

Teaching Financial Mathematics Through a Critical Approach in a University Environment

Celso Ribeiro Campos, Aurélio Hess and Renata Moura Sena

Abstract Financial education shares commonalities with the concept of education for citizenship particularly when it deals with social problems, for example family debts and/or irresponsible consumption. In this chapter, we present an approach, involving constructing a modelling activity in an undergraduate financial mathematics course, to connect financial education with mathematics education. In particular, critical mathematics education elements were incorporated in this activity to explore a critical financial education. Therefore, the aim of this chapter is to explore the affordances of a critical financial education through the adoption of a modelling pedagogy with roots in a democratic dialogic pedagogy, applied in the particular situation of Brazil. We observed an intense student involvement and great interest in the subject. Based on our experience, we claim to have achieved some financial education goals, as well as facilitated the students' participation in a critical discourse when debating themes they themselves proposed.

Keywords Financial education · Critical education · Mathematics education
Financial mathematics · Mathematical modelling

1 Introduction

A concept that has long been commonplace among researchers on education and mathematics education, is an education for citizenship. For instance, Frankenstein (1989) places the teaching of mathematics within a logical foundation that links

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education to a broader consideration of critical citizenship and social responsibility, focusing on how mathematics education could produce a critical citizenship, not just knowledge and awareness but how such knowledge enable critique of those in power and authority. In Brazil, both the Law of Directives and Bases of Education (Brazilian Ministério da Educação e Cultura 1996) and the National Curriculum Parameters (Brazilian Ministério da Educação e Cultura 2000), advocate, at all levels, an education for citizenship, in order to prepare students for an active, reflective and critical life, in which they can exercise their role as citizens aware of social, political, economic and environmental problems that permeate their society. In this context, we see financial education as a field to develop knowledge and information about personal finance that can help improve the life quality of people and their communities. The aim of this chapter is to explore the affordances of a *critical financial education* through the adoption of a modelling pedagogy with roots in a democratic dialogic pedagogy, applied in the particular situation of Brazil. Therefore, to clarify and deepen the discussion on the topic, we discuss the relationship between mathematics education and financial education and analyze interfaces between critical education and financial education. We present an educational project based on a mathematical modelling strategy, in order to put into practice the integration of these pedagogical aspects.

2 Financial Education and an Education for Citizenship

First, we expatiate upon financial education, focusing on some aspects that brings it close to an education for citizenship. Birochi and Pozzebon (2016, p. 268) have pointed out that “there is no single standard definition of the term financial education. Instead, there is a wide range of meanings and correlated terms”. According to the authors, financial education can be broadly divided into two major streams, which they called instrumental and transformative (or critical).

As it can be seen in Fig. 1, the transformative stream is aligned with the idea of an education for citizenship. However, some approaches may combine aspects from both instrumental and transformative streams, such as the Organisation for Economic Co-operation and Development (OECD) publication, which states that financial education is:

the process by which financial consumers/investors improve their understandings of financial products and concepts and, through information, instruction and/or objective advice, develop the skills and confidence to become more aware of financial risks and opportunities, to make informed choices, to know where to go for help, and to take other effective actions to improve their financial well-being. (OECD 2005, p. 26)

Others emphasize the transformative stream. Teixeira (2015, p. 13) points out that:

Financial Education is not only to learn to economize, cut spending, saving and accumulating money, it is much more than that. It is seeking a better quality of life both today and in the future, providing the material security required for any unforeseen.

Financial education streams	Major underpinnings	Objectives	Authors
Instrumental	Financial education should promote efficiency and effectiveness of the financial system, through co-responsibility of the individuals (rights and liabilities). Individuals are consumers.	Financial education should act as a tool to improve the overall efficiency of the financial system, through training programs based on mastering of operational capabilities (knowledge about credit, debit, budget and negotiations).	Cole et al. (2009); Servon and Kaestner (2008); CGAP (2005).
Transformative or critical	Humanitarian and social approach. Individuals have huge socioeconomic constraints. Improvements are achieved by strengthening individual capabilities.	Financial education should aim at social-economic inclusion through strengthening of individual capabilities, targeting individual empowerment and social emancipation.	Cabraal (2011); Landvogt (2006); Sempere (2009); Johnston and Maguire (2005); Mayoux (2010); Augsburg and Fouillet (2010); Fernando (2006).

Fig. 1 Financial education streams. *Source* Birochi and Pozzebon (2016, p. 268)

Research carried out by OECD (2005) in developed and in emerging countries, pointed out a low level of financial awareness and, mainly, a lack of self-consciousness in vulnerable people. According to OECD, many people in different countries not only lacked the knowledge and skills needed to deal adequately with their personal finances but also ignored the very need for such knowledge. This led the OECD to recommend urgency in the implementation of government actions aiming to provide financial education to their population.

In response to the OECD's recommendation, the Central Bank of Brazil (BCB), which has been worried about the financial education of the Brazilian people, has developed a National Strategy for Financial Education (ENEF) to serve several purposes. Thus, BCB has created a Department of Financial Education, which has published a book containing some basic concepts, in order to inform people of some important aspects.

Every citizen can develop skills to improve his/her quality of life and that of his/her family, improving behavioral attitudes based on personal finance management applied to his/her daily life. The Department of Financial Education of the Central Bank hopes that this copybook encourages you to make autonomous decisions regarding consumption, savings and investment, prevention and protection, considering your wishes and current and future needs. (BCB 2013, p. 3)

The BCB's publication deals with citizens' relationship to money, personal and family budget, the use of credit and debts administration, planned and conscious

consumption, saving and investments, prevention and protection,¹ etc. According to BCB (op. cit.), financially well-educated consumers demand services and products that meet their needs, encouraging competition and playing an important role in monitoring the market, since they require greater transparency of financial institutions, contributing to the solidity and efficiency of the financial system. Besides these ideas, a good financial education can provide other important benefits, especially with regard to personal prosperity, self-esteem improvement and achievement of personal financial goals. Additionally, these benefits can spread from the personal to the family, and to communities.

Although BCB's attitude seems to be a good initiative, its extent and possible impact can be questioned. Thus, the National Strategy for Financial Education has elaborated a specific strategy to reach schools, both teachers and students. Along this line, a document was prepared for distribution to schools, containing orientations about financial education and presenting a conceptual model that aims to bring some principles that should guide the actions towards the desired future situation.

In this context, we list some important goals of financial education (adapted from Campos et al. 2015, p. 558) aligned with the transformative stream and which could be implemented in university financial math curriculum:

- (i) Understand the basic functioning of the financial market and how interest rates affect the financial lives of citizens, for good or for bad, considering that people usually does not have deep awareness about how much interest rate really costs.
- (ii) Practice conscious consumption, knowing and avoiding compulsive consumerism. Financial Education can help consumers not only concerning to their budgets, savings, and investments but also to control impulses associate to compulsive consumerism, to develop conscious consumption, associated with quality of goods and services, environment impacts, etc.
- (iii) Be able to conveniently take advantage of the available funding opportunities.
- (iv) Use credit consciously and wisely, seeking to avoid over-indebtedness. In other words, this means to learn how to use credits and how to maximize the funding access benefits.
- (v) Understand the importance and benefits of planning and following up personal and family budget.
- (vi) Understand the role of savings as a means to carry out projects and follow personal aims.

¹Understand the financial risks and the preventive measures and appropriate protection for every situation.

Understand the importance of financial planning for retirement, how the national pension system is structured and what are the advantages and disadvantages of adopting independent strategies, being the manager of your own investments. (BCB 2013, p. 9)

- (vii) Help to disseminate good financial practices among family and friends.
- (viii) Develop a culture of prevention that is, planning for the future considering the mishaps that can happen.
- (ix) Be able to organize and keep a good personal financial management.
- (x) To make a retirement plan, considering that life expectancy has increased and people spend more time in retired condition.

In summary, financial education promises to help citizens to be more conscious of their consumption habits. The organization and discipline required for the practice of financial education may lead to better and more efficient decisions on using one's scarce financial resources. Financial education could transform consumption habits and reduce impulse purchases, bringing control and rationality to consumers.

3 The Impact of Economic Changes in Brazil on Citizens' Financial Behavior

To comprehend the importance of financial education in Brazil, it is necessary to understand the recent history of Brazil's economic scenario. The high inflation that occurred in Brazil during the 1980s and middle of 1990s created important scars in consumer behavior. At that time, it was common for people to focus on spending their money just on the day they received it because the money would lose value daily. According to the BCB, the National Consumer Price Index reached 82% in just one month in March 1990. This index captures the inflation, which precisely affects the low-income population the most.

Economic instability leads people not to formulate long-term expectations, because the inflationary cycle creates uncertainty, even the short run. Thus, there was a generalized preference for liquidity among agents, because it was very necessary to guarantee basic daily consumption.

In the period between 1985 and early 1994, Brazil has experienced six economic plans (*Cruzado*, *Cruzado 2*, *Bresser*, *Verão*, *Collor* and *Collor 2*) and three currency exchanges (*Cruzado*, *Cruzado Novo* and *Cruzeiro Real*), whose economic stabilization attempt has failed. Nevertheless, in mid-1994, a new economic stabilization plan came into force, with a new currency (*Real*), which was finally successful.

The so-called *Plano Real* has pushed prices to stability and has opened the domestic market, leading to increasing imports of goods for supply. This plan substantially reduced the *inflationary tax*, which prejudiced poor people. The price predictability, the employment and income growth brought with it demands from a huge number of people who were outside the market (Fortuna 2008).

From the low-income consumers' point of view, the *Plano Real* had meant access to unthinkable goods and services, like yogurts, meat, and dentists. The plan became known as *yogurt plan* because of the earlier high prices, which limited

access to it, or *chicken plan*, because with just R\$1,00 people could buy 1 kg chicken meat. Consumption increased by 40% (G1 2014).

After the economic stabilization, some typical behaviors from consumers and savers became clearly determined because of the predictability of income, interest, access to the banking system and other factors. From the perspective of microeconomic theory, consumption is a function of income and prices. With prices controlled and increasing income, Brazilian consumers spent a lot of money on several types of products in order to satisfy a huge pent-up demand.

According to de Ferreira (2008), the consumer's decisions reflect the immediate satisfaction possibility, as it seeks to guarantee a present pleasure. From this point of view, consumers choose the present pleasure, instead of saving for buying in the future, because of their inflationary period memory and, fundamentally, because of their not knowing financial control.

One of the reasons that Brazilian consumers do not have the habit to control their budget and the consumption is their memories of the inflation period. However, another reason is the increase in income in recent past years. According to Brazilian Geographic and Statistics Institute, between 2002 and 2015, real average income has increased 14%, minimum wage grew 44%, while families' consumption grew 75%.

Figure 2 shows the growth rates of four variables: minimum wage, average income, GDP *per capita* and household consumption; and shows that consumption grew above income in the period. According to the Keynesian concept of marginal propensity to consume, an extra income causes a variation on spend in consumption and Brazilian consumers confirmed this theory responding greatly and quickly to income variations.

In addition, during the same period, Brazil improved the income's transfer² for poor population, e.g., *Bolsa Familia*, which is a government program benefiting low-income families. This program had beneficiated 13.8 million households, embracing 26% of the population in 2012 (United Nations 2015). The budget of this program corresponded to 0.53% of GDP in 2013 and was fully financed by the social security budget. According to the United Nations' Human Development Report (2015), since the program was launched, Brazil reduced poverty by about 8 percentage points. This introduced more consumers to the economy, people who could not be included before because of poverty.

Another important point related to income and consumption was the decrease in unemployment rates. According to IBGE (2017), Brazilian GDP grew, in real terms by 49.5% and one of the consequences was a reduction on inoccupation rates, e.g., in 2002 the rate was 10.5% and reached 4.3% in 2014, increasing to 6.9% in 2015 (Fig. 3). The employment supply increase has increased income for a population who then transform this reality into consumption.

²Income transfer or cash transfer comprehends government initiatives in order to destine money resources from rich people to vulnerable people, which means to use money from taxes and tributes to pay a monthly amount to poor families. See Medeiros et al. (2007).

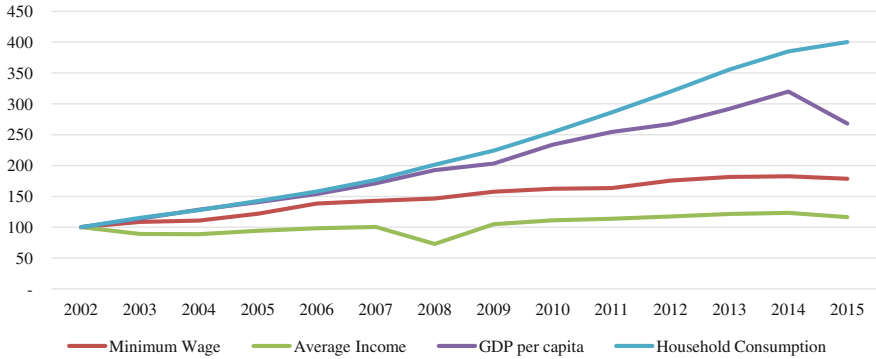


Fig. 2 Growth rates of minimum wage, average income, GDP per capita and household consumption, between 2002 and 2015 (2002 = 100). *Source* Data from Instituto Brasileiro de Geografia e Estatística (IBGE 2016)

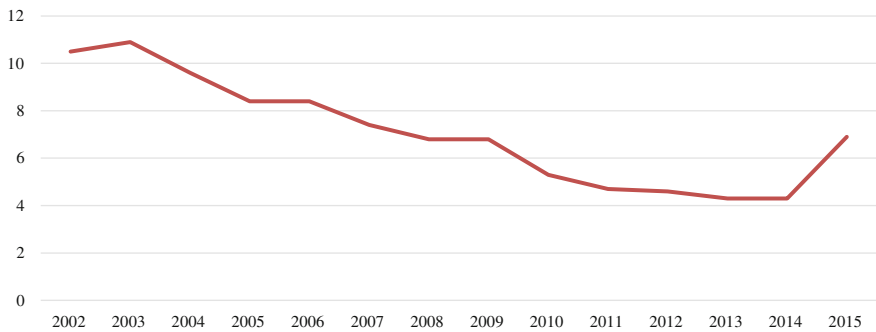


Fig. 3 Inoccupation rate, between 2002 and 2015 (%). *Source* Data from Instituto Brasileiro de Geografia e Estatística (PME IBGE 2016)

The abundance of money led people who were out to get in consumption (entry) and those who were into enlarge their consumption standard. This led to some problems. According to research carried out by Credit Protection Service (SPC), more than 70% of Brazilian people have an incorrect perception about debts. For more than 50%, to have debts meant *just delaying paying of accounts*, which is a wrong perception (Valor Econômico 2015).

Moreover, another study from SPC (2015) concluded that 48% of those interviewed do not have any personal budget control. Around 60% reported much difficulty in controlling monthly income and expenditure, and 33% appealed to credit, like credit cards and banking account limits. According to this study, Brazilian consumers have a relatively poor knowledge and attitude related to financial education.

People do know what they would have to do to reach a financial balance, but do not do anything towards reaching such balance. One way of explaining this

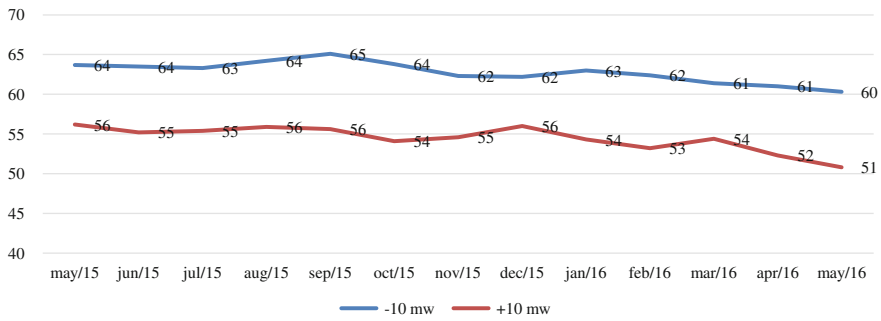


Fig. 4 Percentage of interviewed people that claim to be included in debt's negative register minimum wage reference (%). Source CNC (2016)

behavior is to relate it to financial subject unfamiliarity, lack of habits, lack of discipline and difficulty in seeing how beneficial a good and healthy financial situation can be. An example is that 41% of the Brazilian population do not have savings or another kind of investments, and these are mainly women (49%) and citizens from middle or poor classes (45.5%) (SPC 2015).

Another issue is that of *financial ignorance*, which represents a person's difficulty to do accounts and to use logical thinking. This is a reflection of poor financial and mathematical thinking. According to Gazeta do Povo (2014), the deficit in mathematics' standards is directly correlated with high levels of defaults and high difficulty in saving money.

In order to explain the importance of financial education, especially regarding the most economically vulnerable population, a study carried out by the CNC institute (CNC 2016) showed that the people who live under the 10 minimum wage³ are the most affected by the default problem, as can be seen in Fig. 4.

This research (CNC 2016) has also revealed that the debts are mainly from credit cards, car leasing, hypothec, insurance, etc. Another search carried out by *Serasa Experian* shows that Brazil's north region, which is the poorest region, has the highest percentage of families declared with debts, at 31.1%. Nationally speaking, among people between 31 and 35 years, 29.3% have been declared to be in debt (Serasa Experian 2016). In addition, people's profile revealed that 23% of those defaulting are among young adults living in poor urban areas, as can be seen in Fig. 5.

The economic and political crises that Brazil confronts nowadays, with more than 6% reduction in GDP between 2015 and 2016, and the increase of unoccupied rates (Fig. 3), brings people to renegotiate their debts, control compulsive consumerism and look for ways to save money for emergencies. Despite the financial crisis, these numbers reveal an alarming situation, especially for the most

³A minimum wage in 2016 were of R\$880.00 or around US\$260.00.

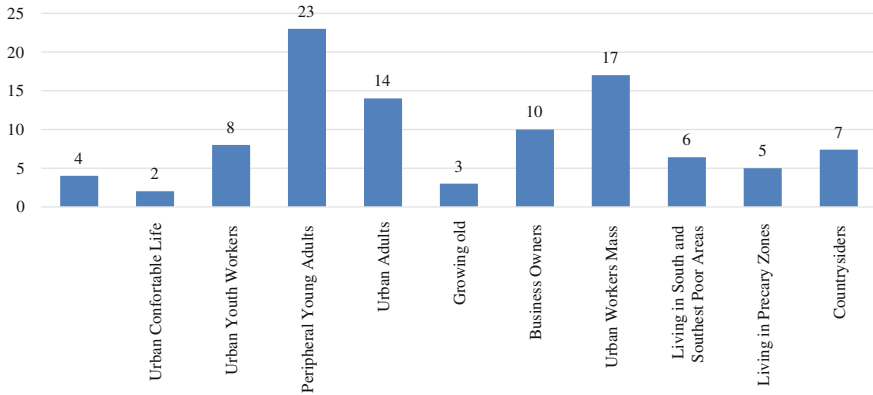


Fig. 5 Percentage of people in debt profile, 2014. *Source* Serasa Experian (2016)

vulnerable population, which perhaps could be less alarming if people had more orientation on how to deal with their personal finances or family budgets.

In summary, spreading financial education concepts could potentially improve people's quality of life as much as the economy as a whole. Practicing conscious consumption, planning personal budgets, saving money for buying something in the future or for retirement tend to help economy to become healthier and richer.

4 A Critical Financial Education

The pedagogical approach that we have developed towards an understanding of some important concepts related to financial education, aligned with the transformative stream, is conceived under the critical education proposals.

The Brazilian educator Paulo Freire substantially contributed to a better foundation of a critical theory of school learning. He emphasized the basis of a true democratic pedagogy, which fights against authoritarian relations through dialogue. The special conditions of Latin America during the 60s and 70s marked his work, but his work remains current until today.

According to Freire (1973), education must have a constant attempt to change attitude, replacing old passivity habits by new habits of participation and interference within student's reality. Campos (2016) points out that Freire shows that the attitude of a critical and criticizing education should lead people to a new position facing the problems of their time and space.

As for Freire, a critical educational science is a process of awareness as it is "[...] the process in which people, not as recipients, but as knowing subjects, achieve a deepening awareness both of the sociohistorical reality which shapes their lives and of their capacity to transform that reality" (Freire 1970, p. 27).

In this line, Freire proposes a problem-posing education, whose reflexive nature involves a constant act of unveiling reality. Thus, in this problem-posing practice, students will improve their power of capturing and understanding the world, which appears to them in their relations with this world, not like a static reality, but as a reality in a permanent process of transformation. According to him, what results is an understanding that tends to become increasingly critical and de-alienated.

In this kind of education, problematization activities take place dialectically:

Dialogue does not depend on the content which is to be seen problematically. Everything can be presented problematically. The role of the educator is not to “fill” the educatee with “knowledge”, technical or otherwise. It is rather to attempt to move towards a new way of thinking in both educator and educatee, through the dialogical relationships between both. The flow is in both directions. (Freire 1973, p. 125, quotation marks from the author)

In short, Freire presents a democratic pedagogy based on dialogue, through an active, critical and criticizing way. Thus, an education based on problematization of the contents, which are presented as relevant to the learners, challenging. This praxis results in a process of reflection-action by the student on his world/reality, activating his/her awareness from the generating themes.

For only as man grasp the themes, can they intervene in reality instead of remaining mere onlookers. And only by developing a permanently critical attitude can men overcome a posture of adjustment in order to become integrated with the spirit of the time. (Freire 1973, pp. 5–6)

5 Features of a Proposed Critical Financial Education Pedagogical Strategy

Skovsmose (2014), a seminal writer in critical mathematics education, emphasizes Freire’s idea of dialogue in the relationship between teacher and students. For him, it is important to break down the figure of the knowledge-owner-teacher and takes effect the presence of the one who teaches and is taught in a dialectical relationship with the students, who become co-responsible for an educational process in which all grow.

The ideas concerning the dialogue and the student-teacher relationship are developed from the general point of view that education must belong to a process of democratization. If a democratic attitude is to be developed through education, education as a social relationship should not contain fundamentally undemocratic features. It is not acceptable that the teacher (alone) has the decisive and prescribing role. Instead the educational process must be understood as a dialogue. (Skovsmose, op. cit., p. 350)

According to Skovsmose (2005, p. 114), “Mathematics education might serve a further development of a concern for democracy and ensure social inclusion. It might, however, provoke exclusion as well. This leads me to consider the importance of critical mathematics education”.

In addition, Skovsmose highlights that an important aspect of critical mathematics education is problem orientation in the teaching-learning process. To select the types of problems for teaching, one should take into account what is really relevant to the student and it must have a close relation to objective existing social problems.

According to Alrø and Skovsmose (2004), critical mathematics education is an approach which values certain mathematics learning qualities.

Critical mathematics education is concerned with the way mathematics in general influences our cultural, technological and political environment and with the purposes for which the mathematical competence should serve. [...] The critical mathematics education is also concerned with issues such as *how learning mathematics can support the development of citizenship* and *how the individual can be empowered through Maths*. (Alrø and Skovsmose 2004, p. 16, emphasis in original)

There are many ways or strategies to carry out critical mathematics education. The *thematization* is more focused on primary and secondary education, while the *organization-in-projects* is more conducive to college education. Skovsmose (2014) does not consider them sufficient or ideal, but just reasonable, and emphasizes, as the most effective strategy, the *problematization*. For it to work as a practical mechanism for critical mathematics education, it is important that students understand the relevance of the problem, which should be related to their experience. Problems should be linked to processes important for society in general and when assuming responsibility for solving them, students must design a political and social engagement.

That said, as practitioners of a critical approach to education, we propose a pedagogical experiment concerning financial education. In mathematics, there are many content topics related to students' daily lives, including financial mathematics, which we believe is a key link to involve the practice of education for citizenship, in financial education and mathematics contents.

In our pedagogical strategy for a critical financial education, we use mathematical modelling. Generally, one may create a model for interpreting and studying natural or social phenomena. The advancement of technology has made the use of virtual models quite common, and these allow a great quantity of simulations. The objective of creating a model can be analytical, explanatory, pedagogical, for forecast, etc. Mathematics is particularly abundant in allowing model creation, when dealing with quantitative variables.

In this perspective, a set of symbols and mathematical relations, which aims to translate a phenomena or a problem from a real situation, is called mathematical model. (Biembengut and Hein 2003, p. 12)

The process that involves obtaining a mathematical model is known as mathematical modelling. Modelling is similar to an art, when creating models for different purposes, and can be seen as a form of creation and expression of knowledge.

For us, mathematical modelling is a method (or pedagogical strategy) that can be employed at various school levels, from elementary mathematics to graduate level. It can be conceptualized as a learning environment to be built in the classroom in which students are asked by the teacher to investigate, through mathematics, situations extracted from daily life or even other sciences.

The mathematical modelling process can be a way to awaken in students the interest in mathematical content, to the extent that they have the opportunity to study, through various investigations, situations that have practical application and value their critical sense. (Campos et al. 2011, p. 47)

Complementing this idea, Campos et al. (2015) have pointed out that teaching concepts of financial mathematics by themselves is not enough to accomplish the objective of forming citizens and promoting critical financial education if they were not contextualized in real or realistic situations, close to the student's life.

Nevertheless, a link between financial education and critical mathematics education can be created, as long as some discussions and debates are to be carried out in order to develop a deeper awareness along with social and economic issues that affect students and their families, friends and communities. A critical consciousness should also include discussions over government responsibilities regarding the country's financial system, its regulation and its decisions, which affect all citizens. This is precisely what we want to call critical financial education.

Critical education aligns itself with the idea of education for critical citizenship, to the extent that it incorporates the tensions and contradictions between what is and what should be in a democratic society that is grounded on equality, freedom and justice. Thus, critical mathematics education reminds us of a social character of pedagogical work, which besides seeking to give meaning to the mathematical content, seeks to do so in a democratic way, encouraging students to develop critical thinking, ethical responsibility and political awareness.

Moreover, Valero (2015) has observed students' decreasing engagement in mathematics, fundamentally caused by a gap between the forms of subjectivity promoted by mathematics as an area of schooling and the forms of subjectivity experienced by them in their everyday life. Thus, we interpret that what Valero stated reinforces the need for approaching mathematical content in a way that assists students to realize its importance and are encouraged to engage themselves in their pedagogical environment, which is precisely what critical mathematics education advocates. Therefore, a critical financial education is in line with the purposes of critical mathematics education, insofar as it aims to bring to the classroom, a discussion about the social problems arising from the mismanagement of personal finances, and aims for a transformation of the harsh reality exposed by the alarming data presented in relation to household indebtedness and excessive consumerism.

6 Example of a Critical Pedagogical Activity in Financial Education

6.1 Description of the Activity

For the purposes of putting into practice the idea of a critical financial education through a modelling strategy, we designed a pedagogical activity, which addressed selected financial education concepts in a financial mathematics class, carried out by the first author in an undergraduate course. We proposed a student research project, focusing on basic ideas from financial education.

The 20 students organized themselves into four groups and chose the following topics:

- (i) Brazil's basic interest rates;
- (ii) Banking interest rates for loans and banking profits;
- (iii) Brazil's budget: debts and incomes;
- (iv) Family budget.

Students were required to make a report and do a presentation on the selected topic. They agreed that a period of two weeks would be sufficient for the task. All presentations would be done in sequence at the same day, and each one of them should be followed by a discussion where everyone should participate.

The first group explained the BCB's basic interest rate (SELIC) and showed its historical series (Fig. 6).

The group reported that considering a 9.2% expected annual inflation (IBRE 2015), the real interest rate would be of 4.62%, which was the world's highest interest rate at that time (Moneyou 2015), since the SELIC was of 14.25% per year. It was also explained that the SELIC rate affects banking interest rates for consumer's loans and, as it presents an upward trend, the perspective for the loan borrowers was not good. The discussion that followed the presentation was mainly about whether government (or BCB) should increase the interest rates or not.

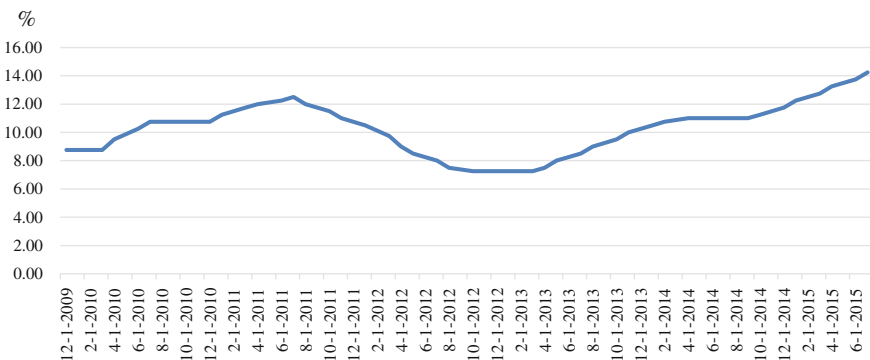


Fig. 6 SELIC historical series. Source Data from BCB (2015, in students' report)

The second group showed data (BCB 2015) revealing that the average interest rate for personal loans (hot money) from the six greatest Brazil's banks was of 11.0% per month, which represents approximately 250% annual rate. As for credit cards, the same banks had even higher interest rates, that is, 12.6% per month, or 316.7% annual rate (average). The group also showed data (Costas 2015) revealing that the Brazil's greatest private bank (Itaú) had obtained profits of R\$20.6 billion in 2014, which represented an increase of 30.2% when compared to the year before. In addition, quoting *Economática Consulting* (apud Costas, op. cit.), they revealed that the profitability of Brazil's banks was of 18.23% in 2014, more than twice the profitability of US banks (7.68%). The debate that followed this presentation had two main subjects: how can Brazilian banks improve their profits along with an economic recession scenario and how people should avoid banking loans.

The third group showed data from BCB, revealing that in 2013, the national account has had a nominal deficit of R\$173.8 billion (BCB 2014), i.e., the outcomes overcome the incomes by about this amount, which represented 3.25% of GDP. They also showed preliminary data from 2014, revealing a primary deficit of R\$32.5 billion and a nominal deficit of R\$343.9 billion, which represented 6.70% of GDP (Exame 2015). After the presentation, students discussed the bad example given by central government, which could not eliminate the country's deficit, and which causes vulnerability in controlling and combating inflation, resulting in an upward trend.

The fourth group presented a fictional family budget, detailing all debts. Based on their own families' expenses, they have built a long list of consumed items and a supposed income, which was not enough to overcome the expenses. Thus, the family had a deficit and the group wanted to discuss how to eliminate it. The question raised in the debate was on how can a family reverse a deficit if they do not make a detailed control like this. They also discussed the need for some items, which some students considered unnecessary or superfluous, especially for an indebted family.

After that, the teacher addressed some important financial education issues like budget planning, excessive consumption and misuse of credit cards, among others. Students pointed out the need for awareness of these concepts and have argued what they could do in order to spread such information.

6.2 Analysis of the Activity

In our point of view, the pedagogical activity that we have described is a modelling project, because:

- (a) Students brought real (or realistic) problems, like an in debt family budget. They discussed the problem and proposed a solution with the help of their colleagues and the teacher.

- (b) Students also brought other problems, which were concerned with the interest rate from private Brazilian banks. They argued about some causes for this issue and showed that there was a poor handling of the country's economic policy, besides the greed of banks for profits, no matter who is harmed.
- (c) In addition, the students were the ones who chose the themes.

The activity is inserted in the context of critical mathematics education, as the students faced a real social economic problem, which affects everyone, but especially the disadvantaged and people that are more vulnerable. Many discussions were carried out concerning this financial problem and the students concluded that a dangerous lack of information was greatly responsible for the problems. Otherwise, they have strongly criticized the government's conduct and that of one of the banks. In facing the problems, they also reflected on what actions they could take to help solve the problem of a lack of information.

In this proceeding with the activity, we have created a democratic environment, to the extent that the students chose the themes and had a voice during the presentations and debates carried out in the classroom. The students engaged in dialogue, discussing and reflecting on solutions to a serious social problem. In other words, this activity seems to support the development of citizenship and students felt empowered through the engagement on this modelling project.

As was pointed out in the fourth group presentation, budget planning is important for any family, in order to administer their finances. The prejudice caused by unnecessary or superfluous consumption was also pointed out. In this line, the debates argued for a qualitatively better world, as long as the students have discussed the importance of spreading information about some financial education concepts. Altogether, bringing public economic policies to the debate, we have performed what Skovsmose called *problematization*. Insofar, students understood the relevance of the problem, which is important for society, and designed a political and social engagement in the debates, thus performing the critical mathematics education.

Moreover, we were able to fight some of the problems that Valero (2015) has stated, namely addressing the gap between the forms of subjectivity promoted by mathematics as an area of schooling and the forms of subjectivity experienced by them in their everyday life, besides the need of approaching mathematical contents linked to reality.

As we have pointed out, for Skovsmose (2014), it is important that students understand the relevance of the problem, which should be related to their experience. In addition, he said that the problems should be linked to processes important for society in general and when assuming responsibility for solving them, students must design a political and social engagement. That is precisely what happened in this pedagogical activity, the students brought the financial problems, whose relevance can be seen through the data that we have shown, that is, the great number of people dealing with debt troubles.

Concerning Freire's ideas (1970, 1973), we have engaged a democratic pedagogy based on dialogue, through an active, critical and criticizing way. This kind of

pedagogy, based on problematizing the contents, which were presented as relevant and challenging to the learners, have resulted in a process of reflection-action by the students on their world and reality, boosting their awareness from generating themes, exactly as Freire has pointed out.

Finally, financial education was also carried out in this pedagogical activity, as students have discussed some of its issues, e.g., family budgets, debts, consumerism, etc. Consonant to the fact that we have done a link between financial education and critical mathematics education, we can say that this pedagogical activity played what we called critical financial education.

6.3 *Student Learning Outcomes*

In this pedagogical activity, we realized that students showed a great involvement in all steps. The debates and discussions over the problems raised from the presentations were noticeable and the engagement of the students was also noteworthy.

We were able to see a critical consciousness emerging on the discussions over government lack of responsibility in managing the country's budget.

Nevertheless, it is relevant to note that some stressing issues played a minor role in this pedagogical approach. Students showed some difficulties in drawing reports, revealing a weak performance caused by their unfamiliarity with this kind of task. In addition, some discussions had to be abbreviated due to the short time available for the accomplishment of the project.

7 Concluding Remarks

The aim of this project was to carry out an activity involving all students in a discussion over some financial education issues within a financial math class, developing a critical approach under a mathematical modelling strategy. There was an intense involvement of the students in all presentations, revealing their interest in the subject. Thus, goals (i) and (v) listed for financial education were assigned in the project, namely: understand the basic functioning of the financial market and how interest rates affect the financial lives of citizens, for good or for bad; and understand the importance and benefits of planning and following up personal and family budget.

The critical approach was mainly revealed during the discussions, when students showed a strong indignation about the issues pointed out by the groups. Many criticisms were about government attitudes, when failing on the management of its accounts and increasing the basic interest rates. The banks were criticized too, as they punish people with high interest rates, while increasing their profits in an economic recession scenario.

Moreover, the last group brought a realistic example closer to the core of the transformative stream of financial education. Their presentation, along with the following debates, led students to realize and feel the importance of taking care of their personal and families' finances.

From our perspective, financial education is of great importance for all citizens and schools should encourage its insertion in mathematics classes at all levels. In addition, a critical education approach mediated by a mathematical modelling strategy has revealed to be worthy for achieving financial education goals in both a schooling and university environment.

Based on what we have experienced with this pedagogical activity, it seems important to connect financial education within financial mathematics classes and curricula, so that students could have a chance to experience its concepts, which are aligned with the idea of education for citizenship.

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