Important and Challenging Issues for Interval Type-2 Fuzzy Control Research

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Abstract The author points out three important issues: (1) when should interval type-2 (IT2) fuzzy control be utilized, (2) how to design IT2 fuzzy controllers, and (3) how to analyze IT2 fuzzy controllers. Discussion is focused on application and practicality.

1 Introduction to Interval Type-2 Fuzzy Control

Fuzzy control is the most active and victorious component of fuzzy systems technology. The first fuzzy controller was developed by Professor E. H. Mamdan at University of London in United Kingdom in 1974 [4]. The primary thrust of this novel control paradigm at the time was to utilize human control operator's knowledge and experience to intuitively construct a controller so that the resulting controller is able to emulate human control behavior to a certain extent. Compared to the traditional control paradigm, the advantages of the fuzzy control paradigm are two folds. First, a mathematical model of the system to be controlled is not required, and (2) a satisfactory nonlinear controller can be developed empirically without complicated mathematics. The core value of these advantages is the practicality—real-word systems are nonlinear; accurately modeling them is difficult, costly, and even impossible in most cases. Proper use of fuzzy control can significantly shorten product research and development time with reduced cost. Since mid-1980s, companies around the world have utilized fuzzy control to make better, cheaper, and smarter products. Many of them are commercial products. All these fuzzy controllers are now called type-1 fuzzy controllers when there is a need to differentiate them from type-2 fuzzy controllers. Nevertheless, they are referred in the literature simply as fuzzy controllers because type-2 fuzzy control did not exist yet when the reports were published. Figure 1 illustrates configuration of a typical type-1 fuzzy controller.

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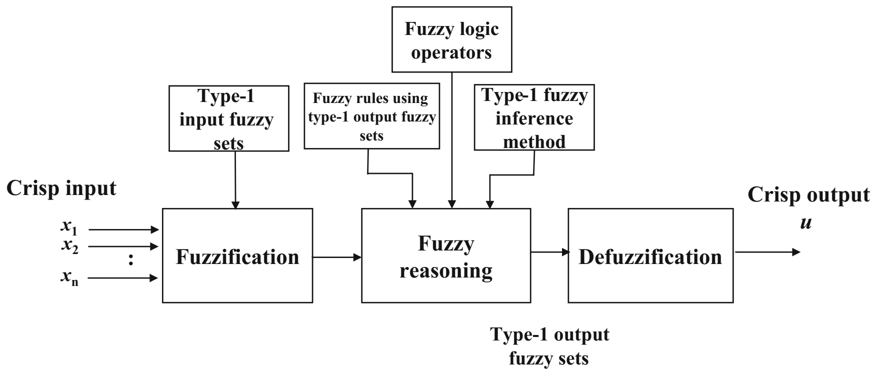

Fig. 1 Configuration of a typical type-1 fuzzy controller

To better reflect complicated nature of expert knowledge, a fuzzy controller may conceivably use a type-2 fuzzy set, which is an extension to a type-1 fuzzy set in that at each value of the universe discourse, the membership value is an interval with another membership function (i.e., secondary membership function) defined over it. A type-2 fuzzy set uses footprint of uncertainty to characterize the region between its upper and lower membership functions. Although the concept of a type-2 fuzzy set was first introduced by Professor L. A. Zadeh in 1975, using it to form a fuzzy inference system is only a relatively recent advance. Professor J. M. Mendel and his coworkers have proposed the first complete type-2 fuzzy inference process, developed various type-2 fuzzy systems, and established their computational principles and foundations since the mid-1990s (e.g., [3, 5, 6]).

With the solid type-2 fuzzy system foundation laid by Mendel and others, researchers extended the notion of fuzzy control to type-2 fuzzy control around the 2000s. The basic idea was to first replace some or all of type-1 fuzzy sets in a fuzzy controllers by (interval) type-2 fuzzy sets, and then added components specific to a type-2 system (e.g., type reducer). Some other modifications were also necessary (e.g., the defuzzification process). Figure 2 shows configuration of a typical type-2 fuzzy controller. The grey boxes spell out the configuration differences between the

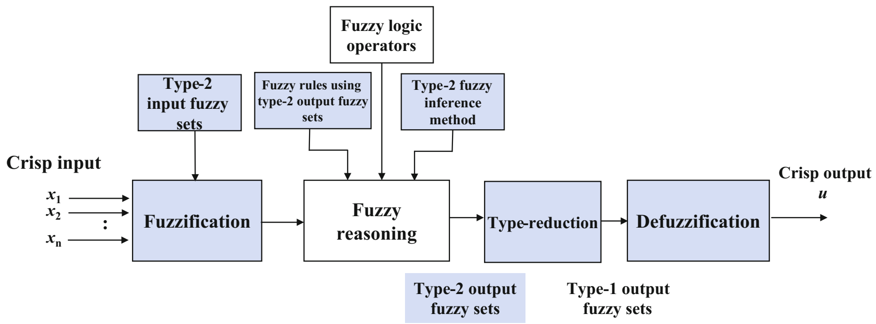

Fig. 2 Configuration of a typical type-2 fuzzy controller

type-2 and type-1 controller configurations in Figs. 1 and 2. Structurally, a type-2 fuzzy controller is more complicated than its type-1 counterpart as the former has more components (e.g., type reducer), more parameters (e.g., footprints of uncertainty of the interval type-2 fuzzy sets), and a more complex inference mechanism.

When the secondary membership function of a type-2 fuzzy set is constant 1, the fuzzy set is an interval type-2 (IT2) fuzzy set. A type-2 fuzzy controller uses IT2 fuzzy sets is called an IT2 fuzzy controller. This chapter focuses on IT2 fuzzy control only as it represents the simplest kind of type-2 fuzzy control and is the most interesting kind at present to the fuzzy control community. Note that an IT2 fuzzy controller degenerates into a type-1 fuzzy controller when footprints of uncertainty of all the type-2 fuzzy sets reduce to 0. Thus, a type-1 fuzzy controller is a special case of this corresponding IT2 fuzzy controller.

2 Research Issue 1: When Should IT2 Fuzzy Control Be Utilized?

Before addressing the issue "when should IT2 fuzzy control be employed to solve a control problem?" let's first discuss the question "when should fuzzy control, type-1 or type-2, be employed to solve a control problem?" Sects. 2.1, 2.2, and 2.4 below are applicable to both type-1 and IT2 fuzzy control.

2.1 Advantages of Fuzzy Control

The biggest advantage of fuzzy control is that it provides an effective and efficient methodology to develop nonlinear controllers without using advanced mathematics. Making a fuzzy controller requires describing human control knowledge/experience linguistically and captures them in the form of fuzzy sets, fuzzy logic operation and fuzzy rules. Fuzzy control can be used to emulate human expert knowledge and experience and is ideal for solving practical problems where imprecision and vagueness are present and verbal description is necessary. Unlike the traditional mathematical-model-based controller design methodology, an explicit system model is not required by fuzzy control. Rather, a system model is implicitly built into fuzzy rules, fuzzy logic operation and fuzzy sets in a vague manner. Fuzzy rules relate input fuzzy sets describing state of output variables of the system to fuzzy controller output. In a sense, fuzzy control combines the system modeling task and the system control task into one task. By avoiding a separate modeling task, which can be more challenging than the control task in many real-world situations, control problems can be solved more efficiently and effectively. Countless applications of fuzzy control around the world have proved this point for type-1 fuzzy control.

Fuzzy control has also created a paradigm for developing nonlinear and multiple-input multiple-output (MIMO) controllers without using sophisticated linear/nonlinear control theory and mathematics. This is in sharp contrast to conventional control technology, especially the nonlinear one. Through manipulating various components of a fuzzy controller, such as the scaling factors, fuzzy sets and fuzzy rules, coupled with computer simulation and/or trial-and-error effort, it is often possible for a non-control professional to build a well-performing fuzzy controller. This advantage makes fuzzy control practical and powerful in solving real-world problems and it explains why (type-1) fuzzy control has especially been popular in industry.

2.2 Disadvantages of Fuzzy Control

A fuzzy controller usually has (far) more design parameters than a comparable conventional controller. To make the matter worse, learning how to construct a good fuzzy controller when the system model is unavailable is, to a large extent, more an art than science. Subsequently, fuzzy controller development may require more tuning and trial-and-error effort. Compared to the industrially dominant PID control that has only three design parameters, the number of design parameters for a fuzzy controller can become overwhelmingly large. They range from the number and shape of input and output fuzzy sets, scaling factors, fuzzy AND and OR operators to fuzzy rules and defuzzifier. Worse yet, there do not exist clear and general relationships between these parameters and controller's performance. The developer need to partially rely on empirical rules of thumb and ad hoc design procedures in the literature to make successful fuzzy control applications. Although there exist a great deal of such knowledge on type-1 fuzzy controllers, it is not sufficient, especially for fuzzy control novices. Fuzzy controllers are nonlinear controllers. As such, the generality of the knowledge is rather limited. Any design and/or tuning procedure can hardly be generalized to cover a broader range of fuzzy control problems. As a result, trial-and-error effort and extensive computer simulation are often necessary. Neither stability nor performance of the fuzzy control system under development can rigorously be guaranteed. This empirical approach, while effective for some applications, is impractical and unsafe for applications in some fields, such as aerospace, nuclear engineering and, particularly, biomedicine.

2.3 Accurate Nonlinear System Models Are Hard and Expansive to Obtain in Practice

Conventional nonlinear control theory is powerful and effective if a nonlinear system model is mathematically available. In order to design a conventional controller for controlling a physical system, the mathematical model of the system is needed. A common form of the system model is differential equation for a continuous-time system or difference equation for a discrete-time system. Strictly speaking, all physical systems are nonlinear. Unless physical insight and the laws of physics can be applied, establishing an accurate nonlinear model using measurement data and system identification methods is difficult in practice.

For any dynamic system modeling problems, linear or nonlinear, two tasks need to be accomplished. The first task is model structure identification, and the second model parameter identification. These tasks are relatively easier for linear system modeling as there have already existed a set of popular linear model structures to choose from, which include AR (Auto Regressive), ARX (Auto Regressive with eXtra input) and ARMA (Auto Regressive Moving Average). They are different types of difference equations and are black-box models. Strictly speaking, a linear system does not exist—a linear model is an approximate model of the nonlinear system valid for a region around one of the system operation points.

Nonlinear system modeling, however, is far more complicated because there exist an infinitive number of possible model structures. Correctly assuming a nonlinear model structure is a hard problem in nonlinear system modeling theory and no general theory exists. Though difficult, different nonlinear system modeling techniques have still been developed, including the Volterra and Wiener theories of nonlinear systems. Such nonlinear system models are black-box models because they only attempt to mimic system's input-output relationship with system measurement data and hence cannot provide any insight on internal structure of the system. Another option is to model a nonlinear system as a (piecewise) linear system. This approach can be over-simplistic in nature and fails to capture diverse and peculiar nonlinear system behaviors, such as limit circles, chaos and bifurcation.

Once the model structure is selected/determined, parameters in the model can be found using system's input-output data and some system optimization procedures (e.g., the least-squares methods), which is the second task.

A linear system model is often adequate for control system development. The whole knowledge base of linear control theory, from linear PID control to modern linear robust control, has been developed based on the notation of linear system models. Once designed, control performance and system stability as well as other properties of the linear control system can usually be examined mathematically. This is because these linear models are difference equations and thus can be analytically analyzed. Whether this linear controller development approach will succeed in practice depends highly on whether the linear model captures the essence of the nonlinear physical system and whether it is a reasonable representation and approximation of the physical system.

In contrast, accurately establishing a nonlinear system model is generally difficult, which significantly limits the application scope of nonlinear control theory.

2.4 When Should Fuzzy Control Be Employed?

There exists literally a countless number of different types of systems in practice. Applicability of fuzzy control, type-1 or type-2, apparently should relate to the strengths and limitations of fuzzy control. In our opinion, fuzzy control is most desirable if (1) mathematical model of the system to be controlled is unavailable but the system is known to be significantly nonlinear, time-varying or with a larger time delay, and/or (2) PID control cannot generate satisfactory system performance.

Given the strengths of fuzzy control, the first criterion is natural and logical. We need to stress the second criterion: It is practically important to know whether PID (including PI or PD control) can solve the control problem of interest before fuzzy control is attempted. PID, PI, PD controllers have been used to control about 90% industrial processes worldwide. PID control techniques are well-developed and numerous control system design and gain tuning methods are available. When the system to be controlled is linear and its mathematical model is available, design and implementation of linear PID control is effective and efficient. Note that using PID control does not necessarily require system model. In the absence of a system model, one can still achieve satisfactory PID control performance in practice by manually tuning, in a trial-and-error fashion, the proportional-gain, integral-gain and derivative-gain. This is true if the system is linear, somewhat nonlinear, or with a mild time delay. Better yet, there exist different types of PID controllers. The most commonly used one is the linear PID controller but often nonlinear ones, such as the anti-windup PID controller, are also employed. Properly adding nonlinearity to linear PID control can lead to desirable control performance. Time has proved that PID control, though simple, is effective and can produce satisfactory results quickly for the majority of control problems, especially those in process control. This is the case even when the system of interest is nonlinear, time-varying or associated with a time delay, as long as they are not too severe.

Fuzzy control should be used, if at least one of the two criteria mentioned above holds. This is the case even if control expert knowledge and experience is unavailable. Practically speaking, it is possible for one to achieve satisfactory fuzzy control of nonlinear systems through extensive computer simulation and trial-and-error effort without expert knowledge. Utilizing available expert knowledge/experience will no doubt reduce development cost and time, particularly when the system is rather complex. But this is not a prerequisite for using fuzzy control.

Even when the system of interest is nonlinear, time-varying or associated with a time delay and its mathematical model is explicitly given, it can often be still advantageous to apply fuzzy control provided that designing an adequate nonlinear controller is more difficult. Unlike linear control theory, there does not exist a general nonlinear control and system theory that is universally applicable to any nonlinear, time-varying or time-delay systems. When a nonlinear system of interest is complicated, or a MIMO one, conventional control theory may be ineffective or even unusable. Furthermore, many of the existing nonlinear control techniques require highly sophisticated control and mathematics background (e.g., differential geometry), which are inaccessible to many of control engineers in the field.

Fuzzy control should not be employed if the system to be controlled is linear, regardless of the availability of its explicit model. For linear systems, there is no advantage to use fuzzy control. PID control and various other types of linear controllers can effectively solve the problem with significantly less effort, time and cost.

In summary, fuzzy control does not and cannot replace conventional control, linear or nonlinear; instead, it complements conventional control rather nicely.

2.5 When Should IT2 Fuzzy Control Be Employed?

In the 1980s, the question “when should fuzzy control be used instead of a conventional controller” was faced by the fuzzy control community. Because the advantages and disadvantages of type-1 fuzzy control relative to those of conventional control were relatively easy to determine and understand, that question was not too difficult to be settled.

A similar question “when should IT2 fuzzy control be used instead of type-1 fuzzy controller” is now waiting the fuzzy control community to answer.

According to Figs. 1 and 2, both type-1 and IT2 fuzzy control methodologies provide a “knowledge engineering” procedure, as opposed to the mathematical approach exclusively adopted in conventional control, to construct $u = f(x_1, x_2, \ldots, x_n)$, where f is a nonlinear and unknown function that represents the control solution being sought. It has been shown that a wide range of type-1 fuzzy controllers are universal approximators in that they can approximate continuous functions arbitrarily well (e.g., [10, 12, 15]), so are various IT2 fuzzy controllers [14]. So, theoretically speaking, IT2 fuzzy control can do whatever type-1 fuzzy control can do, and vice versa.

It should not be difficult to understand that IT2 fuzzy control will not, and cannot, replace either type-1 fuzzy control or conventional control. The three control methodologies are complementary. Arguably, one of the most important research directions is to develop a theory capable of determining whether or not an IT2 fuzzy controller should be used for any given control problem. That is, a theory is needed that can be used ahead of time to determine whether an IT2 fuzzy controller should be employed as opposed to a type-1 fuzzy controller. It is important that such a theory be simple and effective so that it can be used by a control practitioner who may be moderately knowledgeable about type-1 fuzzy control but has little or no knowledge about IT2 fuzzy control (it is not very realistic to assume that someone knowing nothing about type-1 fuzzy control will consider to use IT2 fuzzy control). This theory should not be simulation-based because a system’s accurate mathematical model is, realistically speaking, always nonlinear and thus is very difficult to obtain in practice, as we pointed out above. This theory should also not be heavily reliant on trial-and-error effort because such an approach can not only be costly but also risky to use for safety-critical applications (e.g., nuclear industry and clinical medicine).

In practice, IT2 fuzzy control may have to prove its superiority to both type-1 fuzzy control and conventional control for a particular control problem or a particular class of control problems before it will actually be used. Because type-1 fuzzy control and conventional control are able to deliver satisfactory solutions for so many different practical control problems, defining the niche applications that require the distinct merits of IT2 fuzzy control is a critically important but technically challenging area of study. Another important factor that one has to keep in mind is that a real-world control application typically seeks the simplest and least expensive hardware/software solution that satisfies the technical specifications required by the user. This is why PID, PI and PD controllers, with only two or three design parameters, all of which can be tuned manually in an intuitive manner, have become the most popular controllers since their inception, dating back to the pre-electronic period, despite the availability of numerous more advanced and better (at least in theory) controllers developed in the past dozens of years (e.g., optimal controllers and robust controllers).

An IT2 fuzzy controller should not be used unless its added structural complexity and additional design parameters (as compared with a type-1 fuzzy controller) can be reasonably justified by demonstrated significant gains in control performance (e.g., better transition control response and/or more robust performance in the presence of system noise and/or disturbance). Research has been under way to explore when IT2 fuzzy control can bring substantial performance improvement, and more and more results are appearing.

3 Research Issue 2: How to Design IT2 Fuzzy Controllers?

If, for a given practical control problem, it is decided to use an IT2 fuzzy controller instead of a type-1 controller, the next logical issue is how to design it.

Numerous techniques have been developed in literature for analyzing and designing a wide variety of fuzzy control systems of both the Mamdani type and the TSK type. They are mostly for the type-1 fuzzy controllers for now [2], but a growing number of techniques are developed for the IT2 controllers. The literature can be classified into two groups according to methodology: (1) the model-based approach, and (2) the knowledge-based approach, which is a model-free approach. When the model-based approach is used, the precise mathematical model of the system to be controlled must be assumed explicitly available whereas the knowledge-based approach does not make such an assumption. The model of interest should be nonlinear because a practical system is always nonlinear. While the model availability assumption makes theoretical development mathematically tractable and convenient for the model-based approach, it hardly realistically reflect practical constraints. The fact of the matter is this—it is challenging to attain a reasonable nonlinear mathematical model for most systems in the real world. The pitfall of the model availability assumption holds not only for fuzzy control but also equally for conventional control. Emerging in the 1990s, this approach provides mathematical convenience at the cost of practicality. It has produced a large volume of publications; nevertheless, its usefulness in practice has yet to be established. *In short, without knowing the nonlinear model, most, if not all, of the model-based design methods for the type-1 fuzzy controllers are simply inapplicable.*

IT2 fuzzy controllers are nonlinear controllers with complicated input–output relations. They are certainly more complex than their type-1 counterparts in terms of the mathematical input–output relations and the number of design parameters. Consequently, designing an IT2 fuzzy control system is more challenging than designing a type-1 fuzzy control system. As evident by trends in the recent literature, an important research direction is to extend the analysis and design techniques that have been developed for various type-1 fuzzy controllers and systems to IT2 fuzzy controllers and systems. Interestingly, methodologies available for analyzing and designing IT2 fuzzy controllers and systems are fundamentally the same as those utilized for type-1 fuzzy controllers and systems. For example, the Lyapunov approach, which has been widely used for type-1 fuzzy control systems as well as for conventional nonlinear control systems, is the only general tool that has been used for analyzing system stability or designing a stable IT2 fuzzy control system. To date, there exists no other more effective stability approach for IT2 fuzzy control systems. It is presently the most general and best technique available for IT2 fuzzy controllers and systems, and we believe that it will play a crucial role in the development of future IT2 fuzzy control theory. Note, however, that extending the type-1 fuzzy control techniques to cover IT2 fuzzy controllers can be challenging because, generally speaking, an IT2 fuzzy controller is a more complicated nonlinear controller than is a type-1 fuzzy controller.

An IT2 fuzzy controller, like its type-1 counterpart, is presently viewed and treated by most fuzzy control practitioners and theorists as a black-box function generator that is capable of producing a desired nonlinear mapping between input and output of the controller (i.e., $u = f(x_1, x_2, \ldots, x_n)$ in Fig. 2). The mapping is implicit because $f(x_1, x_2, \ldots, x_n)$ does not spell out the explicit relationship between the input variables and the output variable. In other words, it shows there is a relationship but does not reveal exactly what it is. When the model-based approach utilizes the implicit $f(x_1, x_2, \ldots, x_n)$ to develop a controller design method, it treats the fuzzy controller as the black-box function generator. On the other hand, the knowledge-based approach does not start with $f(x_1, x_2, \ldots, x_n)$. Rather, it relies on a systematic procedure comprising of a number of steps to practically construct $f(x_1, x_2, \ldots, x_n)$ through manipulating, often in a trial-and-error fashion, fuzzy sets, fuzzy rules, fuzzy inference, and other components. For each component, the developer will face choices. For instance, for input fuzzy sets (i.e., the fuzzy sets for fuzzifying input variables), the developer has to decide how many of them should be used, what type should be used (e.g., triangular vs. Gaussian), and whether a mixture of different types should be used. This is just one of the several components that the developer has to specify (other components include output fuzzy sets, fuzzy rules and defuzzifier). Coupled with computer simulation, this approach often suffices for the practitioner to build a satisfactory fuzzy control system as a solution to the real-world problem at hand. Importantly, this tactic usually works even when the mathematical model of the system is not available. Apart from the approach (model-based or knowledge-based), once built, the fuzzy controller remains a black box in that the explicit expression of $f(x_1, x_2, \ldots, x_n)$ is still unknown. The components work together to generate a value for $f(x_1, x_2, \ldots, x_n)$ for any given value of the input variables. Obviously, the explicit expression of $f(x_1, x_2, \ldots, x_n)$ depends on how the components are selected. The implicit nature of $f(x_1, x_2, \ldots, x_n)$ does not change regardless.

4 Research Issue 3: How to Analyze IT2 Fuzzy Controllers

We call the mapping mentioned above the analytical structure of the fuzzy controller. The model-based approach and the knowledge-based approach of fuzzy control, type-1 or IT2, are in sharp contrast to the conventional control theories. In conventional control, once a controller is chosen by the developer according to the system to be controlled, the controller's analytical structure, linear or nonlinear, is always explicitly ready for analysis and design of the control system. The linear and nonlinear control theories are matured with many time-tested analysis and design schemes. The primary technical difficulty for controller design lies in how to first select or design $f(x_1, x_2, \ldots, x_n)$ and then determine its parameter values based on the given system model so that the designed control system performance will meet the developer's performance specifications. $f(x_1, x_2, \ldots, x_n)$ is explicitly known after the control system design is completed. Control system analysis, stability, control performance, and other system characteristics are analyzed and determined based on both the explicitly $f(x_1, x_2, \ldots, x_n)$ and the system model. To bring fuzzy control to the same level of sophistication and acceptance as the conventional control theories, fuzzy control needs to overcome two hurdles pertinent only to fuzzy control and irrelevant to conventional control. The first hurdle is the unavailability of $f(x_1, x_2, \ldots, x_n)$ in an explicit form after it is designed/constructed, and the second relates to the fundamental question of whether $f(x_1, x_2, \ldots, x_n)$ can be an arbitrary nonlinear function. The second issue, referred to as fuzzy systems as universal approximators in literature, has been extensively addressed for the type-1 fuzzy controllers, but has been investigated for the IT2 controllers only in a rather limited scope [14]. To a large extent, mathematically studying IT2 (or type-1) fuzzy control is inherently even more challenging than studying typical nonlinear control problems. Not explicitly knowing $f(x_1, x_2, \ldots, x_n)$ puts both the model-based and model-free fuzzy control approaches in a disadvantageous position.

Studying the analytical structures of both the controller and the system under control can make it possible for the system analysis and design more precise and effective and less conservative. No matter if an IT2 fuzzy controller is theoretically designed using a model-based scheme or is empirically constructed via a knowledge-based method, revealing controller's analytical structure can be significantly beneficial because one can then:

1. insightfully understand how and why an IT2 fuzzy controller works in the same sense as we understand how a conventional controller functions,
2. find a possible connection between an IT2 fuzzy controller and a conventional controller,
3. explore rigorously the differences between an IT2 fuzzy controller and its type-1 fuzzy controller and their relative merits and pitfalls (e.g., control performance and structural complexity),
4. take advantage of the nonlinear control theory to develop more effective analysis and design methods for IT2 control system as the fuzzy control problem has transformed into a nonlinear control problem, and
5. make IT2 fuzzy control more acceptable to safety-critical fields such as clinical medicine and nuclear industry where people are reluctant to employ a black box as a controller.

We stress that the analytical structure of a fuzzy controller should be investigated in such a way that the structure is sensible in the context of control theory. This is to say that deriving the explicit structure is only a first step, after which the structure should be represented in a form clearly understandable from a control theory standpoint to gain the full potential in system analysis and design.

We derived the first analytical structure of a type-1 fuzzy controller in 1990 [9]. The analytical structures of many other type-1 fuzzy controllers have been reported in the literature since then. The benefits of deriving the analytical structures are well documented in the literature for the type-1 fuzzy controllers. As an example, some type-1 fuzzy controllers have been shown to possess peculiar and interesting structures (e.g., nonlinear PID, PI, or PD controllers with variable gains) [9, 13]. This kind of structural information can be used to guide the parameter-tuning process, thus leading to a significant reduction in trial-and-error effort (e.g., [11, 13]).

Challenges associated with analytical-structure derivation depend on the configuration of the fuzzy controller, in particular, which kind of fuzzy AND operator is used. This is the case for both the type-1 and IT2 fuzzy controllers. The product AND operator and the Zadeh AND operator (i.e., min()) are the only two operators that are employed in fuzzy control. Deriving the analytical structure of a fuzzy controller with the product AND operator is relatively simple; however, a fuzzy controller involving the other operator is far more difficult. Structurally, a IT2 fuzzy controller is more complicated than its type-1 counterpart as the former has more components (e.g., type reducer), more parameters (e.g., footprints of uncertainty of IT2 fuzzy sets), and a more complex inference mechanism.

We revealed first analytical structure of type-2 fuzzy controller which used Zadeh AND operator [1]. Subsequently, the analytical structures of a number of other IT2 fuzzy controllers were exposed. (e.g., [7, 8, 16, 17]). We point out that to study a new class of IT2 fuzzy controllers, an innovative analytical-structure-deriving method must be developed first before their analytical structures can be derived because the existing derivation methods can cover only the controller configurations for which they are developed.

In [1], the analytical structures of two Mamdani IT2 fuzzy PI controllers are derived that use the following identical elements—two interval T2 triangular input fuzzy sets for each of the two input variables, four type-1 singleton output fuzzy sets, Zadeh AND operator, and the center-of-sets type reducer. The difference is that one controller employs the centroid defuzzifier while the other a new defuzzifier called the average defuzzifier (whose advantages are established in the context of the analytical structure study in [1]). The resulting analytical structures are linked to nonlinear control. More specifically, the derivation proves explicitly both controllers to be nonlinear PI (or PD) controllers with variable gains (the expressions are different for the two controllers). The characteristics of the variable gains are analyzed and shown to have the potential to yield improved control performance. Taking advantage of the new knowledge, how to determine and tune the design parameters of the IT2 controllers (there are as many as 11 parameters) even when the mathematical model of the system to be controlled is unknown are discussed.

An innovative technique capable of deriving the analytical structure for a wide class of IT2 Mamdani fuzzy controllers is developed in [16]. The configuration of the controllers is typical and quite general—any number and types of IT2 input fuzzy sets, any number and types of general or IT2 output fuzzy sets, arbitrary fuzzy rules, Zadeh AND operator, the Karnik-Mendel center-of-sets type-reducer, and the centroid defuzzifier. One particularly interesting finding is that the analytical structure of a subset of the IT2 fuzzy controllers is the sum of two nonlinear PI (or PD) controllers, each of which has a variable proportional-gain and a variable integral-gain (or derivative-gain) plus a variable offset if and only if the input fuzzy sets are piecewise linear (e.g., triangular and/or trapezoidal). The sum of the two nonlinear PI (or PD) controllers is a new discovery relative to the literature. As an important benefit of knowing the analytical structure, the IT2 fuzzy controllers can now be treated as variable-gain controllers, rather than black-box controllers. The roles of various parameters, such as the footprints of uncertainty of the IT2 input fuzzy sets, play can be clearly understood from control theory standpoint as opposed to from vague and subjective viewpoint of linguistic knowledge representation. Furthermore, the structure information can be used to facilitate control system design. More concretely, for the fuzzy PI (or PD) controllers, because at the equilibrium point, the variable proportional-gain and integral-gain of the IT2 fuzzy PI (or PD) controller become fixed gains. Therefore, one may apply the linear PI (or PD) controller to the system to be controlled with its mathematical model being assumed to be unknown. Tune the proportional-gain and integral-gain (or derivative-gain) of the linear PI (or PD) controller in a trial-and-error fashion to achieve a reasonable system output performance. The gains of the linear controller can be utilized to calculate the scaling factors of the input and output variables quite easily based on the derived variable gain formulas. The detail on the underlying principle is given in [11].

A long-standing fundamental issue is this: how an IT2 fuzzy set's footprint of uncertainty, a key element differentiating an IT2 controller from a type-1 controller, affects a controller's analytical structure. Absence of a general theory, determining a footprint relies on blind search through the trial-and-error method, which is currently widely adopted in the field. Blind searching of a (high-dimensional) parameter space is not only time consuming but incomprehensive with subpar outcome. We address this issue for a particular class of IT2 TS fuzzy controllers in [17] by first developing an innovative technique for deriving their analytical structures. Analyzing the resulting analytical structures reveals the role of the footprints of uncertainty in shaping the structures. Specifically, it is mathematically proven that under certain conditions, the larger the footprints, the more the IT2 controllers resemble linear or piecewise linear controllers. When the footprints are at their maximum, the IT2 controllers actually become linear or piecewise linear controllers. That is to say the smaller the footprints, the more nonlinear the controllers. The most nonlinear IT2 controllers are attained at zero footprints, at which point the IT2 controllers become type-1 controllers. This finding implies that sometimes if strong nonlinearity is most important and desired, one should consider using a smaller footprint or even just a type-1 fuzzy controller. This study exemplifies the importance of investigating analytical structure of an IT2 fuzzy controller because availability of such structure information can lead to comprehensive and insightful analysis and understanding of an IT2 fuzzy controller.

References

1. X.Y. Du, H. Ying, Derivation and analysis of the analytical structures of the interval type-2 fuzzy PI and PD controllers. IEEE Trans. Fuzzy Syst. **8**, 802–814 (2010)
2. G. Feng, A survey on analysis and design of model-based fuzzy control systems. IEEE Trans. Fuzzy Syst. **14**, 676–697 (2006)
3. N.N. Kamik, J.M. Mendel, Q. Liang, Type-2 fuzzy logic systems. IEEE Trans. Fuzzy Syst. **7** (6), 643–658 (1999)
4. E.H. Mamdani, Application of fuzzy algorithms for simple dynamic plant. Proc. IEE **121**, 1585–1588 (1974)
5. J.M. Mendel, *Uncertain Rule-Based Fuzzy Logic Systems*: *Introduction and new directions* (Prentice Hall, USA, 2000)
6. J.M. Mendel, Advances in type-2 fuzzy sets and systems. Inf. Sci. **177**, 84–110 (2007)
7. J.M. Mendel, H. Hagras, W. W. Tan, W. Melek, and H. Ying, *Introduction to Type-2 Fuzzy Logic Control: Theory and Applications* (IEEE Press and Wiley, Inc., 2014)
8. M. Nie, W.W. Tan, Analytical structure and characteristics of symmetric Karnik-Mendel type-reduced interval type-2 fuzzy PI and PD controllers. IEEE Trans. Fuzzy Syst. **20**(3), 416–430 (2012)
9. H. Ying, W. Siler, J.J. Buckley, Fuzzy control theory: a nonlinear case. Automatica **26**, 513–520 (1990)
10. H. Ying, Sufficient conditions on general fuzzy systems as function approximators. Automatica **30**, 521–525 (1994)
11. H. Ying, Practical design of nonlinear fuzzy controllers with stability analysis for regulating processes with unknown mathematical models. Automatica **30**, 1185–1195 (1994)

12. H. Ying, Sufficient conditions on uniform approximation of multivariate functions by general Takagi-Sugeno fuzzy systems with linear rule consequent. IEEE Trans. Syst. Man Cybern. **28**, 515–520 (1998)
13. H. Ying, *Fuzzy Control and Modeling: Analytical Foundations and Applications* (IEEE Press, 2000)
14. H. Ying, A sufficient condition on a general class of interval type-2 Takagi-Sugeno fuzzy systems with linear rule consequent as universal approximators. J. Intel. Fuzzy Syst. **29**, 1219–1227 (2015)
15. X.-J. Zeng, M.G. Singh, Approximation properties of fuzzy systems generated by the min inference. IEEE Trans. Syst. Man Cybern. **26**, 187–193 (1996)
16. H.B. Zhou, H. Ying, A method for deriving the analytical structure of a broad class of typical interval type-2 Mamdani fuzzy controllers. IEEE Trans. Fuzzy Syst. **21**, 447–458 (2013)
17. H.B. Zhou, H. Ying, Deriving and analyzing analytical structures of a class of typical interval type-2 TS fuzzy controllers. IEEE Trans. Cybern. **99**, 1–12 (2016)