Chapter 24
Transient Phenomena in Simple Electrical Circuits

Abstract In this chapter we discuss some transient phenomena in simple electrical circuits (resistive, inductive, capacitive), as an introduction to the later chapters on (local) stability and dynamics of electrical machines and drives.

24.1 Switching On or Off a Resistive-Inductive Circuit

Consider the resistive-inductive circuit (a) in Fig. 24.1. R and L are assumed to be constant (e.g. independent of current or frequency). The circuit is therefore described by the linear time-invariant differential equation

$$v(t) = R \cdot i(t) + L \cdot \frac{di(t)}{dt} \tag{24.1}$$

First, we will examine the case in which the initially current-less circuit is connected to a voltage source $v_s(t) = \hat{V} \cos(\omega t + \varphi)$. The solution of Eq. 24.1 with boundary conditions $i(t) = 0$ for $t \leq 0^-$ and with $v(t) = v_s(t)$ for $t \geq 0^+$ consists of two parts:

- the particular or steady-state solution

$$i(t) = \frac{\hat{V}}{\sqrt{R^2 + \omega^2 L^2}} \cdot \cos(\omega t + \varphi - \arctan \omega L/R) \tag{24.2}$$

- the transient solution

$$i(t) = I \cdot \exp(-t/\tau) \tag{24.3}$$

with $I = -\frac{\hat{V}}{\sqrt{R^2+\omega^2 L^2}} \cdot \cos(\varphi - \arctan \omega L/R)$ and $\tau = L/R$.
Note that the time constant of the system corresponds with the eigenvalue of the system Eq. 24.1, with $i(t)$ considered as state variable (and $v(t)$ as input). This time constant corresponds to the single energy storage of the system, i.e. the magnetic energy in the coil. This solution can also be found using the Laplace transform.

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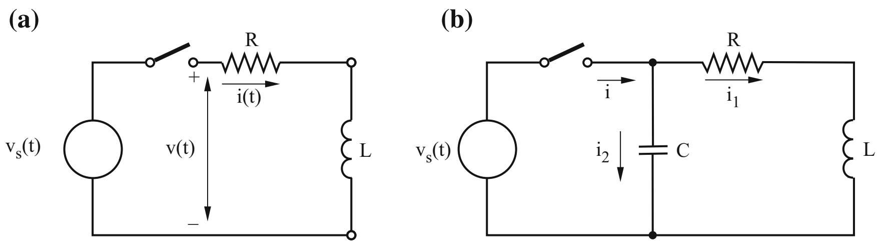

Fig. 24.1 R-L- and R-L-C-circuit

Next, we will analyse the case of an interruption of a (steady-state) DC current I_o in the circuit. The state variable is now the voltage $v(t)$, while the input is $i(t)$. For $t \leq 0^-$, $i(t) = I_o$ and $v(t) = V_o = RI_o$. At $t = 0$, the switch is opened. Suppose that the switch is ideal, i.e. $i(t) = 0$ for $t \geq 0^+$.

In order to use the (one-sided) Laplace transform, we have to transform the variables, i.e. $i'(t) = i(t) - I_o$ and $v'(t) = v(t) - V_o$. In terms of the new variables, the equations and boundary conditions become:

$$v'(t) = R \cdot i'(t) + L \cdot \frac{di'(t)}{dt} \tag{24.4}$$

$$i'(t) = -I_o \cdot u(t) \tag{24.5}$$

with $u(t)$ the unit step function: $u(t) = 0$ for $t \leq 0^-$ and $u(t) = 1$ for $t \geq 0^+$.

The Laplace transform yields

$$V'(p) = R \cdot I'(p) + pL \cdot I'(p) \tag{24.6}$$

$$I'(p) = -I_o/p \tag{24.7}$$

The solution is

$$V'(p) = -\frac{R \cdot I_o}{p} - L \cdot I_o \tag{24.8}$$

and thus in the time domain:

$$v'(t) = -R \cdot I_o \cdot u(t) - L \cdot I_o \cdot \delta(t) \tag{24.9}$$

or

$$v(t) = R \cdot I_o[1 - u(t)] - L \cdot I_o \cdot \delta(t) \tag{24.10}$$

The Dirac term in the voltage is the result of two (unrealistic) assumptions: an ideal switch and a coil with a negligible capacitance between the turns of the coil. In reality, the high voltage between the contacts of the switch will result in a spark, assuring the continuity of the current.

The capacitance between the turns of the coil can (approximately) be modelled by a lumped capacitor as in the circuit (b) in Fig. 24.1. The system equations are now

$$v(t) = R \cdot i_1(t) + L \cdot \frac{di_1(t)}{dt} \tag{24.11}$$

$$\frac{dv(t)}{dt} = \frac{1}{C} i_2(t) \tag{24.12}$$

$$i(t) = i_1(t) + i_2(t) \tag{24.13}$$

with boundary conditions $i_1 = I_o$, $i_2 = 0$, and $v = R \cdot I_o$ for $t \leq 0^-$ and $i_1 + i_2 = 0$ for $t \geq 0^+$.

The solution is now

$$i_1(t) = I_o \left\{ [1 - u(t)] + \frac{R/L - p_2}{p_1 - p_2} \exp(p_2 t) - \frac{R/L - p_1}{p_1 - p_2} \exp(p_1 t) \right\} \tag{24.14}$$

$$v(t) = R \cdot I_o \left\{ [1 - u(t)] + \frac{R/L - 1/RC - p_2}{p_1 - p_2} \exp(p_2 t) - \frac{R/L - 1/RC - p_1}{p_1 - p_2} \exp(p_1 t) \right\} \tag{24.15}$$

with p_1 and p_2 the eigenvalues of this second-order system[1]

$$p_{1,2} = \frac{R}{2L} \left(-1 \pm \sqrt{1 - \frac{4L}{R^2 C}} \right) \tag{24.16}$$

i.e. the zeros of the eigenvalue equation $LCp^2 + RCp + 1 = 0$.

The capacitance between the turns limits the voltage between the contacts of the switch (although sparks are still possible).

24.2 Single-Phase Transformer

If we disregard saturation and skin effects, the single-phase transformer is described by Eqs. 24.17 and 24.18:

$$v_1(t) = R_1 \cdot i_1(t) + L_1 \cdot \frac{di_1(t)}{dt} + M \cdot \frac{di_2(t)}{dt} \tag{24.17}$$

[1] Please analyse the case in which $4L < R^2 C$ and $4L > R^2 C$ and, in particular, the case in which $R = 0$.

$$v_2(t) = R_2 \cdot i_2(t) + L_2 \cdot \frac{di_2(t)}{dt} + M \cdot \frac{di_1(t)}{dt} \tag{24.18}$$

If the turns ratio $a = w_1/w_2$ is known, we may also rewrite these equations as

$$v_1(t) = R_1 \cdot i_1(t) + L_{1\sigma} \cdot \frac{di_1(t)}{dt} + L_{m1} \cdot \frac{d}{dt}[i_1(t) + i_2^{'}(t)] \tag{24.19}$$

$$v_2^{'}(t) = R_2^{'} \cdot i_2^{'}(t) + L_{2\sigma}^{'} \cdot \frac{di_2^{'}(t)}{dt} + L_{m1} \cdot \frac{d}{dt}[i_1(t) + i_2^{'}(t)] \tag{24.20}$$

with $L_{1\sigma} = L_1 - aM$, $L_{2\sigma} = L_2 - M/a$, $L_{m1} = aM$ and with the prime indicating the secondary variables referred to the primary.

The eigenvalues of the free system, with the currents as state variables and the voltages as external inputs (assumed to be zero, for example), are the zeros of

$$\det \begin{vmatrix} L_1 p + R_1 & Mp \\ Mp & L_2 p + R_2 \end{vmatrix} = 0 \tag{24.21}$$

or

$$(L_1 L_2 - M^2) p^2 + (L_1 R_2 + L_2 R_1) p + R_1 R_2 = 0 \tag{24.22}$$

This can be written as

$$\sigma p^2 + \left(T_{m1}^{-1} + T_{m2}^{-1}\right) p + T_{m1}^{-1} T_{m2}^{-1} = 0 \tag{24.23}$$

or also as

$$p^2 + \left(T_1^{-1} + T_2^{-1}\right) p + \sigma T_1^{-1} T_2^{-1} = 0 \tag{24.24}$$

where $T_{m1} = L_1/R_1$, $T_{m2} = L_2/R_2$ are the main field or open-circuit time constants of primary and secondary, $T_1 = \sigma L_1/R_1$, $T_2 = \sigma L_2/R_2$ are the leakage field or short-circuit time constants of primary and secondary, and σ is the total leakage coefficient of the transformer.

For σ that are not too large, the eigenvalues can be approximated by

$$p_1 = -\left(T_1^{-1} + T_2^{-1}\right) \tag{24.25}$$

$$p_2 = -\left(T_{m1} + T_{m2}\right)^{-1} \tag{24.26}$$

These two eigenvalues correspond to the main and leakage fields of the transformer, i.e. the two ways for magnetic energy storage. The eigenvalue p_1 corresponds to that of an R-L-circuit with as resistance the sum of the primary and secondary resistances (referred to the same winding) and the total leakage as seen from this winding:

$$p_1 \approx -\frac{R_1 + R_2^{'}}{\sigma L_1} = -\frac{R_1^{'} + R_2}{\sigma L_2} \tag{24.27}$$

The eigenvalue p_2 corresponds to that of an R-L-circuit with as resistance the parallel connection of primary and secondary resistances (referred to the same winding) and the main field inductance seen from this same winding:

$$p_2 \approx -\frac{R_1 R_2^{'}/(R_1 + R_2^{'})}{L_{m1}} \tag{24.28}$$

To apply this, we will study the transient when a transformer, loaded with a resistor R_{2e} (denoting $R = R_{2e} + R_2$) at the secondary and fed by the grid $(V_1,\ f_1)$ at the primary, is disconnected from the grid at $t = 0$. Just before the switch is opened, the primary and secondary currents are I_{10} and I_{20}, respectively. The secondary current and the primary voltage for $t \geq 0^+$ will be calculated in three ways.

<u>Method n°1</u>: Time Domain
At the secondary side, we have at each instant

$$Ri_2(t) + L_2 \cdot \frac{di_2(t)}{dt} + M \cdot \frac{di_1(t)}{dt} = 0 \tag{24.29}$$

For $t > 0^+$, the primary current as well as its derivative are zero. However, the flux coupled with the secondary has to remain continuous. Therefore

$$L_2 i_2(t = 0^+) = L_2 \cdot I_{20} + M \cdot I_{10} \tag{24.30}$$

or

$$i_2(t = 0^+) = I_{20} + (M/L_2) \cdot I_{10} \tag{24.31}$$

For $t > 0^+$, the secondary current has to satisfy equation 24.29 with $i_1(t) = di_1(t)/dt \equiv 0$. Thus

$$i_2(t > 0^+) = [I_{20} + (M/L_2) \cdot I_{10}] \exp(-t/\tau) \tag{24.32}$$

with $\tau = L_2/R$. For $t > 0^-$, we may write

$$i_2(t > 0^-) = I_{20}[1 - u(t)] + [I_{20} + (M/L_2) \cdot I_{10}]u(t) \cdot \exp(-t/\tau) \tag{24.33}$$

The primary voltage for $t > 0^-$ can be calculated from Eq. 24.17 with $i_2(t)$ according to Eq. 24.33 and $i_1(t) = I_{10}[1 - u(t)]$.

For $t > 0^-$, we have

$$v_1(t) = R_1 \cdot i_1(t) + \sigma L_1 \cdot \frac{di_1(t)}{dt} + M \cdot \frac{d}{dt}\left[i_2(t) + \frac{M}{L_2} i_1(t)\right] \tag{24.34}$$

or

$$v_1(t) = R_1 \cdot i_1(t) + \sigma L_1 \cdot \frac{di_1(t)}{dt} - \frac{M}{L_2} R \cdot i_2(t) \tag{24.35}$$

Using $di_1(t)/dt \equiv 0$ for $t > 0^-$ and substituting $i_2(t)$ from Eq. 24.33 yields

$$v_1(t) = R_1 \cdot I_{10}[1-u(t)] - \sigma L_1 \cdot I_{10}\delta(t) - \frac{M}{L_2} R \cdot I_{20}[1-u(t)] - \frac{M}{L_2} R \cdot [I_{20} + \frac{M}{L_2} \cdot I_{10}] u(t) \cdot \exp(-t/\tau) \tag{24.36}$$

For $t > 0^+$, we therefore have

$$v_1(t) = -\frac{M}{L_2} R \cdot [I_{20} + \frac{M}{L_2} \cdot I_{10}] \exp(-t/\tau) \tag{24.37}$$

The voltage for $t > 0^+$ corresponds to the main flux coupled with the secondary that is fading away with the secondary open-circuit time constant. When the switch opens at $t = 0$, the secondary takes over the magnetising part of the flux corresponding to the primary current (flux continuity). The Dirac voltage for $t = 0$ corresponds to the leakage flux of the primary which is not coupled to the secondary and cannot be compensated by a jump in the secondary current. This Dirac voltage will give rise to a spark in the switch.

If for $t < 0^-$ the transformer was fed by a DC voltage $v_o = R_1 I_{10}$ (with $I_{20} = 0$), we find for the primary voltage

$$v_1(t) = R_1 \cdot I_{10}[1 - u(t)] - \sigma L_1 \cdot I_{10}\delta(t) - \frac{M^2}{L_2^2} R \cdot I_{10} u(t) \cdot \exp(-t/\tau)$$

and for $t > 0^+$

$$v_1(t) = -\frac{M^2}{L_2^2} R \cdot I_{10} \exp(-t/\tau) = -(1-\sigma)\frac{L_1/R_1}{L_2/R} \cdot v_o \cdot \exp(-t/\tau) \tag{24.38}$$

In other words, the initial value is the DC voltage reduced by the factor $1 - \sigma$ and transformed with the ratio of the primary and secondary time constants.

Method n°2: Single-side Laplace Transform
To apply the single-side Laplace transform, we have to replace the currents i_1 and i_2 with fictitious currents $i_1^* = i_1 - I_{10}$ and $i_2^* = i_2 - I_{20}$ which are zero for $t \leq 0^-$. For these new variables, the secondary transformer equation becomes

$$Ri_2^*(t) + L_2 \cdot \frac{di_2^*(t)}{dt} + M \cdot \frac{di_1^*(t)}{dt} = -RI_{20} \tag{24.39}$$

and after the Laplace transform

$$RI_2^*(p) + pL_2 \cdot I_2^*(p) + pM \cdot I_1^*(p) = -R\frac{I_{20}}{p} \tag{24.40}$$

As

$$I_1^*(p) = -\frac{I_{10}}{p} \tag{24.41}$$

we find for the secondary current

$$I_2^*(p) = \frac{1}{R + pL_2}\left(-R\frac{I_{20}}{p} + MI_{10}\right) \tag{24.42}$$

Or, in the time domain:

$$i_2(t) = i_2^*(t) + I_{20} = I_{20}\,(1 - u(t)) + \left(I_{20} + \frac{M}{L_2}I_{10}\right)u(t) \cdot \exp(-t/\tau) \tag{24.43}$$

The primary voltage[2] can be calculated in a similar way.
Method n°3: A variation of Method n°2
Define the new variables $i_1^+(t)$ and $i_2^+(t)$ with $i_1^+(t) = 0$ for $t < 0^-$, $i_1^+(t) = i_1(t)$ for $t > 0^-$, $i_2^+(t) = 0$ for $t < 0^-$, $i_2^+(t) = i_2(t)$ for $t > 0^-$.

The transformer secondary equation is now

$$Ri_2^+(t) + L_2 \cdot \frac{di_2^+(t)}{dt} + M \cdot \frac{di_1^+(t)}{dt} = 0 \tag{24.44}$$

and after Laplace transform

$$RI_2^+(p) + L_2\left(pI_2^+(p) - I_{20}\right) + M\left(pI_1^+(p) - I_{10}\right) = 0 \tag{24.45}$$

With $I_1^+(p) = 0$, we obtain

$$I_2^+(p) = \frac{1}{R + pL_2}\left(L_2 I_{20} + MI_{10}\right) \tag{24.46}$$

[2]Can you also find a Dirac function in the voltage?

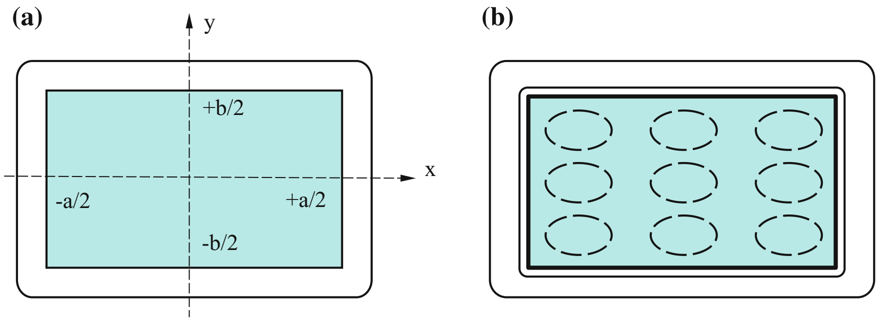

Fig. 24.2 **a** Coil with massive core **b** circulating currents

or, in the time domain for $t > 0^-$:

$$i_2^+(t) = \left(I_{20} + \frac{M}{L_2} I_{10}\right) u(t) \cdot \exp(-t/\tau). \tag{24.47}$$

24.3 Coil with Massive Iron Core

A coil with a massive iron core can be regarded as a special case of magnetically coupled coils. We consider an infinitely long coil with a core with rectangular cross-section and sides a (x-direction) and b (y-direction), as illustrated in (a) in Fig. 24.2. The coil is uniformly distributed along the core length (z-direction).

The general transient solutions have to satisfy the following equations (from Maxwell's laws):

$$E_x = \rho_{Fe} \cdot J_x = \frac{\rho_{Fe}}{\mu} \cdot \frac{\partial B_z}{\partial y} \tag{24.48}$$

$$E_y = \rho_{Fe} \cdot J_y = -\frac{\rho_{Fe}}{\mu} \cdot \frac{\partial B_z}{\partial x} \tag{24.49}$$

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = -\frac{\partial B_z}{\partial t} \tag{24.50}$$

with B the magnetic field (induction), E the electric field, J the current density, ρ_{Fe} the electric resistivity of the iron core and μ the permeability of the iron.

Eliminating the electric field components yields:

$$\frac{\partial^2 B_z}{\partial x^2} + \frac{\partial^2 B_z}{\partial y^2} = \frac{\mu}{\rho_{Fe}} \cdot \frac{\partial B_z}{\partial t} \tag{24.51}$$

As possible (transient) solutions for Eq. 24.51, we propose

$$b(x, y, t) = \sum_{\alpha,\beta} b_{\alpha\beta} \tag{24.52}$$

with

$$b_{\alpha\beta} = B_{\alpha\beta} \cos \alpha x \cdot \cos \beta y \cdot \exp(-\gamma_{\alpha\beta} t) \tag{24.53}$$

Because $x = 0$ and $y = 0$ are symmetry axes, only cosine functions are considered.

We will only take into account cases where the current in the coil is zero for $t > 0^+$. From $t > 0^+$ on, the magnetic field strength $\underset{\rightarrow}{H}$ as well as the induction $\underset{\rightarrow}{B}$ outside the core are zero. Because of the continuity of the tangential component of $\underset{\rightarrow}{H}$, the magnetic field strength and the induction at the boundaries $x = \pm a/2$ and $y = \pm b/2$ are zero from $t > 0^+$ on.[3] Therefore, $\alpha = \alpha_m = m(\pi/a)$ and $\beta = \beta_n = n(\pi/b)$ with m and n odd. Substitution of the solutions of Eqs. 24.53 in 24.51 yields

$$\alpha^2 + \beta^2 = \frac{\mu}{\rho_{Fe}} \gamma_{\alpha\beta} \tag{24.54}$$

and thus

$$\gamma_{\alpha\beta} = \gamma_{mn} = \frac{\rho_{Fe}}{\mu} \left\{ \left(m\frac{\pi}{a} \right)^2 + \left(n\frac{\pi}{b} \right)^2 \right\} \tag{24.55}$$

The proposed solution is

$$b(x, y, t) = b_z(x, y, t) = \sum_{m,n} B_{mn} \cos \alpha_m x \cdot \cos \beta_n y \cdot \exp(-\gamma_{mn} t) \tag{24.56}$$

and consists of the sum of mode (m, n). Each mode corresponds to transient circulating currents, as shown in (b) in Fig. 24.2 for $m = 3$ and $n = 3$. These currents follow from

$$J_x = \frac{\partial H_z}{\partial y} \tag{24.57}$$

$$J_y = -\frac{\partial H_z}{\partial x} \tag{24.58}$$

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[3]This is only the transient solution. For other problems (e.g. switching on a sinusoidal excitation) steady-state solutions should be considered separately. Even with a non-zero current in the coil, however, the induction outside the coil is negligible because of the high iron permeability.

In addition to the boundary conditions in space, there are also boundary conditions in time.

We will investigate the case of a coil with very large diameter (so that its curvature can be disregarded) and with w_0 turns per meter. For $t < 0$ there is a DC current I_0 in the coil, giving rise to a steady-state induction B_0 in the core. The current is switched off at $t = 0$. From $t > 0^+$ on, there is no current in the coil and the magnetic field strength $\underrightarrow{H}$ as well as the induction $\underrightarrow{B}$ outside the core are zero.

The steady-state induction for $t < 0$ follows from Ampere's law:

$$\frac{B_0}{\mu} l = w_0 \cdot l \cdot I_0 \tag{24.59}$$

As a transient solution for[4] $-a/2 < x < a/2$ and $-b/2 < y < b/2$ for $t > 0^-$, we propose Eq. 24.56. The continuity at $t = 0$ then requires

$$b(x, y, t = 0) = b(x, y, 0) = \sum_{m,n} B_{mn} \cos \alpha_m x \cdot \cos \beta_n y = B_0 = \mu \cdot w_0 \cdot I_0 \tag{24.60}$$

Writing the two-dimensional block function B_0 as a product of two Fourier series (one in x and one in y) yields

$$B_{mn} = \left(\frac{4}{\pi}\right)^2 \frac{1}{m \cdot n} (-1)^{\frac{m+n-2}{2}} \cdot B_0 \tag{24.61}$$

with m and n odd. Therefore

$$b(x, y, t) = \left(\frac{4}{\pi}\right)^2 B_0 \cdot \sum_{m,n} \frac{1}{m \cdot n} (-1)^{\frac{m+n-2}{2}} \cos\left(m\pi \frac{x}{a}\right) \cdot \cos\left(n\pi \frac{y}{b}\right) \cdot \exp(-\gamma_{mn} t) \tag{24.62}$$

with γ_{mn} given by Eq. 24.55.

The current densities in the iron for $-a/2 < x < a/2$ and $-b/2 < y < b/2$ are

$$J_x = -\left(\frac{4}{\pi}\right)^2 \frac{B_0}{\mu} \cdot \frac{\pi}{b} \cdot \sum_{m,n} \frac{1}{m} (-1)^{\frac{m+n-2}{2}} \cos\left(m\pi \frac{x}{a}\right) \cdot \sin\left(n\pi \frac{y}{b}\right) \cdot \exp(-\gamma_{mn} t) \tag{24.63}$$

$$J_y = \left(\frac{4}{\pi}\right)^2 \frac{B_0}{\mu} \cdot \frac{\pi}{a} \cdot \sum_{m,n} \frac{1}{n} (-1)^{\frac{m+n-2}{2}} \sin\left(m\pi \frac{x}{a}\right) \cdot \cos\left(n\pi \frac{y}{b}\right) \cdot \exp(-\gamma_{mn} t) \tag{24.64}$$

[4]The edges are excluded as the sum is not uniformly convergent for $t = 0$.

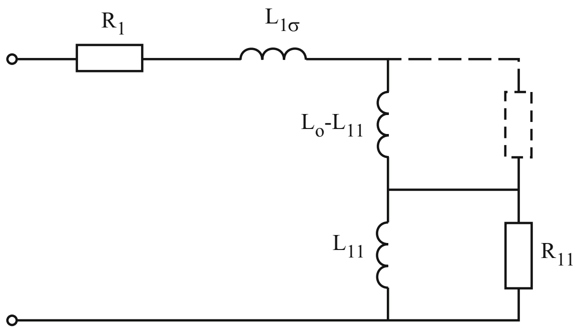

Fig. 24.3 Massive core: equivalent circuit for the fundamental mode

To calculate the flux, we will first have to integrate over $-(a-\epsilon)/2 < x < (a-\epsilon)/2$ and $-(b-\delta)/2 < y < (b-\delta)/2$ and then take the limit for $\epsilon, \delta \to 0$:

$$\Phi_{\epsilon\delta} = \left(\frac{4}{\pi}\right)^2 B_0 \cdot \int_{-\frac{(a-\epsilon)}{2}}^{\frac{(a-\epsilon)}{2}} dx \int_{-\frac{(b-\delta)}{2}}^{\frac{(b-\delta)}{2}} dy \sum_{m,n} \frac{1}{m \cdot n} (-1)^{\frac{m+n-2}{2}} \cos\left(m\pi\frac{x}{a}\right) \cdot \cos\left(n\pi\frac{y}{b}\right) \cdot \exp(-\gamma_{mn}t) \tag{24.65}$$

$$\Phi_{\epsilon\delta} = \left(\frac{4}{\pi}\right)^2 B_0 \cdot \frac{4ab}{\pi^2} \sum_{m,n} (\frac{1}{m \cdot n})^2 (-1)^{\frac{m+n-2}{2}} \sin\left(m\pi\frac{a-\epsilon}{2a}\right) \cdot \sin\left(n\pi\frac{b-\delta}{2b}\right) \cdot \exp(-\gamma_{mn}t) \tag{24.66}$$

and thus

$$\Phi = \lim_{\epsilon,\delta\to 0} \Phi_{\epsilon\delta} = \left(\frac{4}{\pi}\right)^2 \left(\frac{2}{\pi}\right)^2 \cdot ab \cdot B_0 \cdot \sum_{m,n} (\frac{1}{m \cdot n})^2 \cdot \exp(-\gamma_{mn}t) \tag{24.67}$$

The equivalent self-inductance (per meter) of the coil for mode m, n can be calculated in an analogous way as the total self-inductance

$$w_0\Phi_0 = L_0 I_0 \tag{24.68}$$

The inductance L_{mn} results from $w_0\Phi_{mn} = L_{mn}I_0$:

$$L_{mn} = \left(\frac{4}{\pi}\right)^2 \left(\frac{2}{\pi}\right)^2 \cdot ab \cdot \mu \cdot w_0^2 \cdot (\frac{1}{m \cdot n})^2 = \frac{64}{\pi^4 m^2 n^2} \cdot L_0 \tag{24.69}$$

with $L_0 = w_0^2 \mu ab$ the total self-inductance of the coil.

A time constant $1/\gamma_{mn} = L_{mn}/R_{mn}$ corresponds with each mode, with

$$R_{mn} = \frac{64}{\pi^2} \varrho_{Fe} w_0^2 \cdot \left(\frac{1}{n^2} \cdot \frac{b}{a} + \frac{1}{m^2} \cdot \frac{a}{b}\right) \tag{24.70}$$

The largest time constant corresponds with the mode (1, 1). This time constant is commonly called the transient time constant and the corresponding component (mode) the transient component. The other modes, with smaller time constants, are called subtransient components (corresponding with subtransient time constants). Figure 24.3 shows a possible approximate model (*equivalent circuit*); R_1 is the coil resistance and $L_{1\sigma}$ its leakage inductance (which corresponds here to field lines coupled with the coil but outside the iron core); R_{11} and L_{11} correspond with the transient component; the remaining modes are *lumped* in the dashed part.

24.4 Quasi-stationary Modelling of Rotating Machines

For rotating machines, there are not only the electrical transients but also mechanical transients (when the speed is variable).

In principle, electrical and mechanical transient phenomena have to be treated together: the variable speed affects the electrical transients (e.g. via the emf of motion) and the electrical transients affect the torque and, therefore, via the equation of motion, the speed.

In many cases, however, the mechanical time constants are one or more orders of magnitude larger than the *pure* electrical time constants (i.e. those that would occur at constant speed). If these time constants differ by an order of magnitude, it may be permitted to treat the electrical and mechanical transients separately: the electrical transients as if the speed were constant and the mechanical transients as if the electrical circuit were in steady state. For the analysis of the mechanical phenomena, we may then use the steady-state currents, voltages and torques that can be derived from the steady-state equations or equivalent circuits.

An example will be discussed in the next chapter on pulsating loads for an induction machine. As the mechanical time constant of this high-inertia drive is much larger than the electrical time constants of the machine and than the period of the pulsating torque, we may ignore the electrical transients and calculate everything as if the machine were in steady state. For small slip, the torque equation may also be approximated by its almost linear part:

$$T = \frac{3V^2}{\Omega_{sy}} \cdot \frac{R/s}{(R/s)^2 + X_\sigma^2} \approx \frac{3V^2}{\Omega_{sy}} \cdot \frac{s}{R} \tag{24.71}$$

The equation of motion can then be approximated by (using $\Omega_r = (1 - s)\Omega_{sy}$):

$$-J\Omega_{sy} \cdot \frac{ds}{dt} = \frac{3V^2}{\Omega_{sy}} \cdot \frac{s}{R} - T_l \tag{24.72}$$

However, we need to keep in mind that, actually, the electrical transients may give rise to rather large transient torques, which are entirely disregarded here.