Chapter 209
Orbital Dynamics Using Pseudo-Newtonian Potential

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209.1 Introduction

The Pseudo-Newtonian Potentials (PNPs), which are constructed/proposed to replicate few general relativistics features approximately in Newtonian framework, are often used to study inner relativistic dynamics of the accretion flow around spacetime geometries describing black holes. For a general class of static spherically symmetric space time metrics $ds^2 = -f(r)^{\beta} c^2 dt^2 + \frac{1}{f(r)^{\beta}} dr^2 + f(r)^{1-\beta} r^2 d\Omega^2$, where $f(r)$ is the generic metric function, β is an arbitrary constant parameter, the PNP can be written as [1]

$$V_{\text{GN}} = \frac{c^2(f^{\beta}-1)}{2} - \left(\frac{1-f^{2\beta-1}}{2f^{2\beta-1}}\right)\left[\frac{f^{2\beta}-1}{f\left(f^{2\beta-1}-1\right)}\dot{r}^2 + r^2\dot{\Omega}^2\right]. \quad (209.1)$$

The particle trajectories can be obtained by solving the Lagrangian equations for the potential given in (209.1). In the below, we studied particle trajectories for a well known naked sigularity spacetime - Janis-Newman-Winicour (JNW) metric for which $\beta = \gamma$ and $f(r) = 1 - \frac{2r_s}{\gamma r}$ where, $0 < \gamma \le 1$. The geodesic equations are

$$\ddot{r} = -c^2\left(1-\frac{2r_s}{\gamma r}\right)^{3\gamma-1}\frac{r_s}{r^2} + \frac{2\dot{r}^2}{\left(1-\frac{2r_s}{\gamma r}\right)}\frac{r_s}{r^2} + \left[r - \frac{r_s}{\gamma}(1+2\gamma)\right]\left(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2\right), \quad (209.2)$$

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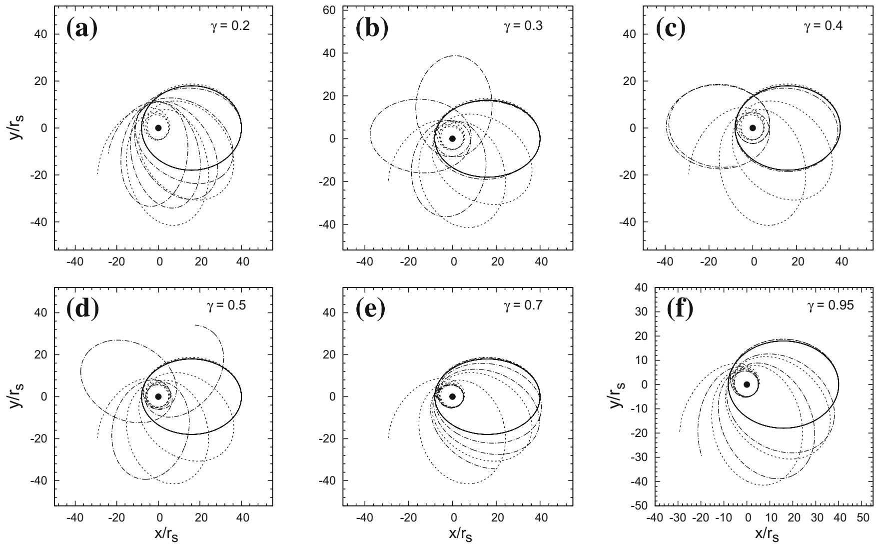

Fig. 209.1 Comparison of elliptic like trajectories of particle orbit in equatorial plane in JNW spacetime with those in Schwarzschild and Newtonian cases projected in the x-y plane. Solid and short-dashed lines corresponding to Newtonian and Schwarzschild cases, respectively. Long dotted-dashed curve in Fig. 209.1a, b, c, d, e, f are for $\gamma = 0.2, 0.3, 0.4, 0.5, 0.7, 0.95$, respectively. The particle starts from apogee with $r_a = 40r_s$ with $v_x = 0.0$ and $v_y \equiv v_{\text{in}} = 0.092$ (in units of c). For $\gamma < 0.2$, no proper well defined elliptic like orbits exist with the orbital parameters chosen here

and

$$\ddot{\phi} = -\frac{2\dot{r}\,\dot{\phi}}{r}\left[\frac{\gamma r - r_s(1+2\gamma)}{\gamma r - 2r_s}\right] - 2\cot\theta\,\dot{\phi}\,\dot{\theta}\,,\ddot{\theta} = -\frac{2\dot{r}\,\dot{\theta}}{r}\left[\frac{\gamma r - r_s(1+2\gamma)}{\gamma r - 2r_s}\right] + \sin\theta\,\cos\theta\,\dot{\phi}^2 \tag{209.3}$$

For particle dynamics along circular orbit, $\dot{r} = 0$ and $\ddot{r} = 0$. The particle trajectories obtained by solving (209.2)–(209.3) are shown in Fig. 209.1.

209.2 Conclusions

The test particle dynamics along circular orbit in JNW space-time departs to those in Schwarzschild geometry. The stated deviation is larger for smaller γ.

References

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