Chapter 5
Mathematical Creativity and Its Subdomain-Specificity. Investigating the Appropriateness of Solutions in Multiple Solution Tasks

Maike Schindler, Julia Joklitschke and Benjamin Rott

Abstract In mathematics education, researchers often talk about mathematical creativity. However, we see a lack of research on the question of whether such an ability exists for mathematics in general; or whether mathematical creativity should rather be viewed subdomain-specifically; for instance, in the contexts of geometry, algebra, or arithmetic separately. In this paper, we present results of an empirical study investigating upper secondary school students' performances in Multiple Solution Tasks (MSTs). First, we elaborate on the notion of appropriateness and its influence on the investigation of creativity; and illustrate implications based on the given data. Second, we give an insight into students' performances along three different MSTs from different mathematical domains and point out correlations between students' performances in two domains: geometry and algebra. Our results do not support the construct of domain-specific or subdomain-specific creativity, but indicate that mathematical creativity should be considered task-specifically.

Keywords Mathematical creativity · Multiple Solution Tasks (MSTs)
Appropriateness · Domain-specificity

M. Schindler (✉)
Faculty of Human Sciences, University of Cologne, Cologne, Germany
e-mail: maike.schindler@uni-koeln.de

J. Joklitschke
Faculty of Mathematics, University of Duisburg-Essen, Duisburg, Germany
e-mail: julia.joklitschke@uni-due.de

B. Rott
Faculty of Mathematics and Natural Sciences, University of Cologne, Cologne, Germany
e-mail: benjamin.rott@uni-koeln.de

F. M. Singer (ed.), *Mathematical Creativity and Mathematical Giftedness*,
ICME-13 Monographs, https://doi.org/10.1007/978-3-319-73156-8_5

5.1 Introduction

Creativity is an ability that is mostly associated with arts or music. Famous artists such as Mozart or van Gogh composed or painted numerous and outstanding pieces of art and are therefore regarded as creative people. Moreover, creativity can be seen as an important aspect in problem solving: Activities such as generating new solutions or elaborating on extraordinary and rare ideas may involve creativity to some extent. With the growing demand of our society for innovation and creative solutions to complex problems in domains such as technology, engineering, or natural sciences—domains in which mathematics plays a crucial role—mathematical creativity is gaining increasing significance. It is important to foster students' creativity (Sheffield 2009, 2013; Silver 1997; Hershkovitz et al. 2009) to both prepare students for their current and future lives in modern societies and to face the needs that our society encounters now and in the future. Whereas research has focused on creativity of outstanding mathematicians and exceptional people for a long time (Hadamard 1954), recent research has increasingly addressed creativity of everyone, especially students' creativity (e.g., Mann 2005).

In mathematics education, research aiming to understand and grasp students' mathematical creativity is so far quite rare (Leikin and Pitta-Pantazi 2013). Approaches to investigate mathematical creativity include problem solving as well as problem posing (Silver 1997; Leikin 2009). Many of the existing studies draw on students' products (e.g., written solutions or drawings) to assess mathematical creativity (Leikin and Lev 2013; Kattou et al. 2015). Performances are quantified along the dimensions fluency, flexibility, and originality, which originally arose from research on intelligence (Guilford 1967). Moreover, Kattou et al. (2015) found that creativity needs to be regarded domain-specific (within mathematics) and not domain-general. However, previous research has not yet sufficiently clarified whether students' performances in different creativity problems correlate and, thus, can be attributed to a single construct called students' mathematical creativity. The question of whether creativity is to be regarded a domain-specific or *sub*domain-specific ability is not yet clarified.

The purpose of this study is to investigate students' creative performances along different fields of mathematics—namely geometry and algebra. For this purpose, we conducted an empirical qualitative study with 21 upper secondary school students. For conducting a thorough data analysis, we first perused the question of which student solutions are to be taken into consideration when investigating creativity: Following Leikin (2013; Levav-Waynberg and Leikin 2012), we discuss the term *appropriateness* of solutions and investigate whether and to what extent solutions that are not correct, have flaws, or lead to a wrong solution may be regarded appropriate nevertheless. Based on these findings, we focus on students' performances along three different MSTs. In this first approximation to investigate the specificity of mathematical creativity, we found that the scores for mathematical creativity seem not to be consistent along different MSTs.

5.2 Theoretical Background

The growing demand in our society and economy for creativity is undeniable. People need skills to solve complex non-routine problems in extraordinary ways. Extraordinary and original ideas are not only important for managers and employees, but also for students in schools and high schools. Especially the fast development of science and technology requires more and more experts who are able to cope with these challenges. Special skills to solve problems creatively become more important for the coming generation, as Kattou et al. (2013) point out: "Given that students, as future citizens, will face problems that are unknown at present, it is especially crucial for them to be creative in order to efficiently tackle the challenges they will meet" (p. 180). Assessing and fostering mathematical creativity has accordingly become an important research field (e.g., Haylock 1987; Silver 1997). The recent development shows that this field is growing; in 2017 at ICME13, there was a distinct topic study group on "mathematics and creativity". The aim to learn more about creativity and the question of how to measure and foster creativity have become important for research in mathematics education. In this line of thought, Leikin (2009) points out: "I consider developing mathematical creativity in school students to be one of the important objectives of school mathematics education. This implies that tools for the evaluation of students' mathematical creativity are needed to realize the students' creative potential and to assess the effectiveness of various mathematical curricula" (p. 129).

In the following, we will outline theoretical aspects regarding mathematical creativity. First, we give an insight into concepts on creativity in general. Second, we focus on creativity in the field of mathematics. Third, we give an overview on methods to quantify products in order to assess mathematical creativity.

5.2.1 What Is Creativity?

So far, researchers are discordant about a coherent definition of creativity (Sriraman 2009). There are various concepts, which cannot all be addressed in this article. However, we give an insight into the beginnings of research on creativity and introduce some prominent concepts.

The current tradition of research on creativity started in the 1940s and 50s when Guildford (1950) conceptualized creativity as one component of intelligence (Guilford 1950). Guilford's psychological model of intelligence was the first one that comprised different forms of creativity. He differentiated, among others, between *convergent production* and *divergent production*: "Convergent production is in the area of logical deductions or at least the area of compelling inferences. Convergent production rather than divergent production is the prevailing function when the input information is sufficient to determine a unique answer. […] For example, if we ask, 'What is *the* opposite of HARD?'" (Guilford 1967, p. 171). In comparison to convergent production, he describes divergent production as "a concept defined in accordance with a set of factors of intellectual ability that pertain primarily to information retrieval and with their tests, which call for a number of varied responses to each test item. […] [These] tests require examinees to produce their own answers, not to choose them from alternatives given to them" (Guilford 1967, p. 138). Divergent abilities are "most relevant to creative performance. [For these abilities a] […] factor of *fluency* […], a factor of *flexibility* […] and [a] […] factor of *originality* materialized. Later, in a study of planning abilities, a factor of *elaboration* was expected and was demonstrated" (Guilford 1967, p. 169; emphasis by M.S./J.J./B.R.). The dimensions fluency, flexibility, originality, and—partly—elaboration are nowadays used in many creativity tests. In this context, *fluency* stands for the ability to come up with a multitude of produced answers. *Flexibility* is to be understood as the capability to generate answers in various ways. *Originality* means the uniqueness of answers, and *elaboration* the level of details of the solutions.

In addition to the question of *what creativity is*, the question of *who or what can be creative* is discussed (Leuders 2010; Liljedahl 2013; Rhodes 1961). Rhodes (1961) describes different strands of research on creativity as *four Ps of creativity* which are *product*, *process*, *person*, and *press*. Concerning *products*, Bailin (1988) states that creativity is reflected in certain achievements or rather products. Liljedahl (2013, p. 255), for instance, focuses on *processes* arguing "that such a use of assessment of end product pays very little attention to the actual process that brings this product forth". The focus on processes goes along with problem solving processes, mostly along the lines of processes as described by Wallas (2014) in his seminal work *art of thought*. Wallas first published his book in 1926 drawing on ideas by the French mathematician Henry Poincaré, describing stages of conscious and unconscious cognitive processes with a moment of illumination. According to Rhodes (1961, p. 308) "the term process applies to motivation, creativity to *persons* —mostly to persons who were considered to be a genius". For example, Kneller (1965) or Ghiselin (1985) investigated various geniuses. Rhodes completes his remarks with the influence of the *press*: "The term *press refers to the relationship between human beings and their environment*" (Rhodes 1961, p. 308). He concludes that these four aspects are strongly interwoven.

In our investigation, we focus on Guilford's components *fluency*, *flexibility*, and *originality* by analyzing students' *products*. We do not focus on the component *elaboration* as this aspect has rarely been considered in mathematics education research as we will point out below.

5.2.2 What Is Mathematical Creativity?

Referring to a product-based conception of creativity (see below) and using a confirmatory factor analysis, Kattou et al. (2015) showed that creativity is not domain-general but domain-specific: "Therefore, psychologists and educators should no longer characterize individuals as creative, but instead, as creative in specific domains" (Kattou et al. 2015, p. 1022). In this respect, Kattou et al. focus on students' products as well as on creativity as a personal trait. Whether one speaks of "students' creativity" depends on the underlying conception and definition of creativity. In mathematics, the term creativity is often considered along the lines of Poincaré's (1948) results regarding the Fuchsian functions. The four-step process described by Poincaré and later expatiated by Hadamard (1954) has become synonymous to mathematical creativity for some researchers (e.g., Liljedahl 2013). Accordingly, Sriraman (2009, p. 15) defines mathematical creativity "as the publishing of original results in prominent mathematics research journals." However, researchers such as Mann (2005) state that not only famous mathematicians can be creative but also everyone else, especially students (see also Sheffield 2009; Hershkovitz et al. 2009). This apparent contradiction has been addressed in the discussion labeled "big C", referring to extraordinary creativity, and "little c", referring to everyday creativity (cf. Sriraman et al. 2014). Our understanding of creativity is not limited to big C or the so-called genius approach on creativity (Hadamard 1954) but includes subjectively new results by students (e.g., Leikin 2009).

5.2.3 Methods to Evaluate Creativity

Guilford proposed ideas to measure components of creativity, for instance, in the form of his well-known Alternative Uses Test (1967). This test is closely related to the Brick Uses Test: Here, the participants are asked to name as many uses for a brick (or another common object) as they can think of in a certain amount of time. With a view to all of the four dimensions fluency, flexibility, originality, and elaboration, the creativity score gets higher, the higher each component is rated. The fluency score depends on the number of solutions. To rate flexibility, the number of different categories of uses is taken into account. If a participant names "building a house", "building a wall", and "building a floor" the flexibility is low, whereas "throw at a cat", "make bookends", and "make a filter" show a high level of flexibility (Guilford 1967, p. 143). The dimension originality is rated based on the relative frequency of the given answer in the focused group. Elaboration refers to the level of detail: Answers such as "use the brick to filtrate tainted water" are more elaborative than "make a filter" and would be rated with a higher score. This task can be managed without a specific content knowledge due to the ordinariness of a brick.

Based on Guilford's theory, Torrance (1974) developed the Torrance Test of Creative Thinking (TTCT). This test contains slightly altered versions of Guilford's test, called "Unusual Uses Activities", as well as additional subtests. Examples of these subtests include verbal items such as the "Ask and Guess"-test where the participants are supposed to ask questions to given drawings. Other subtests are constructed to be non-verbal such as the "Picture Completion"-test, which consists of incomplete figures that have to be completed. This test has earned a widespread acceptance for the analysis of creativity in different components.

5.2.4 Methods to Evaluate Mathematical Creativity

A domain-specific adaptation of Guilford's conception of creativity is suitable to get a more detailed insight into mathematical creativity, especially with a focus on problem solving (Leikin and Lev 2013). Therefore, domain-specific research in mathematics education has focused on these seminal ideas; and has adapted them to mathematics education (see Leikin and Pitta-Pantazi 2013). Leikin (2009; Levav-Waynberg and Leikin 2012) has introduced the concept of Multiple Solution Tasks (MSTs) within the domain of mathematics education: Mathematical tasks that are supposed to be solved in different ways.

Furthermore, Guilford's and Torrance's ideas on measuring creativity have been used: Based on this, researchers such as Leikin and Lev (2013) as well as Kattou et al. (2013) developed tests, which draw on *Multiple Solutions Tasks* (*MSTs*), where the students' products' fluency, flexibility and originality are evaluated. *Fluency* is scored by the number of given answers. For the *flexibility,* the students' solutions are classified depending on their diversity. *Originality* addresses the relative frequency of a given solution in comparison to the reference group of participants. The component *elaboration* is mostly not evaluated "due to the difficulty of determining levels of elaboration in mathematical tasks" (Kattou et al. 2013, p. 174). This approach is linked to the "little c" concept of mathematical creativity: solutions are analyzed with respect to the reference group with comparable prior experience.

In contrast to the brick task, some of the MSTs deal with more or less complex problem solving tasks from different fields of mathematics. As such, they may require a certain mathematical background, which—as pointed out by Leikin and Sriraman (2017)—may lead to a correlation of mathematical creativity and mathematical ability. Especially for evaluating mathematical creativity in cases of complex problem-solving tasks, it appears reasonable to not only take into account solutions that are entirely correct but also those that are *appropriate* (Leikin 2013). Levav-Waynberg and Leikin (2012) point out that also imprecise solutions are scored when assessing creativity; Leikin (2013) explains that the term appropriateness is used instead of the term correctness "to allow evaluation of reasonable ways of solving a problem that potentially lead to a correct solution outcome regardless of the minor mistakes made by the solver" (p. 391). The scores for each appropriate solution (fluency, flexibility, originality) result in a score for mathematical creativity across a set of MSTs. In this chapter, we want to connect to this idea and focus on the question of what makes appropriate solutions appropriate. Following Leikin's work, we elaborate on what kinds of flaws solutions may entail to still being regarded appropriate. Accordingly, our first research question is as follows:

> What kinds of flaws may appropriate solutions entail to still be regarded appropriate? What does appropriateness mean in terms of mathematical MSTs?

Furthermore, there are other approaches to assess mathematical creativity. According to Silver (1997) and Bruder (2001), creativity is not only assessable through problem solving but also through problem posing. The capability to generate subsequent questions to a mathematical phenomenon shows a high level of creativity (Silver 1997). Singer et al. (2017) address problem posing as well: Here, the concept of cognitive flexibility is crucial, which comprises the dimensions cognitive variety, cognitive novelty, and changes in cognitive framing. Singer et al.'s results indicate that a student's cognitive style might be a predictor for their mathematical creativity. In both cases (focusing on problem solving or posing), mathematical creativity is seen as a relative construct because researchers focus on students' performances.

Another—non psychometric—approach is, for instance, the psychodynamic approach (Sternberg 1999), which focuses on the shifts between conscious and unconscious mind. Liljedahl (2013) uses this approach when analyzing students' essays; and draws on a social-personality approach (Sternberg 1999), since he focuses on affective components. Here, mathematical creativity is also seen as a relative construct because students' reports are in the focus and students reflect on their creative moments.

Along these concepts of assessing mathematical creativity, we see that students' performances in certain tasks are taken into regard in order to assess students' mathematical creativity. However, it is not clear whether students' performances along different tasks are consistent. In this chapter, we therefore ask the question of whether one can speak of "students' mathematical creativity" in general—or whether mathematical creativity is rather to be regarded subdomain-specifically. This leads to our second research question:

> To what extent do students' performances vary across different MSTs? Does students' mathematical creativity differ between the sub-domains of geometry and algebra?

5.3 Methods Part I: Investigating Appropriateness

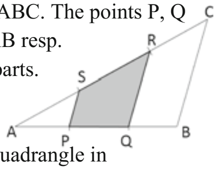
The investigation took place in a project at the University of Duisburg-Essen in Germany called MBF_2, which is a German acronym for *Focusing on mathematical giftedness—for upper secondary school level* (in German: Mathematische Begabung im Fokus—in der Sekundarstufe II). About 21 students (with slightly varying number from lesson to lesson) from different local schools participated in this project. They met every second week for a total of ten times. The students worked on challenging mathematical problems from different mathematical fields such as graph theory, cryptography, and spherical geometry. Furthermore, they worked on special problems that allowed us to investigate mathematically gifted behavior in upper secondary school children (Joklitschke et al. 2017). We focused on mathematical giftedness from different points of view: students' problem solving processes, student's products, and students' behavior. As some researchers see creativity as one component of gifted behavior (Renzulli 2002) we focused on mathematical creativity as well (see, e.g., Rott and Schindler 2017). There were no criteria for a previous selection of the students such as intelligence tests or certain grades in school. All participating students were highly interested in mathematics and attended this extracurricular course in their free time—independently from their regular schooling. In order to estimate students' cognitive abilities, we conducted the Culture Fair Test (CFT20-R) during the project span, which resulted in a measured IQ of 121 on average. Thus, the participating students may be considered above-average intelligent.

To assess mathematical creativity, we used MSTs and evaluated students' products. We used two geometrical problems (Figs. 5.1 and 5.2) and one algebraic problem (Fig. 5.3). For all problems, the wording of the general instructions was the same: "Solve the following problem. Can you find different ways to solve the problem? Find as many ways as possible." The three problems were given in different sessions and the students had a processing time of 30 min for each problem.

Fig. 5.1 MST "triangle" (see Novotná 2017)

Solve the following problem. Can you find different ways to solve the problem? Find as many ways as possible.

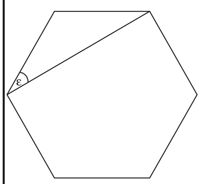
This figure is a triangle ABC. The points P, Q resp. R, S divide sides AB resp. AC each in three equal parts.



What is the area of the quadrangle in comparison to the area of the triangle?

Fig. 5.2 MST "hexagon" (see, e.g., Schindler et al. 2016)

Solve the following problem. Can you find different ways to solve the problem? Find as many ways as possible.



This figure is an equilateral hexagon. How big is the angle ε? Remember, in an equilateral hexagon, all sides have the same length and all angles have the same size, which is 120°.

Fig. 5.3 MST “punch” (see Leikin and Lev 2013)

Lena produces 80 liters of punch for the Christmas market. She fills the punch in equal shares into barrels.
When putting the barrels into the car, she notices that four barrels do not fit into the car and have to be left at home. Nevertheless, she wants to take all of the punch to the Christmas market. Therefore, she distributes the content of the four not-fitting barrels in equal shares to the other barrels.
After this, she realizes that she had added exactly ¼ of the previous amount to each of the barrels. How many barrels did she have in the beginning?

When evaluating students’ products, we used different schemes: First, we only took into consideration *correct* solutions. In a second analysis, we took into account solutions that we considered *appropriate*. Through this approach, we analyzed differences between appropriate and correct solutions in order to investigate appropriateness of solutions thoroughly. Our analyses were based on the rating scheme as used by the research group around Leikin (e.g., Leikin and Lev 2013); once based on correct solutions, once based on appropriate ones (similar to Leikin and Lev 2013). In the first version, which we call “strict” in the following, wrong answers were sorted out and not used in the scoring. In the *second analysis*, we lowered this level to evaluate the products even if they entailed certain flaws, mistakes, or imprecisions.

For considering whether the approaches were considered appropriate, we consensually validated whether the reasoning was sufficient. Based on this, we evaluated whether the approach would be included in the first way of analysis; or only in the second one. In both ways of evaluation, we calculated a total creativity score for each problem and each student. Following Leikin and Lev (2013), we used the following formula:

$$Cr = \sum_{i=1}^{n} Flx_i \cdot Or_i$$

Here, the flexibility score and the originality score are multiplied. Then, all (here n) solutions, respectively approaches (depending on the first or second analysis), presented by each student in one task are summed up to a total score.

5.4 Methods Part II: Students' Performances Across Three MSTs

The results of the first part of our study regarding appropriateness of solutions build the basis for the second part of our investigation, where we study students' performances in the presented problems across the disciplines geometry and algebra.

To analyze to what extent the students' performances vary across the different problems, we need a non-parametric test, which measures the ordinal association between the three MSTs pairwise. Therefore, we use Kendall's Tau Test, which is designed for small sample sizes. This test quantifies the similarity of orderings of students' performances.

5.5 Results Part I: Comparison of Two Evaluation Schemes

A first overview (see Table 5.1) displays how many students worked on the three different MSTs (14–21) and on how many approaches the students worked on average (1.5–3.5 on average).

In this section, we elaborate on the students' performances by illustrating meaningful examples for each of the three MSTs. We furthermore explain which differences occur when choosing the first or the second kind of analysis. In a second part, we compare students' performances by giving an insight into examples from students in all three MSTs.

In the first analysis based on only correct solutions, all non-correct approaches (in the above-mentioned sense) were sorted out in the beginning before calculating a creativity score. In the second analysis, we widened the spectrum of included approaches and included all approaches that were considered appropriate (see above). Table 5.2 displays the number of approaches we took into account in the first and in the second kind of analysis for each task. It displays the total creativity score; not the particular components such as flexibility or originality.

Furthermore, Table 5.2 displays the differences in creativity scores resulting from the first and second way of analysis. The number of students that worked on

Table 5.1 Overview of the number of students and approaches in the three MSTs

	Triangle-problem	Hexagon-problem	Punch-problem
Number of students	21	17	14
Number of approaches	32	59	19
Average number of approaches per student	1.5	3.5	1.4

Table 5.2 Students' performances in the three MSTs (considering correct vs. appropriate solutions)

	Triangle-problem		Hexagon-problem		Punch-problem	
	Correct	Appropriate	Correct	Appropriate	Correct	Appropriate
Number of approaches included	4	21	39	59	13	19
Number of students who have a creativity-score of 0	19	7	1	0	3	0
Number of students with an improvement (2nd vs. 1st analysis)	0	20	0	11	0	9

the triangle problem was 21, whereas 17 and 14 students worked on the hexagon and punch problem respectively. For the hexagon problem, the students found by far the most approaches (i.e. 59). We see that there are major differences in the number of approaches, which were included into the particular analysis. Especially in the triangle MST (4 correct vs. 21 appropriate approaches) and in the hexagon MST (39 correct vs. 59 appropriate approaches), the differences are remarkable. This has, of course, consequences for the students' creativity scores. In the triangle problem, 19 out of 21 students have a creativity score of 0 when the first—strict—analysis is applied (because these students do not have any entirely correct solution), but only 7 students have a score of 0 when the second analysis is used (because other, partly wrong approaches are furthermore taken into account). Moreover, in all three given MSTs, the second analysis leads to an improvement of students' creativity scores.

In the following, we use examples of students' approaches for each of the three problems in order to illustrate what makes students' approaches appropriate even though they may be wrong. We furthermore point out why we think it may be important to include also those approaches that have weaknesses but are comprehensible.

5.5.1 The Triangle Problem

Steven's approach (see Fig. 5.4) shows an algebraic calculation. He uses vectors to represent the corners *P*, *W*, *S*, and *R* of the given triangle, as well as the subdivisions of the line segment *AB*, resp. *AC* in relation to *A* which he chooses as the origin. In the next step, he tries to calculate the surface area of the gray quadrilateral. Therefore, he sees the area as a composition of a parallelogram and a triangle. After substituting the unknown lengths with the known ratios of $\frac{1}{3} \cdot AB$ and $\frac{1}{3} \cdot BC$, respectively, and after having set the lengths of the sides *AB* and *AC* to 1, he is able

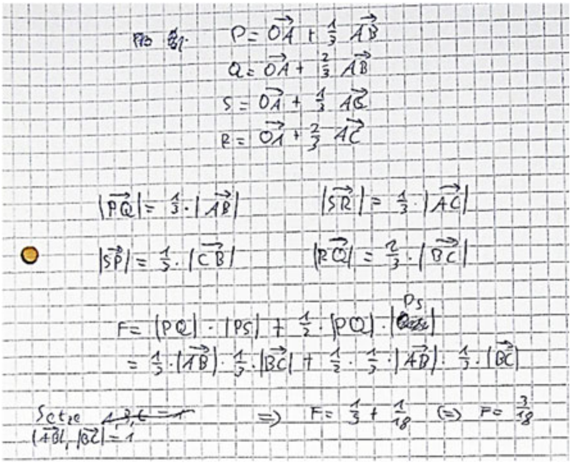


Fig. 5.4 Steven's approach (triangle problem)

to get to the result of 3/18. This outcome, however, does not refer to the triangle *ABC*, but to the parallelogram which can be constructed by adding a second triangle rotated by 180°. The last step, namely a division by 1/2, is missing. Therefore, this approach must be considered wrong. He misses the last step and, moreover, uses a special case to handle the problem. In consequence, if only correct solutions are to be considered, Steven gets 0 points for this approach. However, focusing on appropriateness, we can see a clear and comprehensible strategy. Indeed, the last step is missing and therefore, his conclusive answer is wrong, but his idea is well founded. Interestingly, no one else in our group of participants worked on this problem by using vectors, so that his approach shows a high level of originality. In the second analysis, this approach is included into the calculation scheme and raises Steven's creativity score.

The next example shows *Lilly's approaches* (see Fig. 5.5).

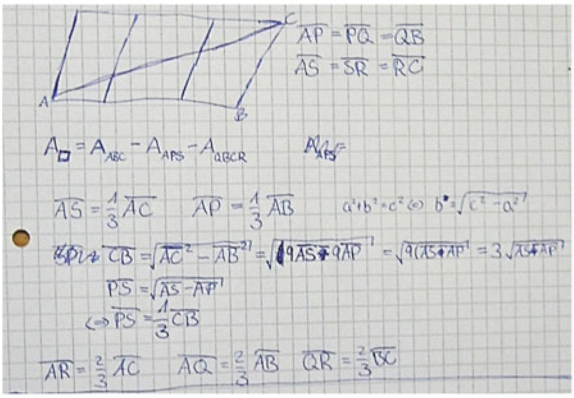


Fig. 5.5 Lilly's approach (triangle problem)

Lilly works on two ideas which are discussed in the following: On the top of her sheet, there is the triangle *ABC*, which she completes to a parallelogram by rotating the triangle at the middle of the line segment *BC*. Furthermore, she extends the lines *PS* and *QR*. To the right of Lilly's sketch, there are two lines of equations, which illustrate the same lengths for the different sections of *AB* and *AC* respectively. In the rest of her notes, she does not refer to this sketch. Of course, she refers to the triangle, but she does not elaborate on the parallelogram anymore. In her notes, she tries to express the gray surface area by building differences (see the line underneath the sketch in Fig. 5.5). Following this line of thought, to determine the particular surface area, she uses the Pythagorean Theorem to determine the lengths of some line segments but does not have any success. Finally, she writes down the proportion of the line segments and the two sides of the triangle. We see two very different approaches with which Lilly tries to solve the triangle problem. In the first one, she has the important idea of building a parallelogram but she does not proceed with this idea, even though she extends the line segments. When only correct solutions are to be considered, this solution is not included into the evaluation scheme, because there are no notes referring to the sketch or a conclusive answer. When appropriate solutions are in focus, we can see that she completes the triangle to a parallelogram. This move constitutes an important and creative part to come to a right solution. Especially the line extensions can be a hint at a reasonable idea. For this reason, we decided to include this approach in the second way of analysis. Lilly's second attempt is about the Pythagorean Theorem. Lilly tries to use this, even though the triangle is not right-angled. Furthermore it is not clear why she tries to calculate lengths of the line segments. We were not able to understand her strategy in this case. This example shows that there were cases in which an approach cannot be taken into account for a further evaluation with neither the strictly dichotomous first nor the broader-viewed second kind of analysis.

5.5.2 The Hexagon Problem

The hexagon problem was the second MST the students worked on. Most students integrated many ideas and approaches within one picture. Therefore, it is difficult to identify the particular approaches. In the following, two examples are presented, which have some similarities.

Figure 5.6 displays many approaches as carried out by Lilly. In the paper at hand, we have a closer look at one particular approach.

In this approach, Lilly draws a line from *A* to *D*. Together with the given line *AE* she writes that her auxiliary line halves the interior angle *BAF* in two angles of equal size, namely 60°. Then, she argues that the given line *AE* leads to a second halving, so that she comes to the conclusion that the angle ε is 30°. However, there is no reasoning provided about *why* the mentioned lines exactly halve the focused angle. Probably she had reasons of symmetry in mind, but we can neither be sure about this nor is symmetry a trivial argument in this problem. The non-trivial

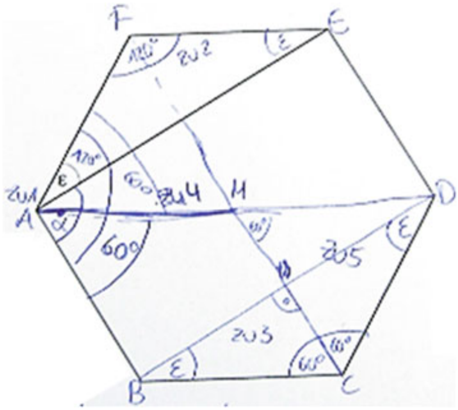


Fig. 5.6 Lilly's solution (hexagon problem). (Students' note "AD divides the angle in 2 equal 60° angles. AE divides one of this in two angles of equal size 30°")

argumentation why *AD* and *AE* are halving the angle ε twice and the fact that she misses to note some explanations may indicate that she is acting intuitively or is not aware of the correct mathematical argumentation. However, she comes to the right solution that *FAE* is 30°. We see that Lilly's approach has the right outcome but her reasoning is insufficient. For this reason, this approach is excluded under the perspective of the first analysis which—because of the high flexibility—leads to a reduction of her creativity score. In the second analysis, we decided to consider this approach appropriate and include it because of the right solution and an understandable way of solving the problem. Even though it may lack proper mathematical reasoning, it is a character of mathematical creativity.

Tina (Fig. 5.7) works on three approaches.

In the first one, she calculates the size of ε from the angle sum of a right triangle. Her next approach, which will be analyzed in the following, is similar to Lilly's solution presented above. Tina also divides an angle: She focuses on the angle *BAE* and divides it into thirds. She then comes to the right solution of 30°. Here, two aspects in the argumentation are missing. First, it is not pointed out why the angle

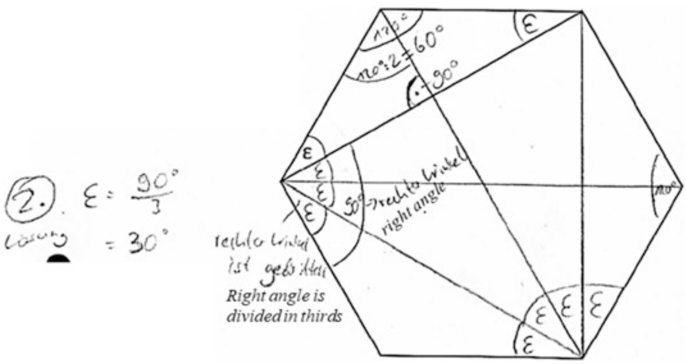


Fig. 5.7 Tina's solution (hexagon problem)

BAE is right-angled. She could argue drawing on the alternate angle, which she used in her first approach. However, she does not mention this. Another explanation could be that Tina does not feel the necessity to motivate this. Second, it is not obvious why the inserted line segments *AC* and *AD* divide the right angle into thirds. This approach is—compared to Lilly's approach—more sophisticated. Tina is probably not aware of the reasons and therefore this approach might be excluded in the first analysis, although her solution is right. However, we consider this approach appropriate, because it shows a clear idea that can be completed. Tina's third approach is very similar to her previous one. She divides the interior angle into four parts like Lilly. As in the approach before, Tina does not give any reasons for why her division is correct. Here, the evaluation follows the same line of argumentation as in Lilly's case: In the first analysis, the approach is excluded, whereas in the second analysis this approach is included. Again, we consider approaches appropriate for assessing students' mathematical creativity that lack certain explication of steps in the student's proof.

5.5.3 *Christmas Punch Problem*

The third problem was an algebraic one. And it seems to be the most complex one of the problems we used: The maximum number of solutions provided by the students was two, but most of the students only worked out one approach. In the following, one example is given.

In Olive's approach (see Fig. 5.8, left), we see some notes at the beginning, which build the central information for solving the problem.

The most interesting lines to understand her procedure are "$3/4 + 1/4 = 1$" and "4 barrels = 3/4 filled". Here, it becomes clear that Olive's idea refers to the wrong underlying set. She assumes that the barrels are filled to 3/4 in the beginning (Fig. 5.8, right). She therefore comes to the wrong solution of 16 barrels in total. In the first analysis, because of this mistake, her approach cannot be integrated into

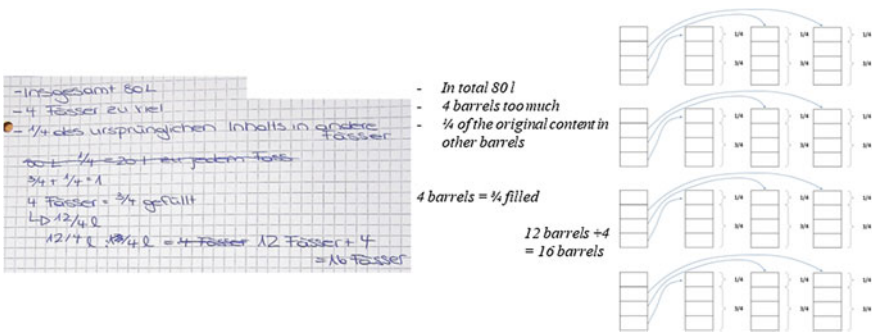


Fig. 5.8 Left: Olive's approach (Christmas punch); right: illustration to Olive's approach

the calculation scheme. However, when focusing on appropriateness, we see an approach which, due to a wrong text comprehension, entails a reasonable strategy to solve the problem. Of course, the mathematical notation is not elaborated but her thoughts are structured and clear. Therefore, this approach is considered appropriate and integrated into the calculation scheme. Because Olive did not work on a further approach, she gets a score greater than 0 only if this approach is taken into account. Nevertheless, many of the students had a very similar solution so that the originality score and thus the whole creativity score is rather low, but higher than 0, in general.

These examples illustrate that under the "strict" scheme of the first analysis, several approaches have to be excluded, which hint at creative thoughts and original ideas. Given that only entirely correct approaches are taken into consideration, we feel that the students' scores are heavily influenced by their ability to produce correct solutions. Therefore, we follow Leikin's (2013) suggestion to replace the notion of correctness with the notion of appropriateness.

In sum, considering our first research question of w*hat kinds of flaws appropriate solutions may entail to still be regarded appropriate and what appropriateness means in terms of mathematical MSTs*, we found that in most cases, the appropriate solutions

- lack certain steps in the reasoning of the proof but still are reasonable and understandable,
- provide an appropriate strategy and are understandable, but include other mistakes (arithmetical, algebraic, or geometrical) leading to a wrong solution, or
- are incomplete but may lead to a correct solution and could be complemented to a correct solution.

5.6 Results Part II: Students' Performances Across Three MSTs

In the next part of our study, we focus on students' performances across different MSTs.

Based on our previous finding that a scoring based on appropriateness gathers a broader spectrum of students' creative approaches, we ranked students' performance in each MST in order to compare students' ranks. Thereby, we aimed at analyzing the consistency of students' performances across three MSTs. By doing so, we wanted to investigate whether students' performances in different MSTs correlate and, thus, can be attributed to a single construct called students' mathematical creativity or not.

Table 5.3 Correlations between each two MSTs

	Problem		
	Triangle	Hexagon	Punch
Triangle	–	r = .233 (p = .280)	r = −.019 (p = −.939)
Hexagon	–	–	r = −.281 (p = .257)
Punch	–	–	–

Calculated with Kendall's tau

Table 5.3 shows the pairwise rank correlation coefficients between each two MSTs which were calculated with Kendall's tau. There is a weak rank correlation between the triangle MST and the hexagon MST (r = .233). This positive rank correlation is not significant (p = .283). If we look at the other two comparisons (triangle MST and hexagon MST; hexagon MST and punch MST), we see no positive correlation (r = −.019, resp. r = −.281). Actually, there is a weak negative rank correlation between the triangle MST and the punch MST, but with p = .939 this correlation coefficient is not convincing. We see that in each comparison there is no informative rank correlation. That means that there is no statistical evidence for a successful or unsuccessful performance along the three MSTs. However, it needs to be taken into consideration that due to the small sample size and the above-average IQ, the group under investigation is not representative; with according consequences for the results of the statistical analysis.

To give a qualitative insight, we focus on three students (Kirsten, Claire, and Phil) in the following and present their performance across the three tasks. We choose these students because their diverse performances are generic for the students in the project.

5.6.1 *Kirsten's Case*

In her approach to solve the triangle problem (see Fig. 5.9), Kirsten divides the grey area into a parallelogram and a triangle by drawing an auxiliary line through S that is parallel to *AB*. Then, she uses the intercept theorem to reason on proportions between different line segments. In comparison to the reference group, her approach is elaborated, as she is the only student who motivates the equality of the lengths of the line segments. Because of the originality of her approach, she receives a high creativity score (Cr_1). Although she does not write down another solution, she gets one of the highest total creativity scores (Cr) in this MST.

In the second geometrical MST (the hexagon problem), her performance is also high (see Fig. 5.10). She works out four approaches. In her first approach, she makes use of the angle sum. Therefore, she focuses on an isosceles triangle, which consists of one interior angle of the hexagon which is given with 120° and two angles ε. Then, she calculates the correct size of the angle. This approach was

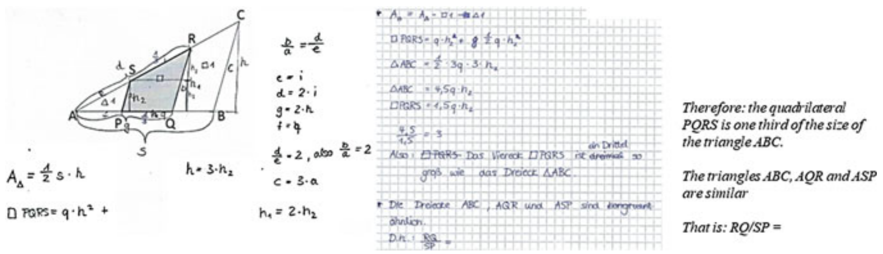


Fig. 5.9 Kirsten's solution for the triangle problem

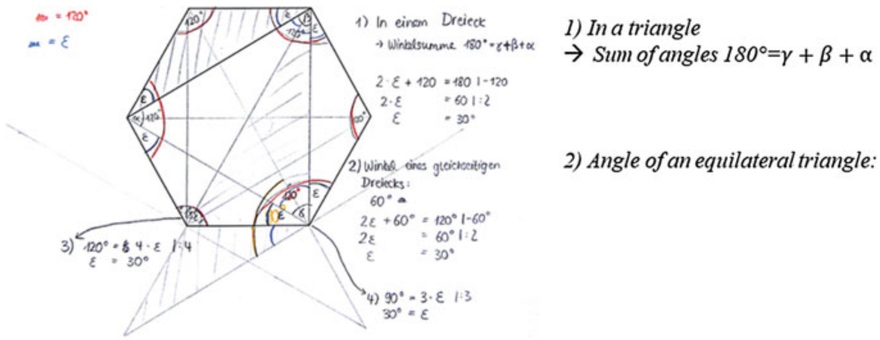


Fig. 5.10 Kirsten's approaches for the hexagon problem

worked out by several students. Therefore Kirsten's originality score is only 1 for this approach. The flexibility is 10, because this approach was her first one. Her second approach is from another category, so that the flexibility is scored with 10 for that approach as well.

The originality is also scored with 10: she inscribes an equilateral triangle and thereby divides the interior angle into 2 angles ε and one angle of the equilateral triangle which is 60°. In her next approach, she divides the interior angle into four angles of equal size. In the analysis, we take this solution into account, although there are missing steps regarding the argumentation (appropriateness). Because she uses the strategy again to divide an angle, the flexibility is 1, but the originality is rated with 10, because this solution does not appear often. In her last approach, Kirsten divides a right angle into thirds. Therefore, flexibility is rated with 1 again. The originality is also rated with 1, because this approach is common. With this performance, Kirsten's creativity score in this task is one of the highest.

In the third MST (the punch problem), Kirsten's performance (Fig. 5.11) is weaker. She works out one insight-based solution. Indeed, her solution is correct, but most students have a solution from the same category so that the originality score is 0.1. Therefore, her creativity score for this MST is lowest among all students participating in this study.

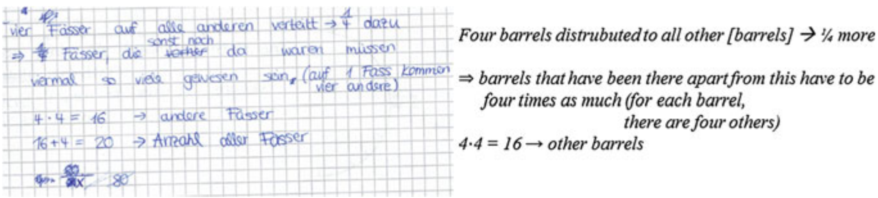


Fig. 5.11 Kirsten's approach for the punch problem

Kirsten's case illustrates that there are students who perform highly in some MSTs but low in other ones. In this particular case, the creativity scores were high in both geometry MSTs, whereas they were low in the algebra MST. Her creative performance appears to be content- or subdomain-dependent. This might indicate, for instance, that her abilities in the domain of geometry are stronger than in algebra; or it might reflect different norms that are established in her regular mathematics teaching in these different domains. It might also indicate that she has more practice with geometrical proofs than with problem solving in algebra. Finally, the differences might rely on difficulties to understand the situation in the last MST or on other influencing factors (concentration, motivation, etc.). However, what we see is that her creative ability does not come to light equally in the three MSTs. Her case indicates that mathematical creativity should possibly be considered subdomain-specifically (here: in geometry vs. algebra) rather than in general.

5.6.2 Claire's Case

Claire is one of the mathematically strongest students in the course. She performs very well in almost all problems given in the course.

In the first MST, her performance is not outstanding, but good (see Fig. 5.12). She works out two approaches, both rated with an originality of 1. In the first one, she completes the triangle to a parallelogram (although she writes "rectangle") and reasons why the grey area is 1/3. Remarkable is her note at the end in which she states that this relation only works with the area in the middle. In her second approach, she divides the given triangle into nine small similar triangles of equal size and determines the ratio.

In the hexagon problem, Claire works out four approaches (see Fig. 5.13).

In one approach, she inscribes a rectangle und then subtracts 90° from the interior angle of 120°. Because of the low originality (rated with 1) she gets 10 points (10 for flexibility). Additionally, she provides two approaches in which she uses the angles sum of triangles—once in an isosceles triangle (similar to Kirsten, see above) and the other time in a right triangle. Both are rated 1 for originality and 10, respectively 1 for flexibility. In her last approach, she halves one interior angle twice. Here, she gets 1 point for both flexibility and originality. Although Claire

If you complement the triangle to a rectangle [she means parallelogram], you can recognize easily the share is one third. The area for the rectangle [not correct but this is what she said] has been multiplied by two. Therefore, the share of the area is also true for the triangle.
But this works only for the piece in the middle.

Parallelogram is just the same as a rectangle [she means the calculation of the area]
because of< …
With this, the big triangle can be divided into several smaller ones of equal size.
In total: 9 small triangles
Selected area: 3 small triangles
Therefore 1/3 of the whole

Fig. 5.12 Claire's approaches for the triangle problem

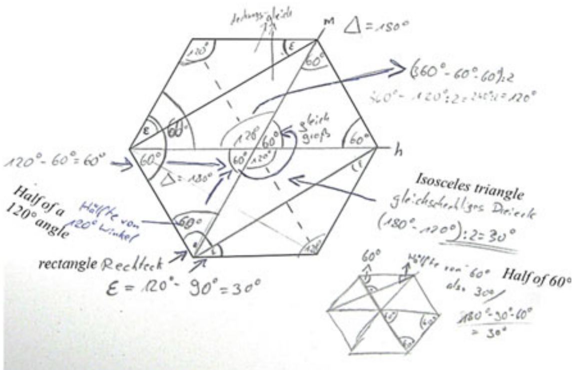


Fig. 5.13 Claire's approaches for the hexagon problem

provides many ideas, her creativity score is below average, because her presented ideas show only a low level of flexibility and originality.

In the punch problem, Claire is one of the few students who works out two different approaches (see Fig. 5.14). Her first approach draws on a diagram, which illustrates the distribution of the barrels. Compared to the solutions of the peers, this solution is original, because only one other student used this idea as well. Therefore, she gets a score of 100. In her second approach, she uses two variables in a system of equation. This is also rated with a score of 100. As in the triangle and hexagon problem, she describes her ideas by giving additional information or writing down exact intermediate steps. In this problem, Claire has the highest score of all participants.

As in Kirsten's case, we see that Claire works on MSTs on different levels. Claire performs outstanding in the triangle MST and in the punch MST. In the hexagon MST, she performs below average as compared to her peers. We can

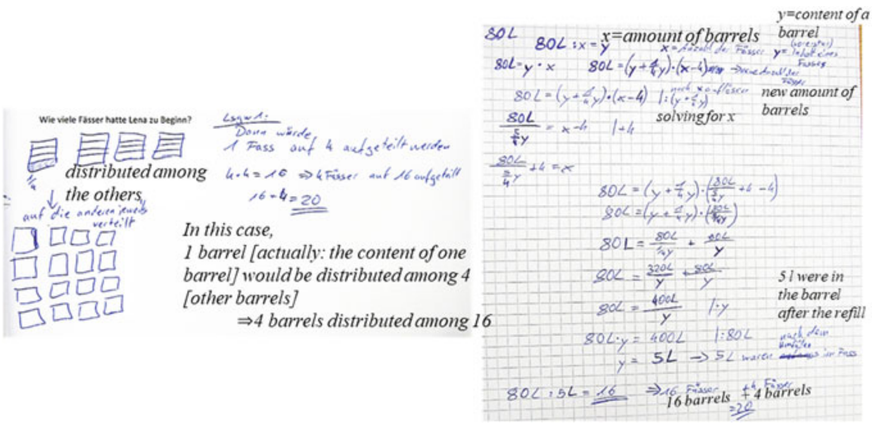


Fig. 5.14 Claire's approaches for the punch problem

assume that she might have a certain affinity to algebraic problems, because here, she shows a very sophisticated solution. An alternative reason could be motivational or affective influences or issues. Claire's case suggests that creativity might not even be conceptualized subdomain-specifically, but only task-specifically, depending on the particular MSTs.

5.6.3 Phil's Case

In the following, we give an insight into Phil's performances, which are lower than in the above-mentioned cases. When working on the triangle problem (see Fig. 5.15), Phil did not come to a conclusive result. We see that he sketched a right triangle as a special case and completed it in another sketch to a square. On his sheet of paper there are also some notes, in which we see rudiments of how to calculate the surface area of the grey area as the sum of a rectangle and a triangle. In all of his notes, we cannot identify a clear strategy that would give insight into his ideas. Even when including appropriate solutions, he gets a creativity score of 0 in this task.

In the hexagon MST, Phil's performance (see Fig. 5.16) is strong. He gets a high score for fluency, because he has figured out four approaches. In the first approach, he divides one of the interior angles into a right angle and ε. This approach is rated with 10 for flexibility and 10 for originality. The second approach is from another category (10 for flexibility), but not very original (rated with 1): He calculates the angle ε by the differences of interior angles of a right triangle. The next approach is similar to the first one: He uses a right angle. This results in 0.1 points for flexibility, but 10 points for originality. In his last approach, he refers to the sum of interior angles of a kite. Even though this approach gets 10 points for originality,

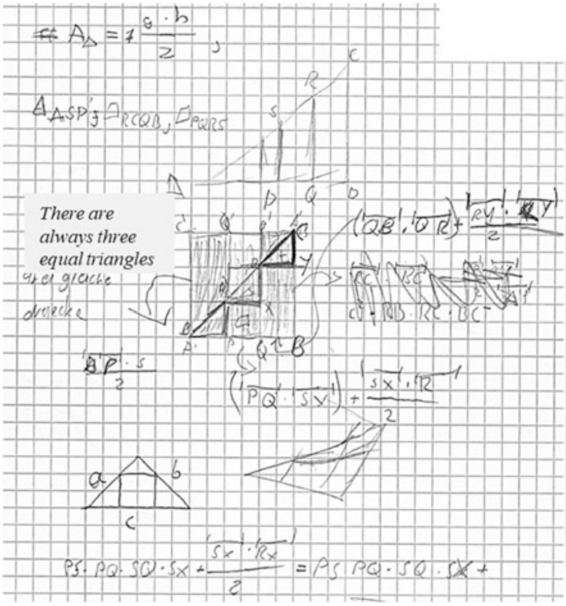


Fig. 5.15 Phil's approach to the triangle problem

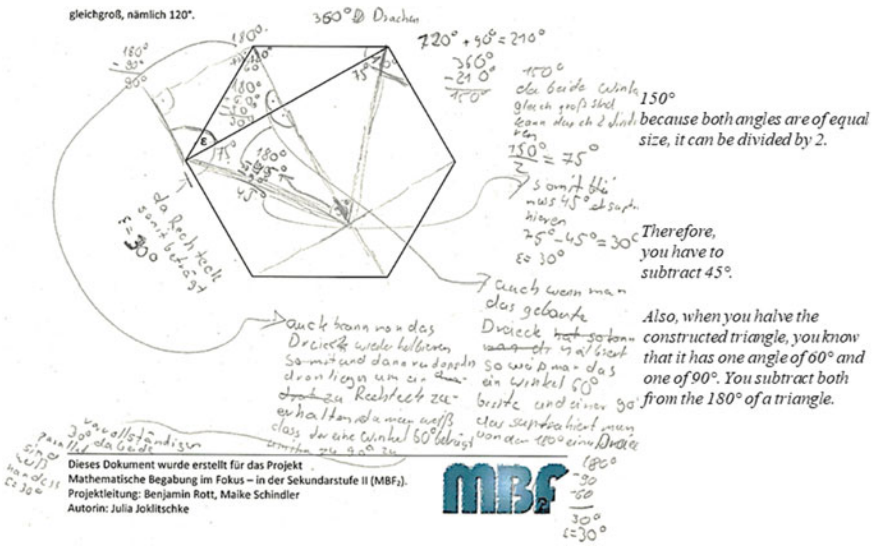
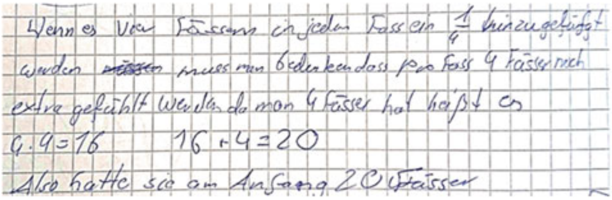


Fig. 5.16 Phil's approaches for the hexagon problem



If ¼ is added to each barrel, you have to consider that for each barrel 4 [other] barrels will be filled [spelling mistake in German]. Because you have 4 barrels, this leads to:
4·4=16

Therefore, she started with 20 barrels.

Fig. 5.17 Phil's approach for the punch problem

the flexibility is rated with 1, because Phil has worked in this category before. A difficulty for rating Phil's work lies in the fact that it does not become clear in which order he noted his ideas. However, a calculation that is based on another order would lead to the same total score.

Phil's approach for the punch problem (see Fig. 5.17) shows an insight-based solution (similar to Kirsten's) which is rated with 0.1 for originality. He does not work on another approach, so that his creativity score for this problem is 1.

Phil only performs on a high level in the hexagon MST. It seems as if he does not find a proper access to the triangle MST. In the punch MST, he works out a common solution. This case is complex because of two reasons: First, Phil does not perform better in one of the disciplines algebra or geometry. Second, he does not improve over the three problems. This might indicate that performances in MSTs depend on the problems themselves and that mathematical creativity is not a question of domain-specificity or even subdomain-specificity.

5.7 Discussion and Outlook

The purpose of our study was to investigate students' creative performances along different fields of mathematics. For inquiring into this question, we first intended to peruse the question of what *appropriateness* of solutions—a concept as offered by Leikin (2013)—may entail and imply in contrast to *correctness* of solutions.

We investigated these questions based on an empirical study with upper secondary school students, who were mathematically interested, showed strong cognitive abilities, and who worked on three MSTs—two geometrical tasks and one algebraic task—that were evaluated quantitatively as well as qualitatively.

A review of the literature in the field of creativity reveals that there is no consistent definition of the term creativity. This goes along with a wide field of ideas about how to conceptualize and operationalize creativity (e.g., Haylock 1987; Sriraman 2005). There are, for example, different assumptions about which persons can be mathematically creative—only professional mathematicians or everyone. This aspect goes along with the distinction between relative creativity and absolute creativity—called "little c" (for relative) and "big C" (for absolute creativity).

We assume that everyone can be creative—at least in a relative way. In mathematics, research focuses on assessing creativity by using problem posing and in problem solving (Silver 1997). Therefore, different approaches are used to assess mathematical creativity. In our study, we focused on a product-based evaluation method where students' products from problem solving tasks were analyzed and rated in the dimensions of fluency, flexibility, and originality (see, e.g., Leikin and Lev 2013; Kattou et al. 2013).

We used the existing and broadly accepted evaluation scheme developed by Leikin and Lev (2013) to quantify mathematical creativity in students' products. As suggested by Leikin (2013), we took into account appropriate solutions. In the first part of this chapter, we investigated the characteristics that appropriate solutions may have. We found that appropriate solutions may either be correct, may lack certain steps in the reasoning of the proof (while still being reasonable and understandable), or include other mistakes (arithmetical, algebraic, or geometrical) leading to a wrong solution while still providing an appropriate strategy.

We found that non-correct appropriate approaches provided us with insights into students' creative potential even though they contained flaws such as lacks in ways of reasoning, mistakes in calculations, or a missing answer. In our paper, we used cases of students to illustrate the value of focusing on appropriate solutions (as suggested by Leikin 2013) for thoroughly evaluating mathematical creativity of students' products. In qualitative analyses, we illustrated that despite their partial incorrectness, these products include creative efforts that would have been disregarded by excluding these approaches.

However, we also experienced that in some cases it is difficult to decide whether a student's approach is to be considered appropriate or not. In these cases, we drew on a consensual validation, in which we discussed whether student approaches impart a comprehensible strategy and can be included into the second analysis or not. Even though this procedure results in high efforts, taking into account appropriate solutions uncovers a broad spectrum of creative approaches which otherwise were excluded, if only correct solutions were regarded. Of course, when appropriate solutions are considered, this has the potential effect that the originality and flexibility scores that are assigned to some approaches are lowered due to a higher number of graded approaches and therefore a higher frequency of solutions.

Furthermore, our study emphasizes that students' prior knowledge should be considered when assessing mathematical creativity. The used MSTs are from the fields of geometry and algebra. To solve these problems in multiple ways, a certain spectrum of mathematical background is necessary. In this respect, Singer and Voica (2017) found that "that creativity manifestation is conditioned by a certain level of expertise" (p. 75). It is therefore difficult to distinguish clearly between mathematical abilities and mathematical creativity. Research results rather indicate that creativity and expertise mutually influence and support one another (Singer and Voica 2017). However, with our understanding of appropriateness of solutions, we see a potential path to face this issue (at least partly). Through including not only correct solutions but also partially correct approaches, the view on mathematical creativity is broadened.

Naturally, the idea of taking into account appropriate solutions (and not only correct ones) is especially useful when the given tasks are complex problems; such as geometrical proofs. If the students solve problems which do not require complex reasoning or proofs, it is eventually more appropriate to focus only on correct solutions. For example, when using a problem in which the students are asked to fill in a number pyramid (see Kattou et al. 2013) it is not necessary and probably not even adequate to lower the criterion of correctness. A modification of the evaluation scheme has to be carefully considered against the backdrop of the given problems, their complexity, and the required reasoning.

From the perspective of test theory, there is an important discussion about the visible *performance* of students and their rather invisible *potential* (Foth and van der Meer 2013). In our study, we were able to have a look at a broader spectrum of students' potential by also taking into account incomplete approaches. Through focusing on appropriate solutions (similar to Leikin 2013) and elaborating on the notion of appropriateness, our study contributes to efforts to bridge this gap between potential and performance. But even when regarding appropriate solutions, it is challenging to grasp students' potential regarding mathematical creativity. To face this issue, we think that a shift in the perspective from products to processes is significant. There is more empirical research needed that inquires into students' mathematical creativity from a process-view. First studies (e.g., Schindler et al. 2016; Schindler and Lilienthal 2017a) hint at the potential that empirical studies focusing on students' processes may have for extending the body of knowledge regarding mathematical creativity. Especially, eye-tracking appears to be a research method with certain potential for investigating mathematical creativity from a process-view in the future (Schindler and Lilienthal 2017b).

We studied students' performances across three different MSTs; two geometric and one algebraic problem. Focusing on rank correlations and using Kendall's Tau Test, we did not find a statistical significant correlation between students' performances in different MSTs. This indicates that MSTs in different subdomains but even within a single mathematical subdomain (geometry) do not require exactly the same competencies. In particular, we hypothesize that it is rather content-knowledge than a creative ability that affects students' performances in these MSTs. A qualitative analysis of the students' products shows a broad spectrum of performances across the three MSTs. There are cases such as Kirsten's who seems to prefer geometric problems, or Claire who might show outstanding results in all domains, or Phil who only performs well in one geometrical MST, but not in the other one.

In the sense of mathematical creativity as a domain-specific construct (Kattou et al. 2015), we would have expected a stronger consistency within the ranks regarding the three MSTs. There are various possibilities to explain our results. First, we investigated a selected group of students. These students were mathematically interested and had—as the results from the intelligence test showed—an above average IQ of 121 on average. Given that high intelligence might correlate with mathematical giftedness (Foth and van der Meer 2013), we assume that a ceiling effect might affect our analysis and disguise possible rank correlations. Second, the students did not have prior experiences working with MSTs. We assume that this might influence the results as well. Accordingly, we recommend to conduct creativity tests with students who are familiar with MSTs. Third, because of the ordinal scaled data in our analysis, a direct comparison was not possible. Therefore, we had to work with ranks and rank correlation, but we have to have in mind that this method does not represent their performances adequately.

Referring to mathematical creativity as a domain-specific concept (Kattou et al. 2015), our results gave hints that the construct of mathematical creativity is not as consistent and homogeneous as it might appear eventually. Possibly, creativity in the field of mathematics should be rather viewed as a subdomain-specific construct (which means that there could be a construct like *geometric creativity* or *algebraic creativity)*; or even as task-specific. This relates to Singer et al.'s (2017) research on cognitive styles. In particular, they found "that cognitive flexibility—a basic indicator of creativity—inversely correlates with a style that has dominance in metric GN [Geometric Nature] and structured CD [Conceptual Dispersion], showing that the detected cognitive style may be a good predicator of students' mathematical creativity" (p. 37). We think that this interesting phenomenon requires more research in the future.

In comparison to studies such as Kattou et al.'s (2015), we cannot present quantitative values such as correlation coefficients or model calculations that are statistically reliable. This relies on the fact that our group of participants was small and not representative. Our intention was rather to share and discuss thorough considerations about mathematical creativity and its evaluation. Finally, we hope that our contribution can lift future scientific discussion on the evaluation of mathematical creativity, the notion of appropriateness, and the question of whether mathematical creativity is to be considered domain-specifically, subdomain-specifically, or rather task-specifically.

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