Shock Wave Development Within Expansive Flows

Beric W. Skews and Randall T. Paton

Abstract The propagation of two-dimensional expansion waves over a corner has received very little attention in the past. It can be studied experimentally in a standard shock tube by placing the test section at the end of the high-pressure section rather than the conventional method with placement at the driven section. It is found that reflected compression and shock waves can form as the wave propagates over the corner. An attached flow separation bubble develops at the corner and for a strong enough wave can develop a region of supersonic flow above it, and, in some cases, a transonic shock wave. Furthermore, for a sufficiently strong expansion wave, the flow behind the trailing edge of the expansion wave becomes supersonic. It has also been shown that when an expansion wave reflects off a wedge, shock waves can also develop due to the induced velocity flowing down the surface of the wedge striking the corner. The development of shock waves can also occur if the expansion wave focuses on a cavity.

1 Introduction

Recent studies of expansion wave dynamics show the development of imbedded shock waves under certain conditions, as described below. The experimental generation of a one-dimensional expansion wave is to place a test section in the driver section of a shock tube. The basic properties of one-dimensional expansion waves can easily be derived from standard shock tube analysis and have been presented by Mahomed and Skews [1]. A shock tube wave diagram is shown in Fig. 1 in order

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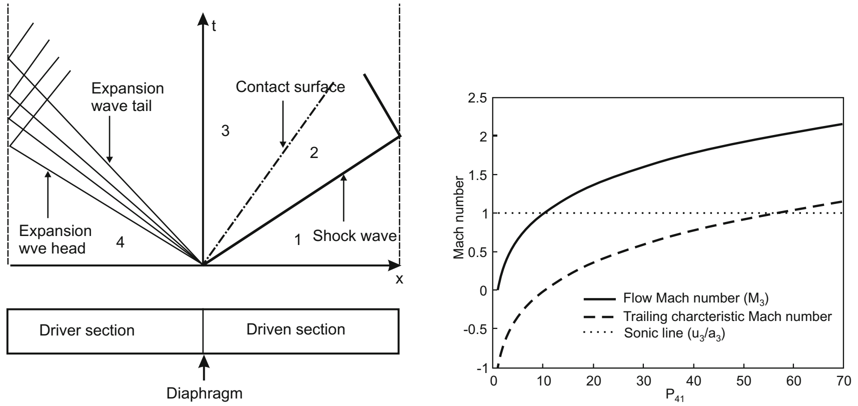

Fig. 1 Shock tube wave diagram, and properties across the expansion wave as a function of diaphragm pressure ratio

to define the flow regions. The head of the expansion wave travels to the left into region 4 and induces flow to the right. The flow in region 3 into which the tail of the expansion moves has the same pressure as that in region 2 behind the shock but is at a lower temperature having passed through the expansion wave. These regions are separated by the contact surface which are then at different Mach numbers even though the velocities are the same.

Relating to considerations of shock wave development, it is of interest to establish conditions for the development of supersonic areas of flow. In the one-dimensional flow case, region 2 becomes supersonic at a shock Mach number greater than 2.07 corresponding to diaphragm pressure ratios above 40. However, in region 3, between the tail of the expansion wave and the contact surface diaphragm pressure, ratios only above 10.4 are required. This is shown in Fig. 1. At this pressure ratio of 10.4, the tail of the expansion will be stationary because it is propagating at sonic velocity in an opposing sonic gas flow. For pressure ratios above this value, the tail of the wave will propagate to the right into the supersonic flow and will have an absolute Mach number as shown in the figure. Two-dimensional expansion wave dynamics is achieved by placing a test section at the end of the driven section in region 4. If further expansion is to occur such as for diffraction over a convex profile, supersonic regions will then occur at lower pressure ratios.

Two basic cases are considered here: reflection and diffraction over two-dimensional concave and convex corners, respectively. In all the examples given, the expansion is propagating to the left thereby inducing flow in the opposite direction.

2 Diffraction over a Convex Corner

The basic flow field for diffraction over a 90° corner is shown in Fig. 2 and has been analysed by [1]. A separation bubble, which remains attached to the corner, is generated as the flow develops. Since the head of the wave moves at sonic velocity, its diffracted portion is a circular arc. However, as the flow expands around the corner, reflected compression waves are generated back into the flow, in the opposite way as a shock wave emerging from a shock tube results in reflected expansions back into the tube in order to satisfy the exit pressure boundary condition.

In this diffraction case, two situations occur when shock waves can be generated [1]. One results from the convergence of the reflected compression waves into a shock wave. The formation of this wave depends on a number of factors: the pressure ratio across the shock tube diaphragm, P_{41}, which determines the strength of the expansion; the distance of the diaphragm from the corner, D, which determines the width of the wave at the corner when the leading edge arrives there; and the time after the start of diffraction, t. A typical result from numerical simulation is given in Fig. 2.

The effect of diaphragm pressure ratio and diaphragm position is illustrated in Fig. 3. The Further the diaphragm is away from the corner; the expansion wave will be wider when it reaches the corner for the same initial pressure ratio. This results in the reflected compression wave spreading out before a shock wave develops. On the other hand, a shock wave develops earlier when the pressure ratio is lower as shown in Fig. 4.

The steepening up of the compression is demonstrated in Fig. 5 by plotting the pressure distribution along a vertical line such as that shown as a red line in Fig. 4. The initial drop in pressure is because the line passes through part of the separation bubble and is due to the low pressure caused by the vortex. As the diaphragm is moved further and further away from the corner, the wave becomes more spread out and thus it takes more time to steepen up. Unfortunately, the domain in the

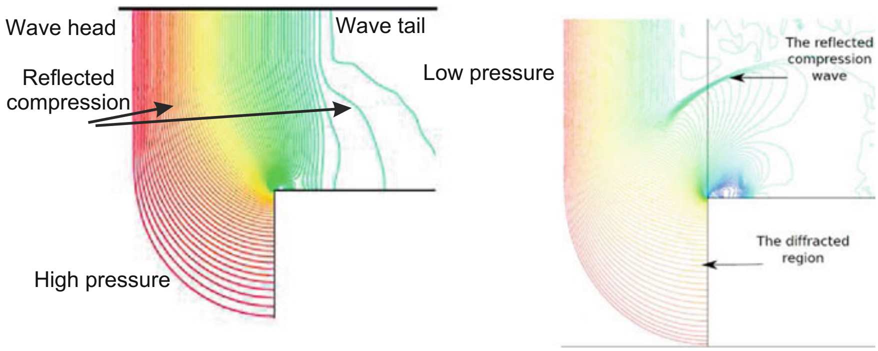

Fig. 2 Expansion diffraction over a corner, with and without shock wave development. Contours of pressure

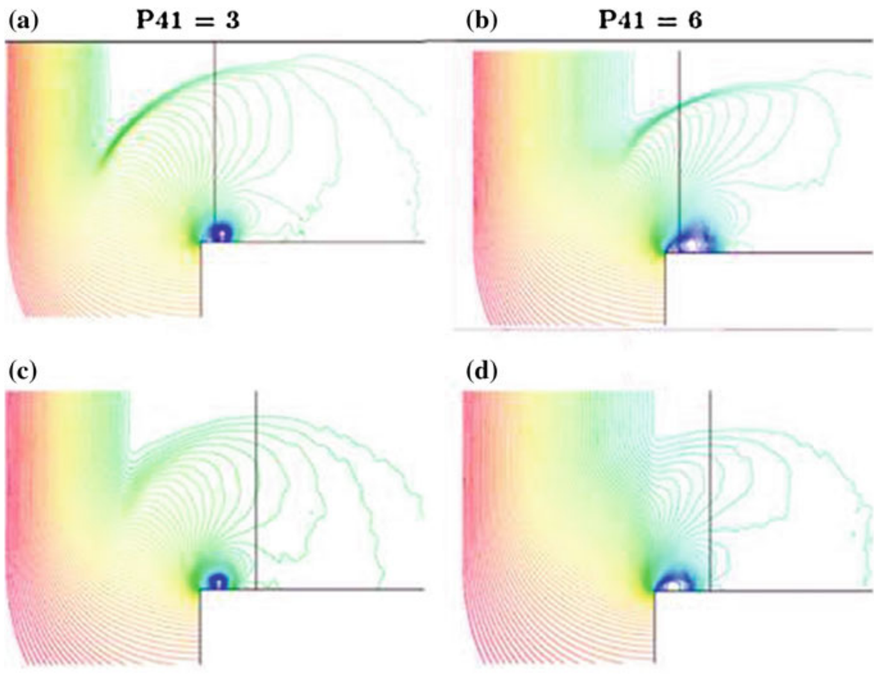

Fig. 3 Effect of pressure ratio and diaphragm position. Top row $D = 10\,\text{mm}$, Bottom row $D = 40\,\text{mm}$, $t = 400\,\mu\text{s}$

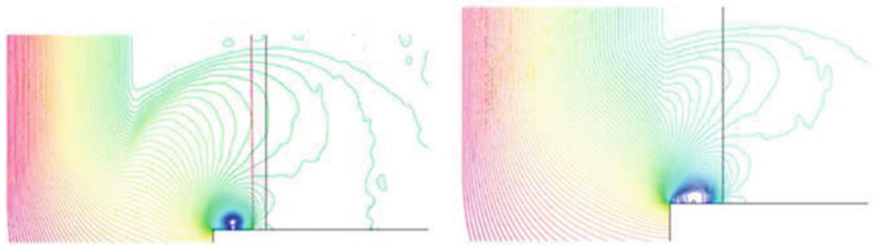

Fig. 4 Effect of pressure ratio. $P_{41} = 3$ and 6, $D = 40\,\text{mm}$, $t = 450\,\mu\text{s}$

experiment was not large enough to capture these waves due to reflection from the boundaries of the test section.

For shock waves to develop within the overall flow, regions of supersonic flow need to develop. The influence of the separation bubble's curvature on the nearby flow field causes acceleration of the oncoming flow which leads to a reduction of pressure (including temperature and density), above the separation bubble. A shock was found to develop, through experiment and simulation, under certain conditions. Two areas of supersonic flow can occur: one as described above due to flow over

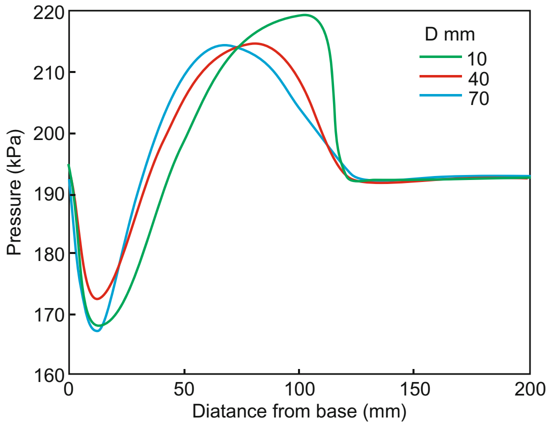

Fig. 5 Vertical pressure distribution 30 mm from the corner. $P_{41} = 3$, $D = 10, 40$ and 70 mm, $t = 400\,\mu s$

the bubble and one due to a sufficiently high diaphragm pressure ratio as described previously.

The development of a shock wave due to the flow over the separation bubble depends on the diaphragm pressure ratio, the position of the diaphragm, and the time from the start of diffraction. In the figures below, the supersonic patch is circumscribed by a red line. For $P_{41} = 3$, a patch only starts to appear at a time of $t = 300\,\mu s$ with $D = 0$ mm and not at the same time for $D = 40$ mm although it does develop then at $t = 400\,\mu s$. Enlarged views in the vicinity of the bubble for $t = 600\,\mu s$ are given in Fig. 6. No shocks are present above the bubble but interestingly there is a small patch within the bubble which may terminate in a shock. The flow within the bubble is complex as shown in the enlargement at the bottom of the figure.

As the patch grows, a shock will develop on the bubble surface as it does for a transonic airfoil, as shown in Fig. 7. The supersonic area continually increases as shown by the sonic line. No shock wave is evident at early times but as the separation bubble grows and the velocity over it increases, a transonic shock wave develops. It is not a normal shock as is evident by the fact that the flow immediately behind it is still supersonic. The obliqueness becomes more apparent at later times.

Some evidences of these waves are evident in the experiment, as shown in Fig. 8, but are not as well defined. Part of the reason for this is that the expansion wave generated does not have the plane front as assumed in the simulation but is curved under the influence of the initial pressure difference across it, so that when it bursts the flow is more complicated and non-uniform, including transverse effects, and the bubble has a more rounded profile. The surface of the bubble is also uneven due to vortices and turbulence.

The other area where supersonic flow can occur is for pressure ratios larger than 10.4, where the flow in region 3 becomes supersonic and the trailing characteristic of the expansion propagates in the same direction as the flow, since it is propagating

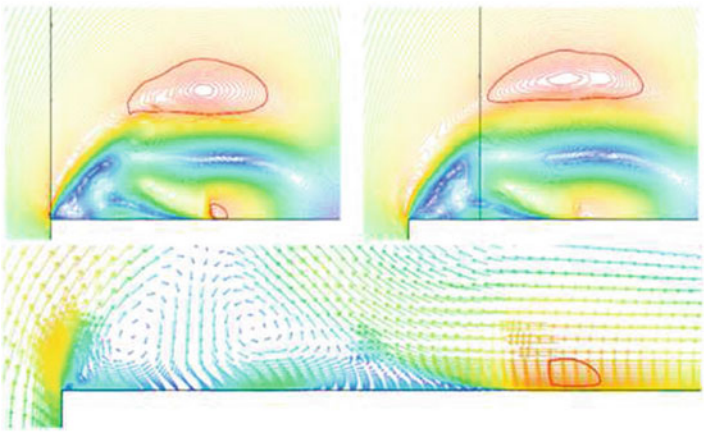

Fig. 6 Mach number contours for $P_{41} = 3$ and $D = 0$ and 40 mm, at a time of 600 μs. Enlargement shows a density contour plot adjacent to the surface

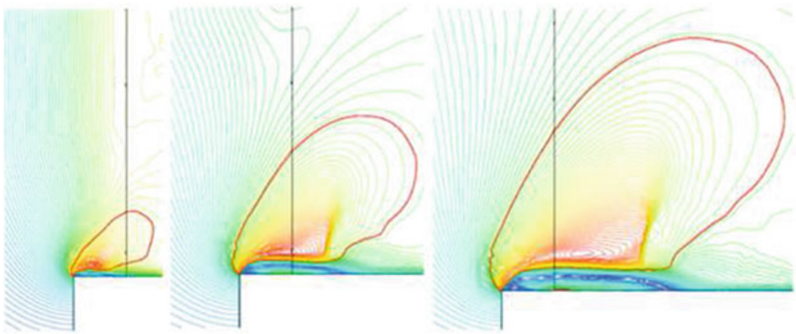

Fig. 7 Mach number contours for $P_{41} = 9$, $D = 10$ and times of 100, 200 and 300 μs

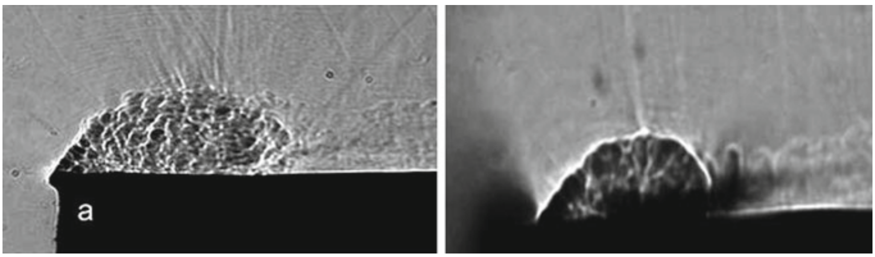

Fig. 8 Shadow and schlieren images of separation bubble showing shocks

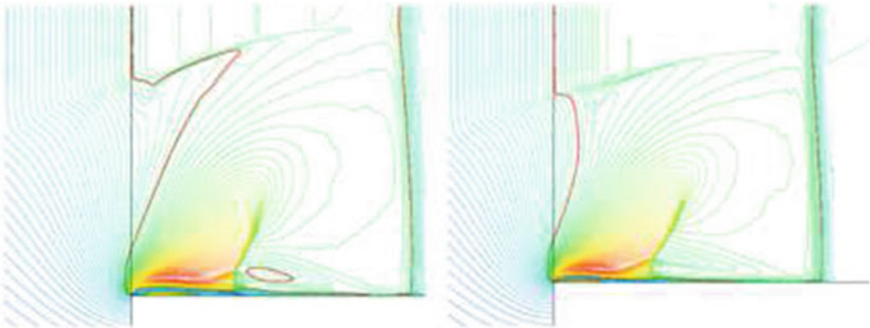

Fig. 9 Mach number contours for $D = 0$ and $P_{41} = 12$, (left) and 15 (right) at a time of 300 μs

at sonic velocity relative to this flow. Two cases are shown in Fig. 9. It will be noted that the sonic line is positioned in the contact surface since the flow on the other side, in region 2, will still be subsonic for the pressure ratios shown. The sonic line to the left of this supersonic region is at the position of the diaphragm in the region where the expansion wave is still undisturbed by the presence of corner. It then becomes distorted as it enters the flow generated by the reflected compression wave from the corner. It is also noted that the shock develops a lambda configuration and in the left-hand case a small region of subsonic flow appears.

3 Reflection Off a Concave Wedge

There has been very little work done for the case of a plane expansion wave, as generated in an ideal shock tube, encountering a plane wedge. As the wave moves up the wedge, the flow is induced downward back towards the corner at the wedge leading edge, and as could be expected for a strong enough wave, a shock wave will develop at the compressive corner, as indicated by the numerical simulation given in Fig. 10. C is the receding contact surface and D is the initial position of the diaphragm. The flow down the surface sees the corner as the entrance to a concave surface and an oblique shock may develop. The flow is clearly complex since the velocity along the surface is continually changing, both because of the properties of the wave propagation up the wall and the fact that the wave is continuously becoming wider. At the same time, the density and temperature are dropping. Thus, the position of the diaphragm and the initial wave pressure ratio will have an influence on the shock strength and its appearance.

Experimentation in the study of such flows is very demanding for a variety of reasons. Visualisation of an expansion wave is particularly difficult because the gas densities are low and the density gradients very shallow, so both shadowgraphy and schlieren require very high sensitivity. For this reason, in order to keep the wave

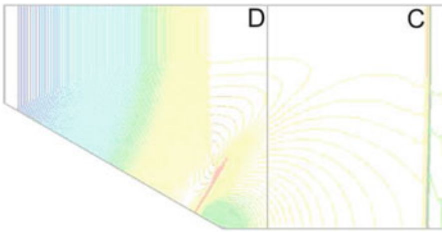

Fig. 10 Shock wave development on a plane wedge at late times

narrow, the diaphragm position needs to be very close to the test section. However, the diaphragm is initially curved under the pressure difference across it which results in an initially curved expansion wave. This is compounded by the finite opening time of the diaphragm, so if the opening starts at the centre and then propagates outwards, the shape of the issuing expansion wave will also be modified. Clearly, detailed studies will need to be undertaken to characterise these effects so that they can be incorporated in the numerical modelling.

Some experimental results are given in Fig. 11 for wall angles of 15° and 30°. The expansion wave is propagated to the left and has already passed over the back end of the wedge. The flow up the rear face separates in a bubble at the rear corner, in the same way as has been found in expansion diffraction tests, and remains attached there. However, even before this happens, a complex layered boundary layer is developed. At these early stages, no shock waves are evident, although these develop at a later time as shown. What is particularly interesting is the way this turbulent boundary layer curves as it approaches the corner. There may even be a circulating region at the corner itself but this has not been identified yet. However, there is evidence of the flow separating at the corner. Furthermore, this does indicate that the shock wave will be initiated by a compression fan arising from this curved flow.

Similar effects occur in the study of an expansion wave propagating into a cavity. These are interesting cases because of the very low pressures and temperatures that can result due to wave focusing. The first image in Fig. 12 shows the wave entering the high-pressure driver section before the leading edge has reached the apex of the cavity. The shock waves from either side propagate up the side walls into the very low-pressure region left behind after wave reflection, and cross over each other forming reflected waves. They subsequently merge and propagate back up the tube. Initial temperature is 300 K and a channel pressure is 83 kPa. With a starting diaphragm pressure ratio of 7, this results in a minimum temperature at focus of 80 K, and pressure of 6.8 kPa.

Similar results occur for a curved wall. Figure 13 shows typical results for a cylindrical cavity. As the head of the expansion engages with the curved surface, flow is induced out from the cavity which then experiences a concave surface and generates compression waves which merge into a shock that propagates up into the cavity.

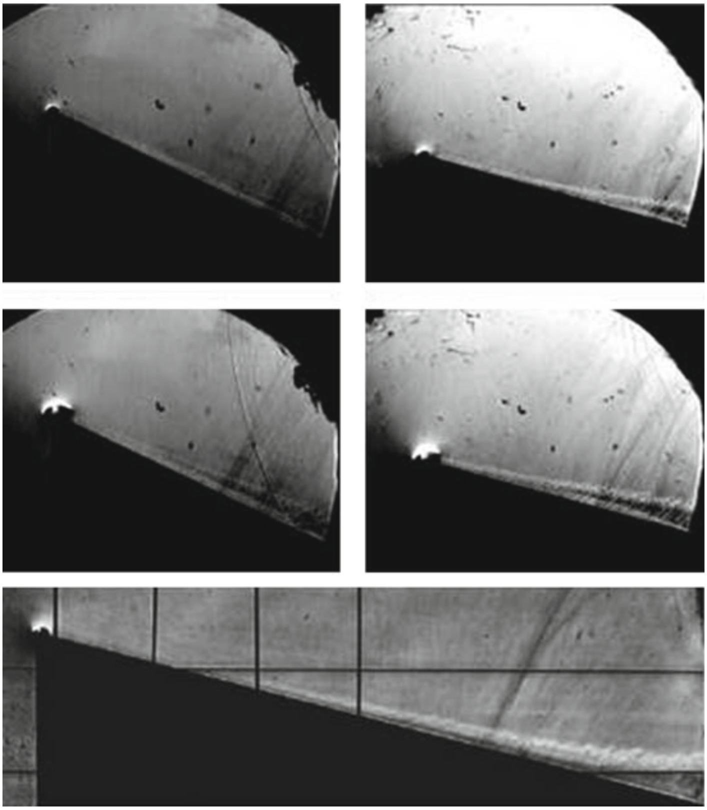

Fig. 11 Experimental images of expansion diffraction up a wedge

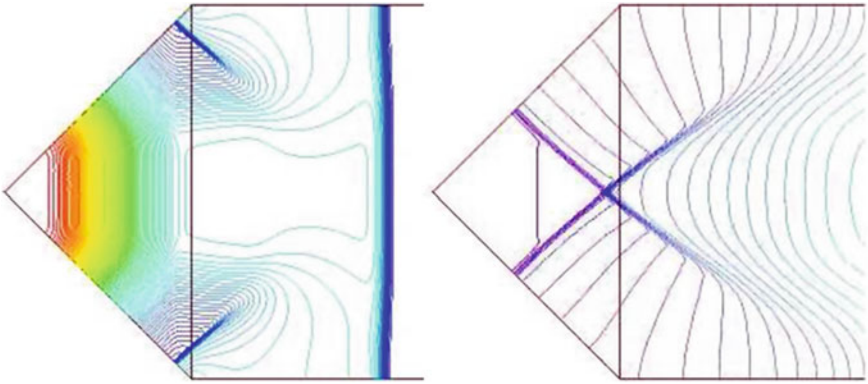

Fig. 12 Expansion wave propagation into a two-dimensional triangular cavity

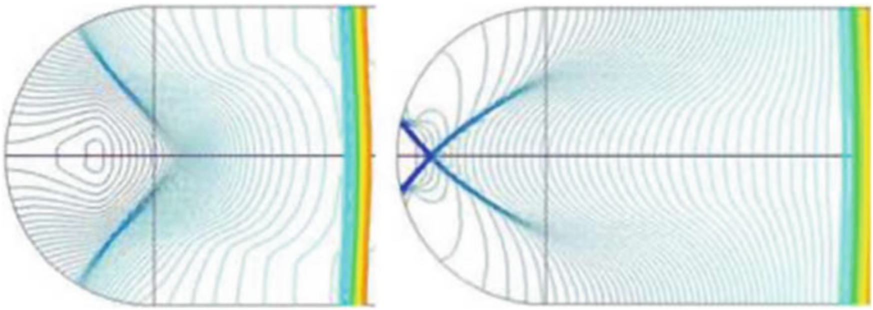

Fig. 13 Expansion wave propagation into a two-dimensional cylindrical cavity

This occurs on either sides of the symmetry plane, and the two shocks then reflect off each other as shown and subsequently interact with the cavity walls and eventually reflect out of the cavity. In this case, minimum pressures and temperatures are 33 kPa and 125 K.

4 Conclusions

It is demonstrated that a plane expansion wave encountering a change in surface slope in two dimensions can generate embedded shocks. These can result from either the development of compression waves which consolidate into a shock or because of local areas of transonic or supersonic flow being generated. Initial experimental studies show a number of features evident in the simulations but for both the concave and convex walls show complex turbulent flow features as the expansion wave propagates. Experimental visualisation of expansive flows presents many challenges so normally only the shocks become evident.

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