Towards an Ethics of Mathematical Application

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Abstract In the light of growing public attention to the influence of algorithms on our lives, this chapter addresses the question of how an ethical perspective on mathematical application could be conceptualised in contemporary late-modern societies. Firstly, we recapitulate some of the recent theoretical developments on the ethics of mathematical application in the field of mathematics education (Skovsmose in Mathematics education in a knowledge market: developing functional and critical competencies. Opening the research text: insights and in(ter) ventions into mathematics education. Springer, New York, pp. 159–188, 2008; de Freitas in Int Electr J Math Educ 3(2):79–95, 2008). Secondly, based on the work of the sociologist Luhmann (Thesis Eleven 29(1):82–94, 1991), we develop theoretical outlines of an ethics of mathematical application as a reflexive theory of moral communication on mathematical application. We then move into the sphere of the social and confront these theoretical considerations with a critique of the ideology of "solutionism". Solutionism refers to a semantics that links the mathematisation of the social to 'the morally good'. This critique leads us to suggest firstly, developing an ideology critique of the underlying semantics as a desideratum; and secondly, a systematic further development of an ethics of mathematical application that could inform moral communication on mathematical application in (critical) mathematics education.

Keywords Mathematical application · Ethics · Mathematization of social process Critical mathematics education

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1 Introduction

Today, it seems to be common sense that 'mathematics is everywhere'. This is not a mere slogan anymore that is solely promoted by mathematics educators to proclaim the relevance of their subject. The number of mathematisations that are 'colonising' the every-day world is "growing exponentially" (Ernest 2001, p. 287). Likewise, the number of reflections which discuss the consequences of this development (most commonly spread by the mass media) is at an all-time high. To name just one example, the article "How algorithms rule the world", published in *The Guardian*, draws on insights from the recent NSA revelations: "The NSA revelations highlight the role sophisticated algorithms play in sifting through masses of data. But more surprising is their widespread use in our everyday lives. So should we be more wary of their power?".[1] In many cases, mathematics appears as a power of its own, changing the world according to its supposed 'own will' and thereby re-programmes the conditions of our lives beneath our consciousness (Han 2017). However, the myth that 'mathematics is everywhere' is contingent upon the fact that humans *apply* mathematics to all spheres of life, including the social sphere.

If we accept the thesis that the social process of mathematisation increasingly influences all different aspects of our lives, reflection on the conditions and consequences of mathematisations becomes more important than ever. Then, one task for mathematics educators and researchers is to draft their possible contributions to the discussion on the "formatting power" (Skovsmose 1994, p. 43) of mathematics. As always, the first step would be to pose a good question. In any case, the evolving forms of interaction and communication will dramatically change the ways in which we see the world and ourselves. This is why the question at stake cannot be about *if* we want mathematisations to regulate the social spheres of life. Instead, it needs to be *how* we can develop a critical stance that allows us to reflexively deal with the mathematisations that in turn shape our lives. Thus, any form of critique that goes beyond a simple rejection of the social process of mathematisation has to rely on a reflexive theory that allows us to confront "what is the case with what is not the case but could become the case" (Skovsmose and Borba 2004, p. 214). Moreover, such a theory would need to exercise this critique from the inside of the object under investigation. In mathematics education theory, that is the point where we enter the domain of critical mathematics education since "reflection is a characteristic of being critical" (Skovsmose 2008, p. 159). In practice, critical mathematics education aims to initiate teaching and learning processes that allow students to turn mathematics against itself. That is, students are encouraged to reflect upon the philosophical grounds of mathematics as well as the conditions and consequences of mathematical applications. Such reflections can reveal political and ethical concerns that are usually not addressed in mathematics classrooms. This chapter shall be read as a contribution to the ongoing endeavour to identify ways to

[1]Retrieved from: http://www.theguardian.com/science/2013/jul/01/how-algorithms-rule-world-nsa on September 8th, 2015.

empower students to become critical citizens. Thereby, we focus on the question of how an ethics of mathematical application, in light of the all encompassing process of mathematisation, could look like. By seeing the mathematisation of the social as a major challenge to both mathematics education research as well as the mathematics education practice, we will firstly recapitulate some of the recent theoretical approaches developed on the ethics of mathematical application (de Freitas 2008; Skovsmose 2008). Secondly, we aim to sketch out the theoretical outlines of an ethics of mathematical application as a reflexive theory of moral communication on mathematical application. Thirdly, we will move into the sphere of the social and use our theoretical work to exemplarily analyse a social phenomenon: The phenomenon that the mathematisation of the social and 'the morally good' seem to enter a peculiar bond in the semantics[2] propagated by the ideological leaders of digitalisation (such as Facebook, Google, Amazon, and the like). Finally, we suggest: (1) the development of an ideology critique of the underlying semantics as a desideratum, and (2) a further development of our sketched framework of an ethics of mathematical application that could inform moral communication on mathematical application in (critical) mathematics education.

2 Ethical Filtration: What Could 'Being Critical' Mean?

In order to better understand the potential effects of the application of mathematics, Skovsmose (2008) investigates "mathematics in action" (p. 163). Therefore, he observers what people do when they mathematise a social practice. With the focus on how mathematics is brought into action for the organisation of the practices, he identifies a phenomenon that he calls "ethical filtration" (ibid.). As soon as a social practice is abstracted to numbers, variables, and the relations between them, all considerations with direct reference to the practice seem to vanish. Immediately, the focus solely resides on the accuracy of the accompanying transformations and calculations. Consequently, this means that the process of mathematisation loses its contact to the concrete situation. Thereby, the model seems to become blind to its own origin in the 'real' social situation (and so does the modeling agent). In other words, the process of mathematisation tends to entail a moment where contingency, subjectivity, and materiality are stripped away (de Freitas 2008) and the (supposedly) immanent logic of mathematics takes over. Nonetheless, ethical filtration itself is not to be understood as a malicious or imprudent (mis)use of mathematics in applications. Instead, it turns out to be "a general feature of bringing mathematics

[2]We conceptualise a semantics as a self-description of the society *in the society* that is articulated in communication. When societies change, so do the available possibilities for the members of societies to communicate about the society they live in, in other words, semantics change. Simultaneously, when semantics change, so do the societies that make use of them. A semantics is thus a pre-condition that shapes communication. In turn, it is shaped by all communications that are recursively producing the society.

into action" (p. 167). Thus, it should be considered as a phenomenon that is *immanent to the process of mathematisation itself*. Due to findings like these, one of the overall aims of critical mathematics education is precisely to counteract such exercising of mathematics without any accompanying reflections. In regard to this, Skovsmose (2008) poses an important question: "What does it mean to establish an ethical perspective on mathematics in action?" (p. 166).

Skovsmose (2008) addresses this question in his report on his project "Family support in a Micro Society". The project aimed to make students "experience how mathematics can be brought into action—how it may be part of a decision making process and, in this way, becomes part of peoples' reality" (p. 166). The participating students were divided into different groups. Each group was assigned to a fictional micro society consisting of 24 families that were further described in essays:

> Each group had to formulate principles according to how they wanted to distribute child benefits among families. The amount of money available was given, but each group could formulate any criteria according to how they wanted to distribute it. Next they had to provide an algorithm for distributing the money. […] In the process of turning the verbally-formulated principles for distribution into functional algorithms, the students experienced how the original principles needed to be simplified. At times the principles were almost ignored when mathematics was brought into operation to do the distribution. The students experienced the general phenomenon that when mathematics is brought into action, a new discourse takes over. (ibid.)

However, the observation that the students "experienced" the phenomenon of ethical filtration should not mislead us about the critical effects that this experience actually unfolded. That is, the experience of ethical filtration does not automatically lead to the development of a critical stance towards the exercised mathematical models in particular and mathematical application in general. It rather opens up a space of possibility that *might* "indicate what 'being critical' could mean in educational practice" (Skovsmose 2008 p. 167). In other words, *there is always a gap between the potentiality of a critical stance evolving from the experience of ethical filtration in the mathematical modelling process and the actuality of a critical stance that is yet to be developed*. This view is supported by the detailed consideration of some students' utterances from the project in Skovsmose's (1994) dissertation. Not seldom with a touch of irony, he depicts how the students are rather "absorbed in the technical task of making the distribution" (p. 138) than being critical to it. For example, one student reflects on his action in an early unit: "I see, the age of the child is missing. Anyway, the family lives in number 13, so the age may as well be 13 too!" (p. 127). Another student, reporting on a late unit where the teacher intentionally initiated a discussion on the differences of the models and their sociopolitical implications, states "We agree that a difference [between different distribution models] exists, but anyway we have made the calculations correctly" (p. 128).

The reflections on the project can be considered as evidence for Lundin's (2012) assumption that the practice of school mathematics is often informed by an ethics that "establishes mathematical knowledge as good, by making such knowledge have beneficial consequences" (p. 81). The reflections on the project can be considered as evidence. In general, this effect can certainly be recorded for any school subject; however, the unique characteristics of school mathematics goes one step further. Mathematical knowledge is not only identified as morally good, but at the same time *allegedly* '*true*' *mathematical knowledge is established as always* '*out of reach*'. That means that students learn to privilege mathematical knowledge by experiencing the beneficial consequences of applying it. They further learn that the mathematics they apply is always just an impure form - a form of mathematics being distorted by the imperfection of school. However, this also presupposes that an application of 'true' mathematical knowledge, which would solve problems in a true and not solely in an approximate manner, is principally possible as long as the limitations of mathematics in action are solely attributed to the limited complexity of school mathematics to fit the complexity of the 'real' world. In other words, students may experience mathematics as a "pure […] and wholly logical knowledge, which […] happens to be useful because of its universal validity" (Ernest 2001, p. 279) not despite, but precisely because of the experience that "the original principles needed to be simplified" (see above) in order to bring mathematics in action within the wider frame of school mathematics, which rewards mathematisation as an end in itself. Instead of leading to a critical stance towards mathematical application, such experience may just as well reinforce an absolutist conception of mathematics.

But, the claim that mathematics represents an eternal body of knowledge which can be stripped of any contingency was also heavily questioned within the mathematical discourse in the beginning of the 20th century: Gödel (1931) showed that it is always possible to construct theorems that are *undecidable from the inside of a formal mathematical system*. In other words, mathematics cannot bootstrap its own conditions of possibility. Thus, since it depends in its very constitution on an extra-mathematical, subjective act that cannot be grounded in mathematics itself, mathematics has to be understood as radically political. Therefore, any ethics that advises us to simply identify mathematics as morally good is actually a "quasi-ethics" because it can only justify the superiority of mathematics as the form of seeing the world by implicitly presupposing an ontological unity between mathematics and being that has become more than questionable. Following this line of thought, we are thrown back into the gap between the potentiality of a critical stance and its yet to be developed actuality where any 'struggle' for a critical stance towards mathematics and its applications seems to take place. In order to unfold its critical potential, an activity like the "Family support in a Micro Society" would thus need to somehow conduct a juggling act: The activity needs to avoid a one-sided moral communication on mathematics (mathematics is morally good in its very structure), while simultaneously, bearing in mind that it is also the initiation of reflective processes themselves that could implicitly reinforce the bond between 'the morally good' and the application of mathematics. Therefore, as a first approximation, we suggest that it is important to: (a) pay attention to not credit students in case that they relativise their models (e.g. "we know very well that *our* model has such and such shortcomings, a professionally developed model, however, could solve the problem"); (b) let students experience genuinely beneficial consequences for subordinating mathematical knowledge to ethical reflections, and (c) allow students to experience such subordination in sufficient frequencies. This last point is particularly important because

> the subjective experience of those hundreds of hours [with mathematical knowledge as the sole warrantor of good consequences] may exceed the ideological parameters whilst remaining in the service of those ideologies by making us believe them through the sheer force of habitual action (Brown, forthcoming).

To provide the possibility that these reflective processes can effectively undermine the 'sheer force of habitual action', de Freitas (2008) suggests the development of a code of ethics of mathematical application that could inform mathematical modelling processes:

> Why not construct an ethics of mathematical application, as we have for medicine? The application oath might simply demand that the mathematical agent (be it a student or a teacher or other) must reflect on the ethical consequences of her/his mathematical actions in the 'real' world, and seek to serve 'real' others in need of assistance through the use of these powerful tools […]. [T]hen time spent in our classrooms on ethical reflection will serve the social justice goals of critical pedagogy (p. 92).

We argue that the first step in the realisation of the ambitious theoretical project to develop an 'application oath' as an ethics of mathematical application is a further clarification of the theoretical concepts at stake. This is why, for the time being, we solely want to sensitise for the necessity of further theoretical considerations by posing three simple, yet not explicitly discussed, questions: (1) *What qualifies a reflection as an ethical reflection? Or with regard to our planed endeavour: What qualifies a reflexive theory as an ethics?* (2) *What is the object an ethics is dealing with? And* (3) *What is the relation between ethics and morality?*

In the next section, we approach these questions by re-contextualising selected works by the sociologist Luhmann (1991) on the relationship between ethics and morality for the field of mathematics education.

3 Towards an Ethics of Mathematical Application

What does it mean if we categorise an action, or a communication as bad or good *as a whole* (as opposed to categorising a particular dimension of it); and what does it mean to evaluate an action or communication as good or bad *as such?* What do we mean if we say that one simply should not act or communicate in this and that way? How is it possible to justify universal judgments like these, or are they even justifiable at all? The specific forms of reflection that are indicated by these questions lead into the sphere of what is commonly known as *ethics* (Tugendhat 1984). Any ethics is a "theoretical reflection of morality" (Luhmann 1991, p. 83). That is, an ethics aims to reflect on the conditions of moral communication. This means that any ethics stands in a theory-practice relationship to moral communication and thus depends on the prevailing concepts of morality that are contingent upon the socio-historical conditions in which they are actualised.

If we understand ethics as a theoretical reflection of the empirical practice of moral communication and if, moreover, the social actualisations of morality are contingent, the first step towards an ethics of mathematical application is to provide *an empirical concept of morality*, which takes into account that the forms of moral communication are changing in time:

> I understand by morality a special form of communication which carries with it indications of approval and disapproval. It is not a question of good or achievements with respects, e.g. as an astronaut, musician, researcher or football player, but of the whole person insofar as s/he is esteemed as a participant of communication. Approval or disapproval is attributed typically to particular conditions. Morality is the useable totality of such conditions at any time. (Luhmann 1991, p. 84)

Firstly, this definition of moral communication does not refer to arbitrary entities but precisely addresses persons and persons only. Further, that is done in a very specific way: Everybody who communicates morally indicates (at least implicitly) the conditions under which he or she can or cannot approve *another person as a whole*. Moral communication thus always expands the range of its validity beyond what it initially sets out to evaluate. It unwittingly expands the approval or disapproval of a person's action, or communication to the person's entirety. Moralisation always entails generalisation. Conceptualised in this way, morality can be considered as the "conditions of the market of approval" (ibid.). Secondly, this empirical shift has the advantage that it limits moral communication to a very specific form of empirically observable communication. Therefore, we can ask

> what happens if conditioning of whatever kind (whether legal, political, racial or of personal taste) is moralized, with the consequence, for instance that X considers he cannot approve of Y and cannot invite him if he has a bust of Bismarck on his piano [or voted for Donald Trump to take a more recent example] (ibid.)

Moral communication is a very *specific* form of communication that, nevertheless, can be *universally* applied: Since we can (and often have to) ground our communications on distinctions different from the distinction between morally good and morally bad, not every communication is a moral communication. This indicates that moral communication is specific. However, when we conduct a moral communication, e.g. by using the moral code to communicate the conditions under which we approve another person, the behaviour which is subjected to moral evaluation can principally origin from all different kinds of realms. In other words, we are able to *morally re-code* all forms of behaviour or communication and evaluate them from a moral point of view. That is what equips the moral code with its universality.

Under the presumption that the moral code is universally applicable, it should be possible to apply it to itself. The question which emerges now is whether the distinction between good and bad is good or bad itself. Here, it is important to note that we *cannot* answer this question from the inside of morality because every binary code results in paradoxes in case that it is applied to itself. We simply cannot decide when we are using the distinction between good and bad to communicate morally if it is morally good or bad that we are doing so. In case that we try to approach this question *from the inside of the moral code*, the only answer we can get is a paradoxical one: The moral code as the distinction between good and bad is good if and only if it is bad.[3]

It seems that it is impossible to guarantee *from the inside of morality* that the application of the distinction between good and bad is itself good. This serves as an indication that there are social situations in which moral communication is rather counterproductive, e.g. when people rule out any contradiction by moralising. This intuition is supported by the acknowledgment of the finding that moral communication tends to provoke "over-engagement of the participants" (Luhmann 1991, p. 86) and thus "is close to conflict" (ibid.) or even violence:

> Whoever communicates morally by making known the conditions under which he disapproves of others and of himself, invests and places at risk his self-approval. (ibid.)

Therefore, an ethics also has the task to define, and thereby limit, the space of applications of any moral communication. This means that an ethics should explicate the conditions under which it is good to use the corresponding moral code to evaluate particular communications and, which maybe is even more important, the conditions under which it is not good to do so. We can exemplify the limitations of moral communication by shortly describing what it would mean for the practice of mathematics as a scientific discipline if it were overdetermined by the moral code. Mathematical communication is organised in the *general medium of proof*. That is to say, every particular theorem which is presented inside the community of mathematicians is always accompanied by a proof. The theorem is then evaluated by means of the distinction between true and untrue due to the validity of the proof. Even if we only presuppose a very weak conception of truth—such as truth is what is counted as true by the community of mathematicians—we immediately see how fatal it would be to identify true mathematical theorems with the side of 'the morally good' and untrue mathematical theorems with the side of 'the morally bad'. It would be fatal because any progress in mathematics is based on the interplay between both *proofs and refutations* (Lakatos 1976). Therefore, we simply cannot brand a mathematician who presents a false theorem as morally bad because this would paralyse the research practice as a whole. This does not mean that mathematical communication is not subject to certain moral conditions that can be articulated in an ethics (c.f. Hersh 1990), but it only means that the practice of mathematics as a scientific discipline simply cannot "be integrated into the social system by means of morality" (Luhmann 1991, p. 85). This implies that scientific mathematical communication is organised by the functional code true/untrue and

[3]Gödel (1931) used this insight and developed a method by means of which it is possible to construct undecidable propositions in any sufficiently rich formal system, e.g. the arithmetic of natural numbers (Incompleteness theorem I).

this code operates at a higher level of *amorality*.[4] Here, amorality does not signify the opposite of "good", but the negation of the distinction good/bad itself. So, in order to be productively applicable, the functional code true/untrue must necessarily remain at a certain distance to the moral code good/bad. Again, that does not mean that moral communication on mathematics is impossible, but it does only mean that the moral code would be highly dysfunctional. It would be dysfunctional as the comprising code of mathematical communication since it would simply undermine the practice of mathematics: Proofs as the exchange medium of mathematical knowledge "emerge through the process of proposal and criticism through which they are improved enough to withstand the critical attitude of mind" (Ernest 2001, p. 278).

So far, what is our interim conclusion with respect to our aim to develop an ethics of mathematical application? Firstly, mathematics is neither intrinsically good, nor intrinsically bad. Therefore, it is important to condemn any ethics that is promoting an all too easy solidarity of mathematics with one side of the moral code. Secondly, an ethics of mathematical application must thematise morality *as a distinction*; that is, moral communication is a form of communication that re-codes decisions in a mathematical modelling process based on an attribution of the label of the morally good *or* bad. Thirdly, we must acknowledge the very specific nature of the moral code as being universal and specific at the same time. Consequently, this means that an ethics of mathematical application should also *reflect upon the limits of the scope of moral communication on mathematical application*.

At this point, we reach a level of abstraction that brings with it specific theoretical challenges because we must rely on a theory that is able to distinguish between the use of different distinctions. In our case, this means that we have to find ways *to agree upon acceptance and rejection values in relation to our moral distinction between good and bad* (Luhmann 1991, p. 85). In other words, we have to find a way to negotiate a consensus of when it is productive to moralise, and when it is unproductive to do so. But how could these acceptance and rejection values look like? And how can we agree about these conditions?

Here, we follow de Freitas (2008), who argued that we always have to consider the possible consequences of mathematical actions for the 'real' others in the 'real' world when we want to evaluate our decision making in the modelling process from a moral point of view. Thus, the ethical task is to transcend our individual position as decision makers in the modelling process (including our individual or commercial interests) and observe the consequences of our models from the perspective of others who could be affected by the decisions within the processes of modelling

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[4]This argument has to be generalised as we are living in functionally differentiated societies where *all* functional systems "owe their autonomy to their individual functions, but also to their binary codes" (Luhmann 1991, p. 85), while in "neither case can the two values of these codes be made congruent with the two values of the code of morality: In case that we, for example, consider the distinction between government and opposition in democratic political systems", we "do not want the government to be declared structurally good and the opposition structurally bad or evil" (ibid., p. 85f).

that were realised in practice. However, these reflections should not simply accompany the modelling process, but instead they should already inform or mediate our practical decisions in the construction of the mathematical model. Although it is incontestable whether the shifting of perspectives is a central theoretical figure to all moral considerations, we should not forget that the reflection of consequences of a mathematical model at the stage of its construction is not a straight-forward endeavour because it demands a very specific form of reasoning. The reasoning that is required can be characterised by the following form: If p then q, although p is not the case. Skovsmose (2008) calls this form of reflection "hypothetical reasoning" (p. 165) because it (re-)inscribes the practical consequences of a theoretical action into the theory itself, but not into practice. It remains theoretical and is thus only hypothetical. In this way, it is possible to reflect upon the gap between theory and practice in theory. However, the immanent limitations of such a theoretical approach, which aims to include the practical consequences of theoretical considerations *in theory* are clearly articulated by Habermas (1973) in his famous book *Theory and Practice*:

> Of course, the objective application of a reflexive theory under the conditions of strategic action is not illegitimate in every respect. It can serve to interpret hypothetically the constellations of the struggle […]. Seen from that anticipated goal, such interpretations are retrospective. Therefore, for strategic action and for the maxims by which the decisions in the discourse that prepares for this action are justified, these interpretations open up a perspective. But the objectivating interpretations themselves cannot claim a justificatory function; for they must comprehend counterfactually one's own action, which now is only being planned […]. (p. 40)

In other words, the first step towards hypothetical reasoning requires that we have to anticipate future consequences. In a second step, this projection allows us to retroactively evaluate our decisions from this *fictive point* in the future, although all decisions are yet to be made in reality (or will possibly never be made). Therefore, "the objectivating interpretations" are bound to a projection of hypothetical consequences into the future and thus, due to their hypothetical nature, "cannot claim a justifying function". Given the case that we cannot justify our actions in this way, on which criteria can we base our decisions then? Since we can only retroactively reflect upon the definite practical consequences of certain theoretical decisions in the modelling process, that is, *after* the decisions as well as the model are put into practice, we are thrown back to the question of the motivation of moralised decisions.

Although we have shown that we cannot sufficiently justify our decisions by hypothetical reasoning, we can still argue that it might be possible to initiate a democratic negotiation process to solve the justification problem—at least with regard to *the moral intention of a decision*. In other words, we could negotiate whether the intention of a decision in a modelling process is morally good or bad. Then, we would commonly decide whether we confirm the decision or not. However, this approach becomes invalid as soon as we take into account that good intentions can have bad consequences and vice versa. Therefore, Luhmann (1991) asks:

> If reprehensible action can have good consequences, as the 17th and 18th century economists assure us, and if inversely the best intentions can lead to bad results, as we can see from politics, then moral motivation blocks itself. Should ethics then counsel good or bad action? (p. 87).

The problem of justification already manifests itself in the motivation of our morally re-coded actions *because the depicted mode of reflection does not provide any criteria that could sufficiently inform our decisions*. Therefore, we pose two questions: (1) Is an ethics that guides moral communication on mathematical application possible at all? And (2) How could it regulate and deregulate moral communication?

To come straight to the point, we do not know the answer to these questions. What we believe is that any serious attempt to develop a positive conception of an ethics of mathematical application has to satisfy certain theoretical minimal conditions that we have, at least rudimentarily, explored in this section. Thereby, we approached the question *ex negativo* in order to show how an ethics of mathematical application could look like. That is, we focused on the identification of selected theoretical dead-ends to illustrate how an ethics of mathematical application *cannot look like* rather than to give a positive draft of it:

1. Any attempt to identify mathematics and its applications with just one side of the moral code can only be condemned as a 'quasi-ethics' because the moral code has to be addressed by an ethics of mathematical application as distinction with two sides.
2. Since it is impossible to justify the application of the moral code from the inside of morality, the limits of moral communication to evaluate processes of mathematical communication and action have to be articulated by the identification of rejection and acceptance values of moral communication on mathematical application (This is what led us to the problem of justification).
3. Neither the consideration of hypothetical practical consequences ('hypothetical reasoning'), nor the reflection on the moral intention could provide us with unambiguous decision-making guidelines that could sufficiently justify theoretical decisions in the mathematical modelling process.

However, we still believe that there simply is no alternative to a reflexive approach to an ethics of mathematical application. That is, an ethics of mathematical application has to remain a *theoretical reflection of the moral communication on mathematical application*. In the next section, we approach the conceptual problems from a more practical point of view by asking the following questions: How is moral communication on mathematical application structured in contemporary late-modern societies outside the realm of schooling? Moreover, which ethics (if any) informs the moral communication on mathematical application in these social spheres?

4 The Ethics of Solutionism

In apparent contrast to our developed thesis that mathematisation is always accompanied by 'ethical filtration', Morozov (2013a) argues that the promotion of the formatting of our social life by means of mathematisations often does not happen in ignorance of moral considerations, but exactly in the ethos of making the world a better, safer, greener, and more equitable place. For example, the former CEO of Google, Eric Schmidt, frames his inspiration: "Technology is not anymore about hardware or software. It is about collecting and analysing enormous masses of data in order to change the world to the better" (cited in Morozov 2013a, p. 9, translated by H.S-P.); Mark Zuckerberg, the founder of Facebook, strikes a similar tone: "We do not wake up in the morning to earn money". Rather, Facebook follows the mission of "making the world more open and interconnected" (cited in Morozov 2013a, p. 9, translated by H.S-P.). Here, it is important to note that the leaders of digitalisation do not position mathematisation as *one* form of regulating social practices amongst others. They do not treat it as a specifically motivated form that is contingent upon a particular distinction which only takes into account those characteristics of a practice that can be quantified successfully. Instead, they paint the mathematisation of the social in the colours of a completely unideological endeavour:

> Out with every theory of human behavior, from linguistics to sociology. Forget taxonomy, ontology, and psychology. Who knows why people do what they do? The point is they do it, and we can track and measure it with unprecedented fidelity. With enough data, the numbers speak for themselves (Chris Anderson, editor in chief of the Wired magazine 2008, cited in Han 2014, p. 99).

In other words, Anderson discredits *any* theory of human behaviour as inadequate to explain human behaviour. He thus aligns the distinction "numerical determination of human behavior"/"(theoretical) explanation for human behavior" to the distinction adequate/inadequate. As Morozov's analysis makes us aware, the distinction adequate/inadequate is, however, already morally coded within the ideology of solutionism. Any theoretically informed explanation of human behavior is thus morally discredited. The 'objective' analysis made possible by 'Big Data' is, in turn, posited as the *unideological opponent*. However, as Žižek (1989) frequently reminds us, the position that declares itself completely free of ideology is the one that we should be most suspicious of because "the idea of the possible end of ideology is an ideological idea par excellence" (p. xxiv).

Any semantics that reifies its own way to describe society within the society to the only one can be labelled as an ideology. Consequently, Morozov (2013b) calls this form of reasoning, which is "[r]ecasting all complex situations […] as neatly defined problems with definite, computable solutions […] if only the right algorithms are in place" (p. 5), an 'ideology of solutionism'. *Firstly*, the semantics is labeled by Mozorov as 'solutionism' because social problems are conceived as sort of puzzles that are, in principle, solvable only if enough data is collected and analysed; *secondly,* the semantics is indicated as an 'ideology' because it disavows its own "political foundation" (Žižek 2000, p. 169); *it reifies one way of seeing the world to the only one*.

At first glance, it could be assumed that the semantics of mathematisation is again grounded in an 'absolutist' conception of mathematics, so that its presupposed universal applicability guarantees the superiority of mathematisation. The sociologists Espeland and Stevens (2008), who developed a sociology of quantification, argue in alignment with this idea when they state that "quantification facilitates a peculiarly modern ontology in which the real easily becomes coextensive with what is measurable" (p. 432). This supposed 'modern ontology' identifies *being* with mathematisation and *nonbeing* with non-mathematisation and thus supports the mathematisation of the social. However, the focus on an ontological conception of mathematics disavows that the mathematisation of the social is embedded in a very specific moral horizon. This moral horizon suggests that any attempt to counteract the solving of problems by means of mathematisation *is a reactionary intervention against the human(e) project of making the world a better place*. In other words, the backside of mathematisation (that is: non-mathematisation), which is standing for the possibility of an extra-mathematical answer to a social problem, is blanked out by its alignment to the backside of the moral code. Consequently, from the inside of the ideology of solutionism, anybody who criticises the *self-referential closure of the social process of mathematisation*[5] is immediately stigmatised as morally bad. Therefore, the moral horizon as well as its self-expression as unideological is a necessary support for the effectiveness of this ideology as it is only by the horizon of the potential realisation of a better world that it can justify that humanity should let a group of supposed pioneers solve its problems, even before these problems have been identified *by human experience,* and before they have been *problematised* in the political arena.

Going one step beyond Espeland and Stevens (2008), we thus argue that we cannot understand the semantics of mathematisation without the consideration of the moralisation it elicits. This moralisation becomes necessary precisely because the classical ontological distinction between *being* and *nonbeing* fails as an all encompassing semantical figure. It appears that it is not possible to simply identify the side of being with the entities that are quantifiable, and the side of non-being with the entities that are not quantifiable. This very failure of the modern ontology as an all encompassing semantical figure effectuated a moral supplementation of the ideology, that is, the linkage of mathematisation with the side of the morally good. However, we have already extensively argued in the section above that an ethics of mathematical application has to thematise the moral code with respect to mathematical application *as a specific distinction* instead of universally identifying it with the side of the morally good (*or* the morally bad). Thus, the ethics that supports the

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[5]This argument of a self-referential closure of the process of mathematisation refers to its circularity: As soon as a mathematisation is implemented in a social practice, it can only be substituted by another, and possibly more sophisticated, mathematisation.

one-sided moralisation of mathematisation is in fact another example of what we have earlier called a ‘quasi-ethics‘.

If mathematical application in school mathematics, and the mathematisation of the social are both in many cases informed by ‘quasi-ethics‘, it becomes necessary to ask: What are the similarities and differences between the two identified quasi-ethics? Moreover, what consequences can be drawn for our project of an ethics of mathematical application?

5 Concluding Remarks

The semantics we have unveiled have tried to establish mathematical application as morally good. This accounts both for the ‘quasi-ethics’ that informs mathematical application in school mathematics on the one hand, and the ‘quasi-ethics’ supporting mathematical application in the sphere of the social beyond schooling on the other hand. However, they do so in different ways: In school mathematics, mathematical knowledge is established as morally good by making the application of mathematics have beneficial consequences for the students. Such identification of mathematical application and the morally good tends to constitute an ‘absolutist’ conception of mathematics because ‘true’, universally applicable mathematics remains as *always out of reach* (cf. Sect. 2). In the sphere of the social, the superiority of mathematisation that is applied to solve social problems *is justified by moralisation*. Any non-mathematical approach to the regulation of the social is marked as morally inferior, so that anybody who is not willing to participate in the mathematisation of the social not only rejects a particular way of seeing the world, but seemingly refuses to participate in the global struggle to make the world a better place for everyone. Within this line of argumentation, we immediately rediscover the two attempts to justify moral judgments that we explored above. The ‘quasi-ethics’ of school mathematics retroactively establishes mathematical application as morally good *by means of its practical consequences*, while the quasi-ethics of mathematisation does so *by means of its moral motivation or intention* (and thereby excludes those who are not willing to share these intentions). School mathematics and solutionism thus differ in the terms by which they establish a ‘quasi-ethics’ of mathematical application.

Nevertheless, what the two depicted ‘quasi-ethics’ have in common is that they promote an alignment of mathematical application and the morally good. Further, in doing so, they also share a common blind spot: The ‘quasi-ethics’ become blind to the primordial distinction that opens up the whole field in which they operate. In both cases, this is the distinction between mathematisation and non-mathematisation. In other words, both ethics are ‘quasi-ethics’ because they are blind to the possibility of a non-mathematical approach to the social. Therefore, we can identify *the distinction between mathematisation and non-mathematisation as the extra-mathematical, political foundation of any application of mathematics.*

This allows us to complement our suggestions towards the implementation of moral communication on mathematical application in the teaching and learning of mathematics. It is not only important that the students are not credited when they relativise their constructed models (cf. Sect. 2), but it might be even more important to "strongly reject any conceit, scientific or otherwise, that measurement provides privileged or exclusive access to the real." (Espeland and Stevens 2008 p. 432). This means that the primordial decision, *whether or not* the application of mathematics is a useful way to treat a certain problem, cannot be predetermined in advance and thus be deprived from the responsibility of the students. The distinction between mathematisation and non-mathematisation has to remain *inside the scope of reflection during the entire modelling process in mathematics education*. This means that a reasonable decision *against* a mathematisation of the problem should not be excluded from the space of 'positively credited' communications about a modelling problem. However, this immediately leads to the following question: How is it possible to evaluate a decision against a mathematical approach to a problem as reasonable or non-reasonable?

The only possible answer here is to rely on distinctions that differ to the one between mathematisation and non-mathematisation. For example, the distinction between morally good and morally bad. In other words, the moralisation of the problem at stake is *one* possibility to identify the cases in which it is productive to subordinate the problem to a mathematisation and the cases under which it might not be productive to do so. In the example of Skovsmose's Micro-Society project, students could be promoted when they refute a mathematisation of social welfare benefits, e.g. by arguing for an unconditional basic income as something that is morally good (i.e., a decision based on a moralisation of the situation that can of course be questioned from a deviating moral horizon).[6]

As we have shown above, to identify something as morally good is never a straightforward endeavour and must be (re-)evaluated case-by-case. Moreover, moral communication cannot be an end in itself, that is, it is impossible to argue for moralisation as the ultimate ground of all decisions with regard to the mathematisation of the social. The moral code itself is also only *one* distinction amongst others which means that we have only shifted the problem of justification. The moral code is a distinction that can inform the primordial distinction between mathematisation and non–mathematisation that is inscribed into any particular application of mathematics. At the same time, it is *unable to limit its own scope of application*. It is precisely here that we enter the field of ethics as a theoretical reflection of the conditions of moral communication on mathematical application. Furthermore, it was one of the identified key challenges to an ethics of

[6]Rejecting mathematisation *as such* is an identification of non-mathematisation as good (and mathematisation as bad) which is structurally completely equivalent to solutionism. However, in a semantic environment in which mathematisation and quantification have become the one and only legitimate sources for moral judgment, even rejecting mathematisation *as such* becomes a political act At least it yields the possibility to argue outside mathematisation (a similar argument would account for a semantic environment governed by "anti-mathematisation").

mathematical application that it has to *warn against moralisation and thus provide orientation to decide under which conditions it is productive to evaluate mathematical applications from a moral standpoint and under which conditions it is not*.

This warning function, we claim, can be particularly productive for those approaches within mathematics education that seek to re-politicise school mathematics, e.g. critical mathematics education. An ethics of mathematical application could be employed as a means for reflexively controlling the necessary moralisation that inevitably comes along with a re-politicisation. In this way, an ethics could serve as a 'reflective warning system' that prevents undesired coalitions with solutionism and thus helps to recover "the meaning of 'critique' in critical mathematics education" (so the title of: Pais et al. 2012).

An ethics developed in this spirit should by no means be confused with a mere relativism in the sense that we simply cannot make any decisions at all but sensitise for the hypothesis that any attempt to identify *rejection and acceptance values of moral communication on mathematical application* can only be grounded in the social conditions of late-modernity itself which need to be investigated from a semantical *as well as* structural perspective. In this chapter, we tried to exemplarily reconstruct the semantics that inform about moral communications on mathematical application in the field of school mathematics and the field of the social. Further, we presented a theoretically, yet rather naïve, analysis of the ideology of solutionism. Recent developments of ideology critique in the field of mathematics education (e.g. Pais 2017; Lundin and Storck Christensen 2017; Straehler-Pohl 2017) appear to provide a profound analytical frame for this purpose. So far, we paid almost no attention to the very specific social structure of late-modern societies that must be considered as well in order to systematically develop an ethics of mathematical application. Such an analysis could inform about moral communication on mathematical application in (critical) mathematics education.

Therefore, we conclude in suggesting firstly, developing an ideology critique of the semantics of solutionism as a desideratum; and secondly, a further development of our outlined framework of an ethics of mathematical application that takes into account both the semantics and social structure of the contemporary late-modern society.

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