Chapter 1
Enhancing Creative Capacities in Mathematically-Promising Students. Challenges and Limits

Florence Mihaela Singer

Abstract The links between research in mathematics education, psychology of creativity and research in gifted education started to gain more attention in the last decade, from researchers and the large public as well. The paper is intended to provide a concise survey of these links, with a focus on: frameworks for studying students' creativity and giftedness in mathematics; domain specificity of creativity; some characteristics of mathematical creativity resulting from its specificity; relationships between mathematical giftedness and creativity from a mind-and-brain perspective; relationships between creativity, giftedness and social inclusion; underlying connections between mathematical creativity and innovation, creativity and metacognition, creativity, giftedness and expertise; and the teaching of mathematically-promising students with a focus on structuring their mathematical competencies. The paper offers also brief reviews of the chapters included in the book, stressing on the benefits of an integrated approach of creativity and giftedness in mathematics education.

Keywords Mathematical creativity · Mathematical giftedness
Mathematically-promising students · Problem solving · Problem posing
Domain-specific creativity · Expertise · Metacognition

1.1 Introduction: Setting the Context

A few years ago, in a comprehensive review study, Leikin (2011) identified a gap between research in mathematics education and research in gifted education. She noticed that in the first ten years of the twenty-first century, seven key journals in the fields of intelligence and giftedness (*American Psychologist, Creativity Research Journal, Gifted Child Quarterly, High Ability Studies, Journal for the*

F. M. Singer (✉)
Faculty of Letters and Science, University of Ploiesti,
39 Bucharest Boulevard, 100680 Ploiesti, Romania
e-mail: mikisinger@gmail.com

F. M. Singer (ed.), *Mathematical Creativity and Mathematical Giftedness*,
ICME-13 Monographs, https://doi.org/10.1007/978-3-319-73156-8_1

Education of the Gifted, The Journal of Secondary Gifted Education, and *Review of General Psychology*) published very few articles devoted directly to mathematical giftedness or creativity. In addition, a search of seven leading research journals in mathematics education (*Educational Studies in Mathematics, Focus on Learning Problems in Mathematics, For the Learning of Mathematics, Journal for Research in Mathematics Education, Journal of Mathematical Behavior, Mathematical Thinking and Learning,* and *ZDM Mathematics Education*) revealed that in that same decade again only few articles were explicitly devoted to mathematical giftedness. A small number of publications in other journals (such as *Journal of Educational Psychology*, *Psychological Science Journal,* and *Journal of Applied Psychology*) have been focused on specific issues related to problem solving and mathematical reasoning in the gifted population, usually with a focus on creativity (Leikin 2011). In addition, a few edited volumes had in focus these aspects (e.g. Sriraman 2008; Leikin et al. 2009).

Things seem to have changed, however, in the last years. Thus, recently, two new special issues of *ZDM Mathematics education* address the topics of mathematical giftedness (Singer et al. 2017a) and psychology of mathematical creativity (Sriraman 2017), completing the series initiated by the special issue devoted to mathematical creativity (Leikin and Pitta-Pantazi 2013). The change is not only in the focus, but it goes deeply into the relationship between psychology of creativity and the applied domain-specific study of creativity and giftedness in mathematics. Moreover, communities of practice contribute to developing these fields on a more systematic base, and their activities were reflected in 2016, during the 13th International Congress on Mathematical Education (ICME-13), through the Topic Study Group 4 (TSG 4) focused on *Activities for, and Research on, Mathematically Gifted Students* and TSG 29, which addressed *Mathematics and Creativity.* TSG 4 published a topical survey before the congress, and this current monograph builds on the work presented in both of those TSGs during the congress.

What looked like to be a far-off goal, i.e. situating mathematics education research within an existing canon of work in mainstream psychology (Sriraman et al. 2013) seems to be today more tangible than ever. Moreover, the call for interdisciplinary views appears to have become the rule rather than approaches that request narrow (frequently ill-defined) conceptualizations. For example, the recent developed model of the Active Concerned Citizenship and Ethical Leadership (ACCEL) for understanding giftedness (Sternberg 2017a) covers aspects related to: teaching for creativity; the role of science, technology, engineering, and mathematics (STEM) in teaching for wisdom; the developmental nature of giftedness; teacher education (Sternberg 2017a, b), and many other integrative perspectives. The present book puts together views underlying, beyond the variety of approaches, strong connections between the psychology of creativity, mathematics education, and the study of giftedness as an interdisciplinary field.

1.2 Frameworks for Studying Students' Creativity and Giftedness in Mathematics

Starting from the well-known work of Torrance (1974), researchers usually explore mathematical creativity through the following parameters: originality, fluency, and flexibility. Adaptations of this approach have been proposed by many researchers, who analyzed students' creativity in problem-posing and/or problem-solving; usually, these approaches are contextualized by the study design. For example, Kontorovich and Koichu suggested a framework based on four "facets": resources, heuristics, aptness, and social context in which problem posing occurs (Kontorovich and Koichu 2009). A refinement of this framework integrates task organization, knowledge base, problem posing heuristics and schemes, group dynamics and interactions, and individual considerations of aptness as parameters in analyzing creativity in problem-posing situations (Kontorovich et al. 2012).

Another set of studies investigated the relationship between students' cognitive flexibility and their abstract thinking using mathematics problem posing as a tool for detecting creative behavior in school children (Singer and Voica 2015; Singer et al. 2013a, b). Here, within an organizational-theory context, creativity has been discussed in terms of *cognitive flexibility*, which can be conceptualized as consisting of three primary constructs: *cognitive variety*, *cognitive novelty*, and *change in cognitive framing*. In problem-posing contexts, *cognitive novelty* was assessed by the "distance" from the initial problem. *Cognitive variety* was measured by the extension and the differences among the posed problems starting from a given one. *Cognitive framing* was assessed through the capacity of generating a pattern of thinking for a class of problems. In this case, creativity refers to the student's ability to change his/her cognitive frame, or even more, to his/her ability of reframing. This model proved effective for detecting some characteristics of mathematical creativity less visible when using other study frameworks.

The traditional psychometric approach seems not to offer today enough consistency for the study and understanding of nature and nurture of giftedness. Scrutinizing some of the deeply held assumptions about the nature of giftedness, Dai and Chen (2014) consider that a contextual developmental approach is more viable than simple psychometric tests, and highlight that understanding high potential and educating youths should be approached beyond the tensions between the gifted-child and talent-development movements, and between excellence and equity in a constructive and comprehensive way.

The issue of designing frameworks for the analysis and the development of students' creativity and students' giftedness raised the interest not only of researchers, but it was seen pragmatically as an issue with political-strategic implications. Thus, some countries and states developed general frameworks for designing differentiated learning experiences for gifted students having in view to complement the official standards and benchmarks. For example, in the US, Florida's Frameworks for K-12 Gifted Learners (2005–2007) stipulate that by graduation from K-12 education, the student identified as gifted will be able to:

critically examine the complexity and apply investigative methodologies; create, adapt, and assess multifaceted questions; conduct thoughtful research/exploration; and think creatively and critically to identify and solve real-world problems. In addition, the student identified as gifted should be trained so that become able to: assume leadership and participatory roles in both gifted and heterogeneous group learning situations; set and achieve personal, academic, and career goals; and develop and deliver a variety of authentic products/performances that demonstrate understanding (Florida's Frameworks for K-12 Gifted Learners (2005–2007).

In some countries, strong emphasis is put on training students for the International Mathematics Olympiad (e.g., China, Russian Federation, Korea), while special programs are devoted to (identified) gifted students in others (e.g. Germany, Israel, Singapore).

In other situations, communities of parents, teachers, and NGOs created groups that assumed theoretical views on giftedness and tried to influence the perception of the giftedness and made a plea for more attention granted to this special social group. The uniqueness of the gifted renders them particularly vulnerable and requires modifications in parenting, teaching and counseling in order to help these children to develop optimally (e.g. The Columbus Group, 1991). Concluding this section, from theoretical frameworks to ad hoc conceptual underpinnings, there is a large gamut of approaches in discussing giftedness in relation to creativity, and the involved special target population needs careful attention from both research and action.

1.3 Domain Specificity of Creativity

From a cultural-anthropological view, each domain of knowledge developed a way of thinking that is intrinsic to that domain. Therefore, it is meaningful to ask if being creative is a general trait or it is a domain-specific feature. Various observers of the theoretical and empirical creativity literature (e.g. Csikszentmihalyi 1988; Gardner 1993; Sternberg and Lubart 2000) assume that the debate might be settled in favor of domain specificity.

In fact, Baer (2010), among others, provided convincing evidence that creativity is not only content specific but is also task specific within content areas. Moreover, research has suggested that transfer across domains is both difficult to achieve and relatively rare (Willingham 2002, 2007). For example, a large-scale study that looked at the possibility of transfer of practiced intellectual skills came to a very negative conclusion. In this 6-week training study, 11,430 participants were trained several times each week on cognitive tasks designed to improve reasoning, memory, planning, visuospatial skills, or attention. "Although improvements were observed in every one of the cognitive tasks that were trained, no evidence was found for transfer effects to untrained tasks, even when those tasks were cognitively closely related" (Owen et al. 2010, p. 775).

If creativity is domain-specific, then the assessment of creativity must also be domain-specific. However, creativity assessment has often assumed domain generality. Thus, the most common tests of creativity have been divergent thinking tests, and the most widely used divergent thinking tests are the Torrance tests of creative thinking (TTCT), with its two components: figural and verbal, although both are used as general measures of creativity (e.g. Kaufman et al. 2012). This finding is in line with evidence Torrance himself offered showing that figural and verbal divergent thinking scores are not correlated, and are therefore measuring two essentially unrelated cognitive abilities. Divergent thinking may be important, but we may need multiple measures of it, domain by domain, for it to be useful (Baer 2012).

Recently, the relationships among domain-general divergent thinking ability, domain-specific scientific creativity, and mathematical creativity have been explored through a study that investigated the relative influences of domain knowledge and divergent thinking ability on scientific creativity and mathematical creativity (Huang et al. 2017). By exposing 187 primary school sixth-graders to The Mathematical Creativity Test (MCT) and the New Creativity Test to assess students' domain-general divergent thinking ability, they found that Mathematical Creativity Test is only modestly positively correlated with the general creativity test and mathematical achievement can effectively explain the variance in MCT performance, but the creativity test cannot. These results imply that there are diverse influences from domain knowledge and divergent thinking ability on creativity in different domains, which, again, supports the domain-specificity of creativity.

Moreover, evaluating potential creativity of 482 children and adolescents, Barbot et al. (2016) found that the contribution of each variance component (thinking-process general, thinking-process specific, domain-specific, task-specific, and measurement error) depends greatly on the task under consideration, and that the contribution of a general creative thinking-process factor is overall limited. Consequently, specialized thinking modalities might be the focus of identifying and developing creativity in a domain-specific approach.

1.4 Mathematical Creativity

As new evidence-based arguments to the debate regarding general versus domain-specific creativity incline the balance towards the second part, it makes sense to consistently discuss mathematical creativity. As highlighted above, there is cumulated evidence that mathematical creativity is of a special type, which distinguishes from other types of creativity. Large part of evidence comes from problem solving studies; the literature in this area is vast, in both time extension and quantity (e.g. Hadamard 1945; Krutetskii 1976; Leikin 2009).

Another set of evidence is brought by problem posing studies, which have gained more terrain in the last decade. Some of these studies investigated the relationship between students' cognitive flexibility and their abstract thinking using mathematics problem posing as a tool for detecting creative behavior (Singer et al. 2011, 2013a, b; Pelczer et al. 2011). In problem-posing contexts, creativity has been discussed in terms of *cognitive variety*, *cognitive novelty*, and *change in cognitive framing*. In a problem-posing context, an indicator of cognitive variety might be the number of different posed problems; *cognitive novelty* was assessed by the "distance" from the initial problem; *cognitive variety* was measured by the extension and the differences among the posed problems starting from a given one. *Cognitive framing* refers to the capacity of generating a pattern of thinking for a class of problems, accompanied by the ability to make changes into that cognitive frame. Singer and Voica (2013) found that, in PP contexts, high achieving students tend to make incremental changes to some parameters in order to arrive at simpler and essential forms needed in generalizing sets of data. It follows that mathematical creativity requires abstraction and generalization, which emerge from gradual and controlled incremental changes in cognitive framing (Singer 2012a, b). Mathematically promising students display a need for consistency that seems to limit their cognitive flexibility to a certain extent. A tension between the students' tendency to maintain a built-in cognitive frame, and the possibility to overcome it (Singer and Voica 2015; Voica and Singer 2012, 2013) is highly visible in the experimental data. These studies revealed that, in problem-posing situations, the students develop cognitive frames that make them cautious in changing the parameters of their posed problems, even when they made interesting generalizations, because of the constraint they self-impose to devise mathematical problems that are coherent and consistent.

Consequently, the training for the development of mathematical creativity should include features that distinguish it from training for creativity development in general. Briefly said, while in the latter, more general case, techniques are to be used for stimulating the free development of ideas, in mathematics learning the variation of parameters should be practiced within a variety of activities where the processes are mindfully controlled and oriented towards abstraction and generalization (e.g. Singer and Voica 2015). More research is needed to address the characteristics of mathematical creativity as compared to other types of creativity and the consequences such studies could have on research and practice.

1.5 Relationships Between Mathematical Giftedness and Creativity—A Mind-and-Brain Perspective

Traditionally, giftedness has been related to a high IQ. However, the IQ concept becomes more and more controversial. Dweck (2006) supports this statement with her research on a growth vs. a fixed mindset and its effect on student learning. In addition, IQs do not remain stable over time. Briefly, a high IQ can be considered necessary but not sufficient for high achievement (Nolte 2012). As Cross and Riedl (2017) underlined, IQ testing should be reserved for finding specific forms of high ability and as a diagnostic tool, not as a gatekeeper that continues to perpetuate the underrepresentation of some groups.

A generous direction in cognitive science and neuroscience brings new data to understanding the development of mathematical talent and innovation in students of all ages and from all backgrounds. Mathematical giftedness started to be conceptualized in recent decades within a context that is sensitive to modern biology, based on studies of cognition within the discipline of educational neuroscience. Thus for example, Woolcott (2011) discussed exceptional performance in mathematics in relation to cognition and performance as a product of internal processing and environmental connectivity of the human organism.

Research that has examined the neuropsychological processes engaged by gifted and talented learners provides insights into how they process information, convert it to knowledge and make connections. It also assists in understanding the creative activity they display. These learners understand, think and know in ways that differ qualitatively from how regular learners perform these activities (Munro 2005).

Data derived from several psychophysiological studies support an important relationship between the specialized capacities of the right hemisphere and mathematical ability. Commonly, associated with giftedness is right hemisphere dominance (Jin et al. 2007), with frontal asymmetry in the right cortical area as a possible physiological marker of giftedness (Fingelkurts and Fingelkurts 2002).

The discussion is sometimes in terms of 'neural efficiency', where gifted functioning involves a more integrated brain with greater cooperation between the hemispheres (O'Boyle 2008), with reduced activity in certain areas as compared with average brains when performing similar tasks—possibly implying that gifted brains spend less time on such tasks. A more recent functional magnetic resonance imaging (fMRI) study using mental rotation to analyze mental capacities of mathematically gifted adolescents (Prescott et al. 2010) seems to confirm this.

Case studies of extremely gifted individuals often reveal unique patterns of intellectual precocity and associated abnormalities in development and behavior. The bulk of scientific inquiries provide evidence of unique patterns of right prefrontal cortex and inferior frontal activation implicated in gifted intelligence, although additional studies suggest enhanced neural processing and cerebral bilateralism (Mrazik and Dombrowski 2010).

Research literature in the area of mathematical ability at a very early age describes various early signs of mathematical giftedness in children (e.g. Diezmann and Watters 2000; Winner 1996). For example, Straker (1983) observed that mathematically gifted preschoolers generally show:

> …a liking for numbers including use of them in stories and rhymes; an ability to argue, question and reason using logical connectives: if, then, so, because, either, or…; pattern-making revealing balance or symmetry; precision in positioning toys, e.g. cars set out in ordered rows, dolls arranged in order of size; use of sophisticated criteria for sorting and classification; pleasure in jig-saws and other constructional toys. (Straker 1983, p. 17)

These signs, recognized in a large gamut of studies, can become abilities in adults developing a career that value them, or not. Still, the connection between mathematical creativity and giftedness can be reconsidered from the perspective of *mathematical promise*. The concept has been developed by the National Council of Teachers of Mathematics (NCTM) with the purpose of maximizing variables such as abilities, motivation, beliefs, and experiences or opportunities (Sheffield et al. 1999) among all students.

The question if giftedness and creativity are synonyms in mathematics has been addressed by Sriraman (2005), and his answer discusses the professional and school realms. The role of creativity in the education of the gifted has been analyzed in numerous studies (e.g. Mann 2006; Koichu and Berman 2005; Sriraman 2003; Chamberlin and Moon 2005; Reed 2004). Many of these studies emphasize the interactions between creativity and the development of mathematical expertise (e.g. Singer and Voica 2016).

Much of the empirical research explores the learning processes of mathematically talented students through problem-solving strategies, noticing their creative approaches (e.g. Amit and Neria 2008). Equally important is the connection between mathematical creativity and giftedness identification in relation to problem posing (e.g. Singer et al. 2015). Problem-posing sessions have shown effective for identifying gifted students. Voica and Singer (2014) found three characteristics that can offer an indication of mathematical giftedness: a thorough understanding of conveyed mathematical concepts, an ability to generalize reasoning moving towards abstractions, and a capacity to frame and reframe content while keeping consistency of new-created problems. In addition, the level of abstraction used to solve a given problem was correlated with the novelty of the newly posed problems starting from the given one; therefore, the abstraction level spontaneously used by a child might be a good predictor of the child‘s creative potential (e.g. Pelczer et al. 2015). More research is needed to see how mathematical giftedness and creativity mutually assist each other in children of various ages.

1.6 Creativity, Giftedness and Social Inclusion

Silverman (2013) has suggested that certain affective traits such as heightened sensitivity, early concern with moral issues, empathy, perfectionism, social maturity, and aesthetic appreciation are evident in gifted children. Beyond considering these as features allowing giftedness identification, these traits may, in a combination of internal and situational factors, put children at psychological risk, leading to interpersonal and psychological problems. Among the issues that may affect gifted children are their asynchronous development, their difficulties of socializing with peers and adults, and their own problems with self-learning (e.g. Singer et al. 2016).

There is also a special category of vulnerable students—twice-exceptional children. These are those who possess giftedness or exceptional ability in one or more areas in combination with special needs, a learning disability or a handicap in other areas. They may achieve high scores on certain intelligence tests but may not do well in school. They may have giftedness in combination with autism, emotional and behavioral disorders, or learning disabilities (dyscalculia, dyslexia, dysgraphia), ADD or ADHD, visual and auditory processing anomalies, or sensory integration and modulation disorders (Chamberlin et al. 2007).

The need for social integration and acceptance is very strong in gifted and talented children, although frequently they seem to isolate themselves from peers. This was identified also in many studies focusing on gifted children social interactions, but also indirectly on problem-posing contexts, where students' posed problems reflect not only a mathematical content, but also proposer's attitudes and affects. Singer and Voica (2015) noticed that both the posed problems and the behaviors displayed by the students in these studies highlighted a social dimension (Singer and Voica 2015; Pelczer et al. 2015).

A review made by Kurup et al. (2013) suggests addressing the following aspects in helping mathematically promising children develop their full potential: the need for talent-appropriate stimulation that is not restricted by the chronological age of the child (Roedell 1989); the need for counseling, acceptance and recognition of talents by peers, parents and teachers (Gross 1998; Silverman 2002); and programs and encouragement to aid the growth and blossoming of their special abilities (Reis et al. 1998).

1.7 Creativity and Innovation

As noted by the US National Science Board (NSB) report, *Preparing the Next Generation of STEM Innovators*, giving every student the opportunity to achieve his or her full potential is critical as we "will increasingly rely on talented and motivated individuals who will comprise the vanguard of scientific and technological innovation" (NSB 2010).

Even for moderately gifted students, research shows that approximately 40–50% of traditional classroom material could be eliminated for targeted gifted students in one or more of content areas, among which is mathematics (Reis et al. 1998). Care must be taken not to skip critical material, however, but to ensure that students are engaged and passionate about the mathematics they are learning, and are not simply memorizing algorithms or accelerating so they can finish taking required mathematics classes early.

The joint NAGC/NCTM/NCSM publication: *Using the Common Core State Standards for Mathematics with Gifted and Advanced Learners* has the following recommendation:

> In order to support mathematically advanced students and to develop students who have the expertise, perseverance, creativity and willingness to take risks and recover from failure, which is necessary for them to become mathematics innovators, we propose that a ninth Standard for Mathematical Practice be added for the development of promising mathematics students – a standard on mathematical creativity and innovation: Solve problems in novel ways and pose new mathematical questions of interest to investigate. The

> characteristics of the new proposed standard would be that students are encouraged and supported in taking risks, embracing challenge, solving problems in a variety of ways, posing new mathematical questions of interest to investigate, and being passionate about mathematical investigations. (Johnsen and Sheffield 2012, pp. 15–16)

1.8 Creativity, Giftedness and Expertise

The fact that the interplay between interests, activities, the environment, and mathematical explorations affect students' mathematical achievement leads to question whether there is a need to distinguish between giftedness and expertise. The need for expertise is a useful analog for many seemingly domain-general theories of creativity. It is true that some degree of expertise is important in many domains—creativity requires some level of knowledge and skill in most domains—but the content of such expertise varies by domain.

The formal school learning aims to expose new generations to the knowledge domains as they have been developed along the cultural history of humanity, by large contributions of communities of practice. As a product of human culture, each domain of knowledge has structured a specific way of thinking and, therefore, a domain-specific training becomes necessary to foster specialized cognitive mechanisms that are commonly activated in the expert of a domain.

Glaser (1988) characterizes expertise on six cognitive dimensions: knowledge organization, complexity of problem-solving representation, changing thinking schemes, goal-oriented procedural knowledge, automatic procedures, and metacognition. In terms of knowledge structure and organization, the expert possesses knowledge organizations that can integrate and structure new information items so that they are rapidly selected from memory in structured units, while novices hold isolated, frequently disconnected elements of knowledge. Regarding the complexity of problem-solving representation, the novice approaches a problem starting from its surface features, while the expert makes inferences and identifies principles underlying the surface structures. In changing thinking schemes, the expert amends his/her own knowledge theories and develops schemes that facilitate more advanced thinking, while the novice manifests rigidity in changing a thinking scheme. In terms of goal-oriented procedural knowledge, the expert displays functional purpose-oriented knowledge, while a novice holds information without clearly understanding the applicability conditions. In terms of automation that reduces the concentration of attention, an expert can focus attention that alternates between basic capacity and higher levels of strategic thinking and understanding, using automated procedures to achieve good performance, while novices have difficulty in sharing attention, and they frequently get lost in details.

An important attribute of an expert in a domain is to identify problems and to tackle them in a knowledgeable manner. In a recent study, Singer and Voica (2016) found that expertise and creativity mutually support each other in the process of building a solution for a nonstandard problem. Consequently, in order to get individual relevant data (for example, to avoid the situation in which one solves a problem because he/she internalized automatized strategies for that category of problems and this is taken as giftedness) the identification of creativity should take place based on tasks situated in the proximal range of a person's expertise and exceeding his/her actual level of expertise, at the time of analysis.

1.9 Creativity and Metacognition

Various experiments that exposed students to problem-posing sessions have shown that participant children become able to: find alternative pathways, predict outcomes or generalizations, note failure in understanding, comeback or plan ahead in order to improve own knowledge. All these are parts of metacognitive capacity developed within the learning process. The metacognitive dimension manifested in a couple of ways. Thus, most of the students were able to analyze critically their own proposals and their own thinking mechanisms, which made them aware of their strengths, and to use these strengths to reinforce a well-defined cognitive frame for a problem. The results show that in the problem-posing process, students develop a genuine philosophy, which refers both to practical actions—embodied in their problem-posing strategies—and to the qualitative form of the posed problems. Typically, students start from a model to which they apply certain constraints based on the philosophy they developed, and they then spontaneously try to get a problem that is mathematically consistent and coherent (e.g. Voica and Singer 2012). Moreover, they get a sense of difficulty or beauty of a problem.

Having in view this capability for metacognitive approaches, Sheffield (1994) recommended that teachers of gifted and talented mathematics students should convey a sense of the beauty and wonder of mathematics in their enthusiasm for both mathematics and for teaching; have confidence in their own mathematical abilities; admit mistakes and enjoy learning along with the students; be continuously involved in professional development; and be willing to let students take over the direction and responsibility for their own learning.

1.10 Mathematical Competencies and Teaching Mathematically Promising Students

Research has shown that students have preferences for some sub-areas of mathematics, or for some problem-solving strategies, which can be relatively easy identified through problem-posing activities. Students' preferences reveal some students' strengths on which teachers can focus in order to develop their mathematical competencies (Singer 2012b; Voica and Singer 2012; Pelczer et al. 2015; Singer and Voica 2015).

In general terms, the mathematics-specific competencies to be developed in students along their school stages refer to: identifying relationships among mathematical concepts/objects; interpreting quantitative, qualitative, structural and contextual data included in mathematical statements; using algorithms and mathematical concepts to characterize a given situation locally or globally; expressing the quantitative or qualitative mathematical features of a contextual situation in order to model it mathematically; analyzing problem situations to discover strategies, to find and optimize solutions; and generalizing properties by modifying a given context or by improving or generalizing algorithms (Singer 2006). These formulations can be particularized, taking into account adequate mathematical content, for a specific age and curriculum trajectory. The competencies acquisition along schooling creates the premises for mathematically promising students to orient their potential towards more expertise in approaching problem situations.

Taking into account the specificity of mathematical creativity, the training for its development should include features that distinguish it from the development of creativity in general. Briefly said, while in the latter, more general case, techniques are to be used for stimulating the free development of ideas, in mathematics the variation of parameters should be practiced within a variety of activities where the processes are mindfully controlled and oriented towards abstraction and generalization (Singer and Voica 2015).

Thus, adequate tasks should develop a domain-specific intuition that favors expert behavior. Through multiple representations, students arrive at seeing their own mistakes, contextualizing them, and explaining them. A focus should be on developing metacognitive abilities in students. From this perspective, the use of problem-posing sessions in teaching is beneficial for students' personal development. Data show that mathematically promising students manifest a strong need for social interaction, which is frequently hidden in certain circumstances. Consequently, social interaction should be part of the teaching-learning process in the class in a consistent way, for example, by means of activities involving posing and solving problems organized in pairs or in small research-teams.

Sriraman (2017) noticed that advances in the study of the psychology of creativity can be consistently linked with mathematics education. Educating for a growth mindset in learning is crucial for the development of creativity.

Our contemporary dynamic society, exposed to unpredictable changes, needs better ways to train students for a changing world. As a practice of learning and thinking, problem posing may play an essential role in stimulating creative approaches and thus preparing students for more dynamic use of their knowledge. Considering that problem-posing research is an emerging force within mathematics education, Singer et al. (2013, 2015) advocate that the use of problem posing in classroom settings: can enhance students' engagement in authentic mathematical activity; develop students' creativity; may generate a positive effect on students' problem-solving achievement and/or their attitudes toward mathematics; and open students' thinking towards new ideas and approaches (Singer et al. 2011, 2013a, b; Ellerton et al. 2015).

The use of multiple representations together with strategies to move from one representation to another can provide explicit scaffolding for the transformation of students' intuitive ideas into domain-specific concepts and procedures, leading to an increase in expertise. These are more related to the cognitive part. Still, other aspects should be equally highlighted. Teachers need a clear goal for gifted education to act educating gifted students purposeful. Purpose is needed for transformational leadership, risk-taking capacity, and skills in ethical thinking. In the dynamic and inter-connected contemporary world, excellence and creativity should be combined with ethics.

The era of information and communication technology creates new social and physical environments and needs. Living in a world where interdependency and dynamics become main features of the global society, young generations have to face unpredictable changes they should learn to cope with. In these conditions, rethinking teaching effectiveness becomes a necessity. There are some capabilities that technology cannot provide yet, and which people need in the present and future society, briefly: higher order synthetic thinking, decision-making capacity under sometimes hardly-predictable situations, transfer capability for solving new problems in new contexts, and a set of values that orient personal behavior in social (and communication) contexts. Within this process, as Tirri (2017) underlined, teachers are seen as key agents in making a significant change in identifying and teaching the gifted. Researchers in gifted education should take the leadership in this change and commit to cooperation with schools.

1.11 Brief Overview of This Book

Within the above discussions, the present book synthesizes the developments presented during two topic-study groups at the 13th International Congress on Mathematics Education (ICME), which took place in Hamburg, Germany, in July 2016. The Topic Study Group 4 (TSG 4), which was focused on *Activities for, and Research on, Mathematically Gifted Students* and the Topic Study Group 29 (TSG 29), which addressed *Mathematics and Creativity*, put together their theories, research, policies and practices to generate a complex integrated approach addressing the issues of giftedness and creativity in mathematics.

The book is structured into four parts, advancing from theoretical underpinnings to practical matters: (1) Frameworks for studying mathematical creativity and giftedness; (2) Characteristics of students with exceptional mathematical promise; (3) Teaching strategies to foster creative learning; and (4) Tasks and techniques to enhance creative capacities.

A brief presentation of the book content follows, emphasizing some lines of thought in connection with the main ideas illustrated above.

An old issue is brought into the contemporary debate by Pitta-Pantazi, Kattou and Christou when discussing four components of Mathematical Creativity: Product, Person, Process and Press (Pitta-Pantazi et al. 2018, this volume). This chapter offers a broad view of various research studies conducted in the field of mathematical creativity which investigated the adaptation of the 1961 Rhodes' 4P model of creativity involving: person (mathematical ability, intelligence, general creative ability, age, gender, culture, personality traits and biographical experiences); product (a novel and useful idea or concept); process (the methodology, or the stages of the creative process); press (teaching environment and the teachers' role, activities and tasks triggering mathematical creativity, new technologies that support mathematical creativity, students interaction/communication). The authors stress that interconnections of the 4Ps are as important as its components. Although these strands can be studied in isolation, when their overlapping and interconnections are considered, the quality of the analysis is much higher and data interpretation leads to conclusions that are relevant for understanding creativity of various groups or individuals.

Under the sign of this conclusion, in the next paper of the volume, Assmus and Fritzlar (2018, this volume) investigate mathematically gifted primary students in their process of creation, in problem-solving and problem-posing contexts. They found that gifted second graders are able to create new mathematical objects. Even if their products are not necessarily true math objects, the chapter contributes with a new vision of young students' capacities. Reviewing a large gamut of situations, Assmus and Fritzlar discuss three instances of the relationship between creativity and giftedness: (mathematical) creativity as a precondition for (mathematical) giftedness, (mathematical) creativity as a possible component of mathematical giftedness, and (mathematical) creativity as a possible consequence of mathematical giftedness, illustrating each instance with examples and theoretical extrapolations. The authors opt for an embedded model of giftedness and creativity in which creativity and giftedness are seen as competence of a person, based on analyzing and confronting data resulting from two categories of samples: primary students gifted and not gifted, who have been exposed to the same types of tasks. Their study also emphasize the idea that tasks like the invention of new mathematical operations encourage primary school students in regular classes being creative with mathematical objects, making stronger an algebraic perspective, even in early grades.

To what extent parameters such as age and training are relevant for the quality of newly created products may seem to have a clear answer. However, it seems that things are not so obvious, and a careful analysis performed by Voica and Singer (2018, this volume) reveal a framework to study creativity by investigating cognitive variety in rich-challenging tasks. Groups of students of different ages and studies (from primary to university) selected based on their interest in mathematics (winners of mathematics competitions, students of faculty of mathematics, professional mathematicians) were asked to start from an image rich in mathematical properties, and generate as many problems related to the given input as possible. The authors found that cognitive variety seems randomly distributed among the tested groups and that, when talking about mathematical creativity, more sophisticated parameters, such as validity, complexity and topic variety, as well as the potential of respondents' products to break a well-internalized frame have to be taken into account. All those are to be balanced against the person's level of expertise in the specified domain.

Consequently, when dealing with concepts situated at the interaction between human knowledge and human psychology, many precautions and careful analysis are needed in order to formulate generalizable conclusions. A study investigating students' performances in multiple-solution tasks (MSTs) brings converging evidence for viewing domain-specificity of mathematical creativity as subdomain-specificity, e.g., in the contexts of geometry, algebra, or arithmetic separately (Joklitschke et al. 2018, this volume). Students' performances along three different MSTs from different mathematical domains such as geometry and algebra show that fluency, flexibility, and originality of the solutions differ consistently between the three subdomains of mathematics, and, therefore, precaution is needed when talking about general mathematical creativity.

The second part of the book starts with a discussion on the characteristics of mathematical giftedness in early primary school age. Here, Assmus (2018, this volume) proposes a comparative study that involves mathematically gifted children and those who are not. The results of the conducted study suggest that the cognitive abilities of mathematically gifted and non-gifted second graders differ in the examined areas. According to her conclusions, the following abilities represent characteristics of mathematical giftedness in early primary school children: ability to memorize mathematical issues by drawing on identified structures, ability to construct and use mathematical structures, ability to switch between modes of representation, ability to reverse lines of thought, ability to capture complex structures and work with them, and ability to use relational concepts and connections. A supportive environment can also have a favorable effect, and therefore, the construct of mathematical giftedness is not reducible to cognitive factors.

Characteristics of mathematically gifted students, such as: unusual quickness in learning; understanding, and applying mathematical ideas, even grasping new ideas before the teacher has finished explaining them; high capability for identifying regularities and complex structures in patterns, extracting them from empirical contexts, and characterizing them in general terms; ability to generalize and transfer mathematical ideas to detect general relationships when observing specific cases; ability to invert mental procedures of mathematical reasoning; flexibility to change from one problem-solving strategy to another if the new one seems to be more useful or easier; development of efficient strategies of problem-solving processes, such as efficiency in using analogical paths in solving various problems—these are characteristics recommended in the specific literature and identified by Gutierrez et al. (2018, this volume) in a gifted student. Through the case of a nine-year-old 5th grader in a primary school, who worked on an experimental pre-algebra teaching unit, the authors test the model of cognitive demand by measuring student's level of intellectual effort as the experiment advanced.

A comprehensive discussion, with intricacies strongly related to cognitive psychology, applied psychology and education is proposed by Nolte (2018, this volume). Her chapter discusses whether the special learning conditions of twice-exceptional students need a differentiated approach than what is usually applied. Furthermore, by means of examples of affected students, the implications for learning processes are illustrated. The paper extends beyond presenting empirical studies, by making a complex discussion of the interactions between mathematical giftedness occurring together with learning disabilities, attention deficit disorders (ADD), attention deficit disorders with hyperactivity (ADHD), and autism spectrum disorders (ASD). The chapter offers an overview of systemic approaches towards giftedness and learning disabilities and disorders, a model for acquiring mathematical competencies including barriers, a thorough discussion about twice exceptionality within the field of mathematics education, about underachievement as a collective term for different disorders and their implications in mathematically promising students, about learning disabilities related to reading, writing, and spelling which affect mathematics learning, including weaknesses in perception as a special learning disability, and explaining why each of them may cause problems to students. In addition, the study raises attention to care-givers and tutors about the masking effect used by these children to hide their giftedness and provides approaches to support students in these conditions. Using four exemplar cases, Nolte succeeds to illustrate the complex aspects presented above and discussed in a large gamut of psychological and educational literature. The study is a good example of how psychology and education can work together for the benefit of clarifying issues in both fields and for finding adequate solutions to cure/improve children's behaviors in a variety of situations.

The third part of the volume emphasizes some teaching strategies to foster creative learning. The research reports presented in this part investigate the effect of various tools on students' creative capacities.

Daher and Anabousy (2018, this volume) discuss flexibility of pre-service teachers in problem posing in different environments and conclude that technology, as well as what-if-not strategy, positively affect students' problem-posing products. However, the combination of technology and the what-if-not strategy positively affected the participants' flexibility in problem posing more than any one of the two tools alone. This finding, qualitatively as well as quantitatively checked, makes a plea for using technology mindfully for a true benefit for students.

Art can also be an important source for creative approaches, stimulating students in multicultural classrooms to engage in mathematics activities. The use of everyday objects like ornaments and the creation of ornaments make the students free to experiment and indulge their imagination (Moraová et al. 2018, this volume). The study found that if pre-service and in-service teachers face a culturally heterogeneous classroom, they tend to be very creative in planning their lessons and at the same time encourage creativity of their students. Thus, cultural heterogeneity may be perceived as an advantage as it may result in breaking out of stereotypes of mathematics classrooms. Moreover, the paper brings into discussion a contemporary issue of teaching: working with unmotivated students, in socially heterogeneous cultures, with migrant students and students from different socio-cultural backgrounds, and provides an effective solution to that. In these circumstances, teachers are naturally motivated to use their creative potential looking for the mathematics that can be discovered and taught in that particular environment and to create substantial learning environments in which the cultural background, the environment are brought into accessible mathematical expressions.

Still, mathematics itself can offer strong inputs for stimulating creative approaches of both teachers and students. This is demonstrated by the study proposed by Friedlander and Tabach (2018, this volume). The learning of algebraic procedures in middle-school algebra is usually perceived as an algorithmic activity, achieved by performing sequences of short drill-and-practice tasks, which have little to do with creative mathematical thinking. The authors provide five instances addressing procedural and conceptual learning, and examine methods of assessing their potential to induce higher-order, and creative thinking in all students. The occurrence of original thinking and students' fluency, originality and flexibility is related to the development of the following mathematical capacities: representing, modeling, interpreting, reversed thinking, generating examples, generalizing, justifying and proving, and thinking divergently.

The last part of the book contains chapters that highlight the benefits of an integrated approach towards creativity and giftedness. Although very different concerning structure and the target population taken into account for investigation, the papers included in this part contain relevant examples of tasks and techniques that can foster students' creativity from kindergarten to university.

The first study compares the methods three mathematically gifted university students used for the resolution of a problem, their strategies, and their transitions from geometrical to algebraic means and vice versa. Poulos and Mamona-Downs (2018, this volume) provided students with a problem to solve, which required the use of software for generating its solution. Observing the solvers' efforts and recording their conjectures in very detailed way, the authors succeeded not only to reveal the solving strategies of these gifted students, but to enrich our understanding of students' attitudes towards 'doing mathematics' in general.

Frequently, high achievers are confronting competitions that reveal their mathematical competencies. Veilande et al. (2018, this volume) propose a paper that analyzes the works of students who have participated in at least three Open Mathematical Olympiads in the 6th, 8th and 9th grades. A set of algebra and number theory problems, whose solving requires high levels of abstract thinking, algebraic reasoning, and an accurate use of the mathematical language were selected for this research. The data collected revealed an interesting result: although very competitive and theoretically well trained, these students showed deficiencies of algebra knowledge in a significant part of their works. Therefore a question is legitimate: Does repeated participation at mathematical Olympiads ensure students' progress in problem-solving? The authors conclude that students need mentors who would help them broaden their problem-solving competencies. In addition, teachers' professionalism is a key prerequisite for developing students' argumentation and justification skills.

If we think that the winners of mathematical Olympiads may become leaders in various domains of social-economical life as adults, then the adequate training of their capacities is more and more important concerning their structured actual specialized knowledge and the development of values and positive attitudes as well.

Special programs for gifted youth, student attendance in interest groups and in clubs, parental support for talent development and collaboration between parents and school are crucial to develop the mathematical abilities of gifted students.

Finally, we move further from Olympiads complex problems to complex open-ended tasks to enrich mathematical experiences of kindergarten students. The chapter proposed by Freiman (2018, this volume) stresses again that at a very young age, some children already manifest unusually strong precocious mathematical abilities that need to be fully developed and nurtured in school. This last chapter investigates in what way a kindergarten curriculum can offer all students a richer mathematical experience by means of open-ended and complex tasks. The data collected during the experiment show challenging situations in terms of the mathematics structures the kindergarten students create during such activities and the strategies they use. While some students struggle with increasing complexity of tasks but still remain engaged and try to overcome obstacles, others seem to exhibit more structured (in terms of mathematical relationships), systematic (in terms of problem-solving strategies), and abstract (in terms of mathematical symbolism) approaches. In addition, all students, even at a very young age, can benefit from a classroom culture of questioning, investigating, communicating, and reflecting on more advanced and meaningful mathematics that can help develop their mathematical minds.

This last paper urges us to think that using more systematic and efficient strategies and encouraging self-control and self-efficacy in young children can further contribute to high mathematics achievement in higher grades. For the readers, it helps increase understanding of the potential of open and complex tasks to enhance the development of mathematical high achievers from a very early age.

References

Amit, M., & Neria, D. (2008). Rising to the challenge: Using generalization in pattern problems to unearth the algebraic skills of talented pre-algebra students. *ZDM - The International Journal of Mathematics Education, 40,* 111–129.

Assmus, D. (2018). Characteristics of mathematical giftedness in early primary school age. In F.M. Singer (Ed.), *Mathematical creativity and mathematical giftedness* (this volume). New York: Springer.

Assmus, D., & Fritzlar, T. (2018). Mathematical giftedness and creativity in primary grades. In F. M. Singer (Ed.), *Mathematical creativity and mathematical giftedness* (this volume). New York: Springer.

Baer, J. (2010). Is creativity domain-specific? In J. C. Kaufman & R. J. Sternberg (Eds.), *Cambridge handbook of creativity* (pp. 321–341). New York: Cambridge University Press.

Baer, J. (2012). Domain specificity and the limits of creativity theory. *The Journal of Creative Behavior, 46*(1), 16–29.

Barbot, B., Besançon, M., & Lubart, T. (2016). The generality-specificity of creativity: Exploring the structure of creative potential with EPoC. *Learning and Individual Differences*. http://dx.doi.org/10.1016/j.lindif.2016.06.005.

Chamberlin, S. A., Buchanan, M., & Vercimak, D. (2007). Serving twice-exceptional preschoolers: Blending gifted education and early childhood special education practices in assessment and program planning. *Journal for the Education of the Gifted, 30,* 372–394.

Chamberlin, S. A., & Moon, S. (2005). Model-eliciting activities: An introduction to gifted education. *Journal of Secondary Gifted Education, 17,* 37–47.

Cross, T. L., & Riedl, J. (2017). Cross challenging an idea whose time has gone. *Roeper Review, 39*(3), 191–194.

Csikszentmihalyi, M. (1988). Motivation and creativity: Toward a synthesis of structural and energistic approaches to cognition. *New Ideas in Psychology, 46*(2), 159–176.

Daher, W., & Anabousy, A. (2018). Flexibility of pre-services teachers in problem posing in different environments. In F. M. Singer (Ed.), *Mathematical creativity and mathematical giftedness* (this volume). New York: Springer.

Dai, D. Y., & Chen, F. (2014). *Paradigms of gifted education: A guide to theory-based, practice-focused research*. Waco, TX: Prufrock Press.

Diezmann, C. M., & Watters, J. J. (2000). Characteristics of young gifted children. *Educating Young Children, 6*(2), 41–42.

Dweck, C. (2006). *Mindset: The new psychology of success*. New York: Random House.

Ellerton, N. F., Singer, F. M., & Cai, J. (2015). Problem posing in mathematics: Reflecting on the past, energizing the present, and foreshadowing the future. In *Mathematical problem posing: From research to effective practice* (pp. 547–556). New York: Springer.

Fingelkurts, An. A., & Fingelkurts, Al. A. (2002). Exploring giftedness. In S. P. Shohov (Ed.), *Advances in psychology research* (Vol. 9, pp. 137–155), Huntington, NY: Nova Science.

Florida's Frameworks for K-12 Gifted Learners (2005–2007), Department of Education. http://etc.usf.edu/flstandards/sss/frameworks.pdf.

Freiman, V. (2018). Complex and open-ended tasks to enrich mathematical experiences of kindergarten students. In F. M. Singer (Ed.), *Mathematical creativity and mathematical giftedness* (this volume). New York: Springer.

Gardner, H. (1993). *Creating minds: An anatomy of creativity as seen through the lives of Freud, Einstein, Picasso, Stravinsky, Eliot, Graham, and Ghandi*. New York: Basic Books.

Glaser, R. (1988). Cognitive science and education. *International Social Science Journal, 115,* 21–45.

Gross, M. (1998). The 'me' behind the mask: Intellectually gifted students and the search for identity. *Roeper Review, 20*(3), 167–174.

Gutierrez, A., Benedicto, C., Jaime, A., & Arbona, E. (2018). The cognitive demand of a gifted student's answers to geometric pattern problems. Analysis of key moments in a pre-algebra teaching sequence. In F. M. Singer (Ed.), *Mathematical creativity and mathematical giftedness* (this volume). New York: Springer.

Hadamard, J. W. (1945). *Essay on the psychology of invention in the mathematical field* (page ref. are to Dover edition, New York 1954), Princeton: Princeton University Press.

Huang, P. S., Peng, S. L., Chen, H. C., Tseng, L. C., & Hsu, L. C. (2017). The relative influence of domain knowledge and domain-general divergent thinking on scientific creativity and mathematical creativity. *Thinking Skills and Creativity, 25,* 1–9.

Jin, S. H., Kim, S. Y., Park, K. H., & Lee, K. J. (2007). Differences in EEG between gifted and average students: Neural complexity and functional cluster analysis. *International Journal of Neuroscience, 117,* 1167–1184.

Johnsen, S., & Sheffield, L. J. (Eds.). (2012). *Using the common core state standards for mathematics with gifted and advanced learners*. Washington, DC: National Association for Gifted Children.

Joklitschke, J., Rott, B., & Schindler, M. (2018). Can we really speak of "mathematical creativity"? Investigating students' performances and their subdomain-specificity in Multiple Solution Tasks. In F. M. Singer (Ed.), *Mathematical creativity and mathematical giftedness* (this volume). New York: Springer.

Kaufman, J. C., Plucker, J. A., & Russell, C. M. (2012). Identifying and assessing creativity as a component of giftedness. *Journal of Psychoeducational Assessment, 30*(1), 60–73. https://doi.org/10.1177/0734282911428196.

Koichu, B., & Berman, A. (2005). When do gifted high school students use geometry to solve geometry problems? *The Journal of Secondary Gifted Education, 16,* 168–179.

Kontorovich, I., & Koichu, B. (2009). Towards a comprehensive framework of mathematical problem posing. In M. Tzekaki, M. Kaldrimidou, & C. Sakonidis (Eds.), *Proceedings of the 33th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 401–408), Thessaloniki, Greece: PME.

Kontorovich, I., Koichu, B., Leikin, R., & Berman, A. (2012). An exploratory framework for handling the complexity of mathematical problem posing in small groups. *The Journal of Mathematical Behavior, 31*(1), 149–161.

Krutetskii, V. A. (1976). *The psychology of mathematical abilities in schoolchildren*. Chicago: University of Chicago Press.

Kurup, A., Basu, A., Chandra, A., Jayan, P., Nayar, S., Jain, G. C., & Rao A. G. (2013). *An introductory reading on giftedness in children: A report prepared as part of the NIAS gifted education project*. National Institute of Advanced Studies. Indian Institute of Science Campus.

Leikin, R. (2009). Exploring mathematical creativity using multiple solution tasks. In R. Leikin, A. Berman, B. Koichu, R. Leikin, A. Berman, & B. Koichu (Eds.), *Creativity in mathematics and the education of gifted students* (pp. 129–145). Rotterdam, The Netherlands: Sense Publishers.

Leikin, R. (2011). The education of mathematically gifted students: Some complexities and questions. *The Mathematics Enthusiast, 8*(1–9). Available on September 29, 2017 at http://scholarworks.umt.edu/tme/vol8/iss1/9.

Leikin, A. B., & Koichu, B. (Eds.). (2009). *Creativity in mathematics and the education of gifted students*. Rotterdam: Sense Publishers.

Leikin, R., & Pitta-Pantazi, D. (Eds.). (2013). Creativity and mathematics education. *Special issue of ZDM Mathematics Education, 45*(2).

Mann, E. (2006). Creativity: The essence of mathematics. *Journal for the Education of the Gifted, 30*(2), 236–260.

Moraová, H., Novotná, J., & Favilli, F. (2018). Ornaments and tessellations—Encouraging creativity in mathematics classroom. In F. M. Singer (Ed.), *Mathematical creativity and mathematical giftedness* (this volume). New York: Springer.

Mrazik, M., & Dombrowski, S. C. (2010). The neurobiological foundations of giftedness. *Roeper Review, 32*(4), 224–234.

Munro, J. (2005). High-ability learning and brain processes: How neuroscience can help us to understand how gifted and talented students learn and the implications for teaching. *Exceptional International Education Journal, 6*(2), 247–251.

National Science Board (NSB). (2010, May 5). *Preparing the next generation of STEM innovators: Identifying and developing our nation's human capital*. NSB-10-33. Washington, DC: NSF.

Nolte, M. (2012). Mathematically gifted young children—Questions about the development of mathematical giftedness. In H. Stöger, A. Aljughaiman, & B. Harder (Eds.), *Talent development and excellence* (pp. 155–176). Berlin, London: Lit Verlag.

Nolte, M. (2018). Twice-exceptional students: Students with special needs and a high mathematical potential. In F. M. Singer (Ed.), *Mathematical creativity and mathematical giftedness* (this volume). New York: Springer.

O'Boyle, M. W. (2008). Mathematically gifted children: Developmental brain characteristics and their prognosis for well-being. *Roeper Review, 30,* 181–186.

Owen, A. M., Hampshire, A., Grahn, J. A., Stenton, R., Dajani, S., & Burns, A. S. (2010). Putting brain training to the test. *Nature*. www.nature.com/doifinder/10.1038/nature09042.

Pelczer, I., Singer, F. M., & Voica, C. (2011). An analysis of relevant hints in problem solving. In B. Ubuz (Ed.), *Developing mathematical thinking. Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education*. Ankara, Turkey: PME. ISBN: 978-975-429-262-6, 1, 370.

Pelczer, I., Singer, F. M., & Voica, C. (2015). When communication tasks become tools to enhance learning. *Procedia-Social and Behavioral Sciences, 187,* 503–508.

Pitta-Pantazi, D., Kattou, M., & Christou, C. (2018). Mathematical creativity: Product, person, process and press. In F. M. Singer (Ed.), *Mathematical creativity and mathematical giftedness* (this volume). New York: Springer.

Poulos, A., & Mamona-Downs, J. (2018). Gifted students approaches when solving challenging mathematical problems. In F. M. Singer (Ed.), *Mathematical creativity and mathematical giftedness* (this volume). New York: Springer.

Prescott, J., Gavrilescu, M., Cunnington, R., Boyle, M. W. O., et al. (2010). Enhanced brain connectivity in math-gifted adolescents: An fMRI study using mental rotation. *Cognitive Neuroscience, 1*(4), 277–288.

Reed, C. F. (2004). Mathematically gifted in the heterogeneously grouped mathematics classroom: What is a teacher to do? *The Journal of Secondary Gifted Education, 3,* 89–95.

Reis, S. M., Westberg, K. L., Kulikowich, J. M., & Purcell, J. H. (1998). Curriculum compacting and achievement test scores: What does the research say? *Gifted Child Quarterly, 42,* 123–129.

Rhodes, M. (1961). An analysis of creativity. *Phi Delta Kappan, 42*(7), 305–311.

Roedell, W. C. (1989). Early development of gifted children. In J. L. VanTassel-Baska & P. Olszewski-Kubilius (Eds.), *Patterns of influence on gifted learners: The home, the self and the school* (pp. 13–28). New York: Teachers College Press.

Runco, M. A. (1994). *Problem finding, problem solving, and creativity*. Norwood, NJ: Ablex.

Sheffield, L. J. (1994). The development of gifted and talented mathematics students and the National Council of Teachers of Mathematics Standards. Storrs, CT: The National Research Center for the Gifted and Talented, University of Connecticut.

Sheffield, L. J., Bennett, J., Berriozabal, M., DeArmond, M., & Wertheimer, R. (1999). Report of the NCTM task force on the mathematically promising. In L. J. Sheffield (Ed.), *Developing mathematically promising students* (pp. 309–316). Reston, VA: NCTM.

Silverman, L. K. (2002). *Upside-down brilliance: The visual-spatial learner*. Denver: DeLeon.

Silverman, L. K. (2013). *Giftedness 101*. New York: Springer.

Singer, M. (2006). A cognitive model for developing a competence-based curriculum in secondary education. In: Al. Crisan (Ed.), *Current and future challenges in curriculum development: Policies, practices and networking for change* (pp. 121–141). Bucharest: Education 2000+ Publishers. Humanitas Educational.

Singer, F. M. (2012a). Boosting the young learners' creativity: Representational change as a tool to promote individual talents (Plenary lecture). In *The 7th International Group for Mathematical Creativity and Giftedness (MCG) International Conference Proceedings* (pp. 3–26). Busan, South Korea: MCG. ISBN: 978-89-98016-10-4.

Singer, F. M. (2012b). Exploring mathematical thinking and mathematical creativity through problem posing. In R. Leikin, B. Koichu, & A. Berman (Eds.), *Exploring and advancing mathematical abilities in high achievers* (pp. 119–124). Haifa: University of Haifa.

Singer, F. M., Ellerton, N. F., & Cai, J. (Eds.). (2013a). Problem posing in mathematics teaching and learning: Establishing a framework for research. *Educational Studies in Mathematics, 1* (83).

Singer, F. M., Ellerton, N. F., & Cai, J. (2013b). Problem-posing research in mathematics education: New questions and directions. *Educational Studies in Mathematics, 83*(1), 1–7.

Singer, F. M., Ellerton, N. F., & Cai, J. (Eds.). (2015). *Mathematical problem posing: From research to effective practice*. New York: Springer.

Singer, F. M., Sheffield, L. J., Freiman, V., & Brandl, M. (2016). *Research on and activities for mathematically gifted students*. New York: Springer Open.

Singer, F. M., Sheffield, L. J., & Leikin, R. (Eds.). (2017a). Mathematical creativity and giftedness in mathematics education. *Special issue of ZDM Mathematics Education, 49*(1).

Singer, F. M., & Voica, C. (2013). A problem-solving conceptual framework and its implications in designing problem-posing tasks. *Educational Studies in Mathematics, 83*(1), 9–26.

Singer, F. M., & Voica, C. (2015). Is problem posing a tool for identifying and developing mathematical creativity? In F. M. Singer, N. Ellerton, & J. Cai (Eds.), *Mathematical problem posing: From research to effective practice* (pp. 141–174). New York: Springer.

Singer, F. M., & Voica, C. (2016). When mathematics meets real objects: How does creativity interact with expertise in problem solving and posing? In R. Leikin & B. Sriraman (Eds.), *Creativity and giftedness. Interdisciplinary perspectives from mathematics and beyond* (pp. 75–103). New York: Springer.

Singer, F. M., Voica, C., & Pelczer, I. (2017b). Cognitive styles in posing geometry problems: Implications for assessment of mathematical creativity. *ZDM Mathematics Education, 49*(1), 37–52.

Sriraman, B. (2003). Mathematical giftedness, problem solving, and the ability to formulate generalizations. *The Journal of Secondary Gifted Education, XIV*(3), 151–165.

Sriraman, B. (2005). Are giftedness & creativity synonyms in mathematics? An analysis of constructs within the professional and school realms. *Journal of Secondary Gifted Education, 17*(1), 20–36.

Sriraman, B. (Ed.). (2008). *Creativity, giftedness, and talent development in mathematics*. Charlotte, NC: Information Age Publishing.

Sriraman, B. (2017). Mathematical creativity: Psychology, progress and caveats, Survey Paper. *ZDM Mathematics Education*, *49*(7), August 30, 2017. https://doi.org/10.1007/s11858-017-0886-0.

Sriraman, B., Haavold, P., & Lee, K. (2013). Mathematical creativity and giftedness: A commentary on and review of theory, new operational views, and ways forward. *ZDM Mathematics Education, 45,* 215–225.

Sternberg, R. J. (2017a). ACCEL: A new model for identifying the gifted. *Roeper Review, 39*(3), 152–169.

Sternberg, R. J. (2017b). Does ACCEL excel as a model of giftedness? A reply to commentators. *Roeper Review, 39*(3), 213–219.

Sternberg, R. J., & Lubart, T. I. (2000). The concept of creativity: Prospects and paradigms. In R. J. Sternberg (Ed.), *Handbook of creativity* (pp. 93–115). Cambridge: Cambridge University Press.

Straker, A. (1983). *Mathematics for gifted pupils*. Harlow: Longman.

Tabach, M., & Friedlander, A. (2018). Instances of promoting creativity with procedural tasks. In F. M. Singer (Ed.), *Mathematical creativity and mathematical giftedness* (this volume). New York: Springer.

The Columbus Group. http://www.gifteddevelopment.com/isad/columbus-group.

Tirri, K. (2017). Teacher education is the key to changing the identification and teaching of the gifted. *Roeper Review, 39*(3), 210–212. https://doi.org/10.1080/02783193.2017.1318996.

Torrance, E. P. (1974). *Torrance tests of creative thinking*. Bensenville, IL: Scholastic Testing Service.

Veilande, I., Ramana, L., & Krauze, S. (2018). Repeated participation at the mathematical Olympiad: Does it ensure the students' progress in the use of problem-solving strategies? In F. M. Singer (Ed.), *Mathematical creativity and mathematical giftedness* (this volume). New York: Springer.

Voica, C., & Singer, F. M. (2012). *Problem modification as an indicator of deep understanding. Proceedings of ICME 12* (pp. 1533–1542), Seoul, Korea. July 8–15, 2012, www.icme12.org/upload/UpFile2/TSG/1259.pdf.

Voica, C., & Singer, F. M. (2013). Problem modification as a tool for detecting cognitive flexibility in school children. *ZDM Mathematics Education, 45*(2), 267–279.

Voica, C., & Singer, F. M. (2014). Problem posing: A pathway to identifying gifted students. *MCG8 Proceedings* (pp. 119–124). Colorado, USA: University of Denver.

Voica, C., & Singer, F. M. (2018). Cognitive variety in rich-challenging tasks. In F. M. Singer (Ed.), *Mathematical creativity and mathematical giftedness* (this volume). New York: Springer.

Willingham, D. T. (2002). Inflexible knowledge: The first step to expertise. *American Educator*, (Winter), 31–33, 48–49.

Willingham, D. T. (2007). Critical thinking: Why is it so hard to teach? *American Educator*, (Summer), 8–19.

Winner, E. (1996). *Gifted children: Myths and realities*. New York: Basic Books.

Woolcott, G. (2011). Mathematics and giftedness: Insight from educational neuroscience. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), *Mathematics: Traditions and [New] practices (Proceedings of the 34th Annual Conference of the Mathematics Education Research Group of Australasia and the Australian Association of Mathematics Teachers)* (pp. 830–838). Adelaide: AAMT and MERGA.