, the methodology and tools inspired by DAD (like the reflective investigation), and Activity Theory (such as the documentation-working mate) will be presented in this section.

9.3.1 *Reflective Investigation and the Tools Developed*

Different from the traditional investigation, reflective investigation involves the teachers as part of the study throughout the whole data collection, with four principles: long-term follow-up; in- and out-of-class follow-up; broad collection of the material resources used and produced throughout the follow-up; and reflective follow-up of the documentation work (Gueudet et al. 2013, p. 27).

To know how the teacher organizes and represents her available resources, and in line with our pilot study (see Pepin et al. 2016), we adapted the tools of "*Reflective Mapping of Resource System (RMRS)*", in which a teacher is asked to draw a map to present her resources in a structured way based on her own reflection, and "*Inferred Mapping of Resource System (IMRS)*", in which the researcher completes some information on the RMRS derived from combining the interview and observation. It should be noticed that the RMRS and IMRS are not final, but will be improved, complemented, and reorganized continuously during the long-term follow up, along with the development of teachers' reflections on their resource systems.

Some other tools were also developed. An online "*Reflective Investigation Box (RI Box)*" was built and shared between the researcher and the targeted teacher, in which the teacher could share her resources used in her activity (such as lesson plans, screenshots of blackboard writing etc.), and respond to the questions (either about resources in RI Box, or any other questions) from the researcher regularly. The software chosen for supporting RI Box depends on teachers' using habits in different contexts, for example, RI Box supported by Dropbox[2] in France (see Chap. 12), and Wechat[3] in China.

Besides field notes of teachers' activity, observation and school visiting from the researcher were also adapted. The combination of field notes and RI Box provides the possibility of a long-term follow-up of the teachers' resource work, for example, during the activity, what resources are integrated, where these resources come from, and how they are integrated.

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[2] Dropbox is a file hosting service, which offers cloud storage, file synchronization, personal cloud, and client software. See more information on: https://www.dropbox.com.

[3] Wechat is a Chinese social media, with the functions like instant messaging (text and voice), hold-to-talk messaging, broadcast (one-to-many) messaging, video conferencing, group chatting, official accounts and moments etc. See more information on: https://web.wechat.com.

9.3.2 Documentation-Working Mate for Understanding Collective Documentation Work

Following the principle of “multi-voicedness” of Activity Theory, a new notion of *documentation-working mate* was proposed here as someone who works closely with the targeted teacher, with mutual influences on their documentation work and DE development. *Mate* in Oxford Dictionary is defined as infers “a fellow member of joint occupant of a specific thing, like table-mate”, with an “origin related to meat (the underlying concept being that of eating together)[4]”. The reason to choose “mate” and but not “peer” as in “peer education” (Turner and Shepherd 1999) is that “mate” breaks the boundary of age and education/professional background. For a given teacher, her *documentation working mate* could be a colleague with similar working experiences in her school, or someone from a totally different working context as an university or research institute etc. In each case of this study, a *documentation working mate* will be selected according to the targeted teacher: they form the smallest but closest collective, and the documentation working mate will be followed in the same way as the targeted teacher.

9.3.3 A Two-Step Case Study Analysis

Designed as a case study focusing on DE from different contexts, two mathematics teachers were selected from two middle schools, one (named Gao) from Shanghai in China, and the other (named Anna) from Lyon in France.[5]

The two teachers’ work is situated in different collectives, TRG in China, and AeP in France. TRG is chosen because it is a widely spread collective schoolwork unit for Chinese teachers since 1952, and AeP is selected because it is an association of French schools linked to the French Institute of Education (IFÉ) which started in 2013 but with rapid expansion. The specific introduction of these two collective work contexts will be presented in Sect. 9.4.

The cities where the schools locate, Shanghai and Lyon, are both developed cities, and the middle schools selected were both located in the city center and they all have close cooperation with the research institutions where the author works in, one is an affiliated school of ECNU and the other is a member of AeP. From the level of students’ performance and teaching technology equipment, they were both ordinary schools, neither top nor bottom.

The two teachers were selected because of (1) their willingness to participate in the research; (2) administrative support from their schools; (3) active participation

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[4] See in https://en.oxforddictionaries.com/definition/mate.

[5] As stated at the beginning of this chapter, the French case of Anna was shared with another Ph.D. student (see Rocha in Chap. 11).

in collective work; (4) rich working experiences and good technology operation skills.

This study is situated in a time of change, which demands the teachers working with resources intensively: a new middle school curriculum reform in France had started since September 2016, and one of the big changes is "algorithmic", which appeared in middle school stage for the first time; while in China, although without any education reforms in curriculum, it was the time (March to May each year) for the novice teachers to prepare open classes with the instructions of the experienced teachers.

The analysis of the cases includes two steps: (1) the teachers' individual resource work analysis, such as their RMRS and IMRS, the collectives they participated in and related working experiences (see the documentational trajectory discussed in Chap. 12), and (2) the teachers' collective work analysis through videos and reflective interviews.

The aim of the first step is to obtain an overall landscape of the targeted teachers' resource work, such as what resources they use, in which collectives they work, how their resources are organized etc. In this phase, a variety of information from the teachers is considered: emails, CVs, published papers and articles, blogs etc. School activity observations and interviews are intertwined as main tools: while observing school activities (such as classroom teaching or conferences), field notes taken by the researcher, and complementary interviews with the teachers are also adapted.

In the second step, three phases including lesson preparation, lesson implementation and the reflective interview were filmed. In Sect. 9.6 of this chapter, a collective lesson preparation of the French case will be analyzed particularly. The collective lesson preparation lasts for one hour. Before their collective work, an email with three questions was sent to them: "What are the difficulties of this activity? What resources do you bring and lack of? Why do you prefer to work together?" The first transcription was shared with Anna through Google document, in which we (the author, Katiane Rocha, and Luc Trouche) marked our confusions and questions in the video, particularly the name of the resources that are unclear for us. Then with the second transcription, we discuss with Anna face-to-face, mainly on the source of the resource appeared in their collective lesson preparation.

9.4 Contexts of Teachers' Collective Documentation Work: TRG and AeP

Working as the contexts of the two teachers' collective work, TRG and AeP will be presented in this section from their cultural backgrounds and historical developments.

With top performances in international tests, such as PISA, TRG is considered as an important factor for Chinese teachers' professional development (Wang 2013).

While remaining a limited network of schools associated to IFE in France, AeP is now a new typical exploration in teachers' collective work in France. They are both school-level collectives with strong institutional supports, closely linking with research institutions, which make them serve as a hub for connecting some professional collectives (both teaching collectives and researching collectives) as well as the resources circulating in the collective activities.

9.4.1 Collective Culture and TRG in China

Working collectively, in China, is considered as essential since Confucius. "Whenever walking in a company of several persons, among them must be someone worth learning from (三人行, 必有我师)". From the view of culture, the school-level working culture in China has been described as *collective* in Yang's study (2013). Research on teacher education in China shows that Chinese teachers are benefiting from some efficient school-based means (Li and Huang 2008): they gain a deep understanding of basic mathematics and adequate pedagogical expertise through the activities of TRG. The word "TRG" firstly appeared in Chinese Ministry of Education regulation in 1952, aiming to "study and improve the way of teaching". In 1957, the property and tasks of TRG were emphasized again and more clearly stated (Wang 2013). Since the 1990s, TRG undertook the work of carrying out post-1990 curriculum reform. From 2001, encouraged to participate in education experiments, TRG slowly gained research components. Now the TRG has become a basic unit for teachers' collective work in each school, a main platform where resources are generated and shared through the regular collective activities.

Generally, a TRG consists of teachers from the same discipline, such as mathematics TRG, or English TRG. And each TRG is composed by several different Lesson Preparation Groups (LPG) based on grade, like a mathematics LPG in Grade 6. In most of the Chinese schools, teachers work full time with their own office or office desk, and generally speaking, teachers from a same LPG are arranged to share the same office (sometimes with other discipline teachers, it depends on the scale and conditions of the school), so that they could communicate with each other conveniently. LPG also works as the basic unit of teaching research activities. Sharing a same office, teachers from the same LPG work collectively very often.

The working modes of TRG could be sorted into "task-based activity" and "operation mode of diagnose-based activity" (Hu and Wang 2014). The former is represented by collective design of resources, such as school-based exercise books, and the latter could be embodied through Chinese open class with several rounds of adjustments during the lesson preparation. Thus, the daily activities in TRG mainly focus on issues about design or implementation of teaching, for but not only for resources.

9.4.2 Collective Experiences and AeP in France

AeP is a network of schools linked to IFÉ (French Institute of Education, see http://ife.ens-lyon.fr/lea/lea-english-version), ENS de Lyon. The first network, comprising 12 schools, was set up in 2011; up to 2015, there are 34 schools in this network, aiming to go beyond the boundaries between basic, clinical and technological research. To be a member of AeP, the school needs to be strongly supported by their administrative staff and to meet the interest of research of a research team in IFÉ. Then the school and the research team will co-propose to IFÉ a joint AeP project which builds on questions emerging from the actor's concerns (Chabanne et al. 2015). Once a school, such as middle school A, becomes a member of AeP, its name will be changed into "AeP A" by other members. AeP builds an explicit association between schools and research institutes, to gain resources and better understanding from interactions between teachers and researchers, or teachers themselves. The short history of AeP does not mean that the collective work among teachers in France is recent. Actually, before AeP, IREM (Institute of Research in Mathematics Education), which gathers teachers and researchers, has existed since 1968, and the origin of teachers' collective work can be traced back to 1900, as the French Dictionary of Pedagogy (Buisson 1911) saying, "Teaching is collaborating".

Different from TRG in China, AeP is not a compulsory choice for neither teachers, nor schools, that is why at the beginning part of this session AeP was introduced as a limited network of schools. However, the teachers who join AeP have compulsory cooperation with the researchers in IFE, because each member of AeP needs to sign a contract based on a common research project, which generally lasts for three years, and could be renewed if it is agreed by both sides. In this way, activities of AeP provide the opportunities for teachers' collective work with both their colleagues inside their school and the researchers outside.

9.5 A First Analysis of Teachers' Individual Documentation Work

As introduced in Sect. 9.3, in France and China, one math teacher was selected from each side as the main teacher to be followed, thus this case study includes two cases, one in China and one in France. Since the author was born and has studied in China, the duration of the two case studies was different: the follow up of the French case took more time, because the author is familiar with the Chinese context already. So more time was spent on the French case when the author was staying in France (from March 2015 to January 2017).

9.5.1 The Chinese Case

Since the thesis is still in process, the Chinese data collection has not finished yet; this is also the reason why the following section will only present the video analysis of the French case. Up to this chapter, I conducted two rounds of data collection in China. The first round included two interviews with Gao and the principle of the middle school. In the second round of data collection, I spent 3 weeks for full-day observation in both Gao's classroom teaching and her school life, including her interactions with other colleagues in office and meetings in TRG. During the observation, I also conducted some informal interviews mainly about her resource usage.

In this school, each Tuesday afternoon was the fixed time for math teachers' collective activities: math TRG activity was held once per month, math LPG activity was held each week. Besides the school activities, teachers in this school were also sometimes arranged to accept training in other schools, in Shanghai or other cities; in return, teachers from other schools could also get trainings in their school.

As a math teacher with 2 classes in grade 7, Gao was also in charge of some class management work of one class. She was the leader of grade group 7 (in charge of administrative management of the whole grade), and the leader of math LPG in grade 7 (mainly for math teaching affairs of this grade). In grade 7, there were 4 classes and 2 mathematics teachers (Gao and Liu) were in charge 2 classes for each. In this school, all teachers from the same grade were arranged to work in a same office, so Gao and Liu shared the same office with all the other teachers who teach grade 7. She needed to give 12 lessons each week (6 lessons in each class), the rest of working time she had to deal with students' homework, attend training or teaching research activities, or prepare lessons etc. Working as the leader of math LPG in grade 7, Gao's responsibility was to unify the teaching progress, or organize discussions when facing some complex problems, and the discussions would be conducted whenever they needed, because "although we have regular teaching research activities each Tuesday, we cannot leave the problems and wait until that day, some of the problems need to be fixed at once, this is also why we keep the teachers in the same grade in a common office, we can communicate in time" (cited from the interview with Gao).

As for the *documentation working mate* of Gao, three teachers were chosen (see Fig. 9.1): Yao, who was a new teacher in grade 6 and supervised by Gao; Liu, a teacher who taught the same grade and shared a common office with Gao; and Zhang, the leader of math TRG in their school.

According to the interview and school observation, the scheme of resources accumulation could be seen in Gao's resources work:

- Goal of the activity: accumulating resources in daily work;
- Rules of generating activity: (1) Marking the students' homework and helping them correct the mistakes one by one, and face to face. For Gao, marking her 70 students' homework, including checking the mistakes and asking the students to

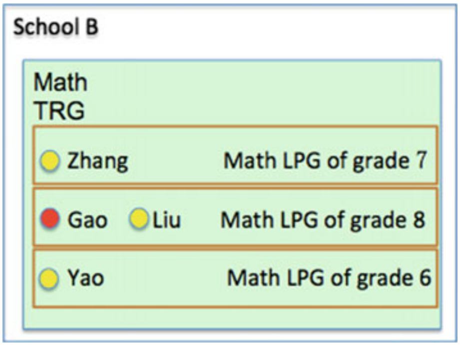


Fig. 9.1 Gao and her *documentation-working mates*

correct them, is a regular job which could cost her two hours each day to deal with their homework, but she insisted on marking the homework of students face to face, and adapts lots of students' feedback into the next lessons. (2) Discussing with other colleagues, both in TRG activities, like open class or teacher trainings, and interactions with researchers; (3) Regularly reading, sorting and writing down the selected exercise items from her self-purchased reference books. Gao has several notebooks for these items, and Gao often sends some of these items to her students as their homework.

- The operational invariants: Gao considers the feedback (like the mistakes made by the students) from the students as the most important resources for her teaching, because "the touchstone for testing whether I have achieved my teaching goal is to see if they have mastered the knowledge, which could be reflected in their homework." Gao also has a clear cognition on her various reference books, some with very detailed explanation on the content are ideal for new lesson preparation; some with basic items and improving items are good for stratified teaching; some are for review lessons. "Learning mathematics needs a lot of exercise, but if the students do not have enough time, we need to offer some selected exercises").
- The possibilities of inference: Gao has built some "organic cycle system" for accumulating resources, which is students-centered. She gathers and selects the available resources, then sends them to the students, and gets their feedbacks as her teaching resources in class, also she exchanges these experiences and items through discussing with other colleagues. This process is beyond accumulation, Gao also experiences the resources integrating, adapting, modification and sharing etc.

9.5.2 The French Case

A 3-month's preliminary follow up of Anna's school activities was conducted from April to July 2015, including her classroom teaching activities observation, in-service teacher training, pre-service teacher mentoring, and school meetings with her colleagues.

Middle school A became a member of AeP since 2013, the first year when AeP was set up. The invitation to join AeP was enabled by a researcher who had close cooperation with Anna in SESAMES (Situations of Science Education: Activities of Modeling and Simulation Evaluation) since 2006. The SESAMES team is composed of researchers and teachers, working for teaching and evaluation resources. The SESAMES Algebra Group that Anna participates in includes 7 math teachers from secondary schools. This group aims to build resources for mathematics teaching, especially for algebra teaching in middle school, providing guidance for teachers.

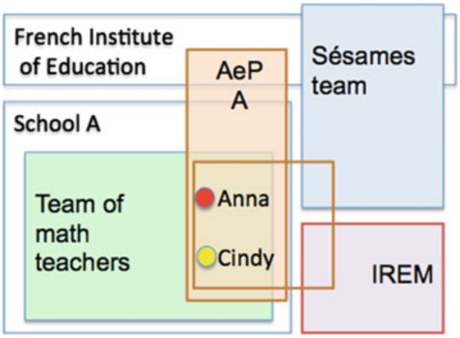
Graduated in 1989, Anna passed her CAPES (Certificate of Secondary Education Professional Qualification) exam in 1990, after one year's pre-service teacher education, she got her first position in a middle school of urban Paris, a "famous" school for the tricky problem students, till 1995. From 1995 to 2005, she worked in a middle school in Lyon. In 2005 she started to work in the current middle school A. She is a math teacher of three classes. She is also in charge of classroom management in one class in grade 6. In 2013, the same year when her school became a member of AeP, she began to work half time in IFÉ, and became a correspondent of this AeP. She works in SESAMES since 2006, APMEP (Association of mathematics teachers of public education) since 1990, and IREM (Institutes of research of mathematics teaching) since 2010.

As Anna's *documentation-working mate*, Cindy (see Fig. 9.2) is a math teacher who works in the same school as Anna, and she is the other correspondent of this AeP A. She is also a member of the SESAMES team and IREM, in which she works closely with Anna.

Based on the observation and interviews with Anna and Cindy, the following scheme of Anna's resource accumulation and management could be found:

- Goal: accumulating and managing the resources meanwhile;
- Rules for generating activity: (1) Taking various technological tools for storing personal resources, like Dropbox or Google Drive used for storing resources; (2) creating a common Dropbox folder to share the resources with all the other

Fig. 9.2 Anna and her *documentation-working mate*



math teachers in her school, and regularly sorting up the structure of the sub-folders; (3) mobilizing the resources from different sources to support their other jobs. For example, when teaching the chapter of "line, segment and half-line", she arranged several activities from IREM website for her students. Then she collected and took pictures of their work, and uploaded to the school webpage for inter-discipline students' masterpieces; and she also adapts these as examples to her teacher-training work, shared and discussed with other teachers.

- The operational invariants: Without fixed office or desks, Anna always took her laptop wherever she went, so she used digital resources more than material ones. For Anna, the reasons why she used to store or share resources with others in such a way are more than "it is my habit", she also had her critical ways in selecting the valuable resources. She emphasized the importance of the analysis and explanations on the content and the order of the exercises to be donated to students: "I need to know the reason why they provide these exercises but not others, and if there is no analysis on the links between textbooks and exercises, I will not use it" (cited from an interview with Anna).
- The possibilities of inference: Anna tried to balance her various roles like teachers, teacher trainers, researchers, users and members in some teacher professional organizations such as APMEP. She tried to organize the resources she accumulated in her working roles in a mutual beneficiation way, which can be evidenced not only in adapting resources from different sources (or collectives) into her different jobs, but also in adapting the same resources into different working roles and get the feedbacks at the same time.

9.5.3 A Preliminary Comparison Result on Documentation Work of the Two Cases

Compared with Anna's resources, Gao relied less on Internet resources when retrieving resources, but more on Wechat, with which the teachers could exchange resources and information with their mobile phones. Reasons may come from the poor construction and maintenance of the official websites, as the principle said, "Frankly speaking, we don't pay too much time on the website or platform construction". Anna had better skills in using non face-to-face collective communication methods than Gao, such as frequent emails exchange, while the Chinese teachers interviewed had no unified and widely accepted communication method. An important reason might be that in China, the teachers have their fixed office, desk, and bookshelf; they can communicate face to face conveniently, which is quite different with the French case: Anna shared a big teacher office (without personal working space) with all the other teachers, and she had to use the digital white board (TBI) often in mathematics classroom: all these conditions compel her to use digital resources and Internet more. Seeing from the different preferences of the two teachers, a priliminary classification on resources could be made: I sorted the resources into two types, the resources work as the explanations for teachers' better understanding or extending the curriculum resources, and resource work as technological or traditional tools for working on curriculum resources like storing and sharing etc.

Some elements for developing teachers' schemes, or DE could also be found:

- Collective work with colleagues. No matter for Gao or Anna, they emphasized the importance of collective work in their interviews. Gao gained lots of reflective ideas from the collective activities in TRG/LPG, such as the open class, even though most of the time, she plays a role of instructors for others. While for Anna, collectives worked as the sources for getting various resources or feedbacks, for example, the resources she gained from IREM (like some exercises or activities) were often adapted to her classroom teaching, while the work and feedbacks of the students were also the resources for her teacher training in SESAMES.
- The competence to adapt to their environments. Anna showed her expertise by choosing different collectives and resources because the French teachers had the "pedagogy freedom", while Gao showed it by doing differently in given collectives and resources because in China generally the important and high quality resources were mainly distributed from superior units to the subordinate units. For Anna, she had the freedom and choices to decide which collectives to join, while TRG for Gao is the main and compulsory collective to work in.
- Reflections. To some extent, the different working modes of the two teachers can be seen as the results of their reflections: Anna has a clear understanding where to get the resources, how to adapt them and get a feedback meanwhile, which could be considered as a new resources for her next activity. While Gao also paved her own way of producing new resources from the feedbacks of her students and the communications with her colleagues. The reflective expertise is also shown in attending research projects, which is a way to combine practice and reflection.

9.6 A Video Analysis of a French Teacher Collective Work

This section will present a video analysis of the French case, because the Chinese videos have not been collected yet up to this chapter. To conducted a deep anaylsis, in this section, the video of a collective lesson preparation will be analyzed through two dimensions: (1) the schemes shown in this collective resource work, and (2) how they gain new elements for their resource systems through their collective interactions.

9.6.1 Background of the Collective Lesson Preparation

After three months of school activities follow up of Anna (individual level), from September 2015, another cycle of collective activities (collective level) follow up was conducted, such as meetings in SESAMES or in their school. In May 2016, a video of collective lesson preparation between Anna and Cindy on algorithmic was filmed, because it was the time to prepare the teaching plans for the following academic year, and they had to decide the textbooks for the new curriculum program.

9.6.2 An Analysis on Schemes

Some schemes of working with resources could be seen in their action. For example, although we asked them not to prepare anything before this collective lesson preparation, Anna made some preparations: she transferred and gathered the resources possibly useful (from Viaéduc[6] to a Padlet[7]), and she brought all the textbooks (13 kinds) provided by the publishers, and a word document with a copy of the contents on algorithmic in the program, and she put it in their common Dropbox folder. This could be seen as a scheme of Anna on preparing a work and resource management: with an aim of preparing a new lesson, she brought the available resources (textbooks, and the related digital resources), she prepared a word document and put it in common Dropbox folders because it is a habit of collective lesson preparation.

The second scheme turns out to be a conscious re-organization of resources. At the beginning of their formal lesson preparation, Cindy proposed to adapt an educational game "le robot" to teach Inter-discipline Teaching Practice (EPI). When Anna wrote this down in the word document, she shared it in a common folder named "les cours", and Cindy suggested that "…we should sort up this folder". This is an evidence that among the math teachers in middle school A, they had already a lived resource pond ("vivier" in French), in which they share their resources (the word document completed in the end), and they reorganize it regularly with the development of the resource inside.

Thirdly, a critical thinking towards the official resources (program) was shown several times:

- When Cindy was reading "re-visit the notion of variable and functions in a different form" with some doubt, Anna said "yes, the problem…is…they cannot

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[6] Viaéduc is a French online platform for teachers' collaborative work, see in https://www.viaeduc.fr.

[7] Padlet is an online platform for both personal and collective resources collection and creative, in the case of Anna, she use Padlet mainly for personal resources collection, see https://padlet.com.

re-visit it, because in grade 7 they haven't learned it…In grade 8 and grade 9, but…no, I want to say, re-visit…they need to already…they haven't learned algebra… for example…they haven't used the algebra variable…They cannot re-visit it". From their dialogue, Cindy proposed her doubt on the text of program, and Anna reflected it on her teaching experience and knowledge of the students' progress, and she concluded that the statement in the program is "impossible".

- About the suggestion of using "Scratch" in the program, Anna disagreed two times: the first time happened at the beginning, when Claire mentioned that the inspectors suggested to use Scratch with the students, Anna said "for me, I do not want to teach Scratch, so algorithmic, for me, it is more a thinking, it is not knowing how to use a software like Scratch; that is to say, a software, we can give it to the students as something non important, they can…" Cindy reacted with "Hum" and "Yes". The second time happened when Cindy comments that the axial position of Scratch, Anna said: "it does not matter if they emphasize Scratch and suggest to teach a lesson on it. Me, I do not want to teach a lesson on it…it is not interesting…".

9.6.3 An Analysis of the Collective Mutual Beneficiation

According to the principle of Activity Theory, we should pay attention to the contradictions of the activity. The interactions between Anna and Cindy thus were analyzed in three types: conflicts, agreements and complements, questions and answers.

- The conflicts could be seen when they hold different ideas, but between Anna and Cindy, there are not obvious or strong conflicts in their dialogues. Taking their first ideas about Scratch as an example: At the beginning, Cindy seems to agree with teaching Scratch according to the suggestions in the program, when she heard the word "but…" from Anna, she tried to remind her that the inspectors also suggested to use Scratch. When Anna explained that algorithmic should be a kind of thinking rather than using a software usage, Cindy seems to change her ideas, she reacted with "Hum" and "Yes". Later she commented that almost all activities suggested in the program are centered in using Scratch, then Anna re-stated her idea that she does not want to teach Scratch. However, in the end, they decided to arrange a computer lesson for the students to let them explore Scratch. This could be seen as a process in exchanging their ideas, and influences on each other.
- There are more agreements and complementation in this collective work, and there appeared more tacit agreements when Anna and Cindy were discussing the textbooks, they read textbooks on their own, they had their division for these 13 textbooks, and they shared the valuable parts, and exchanged the doubtful points.

- In this collective work, the questions and answers happened when one teacher did not know something, and the other explained it. Between Anna and Cindy, Anna often works as an answer-provider. For example, when reading the textbook of Sésamath, Cindy asked Anna: "How do you understand 'some languages are not used in a declared way'?", Anna proposed an example of Python, the equal ("=") is not the equal that we know normally, "it is specific, but it has a different meaning". Also, when Cindy proposed the "idea of dance" in the document of creative computing, she also explained the source and author of the document.

From an overview of this collective work, there are also some evidences or complementation on how they use their resources mentioned in their resource systems. Taking the Dropbox as an example, they all proposed that they had a common folder for sharing lesson plans, but in this video analysis, it was found that the common folder was not only some place for storing resources, they often re-organized the structure of this folder, which made this common folder a "lived pool" for resources.

9.7 Discusion and Conclusion

To explore the definition and the elements of DE, as well as how it is developed in collectives, we defined it at the beginning as the abilities and knowledge in resources integration (retrieving, selecting, organizing, modifying, adapting, creating and sharing off), in order to achieve the teaching goals, or to solve some teaching problems efficiently.

After a preliminary study of the two cases, and a deep study of a collective lesson preparation of the French case, a refined definition of DE was proposed below:

DE is the schemes in resources retrieving, selecting, modifying, adapting, storing and re-organizing, sharing off, in order to solve teaching problems efficiently. For a teacher, DE is developing to integrate her available resources to her understanding of the goals of the activity. In this way, DE is more like the common set of schemes that are suitable for given situations. It could be understood as an adaptive expertise to integrate resources into efficient problem solving in given situations. The "efficient" infers that within the ability scape of the teacher, DE could help her get some efficient and practical solutions, so DE has more an individual nature, because it is based on the understanding of the individuals, and it helps to get the solutions for problems in specific situations with the resources of the individuals. The schemes, or the elements of DE could be:

- The schemes of retrieving resources, which are also based on the schemes of resources management or storage. It could be an ability to make the use of the available resources. For example, due to the different resources environments in

China and France, Gao seldom turns to the Internet for retrieving resources, but she prefers to purchase regularly some specific reference books that she considers good in the bookstore. While for Anna, DE could be traced back to her online working habit, in both the organization and preparation: with various high qualified website resources, she has Google drive to share documents and agendas with her colleagues in the SESAMES team, Dropbox with her colleagues in her school. Meanwhile she stored her personal resources in Dropbox and Evernote, in which the documents are classified by the name of different collectives and projects. She also has some online platform like Pixees and Viaéduc to collect and store her favorite resources so that when she needs some resources she could find them easily.

- The schemes of selecting resources, which rely on the understanding of activity goals, related concepts, and their teaching practices. For Anna, she is clear so that the first lesson preparation of algorithmic should be an introduction with some activities. She has her own understanding of algorithmic, which is different from the explanations in the official program, and this is the basis for her critical thinking on the official resources and the suggestions from the inspectors. The critical thinking in selecting resources also relies on the confidence and proficient knowledge about their teaching practice, for example, when Anna and Cindy were reading the goals of algorithmic in a textbook, they doubted that the goals written ("encourage the students to understand the variables…") impossible, because "it is a notion in information", so "it is better to change the name". What's more, the feedback from the students, which is valued by Gao, could also become an important factor to decide the resources to be used in the following class.
- The schemes of modifying and adapting resources need the teacher's understanding of the situation requirements, and technology skills. Such trends appear more obvious on Anna, she has no personal office space, and so she has to take her laptop all day, which compels her to use digital and online resources more than Gao. According to an interview, as a mathematics teacher in middle school, Anna does not need to learn very complex software, and her first big challenge was the whiteboard when her school equipped it in each classroom, and she had to learn how to use it, which cost her almost one year. She explained happily that her students learned much quicker and often assisted her. This is also an open mind or a kind of curious towards new things, and new changes.
- The schemes of resources sharing, which is not a spirit of contributing others, but an efficient way of mutual benefiting. Taking Anna and Cindy as an example, Cindy used to say directly that when she had some problems in searching information and resources, she will turn to Anna and she always got her answer. And also from the observation of their school meetings or co-training in service teachers, Cindy seems to be strong to propose her ideas, comments, and suggestions in a clear and reasonable way. The sharing off of resources is not only an action of throwing the resources into the common area, but a carefully maintained, regular refreshed and re-organized, just like the common folder named "le cours" shared among Anna, Cindy and other

mathematics colleagues. Working culture or atmosphere, and personal characters could influence these. For example, in the interview with Gao, she also shared her resources, such as books and teaching instruments, with others in her LPG, and teachers in her school exchange resources like messages through Wechat group chatting, but there is seldom processed personal resources such as lesson plans or courseware.

As for the second question of how DE is developed in collectives, evidences could be seen in the interactions between Anna and Cindy:

- Conflicts in understanding or ideas are the entry points to see the influences on each other. For example, the ideas of using Scratch experienced a series of change: at the beginning, Cindy preferred to follow suggestions from program to teach Scratch, then with the arguments of Anna, Cindy changed her attitude and considered the textbooks who suggested to teach Scratch are boring, but in the end, after they finished reading all the textbooks, they both decided to teach Scratch again.
- Agreements and complements could reinforce their common ideas or enrich the current solutions. It seems like to search the hyperlinks with two search energies, which could find the complement information efficiently.
- Questions and answers is a quite direct way to benefit from each other, especially for something unknown to the other. For example, the idea of "danser" proposed by Cindy, it is the first time heard by Anna, but she learned this after it is explained.

Due to the limited space, this chapter cannot present all the three phases (preparation, implementation, reflection) of the whole document generation process. As stated in the methodology part, this case study is a long-term study, paying attention to the whole process of documentation work, and being situated in special moments with contradictions, so after the collective lesson preparation in France, in January 2017, a video of lesson implementation and a video of reflective interview were filmed. Also at the end of June 2017, the second round of lesson preparation on the same topic, algorithmic, was conducted and filmed. On the Chinese side, a series of videos on the Chinese teachers' intense collective lesson preparation, lesson implementation, and reflection discussions were also filmed in April 2017. The analysis will be presented in other papers. Through the two contexts with very different working cultures, institutional systems, and working habits, a final definition of DE, as well as the elements to develop DE in each context are hoped to be found through a synthetic analysis, and suggestions are also expected to be given for benefiting each country.

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Chapter 10
Role of Context in Social Creativity for the Design of Digital Resources

Nataly Essonnier, Chronis Kynigos, Jana Trgalova and Maria Daskolia

Abstract This paper presents a study of social creativity in the collaborative design of digital educational resources within a new socio-technical environment. This environment embeds a communication space for the designers, as well as an authoring tool enabling the meshing of text with dynamic digital widgets. We focus on understanding the processes of social creativity occurring in communities of interest, gathering together members from diverse communities of practice, taking the context of four socio-technical environments seriously into account. Our hitherto achieved results from the design of one digital resource in the French community of interest show a deep interconnectedness between emergent creativity and contextual issues.

Keywords Mathematics teachers' resources · Socio-technical environment
Collective design · Social creativity · Digital resources

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10.1 Introduction

The advent of digital technologies has made the generation of ‘socio-technical environments’ (Fischer 2001) possible. It can provide appropriate settings for teachers and other educational professionals to co-design digital resources for students, i.e. providing a means to learn by design (Mishra and Koehler 2003). With the help of such environments, addressing resource design, as a creative social activity, becomes an emerging and challenging field of research. In our case, the collaborative design process took place in a new genre of socio-technical environment, called C-book technology, an innovative authoring digital environment developed in the framework of a European project titled ‘Mathematical Creativity Squared’[1] (MC2, http://mc2-project.eu/). We aimed to study the new possibilities generated within this environment for collaborative task design and track the design process to explore social creativity occurring in the design community. The first challenging task was to articulate an operational definition of social creativity in resource design. In addition, we dealt with a special kind of resource, a ‘c-book’ (‘c’ for ‘creative’) collaboratively produced via the above-mentioned C-book technology. This technology is innovative in that it embeds a communication space for the designers and an authoring tool which affords meshing narratives with dynamic digital artefacts leading to a generation of new kinds of digital resources for mathematics teaching and learning. Therefore, the purpose of our research was to study social creativity as it emerged amongst designers of c-book resources benefitting from the C-book technology affordances, and to explore how the identity of the collectives and their collaborative design practice were influenced by contextual factors.

In the following sections, we start from setting out theoretical and methodological frames of our research (Sect. 10.2). Then, we describe the context of our research (Sect. 10.3), present the methodology applied, (Sect. 10.4) and analyse a snippet of the design of a c-book (Sect. 10.5) bringing to the fore the influence of contextual factors on social creativity (defined in Sect. 10.2.2), to answer our research question: what are the contextual factors and how do they affect the collectives’ collaborative design practice?

10.2 Theoretical Framework

As we are interested in studying the designers’ choices during the collaborative design of c-books, firstly we present the nature of a collective constituted for designing the c-books (Sect. 10.2.1). Then, we explain our conceptualisation of social creativity (Sect. 10.2.2). Finally, we expose how the documentational

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[1] The focus was both on **social creativity** of groups of designers of digital media and on **creative mathematical thinking** (CMT) developed in their users.

approach to didactics (Gueudet and Trouche 2009) provides relevant lenses to study the genesis of a c-book resource (Sect. 10.2.3).

10.2.1 Communities of Interest

Fischer (2001) defines *communities of interest* (CoIs) as follows: "*CoIs bring together stakeholders from different CoPs* [communities of practice (Wenger 1998)] *to solve a particular (design) problem of common concern*". The collectives constituted for designing the c-books gathered people from different professional worlds, such as researchers, technologists, teachers, artists, etc., with different activity systems (Engeström 1987), to solve a (design) problem, i.e. the collective design of c-books to enhance the creative mathematical thinking potential of their users, constituting CoIs. In order to take into account the context in the MC2 project, the collective design of c-book resources took place within four distinct communities of interest (see Sect. 10.3).

10.2.2 Social Creativity

The 'social creativity' approach views creativity as it springs and grows among designers within an appropriately designed socio-technical environment. More particularly, its 'social dimension' can be identified in the exchange and negotiation processes taking place amongst the designers, leading to the co-construction of novel/new, shared and thus more enriched perspectives of their task at hand (John-Steiner 2000). Different types of creativity can be traced in the related literature. The one informing the theoretical frame of our research is the so-called 'everyday' or 'little-c' creativity (Simonton 2010). This approach is close to what Craft (2000) calls 'possibility thinking', which is when a person realizes a new and improved way to approach an issue or accomplish a task. Boden (1994) uses another term, 'psychological creativity' (p-creativity), to refer to something which is identified as creative at least by its author her/himself.

Apart from the "little-c" theoretical perspective of creativity our approach moves within the "componential" tradition of creativity assessment (Hennessey and Amabile 1999) asserting that creativity is a multi-component and in-context activity. More particularly, our operational definition of social creativity, as identified in the collaborative design of c-book resources fostering creative mathematical thinking taking place within the MC2 socio-technical environment, views creativity as the *generation of ideas* which: (a) stem from a combination of two or more individual 'activity systems', that is the CoI members' knowledge systems or other socio-cultural domains, (b) result from various interactions among the CoI members and with the C-book technology and tools, (c) are externalized in and through specific digital artefacts (such as the c-books) which are not only the final

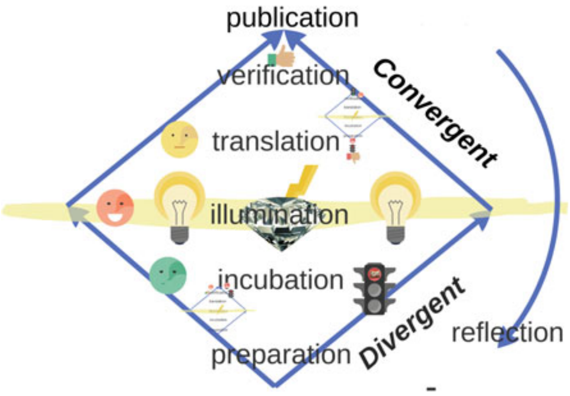

Fig. 10.1 Diamond of creativity [with the authorisation of the author, P. Lealdino Filho, member of the MC2 project (personal communication)]

'products' but they also enable and boost communication, mutual understanding, negotiation and construction of new knowledge among the CoI members during the various phases and activities of the design process, and (d) are considered to be: (1) **novel** (original, unusual or new), at least to the minds of the CoI members who produced them, (2) **appropriate**, that is they conform to the characteristics and functions of the c-books, as defined by the CoI members with regards to their intended target group(s), and (3) **usable**, ready and available to be used in the design of the c-books according to the designers' estimation (Daskolia 2015).

Creativity, whether social or individual, in the design of resources can be modeled as a fractal diamond (Fig. 10.1) comprising several alternating phases, such as preparation, incubation, illumination, translation and verification (Hadamard 1945), representing *divergent* and *convergent* thinking stages (Csikszentmihalyi 1996). Divergent thinking is understood as a generation of novel ideas corresponding to the phases of preparation, incubation and illumination and convergent thinking as a validation of ideas found appropriate and usable, corresponding to the phases of translation and verification. The publication phase corresponds to the implementation of the idea to the resource.

A stage of divergent thinking followed by a stage of convergent thinking is considered as a cycle of individual or collective creativity.

Hence, social creativity is characterised by the reification of creative ideas collectively elaborated, i.e. elaborated by at least two CoI members.

10.2.3 Documentational Approach to Didactics

The collaborative design is studied from the perspective of the documentational approach to didactics (DAD) (Gueudet and Trouche 2009), which introduces the distinction between a *resource*, conceptualized as any "thing" with which the teacher interacts and which re-sources his/her practice, and a *document* resulting from a combination of resources to which a *scheme*[2] *of utilization* is associated. The process of transformation of (a set of) resource(s) into a document is called *documentational genesis*. Likewise the instrumental genesis (Rabardel 1995), the documentational genesis consists of processes of *instrumentation* (resources informing a teacher's action) and *instrumentalization* (teacher's transformations of resources).

Hammoud (2012, pp. 46–47) introduced the concept of *mother resources* to refer to a set of initial resources that a teacher mobilizes to prepare a teaching and which can be identified and explained by the teacher, the concept of *daughter resource* that is the resource implemented by the teacher in her class; a daughter resource is thus a result, the product and the fruit of mother resources, at a given moment, for an implementation in the class. She also introduced the concept of a resource *life cycle* to highlight major evolutions of a resource in relation with the main moments of the teacher's documentational work.

The design of the c-books within the CoIs can be viewed as a documentational genesis. Indeed, the designers bring to the design process resources coming from their own resource systems (*mother resources*), combine and transform them into a c-book resource (*daughter resource*). It is therefore important to consider also the existence of resources coming into play during the design process. The path from mother resources to a daughter resource can be considered as a phase of the resource *life cycle*, and it can be described as a sequence of versions of the resource.

The collective design process viewed as a collective documentational genesis (Fig. 10.2), starting from a resource or a set of resources contributed to the joint enterprise by one of the designers and resulting in a c-book resource. This process benefits from the interactions among the designers and/or with the C-book technology and it includes the *instrumentation* (the resources shaping their users' actions) and *instrumentalisation* (the users adapting the resources) processes.

We assume that between two consecutive versions of a c-book, a creative cycle (Fig. 10.1) occurred. Thus during the design process of a c-book, a number of such cycles have to take place at many different scales, from designers' minds to collective interactions, yielding successive versions of a c-book.

10.3 Context of Research and Research Questions

Following Fischer (2001), four distinct communities of interest (CoIs), rather than communities of practice (CoPs), were set up within the MC2 project, to collectively design c-book resources for mathematics (Kynigos and Daskolia 2014). In order to allow contextual factors impacting the design processes to emerge, the four CoIs were formed in four different participating countries. They brought together

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[2] Vergnaud (1990, p. 48) defines a scheme as *the invariant organization of behavior for a certain class of situations*.

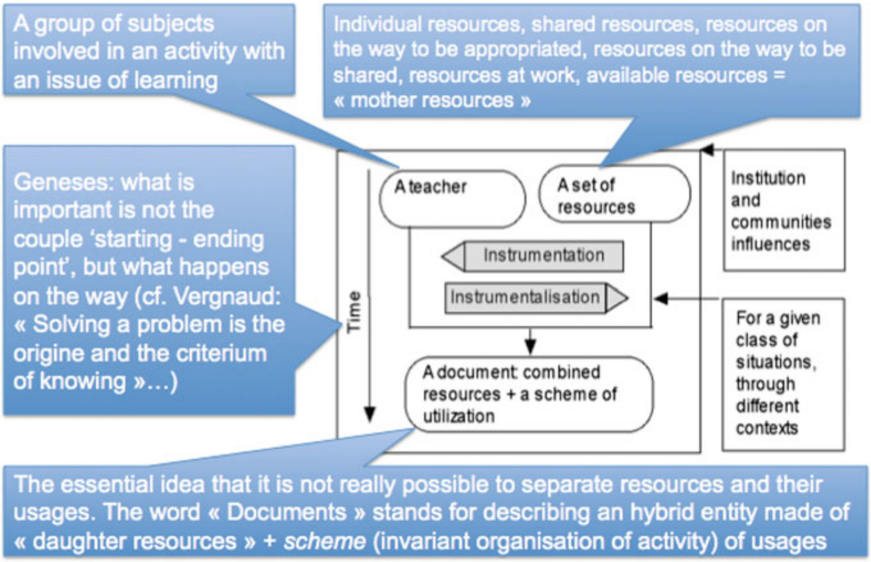

Fig. 10.2 Documentational genesis, adapted from Gueudet and Trouche (2012, p. 26)

resource designers with various backgrounds, such as teachers, researchers, teacher educators, artists, or computer scientists.

The French CoI, involved in the design of the c-book presented and analysed in the Sect. 10.5, comprised 13 members, including two educational software designers, one mathematician and artist, the other members being mathematics teachers, teacher educators and researchers in mathematics or mathematics education. All CoI members shared a socio-constructivist approach rooted in the French didactic tradition of teaching and learning mathematics (Artigue 2016). Several CoI members belonged to communities of practice focusing on some specific issues of mathematics education, such as the use of technology to model everyday situations, or the development of algebraic thinking. The period during which the CoI design activity took place was marked by the announcement of a curricular reform which was about to introduce, in mathematics curricula, a few novelties as interdisciplinary aspects or algorithms.

In studying social creativity processes in the design of c-book resources, we were particularly interested in how the CoI contextual characteristics, either personal, social, institutional, or cultural, affected the CoI's design choices and therefore the resulting resources. We were more particularly curious about exploring how such conditions influenced the choice of the topic, the elaboration of the scenario and the construction of the mathematical content for the c-book, so that 'de-contextualization' (Lagrange and Kynigos 2014) of the design process might become possible making our findings useful more broadly. We can now refine our research question: what are the contextual factors and how do they affect the designers' choices in the c-book resource design (instrumentalisation)?

10.4 Methodology

Our methodology relies on a case study. The analysis focuses on processes of collaborative design of one c-book produced by the French CoI.

Paying particular attention to contextual issues, we developed a set of 'integrating tools' to obtain data about the contexts of the CoIs established within the MC2 project. Two of these tools, the 'CoI profile' and 'CoI moderation' templates, were directly addressing the synthesis of each CoI and the work organization within them. The first collects data about the CoI and its members: the number of CoI members, their profile and institutions, the CoP they represent, their intended role in the c-book resource design. For instance, in the case of the Greek CoI, the 'profile' template reveals that several members stem from CoPs dealing with environmental education, either as a teaching or as a research practice, a fact also reflected in the recurrent choices made by the Greek CoI of environmental topics for their c-books design (e.g., 'Windmills', 'Climate change' or 'Cycling in the city'[3]). The second template describes the moderation strategy, orchestrations of activities and procedures to support and facilitate the CoI's creative performance. The French 'CoI moderation' template, for instance, gives information about the CoI decisions with regards to the design of their c-books, such as the involvement in each c-book design of a small number of CoI members playing different roles (moderator, designers and reviewer). Such division and organization of the tasks at hand, according to the interests and constraints of each member, aimed at sustaining collaboration and thus social creativity.

The main source of data collected for the analysis of the design process was the C-book technical environment, which includes a shared communication space, called 'CoICode',[4] to organize and enhance interactions among designers. CoICode enables each designer to post various kinds of ideas ('contributory', 'alternative', 'objection', 'off task' and 'task organization'), each of them having a specific icon. When a designer posts an idea, the system captures several details: author's name, date, title of the idea, comments, attached resources, hyperlinks, etc. This space was designed to promote social creativity among the designers by enabling exchanges between them (externalization of ideas, sharing resources) and, at the same time, to collect data for both design and research purposes by saving all traces of interaction as a logbook. In CoICode, the discussions can be visualised in form of threaded forum or in a mind-map view (Fig. 10.3), where nodes are ideas, and branches of the tree model the evolutions of an idea.

The analysis of CoICode data is mostly qualitative, based on the content of each contribution. It focuses on the identification and mapping out of the various phases through which the design activity passes, starting from the moment a CoI

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[3] The description of these c-books is available at http://www.mc2-project.eu/index.php/c-book.

[4] CoICode is a communication environment integrated to the C-book technology. It allows creating a workspace within which members of a CoI engaged in collaborative design of c-books can communicate.

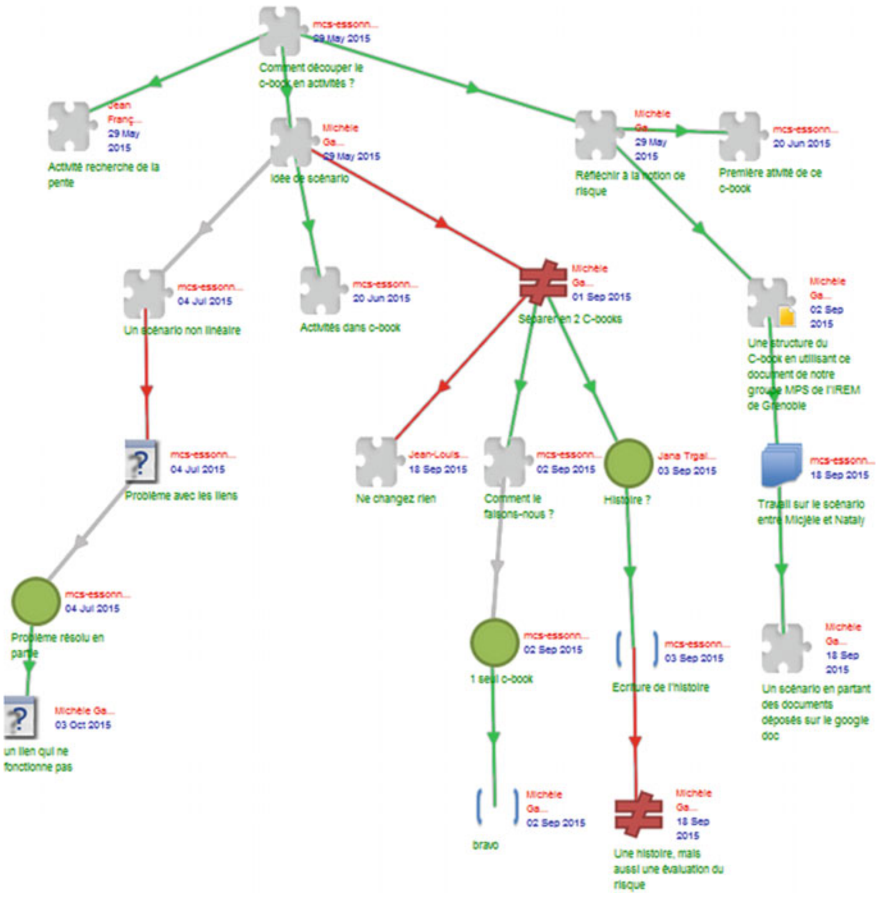

Fig. 10.3 Extract of a CoICode workspace (mind map view)

converged in CoICode workspace on the first design choices and got organised to deal with the design of the resource, till the actual realisation of the c-book. CoICode data allow us to highlight different versions of the c-book constituting a part of its cycle of life. Drawing on the 'diamond of creativity' approach (see Sect. 10.2.2), each version of a c-book corresponds to one design stage. Each stage begins with a phase of divergent thinking, which manifests itself by a multiplicity of contributions during a short period of time, and ends up by a phase of convergent thinking resulting in reification of socially elaborated ideas. The identification of various versions of the c-book enables us to reconstruct the resource life cycle and to explore contextual factors that influenced its design.

10.5 The Design of the "Ski Touring" C-Book by the French CoI

To highlight the influence of context on social creativity in the design of c-books, we have decided to study the design by the French CoI of one c-book called 'Ski touring'.

10.5.1 The Designers' Profiles, Roles and Interests

Seven members (Table 10.1) of the French CoI took part in the design of this c-book (Fig. 10.4). The roles of the moderator, main designers and reviewer were negotiated and distributed during a CoI meeting.

The design period spanned over seven months. The three main designers of the c-book, Zoe, Paul and Jack scheduled regular meetings to discuss design issues and to divide the tasks at hand. Nick, Tom and Leo interacted with the designers during the CoI meetings, mainly through CoICode, posting their feedback and suggestions.

Table 10.1 The designers' profile, role in the design and interest(s)

CoI member	Profile	Role in the design	Interest(s)
Jack[a]	Ph.D. student and college mathematics and physics teacher	Moderator and main designer	Design of resources for teaching
Zoe	Researcher and mathematics teacher educator	Main designer	Introduce students to scientific methods and practice using modelling Learn about C-book technology affordances foreseen as an opportunity to renew and to innovate her own resources
Paul	Teacher educator and former secondary mathematics teacher	Main designer	Use technology for modelling. Interest in interdisciplinary domains for teaching
Leo	Researcher, mathematics teacher educator and expert user of technology	Designer of specific widgets	Use widget factories to create new widgets in a mathematical context for modelling
Tom	Researcher and mathematics teacher educator	Reviewer	Didactics of mathematics and technology
Nick	Teacher educator and former secondary mathematics teacher	Reviewer	Interest in interdisciplinary domains for teaching
Pierre	Mathematician and artist	Reviewer	Interest in mathematics around us, in objects, human body, etc.

[a] Nicknames

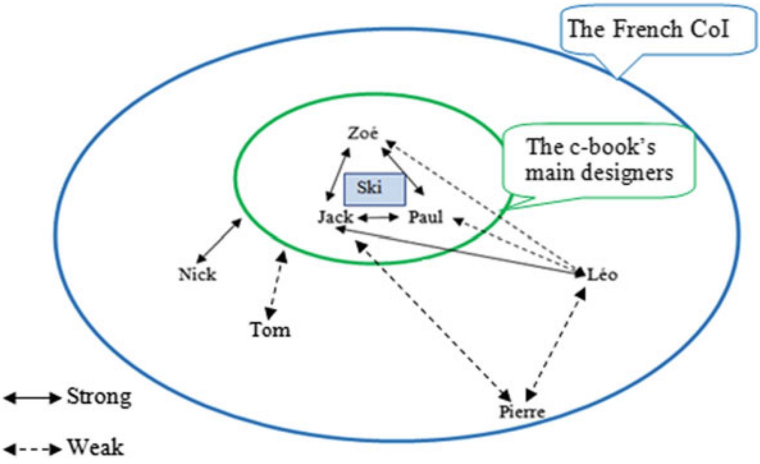

Fig. 10.4 The schematic representation of the interactions among the designers (weak interactions when they are no more than two and strong when there are more than two interactions among the designers)

Leo, because of his technology skills, was solicited at many occasions to work with the designers; whereas Pierre's unique intervention was managed by Jack and Leo during a face to face meeting. Nick and Pierre intervened sporadically, either spontaneously or when asked by Jack; they provided feedback to the main designers' group, bringing new insights and new ideas. The role of the moderator, Jack, during the design process, was central for creating good communication conditions and enabling creativity to emerge. The interactions among the designers are schematically represented in Fig. 10.4.

Leo and Jack had a better knowledge of the C-book technology than the others; they had already used it for designing other C-books. Paul was a newcomer and he was discovering the socio-technical environment. Zoe was also a novice user of the C-book technology as she did not have the opportunity to interact directly with it before (technological context).

10.5.2 Analysis of the C-Book Design Process

From the data collected in CoICode and the notes taken by the researchers during the French CoI or the designers' meetings, we have identified phases of divergent and convergent thinking, which allowed us to reconstruct the life cycle of the c-book, or at least a part of it (Fig. 10.5). A further analysis of these data, drawing on the documentational approach to didactics, enabled us to identify the elements of context, within the CoI, which oriented the design process and thus the final product, the c-book.

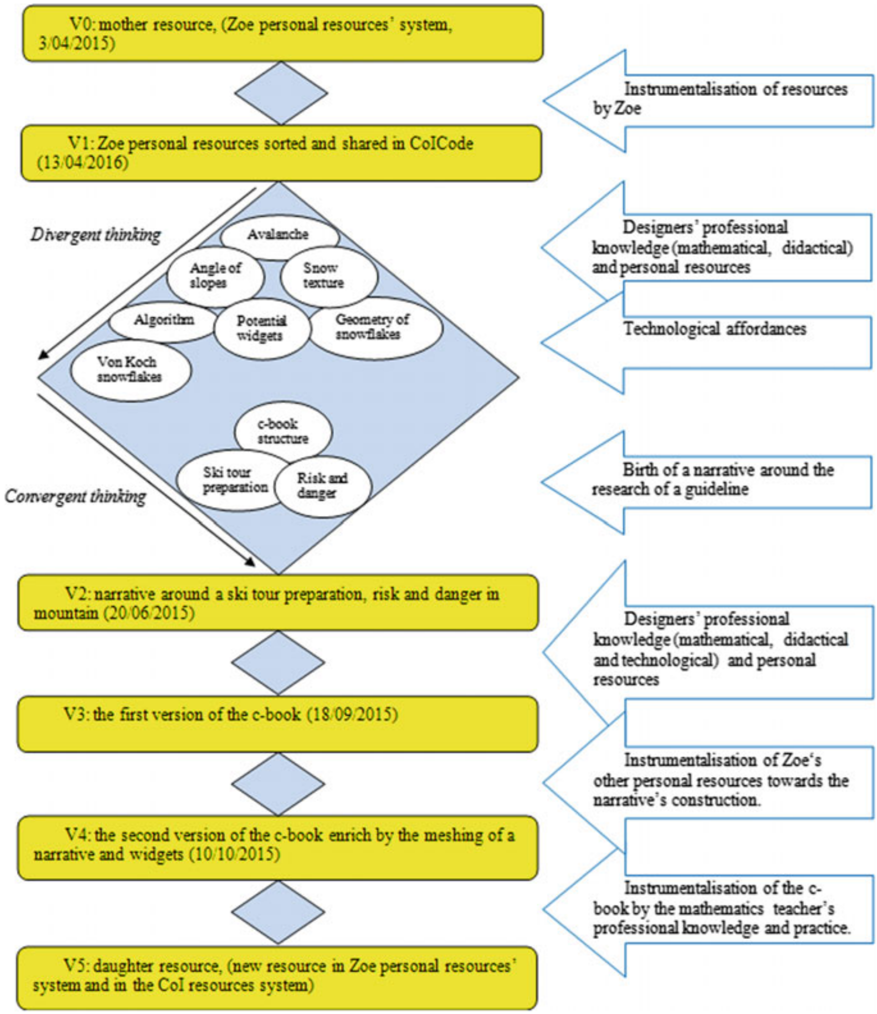

Fig. 10.5 The schema of the resource life cycle, from the beginning of the design process up to a version ready to be used in a classroom

The topic of this c-book was suggested by Zoe during a CoI meeting and matched the institutional context of the moment about interdisciplinary aspects. Zoe lives and works in an area where many students practice ski (socio-cultural context). She had already worked on this topic and experimented teaching sequences based on the use of mathematics for solving problems situated in a scientific context, for Grade 10 students, with a secondary mathematics teacher within an associated CoP. She thus had a set of relevant resources, such as videos, maps, parts of pedagogical scenarios, students' productions, newspaper articles and various other resources. She proposed to design a c-book based on some of these resources expecting their improvement thanks to the affordances of the technological environment, to renew her practice and test the improved resources in the classroom of the secondary mathematics teacher she worked with for several years (personal interest). The combination of these resources, within her personal resource system, constituted the initial version of the c-book which was not yet shared with other designers (v0: mother resources).

The first version of the resource (v1) resulted from the sharing of some of these personal resources by Zoe in CoICode. This version, as driven by Zoe's personal interest, was mainly impacted by two contextual factors: the socio-cultural context of Zoe's geographical situation and her attraction to the C-book technology affordances. It was conceived through manifold posts in CoICode done by Zoe, which can be considered as the instrumentalisation of her resources because she had not posted all the resources she got within CoICode, but only those that, to her view, helped externalise her idea of the future c-book for the other designers.

This was the starting point of a rich discussion, exchanges among the designers and the emergence of eleven new ideas. For instance, Zoe added a **new** idea not fully elaborated about the snow texture. Based on this idea, other designers proposed **new** mathematical directions to explore, such as studying geometric shapes of snowflakes in a plane, sequences based on Von Koch snowflakes, or geometric transformations (symmetry, rotations), later associated with designers' personal resources coming from other CoPs or their own research. These suggestions enlightened the interest of the designers in modelling and were bound with the designers' activity systems, because modelling was also an interest of the associated CoPs. Then, Zoe suggested to study more particularly the growth of snow crystals, which led to the idea of algorithms for drawing snowflakes. This suggestion was motivated by the new mathematics curricula introducing algorithms (institutional context). Other ideas emerged as well, such as geometric shapes in space, modelling with functions, studying the acceleration of an avalanche, because an acceleration widget was already designed for another c-book, or slopes with sand as in a mathematical contest organised by Leo. The incorporation in some pages of a forum tool, epsilonChat (http://www.epsilonwriter.com/fr/accueil.htm), to afford collaboration and mathematical discussions between students and to keep traces of it, came from the designers' socio-constructivist background and was unanimously agreed upon. Seeking for a guideline to better envision the c-book, Jack posted a contribution asking how to structure the c-book, which was the starting point of a phase of convergent thinking. In response, Zoe posted two contributions about the ski tour preparation, and another one related to risk and danger in a mountain (personal concern), whereas Paul seemed more focused on the activity because of his concerns as a teacher educator (**appropriateness**), making a narrative around ski tour preparation and risks emerge. The designers also discussed the features they would have liked to embed into the widgets (dynamics, algorithmic features) to support students' learning and they decided who, between Leo and Paul, would design these widgets, according to their skills (Leo being a specialist of Cinderella dynamic geometry software and Paul being interested in Geogebra and programming). During this phase of convergent thinking, some ideas were further elaborated in some posts in CoICode and the designers' personal resources played an important role in this process (e.g. Paul provided a resource elaborated within an associated CoP in which snowflakes were modelled with triangles, symmetry and rotations). Other ideas were rejected, judged either not **usable**, such as snowflakes shape in space (not associated with any resource), or not **appropriate** because too difficult for secondary school students (Grades 10, 11 and 12), or not directly linked with the notion of snow, risk and danger in a ski tour preparation, such as modelling of the acceleration of an avalanche or modelling of slopes with sand. Moreover, because of time constrains due to various obligations of the designers linked with their primary occupation (institutional context), Jack, already familiar with the C-book technology, was naturally assigned the implementation and reification of commonly agreed ideas in the c-book, constituting the second version of the resource, (v2: emergence of a narrative mixed with technology).

The creative cycle from v1 to v2 started with a phase of divergent thinking when the eleven new ideas emerged bound to the snow and ski context, the socio-constructivist culture of the designers, and/or their interest for modelling, proposing a new perspective of the problem at stake. Meanwhile, the designers attached resources from their personal resource systems to their posts in order to be understood by other teammates. Eight of these ideas were judged appropriate and usable by the designers, and three were rejected. The validation of the eight ideas constituted a phase of convergent thinking. We note that the division of work was done during this phase, to allow reification of creative ideas. We also observe that the role of Jack evolved from moderator to designer in this cycle, because of two contextual factors: time constraints and his familiarity with the technology. The second version of the resource was born around the emergence of a narrative bound to the geographical context used as a guideline and as a support for mathematical activities, and the explicit division of technical tasks at hand due to the technical and institutional contexts, but also taking account of the designers' profiles, interests and knowledge. Likewise, it appeared that the designers mainly kept the ideas closely related to the narrative they chose, or the ones they were able to further elaborate with personal resources, such as resources developed in associated CoPs.

Then Jack, still seeking for a c-book structure to start reification of commonly agreed ideas, proposed a new idea to begin the c-book with an introduction combining photos, videos, texts, tables about ski touring and risks, based on Zoe's and his own personal resources, and embedding hyperlinks to the mathematical activities. He was supported by Zoe who provided an example of a pedagogical scenario, coming from her personal resource system, to externalise her thoughts, because she did not clearly imagine the pedagogical scenario yet (pedagogical concern). Once the scenario was clearer for Jack, he began the implementation of the c-book, exposed technical issues, and solved them. Then, Pierre, reviewing the c-book prototype, raised an issue about students' difficulties to build an appropriate representation of the angle of a slope (pedagogical concern). Hence, he proposed an artefact enabling students to observe a snowy slope with different angles and to evaluate the angle of a slope and the simulation of this artefact was later created by Leo with Cinderella. Later on, some **new** widgets were designed by Leo and Jack following some ideas initiated by Zoe and Paul, enabling students to model snowflakes in a plane using transformations, helping them to find a safe path on a map for a ski tour, and simulating the functioning of an ARVA (a safety device enabling to geo-localise an avalanche victim, https://www.arva-equipment.com/us/) in a playful way to enhance the affective aspects, to foster engagement and to prove that an algorithm is a powerful tool for geo-localisation. The implementation in the technical environment, made by Jack, yielded a third version, (v3: technological and didactical enrichments).

The creative cycle from v2 to v3 started with a phase of divergent thinking yielding a new idea of using the affordances of the C-book technology to initiate a pedagogical scenario for the c-book and another new idea from Pierre about the use of an artefact to facilitate students' understanding. The subsequent phase of convergent thinking allowed the construction of the scenario, its elaboration based on personal resources coming from Jack's and Zoe's personal resources systems and the production of new widgets by Leo and Paul bound to modelling and serious game. We note that the interest of the designers supported their decisions in this cycle and that a serious game was added for fostering the affective aspect of the creative mathematical thinking (CMT).

But the designers found that the mathematical content did not fit with the national curriculum: the content was too wide for one level of class and it would be difficult for a teacher to experiment the c-book in her class. Therefore, a suggestion was made to split it in two separate c-books (institutional context and usability). A phase of negotiation began during a CoI meeting just after the presentation of the c-book, Nick objected: "*It is true that the c-book is rich, but that is what makes it interesting. It is a pity to split it in two c-books. Is it possible to consider a more obvious difference between the levels?*" After discussions, the CoI members agreed with him, and it was decided to find a means to keep the whole content. In addition, Tom suggested to take the opportunity of meshing both text (storyline) and widgets afforded by the socio-technical environment (**novelty**). Likewise, Zoe, still not satisfied with the scenario that she found quite far away from what she had imagined, added two more nodes with some new personal resources to better explain her expectations about the reduction of risk and she proposed to work with Jack on the development of the c-book, which initiated a phase of convergent thinking. This new version of the scenario was crafted by Zoe and Jack, taking into account Nick's and Tom's comments and the curriculum constrains (institutional context), keeping all the activities intact but explaining to the teacher the way to ensure usability by orchestrating the activities, in relation to the students' level and pointing out the reduction of risk, the whole embedded in a story. Jack had helped to reach a consensus by combining all the ideas expressed about the scenario. He was convinced that the storytelling plays a major role in the affective aspects and that it could boost students' motivation and engagement in the activities, (personal beliefs). In addition, some particular features for developing the CMT affordances of the c-book were added such as exercises with limited time and many answers, diverse perspectives on the subject and serious games. A widget developed by the

Greek CoI was embedded in the resource enabling students to work on algorithms in a constructionist way (Kynigos 2015) (v4: the c-book 'ski touring').

The creative cycle from v3 to v4 started with a phase of divergent thinking during which two main ideas emerged: the idea that the content of the c-book did not fit with the curriculum and the one that the pedagogical scenario was not totally appropriate. The subsequent phase of convergent thinking was based on negotiation of the content and the pedagogical scenario, leading to associations of several ideas combining personal and the MC2 project interests.

The next step, not presented in this chapter, was the review of the c-book by a mathematics teacher leading to its last version (v5: daughter resource), and its experimentation in her classes.

10.5.3 Results and Discussion

With respect to our research question (What are the contextual factors and how do they influence the collaborative design practice?), this analysis shows that the designers availed of the possibilities provided by the new digital environment. They used it for pedagogical aims based on a socio-constructivist and constructionist perspectives, the first coming from the French CoI background, while the second being inspired by the work of the Greek CoI. For instance, the designers believed in the meshing of narrative with widgets to motivate the students, to encourage them to try the activities and to engage them in a learning process, in modelling to solve problems, and in mathematical discussions to construct new knowledge. The integration in the c-book of a forum tool, epsilonChat, to foster collaboration and mathematical discussions amongst the students, was also influenced by the fact that the tool developers were members of the French CoI. Indeed, the other three CoIs did not think of using it in their first c-books. Likewise, the widgets were used to help understanding of a notion, building various representations or to support the construction of reasoning, but their design depended mainly on the expertise of one CoI member. The contextual factors appeared critical in the design of the c-book but they did not orient the design process at the same time. For example, the CMT potential of the c-book was taken into account only at the end of the design process to fit with the project objectives but was not the first concern of the designers. One reason could have been that the designers' understanding of the CMT concept became deeper at the final stage of the design process. We also observe that the designers suggested a variety of mathematics topics that were studied in the c-book (geometrical figures, transformations, sequences, functions, trigonometry and algorithms). These topics were developed following the opportunity given by the real life context they were related to (snow texture, avalanche risk etc.) and were not restricted to one class level. We observe the development of tasks around mathematical notions, such as algorithms and transformations, which have recently been added to mathematical curriculum (French education reform in 2016) and which became a design driving force (institutional context). The unity of the c-book was kept by the central theme (ski touring) used as a guideline and its usability was ensured by explicit explanations aimed at teachers how to use it to fit with the French curriculum. Hence, the reification of the pedagogical scenario obliged the designers to be creative and original in order to provide a usable c-book for the teacher.

On the other hand, the reification of ideas in the c-book relied mostly on one designer and his knowledge of the technical environment, which required an ability to understand the other designers and to take into account many different perspectives. Technical expertise of other members, as well as resources coming from the designers' activity systems, also played an important role in the reification of commonly agreed ideas. The technical environment required some technical expertise to be exploited. Hence, the presence of members with technical skills within the designers' team was a key element of the c-book design which would not have been the same without their expertise. In addition, we note that the work done in other CoIs with different cultural contexts, impacted positively the design practice of the French CoI. For example, the constructionist perspective and the idea of meshing narrative with dynamic widgets in the c-books, developed first by the Greek CoI, spread across all CoIs and resulted, in the case of the "Ski touring" c-book, in the development of a storyline and in the integration of a constructionist widget. The review of the different versions of the resource brought new insights and raised many issues, which contributed to enrich the c-book and resulted in a non-linear design process. Thus the reviews of the resource were also key stages in the design process.

Finally, we mentioned that the interactions among the designers were more or less strong. The weakness or strength of the designers' interactions influenced the elaboration of the ideas in the design process and/or the communication amongst them, i.e. the number of exchanges. We observed that people with weak interaction elaborated fewer ideas than the ones with strong interaction, participated less spontaneously and were less involved. On the other hand, weaker involvement of some CoI members had sometimes a positive impact on social creativity in a sense that these members often brought new insights and opened unexpected avenues to the design process.

10.6 Conclusion

Our analysis provides findings highlighting several contextual factors that significantly affected design choices. Thus, the cultural context oriented the topic of the resource, which was used all over the design process as a guideline. The institutional context bound to the middle school mathematics curriculum reform, which added new mathematical notions to teach (e.g., algorithms), led the designers to develop activities addressing these notions. The cultural influence of other CoIs within the project spread across all CoIs and led the French designers to use a constructionist approach and to develop a story-based c-book instead of a purely mathematical one, as was the case of the first c-books produced by the French CoI. The French CoI socio-constructivist background combined with their CMT representation resulted in paying attention to social and affective aspects of students' work by integrating in the c-book collaboration tools and game-based elements to foster students' motivation. Time constrains, personal concerns, curriculum standards, but also the various designers' profiles and interests, constitute other contextual factors that influenced, at different moments, the decisions made in the design of c-book resources. Due to contextual factors, the role of the CoI members may change during the design of a c-book.

The research results reported in this paper show that collaborative design in collectives of individual educational designers with different activity systems, holding different backgrounds and having various personal and professional concerns, supported by an appropriate technology, can bring forth new and alternative ideas, solutions and implementations from the part of the designers, thus leading to a rich design process with strong social creativity. The networking of the documentational approach to didactics with creativity and social creativity perspectives provided a fruitful way to address issues of social creativity and a powerful theoretical lens to better understand the role of resources and the impact of contextual factors on the processes of design of digital resources for mathematics education.

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Chapter 11
Uses of Online Resources and Documentational Trajectories: The Case of Sésamath

Katiane de Moraes Rocha

Abstract This chapter questions the way teachers' collective work helps them to design resources requested by curricular changes. Situated within the framework of the Documentational Approach to Didactics, it addresses this issue in analyzing the case of Sesamath, a French teacher association designing free resources, particularly textbooks, for their colleagues. This analysis uses two lenses, the first focusing on a designer of a Sésamath textbook, the second one focusing on Sésamath textbook users, evidencing the influences of teachers' collective activity on their documentation work. For conceptualizing theses influences, two concepts are proposed: the concept of documentational experience, and the concept of documentational trajectory.

Keywords Mathematics teachers' resources · Documentational approach to didactics · Documentational trajectory · Documentational experience
Collective work · Thought collective · Thought style

11.1 Introduction

New means of communication and digitalization foster the design and sharing of teaching resources. New types of digitalization provide on the one hand, new resources to be used in teaching (tablets, software, smartphones, interactive whiteboards, etc.); on the other hand, new ways of communication facilitate teachers' collective work, their access to a vast set of resources and the sharing of resources (through online platforms, Twitter, Dropbox, forums, sites, etc.). All these possibilities have an enormous impact on how teachers use and design resources.

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In France, these developments have motivated a national research project, ReVEA (Living Resources for Teaching and Learning, 2014–2018, www.anr-revea.fr), that seeks to better understand the processes through which teachers design, share and integrate resources. In this paper we consider two dimensions of the usage of resources: individual and collective. Trouche (2016) presented different collectives that support teacher's usage and design of resources in various ways in France. Hence, in relation to the ReVEA project, we focus on the Sésamath association (http://sesamath.net), seeing their evolving work as emblematic of a broader trend in the current digital evolution (as discussed in Chap. 1).

This research occurs at a critical moment for teaching in France, in the context of the implementation of a new curricular program. This new curriculum was introduced in middle schools in September 2016 (Gueudet et al. 2017) and had involved deep changes. For example, as part of the reform, teaching shifted to a cycle of three years instead of one year; interdisciplinary work was introduced involving all disciplines; new contents (e.g., algorithmic approaches and computer programming to be taught by mathematics teachers); and the introduction of new digital tools. These changes will require the design of new resources by the educational designers as well as by the teachers themselves.

To summarize, these new digital resources, new forms of collective work and curricular changes lead to new research questions about the evolution of teaching resources and of teachers' work with resources. In this context, we look at teacher-participants involved in the Sésamath association as both designers and users exploring the question of *how do teachers' work within collectives inform their work with resources as they grapple with curricular changes.* To this end, we explore two case studies on teachers' documentation work (as defined in Chap. 1). The first focuses on the short term: looking at the design process of a Sésamath 4th cycle textbook (7th, 8th, and 9th grades) incorporating the new curriculum, we analyze *how a teacher's collective work for designing a Sésamath textbook influences her own documentation work when facing curricular changes.* The second case focuses on the long term documentational work of a Sésamath user, analyzing *how teacher's collective work, in using a Sésamath textbook, influences her own documentation work when facing curricular changes.* In both cases, we are interested in how teachers' experiences with resources developed (which specific resources, where did they came from and how were they used). As part of our findings, we articulate in this study two new concepts[1]: *documentational experience* and *documentational trajectories* (defined in Sect. 11.2).

This article is divided into four sections. In Sect. 11.2, we present our theoretical choices. In Sect. 11.3, we show our methodological design. In Sect. 11.4, we present our analyses of a Sésamath designer and of a Sésamath user. And, finally, in the last section, we discuss our conclusions arising from these cases.

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[1] Investigated in the frame of a Ph.D. supervised by Luc Trouche (ENS de Lyon, France).

11.2 Conceptual Framing

In this section, we present our theoretical framework. First, we situate how the documentational approach to didactics nourishes our reflective process and how we in turn continue to develop this conceptual tool. Afterward, we bring to the fore our theoretical choices in analyzing collective aspects of a teacher's practice. And finally, we propose two correlated concepts: *documentational experience* and *documentational trajectory*.

11.2.1 Documentational Approach to Didactics

Our primary interest is to investigate teachers' *professional development* through the analysis of their processes of design and adaptation of resources. We consider teachers' professional development as a process of professional learning. More precisely, according to Gueudet and Trouche (2012), we consider teachers' professional development as the result of an interrelated process of incorporating new resources, developing her knowledge for teaching, and interacting with other participants during this process.

Our work is then grounded in the documentational approach to didactics (DAD) (Gueudet and Trouche 2012; Chap. 1), which centers on a teacher's resource *design* and *use* as her main tasks, involving a process of seeking, selecting and modifying available resources. We use in this chapter the concepts of *resources*, *document, documentation work, documentational genesis*, and *resource system*, already introduced in Chap. 1.

Our approach highlights the importance of a teacher's documentational genesis when adapting to or designing resources. Inversely, over a longer period, we investigate how, beyond the genesis of a given document, the documentational work of a teacher evolves: *where* resources are found, *which* resources are used, and *why* and *how* they are used. As presented by Gueudet and Trouche (2008, p. 7, our translation) a teacher's activity is situated within "a set of institutional and social constraints and forces" that shapes what we call a teacher's *collective work*.

11.2.2 Teachers' Collective Documentational Work

Many frameworks exist for analyzing teachers' activity within a social context (Gueudet and Trouche 2008). In this section, we justify our choice for the frame of thought collectives proposed by Fleck (1981/1934).

Teachers' documentational work happens not only in the classroom or at home, but also during training moments or while speaking with a colleague during a coffee break at school, for instance. Teachers may also participate in a diverse range of collectives with colleagues: stable or transitory, formal or informal, obligatory or voluntary, among other differences. This work involves different types of interactions relating to the exchange of resources or ideas cooperatively or collaboratively: we use the term *cooperation* and *collaboration* as Roschelle and Teasley (1995, p. 70): cooperation "is accomplished by the division of labor among participants" and collaboration "as the mutual engagement of participants in a coordinated effort to solve the problem together". Whatever the nature of collectives or modes of communication, teachers' documentation work happens over time in interaction with other people (colleagues, researchers, families, etc.).

Therefore, we looked for a framework that on the one hand takes into account all the kinds of collectives, interactions, and subjects that teachers might interact with; and, on the other hand, allows us to analyze the boundaries between these collectives. This is the reason we articulate two frameworks:

- firstly, the broad definition of *thought collective* proposed by Fleck (1981/1934) exists when "two or more people are exchanging thoughts" (p. 44) and generates a *thought style* "characterized by standard features in the problems of interest to a thought collective, by the judgment which the thought collective considers evident, and by the methods which it applies as a means of cognition" (p. 99);
- secondly, the concept of *communities of practice* proposed by Wenger (1998) connected with the boundary crossing approach elaborated by Akkerman and Bakker (2011) for analyzing boundaries between members participating in many collectives, i.e. *multi-membership*. We are particularly interested in the notion of *brokers* for designating members that connect different collectives, and *boundary objects*—resources, documents, concepts, and other forms—which connect work between collectives.

Thus, the problems of "how do thought collectivess support teachers' documentational work over time" and "what are the links across the boundaries between these thought collectives" are crucial for our understanding. For this purpose, we will present in the next section two correlated concepts that we developed in our research: *documentational experience* and *documentational trajectory*.

11.2.3 Teachers' Documentation Work Longitudinally: The Development of the Notions of Documentational Experience and Documentational Trajectory

In a teacher's day-to-day activity, many *events* condition her documentational work: when she decides to join a new collective, or encounters new resources, or has to incorporate a change in curriculum, or has to take into account students with special needs. We understand *events* in a large sense as everything that might lead a teacher to transform her resource system and usage.

We believe that over time these events construct teachers' *experience* with resources, in a sense proposed by Pastré (2005) as the accumulation and appropriation of the past by the subject. In particular, we seek to understand this experience as it relates to a teacher's development of their resource system and knowledge, what we call their *documentational experience* (*DE*). Following this thread, we define a teacher's *documentational trajectory* as the set of events through which a teacher constructs her documentational experience as an interplay between individual and collective documentational work. For us, the development of the concept of documentational trajectory helps us to analyze the relationship between resources, collective work, and a teacher's documentational experience.

Having presented our conceptual framing, we articulate our questions: (1) *how does the* thought collective *of the group designing a Sesamath textbook influence a participating designer's* documentational experience *and trajectory?* (2) *how do* thought collectives *influence the* documentational experience *and trajectory of a teacher using a Sésamath textbook as they face the challenges of incorporating curriculum changes?* In the next section, we present the methodology that we are currently developing to support this study.

11.3 Methodology

In this section our methodological reflections are divided into two parts. Firstly, the present our methodological design inspired by the *reflective investigation*. Secondly, we explore our research field, the implications of our methodological choices and elements of data analysis.

11.3.1 Methodological Design

Design and *usage* are dynamic processes that happen in various places over a period of time. Consequently, Gueudet and Trouche (2012, p. 27), in the thread of DAD, propose a specific methodology, the reflective investigation, grounded in some specific principles: "*long-term follow-up*", "*in- and out-of-class follow-up*", "*reflective follow-up*" and "*broad collection of the material resources*", using teachers' view on their own documentational work. In addition, in order to make as clear as possible what is expected of the teacher in this process, Sabra (2011) proposed the notion of a *methodological contract*. Our methodological framework is grounded on these propositions, and aims at following teachers' documentation work first in an individual case, and then, in a collective one. These two ways are interrelated; we have chosen, for the individual case, a teacher strongly involved in her school's collective documentational work. After presenting these primary elements, we present in this section our methodological choices on two topics: one about individual follow-up and another about collective follow-up.

11.3.1.1 Individual Follow-Up

We apply the reflective methodology principles in our following of the teacher's documentational work, meaning: the teacher's work itself, its results, and the teacher's view of this work.

In following the teacher's work on/with resources, we choose, in the frame of the French curricular change, a specific topic, leading the teacher to revise her related document and we video record the lesson preparation, its implementation in the classroom, and other related activities (for example: discussions with colleagues about the topic at stake). In ascertaining the teacher's view of her own documentation work, we employ several tools proposed by the reflective investigation, such as a *logbook filled by the teacher herself* which allows us to follow the teacher's documentation in moments that we can not follow personally and a *guided tour* by the teacher of her resource system.

To glean more insights, from the teacher, relating to her resource system, Gueudet and Trouche (2012) asked her to draw a schematic representation of her resource system (SRRS, see more in Chap. 1). We have tried to improve this tool in three aspects. Firstly, instead of *schematic representation*, we prefer to use the expression *reflective mapping*. We choose the word *mapping* because it better suggests: "a picture or chart that shows the different parts of something[2]", and because it denotes a dynamic and active process of exploring an unknown territory. We use the word *reflective* because we consider essential that the teacher reaches a *productive remembering atmosphere* (Vermersch 1994) in which she explicitly, as precisely as possible, reflects on her teaching past (events and experiences). And finally, we have extended this tool to the teacher's documentational trajectory, terming the process: reflective mapping of a teacher's documentational trajectory (RMDT).

To facilitate the collecting of resources used and produced by a teacher, we use a cloud-based storage tool in conjunction with an application the teacher is already familiar with. From the beginning of our follow up, as indicated in our methodological contract, we asked the teacher to store, in a box all that we share with her, i.e. all the resources she used and produced during the lessons included in our follow up. Moreover, we suggest to the teacher to integrate into this box a file supporting the researcher-teacher dialog about the current documentational work. We call this essential methodological tool a *reflective dialogue box*.

These methodological tools are designed to investigate a teacher's documentational experience and trajectory. In the next section, we introduce our methodological choices and tools for conceiving a *collective* follow up.

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[2] http://www.merriam-webster.com/dictionary/map.

11.3.1.2 Collective Follow Up

We adapt the reflective methodology principles for exploring thought collectives as the participant's design resources, allowing us to investigate the collective process of design and a member's role in this process.

In order to work with the collective, we propose a methodological contract adapted to the collective's interests and to the research questions (Sabra 2011). We chose one specific project or theme—in this case, the textbook reform related to the chapter about algorithms and programming—and we collect data related to the design process and to members' points of view about these design choices.

Data are collected from our research field, combining natural and artificial data (Sabra 2011): *natural data* includes lists, collective discussions items, the collective website, etc., and *artificial* data, i.e. data generated by the application of our methodological tools, such as logbook, mapping, etc.…

In following this collective work, we extend the reflective investigation Principles from individual to *collective reflective investigation*: *long-term* involving follow up on the collectives over the course of different projects; the *broad collection* of resources used and designed by the collectives; the following of specific members *in all possible places* to analyze how the processes of individual and collective documentation feed each other; members' *reflections* about collective work, collected through *interactive follow up*, involving interviews about their collective work and their practice, or *passive follow up,* in which we analyze collective reflection as evidenced by explicit rules and through member reflection in action glean from recordings during the design process.

For analyzing the interplay between an individual's documentational experience and the collective's thought style in the design process, we need to identify the thought styles of the thought collectives. For this purpose, we use the work of Trouche et al. (2014, p. 20) proposing (our translation) several critical points for characterizing a given collective: (1) "missions and stated objectives of the collective", (2) "the collective's relationship to resources", (3) "the organization and functioning of the collective" and (4) "the collective's relationship with the institution". In the case of mathematics, the authors propose to take into account the collective's relationship with mathematics and mathematics teaching.

For characterizing the collective's thought style, we retain all these points, reorganizing them in coherence with Fleck's (1981/1934) definition, in three points (Sect. 2.2): the collective's interest *standard features* (the targeted audience, objectives, missions, types of resources designed, etc.), the collective's *judgment* about mathematics and teaching mathematics (the pedagogical assumptions, point of view about mathematics digital resources, institutional purposes for teaching mathematics, etc.) and the collective's *methods* for creating resources (the functioning mode, member status, type of interaction, etc.). Still following the reflective investigation perspective, we use the collective's self-descriptions (charts, status…).

In the following section, we adapt this general methodological frame to fit the needs of our two research fields.

11.3.2 Research Fields and Related Methodological Choices

In this section, we present our research fields and methodological choices for following, in the case of two teachers, at first Sésamath's design of resources, then Sésamath's usage of resources.

11.3.2.1 The Design Process of a Sésamath Textbook

This section is divided into two parts. Firstly, in relation to our *research field* we detail some elements associating Sésamath's thought style and their process of textbook design. Secondly, in relation to our *methodological tools* we present our methodological contract with the association, the consequences of this contract in relation to our choices and some concerns about data analysis.

Sésamath is well known in France (more information is presented in Chap. 1), and, as an organization, it develops a large number of resources (software, online textbooks, online exercises, online forums, etc.). In our research, we are particularly interested in Sésamath's design of a new textbook integrating the proposals of the 2016 reform, specifical textbooks for cycle 4 (CT4). Given the above, an association where teachers design, collaborative and voluntary, resources seem to us an interesting path for analyzing teacher's documentational work in critical moments of curricular change.

We started to follow a Sésamath's collective, that we name in the followings CT4 collective as it was designing a textbook for cycle 4, in September 2015 to be noticed, the textbook design had started on 16th June 2015. Within Sésamath, each project has a particular organizational structure that is malleable. The CT4, steering committee is responsible for the general organization of the design process (including deadlines, chapter design, textbook layout…). This committee has three members: Helene (Helene was, during this period, Sésamath president), Marie and Sabrina. They had a meeting in June 2016 before starting the design process where they took decisions about the editing process (how many chapters, how many sections in each chapter, etc.). Following that, the committee sent the edit plan to all association members. On the 9th September 2016, the committee sent an email invitation to Sésamath members to join the project.

Currently, the online CT4 collective membership list contains 41 designers, among them the association members designers, old designer-contributors and new designer-contributors. However, only 33 participated (at least once) in the design process. The design process lasted until the 21st December 2016, and almost 3700 emails were exchanged. The data that we study here were collected until the time that the second version was finished, corresponding to the periods where major editorial changes occurred (i.e. organizing of chapters, screening of older Sésamath resources, creating new resources, decisions about how to handle new curricular changes, among others).

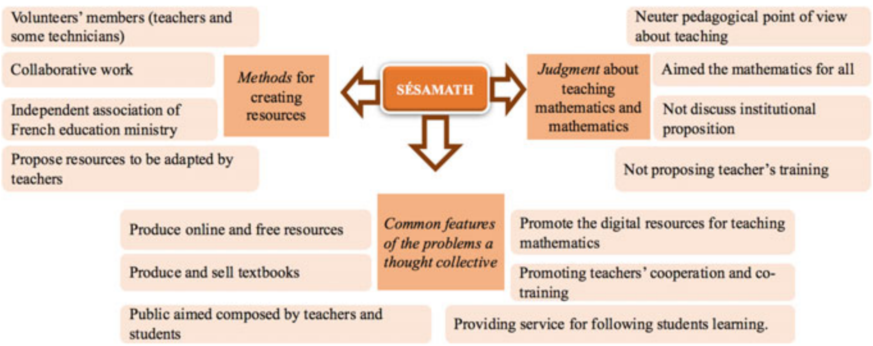

Fig. 11.1 thought style of Sésamath

We draw inferences about Sésamath's thought style by looking for evidence of the three elements presented in the above section based on the association's self-reflection as displayed on their website, see Fig. 11.1.

All of these textual elements were developed over time, since the creation of the Sésamath website in 2000 by the association's members. The online accessibility of Sésamath's resources has greatly contributed to their dissemination in France, making Sésamath a recognized association with thousands of downloads per year. This thought style provides the context for the CT4 collective which exists within the Sésamath association, and enables the more senior members to share their approach (thought style) with the ongoing work of the association. Nevertheless, it is the role of the CT4 steering committee, in particular of Sésamath's president, which we need to explore in Sect. 11.4.1.

In addition, for following CT4, we signed a *confidentiality agreement* with the Sésamath board to manage our rights and restrictions during the design follow up, which is a kind of methodological contract (Sabra 2011). The *confidentiality agreement* left a space for negotiations between the researchers' and the association's intentions, having implicit and explicit rules. In this contract, three aspects are very important. First, we were registered on the CT4 mailing list, meaning we received all emails that would be exchanged during this design process. Second, we were able to contact Sésamath president, Helene (the collective's designer and coordinator) when we needed more information about this process. Third, the association did not allow us to contact others designers, only to conduct a *passive follow up*. Our methodological design is constantly negotiated with the association and adjusted as necessary.

The primary need for adjustment comes from the fact that we cannot analyze the designers' documentational trajectory based on a process of interactive follow up, as we had wanted. We are still trying to negotiate that access with the association's steering committee, but for now, we cannot speak directly with the designers. This has limited our ability to analyze the design process, because there are some unclarities in the discussion that we cannot clarify. Likewise, we cannot investigate the direct relationships between the design process and the designers' history with the resources, and similarly many others aspects that we are interested in cannot be further explored.

However, we have a powerful data set that offers considerable opportunity to try to understand how the CT4 collective has grappled with these new curricular changes. In order to analyze this volume of data including the huge number of emails, we decided to focus on a single critical subject area, algorithmic and programming (constituting the chapter E of the textbook) which is a new addition to the curriculum and thus had no previously created resources.

As seen, the CT4 collective gathered the association's participating designers including experienced designer-contributors and new designer-contributors, to constitute a new thought collective. Experienced members share the association's thought style, but new members needed to incorporate this style into collective work. To access this aspect, we decide to focus our analysis on a new designer. Among the new designers, we choose the individual who had exchanged the greatest number of emails, Sonia. She sent 472 emails during the design process (the second most prolific correspondent sent 183, the third 107 and the fourth 56 emails). We observe that Sonia participated significantly in the design process. Saliently, 144 of the emails sent by Sonia related to chapter E and general design discussions. These emails composed the set of data that we analyze in this chapter.

Our analysis of data followed four steps. First, we read all 144 emails for analyzing how her *perspective evolves in design process*; second, we look for moments that she asked something or presented her doubts to other designers, for analyzing her *interaction with other designers*; third, we look for moments that she reacted to *resources proposed by other designers*. And last, we coded moments where she mentions her documentational experience (DE).

11.3.2.2 Sésamath Users

This section is divided into two parts. Firstly, we present some details about the subject analyzed during this stage of the research. Secondly, we discuss the methodological tools applied and some details about data analysis.

To analyze the use of Sésamath resources, we searched for teachers that were using the Sésamath textbooks as official textbooks in their schools and that had contrasting profiles. Thus, we chose Anna,[3] who has a history of strong participation in collective work outside of her school, and thus had been exposed to different thought styles. We also chose Viviane who has not participated in much collective work outside of her school. In this paper, we will present only Anna's case, because the data related to Viviane is more recent and is still being collected and analysed.

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[3] Anna is also investigated in Chap. 9.

We started following Anna's work in March 2015, and this data collection process is also still ongoing. She is a middle school teacher in Grade 6. Her profile is distinctive due to her active professional life as a producer of resources. She participates in various collectives outside the school, and in Table 11.1 we will present some details about these various collectives as well as a rough sense of their style:

In this table, we present three formal collectives in which Anna participated, but she also participated in a lot of informal collectives. Among them, we want to bring to the fore the *collective Anna-Cindy* (this collective is also analyzed by Wang in Chap. 1). Participation in this collective contributed to Anna's documentational experience. However, analyzing an informal collective's thought style is complex, because normally there is no official document that explains the process and thus

Table 11.1 Some collectives that Anna participates and respective though style

Collective	Common features of the problems a thought collective	Judgment about teaching mathematics and mathematics	Methods for creating resources
Teachers and researchers discussing Algebra (SÉSAMES[a])	– Teachers and researchers thinking about resources for teaching algebra and promoting teacher's training	– Mathematics for solving problems and teaching mathematics basing in the activity of research by students	– Collaborative and voluntary work between teachers and researchers – Principles for creating resources
Teacher's professional association (APMEP[b])	– Gathering teachers teaching mathematics from pre-primary schools to University and promoting teacher's training	– Being a force of proposal for improving mathematics teaching and providing math teachers with rich resources (from a didactical and epistemological point of view)	– Working voluntarily and collaboratively without hierarchy – No specific principles for creating resources, but creating resources with didactical advice
Institute of Research on Mathematics Teaching (IREM[c])	– Articulated work between research and practice looking for diffusing research results and promoting teacher's training	– Mathematics in live and teaching mathematics malleable	– Collaborative work for designing resources – No specific principles for creating resources, but creating resources with didactical advice

[a]More elements in: at http://pegame.ens-lyon.fr/. Accessed 17 February 2017

[b]More elements in: at http://www.apmep.fr/Texte-d-orientation-de-2010. Accessed 17 February 2017

[c]More elements in: at http://math.univ-lyon1.fr/irem/. Accessed 17 February 2017

allows us to create a table such as Table 11.1. To access this informal collective's thought style, we analyze its members in action in Sect. 11.4.2.2.

We hypothesize that a teacher with a strong history of participation in collective work would offer evidence of a rich and complex documentational trajectory. Additionally, her documentational experience offers us access to a large set of resources and events. We suggest that the reflections arising from the analysis of Anna's documentational work allow us to deepen our analysis of other types of profiles. Also, we analyze Anna's work during a critical moment, during the middle school reform in 2016, which allows us to analyze how her documentational experience leads her to face these new changes.

We applied all tools described in Sect. 11.3.1.2: a *logbook* to follow events linked to the 2016 reform, *prepared lessons* to teach algorithms and programing, *class observations* over the course of three months in 2015 and once in 2017, a *reflective box* to collect resources used by the teacher, and interviews to create and explore her *reflective mapping*.

We analyze her documentational trajectory based on the resources she used in her 6th grade classes of the course of three months. We look at this data to explore how she used Sésamath resources during this period. Subsequently, we analyze her *reflective mapping* in relation to her documentational trajectory drawing on data from our interviews with Anna.

We are still analyzing these data, but for this chapter we made three choices. First, we identify in Anna follow-up moments that Sésamath is used or mentioned; second, we analyze the position of her collective work in her documentational trajectory; third, we explore one event presented in Anna's reflective map as having a huge importance for her documentational trajectory, analyzing its impact on her documentation work and her resources. Finally, we repair in her discourse elements about how collective work has supported her documentation work for facing curricular changes.

We present, in the following section, the main results so far of this analysis still in progress.

11.4 Preliminary Results: Thought Style in Action

In this section we present preliminary results of our analysis of the documentation work of teachers *designing* and *using* Sésamath resources. First, we present some preliminary results of our analysis of a new Sésamath designer, Sonia, to explore the question: *how does the* thought collective *of the group designing a Sésamath textbook influence a participating designer's* documentational experience *and trajectory?* Afterwards, we discuss some preliminary results of our analysis of a Sésamath user, Anna, to explore the question: *how do* thought collectives *influence the* documentational experience *and trajectory of a teacher using a Sésamath textbook as they face the challenges of incorporating curriculum changes?*

11.4.1 From the Sésamath Designers Side

In this section, we discuss how Sésamath's thought style as it was embodied by the CT4 thought collective influenced the decisions of a new designer. We present three essential results: the first relates to *Sonia's interactions with others* and how she *capitalizes on her experience*; the second relates to *her interactions with resources* in which she *initiates her learning of language programming*; and the last noteworthy observation is about how her thinking evolves in relation to the design in which *she was strongly engaged.*

Sonia has been a teacher in middle school since 1995 and had previously worked for 2 years in high school. She says that she enjoyed being involved with Sésamath, because she has "*used Sésamath materials for a long time. Participating in some revisions allows me to give partially back to Sésamath what Sésamath gave to me*." At the beginning of the process of designing Chapter E, she mentioned that her competencies made her a mere spectator of the conception. However, she participated significantly in the production phase of the chapter. In her *interactions with other designers*, she presented herself as a neophyte who needed to learn more about programming languages. When Sonia started participating in the design and production of the chapter, Marie, who was the senior designer for the chapter, proposed that they use five different programming languages. Sonia was overwhelmed by the prospect of having to learn so many new programming languages.

Sonia's message: Those like me who have never had the possibility to take advantage of training in algorithms so far will be lost if there are too many different software languages, won't they? Maybe not, it is just a novice's question and maybe it doesn't bother anyone.

Marie's answer: Well… Everyone will take what he needs! This chapter is "complicated" because the teachers will work differently… Thus, we propose some examples, but this cannot be suitable for everybody… We have to make choices (otherwise we will get lost!) The problem is which choice? The problem remains open…

She emphasizes that she had never been trained to teach algorithms. In another message, she mentioned that she had studied this subject 20 years ago in a middle school, but that she knew almost nothing about it (DE). She then asked a lot of questions about the pedagogical aspects of how to teach it (see the extract below), such as how to use the proposed software and how the association proposes to help other teachers that are neophytes such as she is (extract above)?

Sonia's message: It seems that some people have already worked on with algorithms with their students: is it really intuitive for students or did you start with methodological worksheets?

Answer MA3[4]: I am in the second year of my experiments with teaching algorithms in middle school, with Scratch.[5] I am not in favor of "lectures". Through mini-projects, they discover autonomously the different notions. After that, we can make a summary in the form of worksheets, which they can work through as a way to move forward.

This extract offers just one example of her numerous questions. Another noteworthy aspect is her interaction with the other participants, in that she is always very honest about her lack of capacity in relation to the proposed activities. She mentions that, when she reread one activity, she followed the same path as students. Not all the questions she posed were answered, but Marie clearly stressed the importance of these questions for the constitution of this chapter. The association aims at promoting teachers' cooperation and co-training. Marie as an experienced designer keeps alive this thought style, allowing Sonia to feel comfortable during the design process while interacting with other members and capitalizing on their experience.

Regarding her *interaction with the resources proposed by other designers*, Sonia did not know the programming languages originally proposed by the steering group. However, she started to teach herself during the design project through tutorials and she asked the other designers for help when she did not understand.

Sonia's message: Yes, I tested the "technical" exercises just to get settled since I am a novice, that's why I made some comments to both of you about the utility of your exercises. I admit that when it was an option I preferred Python to Scratch because it reminds me of the bit of programming that I studied 20 years ago…

Answer MA3: Ok. And did you try to give them to the class?

Sonia's message: Oh, not yet, I don't think I'm able yet to manage it with students. I have just tested little bits of Scratch with my son who is in Grade 6 to verify if, as I thought, they can quickly master the rudiments (and the answer is yes!)

We can see in these messages that Sonia did not feel able yet to work with some of the programming languages (DE). She was worried that her students would not be able to solve the exercises as well. In other messages, she asked other designers' opinions about the degree of difficulty of Scratch activities for students. At that moment, the MA3 designer replied that they have already used them and that their students solved them. Given her lack of documentational experience with teaching algorithms and programming, Sonia uses two strategies in her preparation:

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[4] This message was written by one experienced designer-contributors, that we named MA3 that means association member number 3.

[5] One of the most popular programming software (developed by the Massachusetts Institute of Technology), proposed by the new curriculum in cycle 4.

Sonia's message: I began to train myself in Python in a self-taught way, it is the same for Scratch and I was able to test my knowledge on the basic exercises proposed by Marie as well as with her course. I am enrolled in training on algorithms with the *In Service Teacher Training*, will this allow me to understand the more complicated exercises? I am skeptical for the moment since there will also be total beginners at this training.

The design process played an important role in her documentation work as she grappled with this new curricular proposal. She shared, via the discussion list, a tutorial that she found interesting about software including good activities according to her (no one answered it). She also tested some activities in her class using Scratch and complex activities. There were some moments when she struggled to resolve activities and sometimes others members helped her to solve them. She re-sourced her system during the design process. And she used her knowledge to solve Sésamath activities, and sometimes she could not solve them. This self-training was necessary, because Sésamath as an association does not aim at training teachers; even if from time to time a fellow member helped Sonia, this is not a goal of the association. It is not a part of the thought style of this collective.

In general, the design process of Chapter E was not as stable as the other textbook chapters. Sonia was not the only one who did not know how to teach programming. The designers felt that curricular program was not clear and did not help to make some choices. In these cases, they continued to struggle until the reification of the first version (V_1). Problems related to *which* resources (software, activities, and projects) would be used and how algorithms and programming would be presented in the textbooks (as sections of courses or only as activities). As a typical example, we present here a problem which happened within the collective during the design of Chapter E.

On the 25th of January 2016, near of the end of the first version of the textbook, Marie was still torn between three software options: Scratch, Algobox and Python. First, she proposed to delete all of the Algobox activities. This instigated an important discussion with 12 messages exchanged among 7 designers. Going outside the collective, Sonia independently sought the opinion of the regional pedagogical inspector (RPI). She reported back to the group:

Sonia's message: I just got the opinion of our RPI for whom nothing is officially chosen and who thinks personally that at the beginning, software like Geotortue[6] or Scratch could be put forward, then that could be followed by the use of other languages.

The designer MA3 who was also participating asked if there were any official resources and said:

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[6] That is a software for teaching geometry, more elements in http://geotortue.free.fr/. Accessed 17 February 2017.

MA3's message: I can tell you that Scratch will be THE language of cycle 4. Even if it doesn't appear clearly in the programs, it will be used a 100% to illustrate the future documents accompanying the programs.

Other designers agreed that Scratch would likely be the software to work with in Cycle 4, as included in the program's accompanying document. In this sense, Marie re-read the program and said that this aspect was not explicit and that the question of progress within Cycle 4 was not clear either. During this discussion, there was an important message from Helene:

Helene's message: Beware, it is too late to change everything, a textbook lasts about 8/10 years, of course they aim to teachers but also to students, therefore they must be able to adapt to any progression
The course is the references, it is absolutely necessary to keep it that way. Next year will be the «Scratch» year, but then the idea is to go toward coding, so we do not have to limit ourselves to Scratch and to do a real course […]

This message provides evidence of the leadership role played by Helene, and also the thought collective of the Sésamath that *resources products should be able to adapt to any teacher's progression*. However, this was not an imposed solution. After that, the designer CO3 commented that when he participated in one training session with his RPI, the latter affirmed that Scratch was the choice of almost all RPI's and that it will be included as a subject in the national middle school evaluation, *Brevet*. In response, Marie explained that they could not impose only one software, and if they did, the direction to do so needed to be explicit. She then said that they were not under any obligation, and she asked "*what do we do? We resist?*" Six other members agreed with her. She deleted Algobox, but she kept Python and Scratch. For us, this moment suggests that the thought style is developed during the design process, and that they maintain the overall thought style of the association, which expects resources to be adapted by teachers.

Looking specifically at the role of Sonia within the collective, she participated actively in trying to resolve this problem along with several others. She defended Marie's course, saying it was very good and needed to stay in the textbooks. Also, she said that it was useless to include only one Scratch course. She said, "*I think the teacher must know a bit more than the strict minimum.*" (DE). We observe here a convergence between the Sésamath thought style and Sonia's own approach.

Considering the *evolution of Sonia's perspective during the design process*, we identify four phases that she went through: exposing her doubts about teaching this topic; learning of the proposed languages and performing the exercises; actively participating in problems encountered during production; and re-reading and revising each exercise assessing them for level and form. We notice a change in her messages over time: she begins to ask fewer questions.

As for Sonia's participation in the design process, we consider the entire design process for Cycle 4 as an event in her documentational trajectory, one that feeds her resource system and furthers her capacity to teach algorithms. We note that the design process leads her to: learn programming languages, exchange experiences with her peers, test new activities with her students, think about the new curriculum program, and try to help other teachers to prepare for teaching it. We hypothesize that designing for CT4 instigated a further evolution in her documentational trajectory; afterwards, she starts others projects with the Sésamath association, new events aimed at designing other resources, further nourishing her documentational experience. All of this leads us to infer that during the process of CT4 designer Sonia appropriated and internalized the Sésamath thought style.

11.4.2 The Sésamath User's Perspective

We present here are results in two parts. First, we present Anna's documentational trajectory in which the work of the collective plays an important role in justifying her resource design choices and uses, practically, in relation to the Sésamath resources. Afterwards, we suggest that her documentational experience, in facing new curricular changes, developed and incorporated thought styles from the diverse thought collectives in which she has participated.

11.4.2.1 Anna's Case: Brokering Thought Styles

In this section, we explore Anna's documentational trajectory to understand both her process of resource production and of resource usage. First, we present some details about her use of Sésamath resources based on our classroom observations and on her reflective mapping. Afterwards, we focus in on the role of collective SÉSAMES in her documentational trajectory and on her role as a broker. In particular, we focus on one resource that she creates in this collective, and how it structures her work across many collectives. We see this resource as a *boundary object*.

During our classroom follow-up, Anna used three different textbooks (including the Sésamath textbook), images retrieved from the Internet, booklets produced in the IREM group, online games, math puzzles proposed by the students, and the software, Geogebra. She used the Sésamath textbook with the class four times with two different objectives: for classroom exercises and for homework for advanced students. (In doing this, she took advantage of two features of the Sésamath resources: that they are available online and are easy to adapt to students' special needs.) Although Sésamath is the official textbook of the school, it appears that she uses it infrequently. In fact, she seems to feel somewhat obliged to use it because her students have easy access to it in class and at home. Some deeper analysis uncovers an underlying reason for this poor use of the Sésamath textbook.

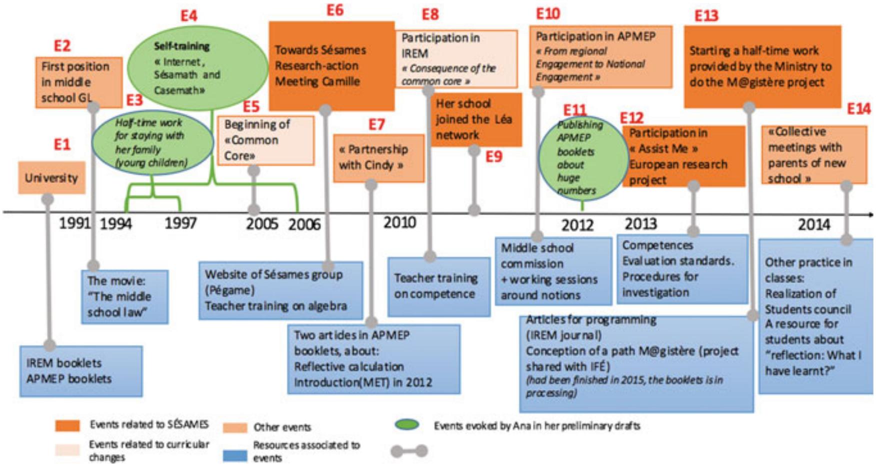

Fig. 11.2 Anna's RMDT (22/02/2016)

To better understand Anna's reluctance to use the Sésamath resources, we need first to understand her documentational experience using and designing resources. Anna's reflective mapping shown in Fig. 11.2 helps us to better understand her perspective.

Anna's reflective mapping indicates three key aspects of her documentational trajectory. First, we note that Anna participates with Sésamath during a period when she did not participate in any collectives outside of her school, and when her objective was self-training. From that time she begins to join other collectives, the Sésamath resources disappeared from her reflective documentational trajectory. This is somewhat surprising because the Sésamath textbooks remain the official textbooks of her school and she had used other Sésamath resources before.

Secondly, Anna has *multiple memberships*. A closer analysis of the events shows Anna's strong involvement in collective work, with eight of the 14 events related to collectives (E6, E7, E8, E9, E10, E12, E13 and E14). As she said, "*I cannot work alone*, *I have worked a lot with people*."

Thirdly, the collective SÉSAMES crosses different collectives, and functions as a reference for them (E7, E9, E12 and E13). Trouche (2016) calls these types of collectives *hub collectives*. In our understanding, key resources from the SÉSAMES collective act as boundary objects and Anna is a broker for this collective promoting SÉSAMES's thought style and resources, as we will describe in the following discussion.

On the SÉSAMES website, Pégame (http://pegame.ens-lyon.fr/) there are some explicit indications of its thought style: they note some principles for teaching algebra, justifying computation throughout algebraic rules, proposing proving activities, and exploiting formulas to introduce the notion of function. They also include some principles for teaching mathematics: providing students with sufficiently rich and open problems, giving them a chance to seek, giving them a chance to speculate, and helping them to make sense of concepts taught. These principles guide their process of collaborative resource design. They were the basis for creating an important resource for the association and for Anna, the *Mise en train* (MET[7]).

This key resource guides Anna's later production of resources as well as her classroom organization. In Anna's classroom practice, we noted two moments: at the beginning (around 15 min of the class) students were given one research activity as a warm-up. (During the rest of the class they followed the regular lesson plan covering new content and training exercises.) This first moment is a manifestation of Anna's work with MET. This is not a punctual and isolated activity, but is rather one part of a sequence of activities to develop a larger concept. Creating a sequence of activities in MET requires the production of activities in which students research and are autonomous in their achievement. During this activity, the teacher has the role of helping the students and at the end of the activity they "putting in common" concepts taught.

The MET acts as a *boundary object* that Anna brokers between various collectives. This resource was created with her involvement during her participation in SÉSAMES and is intended for teaching algebra. It is deep-rooted in their thought style. She uses this resource in other collectives: to teach others this notion in the APMEP collective, for teacher training in IREM, and for preparing her partner lesson with Cindy, among other uses.

Finally, our analysis of Anna's documentational experience suggests that she feels empowered by having participated in the design process of the resources that she uses in class. This collective work was formational for her. This strong personal connection with the MET resource amounts to a preferred approach that does not center on Sésamath. Her design experiences are linked to her participation in various collectives such as APMEP, Collective Anna-Cindy, IREM and SÉSAMES, but among these, SÉSAMES is the *hub collective*, and Anna acts as a *broker* of the thought style and resources. Specifically, the MET resource becomes a boundary object that Anna uses to negotiate her status within various thought collectives.

In the next section, we analyze the relationship between Anna's documentational trajectory, explored above, and how she prepares herself for the 2016 reform.

11.4.2.2 Thought Style in Action Modeling Anna's Documentational Trajectory

In this section, we will explore the role of collective thought in grappling with new curricular changes within the various collectives.

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[7] In the expression *Mise en train*, the word train is an acronym created by Anna for designing Travail de Recherche ou d'Approfondissement avec prise d'INitiative (Research and Deepening Work with Initiative Taken).

Anna works in different collectives each of which must face the new changes. Drawing on what she said in various contexts, we are able to make some inferences about the *thought style* of the various formal and voluntary collectives:

- In the context of APMEP, she read and interpreted curricular changes for all concepts; the association has a critical posture towards the program's proposals; they also held meetings to discuss resources for teaching algorithms and programming.
- Within IREM, they discussed the method of teaching by competencies and they designed new activities for teaching in this new interdisciplinary way.
- Within SÉSAMES they discussed possible calculus programs to be used in teaching algorithms and how to adapt their resources to the new curriculum.

She has had many informal discussions with teachers in her school. This happens face to face and during online work. However, it is difficult to specifically measure how these interactions affect her grappling with new curricular changes. In her logbook, we had hoped to identify these moments, but in the end Anna did not provide much information. Nevertheless, we have found some information about the formal and obligatory collectives in which she has participated:

- In the Teacher's Institutional Training for teaching algorithms, there was training about using Scratch for teaching in Cycle 4.
- She met with her colleges to discuss how to work with the new reform with regards to time organization, creating new progressions, among other issues.
- She met with elementary school teachers to discuss Cycle 3 and the progressions in this cycle.

An important informal, voluntary collective in Anna's documentational trajectory is her partnership with Cindy. They often prepare their lessons together. We recorded their lesson preparation, and in the following discussion we present some of what we have inferred about their thought style for creating resources together (Fig. 11.3):

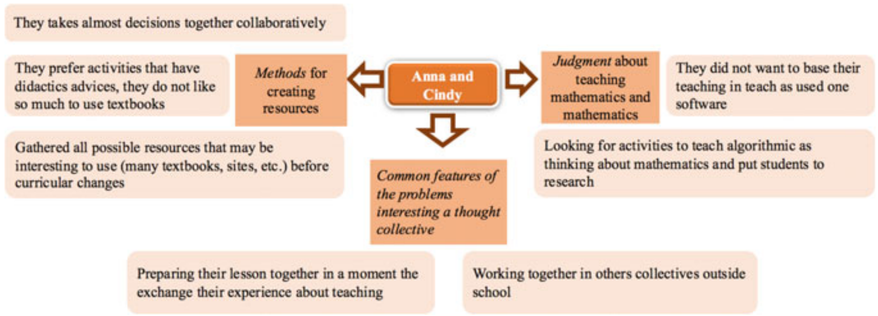

Fig. 11.3 thought style of Anna and Cindy's collective

In Anna's documentational work preparing lesson plans for teaching algorithms, she did not encounter problems in finding activities or with programming languages. She said there are a lot of activities online to support her teaching. The problem identified by her is that there are not many resources that explain why, from an educational point of view, one activity is more interesting than another. The first year, Anna and Cindy decided to use the same activities in all their classes, because the students have never seen this subject. They have still not prepared yet a progressive plan for teaching algorithmic and programming and they have not delineated a clear goal for each year. During the first year of curricular reform they only taught one intuitive idea of what algorithms and programming are through the activities. They plan to clarify this next year.

The follow up of Anna's documentational trajectory is ongoing, but the analyses presented here allows for some general discussion about her preparation for the new reforms. We present this in the next section along with the related discussion of Sonia's case.

11.5 Discussion

This paper presents part of the data analysis of a thesis in progress, so the theoretical framework is under construction and the data are still being collected. Thus, we will present some partial conclusions based on the current state of our work.

In Sonia's case, we posed the research question: *how does the* thought collective *of the group designing a Sésamath textbook influence a participating designer's* documentational experience *and trajectory?* We could see that Sonia did not have the experience to teach teachers how to approach algorithms and programming. She used the design process to develop her own understanding. She used e-textbooks, chapter exercises, and lessons to begin to learn the needed programming languages. She interacted with others designers to better understand the curricular changes. She participated of all of the Chapter E steps, and over time she began, as she gained confidence, to ask fewer questions about the curricular changes. She also participated in designing other chapters, not detailed here. She went on to participate in other new projects. The Sésamath thought style seems to have nourished her documentational experience. Regarding Sonia's documentational trajectory, we hypothesize that the Sésamath design process is an event that contributed to her documentational experience.

In Anna's case, we explored the research question: *how do* thought collectives *influence the* documentational experience *and trajectory of a teacher using a Sésamath textbook as they face the challenges of incorporating curriculum changes?* Anna did not have experience teaching algorithms and programming. She did not appear preoccupied with learning programming languages, and was more concerned with which activities to choose and why. She participated in many thought collectives to prepare herself for the new curricular changes. In these collectives, she discussed new changes and prepared new resources. For example, she read curricular proposals and organized yearly content in APMEP; she prepared resources for interdisciplinary teaching at IREM; she thought about calculus programs to teach algorithms at SÉSAMES; she prepared her class lesson with her colleague Cindy; and she examined cycle progressions with the school commission for Cycle 3, among other activities. Regarding Anna's documentational trajectory, we observed that she has a strong history of creating resources. She has *multiple memberships* and works with various collectives. Among them, SÉSAMES is a *collective hub* that links many of the collectives she participates in. In this collective she created one key resource, the MET, which has served to structure her subsequent documentational work. We suggest that this resource is a vehicle for SÉSAMES's thought style. We suggest that Anna is a *broker* of SÉSAMES's thought style and resources, and that in this context the MET can be understood as a *boundary object* between different collectives.

After, analyzing these two teachers' experiences, we return to the central question*: how do teachers' work with collectives inform their work within resources as they grapple with curricular changes?* Both of these teachers' involvement with collectives helps them to prepare for the new changes. We saw that Sonia joined a new collective while Anna used her existing connections with collectives to prepare herself. Sonia learned about new programming languages while Anna reflected about new curricular propositions. Sonia used Sésamath's resources to learn while Anna created resources in many collectives to support her own teaching and did not use Sésamath's resources. Sonia had a lot of questions about curricular changes, and she used the design process to develop her own understanding. Anna used her *multiple memberships* to help develop her understanding of the curricular propositions. In general, both for the designer and the user of Sésamath's resources, their work in collectives is a powerful source not only of resource production but also of opportunities to reflect on new curricular changes. Throughout our study we have seen teachers gaining documentational experience while grappling together to try to figure out how best to teach within an evolving field.

Our research is grounded in diverse frameworks. Among them, the documentational approach of didactics nourishes our reflections about teachers' work with resources, as one process in the genesis of knowledge. Much recent research analyzing teachers' design and use of resources is based on this perspective. However, less research has investigated teachers' documentation evolution longitudinally. We have a particular interest in understanding a teacher's professional development working with resources over time. This interest has led us to think about how teachers develop through their experiences and to trace their trajectory working with resources throughout their careers. We have introduced several new concepts to investigate this phenomenon: *documentational experience and trajectory*. This process of conceptualization is still underway. However, some aspects seem clear in this process, such as the collective dimension in teacher's documentational work, and how this can structure teachers' use and design of resources. The frameworks proposed by Fleck (1981/1934) and Wenger (1998) offer us analytical support, which we will explore more in the course of our research. The open conceptualization of the *thought collective* and the connections between collectives (as *boundary objects*, *multiple membershi*ps and *brokers*) provide key tools for analyzing this aspect.

For analyzing teachers' experiences and trajectories, we consider teachers' past, present and future. This is a complex phenomenon, which requires the creation of a specific methodological framework. We have had some advances as well as problems on this issue. The concept of *mapping* helps us to create a panoramic view of teachers' documentational trajectories. The process of *reflective mapping* enhanced with interviews helps to create a *productive remembering atmosphere* (Vermersch 1994) for teachers to make explicit how they appropriate their past in their ongoing development. However, sometimes these maps were overloaded with information and did not clarify the main features of a teacher's documentational experience and trajectory. We also had some difficulties following teacher's work in the present. We relied on a logbook but it was not well integrated into all subjects, hence we need to improve our tools for collecting teachers' daily reflections.

Finally, the results offered here are based on two case studies that have allowed us to start thinking about teachers' documentational work as it evolves over time. However, in order to enrich this conceptualization and our understanding of this phenomenon, more cases are needed. To accomplish this, we should design new tools for mapping in order to make more explicit how teachers develop their documentational experience and trajectory.

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Part IV
Teachers' and Students' Interactions Through Resources

Chapter 12
Instructional Activity and Student Interaction with Digital Resources

Kenneth Ruthven

Abstract This chapter examines selected recent studies of the design and use of digital curriculum programmes and dynamic mathematical tools in school mathematics. The examples chosen bring out diversity both in the types of digital resources which are being adopted for teaching school mathematics and in the ways in which these are being taken up in instructional activity. These examples also show how any particular resource can be used in very different ways, and in ways quite different from those espoused by its advocates or intended by the designer. Digitised versions of traditional textbooks are cautiously innovative while individualised learning designs promote more 'personalised' instruction. Use of dynamic digital mathematical tools can support exploratory patterns of mathematical activity, underpinned by feedback from students' interaction with these tools.

Keywords Mathematics teachers' resources · Curriculum programmes
Digital resources · Dynamic tools · Instructional activity · Interaction patterns
School mathematics · Task environments

12.1 Introduction

The development of digital technologies is changing the media employed in doing, learning and teaching school mathematics. Although non-digitaltools and resources continue to be widely used, there is a shift towards their digital counterparts whether—by way of example—that be from ordinary to interactive whiteboards, or from graph paper to graphing software. These new media do not simply replicate the functionality of the old with increasing efficiency (although that is often how users initially view them); they make possible qualitatively different forms of interaction between user and medium, based—for example—on the introduction of

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new types of user interface or on the provision of instantaneous feedback on user actions.

This chapter examines some key current exemplars of instructional activity in school mathematics mediated by digital resources, focusing, in particular, on the types of interaction in which students are involved. Again, while digital resources are often, at least initially, assimilated to established patterns of instructional activity, they have the potential to reorganise such activity in significant ways. For example, the idea of the 'flipped classroom' proposes an inversion of a widely used pattern of instructional activity. Rather than starting with a lesson in school involving teacher exposition of new material through a whole-class presentation, followed up by some form of student practice of that material under teacher supervision through an exercise to be finished off after the lesson and typically at home, the idea of the 'flipped classroom' proposes a pattern of instructional activity which starts with students viewing a video-recorded exposition of new material at home (or otherwise outside lesson time) and continues with some form of class discussion and/or supervised practice during lesson time. It should be noted, however, that the two forms of exposition do not afford the same opportunities for interaction. On the one hand, a student can pause, review or advance the video-recorded version but not pose a question; on the other hand, the teacher can make their in-lesson exposition to the class more interactive and responsive, but while students can in principle pose questions, in practice this opportunity is limited and an individual student has little control over the pace and direction of the exposition. Equally, different forms of, and locations for, practice afford different opportunities for interaction. A student completing a paper-based exercise on their own at home can review material in the textbook, perhaps check answers against those given at the end of the text, and—*in extremis*—consult a family member or phone a friend. In class, as the 'flipped classroom' model recognises, there are possibilities for interaction with peers and the teacher that are not available at home. And, whether at home or in class, if the exercise is being undertaken on some kind of responsive and/or adaptive digital system, then the provision of automated guidance, feedback and customisation greatly changes the task environment and so the potential for student interaction.

12.2 Instructional Activity with Digital Curriculum Programmes

As its privileged mention in the title of this book acknowledges, the textbook has generally been the pre-eminent type of curricular resource for mathematics teaching. It is natural, then, for this chapter to give first consideration to the still evolving digital analogues of the printed textbook, commonly referred to as e-textbooks or digital curriculum programmes. Proponents of such programmes draw attention to the affordances they provide not just for production and distribution costs to be reduced and for material to be updated regularly, but—more significantly in pedagogical terms—for multimedia resources to be incorporated, for instruction to be customized, and for users to connect to virtual communities (Choppin et al. 2014).

While such developments are taking place around the world, it is in the United States that they have acquired a particular impetus in recent years which has led them to be subject to relatively extensive research. For that reason, I draw on two recent studies conducted in the US which have examined the emerging characteristics of digital curriculum programmes and their patterns of use in schools. In line with the aim of this chapter, my focus will be on the types of instructional activity associated with such programmes, including student interaction with and through their resources: interaction between student and programme system, interaction between students while using the system, and interaction between student and teacher in association with use of the system.

12.2.1 Choppin et al.'s Study of Digital Curriculum Programmes

This recent study examined the range of digital digital curriculum programmes emerging in the US: programmes designed to substantially supplement or entirely replace traditional printed textbook series (Choppin et al. 2014). The researchers found that such programmes were broadly of two types. Characteristic of the major educational publishers were what the researchers termed 'digitized versions of traditional textbooks', having structure and content similar to existing textbooks but taking a digitized rather than printed form, and intended to be used in much the same way as traditional textbooks, under the direction of a teacher. What the researchers termed 'individual learning designs' were designed to be used more directly by students as individualised study programmes, largely independent of the teacher, often with built-in assessments used to adjust the pacing and sequencing of content to the individual student user. These programmes can be seen as seeking to bring into the educational mainstream the type of approach pioneered by earlier traditions of paper-based programmed learning (Gagné and Paradise 1961) and individualized instruction (Hirsch 1976), followed by computer-based intelligent tutoring systems (Wenger 1987) and integrated learning systems (Becker 1992).

The researchers selected six programmes for more intensive study, mostly of the latter type (because these appeared to vary more in their characteristics) but including one programme of textbook type (representing what appeared to be many programmes with very similar characteristics). A substantial sample of the curriculum materials for each of these programmes was analysed in terms of themes derived from prior literature review. The theme which is of particular interest for this chapter concerned factors affecting student interaction with these programmes, conceptualised in terms of the types of learning experience (i.e. instructional activity) provided; the mechanisms provided for individualization and differentiation; and features aimed at virtual communication between students, teacher and others.

The study found that digital digital curriculum programmes of both types tended to emphasise what it characterised as ‘passive’ types of learning experience, such as viewing recorded presentations, following model demonstrations, and then completing related examples. Typically, the use of multimedia did not extend beyond videotaped presentations or narrated PowerPoint files. Thus the term ‘passive’ seems intended to highlight the limited possibilities of interaction with the system available to students. Nevertheless, a minority of programmes did take greater advantage of the learning potential of multimedia by employing interactive applets to introduce mathematical ideas with students assigned a more ‘active’ role in manipulating representations of a scenario so as to solve mathematical problems set in that context.

In terms of individualization or differentiation of activity, programmes ranged from those with some form of adaptive assessment built in, automatically assigning new tasks to students on the basis of their prior performance, to those which tracked student performance but, rather than using the results to set new tasks, either made suggestions to students about suitable tasks or provided the teacher with reports intended to inform their decisions about assigning tasks. At its most sophisticated, automated adaptivity introduced a high quality of interactivity between system and student, responding not just to the accuracy of students’ performance on tasks but to the speed and facility of their handling of virtual manipulatives. In this respect, then, some of these e-textbooks introduced an important degree of responsiveness to the student, lacking from conventional textbooks.

As regards facilities for virtual communication, some programmes provided a facility for teachers to comment on student work or offered a messaging or mailbox system enabling one-way (teacher to student) or two-way (student to teacher as well) communication. Other programmes did not make any provision of this type. Indeed, in most of the programmes, the emphasis appeared to be on individualised learning activity, involving each student working independently on the system at their own machine, with little interaction with the teacher or other students envisaged in the design of the materials themselves. The researchers also commented on the absence from the programme systems of discussion boards to enable students to exchange ideas about tasks. It seems that mechanisms for reciprocal interaction between students were rarely engineered into the systems themselves, although the way in which such systems were used in practice could introduce such interaction externally.

This study provides a useful overview of the types of digital curriculum programme currently available, based on studying their materials. In particular, it identifies the kinds of instructional activity and the forms of student interaction anticipated by these designs. However, mindful that designers’ intentions are not always reflected in users’ implementations, it will be useful to complement this study with one examining the use, adaptation and development in practice of one of these digital curriculum programmes.

12.2.2 Murphy et al.'s Study of School Use of Khan Academy

Murphy et al. (2014) examined the evolving use of one of the digital digital curriculum programmes included in Choppin et al.'s study: Khan Academy. Khan Academy originated as a website providing short videos showing the process of solving standard types of mathematical problem on a blackboard with a voiceover explaining each step. A natural extension was to provide associated sets of problems suitable for practicing the procedures demonstrated, interactive to the extent that users can check an answer or request a hint. Further facilities were added to generate reports on users' coverage of, and performance on, problem sets, as well as to introduce game-like features allowing users to gain points and badges.

Recently, the developers of Khan Academy have sought to strengthen its capacity for use in schools, undertaking a project in which they worked with researchers from SRI (who conducted this implementation study) to explore use of the product in a number of volunteer schools, refining the design responsively and developing professional guidance. The study treated Khan Academy as a generic example of a much wider class of similar digital learning tools and resources intended to support *personalized learning* of mathematics, tailored to the student user. The study took place over 2 consecutive school years and across 9 sites (school districts, charter management organisations, or individual schools). In each school year, data were collected—using both structured and semi-structured methods—through site visits, classroom observations, interviews with organization and school leaders as well as teachers, parents and students, surveys of teachers and students, and students' user log files. The approach to analysing the resulting data corpus could broadly be described as combining systematic survey and multiple-case study methods, including various forms of triangulation. Again, my focus here will be on those aspects of the study bearing on instructional activity and student interaction.

Khan Academy has become associated in the popular imagination with a 'flipped classroom' model in which the teacher assigns students to view, as homework, a video covering new material in advance of a follow-up classroom lesson. However, this study found that few teachers asked students to watch the Khan Academy videos, either inside or outside of school lessons. Considerations influencing teachers were not just logistical ones such as the feasibility of students being able to access the videos out of class, but pedagogical ones such as students being unable to raise questions—and more generally interact with a teacher—when meeting new material through viewing videos. Rather, teachers preferred to themselves continue to introduce the class to new mathematical material through conventional teacher-led classroom instruction. Nevertheless, the teachers did make extensive use of another component of the Khan Academy system, assigning its problem sets to give students practice relating to material that had either recently been introduced by the teacher or that the teacher had identified as requiring revision on the part of some students. The study reported that both teachers and students particularly appreciated the student-system interaction associated with these problem sets whereby the system provided immediate feedback when the user entered an answer to be checked. The study also reported that sites and teachers varied in whether they encouraged student-student interaction while working with the programme.

When students did choose to access videos, this was to view material relevant to the problem set they were currently working on. Nevertheless, the study found that the overwhelming majority of the time that students spent logged on to Khan Academy was devoted to working on problem sets rather than viewing videos. Equally, the study noted that when students were having difficulty with a problem they tended either to seek help from teacher or peers or to use the hint and step-by-step features in Khan Academy rather than viewing or reviewing the related video. Consequently, to support use of the videos as a resource by students, the developers made changes to the system: positioning links to relevant videos so as to make it easier for students working on a problem set to access them; adding a facility to fast-forward videos during playback so as to locate information more efficiently.

This study, then, highlights a two-way process of adaptation. First, teachers and students selectively appropriated those components of the original Khan Academy system that they perceived as enhancing existing forms of instructional activity. In particular, they embraced the use of problem sets, attracted by the supportive forms of feedback that the online system made available, and the consequent enhancement of student-system interaction within the established instructional activity of working on practice exercises. Then, the designers modified the system to improve its appropriability, particularly that of underused core features. In particular, the curriculum alignment and user interface of the video resources were improved in ways intended to make them more readily appropriable by, and valuable to, school users. At the same time, further modifications appear to be intended to encourage a shift in instructional activity towards the designers' vision of a personalised approach to learning mathematics. In particular, features were added to support more independent student use of the system: a search capability to quickly find videos and problem sets by topic; and a goal-setting feature to allow students to specify specific videos and/or problem sets to view and complete.

12.2.3 Discussion of Instructional Activity with Digital Curriculum Programmes

These studies suggest that the perceived quality and additionality of student-system interaction is likely to be a key factor influencing teacher and student decisions about whether to embrace innovative features that current digital digital curriculum programmes bring to a traditional instructional model of exposition and practice. In the case of Khan Academy, for example, where exposition of material was concerned, the student-system interaction associated with video presentations was generally viewed as inferior to that available through teacher-student interaction, and so was not embraced; conversely, where practice of material was concerned, the student-system interaction associated with checking answers to problem sets was viewed positively, and so was embraced.

In terms of the three aspects of interaction set out at the start, it seems that development to date of digital digital curriculum programmes has placed greater emphasis on, and had greater success with, the first of the aspects of interaction highlighted earlier—interaction between student and programme system—rather than the other two—interaction between students while using the system, and interaction between student and teacher in association with use of the system. Indeed, it seems that many current digital curriculum programmes aspire to individualise instruction and ‘personalise’ learning. Particular strengths of such programmes are their use of adaptive assessment to tailor the content presented by the system to the response history of the student, and/or their provision of reports through which the teacher can monitor student progress and adjust provision accordingly.

A particular weakness, however, of many of these programmes is lack of attention to peer interaction between students. Generally, facilities for such interaction are not engineered into the delivery system, nor are curricular tasks designed with it in mind. This interpersonal dimension is, then, an important topic for future research on the design and use of digital digital curriculum programmes. Bearing in mind, too, the risk that students come to see their responsibility as getting schoolwork done efficiently rather than as learning mathematics deeply, one can easily envisage ways in which overly instrumental use by students of a system’s provision of hints and checking of answers could lead to degeneration in quality of learning. Indeed, such phenomena bedevilled previous generations of individualised learning systems (Erlwanger 1975; Hativa 1988).

The study by Choppin et al. (2014) also examined the extent to which digital digital curriculum programmes provided students with learning experiences in which they could change parameters in figures or equations to explore dynamic relationships between quantities, or choose or manipulate tools or representations to solve problems. It found such provision in only a minority of the programmes, and commented on the absence from any of the programmes of resources which exploited the dynamic coordination of graphical, numeric and symbolic representations. This points to a further important way, now to be examined, in which digital resources potentially modify instructional activity and student interaction through changing the task environment for mathematical activity.

12.3 Instructional Activity with Dynamic Mathematical Resources

Just as the provision of hints and checks changes the task environment for undertaking practice exercises, the use of digital mathematical tools changes the task environment for tackling tasks which require the construction, manipulation and coordination of mathematical representations. In recent years, digital tools for such purposes have started to be taken up in school mathematics, typically to support the inclusion in the curriculum of more challenging investigative and problem-solving tasks, and often with the intention of developing a more inquiry-based approach to instruction. Thus, introduction of digital technologies may also influence instructional activity and interaction between teacher, students and resources through change in the classroom working environment and the mathematical tool system in play. To examine this issue, I draw on recent studies which illuminate features associated with the use, first, of dynamic mathematical tools, and then also of networked classroom technologies.

12.3.1 Ruthven et al.'s Studies of Instructional Activity with Dynamic Mathematical Tools

Recent years have seen considerable interest in, and increasing use of, various forms of dynamic mathematical software—either computer- or calculator-based—in mainstream school mathematics. My research team has conducted a number of collective case studies of teaching practices incorporating the use of such tools. These studies have gathered data through lesson observation and teacher interview, and employed both emic and etic modes of thematic analysis to analyse the teaching practices and the thinking behind them. A general finding is that teachers regard the use of such tools as supporting more investigative classroom approaches by enabling mathematical processes to be carried out more easily and efficiently, making them more open to replication and revision, and so supporting a more experimental style of working mathematically.

One study examined teaching practices involving use of dynamic geometry software (Ruthven et al. 2008). All the teachers involved indicated that they valued dynamic geometry for the contribution it could make to guiding students to discover mathematical properties for themselves, but the practical expressions of this idea were very varied. Correspondingly, while all of the teaching practices observed in this study exploited the dragging of figures to identify mathematically significant properties, beyond that there were important differences of approach to instructional activity. First, teachers differed in the degree and type of interaction with the software that they saw as being valuable for students. At one extreme, the software, projected to the whole class, was used only by the teacher as a presentational tool; more typically, students, working individually or in pairs at their own machine, were given the opportunity to manipulate prepared figures; but only occasionally did teachers see value in students learning to construct their own dynamic figures. One factor influencing these choices was the teacher's view of how best to manage mathematical complexities associated with the software: while some teachers sought to avoid exposing students to what they saw as unhelpful difficulties, other teachers welcomed opportunities to interact with students to help them recognise and resolve what they saw as challenges capable of generating mathematical insight. Another important factor was whether teachers saw students' own use of software as providing experience of mathematically disciplined interaction: teachers who took this view saw interacting with students to debug their constructions as a productive way of supporting learning.

Indeed, in one case, the teacher emphasised the way in which the dynamic software created a distinctive task environment for geometric work, enhancing opportunities for interaction both between student and system and between teacher and student. First, he saw as a key characteristic the way in which getting his students to make use of the software required them to develop the capacity to give clear instructions in mathematical terms. As students worked on their constructions, the teacher could help them to analyse and overcome difficulties they encountered and to express these in suitably mathematical terms. This teacher also noted the crucial part that he played in making key mathematical properties notable to students by prompting them to drag figures. Finally, he had identified how getting students to make use of text boxes to accompany their dynamic figures could help to sharpen the precision with which they expressed their procedures and findings in writing, because the provisionality of digital text made revision much easier.

Another study examined teaching practices involving use of graphing software (Ruthven et al. 2009). Here, one interesting common feature was the emergence of types of task structure, dependent on use of the graphing software by students to generate new information which they then had to find ways of interpreting so as to throw light on the fundamental mathematical question being addressed. In particular, this type of task structure and environment supported the teacher in taking on roles as co-enquirer with, or coach to, students.

To take the example of one lesson, the use of graphing software was crucial in underpinning the two related task formats in play. The focus of the lesson was on the graphs of quadratic algebraic forms. The first task that students were given was to use the graphing software to explore the effect of altering each of the coefficients of a quadratic form on the shape and location of the resulting curve. The second task, referred to by the teacher as "target practice", was to find equations for quadratic curves which would pass, in the first instance, through a single specified point, and then through a pair of specified points. Although it would clearly not be impossible for students to use other strategies, the intention was that they should tackle the first task by using what might be termed a *vary-and-infer* strategy based on finding a relevant pattern linking variation of the particular coefficient in the quadratic form to change in some property of the corresponding curve, and the second task using a *trial-and-improve* strategy based on iteratively trialling some speculative quadratic form and then successively refining it in the light of the fit of the resulting curve to the target point(s). What is fundamental for both strategies is the interaction between student user and graphing software; in particular, the information that the graphing software provides through displaying the graph of the expression that the student has entered; information which the student then needs to interpret mathematically to provide feedback relevant to the particular task.

12.3.2 Clark-Wilson's Study of Instructional Activity with Networked Dynamic Mathematical Tools

Over recent years there has been a progression in the working environment for making use of digital resources in school mathematics. During a period when students and teachers typically had access only to handhelds or workstations designed for personal use, instructional activity with digital resources was largely restricted to working individually or in pairs or small groups. The introduction of data projection and interactive whiteboards made whole-class activity with digital resources much more viable. Ideally, however, a mathematics classroom would provide scope, not just for activity at both scales, but for ease of switching from one to the other and of sharing the results of work. Thus, linking the various forms of digital technology in play through a digital network helps to create a more integrated working environment which facilitates the storage, retrieval and exchange of information, and its collective organisation and analysis.

Clark-Wilson (2010) examined development in teaching practices over an initial period of a few months following the introduction of a networking facility to mathematics classrooms where teachers and students were already experienced in using hand-held devices providing a range of dynamic mathematical tools. The networking facility linked students' individual hand-held devices to a central computer providing network management software, connected in turn to a classroom data projector or interactive whiteboard for public display. This central management software enabled the teacher to project the screen displays of all or some of the student handhelds as well as to distribute resource files to and from the handhelds.

The study employed data collected from a wide range of sources including teachers' own records and lesson logs, lesson observations, teacher interviews and questionnaires, student interviews, and e-mail correspondence. This was then analysed by combining two waves of coding: the first focusing on features of the technology, the second on teachers' descriptions of 'desirable' features and 'enhanced' student engagement and achievement. Clark-Wilson reports on the three main functionalities of the networked system which were taken up by teachers.

The Screen Capture facility allowed the teacher to display the state of all the handheld screens. This was the facility most widely used by teachers, and associated with a wide range of pedagogical purposes and forms of instructional activity. These included "monitoring students' activity during the lesson; supporting teachers to know when to intervene; promoting and initiating whole-class discourse; promoting and supporting peer and self-assessment; privileging mathematical generalization; … and enabling mathematical sorting" (pp. 752–753).

In particular, it seems that use of the Screen Capture facility led to further adaptation of the form of instructional activity which Drijvers (2011) has termed Work-and-walk-by. Drijvers argues that changes occur in the process whereby a teacher circulates observing different students at work as a result of that work being displayed on their computer screens rather than in exercise books. In particular, he suggests that the greater visibility of the screens makes it easier for the teacher both to establish a global view of work across the class and to follow up the work of particular students. Use of the Screen Capture facility brought this accessibility to bear on the smaller (and so less visible) screens of handheld devices, and made it still easier for the teacher to monitor a range of individuals and form an overview because of the simultaneous availability of the screens of all students, either displayed on the central computer or projected at the front of the class.

Equally, use of the Live Presenter facility, which allowed the teacher to select one handheld device for public projection, showing to the whole class the key presses and screen action from that device, appeared to support a form of instructional activity, termed Spot-and-show by Drijvers (2011), in which one student's work is demonstrated to, and then possibly discussed by, the whole class. This Live Presenter facility was used reasonably regularly by participating teachers, and again associated with a range of pedagogical purposes and forms of instructional activity: "teacher and student use to support the use of the … handhelds; teacher use to introduce and develop mathematical tasks; teacher use to generate data for use by the class; and student use to share mathematical observations, outcomes and insights" (p. 753).

The third facility used reasonably regularly by teachers was Quick Poll. This enabled the teacher to interrupt activity on all the students handhelds with a pop-up question accompanied by a forced choice of answers. The class set of responses could then be publicly displayed, with or without the students' names, and analysed in several ways. The reported range of uses covered: "as a focusing act to initiate the start of lesson activities; the generation of data for use during the lesson; prompting class discussion on a particular mathematical feature, concept or fact; and checking students' understanding of a particular mathematical feature, concept or fact" (p. 753).

Clark-Wilson concludes that exploiting these networking facilities supported change in instructional activity and patterns of interaction between teacher, students and resources. In particular, these facilities enabled development of teaching practices which enhanced formative assessment and were mathematically innovative. In terms of formative assessment, the public sharing of responses and screens through use of these facilities promoted more thoughtful teacher intervention and student discussion. These developments, in turn, increased opportunities for purposeful self and peer assessment by students. In terms of innovative mathematical tasks and approaches, amongst the examples offered are the use of a range of results from multiple handheld screens accessed through Screen Capture "to support mathematical generalizations; … as objects that can be sorted according to mathematical criteria; and … to increase the sample space of data or ideas" (p. 758).

12.3.3 Discussion of Instructional Activity with Dynamic Mathematical Resources

In terms of the aspects of interaction highlighted earlier, the first group of studies of dynamic mathematical resources emphasises forms of instructional activity in which distinctive types of interaction between student and digital system underpin complementary forms of interaction between teacher and student: here, the feedback that students gain from the system in response to their actions lies at the heart of instructional activity, allowing the teacher to focus on supporting students in interpreting this feedback and deciding how to act on it.

The second study focuses particularly on changed forms of collective interaction between teacher and students, underpinned by new forms of interaction between both teacher and student users and networked systems, and associated with a pedagogical shift towards practices of formative assessment. Clark-Wilson summarises these new forms of interaction with networked systems as promoting purposeful classroom discussion through which the teacher's awareness of students' current mathematical reasoning was enriched; providing teachers with fresh insights enabling them to provide thoughtful interventions during lessons; and supporting strategies for peer assessment and self-assessment by students.

Bearing in mind the dangers noted earlier of students treating mathematical tasks as work to be done rather than problems to be thought through, this interpersonal dimension seems crucial to avoid the risk of the more experimental approach often associated with the use of digital tools degenerating into unreflective trialling. What all these studies bring out is the still crucial role of the teacher in scaffolding the interaction between students and digital resources so as to increase the depth of reflection on results and the quality of mathematical interpretation.

12.4 Conclusion

I chose the examples of research which have been examined here so as to bring out some of the diversity both in the types of digital resources which are being adopted for teaching school mathematics and in the ways in which these are being taken up in instructional activity. Perhaps the first important lesson that can be drawn is that any particular resource can be used in very different ways: as illustrated—in the Clark-Wilson study—by the multiplicity of usages of each of the three system functionalities, and—in the Ruthven, Hennessy and Deaney study—by the differing forms of instructional activity that teachers employed in incorporating dynamic geometry into their classroom practice; notably the contrasting degrees of direct student interaction with the software. In particular, this signals that teachers may make use of a digital resource in ways quite different from those espoused by its advocates or intended by the designer: as illustrated—in the Murphy et al. study—by the way in which teachers tended to favour those Khan Academy facilities which they saw as enhancing their established pattern of instructional activity rather than shifting either to the ‘flipped classroom’ approach popularly associated with the product or to the more ‘personalised’ instructional model influencing the system designers.

Against this background, the cautiously innovative characteristics of the digitised versions of traditional textbooks—as studied by Choppin et al.—are not surprising from established publishers that currently dominate the market for curriculum resources. Equally, the drive from insurgent enterprises to promote individualised learning designs in terms of the more ‘personalised’ instruction that they make possible has its own commercial rationality. Clearly, these individualised programmes could draw further on the now well established tradition of research on intelligent tutoring systems as well as capitalising on continuing technical developments to enhance the interactive and adaptive functionality of such products and reduce their cost. Already—as the Murphy et al. study illustrates—the practice exercise components of current systems are seen by teachers and students as sufficiently advantageous for them to be routinely incorporated into instructional activity.

Nevertheless, it appears that a number of barriers remain to the widespread adoption of individualised learning designs. The first is that the expository components of such systems, while they largely follow established classroom conventions of narrated written presentation, are perceived to be less well adapted to curricular and pedagogical requirements than in-class exposition by the teacher. A second barrier is that most of these systems have not yet adequately incorporated the new kinds of digital mathematical tool and dynamic software which are increasingly used in school mathematics, although—as the Choppin et al. study establishes—some are making moves in this direction. A third barrier is the limited range of types of interpersonal interaction that present programmes appear to be able to foster: in part this is inherent in the attempt to individualise instruction, but it also reflects the limited aspiration or achievement of both types of digital curriculum programme in fostering forms of interaction through which students can productively exchange and discuss mathematical ideas with their peers or with a teacher (although, of course, such interaction could be organised off-line). Finally, there is the barrier of what teachers may see as a diminution of their role: towards manager and adviser of learning, rather than as more active initiator and director: indeed, the current attractiveness to the teaching profession of the re-sourcing of their own curriculum materials (Ruthven 2016) is, at least in part, attributable to the greater opportunities that such an approach confers on teachers to originate and curate their own resources and customise them to their situation and preferences. Thus a more realistic niche for individualised learning designs may be as a complement to teacher-led forms of instruction rather than as a replacement for them.

In general—in the studies by Clark-Wilson and by Ruthven et al.—the use of digital mathematical tools and accompanying resources appeared to support relatively exploratory patterns of mathematical activity and inquiry-based approaches to learning. Underpinning this was the way in which students' interaction with digital systems could provide them with feedback. A relatively open task structure and the responsiveness of a digital task environment appeared to support teachers in adopting roles as co-enquirer with, or coach to, students, although such patterns were not uniform. Feedback from the digital system, discussion between students and metacognitive scaffolding by the teacher generated a rich base for students to engage in formative assessment.

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Chapter 13
Resourcing Teachers in Transition to Plan for Interactions with Students' Ideas

Jana Visnovska and Jose Luis Cortina

Abstract We explore how resources support teachers' work broadly, and their preparation for interactions with students' ideas specifically. We draw on data from two professional development design experiments aimed at supporting teachers in making students' mathematical reasoning central to their instructional decisions. The forms of support that traditional resources and expectations provided were no longer present when teachers transitioned to proactively planning for classroom interactions. We identify new forms of support that designed instructional sequences can provide for teachers by (a) specifying simple initial goals for students' reasoning, (b) supporting teachers' design for classroom interactions, and (c) increasing the likelihood that these designs would do useful work in classrooms.

Keywords Mathematics teachers' resources · Instructional sequence
Learning trajectory · Ambitious teaching practices · Teacher support

13.1 Introduction

Supporting teachers' development of classroom practices that aim at ambitious goals for students' mathematical learning is a complex undertaking (Lampert et al. 2010). These practices place student mathematical reasoning at the center of a teacher's decision-making and foreground classroom interactions in formats such as project-based, inquiry-based, or problem-based learning. These practices are currently not typical, and involve substantial teacher learning (Maaß and Artigue 2013).

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Material resources such as online and printed textbooks, and instructional sequences are necessarily a piece in a puzzle of facilitating teachers' transition to ambitious teaching (e.g., Remillard 2012). The question of how material resources can and should support such transition has captured the interest of researchers, resource developers, and professional development (PD) practitioners. Davis and Krajcik (2005) advanced a vision of educative curriculum materials that should ground teacher learning "in specific instances of instructional decision making" while helping teachers "develop more general knowledge that they can apply flexibly in new situations" (p. 3). These and other researchers have since explored ways in which educative curriculum materials are used and how they could be designed and continually improved to support teachers' engagement in ambitious teaching (Davis et al. 2014; Stein and Kim 2009). Others have further theorized teacher-resource relationship and specified characteristics of both teachers and resources that come to play as teachers use resources to design their teaching (Brown 2009; Gueudet and Trouche 2009, 2012; Remillard 2005, Chap. 4). The work reported in this chapter confirms that material resources, indeed, have a potential to productively shape teaching of individual teachers and teacher groups, especially if ample time and adequate support is provided. We also show that even when materials are carefully constructed to communicate specific instructional rationales, and portray and justify a particular vision of teaching and learning mathematics, teachers' interpretations and uses of these materials will vary.

Teaching practices within which instructional decisions are based in how students reason mathematically had long been of interest to mathematics education research community. Teachers are envisioned to anticipate, understand, and respond to their students' ideas, thus proactively scaffolding the reasoning that occurs in their classrooms (Fennema et al. 1996; Smit et al. 2013). Analyses show that effective teachers indeed support, elicit, and extend their students' ideas (Fraivillig et al. 1999) and that learning to do so requires support (Franke and Kazemi 2001). There is a recognition that educative curriculum materials need to aim to support teachers' work with students' reasoning (Davis and Krajcik 2005) and they most frequently do so by providing examples of student solutions and snippets of classroom conversations. While these examples are practical and teachers recognize them as useful, they, inevitably, provide only limited guidance to organizing productive classroom interactions. They help teachers in transition to recognize the broad strokes and envision some possible classroom interactions. However, teachers often feel lacking the specific guidance, in particular when students' responses do not match those in provided examples, or when mathematical goals remain implicit (Grant et al. 2009).

Our aim in this chapter is to explore some of the *functions* that resources need to fulfill in supporting the teachers in transition as they work on changing the nature of interactions in their classrooms. We approach this task through our work as design researchers, oriented by the design theory of Realistic Mathematics Education (RME; Gravemeijer 1994). We draw on two PD design experiments, in which instructional sequences, previously developed in classroom design experiments, were used and revised (Gravemeijer 2004). We draw on data from the first design experiment to document the uncertainties the participating teachers voiced and use these to indicate the forms of support that ‘traditional’ curricula, textbooks, and expectations provide, but that are typically not sufficient when teachers transition to working with innovative teaching approaches and materials.

We then document our learning about supporting teachers’ reconstruction of the rationales that underpin instructional sequences. We identify, in particular, the supports that the instructional sequence can provide for the teacher by (a) specifying relatively simple initial goals for students’ mathematical reasoning, (b) supporting teachers’ design of means of supporting students’ reasoning, and (c) increasing the likelihood that the designed means would do useful work in classrooms. We propose that, with appropriate support, the demanding in-the-moment decisions about guiding classroom interactions can become more manageable for teachers, making ambitious teaching centered in students’ reasoning a reality.

13.2 A Design Perspective on Resources and Teacher Learning

Design experiments, as developed by Cobb, Gravemeijer and colleagues (Cobb et al. 1997; Gravemeijer and Cobb 2006), are a research methodology of mathematics education that involves developing both instructional designs to support particular forms of learning, and explanatory theoretical constructs to account for how this learning was supported. We adopt this version of the design experiment methodology, and formulate our instructional designs following the RME design theory (Gravemeijer 1994) and its more recent adaptations (Cobb et al. 2008), some of which are in particular relevant to establishing our theoretical background for this chapter.

Within the RME theory, instructional design is guided by a set of positive heuristics, which initially aimed specifically at supporting ambitious student learning. Two adaptations to RME were later introduced, acknowledging the mediating role of the teacher in students’ learning. First, the instructional designs began to aim for supporting teachers in achieving the envisioned instructional agendas. In other words, researchers’ design decisions about means of supporting students’ learning now take into account whether and to what extent the designed tasks and tools would be usable by teachers, and in particular by *teachers in transition*, as they organize for their students’ learning.

For example, attention is not paid solely to designing tasks and tools that would open up possibilities for student reasoning so as to enable productive classroom discussions led by a researcher or by an expert teacher. It is considered equally important that the designed tasks and tools *productively* constrain (cf. Wertsch 1998) the number of mathematically different ideas that students might propose, so that the teacher in transition is aided in anticipating a reasonably small number of possible directions for classroom discussions that would advance their instructional agenda.

Second adaptation to RME theory is aligned closely with the notion of educative curriculum materials and concerns the potential contribution of designed instructional resources as a means of supporting teachers' learning. In particular, designed resources aim to provide opportunities for teachers to reconstruct the rationales for instructional sequences, which often differ considerably from the rationales that underpin instructional decisions of teachers in transition. Once instructional sequences are developed and refined in classrooms, we engage with teachers in a variety of PD and classroom teaching activities grounded in the instructional sequence, aiming to make visible, interrogate, and refine over time, the rationales that underpin the instructional decisions (Cobb and McClain 2001). Insights form experimenting in PD context further shape both the instructional sequence specifically and design heuristics for supporting teacher PD more broadly. In this chapter, we illustrate the sources of several such insights and combine these into a retrospective story, and a conjecture, of one way in which instructional sequences could be designed to provide a feasible starting point for teachers' transition.

13.3 Background to Design Experiment Illustrations

Illustrations for our discussion come from two rather different PD design experiments, in which both authors were involved. The two studies share the specific design research methodology (Cobb et al. 2008), and conceptualization of both goals for and means of supporting teacher learning (Visnovska et al. 2012). They allow us to explore the trajectories of teachers in transition as they worked to reconstruct design rationales of an instructional sequence, and plan for interactions with their students' mathematical ideas.

Data for the discussions of the complexities related to teachers' *expectations* for support come from years 3 to 4 of a 5-year PD design experiment with a group of 12 middle school mathematics teachers, conducted in the southeastern USA.[1] One of the primary goals was to support the teachers' development of instructional practices in which they would induct their students into the ways of reasoning of the discipline by building systematically on their current mathematical activity. At the beginning of the study, the teachers' practices were rather homogeneous and could be characterized as traditional (Dean 2005).

The PD group met for 6 full-day sessions dispersed through the school year and for a 3-day summer workshop every year. A statistics sequence designed in two prior classroom design experiments (Cobb et al. 2003; McClain and Cobb 2001) was a primary means of supporting teachers' learning. In years 1 and 2, the aim of

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[1] The research team included Paul Cobb, Kay McClain, Chrystal Dean, Teruni Lamberg, Qing Zhao, Melissa Gresalfi, Lori Tyler, and the authors.

PD activities had been to foster the teacher community, and to deepen teachers' understanding of central statistical ideas. In years 3 and 4, PD activities focused increasingly on the teachers analyzing their own and others' pedagogical practices with particular attention to student learning opportunities. The teachers usually participated in a statistical activity as learners during PD session, collectively planned a lesson based on the same activity, taught it to their students, and brought students' work or classroom video back to the next PD session.

The data for our discussion of means of support that an instructional sequence can provide for designing classroom interactions come from a collaboration with Irene, an experienced Mexican teacher who agreed to conduct a classroom design experiment to test a fractions sequence (Cortina et al. 2014) with her fifth grade students. At the time of the experiment, Irene was enrolled in a Master's degree program on educational development at a local public university. She also worked as a full-time teacher in an urban elementary school that serves children living in unfavorable social and economical conditions, with parents' irregular access to employment. Prior to this collaboration, Irene's teaching could be characterized as traditional. In a series of six one-hour meetings with the second author, Irene first became acquainted with the instructional sequence, including its rationale, and how this sequence was developed and used in prior design experiments. She then worked with the instructional sequence during 18 dedicated weekly classroom sessions, about 35 min each. After each session, she met with the second author to analyze the classroom events, and collaboratively plan for the upcoming session. Throughout the collaboration, the two authors analyzed and planned for supporting Irene's learning. Gravemeijer and van Eerde (2009) characterized this type of research design as *dual design research*, because both the teacher's and her students' learning are the goal of systematic classroom experimentation.

Instructional sequences with which we engage the teachers in our studies instantiate how learning in a specific mathematical domain can develop in a classroom setting, and how the outlined developments can be supported. The design rationale for these sequences is grounded in analyses of how this process unfolded in actual classrooms, and is expressed in the form of a conjectured learning trajectory (Simon 1995). In it, the designers specify the key shifts in classroom mathematical practices (Cobb et al. 2001) and how each of those shifts might be supported at a classroom level. The supports include establishing productive norms of classroom interactions, engaging students in specific instructional activities, making particular issues the focus of whole class discussions, and the use of particular tools and inscriptions. These kinds of instructional sequences are different from collections of instructional tasks that address a particular mathematical construct, but do not specify (a) the mathematical insights students are expected to develop by engaging in the tasks, (b) how might those insights be related to each other in a learning process and, importantly to our present discussion, (c) how to instructionally support the emergence of a new insight from the prior emergence of another.

Our goal as we worked with the teachers was that they would examine issues of teaching and learning statistics and fractions, respectively, as they adapted, tested, and modified the sequences in their classrooms. We did not focus on specific teacher moves, but instead pressed the teachers to justify the moves and actions that they chose by considering opportunities for student learning. Many of these PD discussions specifically attended to classroom interactions, as the nature of classroom discourse and organizing whole-class discussions are among the key means of support within the instructional sequences.

13.4 Methodology

The data in the 5-year PD design experiment included video-recordings and field notes of all PD sessions, transcripts of the key episodes, copies of the teachers' work, and a debriefing and planning research log. Additional data included student work from the teachers' classrooms, the video-recordings of the statistics lessons taught by two of the teachers, and annual modified teaching sets (Simon and Tzur 1999) comprising a video-recorded lesson in each teacher's classroom and a follow-up audio-recorded teacher interview that focused on issues that emerged in the course of the lesson.

The data from the collaboration with Irene who conducted a classroom design experiment included video-recordings and field notes of all classroom sessions and copies of students' work. In addition, two logs were produced, first by the teacher. Prior to each classroom session, the teacher recorded the goals she pursued, the activities she planned to conduct, and her expectations for classroom interactions and outcomes. After each session, she reflected on what actually happened in the classroom, and outlined what needed to be done next. This log played a central role during the debriefing sessions with the second author.

The second author produced a *research log*, which included design conjectures and notes related to both students' and Irene's learning. First, the log documented the second author and Irene's conversations during weekly debriefing and planning meetings, in which they relied on Irene's notes, classroom video, and copies of student work to understand students' learning progress. Second, this log documented weekly to bi-weekly debriefing sessions between the two authors, which focused on Irene's teaching and planning, and on the ways in which her work was supported.

In both cases, we analyzed the data using an adaptation of constant comparative method described by Cobb and Whitenack (1996) that involves testing and revising tentative conjectures while working through the data chronologically. As new episodes are analyzed, they are compared with conjectured themes or categories, resulting in a set of the theoretical assertions that remain grounded in the data. The thorough analysis of the PD study is available in the author's dissertation (Visnovska 2009) and the analysis of the dual design experiment on fractions learning is ongoing.

When we collected and analyzed data in statistics PD design experiment, we conceptualized learning as participation in communities of practice (Wenger 1998). Our focus was on documenting the development of the teachers' views and uses of students' reasoning (Visnovska 2009). These analyses provided insight into the teachers' learning as related to work with the instructional sequence over time. For this chapter, we were in particular interested in how the teachers in transition actively pursued questions of *their pedagogical interest*, irrespective of whether these were aligned with our PD agenda. We therefore analyzed the data corpus to document the pedagogical concerns the teachers in the PD group voiced in years 3 and 4, when the PD activities focused on pedagogical practices. This involved searching the retrospective analysis log for the questions the teachers brought up that diverted from the theme of PD group conversation at the time.

We categorized these questions as *pedagogical* or *other*. We aimed to gain a sense of frequency of teachers' focus on pedagogy, as opposed to focus on non-pedagogical issues such as their own mathematical learning, or institutional context of their work. The two latter foci were ongoing topics of PD conversations, and were prevalent in the first two years of PD interactions (Dean 2005). Within pedagogical questions, we noted whether teachers inquired about enactment (the *how*) or rationale (the *why*) of teaching. We use the summative view of these unsolicited teacher contributions to speculate about the nature of support that teachers sought and what this reveals about supporting the needs and learning of teachers in transition.

In the fraction dual design experiment, we conducted a retrospective analysis using as a guide the log produced by the second author. We then checked the formulated conjectures about the evolution of the teacher's learning against the teachers' log and the rest of the collected data, looking for inconsistencies, and refining the conjectures whenever necessary.

13.5 Uncertainties and Needs of Teachers in Transition

Requirements frequently cited for educative curriculum materials are that they include features that help teachers "recognize both the rationales for recommendations and the ways in which they can productively adapt the recommendations in their own classrooms" (Davis et al. 2014, p. 26). Indeed, teachers are much more likely to productively engage with teaching materials when they have opportunities to understand the underlying rationales and can use these to design meaningful adaptations. However, evidence abounds to suggest that it is non-trivial for teachers to recognize designers' rationales, even where explicit efforts were made to make such rationales visible in the teaching materials (e.g., Chval et al. 2009; Stein and Kim 2009).

We draw on the statistics PD design experiment to illustrate difficulties in supporting teachers in transition to reconstruct the rationales for instructional decisions. We first introduce the instructional sequence and structure of PD sessions relevant to our discussion. We then follow the teachers' agendas, and illustrate the questions and clarifications they independently brought up in PD sessions in years 3 and 4, when the focus was primarily on pedagogical issues related to teaching exploratory data analysis. Finally, we outline the teachers' learning in relation to our PD agenda and illustrate what was involved in supporting the group in the reconstruction of the design rationales that underpin the sequence.

13.5.1 Statistics PD Design Experiment

The focus of the *statistics sequence* was on supporting students to reason about univariate and bivariate distributions of data, often in activities that involved making recommendations based on comparing two or more sets of data. The intent of the instructional activities was that students would conduct genuine data analyses in order to address problems that they considered significant. The initial phase of the classroom activities, in which the problem situation was introduced and students discussed how useful data could be generated, was of considerable importance. The data was then introduced as being generated by this process. Three computer tools provided the students with a variety of options for organizing data sets with data represented graphically on a computer screen (for descriptions and analyses of these tools see e.g., Bakker and Gravemeijer 2003). In the classroom design experiments in which the instructional sequences had been developed, students compared their recommendations in classroom discussions and justified them by explaining how they had analyzed data.

To organize classroom activities productively, the teachers needed to monitor how their students reasoned about the data as they came to understand the problem scenario and conducted data investigations. They needed to plan classroom discussions of their students' analyses and make decisions about subsequent tasks and statistical learning goals.

The PD goals in years 3 and 4 included supporting teachers in making sense of individual students' statistical interpretations and solutions, and in adapting the statistics sequence to the needs and constraints of their classroom situations (Cobb and McClain 2001). Our role during the PD sessions was to link the teachers' insights, comments, and questions to the bigger pedagogical issues that were the focus of particular activities (e.g., how to make decisions about when to "move on" in instruction).

13.5.2 Teachers' Focus on Enactment

In both conducting and analyzing PD sessions, we were aware of teachers' strong inclination to focus on the enactment of the statistics sequence activities. While our attempts at bringing students' reasoning to the fore were met with a degree of confusion, teachers periodically steered the conversation to issues such as *How do we know when to move on (within an activity, to the next type of task,* etc.*)?* and *When do we introduce vocabulary?* This led us to search PD data for instances where, in our view, teachers' questions diverted from the theme of conversation at the time. In some cases, a teacher introduced an unexpected theme. In others, teachers asked for clarifications within a theme that, we assumed, had already been resolved and concluded. The summary of the results is presented in Table 13.1.

To interpret the table it is important to first clarify that the overall participation patterns in years 3 and 4 were largely similar (Visnovska 2009). What seemed to differ is the degree to which the teachers shaped the themes for the PD conversation in year 4.

Enactment. While the *When to move on?* and *When to introduce (vocabulary, median, box plots, calculations)?* questions periodically emerged during this period, some enactment questions in year 4 became more refined and started to relate to specific elements within the sequence. For instance, teachers asked about how to conclude a lesson where students proposed incompatible problem resolutions, and how to find out what students understood.

Rationale. Typical questions in this category included teachers seeking clarification for purposes of pedagogical actions such as *launch* and *re-teaching*. In the second half of year 4, three questions were genuinely related to the rationale of the instructional sequence. The teachers discussed where would a specific activity best fit within the sequence, asked about purposes for moving on to the next computer tool, and elaborated on a specific example from their teaching, explaining that the sequence cannot be a set collection of tasks but must reflect what students do in the classroom.

Other. Illustrative examples of questions that did not have specifically pedagogical focus included asking whether the median always indicates where most of the data are, or whether materials created within PD sessions will be used by other teachers in the school district.

The analysis supported two observations. First, the teachers, unprompted, asked substantially more pedagogy-related questions in year 4. At the time, the teachers were more familiar with the sequence and began to appreciate its impact on their students' motivation and engagement. Elsewhere (Visnovska and Cobb 2013; Visnovska et al. 2012) we conceptualized the developments of the teacher group as a process of community documentational genesis (Gueudet and Trouche 2009, 2012), highlighting how both teachers and resources they used were transformed in

Table 13.1 Distribution of different types of questions brought up by the teachers in PD sessions in years 3 and 4

	Pedagogy		Other (mathematics, institutional context)	Total
	Enactment	Rationale		
Year 3	3	5	3	11
Year 4	32	7	8	47
Total	35	12	11	58

the course of their interactions. We illustrated how the instructional sequence came to have significantly different meanings in the activities of PD group over time. Present analysis suggests that the teachers' initial interactions with the sequence and their use of instructional activities in their classrooms supported teachers in taking a greater initiative and actively pursuing pedagogical learning during the later PD sessions.

Second, in the majority of the unprompted questions, the teachers sought enactment advice centered in issues such as timing of specific teaching moves. In most of these instances, our efforts at re-orienting teachers to consider their students' reasoning when deciding about enactment were unsuccessful. For instance, we prompted teachers to consider whether their students had created new meanings that called for new vocabulary. Suggestions of this kind were not constituted as adequate or relevant to the question at hand. Instead, the teachers seemed to seek recommendations that were absolute and independent of the messy details of what was happening in the classroom. Only towards the latter part of year 4, some teachers started to pursue clarifications of rationales for pedagogical decisions that underpinned the sequence, and took student reasoning in consideration.

It is important to consider that the teachers in the PD group had long histories of participation within instructional practices that center on their enactment of lessons and on the students' completion of specific tasks. When the lesson objectives are stated in terms of content coverage, or as mathematical concepts and relationships that a teacher needs to explain to their students, the teacher has a rather good control over whether or not these objectives are addressed in the classroom. When the focus is on teacher's actions, it is also relatively easy for the teacher to assess whether specific lesson objectives were met, and therefore, whether it is in order to move to the next lesson.

When teachers aim to transition to working with objectives that are stated in terms of specific forms of students' reasoning that are expected to 'emerge' in their classrooms, the relationship between teachers' actions and whether or not a lesson objective is met becomes a lot more complicated. What teachers do still shapes students' learning, but it no longer determines whether any particular lesson will be deemed successful. Determining whether lesson objectives were met becomes non-trivial, largely because there is no longer a set of 'the right things to do' that would guarantee meeting the objective.

Re-occurrence of enactment-focused questions illustrates some of the expectations that teachers in transition have of the supports provided by new teaching materials, and of their learning about a new pedagogical approach. We contemplate that these are the issues for which teachers would read selectively in written instructional resources and guidance materials. The question then remains whether and how could such materials bring to the fore the rationales for pedagogical decisions so that these would, over time, become a relevant element in teachers' search for an enactment advice.

Let us now further illustrate how the reconstruction of design rationales is shaped by teachers' existing interpretations and is thus in principle a non-trivial matter. We then move to the more recent dual design experiment on fractions as measures and describe some progress we made with addressing the teacher's concern of assessing the success of a lesson, or *how to know when to move on*.

13.5.3 Supporting the Reconstruction of Design Rationales

Our work in the statistics PD design experiment illustrates the demands of reconstructing designers' rationales even when the designers can talk to the teachers directly and provide PD activities to proactively support such reconstruction. In years 3 and 4, we engaged the teachers in a variety of activities that focused on different aspects of teacher's role in the classroom, including how to conduct data generation discussions (see also Visnovska and Cobb 2013). Data generation discussions serve to introduce the context of the problem in the classroom, establish its relevance, consider what data could be collected that would allow for developing insight into the question under investigation, and consider specific details of how this data could be collected. These discussions play a significant role in shaping the ways in which students interpret data.

Early in our collaboration, the teachers recognized that data generation discussions were an important aspect of instruction. However, it became apparent that, from their perspective, effective instructional activities involved a scenario that was immediately interesting and personally relevant to students. For example, the teachers considered that activities that involve soft drinks or roller coasters were instructionally more promising than those that focused on issues of broader social significance (e.g., driving safety). They therefore understood the importance of data generation discussions primarily in terms of capturing students' interest and enticing students' engagement in instructional activities. It was also apparent that, from the teachers' perspective, data generation discussions made little if any contribution to the ways that students interpreted and analyzed data. These understandings were shaping significantly teachers' planning decisions and how they guided classroom discussions when they introduced new activities.

During our ensuing collaboration with the teachers, we engaged them in a number of activities that focused on data generation discussions. For instance, we introduced a statistical activity, in which the teachers were to act as students. We described the context of the problem, but we did not press the group to propose or clarify the process in which relevant data could be generated. When the teachers received data sets to analyze, they raised a number of questions related to the meaning of the data and its suitability as a basis for making claims about the situation at hand. Instead of responding to teachers' questions, we asked them to create a list of these questions and suggest whether and how these could have been addressed in the initial data generation discussion. We also asked the teachers to trial problem scenarios that they did not initially see as exciting for their students (e.g., addressing speeding on a local highway) and monitor how their students responded.

This approach was reasonably successful in that the teachers came to view a broader range of problem scenarios as potentially productive and saw it as their responsibility to develop the significance and relevance of problem situations with students, rather than limiting problem scenarios to those that they judged as familiar and exciting to students. Towards the end of our collaboration with the teachers, there were strong indications that the teachers had become aware that the students' understanding of the process by which the data were generated influenced how they interpreted and analyzed the data. In particular, they explicitly linked the issues that they addressed (or failed to address) while conducting data generation discussions to students' subsequent analyses.

This development would have been unlikely had we not made the rationale for the instructional sequence an explicit focus of professional development activities. We had to exert considerable effort at shaping teachers' planning decisions, including which problem scenarios were worth trialing in classrooms. Given the ongoing PD relationship, the teachers on occasion gave us benefit of doubt and tried the suggested activities in their classrooms, but they often did so *against their better judgment*. Nevertheless, these trials, and reflections on what happened in classrooms, were instrumental in allowing teachers to start questioning their instructional rationales and reconstructing those that underpinned the instructional sequence.

Supporting teachers' reconstruction of instructional rationales that their resources aim to advance is even more important in light of other contributions to this edited book. Remillard (Chap. 4) explores how teachers read resources in order to use them, while Kim (Chap. 15) and Leshota and Adler (Chap. 5) look into the patterns related to teachers' adaptations, omissions, and injections of classroom activities and their sequences. It would appear that without a sustained support, teachers in transition are rather likely to omit, or substantially alter those parts of resources that do not align with their current, possibly largely implicit instructional rationales.

13.6 Designing Resources to Support Teachers' Transition

Some of the illustrations from the PD design experiment we reported in the previous section can be easily seen as portraying teachers in transition as being captive to the very instructional practices they aim to abandon. This appears to be the case even when the PD activities are designed with an explicit goal of exploring instructional rationales, and situated within the overarching goals of the sequence. Such interpretation, of course, is inaccurate given that some teachers, indeed, have been known to accomplish the transition.

The illustrated complexities inspire us to look for understanding of the trajectories of transition and identifying elements that made these successful. Could we, for instance, provide teachers in transition with initial, *local*, well-defined goals and foci that would meaningfully orient their planning and classroom decision making? Could we do it in ways that bring students' reasoning into the picture and harness teachers' interest even when the big picture of ambitious practices is still somewhat elusive? What might such goals and foci look like? What are the mathematical contexts in which this can be done most productively?

Our analysis of teachers' questions suggests that supporting teachers to judge the progress within the sequence, and to make decisions about ways to continue would be of particular importance. These were some of the background considerations that guided the dual design research project in which a fifth grade teacher Irene collaborated with us to further develop the instructional sequence on fractions as measures. We illustrate how we engaged Irene in discussion of different ways to judge progress within the sequence, and how co-planning to help students' problematize an established strategy or idea was central to her transition.

13.7 Transition as Co-participation

13.7.1 Fractions Dual Design Experiment

The fractions as measures sequence (Cortina et al. 2015) is set within a narrative about the ways in which a group of ancient peoples, the Acajay, measured. Before the students encounter a situation, in which they explore lengths as related to the notions of unit fraction and proportion, measurement is approached more broadly. Within the narrative, Acajay people initially measure with body parts. The students engage in measurement problems using the same technology until they—like the Acajay—realize that this is at times problematic. Once they develop a need to standardize the measurement unit, a traditional measurement tool, the wooden *stick*, is introduced.

While the stick is initially a solution to an earlier problem, its use is subsequently made problematic by engaging students in situations where they develop the need for measuring lengths more accurately than what the stick alone allows. At this point, students learn that Acajay elders solved this problem by introducing smaller length measures, *smalls*, lengths of which represent unit fractions of the length of the stick (Fig. 13.1). The overarching aim of the sequence is that students develop understandings of fractions as quantities (Thompson and Saldanha 2003), in particular in linear measurement situations.

Similar to working with the statistics sequence, to organize classroom activities productively, a teacher needs to monitor how their students reason about measurement, and later about relative lengths of different smalls. They need to plan classroom discussions in which measurement methods that were previously acceptable would become problematic from the students' point of view. To accomplish this, the teacher constantly makes decisions about suitable tasks, and the foci and goals for the classroom sessions, based on the actual reasoning of their students.

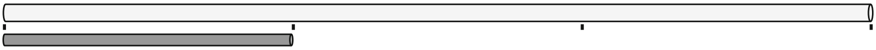

Fig. 13.1 *Small of three* rod with such a length (1/3) that three iterations of the rod cover the same length as the *stick* (reference unit)

13.7.2 Supporting Irene's Reconstruction of Design Rationales

When testing the sequence in her classroom, Irene initially followed a rationale in which she tried to be as faithful as possible to the instructional activities, in the way she understood they had to be enacted. At this point, Irene was much more focused on what she considered she had to do, than on how her students were reasoning. This became apparent after the first classroom session in which Irene engaged her fifth graders in an activity that entailed measuring lengths of different items with their body parts. She noticed that her students engaged in the activity enthusiastically, measuring and recoding the measures they took in their notebooks. Irene then tried to orchestrate a whole class conversation in which the advantages and disadvantages of measuring with body parts were to be discussed. Such a conversation was contemplated in the planed activity. However, when Irene asked her students about the disadvantages of measuring with body parts, none of them regarded measuring in this way to be problematic.

Irene knew that the ensuing activities in the instructional sequence involved using the stick (the standard unit of measure). She also knew that, within the sequence rationale, it was expected that students would regard using this tool as a means to overcome the limitations of measuring with body parts (i.e., inconsistent measures produced for the same length). She was therefore unsure about why none of her students saw measuring with their body parts as problematic. Irene considered whether the way she guided the activity was to blame, or even the students themselves, as they came from low-income families and performed poorly on standardized tests. Even though she had a clear sense that something had not gone according to her plan, the most reasonable course of action, in her view, was to move on to the next classroom activity. She planned to introduce the stick to students, ask them to measure with it, and hoped for the best.

In the debriefing that followed this first classroom session, the unexpected situation that Irene had faced became an opportunity to discuss how progress made in a classroom can be assessed. In particular, the second author used Irene's experience to contrast two ways in which one can make instructional decisions: trying to faithfully enact an instructional activity on the one hand, and progressively supporting students to reason about specific issues, in particular ways, on the other hand. The major difference between the two, they agreed, was in deciding when the instructional goals for a classroom session had been accomplished. Irene recognized that she was attempting to enact the first activity faithfully and when she completed the enactment, she considered herself, and the classroom, to be ready to move to the next type of activity. Seeing that her mentor had a different way of proceeding in mind, she agreed to instead explore whether she could help at least some of her students to realize that measuring with body parts had some limitations.

Irene and the second author designed several problem scenarios aimed at helping her students recognize how measuring with body parts could be unreliable, as it would render different numbers for the same length. In one of them, different students were to be asked to use their hands to measure a paper strip that was placed on the whiteboard. A conversation would follow in which the students would discuss why everyone did not obtain the same number and whether this meant that some of them made a mistake.

In the following debriefing with the second author, Irene commented that she had been successful in helping students recognize the different complications that measuring with body parts could cause. However, she was not satisfied with this result and wanted to make sure that *all* her students were aware of the limitations. From the sequence design point of view, developing such awareness was important so that all the students would see the standard unit of measure, once introduced, as a meaningful innovation. Irene thus decided to design additional problem scenarios and use them in the following teaching session.

Planning in this way represented an important shift in the rationale Irene employed for making instructional decisions. She no longer focused on which activities she needed to enact, when, and how. Instead, she now focused on the mathematical issues she wanted her students to discuss and understand, and viewed the problem scenarios as the means that could support the students in doing so. By and large, Irene kept focusing on learning goals, in terms of forms of students' reasoning she aimed to elicit, throughout the rest of the classroom design experiment.

Retrospectively, a number of issues have been critically influential in helping Irene shift her perspective. Several of them are related to the nature of the instructional sequence that was being tested, in which the learning goals were clearly specified and sequenced. First, the learning goal that was pursued at the beginning of the sequence was specific and relatively simple; to help students recognize the shortcomings of measuring with body parts. Had the sequence started directly with creating and comparing unit fraction lengths, we imagine the teacher would have been more inclined to follow a (perceived) enactment script.

Second, the problem scenarios Irene and the second author co-designed to help students uncover the problematic nature of measurement with body parts were also relatively uncomplicated. These built rather directly on Irene's awareness of situations in which measuring with body parts breaks down. The co-planning made her aware of a possibility of re-creating such situations in her classroom, thus creating opportunities for her students to notice and discuss the problematic results.

Third, and most importantly, these relatively simple means of support had been immediately effective in helping some of Irene's students develop forms of reasoning for which she was aiming. Rather than being incidental, these elements are products of prior classroom design experimentation, during which the capacity of the designed resources to support the work of teachers in transition was a primary consideration. We believe that without these supports, it is unlikely that Irene would have shifted her focus for making instructional decisions with such ease.

Nonetheless, we do not think that Irene would have made this shift had she been introduced to the sequence and then left to her own devices. It was critical that she had opportunities to regularly discuss the developments in her classroom with someone well acquainted with the sequence, and to receive feedback and support in the form of co-planning. She needed support in both understanding *that* she could proactively help students reason in specific ways, and developing images of *how* she could go about it in her classroom. Her initial planning ideas suggest that without this support she would have resorted to the ‘covering the content’ strategy.

13.8 Conclusions

Teachers have a need to know where they are going with their teaching: What is it that they are trying to achieve? Often, they solve this issue by focusing on what *they* need to teach, and base their instructional decisions on the content that needs to be covered, the activities that need to be implemented, and the work that students need to produce. The supports available in many curriculum materials, textbook resources, and teachers’ workplaces often encourage, or at the very least, align well with this particular view of teaching goals and aims.

Transition to practices where students’ reasoning is central entails a huge change for teachers. This goes beyond having to engage in a new kind of teaching, where problem solving plays a central role, and where students work collaboratively and share their ideas. Teachers also need an alternative way to keep track of progress and to guide their teaching, in issues such as “What comes next?” and “How much time should I spend on this?”

In this chapter, we first illustrated that teachers whose transition to ambitious teaching we facilitated in the statistics PD design experiment indeed felt somewhat under-supported around these issues. Importantly, this was the case even once the teachers’ own mathematical understandings were reasonably strong and even in the PD program where care was taken to provide them with supports in responsive manner. We were eventually reasonably successful in supporting the PD group’s reconstruction of instructional rationales for the sequence (Visnovska et al. 2012), but this success relied heavily on our ongoing co-participation in planning of and reflecting on the learning that ensued in the teachers’ classrooms.

The case of Irene’s participation in dual design experiment on fractions as measures allowed us to zoom in on several aspects of support that the sequence provided for her transition. First, it specified relatively simple initial goals for students’ reasoning (i.e., coming to view measuring with body parts as insufficient), while at the same time creating opportunities for PD conversations in which two different rationales for deciding a specific course of teacher’s action could be compared and contrasted. We recognize that holding such conversations is not the answer to supporting teachers’ transition and that PD activities in which such comparisons would become meaningful to specific groups of teachers remain a non-trivial design challenge. However, targeting teacher’s enactment-related decisions—an issue that is among their primary concerns—seems to be a direction worth further testing and development. We now plan PD activities so that they would allow us to intentionally initiate similar conversations with teachers early in their classroom work with instructional sequences, while grounding this work in relatively simple mathematical contexts.

The initial goals within the instructional sequence also directed Irene's attention to a more general view of *her role* in instigating the progress within the sequence, by helping her students problematize previously established classroom practices. This, however, would be likely less successful had Irene not been supported to *design* her own means of supporting student learning (in this instance tasks) that she tailored to the specific circumstance of her classroom. Indeed, if we accept that "teaching by design" is teachers' inevitable reality, not their choice (Brown 2009, p. 19), then the resources at teachers' disposal have to adequately support them in the design aspect of their work.

Related to this is the observation that the new tasks Irene co-designed and later designed independently for her classroom use, did useful work for her. This can be seen as a result of extensive prior experimentation in classrooms that the sequence reified. But equally importantly, Irene interpreted students' work with the designed activities as successful because her instructional agenda now oriented her to the specific forms of student reasoning she knew to expect, elicit, and reinforce during classroom interactions.

In discussions of educative material resources, teachers are often viewed as relatively independent learners, even when positive contribution of collaboration with other teachers or more experienced mentors is acknowledged. Our current view of a useful material resource inherently involves co-participation, but we find it equally worthwhile to explore ways in which some of the supports highlighted in this chapter can be embedded in material resources and thus support both teachers in transition and PD facilitators.

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Chapter 14
Prospective Teachers' Interactions with Interactive Diagrams: Semiotic Tools, Challenges and Well-Trodden Paths

Elena Naftaliev

Abstract The goal of the study was to analyze prospective teachers' interactions with interactive texts and to understand the affordance of the texts' design, which was conducted within the semiotic framework, on the different stages of the interactions. The findings of the empirical study shed light on awareness of design functions of interactive text in the teachers' interactions with the materials: they developed teaching plans and scenarios of student-textbook-teacher interaction that included similar tasks distinguished by the designed functions in different stages of the interaction and defined, for each task, different goals for teaching. However the teachers did not always take an advantage of the wide variety of options available with the interactive texts and preferred the familiar paths in teaching.

Keywords Mathematics teachers' resources · Interactive curriculum resources
Student-textbook-teacher interactions · Semiotics · Tasks design
Prospective teachers

14.1 Introdcution

One of the challenges that educators of prospective teachers (PTs) face today is to support PTs in using e-textbooks for the teaching-learning processes. By e-textbooks we mean interactive textbooks. A textbook is an educational form which represents curriculum material for teaching and learning and can serve as a learning material for both students and teachers. Textbooks present specific views concerning mathematical content and also serve as a primary influence as to how teachers should teach such content. In traditional printed textbooks, the content is displayed in a static mode and users are invited to interpret it with limited

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possibilities of interaction (e.g., pointing to figure, tracing with a pencil). New technologies allow the production of interactive textbooks that enable broader interaction between the users and the content. Interactive textbooks are far more complicated than static textbooks; they include various interactive representations, tools that allow linking between the representations in the text, as well as tools that allow for construction, manipulation and activation of new examples. In this case, users are invited to interpret the content of the textbook by interacting with it. Interactive textbooks explicitly invite the reader to take action, that is, to activate or change the text within given limitations. For teachers, adopting interactive textbooks that are tailored to deliver curriculum and are designed to accommodate the inquiry situation poses new challenges. To allow students' engagement with interactive texts, teachers must figure out how to promote and guide the exploration while bridging the tensions between standards required by the curriculum, opportunities for students' active personal learning, and teachers' orientations (beliefs, values and preferences). It requires examination of the interactions between: (a) the mathematical content to be taught and learned, (b) the e-textbooks to be served as a guidance as to how to teach the content, (c) the instructional practices of the teacher, and (d) the students' work and experiences within a particular educational setting.

14.2 Theoretical Framework

A textbook is an evolving pedagogical form, a changing medium consisting of smaller media components (e.g., images, tables and verbal prompts) (Friesen 2013). These components have changed not only through technological innovation, but also with larger cultural and epistemological developments. The limitations and the potential of the presentational media were always part of the mathematics culture. Describing the communication of Greek mathematicians, Netz (1999) analyzes forms of mathematical presentations in relations to the media available to the Greeks. The transformation from the mathematician thinking to himself to the act of communicating was both supported and limited by the available media: wetted sand, dusted surface, papyri and wax or clay tablets. Netz (ibid.) suggests that the limitations of the media available were essentially similar to those of printed books: "Diagrams, as a rule, were not drawn on site. The limitations of the media available suggest rather, the preparation of the diagram prior to the communicative act- a consequence of the inability to erase" (p. 16).

Technological development has caused changes in learning environments in general and textbooks in particular. The mathematical activities themselves have transfigured since the appearance of technological tools enabling "playing around" with mathematical objects (Davis 1995). In particular, dynamic tools, interactive books and various interactive platforms enabling one to experiment with mathematical objects not only have their impact on school mathematics, but also have become a part of professional mathematicians' ways of work (Borwein 2016). Teaching-learning processes should create the opportunity for students to acquire the sense of the discipline, the mathematical cultural traditions by their own activity (Freudenthal 1973; Schwartz 1999; Schoenfeld 1992; Lampert 1990). Interactive textbooks are envisioned as allowing the learner and teacher to experiment with mathematical objects, to approach texts in an exploratory mode, rather than simply receiving and interpreting it in a fixed, prepackaged form.

The mathematical activities in interactive textbooks are presented by software applications, which we call interactive diagrams (IDs). IDs can be used for different purposes: an exposition, a task, an exercise, etc. The IDs' components are: the given example, its representations (verbal, visual and other) and interactive tools. The difference between an ID and other interactive tools is that an ID is built around a pre-constructed example to carry a specific mathematical activity.

Mathematical learning, doing and thinking and mathematical communication often involve the use, change or construction of diagrams. Textbook or assessment tasks often include diagrams to be interpreted and modified by the reader. As such, the use of paper diagrams draws the attention of educators and scholars investigating strategies of using diagrams in mathematics education. Following Vygotsky, Murata (2008) noted that one of the affordances of diagrams is to structure students' ideas (to make them meaningfully visible and concrete) so that students can focus on core aspects of the problem and engage in their own sense-making process. Duval (1995) suggested that to work heuristically in problem solving, students must apprehend the given figure in an "operative" way—they must be able to modify the figure mentally or physically. Bremigan (2005) maintained that the modification of given diagrams or the construction of new diagrams in students' solutions were found to be related to success in problem-solving. Nunokawa (1994) suggested that in many cases paper diagrams do not stimulate reasoning and do not function as problem-solving tools because the mathematical structure of the situation is not sufficiently apparent in them.

To the extent to which static diagrams which are part of a printed mathematical text are intended to present certain information and a point of view (and could implicitly engage the viewer in meaningful interpretations), IDs offer viewers more explicit options for manipulating the diagram within given limitations. An ID includes the characteristic features of a static diagram and of a tool. Each ID has a unique pedagogical function in the teaching-learning process. Following Kuhn's analyses of knowledge "paradigms", Friesen (2013) concludes that textbook's features provide an essential animating pedagogical function. Visions of the future of the interactive textbooks raise questions about the pedagogical functions of this educational form.

14.2.1 The Semiotic Framework for Pedagogical Functionality of IDs

To address the pedagogical issues concerning the components of interactive textbooks, Yerushalmy (2005), Naftaliev (2012) and Naftaliev and Yerushalmy (e.g. 2011, 2013, 2017) have developed and elaborated a framework for analyzing the pedagogical functionality of IDs. The framework for pedagogical functionality is a set of categories that help educators (teachers, designers, researchers, etc.) to decide how to use, design, or choose an ID to support their educational goals. The semiotic framework proposed three dimensions (Table 14.1) for defining the pedagogical functionality of IDs that address a variety of learning and teaching settings: presentational (refers to type of example in the ID), orientational (refers to mode of representations in the ID), and organizational (refers to the connection between all the components of the ID).

14.2.2 Presentational Functions of IDs

Although examples in an ID are usually designed to be modified by the user, the example that initially appears in the diagram determines the nature of the *presentational* function of the example. Three types of examples are widely used in IDs: specific, random, and generic. Specific examples serve as a dynamic illustration that helps analyze the situation without being able to change the information. Random examples are specific examples generated within given constraints. There are two characteristics of random examples which differ the last from specific examples: (a) random examples present different information for different readers while they access the same ID and every time the reader accesses the book; (b) random examples are designed to create an experience with various cases of the same presented domain. The random examples, which provide various cases, may not be helpful in carrying a systematic inquiry, the process which provides the basis for generalization. In a generic example, the ID is structured to be representative; it presents a specific example as a part of the given task, but it is not intended to present the specific data of the activity but to help learners become aware of the representativeness of the example through a process of inquiry. Mason and Pimm (1984) noted that generic examples are transparent to the general case, allowing one to see the general through the particular: "A generic example is an actual example,

Table 14.1 The semiotic framework: three types defining the functionality of IDs

Presentational function	Orientational function	Organizational function
Specific	Schematic	Illustrating
Random	Metric	Elaborating
Generic	Schematic and/or metric	Guiding

but one presented in such a way as to bring out its intended role as the carrier of the general; this is done by means of stressing and ignoring various key features, of attempting to structure one's perception of it" (p. 287). The art of designing generic examples consists of finding ways to place the focus on generality or representativeness, as elaborated by Goldenberg and Mason (2008). It is not usually the case, however, that the generic nature of the example is visible to the learner, and often the example remains a particular case. Davydov (1972/1990) articulated the difficulty: "The real problem is precisely in finding a form for a concept in which the derivation of properties would be possible" (p. 35). Design that offers ways to systematically generate multiple and varied examples, and to preserve and reconstruct processes, provides the basis for conceptual construction of knowledge by generalizations and conjectures. The design of a generic interactive example should encourage the learner to take personal actions within a well-defined domain. It is only when the viewer becomes aware of the generality in the specific example that its mission is achieved. Our studies provide evidence that that the process of constructing new examples by the students or their interacting with the components (the representations, as well as the linking and control tools) of the given carefully designed repertoire of examples were found to be crucial for unfolding the representativeness of the examples as carriers of the general meaning (e.g., Naftaliev and Yerushalmy 2011, 2017).

14.2.3 Orientational Functions of IDs

The tone in which the text addresses the learner is subject to design decisions having to do with the *orientational* function. Schematics versus Metrics of the diagrams is an important factor in reader orientation. Netz (1999) identified a connection between types of diagrams and the practices of ancient Greek mathematicians regarding their use. "The most significant question from a mathematical point of view is whether the diagram was meant to be metrical: whether quantitative relations inside the diagram were meant to correspond to such relations between the object depicted. The alternative is a much more schematic diagram, representing only the qualitative relations of the geometrical configuration. … they very often seem to be schematic in this respect" (ibid, p. 18). Based on Fish and Scribner (1990), Mason (1995) drew attention to the importance of sketches rather than paintings as a metaphor for providing stimulus to students: "A painting has richness of detail, but its completeness of detail means that the observer has to work in order to see through the whole, to make contact with and examine details and yet retain a sense of connection to the whole; a sketch provides just enough to invoke Gestalt powers of closure and to initiate a process of construal" (Mason 1995, p. 386).

An example that appears in an ID can have an accurate metric appearance and communicate in a strict tone. For example, a graph drawn on paper indicating coordinate values and scale would be interpreted as a specific case (Fig. 14.1). The example can adopt a schematic tone. For example, it may not attempt to provide the

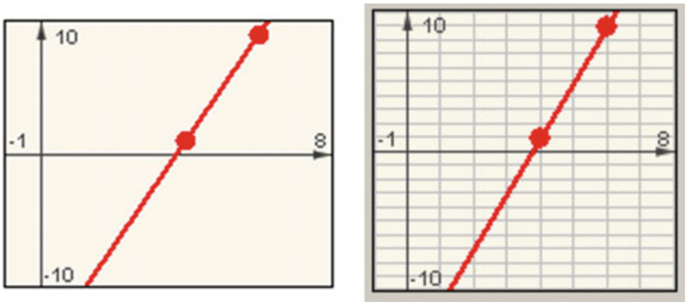


Fig. 14.1 Schematic versus metrics

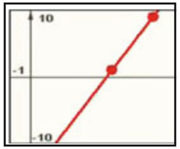
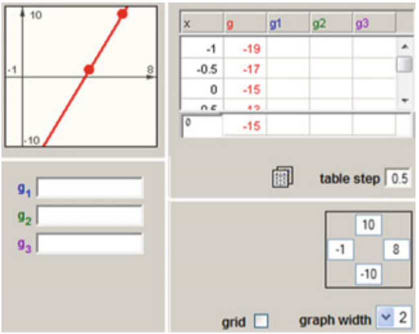
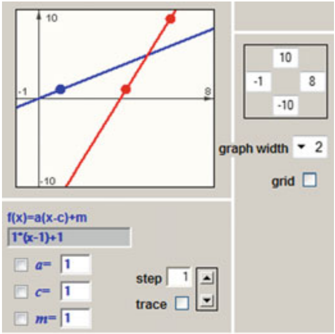
complete picture, but rather to highlight important elements, so that it can be used as a plan for a variety of final products that share the same idea or structure. The IDs' design made it possible to address the given graphs as a sketch, but at the same time the sketch can be interactively unfolded into a detailed metric diagram, which causes students to change their focus from data testing to choosing the necessary data (Naftaliev and Yerushalmy 2011). Our findings (Naftaliev and Yerushalmy 2011, 2013, 2017) provide evidence that the multiplicity of viewing shapes mathematical activity and thinking, enables the students (1) to seize the qualitative properties of objects and relationships; (2) to highlight important elements; (3) to "picture" their thinking (Siegel 1995) (for example, dynamics of mouse tracing in ID sketch helped students start to consider the idea of rate); and (4) to invent or to generate their own fuller story. This pattern resembles the process advocated by Leonardo da Vinci: "the use of untidy indeterminacies for working out composition, because he believed that they stimulated visual invention regardless of the subject" (Fish and Scrivener 1990, p. 117). The processes of students inventing their own fuller story were supported and stimulated by IDs' design.

14.2.4 Organizational Functions of IDs

The *organizational* function refers to the connection among all the components of the ID: representations, tools, examples, etc. IDs can be organized in three ways: illustrating, elaborating, and guiding. The three types differ in their settings, each characterized by its own constraints and resources, and intended for a different aspect of inquiry.

Illustrating IDs are simply-operated, unsophisticated representations. For example: the ID allowed only viewing of the given example and permitted only a limited degree of intervention such as providing the values of ordered pairs for any point on the plane (Table 14.2). We found that even the minimal interaction designed in the illustrating ID can be helpful in consolidating relevant knowledge that is not adequately structured yet (e.g., Naftaliev and Yerushalmy 2011, 2017). Students who

Table 14.2 Comparative view of the three settings of IDs based on the organizational, presentational, and orientational functions

The task: write an expression describing the given line graph containing the two points marked on the graph			
		 x g g1 g2 g3 -1 -19 -0.5 -17 0 -15 0 -15	 f(x)=a(x-c)+m 1*(x-1)+1 ☐ a= 1 ☐ c= 1 ☐ m= 1
Organizational functions	Illustrating ID	Elaborating ID	Guiding ID
Presentational functions	Specific example; the design of the ID doesn't offer ways to generate examples	Specific example; the design of the ID offers ways to generate multiple and varied examples	Specific example; the design of the ID offers ways to systematically generate varied examples
Orientational functions	Schematic and/or metric; the design of the ID provides the values of ordered pairs for any point on the plane		

worked with the ID looked for ways to bypass the designed constraints: they changed the representation of the data in the given example and expanded the given representations or built new ones. The students who could not change the representation of the data in the given example or construct their own representations and tools were not able to proceed with the activity and to complete it.

Elaborating IDs provide the means that students may need to engage in activities that lead to the formulation of a solution and to operate at a meta-cognitive level. For example, the same graph that serves as an illustrating ID can be part of an elaborating ID when set within other tools and representations such as the option of typing any function expression in any structure (Table 14.2). Linked graphic representations and a table of values provided interactive feedback. The various linking tools and representations in the elaborating IDs lead to different problem-solving processes and a variety of solutions. Students did not always take advantage of the wide variety of options and representations available.

We use the term *Guiding IDs* in relation to guided inquiry. This kind of diagram provides the means for students to explore new ideas. In addition to providing resources that promote inquiry, they also set the boundaries and provide a framework for the process of working with the task. The *Guiding IDs* are designed to call for action in a specific way that supports the construction of the principal ideas of the activity and may serve to balance constraints and open-ended explorations and support autonomous inquiry. For example the guiding ID in Table 14.2 was designed to lead to a solution through the use of specific tools: changing the values of parameters in the given parametric expression of a linear function in the form $f(x) = a(x - c) + m$, where a describes the slope and (c, m) are the coordinates of a marked point located on the function graph; and an additional line graph that reflects the change in the graphic representation resulting from the parameters' changes. The design of this diagram attempts to provide a setting in which to consider the general form of a linear symbolic expression and to enable learning by comparison. We found that guiding IDs can be a form of instruction toward development of new scientific concepts (Naftaliev and Yerushalmy 2011, 2013).

Curricular resources are part of teacher learning and not meant to be used by students directly and independently of the teacher (Remillard and Bryans 2004). The teacher–students-curriculum interaction is characterized by negotiation, in which the teacher' students and the curriculum shapes and is being shaped by each other through the teacher's and students' experience. The main purpose of teachers' interactions with curriculum material is to guide students' learning of subject matter during instruction processes. Across our studies we had found that similar tasks with different IDs should be considered as different learning settings (e.g., Naftaliev 2012; Naftaliev and Yerushalmy 2011, 2013, 2017). The elements of the framework were valuable in explaining students learning with various IDs in different contexts of algebra tasks. They were also valuable in guiding the design of new instructional resources, and it remains to be explored whether this framework is valuable and productive as a guide for teaching processes and as a tool for teachers' professional development. It requires examination of the interactions between interactive materials, instructional practices of teachers and students' experiences in particular educational situations. In such interactions the teachers play an important role drawing on their knowledge.

Studies of the last years indicate that integration of technological interactive curriculum resources, and changes in mathematics teaching associated with these resources are challenging processes for a teacher (e.g. Hoyles and Lagrange 2010; Clark-Wilson et al. 2014; Trouche et al. 2012). Reasons for the teachers' reluctance to use the new approach include (a) teachers' proficiencies in mathematics and their perceptions of the nature of mathematical knowledge and how it should be learned with technology; (b) their understandings of the pedagogical principles required to teach mathematics with interactive resources (e.g. Pepin et al. 2013; Monaghan and Trouche 2016). Studies based on extensive research populations have accumulated vast knowledge on mathematics teachers education (see e.g. Rösken-Winter et al. 2015), though little of this knowledge is related to the design of teachers education programs involving learning technologies uses, and little is known on the impact of such programs on teachers' professional education. The goal of our research was to look closely at PTs' interactions with curriculum interactive materials (IDs) and to understand the affordance of the IDs' design, which was conducted within the semiotic framework, on the different stages of the interactions.

14.3 Methodology

This study implemented LessonSketch media-rich environment (https://www.lessonsketch.org). LessonSketch allows creating experiences around classroom scenarios performed with cartoon characters in the form of a slide show. "The range of controls available to comics makes them especially useful for representing scenarios that seldom exist" (Herbst et al. 2011, p. 94). These were particularly important for our study where we have dealt with innovation materials in student-curriculum-teacher interactions.

Depicting the interactions by using the comic (Fig. 14.2) allows us to present it for the PTs' interpretation and allows the PTs "to project their own circumstances onto an interaction, to share their own perspectives" (Chazan and Herbst 2011, p. 12). The environment played two roles in our research: as a professional development tool and as a research tool that allowed us to examine the PTs' interactions with the IDs.

We used the semiotic framework of pedagogical functions of interactive texts to design the IDs in the study. Three ID settings were designed around the same linear function task (Table 14.2). The IDs' design was conducted within the three organizational functions: illustrating, elaborating, and guiding.

Twenty-five PTs participated in the study. They were involved in three stages of interactions (Table 14.3, Figs. 14.3, 14.4 and 14.5) with the IDs. The stages were designed according to the following stages of teachers' interactions with the curriculum materials (Ball and Cohen 1996; Remillard 2005; Stein et al. 2007): (a) Intended curriculum refers to the teachers' plans for instruction with the

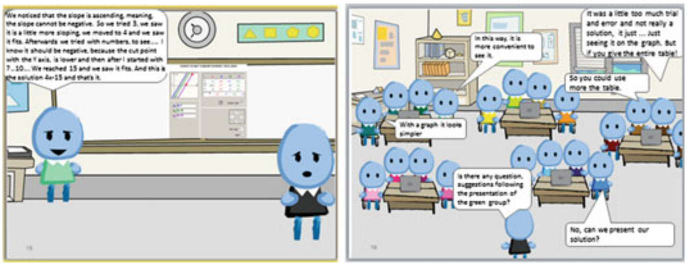


Fig. 14.2 Fragment of the comic presenting the classroom scenario

Table 14.3 Three-step research process

Step	Purpose	Materials and tools
1. Developing intended curriculum	Analyzing interpretation of designed IDs' functions in the intended curriculum	Paper task and three versions of the same task distinguished by the type of ID
2. Analyzing experienced curriculum	Learning about PTs' awareness of the impact the curriculum materials have on student learning	LessonSketch environment: the comic which presents the classroom scenario demonstrating experiences with the IDs
3. Representing enacted curriculum	Learning the imagined, enacted curriculum that is presented by comics	The IDs and LessonSketch environments: design instruction and classroom scenario with the curriculum materials for the class; representation of the scenario by comic

materials; (b) Enacted curriculum refers to the curriculum as implemented in classrooms; (c) Experienced curriculum describes the impact the enacted curriculum has on students. The PTs were involved in the next three steps of the research process: (a) developing teaching plans based on similar IDs distinguished by the designed semiotic functions (developing Intended Curriculum); (b) analyzing classroom scenarios demonstrating interactions with the IDs (analyzing Enacted and Experienced Curriculum); and (c) developing representations of scenarios about classes engaged with the IDs (imagination of Enacted and Experienced Curriculum). The relevant dimensions of the interactions are highlighted in Figs. 14.3, 14.4 and 14.5.

At the first stage of the interactions (Fig. 14.3) the PTs were asked to get familiar with each version of the linear function task, to consider which purpose each version can be applied to and to develop a teaching plan using one or more versions of the task.

At the second stage (Fig. 14.4) the PTs analyzed the classroom scenario presented by a comic, the scenario included representation of students' engagements

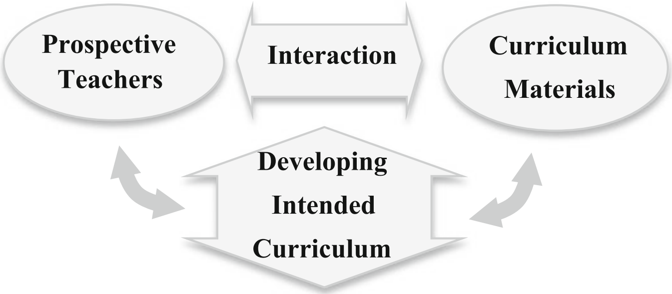


Fig. 14.3 The first step of the research process

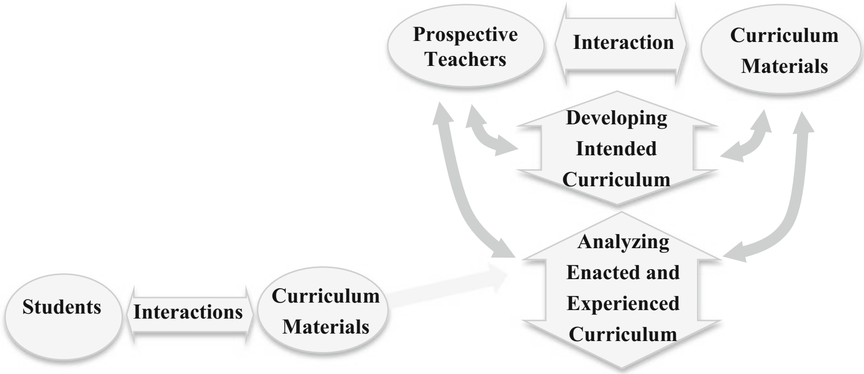


Fig. 14.4 The second step of the research process

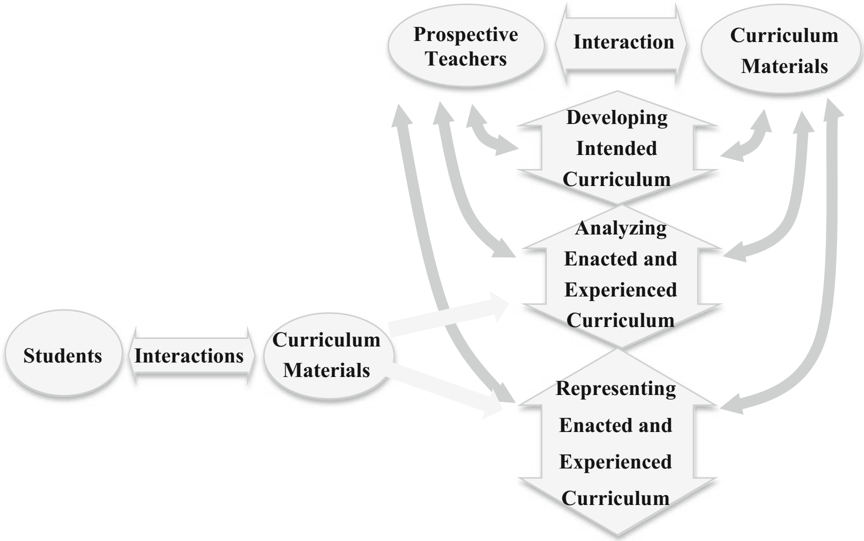


Fig. 14.5 The third step of the research process

with the tasks presented by each ID. The scenario was built on the basis of the data gathered in our previous studies on students-IDs interactions (e.g. Naftaliev and Yerushalmy 2011).

At the third stage (Fig. 14.4) the PTs developed their own classroom scenarios which were represented by comics. The scenarios presented the PTs' imagined continuous development of the situation which was analyzed by them in the second stage.

The data in the research included the materials developed by the PTs during each interaction stage. We analyzed the data from the two perspectives: mathematical and pedagogical. In the next section we will illustrate the analyses of the materials from each stage of interactions using the lenses of the perspectives.

14.4 Results

14.4.1 Developing Intended Curriculum

Analyzing the PTs' developing intended curriculums we can see that designed functions of the IDs were transparent in the PTs' work. For example, one of the plans designed by the PTs:

"Part 1 (with Illustrating ID):	Work with a specific example in order to calculate a slope by the two points given in the example and to write an expression of the function.
Part 2 (with Elaborating ID):	Construct collections of examples and generalize the links between the representations. We will demonstrate 3 functions along with the demonstration of our given red line function. We will change each one of the 3 functions and then see how the change affects the transformation of the function graph and changes of values of y according to the given x values in the table.
Part 3 (with Guiding ID):	Learn what m and c symbolize in the given linear function in a new form of expression $f(x) = a(x - c) + m$ by systemic work with the parameters: We will enlarge 'a' until there is a slope of 4, which is the slope of the given red function line (calculated in part 1). 'c' is the value of the x coordinate of the blue point C(1,1) on the blue line. The more we enlarge it then the x coordinate values will grow larger and the line will move right and closer to the red line until we reach the value of x coordinate of the points A(1,1) on the red line. 'm' is the value of y coordinate of the point C on

the blue line. It exactly equals 1 just like the value *y* coordinate of the red point A(4,1) of the line and therefore there is no need to change it."

At the first step of the plan the PTs used the Illustrating ID for the presentation of new activity and for solving it by writing the linear function expression of the specific given example. The second step included open inquiry with the elaborating ID. The focus of the second step was to build a collection of examples and to examine the meaning of the parameters in the expression and their connection with the characteristics of the functions. The third step defined for systematic inquiry of the new form of linear function expression in a process of systematic change and comparison. In the three steps the PTs were suggesting various paths for using the three versions of the same interactive task in mathematical outcomes. The similarity in the steps was that according to their plans the PTs were suggesting that each of the described processes would be done and demonstrated by the teacher in the class. So they expressed themselves in the plan's example above as in the plans of the other PTs: "We will demonstrate…"; "We will change…"; "We will enlarge…" and so on. The students were assigned a passive role as the plan did not present the intentions of the PTs to allow the students in the class to interact with the IDs. Analyzing mathematics in the PTs' plans, we can see that the PTs were open to the non-conventional mathematical ideas in planning their teaching. They suggested not only using a known algorithm to write the familiar form of linear function expression, but also using guiding ID for systematic inquiry of the new form of linear function expression. As a result, they used the new expression form to solve the task. The developing of the ideas followed the PTs' interactions with the IDs and was afforded by the IDs' design.

14.4.2 Analyzing Enacted and Experienced Curriculum

While analyzing the scenario of the classroom situation, the PTs got involved with the ways of students' engagement with the IDs. The PTs discussed the different ways of the students' solving processes and analyzed the students' mathematical thinking. The scenario presented processes from the data gathered in our previous studies on students-IDs interactions (e.g. Naftaliev and Yerushalmy 2011). With the illustrating ID the students in the scenario followed the changes of the coordinates along the line and organized values of consecutive integer coordinates in a table on paper. They calculated the differences between the values in the table and the ratio between the differences to find the slope. To obtain the constant term they estimated where the line would cross the *y*-axis and suggested an approximate solution. The PTs' feedback to the process was: "The students did not solve the task correctly, I cannot understand their way of looking for *b* [constant term in expression $y = ax + b$]"; "It is evident that some students did not understand the meaning of the learning materials, did not find an exact solution, but only found an approximate solution."

With the elaborating ID the students in the scenario used the ID to compare the given example with the new examples they created using the ID while searching for the right expression. When they recognized a visual characteristic, such as parallelization, relative tilting, or relative Y-intersection values, they started working systematically. The students determined the roles of the expression's parameters concerning the characteristics of the graph. Some of the PTs' feedbacks on the described process were: "We saw that the students presented their conclusions based on trial and error strategies and not on mathematical knowledge"; "Attempts were made to guess the function and not to reach a mathematical way of solving the problem."

The students in the scenario used the guiding ID for systematic inquiry by following the parameter changes in the given expression form, which was new to them. In the process of changing the parameters, students investigated the effects of changes in the new forms of the expression on the graph and reached a generalization. Some of the PTs responded by asking the question concerning the process: "According to the curriculum, do we need to teach the new linear functions form expression which the students found (about the form $f(x) = a(x - c) + m$)?"

To summarize, the PTs were challenged to deal with the students' ways of developing mathematical ideas while engaging with the IDs. Some of those processes in the presented scenario were new for them and the PTs wondered whether the processes reflected what the students should learn according to the curriculum. The additional challenge for the PTs was to be able to deal with students' knowledge which was developed while engaging with the IDs: approximating solutions as a possible way of solving math problems; a trial and error method as one of the problem solving methods allowed in math; developing knowledge about a new form of algebraic expression which is not defined by curriculum.

14.4.3 Representing Enacted and Experienced Curriculum

We learned about the PTs' imagined, enacted and experienced curriculum from their presentations of classroom scenarios by comics. The PTs were directors of the scenario which allowed us to learn about how they saw the enacted and experienced curriculum. On one hand, we noted that some of the PTs focused on the different pedagogical functions of the IDs in their scenarios. The PTs' comics showed various processes of the classes' engagement with the different versions of the same task as well as the role of the teachers in the scenarios as promoting such processes. On the other hand, some PTs didn't base their teaching instructions on either of the interactive possibilities provided by the IDs or the students' ideas developed with the IDs, but rather used conventional instructional practices (e.g. saying facts and presenting algorithms) with facts taken from Wikipedia and put in teachers' scripted speech for the scenario they developed.

14.5 Discussion

We analyzed PTs' interactions with the IDs and the affordance of the IDs' design on their instructional practices in the different stages of the interactions. Our findings include evidence that the PTs were aware of the different IDs' functions in the three stages of the interactions with the materials. They developed teaching plans that included each of the similar IDs in different steps of the plans and defined different teaching-learning goals for each ID. The PTs used the Illustrating ID only for the presentation of a new activity around the specific given example. They suggested open inquiry with the Elaborating ID to develop generic example for the familiar form of linear function expression. The Guiding ID was used for systematic inquiry of the new form of linear function expression. The PTs were also aware of different learning outcomes of the students with the different IDs in the classroom scenario. So the semiotic framework for pedagogical functionality of IDs was productive as a guide in the PTs interactions with the IDs and may serve as a tool for teachers' professional development. But at the same time the PTs did not always take an advantage of the wide variety of options for teaching-learning processes available with the IDs and some of them preferred the well-trodden paths in teaching. The PTs faced a number of challenges while interacting with the materials. One of the challenges was correspondence of the IDs' orientations to those of the PTs. The PTs were aware of various pedagogical possibilities of the IDs but some of them continue to use conventional instructional practices in their interactions with the materials. Another challenge the PTs dealt with involved discussing a balance between what the students learnt by engaging in the variety of interactions that the IDs offer and what the students should learn according to the teachers' learning goals. The third challenge was that the PTs had difficulties dealing with students' knowledge developed while engaging with the IDs and designing teaching learning processes to help them progress.

The literature describes that teachers have a mode of interactions with textbooks, which have developed along their use of various textbooks, and conduct their using of a new textbook, independently of the features of this textbook (e.g. Pepin et al. 2013). Moreover, an important condition for adoption of curriculum materials is its potential integration into the teacher's 'normal' practice. Teaching with an interactive textbook should be considered more than a technological change; indeed, it is an attempt to create new paths for the construction of mathematical meaning. The teaching-learning processes with interactive textbooks aim to create the opportunity for students to acquire the sense of the discipline, to get experience with mathematical objects and problem solving. The teachers' decisions should encompass the students' ideas as the teachers try to steer the students' progression toward the intended learning goals. The multiplicity of students contributions, the new ideas developed by them with the interactive materials that diverge significantly from the teachers' learning goals or from the standards required by the curriculum, as well as the expectation of the teacher to build on students' ideas during the lesson, can place increased demands on the teacher for improvisation and in-the-moment decision making (Stylianides and Stylianides 2014). Developing the new practices and experiences in teaching-learning with interactive textbooks is a necessary condition for PTs' implementation of these sources in their future teaching. The PTs in the study were involved in developing new practices and experiences during their interactions with the IDs and the students' outcomes. The three steps procedure of the interactions served as an intermediate stage between what normally may happen in the classroom according the initial PTs' orientations (beliefs, values and preferences) and the implementation of the innovative practices in their future teaching with the new kind of materials. The semiotic framework for pedagogical functionality of IDs and the three steps procedure enabled facilitating the PTs' design processes, to share, to discuss and to modify their decisions.

The present study is a pilot of a larger attempt to provide practical understanding of possible ways for teachers' and PTs' supporting the development of new practices for teaching- learning processes with the new kind of materials. In the attempt to connect theory and practice, we expect the results of this study to support in-service teachers' and PTs' professional development, especially in the area of pedagogical and curricular content knowledge, and to help further studies concerning teachers' ways of interactions with the materials. Studying such attempts is becoming increasingly important, as the feasibility of using interactive textbooks and similar resources is on the rise.

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Chapter 15
Teacher Decisions on Lesson Sequence and Their Impact on Opportunities for Students to Learn

Ok-Kyeong Kim

Abstract When using existing resources to plan and enact a series of lessons, teachers make various decisions, one of which is whether to follow or modify the sequence of tasks and lessons presented in the resources. One important question to ask is how teacher decisions on lesson sequence affect the quality of instruction and opportunities for students to learn. I examined ways in which teachers, using three different curriculum programs, sequenced tasks and lessons, and whether these sequences provided opportunities for students to engage with mathematical points of the lessons and a mathematical storyline through a proper learning pathway. Findings of the study have implications for teaching, teacher education, and curriculum development.

Keywords Mathematics teachers' resources · Curriculum · Teacher decision
Lesson sequence · Opportunities to learn

15.1 Introduction

When using existing curriculum recourses, teachers make a range of decisions for various reasons. One of the decisions teachers make is whether to use various elements in the resources and how to use them. Such decisions can influence lesson enactment significantly (Kim 2015; Kim and Atanga 2013, 2014). In this study, I examined ways in which teachers use existing curriculum resources to sequence tasks or activities within and across lessons and their impact on opportunities for students to learn important mathematical ideas and concepts.

Curriculum designers have specific intentions and mathematical goals to achieve through a series of lessons. These intentions are communicated through various kinds of support for teachers regarding how to enact the tasks and lessons, including

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a unit overview that provides the sequence of the tasks and mathematical ideas and concepts, and how these are connected to each other and developed across lessons. Moreover, each lesson outlines what is expected to take place so that students' learning of mathematical ideas and concepts can progress in a series of components of the lesson. This sequence of lessons and components provides a *curricular trajectory* to project the progression of a set of related mathematical ideas and concepts in student learning (Sleep 2009).

Teachers decide whether to follow or modify the sequence of components of the lessons provided in the curriculum. Such teacher decisions indicate various possible adaptations teachers can make as they use written lessons to design instruction. One important question to ask is how such decisions impact the quality of enacted lessons, or the quality of the transformation from the written to the enacted, and shape opportunities for students to learn the mathematics they are supposed to. Whereas teachers can improve student learning by making alternations in the sequence provided in the curriculum, modifying the sequence of tasks and lessons may be critical to the quality of the enacted lessons and student learning. Whether following or modifying the sequence in the resources, teachers need to make a well-developed plan for a proper trajectory for student learning.

15.2 Theoretical Perspectives

15.2.1 Teachers' Reasoning with Curriculum Resources

Researchers view that teachers actively engage in curriculum design through interactions with the curriculum resources that they use, rather than passively following them (e.g., Remillard 2005). When using existing resources to teach mathematics, teachers read, evaluate, and adapt the resources, and their reading and evaluation lead to various adaptations (Sherin and Drake 2009). Brown (2009) uses the notion of *pedagogical design capacity* (PDC) to explain the teacher capacity needed for productive curriculum use that helps achieve instructional goals. According to him, PDC is "a teacher's capacity to perceive and mobilize existing resources in order to craft instructional episodes" and "a teacher's skill in perceiving affordances [of the resources], making decisions, and following through on plans" (Brown 2009, p. 29). I argue that teachers are engaged in significant reasoning with curriculum resources in the process of reading and making sense of the resources, recognizing the affordances, and making decisions about what to use and how to use.

Researchers have articulated teacher knowledge actually used in teaching. Based on the assumption that teachers use any form of resources to teach a lesson and the view that teaching is a process of reasoning, Shulman (1987) elaborated aspects of *pedagogical reasoning and action*, which includes a cycle of comprehension, transformation, instruction, evaluation, and reflection. Following Shulman's approach to teaching and teacher knowledge, Rowland and his colleagues (Rowland 2013; Rowland et al. 2005) proposed *knowledge quartet* with a set of units (i.e., foundation, transformation, connection, and contingency) to describe ways in which teachers draw on their knowledge. Whereas the first unit describes knowledge base or propositional knowledge, the other three indicate situations in which teachers draw on various forms of knowledge to make instructional decisions. Also, Remillard and Kim (2017) conceptualized *Knowledge of Curriculum Embedded Mathematics* (KCEM, the mathematics knowledge activated by teachers when reading, interpreting, using mathematical tasks, instructional designs and representations in mathematics curriculum materials) to articulate the kind of knowledge teachers need to draw on in order to make sense of the mathematics presented in the written lessons to design instruction, and proposed four dimensions of KCEM: foundational mathematical ideas, representations and connections among these ideas, relative problem complexity, and mathematical learning pathways. All of the three notions mentioned above (i.e., *pedagogical reasoning and action*, *knowledge quartet*, and *KCEM*) illuminate the significance of teachers' reasoning with curriculum resources in designing instruction.

15.2.2 Mathematical Storyline and Lesson Sequence

Curriculum resources provide tasks and activities to support students' learning of mathematical points, and a proposed learning trajectory in their lessons, which can eventually help develop a coherent mathematical storyline—"a deliberate progression of mathematical ideas" (Sleep 2012, p. 954)—in a series of lessons. Individual tasks, lessons, and chapters are organized into a sequence to develop students' understanding of mathematical concepts and ideas, and build a mathematical storyline around a topic and across topics. Teacher decisions on whether to follow or modify the sequence in the curriculum can affect students' learning of mathematical points and the development of a mathematical storyline in the course of lessons.

In Shulman's notion of *pedagogical reasoning and action*, comprehending purposes and subject matter structures, and transforming them for students to learn are closely related to teachers' decision on the sequence of activities and lessons. Rowland and his colleagues (Rowland 2013; Rowland et al. 2005) also emphasized that teachers need to understand the mathematics that they teach and make necessary connections to design instruction. In particular, "Within a single lesson, or across a series of lessons, the teacher unifies the subject matter and draws out coherence" (Rowland et al. 2005, p. 265). Proposing principles for using curriculum in preservice teacher education, Drake et al. (2014) emphasized that teachers need to "examine multiple lessons and units in order to identify and understand the development of content over time" (p. 159). In addition, Sleep (2009, 2012) elaborated teachers' work of articulating learning goals of activities and lessons at the micro and macro levels to understand how they are connected and enact the activities and lessons toward the goals.

Two of the dimensions of KCEM, *relative problem complexity* and *mathematical learning pathways* are directly related to sequencing tasks and activities within and across lessons (Remillard and Kim 2017). Teachers need to carefully examine the proposed trajectory for student learning of the mathematical points in a series of lessons and how various tasks and activities within and across lessons support students' development of the mathematics in the lessons. Teachers certainly can decide to add new elements or omit existing components of the lesson to design instruction in order to better support the anticipated learning trajectory. Before making a decision, however, they need to examine whether the alterations affect the students' learning trajectory and, if so, whether they can enhance student learning through the revised learning trajectory. Remillard and Kim argue that "when using curriculum materials, being able to recognize learning pathways and their goals at different levels of focus allows teachers to find themselves at any moment on a broader curriculum map."

Using terms such as *mathematical purposing* and *focusing*, Sleep (2009) elaborated the complexity of teaching to the mathematical point, for which teachers have to attend to multiple learning goals and intentionally scrutinize a curricular trajectory in relation to a mathematical trajectory. Placing the importance and necessity of articulating the mathematical point from both mathematical and instructional perspectives, she described mathematical point as "a connected package of mathematical goals and instructional purposes, with depth and weight and time" and teachers' work of articulating the mathematical point as "articulating the intended mathematics and how the instructional activity is designed to engage students with it" (p. 14). She highlighted teaching for coherence, connections, and learning progression in the trajectory. Although there are other issues influencing the development of a mathematical storyline (Sleep 2009), properly sequencing activities within individual lessons and across lessons seems to be the starting point to develop a coherent mathematical storyline.

In this study, I define mathematical point as eventual goal(s) to achieve in the lesson (s), which may or may not be stated explicitly in the written lessons. Different curriculum programs use different terms (e.g., objectives, focus points, and math content) to indicate these goals, but reading the entire lesson including activities and guidance for teaching (i.e., articulating mathematical point) may illuminate something fundamental for student learning, but not explicitly stated as a goal or objective.

15.2.3 Teacher Decisions and Opportunities for Students to Learn

The National Research Council (NRC 2001) points out that opportunity to learn (OTL) is "the single important predictor of student achievement" (p. 334). Hiebert and Grouws (2007) explain that OTL depends on both teacher and curriculum materials. They further argue that creating moments in classrooms where students learn goes beyond exposing them to subject matter and learning goals. Stein et al. (2007) argue that curriculum materials can influence students' learning, as they contain different types of mathematical tasks that require various student engagement with the mathematics content embedded in them. They may also contain a well-developed sequence of tasks and lessons to support student learning. However, whether the tasks and the sequence are used as intended depends on the teacher. This may indicate that even though both curriculum materials and teachers are significant in creating opportunities for students to learn, the teacher's role seems even more critical. Many elements that can help create opportunities for the student to learn may be present in curriculum resources, and yet they may be inert if not deliberately pursued by the teacher (Kim 2015).

Teachers need to recognize the affordances of the resources they use in order to make a proper instructional decision (Brown 2009). It was observed that when teachers were not able to notice such affordances and made a poor decision, they created students' difficulty with learning the mathematical points of individual and multiple lessons (Kim 2015; Kim and Atanga 2014). For example, whereas the written lesson includes helpful intervention suggestions for struggling learners, the teacher, not using them, mainly repeated the same procedural explanations to the students in confusion (Kim 2015). Son and Kim (2015) also reported that two teachers enacted the same lessons from an inquiry-based curriculum program quite differently, which resulted in dissimilar learning opportunities for students. In enacting the lessons, the two teachers basically asked questions provoking different kinds of student thinking. Their goals of the tasks were different and one of the teachers failed to articulate the mathematical point of the tasks.

In this study, drawing on data from classroom teachers using curriculum programs with either a directing-teaching or student exploration model, I examine teachers' adaptions of lesson sequences and their impact on student learning. Research questions of the study are:

1. In what ways do teachers sequence lessons and activities from the existing resources?
2. What are the impacts of such decisions on opportunities for students to engage with the mathematical points and mathematical storyline of the lessons?

First, comparing and contrasting the sequence of written lessons with that of corresponding enacted lessons, I explored ways in which the participant teachers sequenced tasks and lessons from the curriculum resources they used. Then, I examined whether the sequence of the enacted lessons supported and enhanced opportunities for students to experience the mathematical points of the lessons and progress through a coherent learning pathway. The details of the methods are described below.

15.3 Methods

This study is part of a larger project investigating elementary teachers' use of mathematics curriculum resources in the United States, the *Improving Curriculum Use for Better Teaching* (ICUBiT) Project.

15.3.1 Participant Teachers and Curriculum Programs

I drew on data from 11 teachers in grades 3–5 who used three different curriculum programs (four, three, and four in each program, respectively). The participant teachers had at least three years of teaching experience (ranging from 3 to 25 years) and at least two years of using the current curriculum program (ranging from 2 to 14 years). One of the three programs, Scott Foresman–Addison Wesley *Mathematics* (Charles et al. 2008) was a traditional curriculum program with a direct-teaching model, which was commercially developed. One other program, *Math in Focus* (Singapore Ministry of Education/Marshall Cavendish International 2008), was also based on a direct-teaching model, but it emphasized conceptual foundations along with representations throughout the lessons. This program was developed in Singapore and had gained popularity in the United States. The lessons of the two programs with a direct-teaching model typically had components of teacher explanation/demonstration and student practice. Finally, a third program, *Investigations in Number*, *Data*, *and Space*, was a reform-oriented one with a student exploration model, primarily based on the recommendations by the National Council of Teachers of Mathematics (1989, 1991, 2000). The lessons of this program typically included components of group/pair work and whole group discussion after student work.

15.3.2 Data Sources

The data used in this study include Curriculum Reading Logs (CRLs), classroom observations data (videotapes, transcripts, and field notes), and teacher interviews (introductory and post-observation). Each participant teacher completed CRLs for a set of lessons that were observed; on a copy of the written lessons, using different colored highlighters, the teacher indicated which parts he/she read as he/she planned instruction, which parts he/she planned to use, and which parts that influenced his/her planning. CRLs helped me see teachers' plans for instruction and compare written and enacted lessons. Each teacher was observed for three consecutive lessons in each of two rounds. These enacted lessons were videotaped and transcribed. Also, each teacher was asked questions about his/her teaching experience and overall curriculum use at the beginning of the study, and then asked about specific teacher decisions in the observed lessons after each round of three observations. These interviews were audiotaped and transcribed.

15.3.3 Data Analysis

Data analysis began with identifying the sequence of the written lessons along with mathematical points (MPs) and the mathematical storyline. Sleep (2009) provided detailed examples of classroom episodes along with specific MPs she identified from written texts. I followed a similar process, but focusing on the development of MPs and mathematical storyline rather than individual MPs. By reading the entire individual lessons carefully, including objectives, key concepts, key ideas, tasks and activities, mathematical explanations, and instructional guidance, two researchers (including the author) identified the MPs within and across lessons and determined the proposed mathematical storyline in the sequence of multiple written lessons. In articulating the MPs for the purpose of analysis, the researchers attended to two separate but related aspects: conceptual and procedural goals. Next, I listed each teacher's sequence of tasks and lessons from the lessons observed, and compared the sequence of the written lessons with that of the enacted lessons. In comparison, the focus was given on whether the sequence from the observed lessons was significantly different from that of the written lessons in terms of the development of the MPs and mathematical storyline over the lessons, and if so, ways in which the sequence was modified. Then, I examined overall opportunities for students to engage with the MPs and mathematical storyline identified in the enacted lessons and whether the student learning opportunities were *enhanced* (better opportunities for student engagement with the MPs in the enacted mathematical storyline), *maintained* (the same level as in the written lessons or not much difference between written and enacted lessons in terms of student engagement with the MPs and mathematical storyline), or *reduced* (limited opportunities for student engagement with the MPs in the enacted mathematical storyline), compared to those proposed in the written lessons. Although single incidences, such as using an additional activity focusing on conceptual support to bridge a gap in student understanding, and omitting an important activity that is important in developing a proper mathematical storyline, were critical in the coding decision, the determination of *enhance*, *maintain*, or *reduce* was based on overall student learning opportunities in the course of the enacted lessons rather than discrete moments. Teacher interviews were analysed to see teachers' general approach to using their curriculum programs and their intentions and rationale for specific instructional decisions. These include explanations for why they omitted certain activities, added new elements, or made any other alterations to the proposed sequence. After examining individual teachers, I searched for patterns in teacher decisions on lesson sequence and their impact on lesson enactment within and across three curriculum programs. The patterns across programs were compared to account for characteristics of teacher decisions within each program. This was to search for a possible association between teachers' sequencing decision and the nature of the program.

15.4 Lesson Sequence, Mathematical Points, and Mathematical Storyline

In this section, I describe the participant teachers' sequencing of the lessons within each program in general and then two particular teachers' cases to illustrate specific ways they sequenced their lessons and how their sequences affected opportunities for students to learn mathematical points of the lessons and progress in the learning trajectory.

15.4.1 Patterns of Sequencing

Overall, teachers using the reform-based program, *Investigations in Number*, *Data*, *and Space*, made various decisions deviated from the curriculum in terms of sequencing lessons and activities. Teachers using the programs with a direct-teaching model showed different patterns; those who used Scott Foresman–Addison Wesley *Mathematics* seldom changed the sequence from the written lessons whereas those using *Math in Focus* altered the sequence of the written lessons significantly.

Four teachers using Scott Foresman–Addison Wesley *Mathematics* in this study added or omitted a short activity in a lesson occasionally, but this did not significantly alter the kind of opportunity for students to learn in terms of the content and the way they experienced the content. Lessons in Scott Foresman–Addison Wesley *Mathematics* had a typical format that included a short warm-up, teacher explanation of procedure or concept, and then a large set of practice problems for students. Often, the teachers omitted the warm-up activity but followed through the other two main parts of each lesson deliberately. Warm-up activities were composed of a small set of skill-based problems, omission of which seldom affected students' learning of the mathematics in the lesson because they were often not related to the main mathematics of the lesson. For example, the warm-up in a lesson whose objective was "tell time to the nearest 1 minute or 5 minutes using analog and digital clock, and identify times as A.M. or P.M." is "Write the number that is ten more than each number. 24, 56, 32, 98" (Charles et al. 2008, p. 190).

The four teachers also followed lessons as sequenced in the curriculum. Each lesson in this program had a narrow, focused content in a step-by-step order. For example, titles of lessons on division in grade 4 were as follows: dividing with remainders, two-digit quotients, dividing two-digit numbers, interpreting remainders, dividing three-digit numbers, zeros in the quotient, and the like. Since the focus of the lesson was narrow and each lesson had limited components (i.e., mainly teacher demonstration and student practice), teachers had little room to change the sequence of components and lessons, although they could have added student exploration or discussion, or combined lessons, such as those for "dividing with remainders" and "interpreting remainders." Mostly, using Scott Foresman–Addison Wesley *Mathematics*, teachers determined what to show and explain, and then what problems to assign to students. No significant modification was evident in their sequence of lessons and components and mathematical storyline. As a result, the opportunities for students to learn the mathematical points and their progression in the projected learning pathway mainly remained the same as in the written lessons. Overall, conceptual aspects of the mathematical points were largely missing in enacted lessons, as these were not explicitly pursued in the written lessons.

Three teachers using the other program with a direct-teaching model, *Math in Focus*, had one additional lesson component to enact, compared to those using Scott Foresman–Addison Wesley *Mathematics.* As mentioned earlier, lessons in *Math in Focus* usually included a specific, explicit, deliberate component for conceptual foundation, which unpacks the mathematical concepts and ideas to be used in the subsequent procedural tasks and problems. Scrutinizing lessons from Scott Foresman–Addison Wesley *Mathematics* revealed that this program also had the potential and the necessity for such conceptual foundation, but that was not explicit in lesson components; especially, explicit students' engagement in such conceptual foundation was usually not expected in the lesson segment. In contrast, lessons from *Math in Focus* began with conceptual foundation and then moved to procedures that students need to follow and practice. Therefore, enacting this component is critical in student learning because it affects students' learning of the mathematical point and progression in the leaning pathway.

The three teachers using *Math in Focus*, however, dismissed lesson components for conceptual foundation in teaching their lessons. Two of the teachers explained procedures step by step, mostly using the practice problems only. They did not use base-ten blocks to illustrate multiplication or division, although these materials were explicitly used in the written lessons. Whereas the written lesson attended to place value in division (e.g., $810 \div 9 = 81$ tens $\div 9 = 9$ tens $= 90$), one teacher constantly made a comment, such as "add a zero at the end," without using suggested terms including tens and hundreds. The other teacher asked many questions about "why" and attempted to support students' understanding, but still without conceptual foundation components, she limited opportunities for students to make sense of the procedures they went through and do subsequent problems with meaning. In sum, the three teachers did not utilize the affordances of the written lessons (i.e., the conceptual foundation components) and focused on the practice problems for the procedures students were asked to do to find answers. The two teachers who were giving step-by-step explanations of procedures throughout the lessons did pick and choose practice problems from the written lessons and reorganized the lessons around the procedure practice. Their sequence of the lesson components was primarily related to what problems to provide and in what order. They also used practice problems outside the curriculum as they thought their students needed more practice with the procedures that they learned. They made the lessons even more teacher-centered than the original lessons.

Four teachers using the reform curriculum program, *Investigations in Number*, *Data*, *and Space*, adjusted the sequence provided in the curriculum significantly. The most common was omitting an activity or a lesson and combining multiple activities into one. Their rationale for the sequence change varied: redundancy or content similarity, student response, lack of time, assessment, and past experience. Apparently, the reform curriculum program placed a lot more demand on the teacher than the ones with a direct-teaching model. Although the lessons had usually two or three main activities, including individual or group work and whole-group discussion, the ways students were expected to work on the tasks/activities were not uniform in this program. Depending on the content explored and the representations or materials used, activity formats changed for student exploration. Some activities (e.g., a game as a choice for practice time) occurred in more than one lesson; also, often the same math focus points appeared in multiple lessons. For example, math focus points, such as "*finding fractional part of a rectangular area*," and "*identifying fraction and percent equivalents through reasoning about representations and known equivalents and relationships*," appear in several lessons on fractions, decimals, and percents in grade 5 (see Table 15.1 for more detail in the following section). These indicate that the foci of individual lessons were not as narrow as those in the other two programs with a direct-teaching model. Using this program required teachers to articulate mathematical points within and across lessons more carefully. When planning a lesson, however, the four teachers tended to focus on student pages for individual work to see the content of the lesson, which indicates that they prepared less for whole group activities and discussion that were important in the sequence of lessons and student learning through the anticipated trajectory.

The ways in which the teachers in this study altered the sequence of lessons in the resources include:

- Omitting a lesson component (activity/task)
- Omitting an entire lesson
- Combining lesson components within and across lessons
- Adding a new component or lesson.

Switching the order of lesson components or lessons was not observed in these teachers' lessons although they could have chosen to do so. Omitting and combining components and lessons reduced opportunities for students to learn the mathematics of the lessons. In the remainder of the chapter, I focus on two teachers' cases to illustrate these various ways in which the teachers modified the sequence of the tasks and lessons in their curriculum program, the reasoning behind their decisions, and how their decisions influenced the articulation of mathematical points (MPs) in student learning pathways and the development of a mathematical storyline.

Table 15.1 Sequence of the written lessons (Becca)

Lesson (MP)	Math focus points	Components
1.1 Everyday uses of fractions, decimals, and percents (Students understand everyday use of fractions and percents, and find fractional parts of a whole or of a group and a percentage of a whole or a group.)	• Interpreting everyday uses of fractions, decimals, and percents • Finding fractional parts of a whole or of a group (of objects, people, and so on) • Finding a percent of a group (or objects, people, and so on)	A. Uses of fractions, decimals, and percents (In the whole class, the teacher leads a discussion in which students talk about fractions, decimals, and percents used in everyday situations and their relationships.) B. What do you already know? (Students work on problems that relate fractions and percents, which helps the teacher assess students' prior knowledge.) C. Fraction and percent problems (Students share how they solved the problems in B, focusing on 2–3 problems.)
1.2 Relating percents and fractions (Students understand equivalents are fractions, percents, and decimals that represent the same amount, and identify percent equivalents of fractions and fraction equivalents of percents.)	• Finding fractional parts of a whole or of a group (of objects, people, and so on) • Finding a percentage of a rectangular area • Identifying fraction and percent equivalents through reasoning about representations and known equivalents and relationships	A. Introducing guess my rule (Students use fractions and percents to write statements about a group of students in front of the class [e.g., 50% are wearing buttons], and identify the characteristic of the students given a fraction or percent.) B. Writing equivalent percents and fractions (In the whole class, the teacher leads a discussion on what 50% means and its fraction equivalents and then other percents and their fraction equivalents.) C. Grid patterns as percents and fractions (Given shaded grids, students determine the percent and fraction of the shaded portion of each grid.)
1.3 Finding percents of an area (Students understand how percents and fractions are related, and find percent equivalents of fourths and eighths, by using area representations of fourths and eighths, and what they know about fraction relationships and equivalents.)	• Finding fractional part of a rectangular area • Finding a percentage of a rectangular area • Identifying fraction and percent equivalents through reasoning about representations and known equivalents and relationships	A. Percents for fourths and eighths (In the whole class, students share how they shade 1/4 of a 10 × 10 grid and determine the equivalent fraction with a denominator of 100 and the percent. Then, students individually use grids to shade 2/4, 3/4, 1/8, and 3/8, and write a fraction

(continued)

Table 15.1 (continued)

Lesson (MP)	Math focus points	Components
		with 100 as the denominator and its equivalent percent for each number.) B. What percent is 3/8? (Students explain how they found 3/8 of a grid and how they knew it equaled 37 1/2%.) C. Fraction and percent equivalents (Students individually or in pairs record the percent equivalent for each fraction for halves, fourths, fifths, eighths, and tenths, and get ready to explain how they figured them out.)
1.4 Percent equivalents for thirds and sixths (Students understand relationships between percents and fractions, and use these relationships, known equivalents, and representations to determine fraction equivalents of thirds and sixths.)	• Finding fractional parts of a rectangular area • Identifying fraction and percent equivalents through reasoning about representations and known equivalents and relationships	A. Reasoning about fraction-percent equivalents (Students share how they found the percent equivalents for halves, fourths, fifths, eighths, and tenths.) B. Finding thirds and sixths (Students find percent equivalents for thirds and sixths and show they figured them out by using 10 × 10 grids.) C. What percent is 1/3? (Students share how they found 1/3 = 33 1/3% by using a grid, and then percent equivalents of 2/3, and sixths.)

15.4.2 A Case of Becca with *Investigations in Number, Data, and Space*

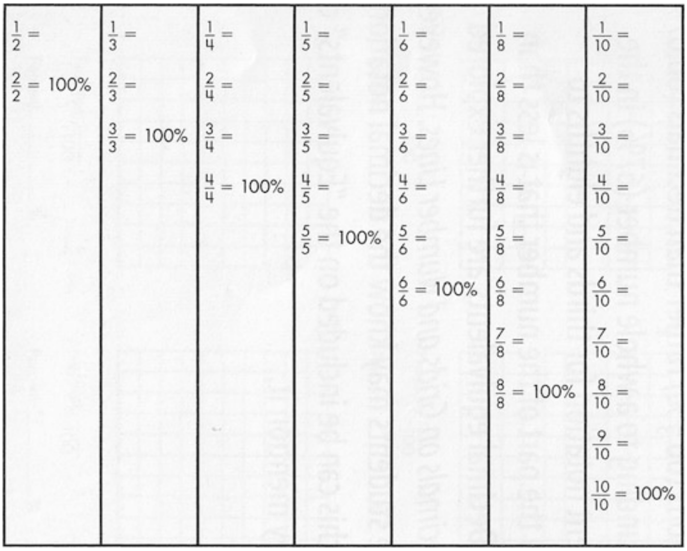
Becca had about 15 years of teaching experience and had used various curriculum programs. She had taught *Investigations in Number*, *Data*, *and Space* for 6–7 years by the time she was observed. She was confident in using the curriculum and had an established practice of using it. She also mentioned that using the curriculum helped her understand the mathematics she taught and made her gain confidence in teaching mathematics. Her sequence of the lessons, however, was far from articulating MPs of the lessons. Overall, her enacted lessons did not maximize opportunities for students to explore the MPs of the written lessons, let alone the mathematical storyline intended in the curriculum. In fact, Becca was the one who modified the sequence of tasks and lessons most drastically among the four teachers using the program in this study. She not only omitted tasks, but also added a new component and reorganized the tasks from multiple written lessons.

In the example described below, she taught three lessons on fractions, decimals, and percents by using four written lessons. Each written lesson was for 60 min, and all of the observed lessons lasted 60 min each as well. Table 15.1 presents details about the four written lessons, including specific lesson components and *Math Focus Points*, which is the term the curriculum program used to indicate objectives of lessons (TERC 2008). Please note that the content of the table is excerpted from a few pages of the curriculum, except for the MPs in the first column and the summary of lesson components in the last column in parentheses.

The very first written lesson (1.1) encourages students to think about everyday use of fractions, decimals, and percents. They review what they already know about fractions, decimals, and percents, and create a chart that lists how fractions, decimals, and percents are used in everyday situations. The second lesson (1.2) leads students to relate fractions, decimals, and percents, and introduces 10 × 10 grids, which represent fractions and percent equivalents (e.g., 1/2 = 50% = 3/6 = 10/20 = 25/50 = 50/100). Students also identify the percent and fraction of each 10 × 10 grid already shaded. The third lesson (1.3) has students use 10 × 10 grids to show fourths and eighths and find their percent equivalents. For this task, students use the area representation and what they know about relationships of fractions and equivalents to determine percent equivalents for fourths and eighths. The last lesson (1.4) finally extends to percent equivalents for thirds and sixths. Students discuss the fraction-percent equivalents they have found so far, find percent equivalents of thirds and sixths using the 10 × 10 grid, and explain the reasoning they used to find percent equivalents of thirds and sixths.

The MPs of the lessons are summarized in parentheses in the first column of Table 15.1. Examining the sequence of the lessons and their components reveals the progression of anticipated and projected student learning in the four written lessons. Students are expected to (1) activate their prior knowledge of fractions, decimals, and percents in the first lesson, (2) explore relationships among fractions, decimals, and percents with easy numbers, such as halves, fifths, and tenths, and start to use 10 × 10 grids in the second lesson, (3) extend to percent equivalents for fourths and eighths in the third lesson, and (4) finally move to percent equivalents for thirds and sixths. In this way, students could use what they know to develop a deeper understanding of relationships among fractions, decimals, and percents as lessons progress. By the end of the four lessons, students are expected to complete a chart for fraction and percent equivalents (see Fig. 15.1).

In contrast to the written lessons, Becca already asked students to use 10 × 10 grids to show percents in the first enacted lesson. In the subsequent lesson, she made students create and shade their own 10 × 10 grids and name the percents that the grids represented, by counting the number of shaded squares basically and



$\frac{1}{2}$ =	$\frac{1}{3}$ =	$\frac{1}{4}$ =	$\frac{1}{5}$ =	$\frac{1}{6}$ =	$\frac{1}{8}$ =	$\frac{1}{10}$ =
$\frac{2}{2}$ = 100%	$\frac{2}{3}$ =	$\frac{2}{4}$ =	$\frac{2}{5}$ =	$\frac{2}{6}$ =	$\frac{2}{8}$ =	$\frac{2}{10}$ =
	$\frac{3}{3}$ = 100%	$\frac{3}{4}$ =	$\frac{3}{5}$ =	$\frac{3}{6}$ =	$\frac{3}{8}$ =	$\frac{3}{10}$ =
		$\frac{4}{4}$ = 100%	$\frac{4}{5}$ =	$\frac{4}{6}$ =	$\frac{4}{8}$ =	$\frac{4}{10}$ =
			$\frac{5}{5}$ = 100%	$\frac{5}{6}$ =	$\frac{5}{8}$ =	$\frac{5}{10}$ =
				$\frac{6}{6}$ = 100%	$\frac{6}{8}$ =	$\frac{6}{10}$ =
					$\frac{7}{8}$ =	$\frac{7}{10}$ =
					$\frac{8}{8}$ = 100%	$\frac{8}{10}$ =
						$\frac{9}{10}$ =
						$\frac{10}{10}$ = 100%

Fig. 15.1 Fraction and percent equivalent chart

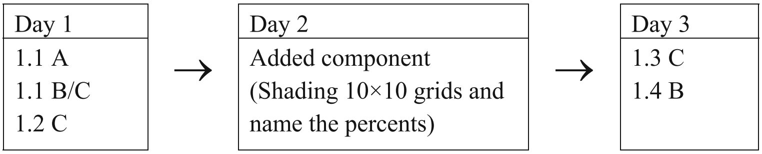


Fig. 15.2 Sequence of the observed lessons (Becca)

without necessarily relating percents with fractions. On the third observed lesson, the teacher had students find percent equivalents for fourths, eighths, thirds, and sixths all in one lesson. Figure 15.2 summarizes Becca's sequence of the lessons and components. She shortened components in Lesson 1.1 and squeezed in student work and sharing of Lesson 1.2 C to the first lesson. Then, she skipped Lesson 1.2 A and B; instead she did the grid shading activity in the second lesson. Again, she skipped most components of Lessons 1.3 and 1.4, and combined Lesson 1.3 C and Lesson 1.4 B in the last observed lesson. In other words, she combined the mathematical explorations of two lessons (Lessons 1.3 and 1.4) into one, in which she ended up showing the completed chart for fraction and percent equivalents of halves, thirds, fourths, fifths, sixths, eighths, and tenths at the end of the lesson because of a lack of time.

As she rushed through the lessons along with the added component on the second day, she limited opportunities for students to explore percent equivalents of fractions, such as 1/4, 1/8, 1/3, 1/6, and related fractions. The written lessons allotted one day for fourths and eighths, and another day for thirds and eighths,

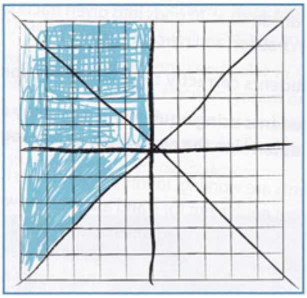


Fig. 15.3 Area representation of 3/8 on 10 × 10 grid

given the complexity of the fractions and their present equivalents. Students were expected to use 10 × 10 grids and relationships they already knew to find the percent equivalents for fourths and then move to other harder fractions. Figure 15.3 presents the grid use to find the percent equivalent for 3/8 as an example. She did not use the grid sufficiently for percent equivalents of fractions. In contrast, she spent one entire lesson for shading grids (Day 2), which was not related to the MPs of the lessons. Her students "designed" 10 × 10 grids on their own and determined percents and fractions of those grids (e.g., 78% or 78/100, and 43% or 43/100) by counting the number of shaded squares basically. Using the grid in this way was not related to the target fractions of the lessons, such as fourths and eighths. Moreover, the students were not asked to relate fractions with percents at all in the way the written lessons outlined.

Becca explained why she modified the sequence in the way she did. Her reason for skipping some activities and introducing the grid in the first lesson was: "I want them to make the grid and be comfortable with a fraction first, before I try to get them to jump into the percentages. … because those grids for them to color makes it easier then to figure the percentages." In contrast, the written lessons encouraged students to explore fractions and percents together to see their relationships, rather than one at a time. The focus of the written lessons was on the relationships between factions and percents whereas Becca treated them separately. She also explained why she did the activity of shading grids on the second day although it was not in the written lesson.

> I've noticed over the years, kids, because they enjoy that, they don't see it as learning. "Oh, I get to color in a grid!" And it's more fun for them and it helps them transition better into the other activities. … I go "Okay, do you guys remember the grids you made?" "Oh, yeah! Those were easy." "Okay, this is just like that, only—" So it's something to tie back to.

Spending the entire second day on the added activity and skipping important activities and discussions on halves, fifths, and tenths and their percent equivalents, however, she created a big gap in students' learning in her enacted lessons. She explained why she organized the third lesson in the way she did as follows:

> … kids know fourths because of quarters, and so I always relate fourths to quarters and most of them get that. Eighths, I saved until we had the percent equivalent chart, and then I had them go figure it out on those grids, what the percentage for eighths would be. … to

> stop a chart [for fraction and percent equivalents] like that and then have them come back to try and get them back in the mode of that thinking, actually takes more work than to just extend the lesson. It was a long lesson and they had to do a lot with it, but I like to get that chart done in one day.

It was evident that it was too ambitious to cover the mathematics content of two lessons in a one-hour lesson. Given the complexity of thirds and sixths, the curriculum deliberately saved those fractions and their equivalents for the last, separate day although students were expected to start filling in the fraction-percent chart (Fig. 15.1) in the second written lesson. The way Becca sequenced the lessons did not allow her students sufficient time to fully explore percent equivalents of eighths and sixths. It was evident that she did not clearly articulate the MPs of the lessons in the projected learning pathway.

To summarize, reorganizing tasks and lessons and adding a new activity, Becca limited opportunities for students to learn the MPs of the lessons (i.e., relationships among fractions, decimals, and percents) and develop the coherent mathematical storyline that the curriculum carefully laid out. The teacher created a sequence that students had difficulty following through. Without sufficient foundational and intermediate work, her students struggled with the task of finding percent equivalents for fourths, eighths, thirds, and sixths all in one day.

15.4.3 A Case of Kate with Math in Focus

As described earlier, *Math in Focus* is a curriculum program whose typical lesson format is teacher demonstration/explanation and student practice, which is similar to Scott Foresman–Addison Wesley *Mathematics*. Unlike Scott–Foresman Addison Wesley *Mathematics*, however, *Math in Focus* deliberately provides a conceptual foundation for procedures in every lesson, although this foundational work is primarily based on teacher demonstration/explanation. Building the foundation of the procedure in the lesson helps students know why they go through certain steps in particular problems. These foundations are usually built along with representations that illustrate the core mathematical idea embedded in a set of problems that follows. Therefore, using representations to build the foundational work in each lesson or a series of lessons is critical in using *Math in Focus*.

In the example below, I describe one third-grade teacher (Kate)'s case with lessons on fractions in grade 3. She enacted two two-day lessons (two lessons for four days) on improper fractions and mixed numbers (see Table 15.2). Throughout the lessons, conceptual components are prevalent. The MPs of the lessons are summarized in parentheses in the first column of Table 15.2. In the first two-day lesson (6.5), students are expected to understand the relationship between improper fractions and mixed numbers, and use multiplication and division to rename improper fractions and mixed numbers. Then, in the following two-day lesson (6.6), students use the relationship between improper fractions and mixed numbers to add two or three fractions to get a mixed number and subtract a fraction from a whole

Table 15.2 Sequence of the written lessons (Kate)

Lesson (MP)	Objective	Components
6.5 Renaming improper fractions and mixed numbers (Students understand the relationship between improper fractions and mixed numbers, and use multiplication and division to rename improper fractions and mixed numbers.)	• Use multiplication and division to rename improper fractions and mixed numbers	Day 1 A. Use models to rename improper fractions as mixed numbers or whole numbers (The teacher explains how to rename improper fractions as mixed numbers along with a representation and students do a "guided practice" problem.) Day 1 B. Use division to rename improper fractions as mixed numbers or whole numbers (The teacher explains how to rename improper fractions as mixed numbers by using "division rule" and students do "guided practice" problems.) Day 1 C. Roll and rename! (Students play a game in groups, where they roll two dice to form an improper fraction and rename it as a mixed number.) Day 2 A. Use multiplication to rename a mixed number as an improper fraction (The teacher explains how to rename a mixed number 3 3/4 as an improper fraction using the number line and introduce the multiplication rule for converting the mixed number to the improper fraction. Then, students do "guided practice" problems.) Day 2 B. Another way to use the multiplication rule (The teacher explains a shorten version of the multiplication rule with a representation and students do "guided practice" and practice problems.)
6.6 Renaming whole numbers when adding and subtracting fractions (Students understand the relationship between improper fractions and mixed numbers, and use the relationship to add fractions to get a mixed number and subtract fractions from whole numbers.)	• Add fractions to get mixed-number sums • Subtract fractions from whole numbers	Day 1 A. Add two fractions to get mixed numbers (The teacher explains how to add two unlike fractions and students do guided practice problems.) Day 1 B. Add three fractions to get a mixed number (Students explain how to add three fractions, such as 3/4 + 1/8 + 5/8, in teacher-led solution process and do guided practice problems.) Day 2 A. Subtract fractions from whole numbers (The teacher explains two methods for subtracting a fraction from a whole number with a bar model, as shown in Fig. 15.6 and students do "guided" practice problems.)

number. The conceptual components of these lessons are to support students' thinking in the procedural tasks.

The first lesson (6.5) is about renaming improper fractions and mixed numbers. On Day 1 of this lesson (6.5 Day 1 A) teachers are expected to "use fraction circles or pictures to show students how an improper fraction can be renamed as a whole number" (Singapore Ministry of Education/Marshall Cavendish International 2008, p. 243) including the following examples:

$$\frac{3}{3} = 3\,\text{thirds} = 1\frac{6}{3} = 6\,\text{thirds} = 2\frac{9}{3} = 9\,\text{thirds} = 3\frac{12}{3} = 12\,\text{thirds} = 4$$

$$\frac{5}{5} = 5\,\text{fifths} = 1\frac{10}{5} = 10\,\text{fifths} = 2\frac{15}{5} = 15\,\text{fifths} = 3$$

This component of the lesson (6.5 Day 1 A) also suggests the teacher "demonstrate how to rename 4/3 as a mixed number by separating 4/3 into a whole and a fractional part" (Singapore Ministry of Education/Marshall Cavendish International 2008, p. 243) and includes an illustration as seen in Fig. 15.4.

The explanations and the representation help students see what part of the improper fraction becomes a whole number part of the mixed number and why. A guided practice problem that follows also includes a similar representation with *fifths* to support the process to determine a mixed number for 13/5. Using words, such as 3 fifths and 5 ninths, instead of 3/5 and 5/9, is throughout Lesson 6.5 Day 1.

Day 2 of Lesson 6.5 (6.5 Day 2 B) also includes a conceptual explanation of a procedure ("the multiplication rule") using a representation (see Fig. 15.5), which unpacks the steps of "multiply the whole number by the denominator and add the product to the numerator" (e.g., $3\frac{1}{2} = \frac{3\times2+1}{2}$) for renaming a mixed number as an improper fraction.

The teacher's guide includes the following elaboration of the multiplication rule: "First, multiply the whole number by the denominator. 1 whole = 2 halves, 3 wholes = 3 × 2 halves = 6 halves. Then add the product to the numerator (6 + 1 = 7)" (Singapore Ministry of Education/Marshall Cavendish International 2008, pp. 247–248). This component of the lesson conceptually supports students' sense-making of the rule for converting a mixed number to an improper fraction.

$$\begin{aligned} \frac{4}{3} &= 4\ \text{thirds} \\ &= 3\ \text{thirds} + 1\ \text{third} \\ &= \frac{3}{3} + \frac{1}{3} \\ &= 1 + \frac{1}{3} \\ &= 1\frac{1}{3} \end{aligned}$$

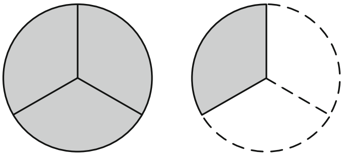


Fig. 15.4 Renaming an improper fraction as a mixed number

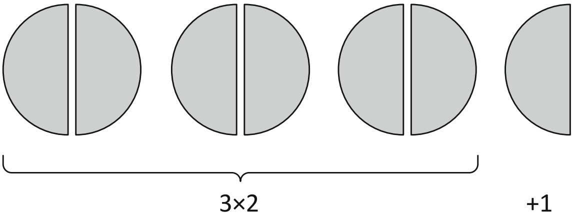


Fig. 15.5 "Multiplication rule" to rename a mixed number as an improper fraction

Day 2 of Lesson 6.6 also provides a conceptual support for subtracting a fraction from a whole number or a mixed number. The lesson introduces two distinct, but related methods for the operation seen below, including a representation for Method 1 (see Fig. 15.6).

$$\text{Method 1}: 3-\frac{4}{9}=2\frac{9}{9}-\frac{4}{9}=2\frac{5}{9}$$

$$\text{Method 2}: 3-\frac{4}{9}=\frac{27}{9}-\frac{4}{9}=\frac{23}{9}=2\frac{5}{9}$$

As seen in Fig. 15.6, the written lesson uses a bar model to represent 3 − 4/9 visually and conceptually—what it means to subtract 4/9 from 3 and what is left as a result of the operation. If earlier conceptual approaches are employed, Method 2 is basically counting how many ninths are left after taking 4 ninths away from 27 ninths (=3 wholes): 27 ninths − 4 ninths = 23 ninths.

Overall, the lesson components for foundational work described above are to establish the relationship between numbers (improper fractions and whole numbers, or improper fractions and mixed numbers) that students will use later to solve problems and practice the procedures, and to support students' meaning making over four days of the lessons. Kate taught three lessons by using the two two-day lessons. She, however, did not use the conceptual lesson components in her instruction. Her sequence of the lesson components for three days is summarized in Fig. 15.7.

Kate skipped Lesson 6.5 Day 1 A, Lesson 6.5 Day 1 C, Lesson 6.5 Day 2 B, and the conceptual foundation portions of Lesson 6.6 Day 1 and Day 2. Basically, she

Fig. 15.6 Bar model used to illustrate Method 1

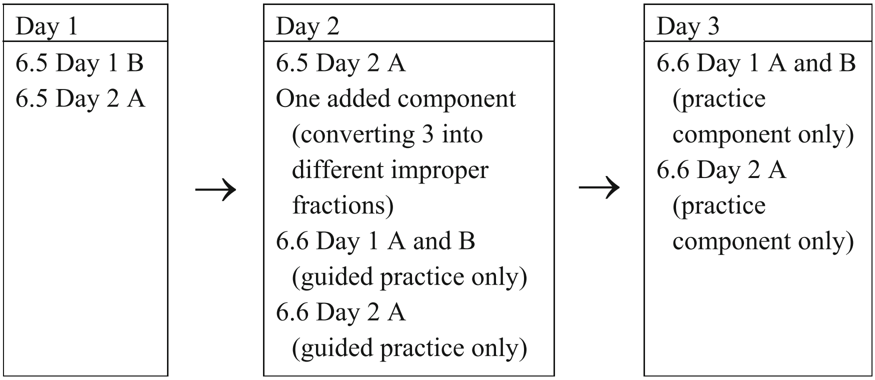


Fig. 15.7 Sequence of the observed lessons (Kate)

omitted lesson components that support for conceptual foundations for students' understanding of fraction operations involving whole numbers and mixed numbers. In contrast, she kept most of the teacher explanations (without conceptual component) and practice problems. Also, she added a short component, renaming 3 in different ways, such as 9/3, 15/5, 2 and 9/9, before introducing the procedure for subtracting a fraction from a whole number.

She assumed that previous work was sufficient for students to add or subtract fractions, and omitted lesson components to build conceptual foundations for procedures students needed to do. By eliminating the conceptual foundation work laid out in the written lessons, she reduced her students' accessibility to the mathematics in the procedures of adding and subtracting fractions. She repeated the practice portions of Lesson 6.6 on the third day observed, but her students still had difficulty adding two or three fractions to give the final answers in the form of mixed numbers, or subtracting a fraction from a whole number or a mixed number. Students were supposed to use what they learned from Lesson 6.5 (understand the relationship between improper fractions and mixed numbers, and rename improper fractions and mixed numbers) to do the operations. Without a solid foundation, however, her students struggled to follow through the procedures that the teacher explained.

In particular, without using the bar model (Fig. 15.6), Kate verbally explained renaming of 3 in different ways (e.g., 2 and 9/9, and 27/9) in order to subtract 4/9. The bar model clearly shows why they needed to change 3 to 2 9/9 (or 27/9) in order to subtract 4/9, but her explanations of renaming of 3 without the model kept the concept on an abstract level and did not help the students see the rationale for the procedures. The students still struggled to use the two methods in other problems during the lessons observed, and not being able to relate the two methods, many of them chose only one of the methods to solve other problems. In fact, some students suggested they use the bar model ("I can draw a picture on the board"), and yet the teacher was reluctant to do so ("No, that's okay. If somebody needs a picture, we will add that. I don't want to confuse anybody."). The teacher strongly believed that the model would confuse students rather than helping them see why the procedure works and explained the renaming repeatedly.

During the interview, Kate said, "I read this [lesson] to see if it is appropriate" to determine whether her "students will be able to make sense, or if I need to do something else, share another example." Her evaluation of the lessons led her to focus more on explanations with examples and remove visual representations. She said, "… sometimes if they start with the picture examples, those are too simplified and I just don't write through the number examples or the computation part because it tends to confused the kids sometimes if the pictures are involved." Kate was not in favour of using the representations (fraction circles or pictures, bars, and number line) the written lessons included, and did not see their mathematical significance and instructional affordances. Because of that, she omitted most of the conceptual components of the lessons. As many other teachers mentioned in this study, however, her rationale for omitting some lesson components was a lack of time. She said,

> They give us the time, the pacing, and then they give us a ton of activities, and like we've talked about, the games, the thinking, and the extra pages in the workbook that give you extra material. If you used all those you definitely wouldn't finish the lesson in a certain amount of time, but you have that option if you need it.

While removing lesson components for the reasons of limited time and student confusion, Kate added one lesson component (i.e., renaming 3 in different ways) that was not specified in the written lessons. She provided her rationale for this addition as follows:

> Well, I think the first time we did three as a whole number renaming if it came up as 9 I think. But then they kept using 3 with different denominators so you know if it was 10, how could you make 3 with a fraction with 10 in the denominator? And so I felt like it was important. First of all, fractions is something that they don't all grasp all the time. They look different and even with the picture representations early on in the chapter. They would look at 1/2 and 2/4 and not really think that they were the same thing. Equivalent fractions were just kind of out there, and I think it was important to show that a whole number could have different names according to what denominator you put it in. And that kind of goes along with multiplication and division and stuff too, how they get those equivalent fractions. So I thought it was important. And I'm big on connections with different topics. Fractions are not by themselves. You need to connect those with something.

As described earlier, the bar model could have helped students see why they needed to change 3 to 2 9/9 in order to subtract 4 ninths, without asking students to rename 3 in different ways and mechanically explaining that since "the denominator of 4/9 is 9" they needed to change 3 to 2 9/9, not 2 6/6, 2 12/12, or something else. Although during the interview she claimed that she emphasized connections, she did not see how the representations that she did not use could have helped students make the connections in the lessons.

To summarize, Kate created a lesson sequence quite different from the one laid out in the written lessons. She removed important lesson components for students' learning of the MPs (i.e., how improper fractions and mixed numbers are related, and how this relationship can be used in operations), which serve as building blocks in developing the mathematical storyline of the lessons. Her articulation of the MPs and learning trajectory did not accurately capture the affordances provided in the written lessons. As a result, her students had limited opportunities to learn the relationship between improper fractions and mixed numbers and do related operations with meaning.

15.5 Discussion

Analyzing a small set of teachers using each of the three elementary mathematics curriculum programs described above, this study explored teachers' decisions on lesson sequence and their potential impact on student learning. Although the patterns identified in the study cannot be generalized to all other teachers using the same programs, the ways the participant teachers enacted the lessons in the sequence are quite feasible in other teachers' classrooms and provide implications for teaching, teacher education, and curriculum development.

Teachers using Scott Foresman–Addison Wesley *Mathematics* tended to follow the sequence provided in the curriculum, whereas teachers using *Math in Focus* and *Investigations in Number, Data, and Space* often modified the sequence in the curriculum. It seems that the demand on the teacher is higher with programs incorporating conceptual support (ICUBiT Project 2011). Especially, using a program with a student exploration model requires more careful reasoning about the mathematics in instructional activities; teachers need to make sense, evaluate, and use various resources in the curriculum to sequence the lessons and tasks properly to support students' learning and development of the MPs and mathematical storyline over a period of time. It can be hard for teachers to see the connections in tasks/lessons, and it may be even harder to sequence them in a way that highlights the mathematical coherence (Sleep 2012). It is important for teachers to understand in the various given resources what MPs are addressed within and across lessons and in what ways the MPs are further developed to build a coherent mathematical storyline.

Then, is the curriculum program with a direct-teaching model easier to teach toward the MPs? We cannot answer just based on how lessons are sequenced alone. There are other aspects and elements of the programs that support or limit teaching to MPs. In fact, conceptual aspects of the MPs were not explicit in lesson components in Scott Foresman–Addison Wesley *Mathematics*, which led teachers to mainly focus on procedural aspects in their instruction. It is hard to claim that procedural aspects of the MPs alone can build a proper mathematical storyline. Noticing this limitation of the program, teachers may try to make up the gap, which is not an easy task.

In the cases of Becca and Kate, there was a significant gap between written and enacted lesson sequences in terms of the MPs and mathematical storyline within and across lessons. It seemed that both Becca and Kate failed to articulate the intended mathematics and how the instructional activities were designed to engage students with it. Using existing resources, teachers need to decide how to do so based on sufficient knowledge and capacity. It is likely that Becca and Kate lacked significant aspects of such knowledge and capacity, some of which were elaborated in the notions of *knowledge quartet* (Rowland et al. 2005), *knowledge of curriculum embedded mathematics* (KCEM, Remillard and Kim 2017), *pedagogical design capacity* (PDC, Brown 2009), *pedagogical reasoning and action* (Shulman 1987), and *mathematical purposing* and *focusing* (Sleep 2009). Teacher education and curriculum design need to support teachers' reasoning with resources and help them build a capacity required to enact lessons productively.

Teacher education should provide teachers with opportunities to use knowledge in various situations for decision making, in particular, articulating the mathematical goals of activities and lessons to develop a proper mathematical storyline. For this reason, lesson planning needs to be done in relation to multiple prior and subsequent lessons, and lessons in grades before and after. Teachers need to situate individual lessons in a broader context and understand how activities and lessons are weaved into mathematical pathways. Describing curriculum use for preservice teacher education, Drake et al. (2014) emphasized teacher learning about and from curriculum resources. They argued, "Learning to read and interpret the features of curriculum materials in ways that leverage the educative potential of those features seems particularly important" (p. 158). Teacher educators need to examine ways in which curriculum resources can be systematically used to support teachers' reasoning with the resources.

Reasoning *in* the resources (e.g., the intent of lessons and activities and a proposed mathematical storyline) needs to be transparent to teachers in order to support their reasoning with the resources. Curriculum programs have various ways to communicate the MPs of lessons to teachers, such as listing lesson objectives, describing activities and tasks, listing vocabularies and key content, and even explaining the MPs directly to teachers in a separate place (e.g., notes for teachers). However, it was observed that some written lessons failed to specify the core mathematical ideas of the lesson/tasks (ICUBiT Project 2011). In fact, the lessons on fraction and percent equivalents in *Investigations in Number, Data, and Space* could have made the MPs specific and clear in each lesson, rather than stating the same broad "math focus points" in multiple lessons. For example, instead of including, "Identifying fraction and percent equivalents through reasoning about representations and known equivalents and relationships" as one math focus point in three consecutive lessons (see Table 15.1), focused fraction and percent equivalents (e.g., fourths and their percent equivalents) can be specified in the math focus point of each lesson to support teachers to better understand how the math focus point can be met in a series of lessons. Moreover, the lessons from *Math in Focus* need to make conceptual aspects be part of lesson objectives so that teachers can attend to conceptual foundation components of the lessons. The examples suggest that curriculum designers attend to ways to make MPs and mathematical storylines explicit. It is notable that *Investigations in Number*, *Data*, *and Space* lists focus points of discussion segments explicitly, which will help teachers attend to the main ideas during discussion. Especially, reform-oriented curriculum programs that include various resources for teachers may bury MPs and mathematical storylines in those resources, rather than making them transparent. Curriculum designers need to provide a clear picture of how lessons are weaved to introduce and develop MPs and a mathematical storyline in a series of lessons, in a unit/chapter, within and across years.

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Part V
Concluding Remarks

Chapter 16
Present Research on Mathematics Textbooks and Teachers' Resources in ICME-13: Conclusion and Perspectives

Sebastian Rezat, Jana Visnovska, Luc Trouche, Chunxia Qi and Lianghuo Fan

Abstract This chapter summarizes and discusses the state of the art of research on mathematics textbooks and other resources and their use as depicted in this volume. The discussion focuses on three questions: First, what are the theoretical and methodological tools used in order to conceptualize and investigate textbook and resource use? Second, what can we learn about the use of textbooks and resources by teachers from the chapters in this volume? Third, what do we learn from comparing teachers-resources interactions across different countries? Based on the discussion, conclusions are drawn regarding perspectives of future research on mathematics textbooks and resources and their use.

Keywords Mathematics teachers' resources · Mathematics textbooks
Use of resources · Methodology · Cross-cultural comparison

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16.1 Introduction

A first look at the chapters of this volume already shows that the field of research on textbooks and teachers' resources has changed. While there used to be a clear focus on the textbook and textbook analysis used to be the dominant theme in textbook research (Fan 2013; Rezat and Sträßer 2015), a shift of focus to the design and use of textbooks and other resources is noticeable. The textbook is seen to be but one resource among many others that teachers are using. One chapter in this volume presents a textbook analysis from a comparative perspective (Fan et al., Chap. 3). The remaining chapters present many examples of teachers designing and using textbooks and other resources and aim at understanding related processes. Furthermore, in crafting lessons—one of the teachers' core activities—design and use of resources are even regarded to be intertwined. Altogether, the chapters in this volume take different perspectives on use and highlight that the design and use of textbooks and other resources is a multifaceted issue.

This chapter of conclusion attempts to connect the insights of the different chapters in this volume in order to draw a bigger picture. We focus on three issues in the following sections:

Section 16.2:	What are the theoretical and methodological tools used in order to conceptualize and investigate textbook and resource use?
Section 16.3:	What do we learn about the use of textbooks and resources by teachers from the chapters in this volume?
Section 16.4:	What do we learn from comparing mathematics teachers-resources interactions across different countries?

We then conclude the chapter by formulating several directions for future research.

16.2 Theoretical and Methodological Considerations in Research on Textbook and Resource Use

Within this overview, it is not possible to summarize all different frameworks and methods that are used in order to understand textbook and resource design and use. Therefore, we will only summarize some trends that can be observed across the chapters of this volume. First, we will generally consider the issue of terminology and methodology. Second, we will focus on particular theoretical frameworks and related analytical tools that are used across different chapters and thus seem to be influential in the field.

A first look at the chapters already reveals that different terms are used to refer to the object of study. This seems to be mediated by different traditions and perspectives in the different parts of the world, but at the same time implies that the object of study varies among the contributions—very slightly in some cases and more apparently in others. While the title of the volume refers to mathematics textbooks and resources many other terms are used in the chapters of this volume: textbook, curriculum resources, instructional resources, curriculum materials, resources, e-book. Remillard (Chap. 4) offers a clarification of terminology in her chapter and at the same time introduces the notion of curriculum resource genre, which highlights that curriculum resources are but one part of a larger class of written and visual communication with its particular "zone of expectation".

The predominant methodology that is used in the studies in this volume is a case study methodology within the qualitative research paradigm. The case under study is either a single teacher or a collective of teachers. Besides this methodological homogeneity, the methods of data collection and the theoretical perspectives taken vary among the studies. While the majority of studies uses video recordings of lessons, interview- and log-data, some of the studies refer to more specific methods of data collection, which combine some of these methods in a specific way, in order to fit the theoretical perspective taken.

Some theoretical notions and frameworks are used repeatedly across the chapters in this volume. These are the documentational approach to didactics (Gueudet and Trouche 2009), the notion of pedagogical design capacity (PDC) as introduced by Brown (2009) and the notion of communities of practice (Wenger 1998), which proves to be fruitful in order to understand teachers' collaborative work. We now discuss how these conceptual tools relate to and provide particular value for research on mathematics teachers' resource use in this volume, and summarize the contributions of chapters in this volume to the further development of these frameworks and notions.

16.2.1 Documentational Approach to Didactics

It is apparent in this book that the documentational approach to didactics (Gueudet and Trouche 2009) is both an established and evolving framework in the field with a related methodology. In the chapters of this volume the documentational approach is enriched and further differentiated by the introduction of new theoretical concepts, such as documentational experience, documentational trajectories, and documentational expertise. While the grounding concepts of this framework such as instrumentation and instrumentalization referred to the processes of documentational genesis, in particular focusing on the development of a resource into a document, the new concepts either emphasize particular aspects of the documentational genesis or introduce new perspectives within this process.

The notions of documentational experience and documentational trajectory (Rocha, Chap. 11) introduce a time dimension into the documentational approach to didactics in order to enable a longitudinal perspective on teachers' documentation work. This time dimension was only implicit in the concept of scheme before. According to Vergnaud (1996), schemes are considered to be invariant organization of behavior for a given class of situations. In order to become invariant, they need to evolve over a period of time, in which they are repeatedly applied. Rocha (Chap. 11) is taking a broader perspective that is not focusing on the development of schemes, but on the development of a teacher's resource system. She introduces the notion of documentational experience and defines it as teachers' experiences with resources over time. The experience is conceptualized in terms of events, while an event is understood as everything, which might lead to a transformation of a teacher's resource system. The set of events at the interplay of teacher's individual and collective documentation work that makes up a teacher's documentational experience is called a documentational trajectory.

Documentational expertise (Wang, Chap. 9) coordinates the instrumentation processes with the related development and refinement of utilization schemes of teachers. Wang proposes to define documentational expertise (DE) as "the schemes in resources retrieving, selecting, modifying, adapting, storing and re-organizing, sharing off, in order to solve teaching problems efficiently. For a teacher, DE is developing to integrate her available resources to her understanding on the goals of the activity" (p. 196). This definition emphasizes that teachers' work with resources is an important aspect of their professional expertise. The author also builds on work of Pepin et al. (2017) to clarify that the notion of DE is broad, and includes aspects of teachers' work beyond their design and design capacity.

In addition, a distinctive set of purpose-built methodological tools has been developed within the documentational approach to didactics to aid investigations of teachers' documentational geneses. Reflective investigation with its four principles of "long-term follow-up", "in- and out-of-class follow-up", "reflective follow-up", and "broad collection of the material resources" (Rocha, Chap. 11) is the core of the methodology. Given that the methodology builds on teachers' own views about individual and collective aspects of their documentational work, specific tools are developed and revised for teachers to record their resources, and map their resource systems. Other tools, such as documentational working mate (Wang, Chap. 9), aim to create a multifaceted perspective of a teacher's documentational work by including an influential colleague's perspective.

16.2.2 Pedagogical Design Capacity

The phenomena directly related to teachers' design are of interest to a suite of chapters in this volume, authors of which conceptualize teachers' interactions with textbooks and resources by drawing on the notion of pedagogical design capacity (PDC). According to Brown (2009), PDC orients us to understand what teachers bring to their interactions with textbooks and curriculum materials, and it is frequently defined as "teacher's capacity to perceive and mobilize existing resources in order to craft instructional episodes" (p. 29). Concerning this definition, authors in this volume both operationalize teacher's PDC in relation to purposes followed in their analyses, and elaborate on the theoretical aspects of the notion.

From the analysis of the case of one teacher, Remillard (Chap. 4) convincingly argues that PDC needs to be conceptualized as an interaction between affordances of the resource and the interpretive capacities of the teacher, as both of these shape what becomes possible in a classroom as a result of the teacher's design work with resources. Methodologically then Remillard suggests that "studying or assessing PDC involves examining the teacher's interpretive interactions with the resource and accounting for characteristics that both the teacher and the resource bring to and leverage in the interaction" (p. 73).

Chapters by Leshota and Adler (Chap. 5), and Kim (Chap. 15) both empirically support Remillard's conceptualization of PDC by documenting how teachers either follow or modify designed resources when planning for and enacting classroom instruction. In both chapters, two issues related to teachers' interpretive interactions with the resource seem crucial to determining PDC: the quality of learning opportunities that arise from teachers' designs, and the degree to which teachers recognize the intentions of resource designers, the mathematical goals pursued, and related affordances of designed tasks and activities. The contribution of characteristics of the resource to PDC is also evident in Kim's report on design work of teachers who used resources that did not encourage students' conceptual understanding. Even though these teachers' interpretations of used resources aligned closely with those of the designers, the quality of the resulting student learning opportunities, and thus the PDC demonstrated in this setting, remained rather low.

Inquiring into Design Capacity for Enactment (Brown 2009), Qi et al. (Chap. 2) built on the framework for approaches to textbook use proposed by Nicol and Crespo (2006), and distinguished between teachers adhering to, elaborating on, and creating new content. These authors related different levels of resource use to differences in teacher's roles.

16.2.3 Communities and Teachers' Resources

In order to understand the collaboration of teachers (and designers) within the processes of design and use of resources, the reference to Lave and Wenger's (1991) and Wenger's (1998) notion of communities of practice is most prominent in this volume. In contexts, where people from different disciplinary and professional domains are collaborating, a preference for the notion of communities of interest according to Fischer (2001) is noticeable. Rocha (Chap. 11) prefers to use the notion of thought collectives according to Fleck (1934/1981). The aim across different chapters is to understand processes by which the thought collectives, and interactions between communities of practice, and within communities of interest shape teachers' resource systems. The notions of *boundary objects* and *boundary crossing* (Akkerman and Bakker 2011) appear to be particularly helpful to this end.

Steenbrugge et al. (Chap. 8) focus on teacher's collaborative learning from curriculum resources. In order to understand how curriculum resources mediate the learning of a group of teachers within a professional development program, they combine an analysis of the negotiation of meaning within a community of practice according to the framework proposed by Wenger (1998) with an analysis of the meaning potential of the curriculum resources based on Halliday's social semiotics (Halliday 1978, 1985).

Kynigos and Kolovou (Chap. 7) as well as Essonnier et al. (Chap. 10) aim at understanding the creativity within the processes of teachers' collective design of resources. They capture creativity through a social creativity approach, which enables them to unveil creativity as it emerges in the interaction of members of a community of interest working within a technological environment. Essonnier et al. offer an operationalization of social creativity within technological environments, which focuses on the origins, negotiation, and materialization of ideas among the designers. The ideas of interest are classified as being emergent, novel, appropriate, and usable ideas among the designers.

16.3 Teachers' Textbook and Resource Use

The chapters in this volume focus on different actions with textbooks and resources as well as on mediation of textbooks and resources within different activities. Wang (Chap. 9) summarizes the most apparent actions in her definition of documentational expertise: retrieving, selecting, modifying, adapting, storing and re-organizing, sharing off are all actions to be carried out with textbooks and resources within teachers' professional work. This list is likely to be expanded. We will elaborate further on four aspects of teacher's use of resources, namely teachers' selection, implementation, and design of textbooks and resources, as well as the role of collectives related to resource design and use.

16.3.1 Teachers' Selection of Resources

The first question that arises when focusing on teachers' use of textbooks and other resources is related to the mechanisms of choice: If teachers have different options, which materials do they choose and why do they choose them? These two questions are tackled by Siedel and Stylianides (Chap. 6). They interviewed 36 secondary mathematics teachers in England. They argue that the case of England is of special interest because there is a low use of textbooks and a high degree of teacher choice. Their results show that teachers' resource packages are highly individualized and are best characterized by "plurality and variation" (p. 130). Altogether, the 36 teachers mentioned more than 70 specific resources with NRICH, the Times Educational Supplement, MyMaths, and YouTube being the most popular. Furthermore, three generic resources were mentioned by nearly all teachers: student textbooks, colleagues, and materials at school (other than student textbooks). In terms of reasons for teachers' choices Siedel and Stylianides identify six key themes: "student-driven selection, teacher-driven, mathematics-driven, constraints-driven, resource-driven, and culture-driven" (p. 132).

Wang (Chap. 9) understands the process of selection of resources in terms of schemes, and regards it as an important aspect of a teachers' documentational expertise. According to Wang, the schemes of selection rely on the understanding of the goals of the activity and related concepts as well as on the teachers' teaching practices.

The chapter of Ruthven (Chap. 12) is in some sense the complement to Siedel's and Stylianides' chapter. While Siedel and Stylianides focus on reasons that explain teachers' choices, Ruthven tries to understand the range of possible ways that digital curriculum programs on the one hand and dynamic mathematical tools on the other hand can be expected to shape interactions in a classroom setting. Based on a discussion of four recent studies on the use of digital curriculum programs and dynamic tools, he documents strengths of these programs that are an addition to more traditional classroom instruction, such as rapid feedback to learner and to teacher. He also points out the rather surprising lack of opportunities for dynamic coordination of different representations by students, noting that even though such opportunities are well within technical capacities of interactive systems, they are included only infrequently. Finally, Ruthven identifies four key barriers that hinder the widespread adoption of individualized learning designs:

(1) The expository components of individualized learning designs are perceived to be less well adapted to curricular and pedagogical requirements than in-class exposition by the teacher.
(2) The new kinds of digital mathematical (dynamic) tools are not yet adequately incorporated in individualized learning designs.
(3) Individualized learning designs foster a limited range of types of interpersonal interaction.
(4) Teachers perceive digitized learning designs as a diminution of their role: From active initiator and director towards manager and adviser of learning.

Chapter 1 by Trouche et al. also tackles the question of choice indirectly. These authors start from the assumption of an established and more or less stable resource system of teachers and ask to what extent the availability of open educational resources, which they call "resources-on-offer", leads to an opening up of teachers' resource systems. They aim to identify the mechanisms of change in the resource systems of teachers. Studying the case of one teacher using a particular resource, the authors find that the integration of new resources into the resource system is driven by the teacher's perception of a lack of resources that are regarded as appropriate for achieving the intended goal. The authors also describe the tendency that newly discovered resources seem to complement the ones in the teacher's resource system rather than to replace them. Nevertheless, the adoption of new resources leads to a reorganization of the teacher's resource system. And finally—especially in the case of digital open resources—the use and the (re-)design of resources are intertwined processes that cannot be separated. This leads to a second focus in research on the use of resources: the design of resources.

16.3.2 Teachers' Use of Textbooks and Resources in Teaching and Learning

The fidelity of implementation of textbook content and the resulting opportunities to learn are still an important matter of interest. This is especially the case in contexts where the textbook is the only, or the most prominent, learning resource for both students and their teachers (like in the case of South Africa), or when the textbook is conceptualized as the major agent in supporting the change of teachers' practices (like in the case of some reform initiatives in the US). One aspect of fidelity relates to whether opportunities to learn that were intended by textbook designers actually emerge in the course of classroom sessions. Leshota and Adler (Chap. 5) conceptualize pedagogical design capacity in terms of omissions and insertions/injections. They illustrate how omissions and insertions of content from a textbook affect students' opportunities to learn, especially, if they critically change the intention of the written lessons.

Besides omission or insertion, the sequence of opportunities to learn as a whole is an important design aspect of curriculum materials. Kim (Chap. 15) focuses on the fidelity of implementation of the sequence. She analyses how 11 teachers implemented the lesson sequences suggested in three different US textbooks. From this perspective, she provides once more evidence that fidelity hardly occurs in the use of curriculum resources. She finds that teachers deviate from the suggested lesson sequence even changing the core mathematical idea of the lesson. However, her data indicates that this seems to be depended on the curriculum material used. Specifically, the more the material challenged, or aimed to reshape, teachers' existing practices, the more likely the teachers were to introduce adaptations to the sequence that deviated from the intended design, and instead pursued teaching goals that they already recognized as legitimate.

Based on video data from one lesson, Qi et al. (Chap. 2) analyze how 6 Chinese teachers use the mathematics textbook in terms of the three levels of enactment: "adhering", "elaborating" and "creating" (Nicol and Crespo 2006) related to different teachers' roles, e.g. as a mediator and validator of mathematical knowledge, a source for mathematical problems, a communicator with students, and promoter of self-regulation (Trigueros et al. 2014). Among the six teachers, they found a tendency that in the Chinese context the teachers do not solely adhere to the textbook while teaching geometric transformation, but the use of the textbook reaches the levels of elaborating and creating with an emphasis on elaborating. The application of technology seems to be the main approach to using textbooks creatively. While the problems for practicing mostly stem from the textbook and the exploration of new concepts always followed the textbook procedures, teachers used multiple sources for introductory problems, including the internet, real life, and campus activity. Compared to a previous study, Qi et al. concluded that their study of 6 teachers indicated a decrease of Chinese teachers' reliance on textbooks: "Teachers turn their attitudes from 'teaching textbooks' to 'making better use of textbooks'" (Qi et al. Chap. 2, p. 46).

Another important issue related to the implementation of textbooks and resources is the question how the characteristics of the resources themselves support teachers' understanding and guide thoughtful implementation. This issue is especially related to innovation and curricular reform. Naftaliev (Chap. 14), Visnovska and Cortina (Chap. 13), and Steenbrugge et al. (Chap. 8) address this issue in their chapters.

In a design experiment, in which prospective teachers were involved in (a) developing opportunities to learn using interactive diagrams, (b) analyzing students experiences with similar opportunities to learn, and (c) the imagination of the enactment of the designed opportunities to learn, Naftaliev (Chap. 14) aims to understand the affordances of the design of the interactive diagrams. She finds that the prospective teachers in her study became aware of the different functions of interactive diagrams and, to a lesser extent, of related pedagogical possibilities. In this sense, the new type of resource for teaching provided valuable learning for teachers. While some of the prospective teachers incorporated various versions of the interactive diagrams into their representations of classroom learning situations, others continued to prefer conventional instructional practices and avoided interactive diagrams altogether. Naftaliev reports that prospective teachers' difficulties in dealing with the unexpected mathematical ideas the students would develop in course of engaging with interactive diagrams could have been an important factor.

Visnovska and Cortina (Chap. 13) explore the support teachers get from instructional materials that were designed for innovative teaching approaches. They analyze data from two professional development design experiments and draw conclusions about design principles of these materials. Hence, they connect a perspective that focuses on the use of teachers' resources with a perspective that puts the resources and their attributes in the center. They propose that traditional curriculum materials, with which the teachers are typically familiar, are reassuring in that they support teachers' understandings of what they need to teach. Such materials align well with practices in which instructional decisions are based on the content that needs to be covered, the activities that need to be implemented, and the work that students need to produce. The authors point out that innovative teaching materials, in contrast, do not foreground content coverage, and do not prescribe the number and type of tasks to be completed—these should now be determined by the teacher based on their assessment of ways in which their students reason. The three design-principles authors propose for supporting the teachers in transition in overcoming the difficulties they are likely to face are: (1) Specification of relatively simple initial goals for students' mathematical reasoning, (2) Creating early opportunities for conversations and co-participation among teachers geared towards making visible the rationales for instructional decisions, and (3) Providing guidance for teachers' design work in response to specific forms of reasoning present in their classroom. The first of these design principles relates to the issue identified by Naftaliev, and aims to support teachers' work with novel resources by productively constraining the mathematical difficulty of students' ideas that initially emerge in the classroom.

Initial findings by Steenbrugge et al. (Chap. 8), who explore the processes of constructing and negotiating meanings in teacher communities around specific resources, also relate closely to a similar set of ideas. Like the previous two studies, these authors report that resources can and ought to support teachers' understanding of ideas that are central to productive mathematical and pedagogical practice. They too share an observation that the closer these ideas are to teachers' established practices, the more productively the groups of teachers seem to engage in negotiations of meaning related to these ideas. Lastly, Steenbrugge et al. also highlight the learning potential of the provision, in the resources and in facilitation of the teachers' collective learning, of specific prompts for collective reflection, both on the introduced central ideas, and on the practical implications of these ideas for participating teachers' instruction.

16.3.3 Teachers' Design of Textbooks and Resources and the Role of Communities and Collectives

Notions like PDC (Brown 2009) highlight that teaching is always a design process, in which teachers interact with different resources. Especially in relation to digital resources, the design processes of teachers have become a matter of interest. On the one hand, this might be due to the fact that some digital resources facilitate the design of opportunities to learn or are even developed with the intention that teachers become designers themselves. On the other hand, digital resources enable researchers to collect data more easily about the ongoing processes of design so that researchers are actually able to get an insight into these processes. Furthermore, digital resources afford that teachers' collaboration no longer needs to be synchronous and does not need to take place in the same physical space. In conjunction with these new possibilities for teachers to collaboratively design textbooks and other curricular resources the question of the role of collectives/communities comes to the fore. In this volume, research on the role of collectives and communities aims to understand how they affect teacher's design, use of resources, and learning related to use of resources. In addition to the previously discussed studies by Steenbrugge et al. and Visnovska and Cortina, communities and collectives play an important role in teachers' learning and in teachers' collective resource design work, as reflected in other chapters in this volume.

Kynigos and Kolovou (Chap. 7) as well as Essonnier et al. (Chap. 10) in their chapters analyze the creativity within these design processes. Essonnier et al. identify time constraints, personal concerns, curriculum standards, and the designers' preferences and inclinations as contextual factors that influence the design of technological resources. The design process itself is fostered by collaborative design in communities that combine different professional backgrounds, inclinations, and preferences combined with an appropriate technology. Kynigos and Kolovou view boundary crossing (Akkerman and Bakker 2011) as the driving force of creativity in such communities. In particular, they found in their study that social creativity was enhanced by coordination, or in other words, by communicative connection between diverse practices, and reflection on the differences between these practices.

Wang (Chap. 9) summarizes that the two cases in her study emphasize the importance of collectives for their work. The collectives serve as a source and a forum for exchanging ideas, resources, and feedback.

Rocha (Chap. 11) investigates how teachers' work with resources is informed by collectives as they face and implement curricular changes. From the analysis of two cases she concludes that the two cases' involvement in collectives helps them to prepare for the changes though in different ways. One case uses the design process itself to develop her own understanding of the curricular changes, while the other case uses her multiple memberships in collectives for the same end. This indicates that collectives themselves are resources that teachers instrumentalize for their goals according to individual utilization schemes.

However, understanding the role of collaboration in collectives or communities of practice/interest seems to be in a phase of exploration. Research on the role of collaboration in collectives in this volume focuses on the application and development of theories that are suitable to understand the role of collectives and communities in resource design and use. So far, the results indicate very generally that collectives and communities trigger and foster professional development of teachers, but the understanding of the actual mechanisms of these processes is still at the outset.

16.4 Comparing Teachers-Resources Interactions Across Different Countries

The chapters in this book provide examples of teachers-resources interactions from many different countries, including China, France, Israel, South Africa, Mexico, Indonesia, Saudi Arabia, Greece, United Kingdom, and United States of America. Thus, the analyses in this book span over a wide range of social, political, institutional, economic and cultural contexts. While some of the chapters actually focus on the contextual differences (Chaps. 3, 9 and 12) the role of the context remains rather implicit in other chapters and only becomes apparent by comparing teachers-resource-interactions across the chapters. In this section, we will focus on questions that arise when comparing mathematics teachers-resources interactions across different countries. First, we focus on the chapters that explicitly deal with such comparisons:

Fan et al. (Chap. 3) compare different presentations of geometric proofs in mathematics textbooks from China, Indonesia and Saudi Arabia. They find important variations in geometry teaching regarding the number of examples, the distribution of contents, and the types of proof. As they mention, an issue for further exploration could be to find out if these variations might be explained by cultural differences in the three countries.

Wang (Chap. 9) confronts different traditions of teachers' collective work in China and in France. She traces differences in resource selection and use with regard to contextual factors and points to some variations in the way teachers' collective work affects their professional development.

Ruthven (Chap. 12) confronts two types of digital resources usages for learning mathematics: on the one hand, digital curriculum programs in the US; on the other hand, dynamic mathematics tools in the UK. Although Ruthven does not provide any cultural/contextual arguments for this link between the countries and the resources, it would be interesting to explore in more depth if the interest for these different environments is due to the variation of school mathematics contexts in the two countries, or to the variation of scientific interests of the mathematics education communities.

Beyond these chapters, the whole book provides opportunities for comparing school mathematics context in various countries, because each case, which is analysed, is grounded in and affected by a given social, political, institutional, economic or cultural context.

Trouche et al. (Chap. 1) present the development of the mathematics teacher association 'Sésamath' that aims to collaboratively design resources for teaching in a 'mathematics for all' spirit at a large scale. This could be a peculiarity linked to the national history of mathematics education in France, or a precursor of phenomena to appear in other countries in the thread of digitalization.

Qi et al. (Chap. 2) trace the evolution of Chinese teachers' relationships with textbooks according to their teaching experience, leading them to improve their usages of these resources. An interesting question to ask is, is this a peculiarity linked to the central role of textbooks in China, or a general feature of teachers' professional development and experience with resources?

Remillard (Chap. 4) discusses terminological issues with regard to teachers' resources and their respective purposes and relations. Based on this discussion, she proposes a typology of instructional resources, and highlights the role of teachers' guide, student text and teachers' documents in analyzing teachers' PDC. The question is, is this a typology, which is particularly linked to the context of the US—as we know that teachers' guides could vary a lot according to the national curriculum and institutional requirements—or is it relevant for other countries as well?

The role of the cultural context is not only important in order to understand its influences and teachers-resources interactions, but also matters for the development of theoretical ideas that conceptualize such interactions. For example, Leshota and Adler (Chap. 5) and Kim (Chap. 15) both refer to and further develop the notion of pedagogical design capacity (PDC) as introduced by Brown (2009), the first one in the context of South Africa, and the second one in the context of the US. Interestingly, in both environments, teachers' designs and enactments appear to be expected to remain reasonably close to the learning opportunities as intended by textbook designers. It is important to note that analyses used in these chapters (of content omissions and inclusions, and of teachers' adaptations of sequencing) would be both more difficult to conduct in contexts like the UK, where teachers only rarely rely on a single resource, and frequently redesign the sequencing and aims of adapted activities (e.g., see Siedel and Stylianides Chap. 6). However, underlying notions of deliberateness in use of resources and coherence of enacted mathematical storylines (Sleep 2012) remain a valid issue of interest across the contexts.

Finally, looking at teachers work through the lens of the resources they use, design, and share leads to taking into account the whole of their teaching reality. Similar to the role of artefacts within mediated action in the context of sociocultural theories (e.g. Wertsch 1998), resources can be considered as "crucial interfaces between culture, policy and teacher curricular practice" (Pepin et al. 2013). This calls, as Chap. 3 authors in their conclusion note, for extending the current studies, and taking into account the institutional, cultural, and social aspects of the teaching environments. This also relates to the general validity of concepts and methodologies used: Are the concepts and methodologies appropriate for the context in which they are applied or do they need to be transformed as we adapt them to new contexts? This 'comparative attitude' is then a means for making theoretical concepts and frameworks better fitted to a broader scope of context-specific realities.

16.5 Perspectives

So far, we have focused on contributions of this volume to methodological and theoretical developments in research on the design and use of mathematics teachers' resources, discussed the new insights related to resource selection, use, and design, and reflected upon the role of different national contexts in this research. We can see that the overall understanding of the use and design of textbooks and other resources is evolving. Nevertheless, the resulting picture comprises areas that could be more illuminated by further research.

Although the field of textbook research seems to shift focus from textbooks analysis to the investigation of the use of textbooks, Fan et al. show in their chapter that many questions related to the content and how it can be productively presented in mathematics textbooks remain. The ways in which mathematics textbooks should contribute to creating opportunities to learn different mathematical topics and proficiencies are not well established, and this becomes apparent in sometimes striking differences in goals and aims pursued in textbooks in different countries. However, an understanding of the contents of textbooks and other resources and how these contents are being shaped by social, political, institutional, economic, historical, and cultural contexts is essential for a deep analysis of teachers' resources.

Our discussion in this conclusion chapter shows that many theoretical notions and concepts are developed in this volume based on case studies conducted in particular social, political, institutional, economic, and cultural contexts. On the one hand, the different studies illustrate how these theoretical concepts are useful in illuminating processes of textbook and resource design and use. On the other hand, the dominance of case studies in this volume with only a very small number of cases highlights that there is a need for large scale, quantitative investigations that would explore whether and how these concepts and theories might be useful as we explore teachers' resource use at scale and in diverse social, political, institutional, economic, and cultural contexts. One example here might be the notion of teachers' PDC. Authors in this volume agree that it is an important aspect of how teachers' professional expertise gets realized vis-a-vis affordances of the resources used. As such, it would be an interesting object of a large scale investigation of how these relationships play out in different cultural and institutional contexts across the world. This idea calls for an operationalization of teachers' PDC, which on the one hand would be suitable for quantitative evaluation of teachers' resource interpretations and resource affordances, and on the other hand would remain sensitive to social, political, institutional, economic, and cultural contexts in which participating teachers would work.

The focus of the contributions in this volume is on teachers' use of resources. Resources are regarded as something that teachers use in order to design instruction, and even as something teachers learn from (e.g. Steenbrugge et al. Chap. 8). The students' perspective is less prominent. Only in a few chapters the students are seen to shape teachers' decisions, for instance in teachers' "student-driven selection" of resources (Siedel and Stylianides Chap. 6) and in studying resource features important in helping teachers base their instructional decisions on their students' mathematical reasoning (Visnovska and Cortina Chap. 13). Even in Kim's chapter, in which students' opportunities to learn serve as the main point of reference, no data on students' actual learning is collected. Eventually, it is the researcher's evaluation of the quality of the opportunities to learn that is used as a criterion for the quality of the teacher's implementation of the curriculum and not students' actual learning.

Finally, as we can see from the reflections regarding the role of social, political, institutional, economic, and cultural contexts (Sect. 4), the understanding of the influences of these contexts on the design and use of resources has not been a major point of focus to this point. In order to understand the processes of resource design, and use and to learn from insights generated in other countries, more insight into the role of the context is needed. This calls for approaches, which incorporate the social, political, institutional, economic, and cultural contexts in their analysis like the Socio-Didactical-Tetrahedron introduced by Rezat and Sträßer (2012). Furthermore, we maintain that the development of theories in the field can be strengthened when we strive to understand how theoretical concepts developed in one context can be adopted, adapted, or modified to become useful in other contexts.

We hope, that the threads laid out at the TSG 38 *Research on resources* (*textbooks, learning materials* etc.) at ICME-13 as summarized in this volume might provide a starting point to be further taken up and woven at the upcoming events focusing on resources in mathematics teaching and learning: The Re(s)sources 2018 conference in Lyon (France), The Third International Conference on Mathematics Textbooks Research and Development in 2019 in Paderborn (Germany), The Fourth International Conference on Mathematics Textbooks Research and Development in 2021 in Beijing (China), and finally, at a Topic Study Group focusing on teachers-students-resources interactions at ICME-14 in Shanghai (China).

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Chapter 17
Enhancing Teacher Learning with Curriculum Resources

Birgit Pepin

Abstract In this commentary paper I start from the chapters of the book (and related research), and develop an argument for a complementary line of research, which (in my view) deserves additional (and increased) attention in mathematics education research. Whilst curriculum materials have been examined in terms of their quality and support for student learning, less research has focused on teacher learning with educative curriculum materials. This is especially relevant, as teachers are increasingly expected to design their own materials, in particular in times of curriculum change and profusion of digital materials on the web. In this chapter I discuss design criteria and suggest selected design specifications, and their functions, for educative curriculum materials. Moreover, I argue for 'flexible design criteria' aligning my suggestions with my earlier work on teacher design capacity.

Keywords Mathematics teachers' resources · Curriculum resources/materials Teacher design · Teacher design capacity · Educative curriculum resources/materials

17.1 Teachers' Interaction with (Digital) Curriculum Resources

Recent conferences (e.g. ICMT textbook conferences in Southampton and Rio) and publications (e.g., Fan et al. 2013) reflect a renewed, or ongoing, interest of the mathematics education community in textbooks, acknowledging them as a crucial factor in shaping the curriculum that is taught in classrooms (Valverde et al. 2002). At the same time it is known from previous studies (e.g. Pepin and Haggarty 2001) that textbooks relate to the intended curriculum, and do influence but not determine the taught curriculum. It has been known for some time that teachers interpret

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textbooks and other curriculum materials (e.g. Pepin and Haggarty 2001; Remillard 2009). Hence, the relationships between the teacher, his/her teaching, and curricular materials have been becoming more prominent which is evident by much recent research (e.g. Pepin et al. 2013; Remillard, Chap. 4, this volume). With the availability of technology, and the coming of age of the e-textbook, the nature of teachers' interactions with textbooks is changing (Pepin et al. 2016).

Even though printed textbooks (sometimes combined with digital resources on CD roms) are still the dominant vehicle for the curriculum (e.g. Qi et al., Chap. 2, this volume; Fan et al., Chap. 3, this volume), many teachers have started to integrate interactive computerized activities in their teaching. Whilst there is (in many contexts) an institutional expectation that teachers "integrate technology" into their teaching, there are many examples now where the technology supports, or is designed/used to support, well-defined pedagogical goals of individual teachers or teaching groups. In this volume there are many examples for this: for instance Ruthven (Chap. 12, this volume) regarding instructional activity and student interaction with digital resources; Kynigos and Kolovou (Chap. 7, this volume) and creative mathematical thinking; Essonier et al., Chap. 10, this volume) and the role of context in social creativity for the design of digital resources; Naftaliev (Chap. 14 , this volume) and interactive diagrams for students' mathematical activity. And elsewhere we find more (e.g. Yerushalmy 2016, for inquiry learning).

In many countries particular digital technologies are provided and supported by national agencies. At the same time the web is flooded with interactive learning resources, searching for suitable existing activities is what teachers most commonly choose (rather that developing activities from scratch). Yet searching the web may be a frustrating experience for critical teachers. Research has demonstrated that "the right fit between the technology, the software, and the instruction is essential for implementation" (Hooper et al. 2015, p. 76), but existing search engines are not designed to help achieve such a fit. Teachers are often limited to rather coarse-grained searches—by mathematical topic, possibly including some keywords such as "inquiry" or "grade … students", which may or may not prove to be useful in focusing the search—and will typically consider only the most high-ranking (i.e., popular) results of their search. More importantly, in this state of affairs, teachers are typically not explicit (or knowledgeable?) about the kind of interactions they would like their students to have with the mathematics.

Changes in the way developers/designers and researchers conceptualize mathematics textbooks may also have an impact on teachers' relationship with textbooks: Is the teacher/practitioner the "enactor/implementer" of the textbook, or is s/he perceived as the "expert practitioner" who designs learning sequences (e.g. Kim, Chap. 15, in this volume) with the support of the textbook? Is the teacher the "expert", or is the teacher educator/didactician/inspector the "true expert"? Several researchers (e.g. Chazan and Yerushalmy 2014; Gueudet et al. 2012) have argued that changes in the role of textbooks, teachers' relationship with textbooks, and changes in the processes for authoring and publishing textbooks, can have important influences on the role of teachers in the curriculum development process.

This is particularly true as e-textbooks become the norm, albeit not all of them truly interactive (Pepin et al. 2017b). Pepin et al. (2016) distinguish between three models of currently available e-textbooks—dynamic, evolving or "living", and interactive. In the "dynamic" model, a static textbook (traditional or digital) is linked to other learning objects. In the "living" model, textbooks are dynamically and cumulatively authored by a community—often a community of teachers (e.g., Gueudet et al. 2013). The third model of e-textbooks—interactive—is based on a toolkit model, and is anchored in a set of learning objects, where tasks and interactive materials can be linked and combined in different ways. Confrey and her team (e.g. Confrey et al. 2017) have designed tools and materials to help teachers develop learning trajectories through such a "bag of resources" in alignment with particular standards (in this case US Common Core State Standards). Rocha (Chap. 11, this volume) describes the difficulties mathematics teachers have to design their instruction, with established and digital resources, in a period of curricular changes. She coins the terms *documentational trajectories* as a tool to understand teacher learning over time.

In the first two models of e-textbooks, teachers must find (or author) learning objects, and in the third model (interactive) they must sequence learning objects. In all cases they face the challenge of maintaining a more or less coherent approach to teaching mathematics. Pepin et al. (2016) have elaborated the notion of *coherence* in the context of producing a textbook. Drawing on Gueudet et al. (2013) and Yerushalmy and Chazan (2008), they distinguish between two types of coherence. Coherence of the design of a textbook encompasses aspects such as mathematical correctness, epistemological stance to mathematical topics, sequencing that avoids gaps in the mathematical progression, consistent handling of mathematical objects, and consistency with national curricula. These aspects of coherence are constituted in the textbook's expositions, its tasks, and ways in which technology is made available to students. Coherence in use is the coherence of what teachers actually propose to their students, drawing on the textbook, or on other curricular material. The e-textbook is changing the boundary between coherence of design and coherence in use. Issues pertaining to sequencing and availability of technology, which have been considered aspects of design of a linear textbook, are becoming aspect of coherence in use, as teachers re-design the textbook. Regardless of where we draw the line between coherence of design and coherence in use, Cai et al. (2014) have shown that teachers may have very different notions of instructional coherence, and these notions may be quite different from those of the author of their textbook.

To summarize, teachers' interactions with curriculum materials are changing, as the access to and nature of such materials are changing: from traditional text based resources for teaching and learning (e.g. textbooks), to web based and interactive materials (e.g. Trouche et al., Chap. 1, this volume). Studies such as the TIMSS report (Hooper et al. 2015) show that although mathematics teachers in most countries still rely on a textbook for most of their curricular decisions, it is quite common for teachers to supplement the curriculum with learning resources available on the Internet. In some contexts, teachers rely heavily on a single textbook, and they might follow it “from cover to cover” (e.g. “teaching by the book”); in others they use a range of textbooks, and pick and choose particular chapters for particular mathematical topic areas (as they believe that these suit the needs of their pupils better), or they combine a textbook with particular internet resources (that they deem suitable); in again other contexts teachers do not have a single textbook to rely on, and they are responsible for the construction of teaching sequences, or indeed they deem the textbook not to be suitable at all, and they design their own materials for the teaching and learning of mathematics.

However, this purposeful selection process is not an easy task for teachers (see Seidel and Stylianides, Chap. 6, this volume), in particular as it would assume that teachers have clear selection criteria, perhaps based on the quality of the materials (e.g. Gueudet et al. 2013), or their alignment with agreed standards or national curricula and guidelines (e.g. Polikoff 2015), or indeed their fittingness with their own ideas of instructional coherence (e.g. Cai et al. 2014).

At the same time, whilst there is an immensity of potentially suitable materials on the web, the web does not provide suitably sensitive search engines for semantic information about content, which would be necessary if one wishes to search for particular (perhaps interactive) learning resources that combine with other (e.g. textbook) resources in their subtle epistemic or didactic features. In other words what is provided for teachers is often “a pile of bricks”, without being given guidance on how these bricks might be put together to develop a coherent learning paths for their students. Whether searching for tasks to supplement a given learning sequence, or planning learning paths through a flexible e-textbook, teachers will require professional support to help them develop a sensitivity to mathematical and pedagogical aspects of learning objects, and will need tools to support such professional training, and to help them make effective use of this sensitivity (e.g. van Steenbrugge et al., Chap. 8, this volume) and develop expertise (see Wang, Chap. 9, this volume, on *documentational expertise*).

17.2 Teachers as Designers

In this book there are several chapters alluding to the notion of Pedagogical Design Capacity (PDC) (e.g. Leshota and Adler, Chap. 5; Kim, Chap. 15; Remillard, Chap. 4) and I would like to comment on and add to this in particular ways.

Elsewhere (Pepin et al. 2017b) we have argued that teaching can be regarded as design. This is in line with Brown (2009) who explains that “teachers must perceive and interpret existing resources, evaluate the constraints of the classroom setting, balance tradeoffs, and devise strategies—all in the pursuit of their instructional goals” (p. 18). He claims that the interpretation of teaching as design, and the notion of teachers as designers, is fitting with a range of cognitive theories that “emphasize the vital partnership that exists between individuals and the tools they use to accomplish their goals. … And it is not just the capacities of individuals that dictate human accomplishment, but also the affordances of the artifacts they use” (p. 19).

Hence, Brown (2009) sees this relationship in the same way as we do, as an interrelationship: that is, the activity of “designing” is not only dependent on the teacher’s competence, but that it is an interrelationship between the teacher/s and the (curriculum) material/s, the teacher-tool relationship, that is at play here, and hence the affordances of the curriculum materials influence this relationship. This is in line with recent theoretical frames (e.g. the Documentational Approach to Didactics (see Gueudet and Trouche 2009; Gueudet et al. 2012); or the framework for enactment by Remillard and Heck 2014).

As mentioned elsewhere (Pepin et al. 2017b), we also understand this notion of “design” (a) as the practice of designing for teaching, as in lesson preparation (that is design before enactment), (b) as well as in teaching, what we labeled as “design-in-use” that happens during enactment of the resources/materials (e.g., Pepin et al. 2013). However, we emphasize that any understanding of teacher as designer must include a **conscious/deliberate act** of designing, of creating ‘**something new**’ (e.g., combining existing and novel elements) in order to reach a certain (didactical) aim. This must be true for both (a) and (b). Hence, any act of designing (in my view) would include the teacher being explicit (and conscious), in particular about the following:

(1) Why s/he is designing in this particular way;
(2) Which aims/goals s/he has in mind;
(3) Who is his/her audience;
(4) Which resources and tools is s/he using for the design;
(5) How s/he plans to evaluate the designed.

This brings us to what is coined in the literature as ‘pedagogical design capacity’ (PDC) (Brown 2009) or ‘teacher design capacity’ (TDC) (Pepin et al. 2017b). Brown describes PDC as the capacity to utilize and transform existing curricular resources effectively, and to design/create new materials, for the purpose of effective mathematics instruction. He identified three types of curriculum use: offloading, adapting, and improvising, and proposed a framework for describing teachers’ capacity to design with curriculum materials (Design Capacity for Enactment Framework) which was later further developed by Remillard (e.g. Remillard and Heck 2014). Pepin et al. (2017b) describe TDC in a more goal-oriented way by emphasizing that TDC should include the following component:

- A goal, an orientation, or point/s of reference for the design;
- A set of design principles, which must be firm but flexible;
- A “reflection-in-action” type of implicit understandings and realizations (most visible in action, that is during teaching as “design-in-use”).

(for more explanations refer to Pepin et al. 2017b, p. 802/803)

It should be noted that (1) both conceptualizations/definitions are, in my view, complementary—they do not oppose each other; (2) both are theoretically underpinned by socio-cultural theory, in particular the literature on the mediating role of artifacts (e.g., Wartofsky 1973); and (3) it is clear that design/ing is a purposeful activity, and should not be interpreted as an 'subconscious' act.

17.3 From Artifacts to Instruments

In this section I would like to briefly refer to socio-cultural theory underpinning the conceptualization of teachers' interactions with curriculum resources, and how 'artifacts' develop into 'instruments'. I argue that this has implications for the conceptualization of 'teacher design', and in turn what is needed to develop teacher design capacity.

Cole (1996) wrote:

> By virtue of the changes wrought in the process of their creation and use, artifacts are simultaneously *ideal* (conceptual) and *material.* They are ideal in that their material form has been shaped by their participation in the interactions of which they were previously a part and which they mediated in the present. (p. 117)

This focus on *artifacts* underscores the historical, human-made quality of tool/instrument design and use. Instruments/tools carry the residue of prior needs, questions, problems, and solutions, offering both affordances and constraints for the activity. As Pea (1985) stated,

> The design of artifacts, both historically by others, and opportunistically in the midst of one's activity, can advance that activity by shaping what are possible and what are necessary elements of that activity. (p. 50)

That is, when introducing new tools, this is likely to provoke changes in the (artifact-using) person/s (Wertsch 1998).

As mentioned earlier, teachers' artifacts for mathematics instruction include curricular materials, such as textbooks that they engage with in the process of planning lessons (e.g. Shield and Dole 2012). According to socio-cultural theory, artifacts are defined as materials created by humans, in this case, the materials are used for informational purposes (Wartofsky 1973). As teachers use, shape, and mold these artifacts to plan their lessons they generate a relationship with the curriculum resources that influences how they view and perceive resources as a means for instructional implementation. In this process, "the ways teachers attend to curriculum resources is influenced by … their capacity to competently use curriculum materials to enact particular forms of instruction" (Choppin 2011, p. 333).

Let us turn to 'instrument/s' now, in order to better understand the relationship between *artifact* and *instrument*, and the processes that develop when teachers interact with curriculum resources: the interrelated processes of *instrumentation* and *instrumentalization* (e.g. Trouche 2004). Rabardel and his colleagues (Rabardel 2002; Verillon and Rabardel 1995) explain that the *artifact* is the 'bare tool': in our case the 'material' curriculum resource, which is available to the teacher, for example, to prepare and enhance a lesson. Unless the teacher has the goal of improving this lesson, and the ‘intuition’ (and intention) that this particular resource can enhance his/her lesson, it would not be seen as useful for him/her. So, only after the teacher has become aware of how the artifact/tool can extend his/her capacities for the enhancement of his/her lesson, and after s/he knows how to use this resource for her/his lesson (improvement), only then becomes the resource/artifact, from this teacher’s perspective, a valuable and useful instrument that mediates his/her activity of teaching. Typically, the notion of instrument can be summarized as

Instrument = Artifact + Schemes of usage

Hence, the instrument encompasses both the artifact (e.g. the ‘bare tool’) and the accompanying ‘schemes’ that the teacher/teachers develop/s as they use the curriculum material/s for accomplishing specific kinds of tasks (e.g. designing a new lesson/series). This implies that instruments are always bound to an individual user, or a group of users. It has to be noted that the artifact does not need to be a material object, but can also be a digital one, for example. It is also clear from this explanation that an artifact only develops into an instrument when combined with purposeful use scheme/s (of the user). In other words, the development of an instrument requires a process of ‘intended use’, or interaction, or appropriation, which would allow the artifact to mediate the activity (a process called *instrumental genesis*). So, in the same way as the affordances and constraints of the artifact shape and influence the (thinking of the) teacher (the *instrumentation* process), the teacher’s understandings and preferences shape the ways s/he uses the artifact—the artifact is shaped by the teacher (the *instrumentalization* process). This theoretical link is used in the *Documentational Approach to Didactics* (DAD; see Gueudet and Trouche 2009; Gueudet et al. 2013; Trouche et al., Chap. 1, this volume). This interactive relationship is also well expressed in the term “mutual adaptation” in the curriculum design literature (e.g. Berman and McLaughlin 1977; Fullan 2007).

17.4 What Does This Mean for Teachers as Designers and Teacher Learning?

What we have learnt from the research literature and the chapters in this book: whether searching for tasks to supplement a given learning sequence, or planning learning paths through a flexible e-textbook, or adapt a given learning sequence to specific contingencies of their classroom (e.g. Visnovska and Cortina, Chap. 13, this volume), teachers will require help and support, in other words professional development, often provided in teacher collectives (e.g. van Steenbrugge et al., Chap. 8, this volume; Wang, Chap. 9, this volume; Visnovska and Cortina, Chap. 13, this volume). This is particularly relevant at times of curriculum change (see e.g. Rocha, Chap. 11, this volume), as Ball and Cohen (1996) have argued—they regarded curriculum materials as a lever for effecting change in classrooms.

What I argue in this chapter is that this kind of support needs to be deliberately and systematically addressed and organized, in order to help teachers develop a didactical sensitivity (e.g. Pepin 2015; Visnovska and Cortina, Chap. 13, this volume) to mathematical and pedagogical aspects of teaching and learning resources. Furthermore, they will need tools and educative materials to support their professional development, and to help them make effective use of this sensitivity. In other words, in order for a mathematics teacher to become a creative and didactically effective designer/developer of suitable curriculum resources (for his/her teaching), s/he cannot only rely on curriculum materials (developed for student learning, e.g. commonly used textbooks, whether they are digital or traditional texts), but also needs *educative curriculum materials* (Davis et al. 2017; Davis and Krajcik 2005; Krajcik and Delen 2017) in addition to professional didactical support in teacher collectives (Loucks-Horsley et al. 2003).

In theoretical terms this means that we (as researchers/teacher educators) do not only have the responsibility to investigate/attend to the *artifact* (see definition of instrument/document above), the “bare tool”, ‘as is’ (e.g. investigations of (e-) textbooks), but we have to outline the tools’ affordances (and constraints) for teacher learning, in other words their educative potential for teachers. What we have done to date (in mathematics education) is mainly researching their potential for student learning, often with less attention to teacher learning (or assuming that teachers learn with any curriculum resource). One of the few exceptions is the work of Cobb and Visnovska (e.g. Visnovska et al. 2012; Visnovska and Cortina, Chap. 13, this volume). Moreover, and as importantly, it is not only our responsibility to investigate teachers’ interactions with curriculum resources ‘as is’ (in theoretical terms, attend to the schemes of usage, that is how they use these materials at present), but also suggest/design/develop ways of how teachers can usefully work with these chosen ‘tools’. This brings us to *educative curriculum materials* for mathematics teachers.

The literature (in science and general education) has outlined *design heuristics* (e.g. Davis and Krajcik 2005) as design guidelines, and at a later stage *design principles* (e.g. Davis et al. 2017). In my view design principles also have a heuristic (not algorithmic) nature; at the same time, of course those principles will become more credible when they have a stronger empirical base.

They also distinguish between *curriculum materials* (i.e. resources designed to be used by teachers in classrooms to guide their instruction, e.g. Stein et al. 2007), and *educative curriculum materials* as those curriculum materials designed to support teacher learning as well as student learning (Davis and Krajcik 2005). Hence, educative features refer to “the elements in curriculum materials specifically intended to provide support for teacher learning” (p. 294, Davis et al. 2017).

The results of their study (Davis et al. 2017) showed the importance of educative curriculum materials for teacher learning (in science). They provided six “design principles” for educative curriculum materials (p. 301, Davis et al. 2017):

1. Teachers will adapt curriculum materials: these adaptations are likely to be informed by teachers’ concerns about time and student capabilities. By anticipating these adaptations, educative features can facilitate principled and

productive adaptations. Hence, educative features should provide suggestions for adaptations of lessons that would take different amounts of time and meet a range of students' needs.

2. Provide educative features that provide representations of practice in order to support teachers' uptake of the ideas in the features.
3. Teacher subject knowledge: designers should use multiple forms of support for highlighting important content.
4. Different teachers' needs: designers should develop a constellation of educative features that have the potential to meet these various needs.
5. Teachers' scientific explanations: educative feature should help teachers (a) appreciate the definition, intention, and value of constructing scientific explanations, and (b) learn how to support students engaging in explanation construction and argumentation.
6. Scientific practices (e.g. making and recording observations, making and justifying predictions): educative features should support easier-to-enact scientific practices. Designers should connect to teachers' existing teaching practices, to create leverage points while helping teachers recognize salient points.

Turning to mathematics education, Hill and Charalambous (2012) synthesized a set of related case studies (see their special issue in 2012), to develop insights about the relationships among curriculum materials, teacher knowledge, and mathematics instruction (in primary mathematics). However, their focus was not on educative curriculum materials; their focus was on teacher knowledge and quality of mathematics instruction. Choppin and his team (e.g. Choppin et al. 2014) also developed a typology for mathematics curriculum materials. More recently, in a special issue Pepin et al. (2017a) have examined issues related to e-textbooks and other types of digital curriculum resources, which have become more prevalent in mathematics classrooms, and which can potentially be regarded as educative materials for teacher learning. One of the sections relates to the analysis and design of mathematics curriculum resources, and it is interesting to understand the different points made by researchers and designers here. If teachers are to become designers, it is crucial that they develop insights not only into the different features of curriculum materials, but also in which ways these may influence, afford, or indeed may transform, particular educational processes and practices- educative curriculum materials are the link to designing more "new" materials.

Leaning on research presented in this book, and my experiences and development/research in selected European countries (e.g. in France and Norway—Pepin et al. 2017b), I suggest that mathematics education research has to provide its own design principles for educative curriculum materials to support teacher learning. At the same time, and as I have done in previous articles (idem), I lean on literature from the curriculum design/development knowledge area: in my view this is particularly beneficial, as over the past 30 years (in particular in the 1990s) much research on educative materials has been done in this knowledge field (e.g. van den Akker 1988).

First, what I learnt from working with teachers and readings in the field of curriculum development, the **functions** of the educative curriculum materials should be made explicit. For example, in general terms (and linking to the preparation, enactment and evaluation of a lesson) the following are pertinent functions of educative materials:

- With the educative materials the teacher should be supported to create a clear image of the student learning trajectory and his/her own role in supporting this learning, and hence to develop his/her own plan of action for the enactment.
- The educative materials should help the teacher to enact the lesson/s successfully, and in accordance with the original intentions (e.g. of the changed curriculum).
- The educative materials need to contribute to the teacher's learning process by providing support for reflection on how the lessons went, on his/her own role in the processes, and on his/her attitude to this proposed curriculum idea and sequence.

Second, in order to fulfill these functions, four groups of elements for educative materials can be proposed: those that concern (1) Lesson preparation; (2) Subject matter/content; (3) Pedagogical/didactical role (about the teacher's actions); and (4) Learning outcomes. In the following I suggest items that might be concretely considered for each of the four; I call them **design specifications** (amended from van den Akker 1988):

1. In view of lesson preparation, educative materials would have the function to support teachers to prepare a lesson trajectory on a particular topic, and what his/her role is in the lessons. Concrete design specifications could include:
    - Time estimations of subsequent parts of the lesson(s)
    - Suggestions for "shopping list" of concrete materials needed for each lesson
    - Reference to sources for further development of certain parts of the lesson
    - Suggestions for task orientations (that students need to do)
    - Support to be able to study lesson suggestions carefully, in order to anticipate student reactions
    - Suggestions for how to group students
    - Anticipations of what students can learn from tasks and lessons and expectations about learning results
2. In terms of subject matter/content, educative materials would have the function to provide sufficient support for the mathematical content base, in order to be able to anticipate problem areas/misconceptions and how to address those (e.g. when enacting and evaluating the lessons). Concrete design specifications could include:
    - Factual information on further sources on content area
    - Information on what are the core ideas, and what are the 'side aspects'; which activities relate to which concepts

  - Anticipation of potential student questions, and answers
  - Estimation of what is relevant for students, and what is ‘doable’
  - Information on which tools to use for which task/concept

3. In terms of the didactical role of the teacher, the function of educative materials would be to provide the teacher with directions about the ‘what & how’. Concrete design specifications could include:

   - Directions for grouping, role division, task instructions
   - How to distribute which materials
   - Lesson phasing/staging in terms of activities, and “fillings” (question/answers; suggestions for conversations; how to start activities; how to address problems in learning, etc.)
   - Outlining potential variations in the teaching-learning process, various possible scenarios

4. In terms of learning effects, the function of educative materials would be to provide the teacher with points of reference to adjust his actions. Concrete design specifications could include:

   - Giving examples of learning effects (mid term, final)
   - Suggestions to make those learning effects visible (e.g. assignments, tests, portfolios).

17.5 Towards Flexible Design Principles for Educative Curriculum Resources

Returning now to the **design principles** as presented by Davis et al.’s (2017), I find those ‘principles’, although they do contain a number of relevant tips, unsatisfactory in the sense that for me a ‘principle’ is a proposition that serves as the foundation for a chain/line of reasoning, that is it has

- A goal orientation (what do I want to design and why);
- An explanation and specification of what the design should look like; and
- A justification in the sense of why the design was developed as it was, and whether it worked as intended.

This notion links closely to the work of Visnovska and Cortina (Chap. 13, this volume), and what we outlined earlier for mathematics teachers’ design capacity (Pepin et al. 2017b).

In my understanding, design principles for mathematics education materials can be developed from aspects of curriculum theory (e.g. Thijs and Van den Akker 2009). First, we know from curriculum theory (and international comparative studies) that the curriculum can be viewed at different *curriculum levels* (e.g. nano, micro, meso, macro, supra levels) and at each level there are different *curriculum*

Table 17.1 Forms of curriculum (Thijs and van den Akker 2009, p. 10)

Intended	Ideal	Vision (rationale or basic philosophy underlying a curriculum)
	Formal/ written	Intentions as specified in curriculum documents and/or materials
Implemented/ enacted	Perceived	Curriculum as interpreted by its users (especially teachers)
	Operational	Actual process of teaching and learning (also: curriculum-in-action)
Attained	Experiential	Learning experiences as perceived by learners
	Learnt	Resulting learning outcomes of learners

products. In terms of teacher design we work at the so-called micro level, that is the teacher in the mathematics classroom, and the curriculum products are, for example, the teaching plan, instructional materials, modules, courses, textbooks. A second, clarifying distinction concerns the different forms in which curricula can be represented. Typically, there are three levels (i.e. the intended, the implemented/ enacted, and the attained curriculum), which can be split up in further two each, to make up six forms (Table 17.1).

The division into six representations, building on the work by Goodlad (1979), can be especially useful in the analysis of the processes (and the outcomes) of curriculum innovations. For our purpose, we stay mainly at the 'enacted' form (as we are concerned with teachers). However, educative curricula would support teachers developing insights at all levels, as a teacher designer would need to understand how what s/he is designing is embedded in the larger picture.

Leaning on our earlier definition of 'principles', we now want to go further and consider which aspects need to be considered for curriculum design, in order to refine our notion of 'design principles'. A clarifying way to visualize the relationship between the various aspects of the curriculum (see Table 17.2) is the so-called *curricular spider web* (van den Akker 2003).

The rationale serves as a central link, connecting all other curriculum components in the web. The spider web illustrates a familiar expression: every chain is as strong as its weakest link. In other words, if the educative material attends only to the content (and not to the other parts), the spider web might be pulled into one direction, and finally break. At the same time a spider web is relatively flexible. Hence, in order to develop consistency and coherence (in the curriculum material), ideally, all aspects should be connected to each other.

We have used these components (and questions) in a previous paper (Pepin et al. 2017b), to analyse how teachers, and professional designers, might design. Here I suggest that these ten questions together can lead to *flexible design principles* that should underpin each educative (and general) curriculum material. If the teacher designs a new learning sequence, these are the questions s/he has to answer for the design. If the educative material (designed for teacher learning) is to help teachers

Table 17.2 Curriculum components in question form (amended from Thijs and van den Akker 2009, in view of educative curriculum materials)

Component	Core questions
Rationale	Why are students expected to learn *this*?
Aims and objectives	Towards which goals are students expected to learn?
Content	What are students expected to learn in this module/trajectory?
Learning activities	How are students expected to learn *this*? (What learning experiences would make this learning possible for students?)
Teacher role	What is the teacher's role in this process?
Materials and resources	Which resources do students (and teachers) need to facilitate the accomplishment of the learning goals?
Grouping	With whom are students expected to learn?
Location	Where are students learning?
Time	When are students learning?
Assessment	How will student learning (on short and long term) be assessed?

to design, it has to help the teacher to answer these questions, and subsequently to apply them in the design.

At this point I want to close, although it would be useful to illustrate my points, in particular the design specifications, with concrete mathematical examples (e.g. from the PRIMAS project, see Pepin et al. 2017b). It would also be helpful to take reports from other design projects, to see how other subject areas have attended to the issue of design principles to support teacher learning. Would they be different from subject area to subject area? I suspect that, if they are 'principles', they might be similar if not the same, but that the design specifications might look quite different from subject area to subject area.

To conclude, I suggest that the book has contributed significantly to further our knowledge on mathematics curriculum resources, a milestone in the right direction for researchers as well as practitioners. For me personally, it has highlighted the different ways of perceiving curriculum resources, their design and enactment. Moreover, it has emphasized for me that as a mathematics education research community we might need further efforts to study educative curriculum resources, those that support teacher learning.

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