Integrating the Sociocultural and the Sociopolitical in Mathematics Education

Murad Jurdak

Abstract The purpose of this chapter is to seek an integration of the sociocultural and sociopolitical perspectives in mathematics education by integrating a locally attuned version of Bourdieu's field theory (Ferrare & Apple in Camb J Educ 45 (1):43-59, 2015) and activity system (Engeström in Learning by expanding: an activity-theoretical approach to developmental research. New York: Cambridge University Press, 2015) to disrupt the separate development of the two perspectives. I combine the two theories using modular integration. Next, the chapter discusses the implications of this integration to mathematics education research, practice, and policies. I conclude with a personal narrative on my theoretical journey to sociopolitical mathematics education.

Keywords Activity theory · Bourdieu field theory · Mathematics education
Sociocultural · Sociopolitical · Integration

1 Introduction

Up to the 1960s, the social dimension of the mathematics education discourse had witnessed a recurring tension between school mathematics for social utility and school mathematics for the intellectual development of the individual. The sixties of the past century represented the climax of a movement that considered school mathematics as a cornerstone for not only the intellectual development of the students but as a necessary tool for student academic progression and as a basis for science and technology and hence for the socioeconomic development of countries.

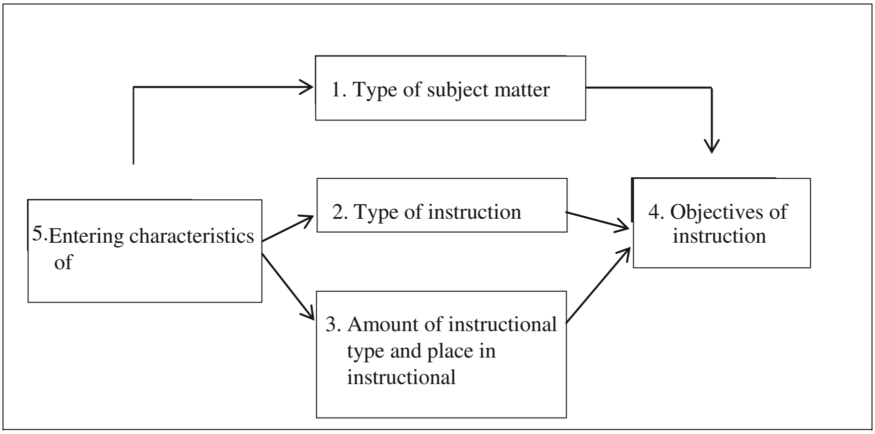
One example, that of Shulman (1970), reflects the dominant conception of teaching mathematics in the sixties of the last century. Shulman's model (Fig. 1) featured teaching as a one-directional system in which the input (entering

M. Jurdak (✉)
American University of Beirut, Beirut, Lebanon
e-mail: jurdak@aub.edu.lb

M. Jurdak and R. Vithal (eds.), *Sociopolitical Dimensions of Mathematics Education*, ICME-13 Monographs, https://doi.org/10.1007/978-3-319-72610-6_2

characteristics of learners) is to produce output (objectives of instruction) through the process of instruction. The emphasis on subject matter and teaching are obvious from the examples given in Fig. 1. Also, the interaction among "type of subject matter," "type of instruction," and "amount and sequence of instruction" is not made explicit in this system. Notably, no mention is made of the role of the broader socioeconomic and cultural context of teaching and learning.

By the 1980s mathematics education witnessed what Lerman (2000) termed a *social turn* in mathematics education to refer to a paradigm shift in the conception of teaching and learning by stipulating that learning and teaching are products of social activity. The first phase of the social turn recognized the social and school material contexts as core components of instruction. It considered the context as a "given," which may constrain or support the components of the system contextual dimension to teaching. However, the social turn did not challenge the one-directional system which assumes that the process of instruction acts on the input (entering characteristics of learners) to produce the output (instructional objectives). According to Cobb (2006), the social turn was characterized by approaches, which "accounted for learning in terms of internal cognitive processes, but acknowledge that cognition is influenced by social interactions with others and, to a lesser extent, by the tools that people use to accomplish goals" (p. 189). Vygotsky's assertion (1978) that cultural products, like language and other symbolic systems, mediate thought marked a shift in psychology from treating cognition and culture as separable to the view that "cognition and culture are no longer regarded as divisible" (Lucariello 1995, p. 1).



1 Mathematics, foreign languages, social studies (subject matter is defined in task terms)
2 Expository-discovery (degree of guidance); inductive-deductive
3 Number of minutes or hours of instruction; position in sequence of instructional types
4 Products; processes; attitudes; self-perceptions
5 Prior knowledge; aptitude; cognitive style; values

Fig. 1 Theoretical generalization about the nature of instruction (Shulman 1970, p. 63)

Later, Lerman (2006) introduced the term *strong social turn* to refer to a theoretical trend that argued for "the situatedness of knowledge, of schooling as social production and reproduction, and of the development of identity (or identities) as always implicated in learning" (p. 172). The strong sociocultural approaches followed Vygotsky's argument that social and cultural processes do not merely condition internal cognitive processes, but rather form learners' minds as they engage in social and cultural practices. Skovsmose and Greer (2012) describe the social turn as "manifesting the humanization (a word frequently used by Freire) of mathematics and mathematics education, encapsulated in the phrase "mathematics as a human activity" (p. 4).

In contrast to the sociocultural developments, Gutiérrez (2013) used the term *sociopolitical turn* to signal "the shift in theoretical perspectives that see knowledge, power, and identity as interwoven and arising from (and constituted within) social discourses" (p. 40). The political turn in education was ushered by the Marxist idea that the production and transmission of knowledge serve the interests of the ruling class that controls the means of material production. Until the eighties of the past century, the prevailing idea was that mathematics learning is universal and does not lend itself to the Marxist idea. The political dimension of mathematics education grew out of the realization that mathematics, as enacted in schools or produced by research, is not immune from the power structure and its distribution in the society. That idea led to a growing awareness that economically and socially advantaged groups created and maintained a schooling system that systematically favors students with privileged backgrounds by adopting a seemingly class-neutral education. The perception of mathematics as a formal language capable of imparting any meaning, promoted the idea that mathematics may act as an instrument for exercising power by imposing meanings on students. Two of the leading sociopolitical theories in general education included emancipatory education (Freire 1970/2013) and Bourdieu's field theory (Bourdieu and Passeron 1990). Skovsmose's (2011) critical mathematics education was one of the leading sociopolitical in mathematics education. The political in mathematics education was described by Skovsmose and Greer (2012) as (re)humanizing mathematics and mathematics education, which "are inextricably political activities" (p. 4).

The literature indicates that the sociocultural and the sociopolitical perspectives in mathematics education have been generally developing along separate paths. Following a mathematics education review, Pais (2012) concludes that exclusion and inequity "within mathematics education, and education in general, are integrative parts of schooling and cannot be conceptualised without understanding the relation between scholarised education and capitalism as the dominant mode of social formation" (p. 51). By pushing this argument to its extreme, he arrives at a deadlock which states "that exclusion is something inherent to the school system we realise that to end exclusion means to end schooling as we know it" (p. 82). My premise is that there are theoretical tools to disrupt the separate development of the sociocultural and the sociopolitical perspectives in mathematics education.

The purpose of this chapter is to disrupt the separate development of the sociocultural and the sociopolitical perspectives in mathematics education by integrating a locally attuned version of Bourdieu's field theory (Ferrare and Apple 2015) and Engeström's (2015) activity system. I use a modular integration approach (Markovsky et al. 2008) to bring together the two theories. Modular integration "treats two or more theories as integral modular components that can be used separately or jointly, as needed, much like different modularized electronic components can be used either alone or together in an integrated circuit for specific applications" (Kalkhoff et al. 2010, p. 3). This integration brings the two theories together to construct a common theoretical foundation for sociopolitical mathematics education. Each theory by itself does not explicitly explain the complexity and interaction of social and political dimensions of mathematics education. On the one hand, Bourdieu's field theory, which views the process of education from the perspective of power through cultural reproduction, does not address education as a socialization/acculturation developmental process. On the other hand, CHAT, which views education as a developmental collective purposeful activity embedded in a sociocultural context, is silent on the issue of power in the educational field.

The rationale for seeking an integration of the separate development of the sociocultural and the sociopolitical perspectives in mathematics education by integrating a locally attuned version of Bourdieu's field theory (Ferrare and Apple 2015) and Engeström's (2015) activity system is to disrupt the separate development of the two perspectives. On one hand, sociocultural theories view school mathematics education as a collective human activity whose purpose is to engage students in socially and culturally relevant mathematical experiences. On the other hand, sociopolitical theories propose that in school mathematics education, advantaged social groups have the opportunity to exchange and disguise their possession of different forms of power to dictate policies, practices, and beliefs in mathematics education institutions and actors, thus reproducing inequities and exclusions in mathematics education. The chapter proposes a theoretical way to disrupt the separate development of the sociocultural and the sociopolitical in mathematics education by integrating the two theories of a locally attuned version of Bourdieu's field theory (Ferrare and Apple 2015) and Engeström's (2015) activity system. The integration of the two theories is intended to provide support to the idea that mathematics education is valued as a socially and culturally enterprise and simultaneously may help mitigate the impact of power in mathematics education by providing "a space to interrupt the arbitrary and inequitable valuation of certain cultural forms over others" (Ferrare and Apple 2015, p. 54).

In what it follows, I frame this chapter within my own epistemological interests and preferences. The focus of this chapter is, by design, mathematics education whose object is the learning and teaching mathematics, mainly in school context. My choice of activity theory as one candidate for a possible integrated sociopolitical mathematics education theory is based on my epistemological orientation as reflected in my research work. My interest in Bourdieu's field theory, which is rather recent, is motivated by a desire to intensify my engagement with the political dimension of mathematics education.

2 Engeström's Cultural Historical Activity Theory (CHAT) and Mathematics Education

According to Engeström (2001), CHAT developed in three stages: basic individual human activity (Vygotsky and Leont'ev), collective human activity (Leont'ev and Engeström), and interacting activity system (Engeström). The first generation was ushered by Vygotsky's (1978) idea of cultural mediation of actions, in which the conditioned direct connection between stimulus (S) and response (R) is transcended by "a complex mediated act". Vygotsky's idea of cultural mediation of actions is commonly represented as a triangle with subject, object, and mediating artifact as vertices (Fig. 2)

2.1 Individual Human Activity

According to Leont'ev (1978, 1981), the individual human activity involves a person or group of persons who engage in an action, using tools (artifacts), to achieve an outcome embodying the intended goal. Leont'ev distinguishes three activity-related concepts: activity, actions, and operations—and he relates these concepts respectively to the motives, goals, and conditions under which the activity occurs.

The starting point in any activity is that the person who engages in the activity should have a motive, without which the activity fails to initiate. According to Leont'ev (1978), "unmotivated" activity is an activity in which the motive is not subjectively and objectively explicit. The motive is concretely translated into a possible achievable goal. The desire to achieve this goal generates actions, which are not random but subordinated to a conscious purpose on the part of the person engaged in the activity. Just as the concept of activity is subordinate to the concept of motive, the concept of action is subordinate to the concept of goal. The artifacts (material and symbolic tools) that are accessible under the objective conditions of the specific social-cultural context mediate the actions.

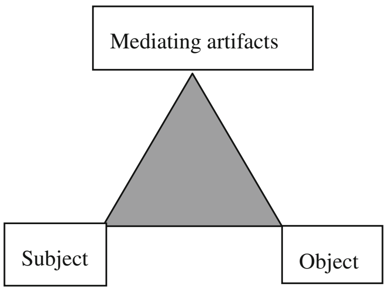


Fig. 2 Individual human activity

The activity of learning mathematics in school is an exemplar of individual activity. The learner, motivated to learn mathematical competencies and concepts, takes actions to achieve the intended mathematical goal, using operations, mediated and constrained by the accessible material and symbolic artifacts that exist in the objective conditions of the social-cultural context of the school.

A core premise of activity theory is the centrality of the learner's agency in the learning activity. For the learning activity to start, the activity goal has to be meaningful to the learner to evoke engagement in taking action toward realizing the goal. The choice of actions as well as the artifacts is contingent on the learner's choices and consciousness of the potential effectiveness of the actions in realizing the intended goal.

2.2 Collective Human Activity

The second generation was ushered by Leont'ev (1981) who expanded the concept of individual activity to a collective activity by introducing the element of division of labor as an essential component of collective activity. Engeström (2015) formally introduced and represented the collective activity as an activity system (Fig. 3).

The activity system is a collective purposeful activity in which a subject (or subjects) is engaged to attain an object shared by a community, using mediating artifacts, where responsibilities are assigned collectively among members of the community (division of labor) according to organizational rules and socio-cultural norms (rules). School mathematics is an example of a collective learning activity. Figure 3 represents the activity system of school mathematics.

The activity system of learning mathematics in school is a social space where students are motivated to engage in learning mathematics using appropriate

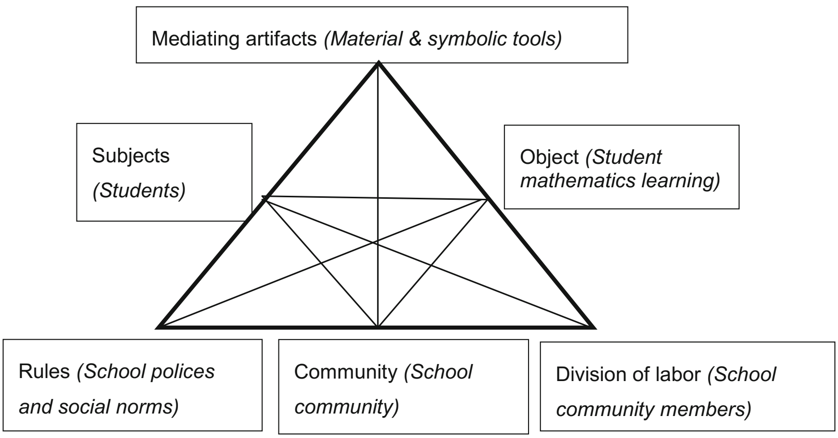


Fig. 3 School mathematics as an activity system

artifacts, which include symbolic tools such as language and mathematics as well as material tools manipulative learning materials and computers. The collective activity introduces the concept of "community" to the individual activity and thus triggers profound consequences to the dynamics of the individual learning activity. The learner in the collective activity does not assume only the identity of an individual but also that of a member of a community, which shares the same object (learning mathematics). Thus, the learner's actions and interactions are not simply individual behaviors linked to individual motives, but also moderated by the motives, actions, and interactions of the community members.

The mediating artifacts available to the learner are also communal cultural objects closely linked to the historical-cultural development of the school system. The existence of the community calls for the need for "division of labor" among the community members (student, teachers and staff, parents), and for 'rules' to govern the actions and interactions within the collective activity; regulatory rules that are explicit and public, while the social and cultural norms of the broader community of the school that are implicit and invisible.

2.3 The Role of Culture in the Activity System

Culture plays a central role in shaping the social space of school mathematics education. The school community in the activity system of school mathematics is a microcosm of the broader community, which the school serves. The social and cultural carryover from the broader community to the school community seem to affect all the nodes of the activity system of school mathematics. The process of acculturation, i.e., acquiring and appropriating the culture of the community, affects the pedagogic mode of acculturation of school mathematics. According to Jurdak (2016a), an acculturation mode of *transmission* views the pedagogic mode of teaching mainly as deficit filling. The *participation* mode of acculturation, however, views the pedagogic mode of teaching as constructing and negotiating learners' meanings. The *inculcatio*n mode views the pedagogic mode of teaching mainly as 'imposing' knowledge and values on students. The *mediating artifacts,* particularly the symbolic tools such as language and mathematical practices, used in learning and teaching mathematics are also cultural tools that belong to cultural contexts of the broader community. The *rules* of the activity system of school mathematics, which includes cultural and social customs and traditions of the broader school community, tend to mediate the actions of students in an invisible way. The *division of labor* in the activity system of school mathematics involves the assignment of roles of students, teachers and administrator in the teaching learning process. The division of labor in school mathematics reflects also the communal beliefs regarding authority and individual agency. Such beliefs range from a total respect to authority to a recognition of free individual agency. The object of the activity system is not only an individual goal but also a communal object, determined by the broader community in the form of a curriculum or standards.

2.4 Mathematics Education Research and CHAT

During the last two decades, the use of activity theory in mathematics education research has intensified. Almost all studies focused on the impact of sociocultural contexts on mathematics learning with little engagement in the role of power in such contexts. Stone and Gutiérrez (2007) conducted a study involving an instructional design in which multi-aged participants who are members of various cultural communities are encouraged to coordinate their efforts on educational tasks. The study examined how the multiple activity systems of the undergraduate course and the school and university communities, all organized around cultural-historical activity theories of learning and development; promote learning among undergraduate and elementary school children. The authors concluded that the way individuals interpret and articulate problems, the mediation strategies they use, and how they define and negotiate their roles and responsibilities for knowledge production relate to their local community and its history of practices. Jurdak (2006) contrasted theoretically and empirically the problem solving of situated problems in school and the real world. Thirty-one last year high school students in the scientific stream solved three potentially experiential problem tasks. The results indicated that there are fundamental identifiable differences among the activities and the activity systems of problem solving in the real world, situated, and school contexts. Jurdak and Shahin (2001) compared and contrasted the nature of spatial reasoning by practitioners (plumbers) in the workplace and students in the school setting while constructing solids with given specifications, from plane surfaces. The results of this study confirmed the power of activity theory and its methodology in identifying and explaining differences between the two activities in the two different cultural settings.

In general, sociocultural mathematics education research focused mainly on understanding the learning of mathematics in different sociocultural contexts. However, a close examination of the details of these studies show a little engagement with the role of power in influencing the learning of mathematics in different cultural settings.

3 Bourdieu's Theory and the Political Dimension of Mathematics Education

The ICME-13 Topical Survey on *Social and Political Dimensions of Mathematics Education, Current Thinking* (Jurdak et al. 2016) identified a number of factors that may account for inequities in access and distribution of mathematics education. Socio-economic include socioeconomic status, ethnicity, gender, material conditions within which mathematics education takes places, and nature of a society's economic structure. Mathematical factors include the nature of mathematics as a discipline and mathematics as a regime of truth. Ideological factors include ideologies/philosophical underpinnings underlying state policies and practices of actors in mathematics education. One thesis of this chapter is that the power construct in Bourdieu's field theory may provide an explanatory framework to account for inequities and student marginalization in mathematics education.

3.1 Bourdieu's Construct of Power

Bourdieu's construct of power rests on a complex interplay of the concepts of field, habitus, and capital. A ***field*** is a social network or configuration, which has an object and exists in specific location. It consists of a structured space of positions and a space of positions-takings (Bourdieu 1993). According to Bourdieu, a field is not only a social field but also a *field of power* since the state of the relations between positions in the field is the result of agents striving—intentionally and unintentionally—for goods and resources (i.e. capital) that are specific to the field. ***Habitus*** is a complex interplay between *individual internalization of past socializations and those of present* (Bourdieu 1990). A social agent, not only acts on current circumstances, but also internalizes them to become another layer to add to those from earlier socializations. Habitus undergoes continual restructuring. ***Cultural capital*** is the product of education and refers to forms of knowledge, skills, education and academic credentials, etc. ***Social capital*** refers to resources based on group membership. ***Economic capital*** *refers* to material wealth and time, which can be cashed in any part of society. ***Symbolic capital*** is nothing other than capital, in whatever form, when perceived by an agent, without questioning the basis of its existence and basis of that capital, as evident and legitimate.

The essence of Bourdieu's construct of power is exchanging capital from one form to another and disguising that in the form of symbolic capital. For example, those who have the advantage to exchange and disguise their possession of economic capital, and subsequently cultural or social capital, in the form of symbolic capital, that is honor or prestige, assume the power to dictate systems of meaning on those who do not have that advantage. Applied to mathematics education, Bourdieu's construct of power proposes that advantaged social groups have the opportunity to exchange and disguise their possession of economic, cultural or social capital, in the form of symbolic capital (honor or prestige), assume the power to dictate policies, practices, and beliefs on mathematics education institutions and actors. Bourdieu's construct of power may account for the inequity and marginalization in mathematics education attributed to socioeconomic factors. Economically advantaged social groups may exchange and disguise their possession of economic capital to impact policies (curriculum, for example), practices (use of technology), and beliefs (student self-concept). Socially advantaged social groups such as male-dominated or ethnic majority dominated social groups may similarly exchange and disguise their possession of social and or cultural capital to impact policies (racial-based school compositions), practices (gender discrimination), and beliefs (student alienation). The state may exchange and disguise its possession (sanctioned by legitimate popular mandate) of economic, social, and cultural capital, to enact ideologically motivated policies that may disadvantage certain social groups. Mathematics educators may exchange and disguise their possession of their cultural capital (knowledge of mathematics and its pedagogy) to impose mathematical meanings that may exclude certain groups of students.

3.2 Mathematic Education Research Using Bourdieu's Theory

Recently, a few research studies have addressed the role of power in mathematics education, using Bourdieu's filed theory. All these studies focused on the power rather than the sociocultural dimension. For example, Jorgensen et al. (2014) examined two children's mathematical learning trajectories to highlight how school mathematics practices allow greater or lesser access to school mathematics depending on the cultural backgrounds and dispositions of the learners. Their findings were consistent with Bourdieu's original field theory. Nolan (2016), using Bourdieu's social field theory, explored discourses of school mathematics classrooms as experienced by two novice secondary mathematics teachers. The data reveal that the ways in which the two novice mathematics teachers carefully negotiate space for enacting agency amid school social structures, are consistent with Bourdieu's social field theory in that the social structures of a field both "constrain and (re)produce the becoming teacher" (p. 328). However, Nolan noted that the letters from the field written by the two novice teachers during the study provided discourses which competed with the discourses offered by their teacher educators/researchers in their teacher education program.

In general, Bourdieu-inspired mathematics education research attempted to establish that even mathematics education is not immune from the impact of power that enable advantaged groups to use their possession of capital to reproduce existing inequities in mathematics learning. However, these studies show little engagement with the role of sociocultural factors in shaping mathematics learning.

4 A Locally Attuned Version of Bourdieu's Field Theory and Mathematics Education

On its surface, Bourdieu's field theory may lead to the pessimistic conclusion that education is bound to reproduce existing inequities because it considers culture a carrier of capital through the accumulated internalized socializations of the habitus of the social agent. Ferrare and Apple (2015) suggest that Bourdieu's field theory needs not arrive at that conclusion and propose an elaboration of Bourdieu's field theory to be more attentive to understanding of how actors construct experience and struggle over meanings in local contexts such as individual schools and universities.

In the following paragraphs, I first introduce Ferrare and Apple's (2015) locally attuned version of Bourdieu's field theory followed by my interpretation of this theory as it applies to mathematics education.

Ferrare and Apple (2015) argue that "the most important problem inherited from Bourdieu's field theory stems from his disinclination—shared by many in sociology—to venture into the realm of individual perception and experience" (p. 45). They argue that:

> Bourdieu's primary emphasis on the macro view of cultural fields obscures an understanding of how educational actors directly experience and make sense of the pedagogic qualities—what we will later call 'affordances'–inherent in local field positions, practices and meanings.
>
> (p. 45)

To build upon the problem inherited from Bourdieu's field theory, Ferrare and Apple have drawn upon insights from social psychological field theory, ecological psychology and in relational sociology.

> The first insight suggests the need to consider a greater degree of complexity in the mental structures constituting the individual (or group) life space. Our second point makes the case for extending the affective dimension of field theory to include the values that inhere in objects, not just an individual's habitus. Finally, we suggested that it is important to consider fields from a phenomenological perspective, which means that we must be attentive to the local institutional positions in which actors experience their day-to-day lives. (p. 52)

According to Ferrare and Apple (2015), Bourdieu constructed his version of field theory "in which social actors experience fields as both arenas of force and arenas of struggle". (p. 48). They note that Bourdieu emphasized and developed fields more as arenas of force than that of struggle. Social actors experience fields as an arena of force in the sense that fields are rules that "direct normative values, regulate actions, reward ontological complicity, and place sanctions on transgressors" (p. 48). Social actors experience fields as an arena of struggle in the sense that actors who possess "an advanced feel for the game (i.e. a *habitus attuned to the situation*), have more opportunities to adapt and improvise strategies to achieve success in the field. It is this feel for the game that enables some actors the freedom to know when to take risks—to engage in subversion strategies—and when to 'dig in' and fight to conserve the present rules of engagement" (p. 48) [*italics are mine*].

Based on these insights, Ferrare and Apple (2015) proposed a locally attuned version of field theory that extends the concept of habitus to better account for the information that inheres in local field positions in educational contexts as well as recognizes how students perceive this information as constraints and affordances related to their educational experiences, goals and aspirations. This locally attuned version "shifts the deficit from the individual to the field itself, and provides a space to interrupt the arbitrary and inequitable valuation of certain cultural forms over others" (p. 54).

The implications of the locally attuned version of Bourdieu's field theory to school mathematics are far-reaching. Historically, the dominant public view of school mathematics is that mathematical ability is genetically endowed or culturally inherited. The locally attuned theory challenges that view by enabling the habitus of the individual student to interrupt the complicity between the advantaged groups and school mathematics practices. This theory shifts the deficit in mathematics learning from the individual student to what is lacking in terms of democratization of school mathematics education. By doing so, students are provided with more opportunities to best use their own perceptions of the local constraints and affordances related to their experiences, goals and aspirations to adapt and improvise strategies to achieve success in mathematics learning.

5 Modular Integration of the Locally Attuned Version of Bourdieu's Theory and Engeström's Activity System

This is not the first time that CHAT's tradition and Bourdieu's field theory are proposed to be combined in one theory. Williams (2012) proposed to extend CHAT to incorporate Bourdieu's sociology in order to bring together theories in "a joint theory of education as both development and re-production of labour power, in which use and exchange value both have their place (in commodity production)" (p. 57). This extension can incorporate the "CHAT perspective on the 'cultural development of the mind' to "the use of mathematics as a tool for the critical, scientific examination of society" (p. 70). This chapter proposes to integrate Bourdieu's field theory, as extended by locally attuned version of Bourdieu's field theory (Ferrare and Apple 2015) and Engeström's activity system, in order to seek a common foundation to sociopolitical mathematics education.

The modular integration connects existing theories without replacing them or subsuming one within the other. "The original theories are treated as modules, pulled off the shelf and plugged into one another as needed, and still available in their un-integrated form for use in other integrations. This is deeply analogous to the integrated circuit in electronics" (Markovsky et al. 2008, p. 347).

5.1 The Two Theories as Candidates of Modular Integration

At a practical level, modular integration means that two integrated theories, plugged into one another, are able to provide better explanations than they could individually. According to Kalkhoff et al. (2010), to be candidates for modular integration, theories should be coherent and clear, empirically supported, and share concepts and constructs by which they can be integrated. Bourdieu's field theory and its locally attuned version (Ferrare and Apple 2015) as well as Engeström's activity system are well-established theories as explained earlier. These two theories are empirically supported in the area of mathematics education (see Sects. 2.4 and 3.2). Next, we explore the common concepts and constructs between the two theories.

Both CHAT and Bourdieu's locally attuned version share the concepts of *culture* and *history*. CHAT assumes that the understanding of human activity can only be within the communal collective meanings of the cultural context in which the activity is enacted. In Bourdieu's field theory, culture plays a pivotal role by being a carrier of capital through the internalized socializations of the habitus of the social agent. CHAT and Bourdieu's field theory posit that the *historical* dimension is an indispensable ingredient of their theoretical foundation. According to Roth et al. (2012), CHAT assumes that human activity occurs in a historical context in the sense that the historical context contextualizes the activity itself. Therefore, what we are observing today is different from what we might have observed in the same context ten years earlier. According to Bourdieu (1990), habitus refers to individual history. Habitus undergoes continual restructuring. The habitus acquired in the family is at the basis of the structuring of school experiences and the habitus acquired in school is in turn at the basis of all subsequent experiences. Bourdieu's focus on the agency of the social actors, whose habitus is more attuned to the situation in their local contexts, was obscured by his tendency to focus more on the structure of macro-level field positions. Ferrare and Apple (2015) contribution is in their attempt to bring into prominence an under-developed concept of Bourdieu i.e. the nuanced "understanding of how actors construct, experience and struggle over meanings in local contexts such as individual schools and universities" (p. 45).

5.2 Applying Modular Integration to the Two Theories

According to Markovsky et al. (2008), the "connecting threads link theoretical elements" (p. 348) hold together theories that form the fabric of knowledge. Some of those threads are terms, definitions, propositions, arguments, and scope conditions. The two theories under consideration have many terms in common such as culture and history. However, the definitions of those terms are different in the two theories and hence are not good candidates to be 'connecting threads'. On the other hand, 'agency of the social actor' and 'local contexts' are two terms which have similar definitions in both theories. In activity theory, the choice of actions as well as the artifacts is contingent on the learner's choices and consciousness of the potential effectiveness of the actions in realizing the intended goal. Moreover, local temporality is involved in an activity (Roth et al. 2012). In the locally attuned version of Bourdieu's theory, social actors experience fields as an arena of struggle in the sense that actors who possess "an advanced feel for the game (i.e. a habitus attuned to the situation), have more opportunities to adapt and improvise strategies to achieve success in the field (p. 48).

In summary the agency of the social actor and the local contexts are the two connecting threads that bring the two theories together. The integrated theory may account for both the social and political dimensions of mathematics education.

Specifically, on the one hand, it provides a lens that enable mathematics educators to see 'both sides of the coin' of sociopolitical mathematics education: The integrated theory supports the view of school mathematics as both a social cultural activity system as well as a field of force and of struggle. On the other hand, the integrated theory provides a perspective that may open a window to avoid the deadlock that arises from the assumption that exclusion in school mathematics is something inherent to the school system and to end school mathematics, as we know it (Pais 2012). Because it shifts attention "toward the qualities that inhere directly in school-level social structures, practices and meanings", the locally attuned version of Bourdieu's field theory (Ferrare and Apple 2015) shifts the deficit from the individual to the field itself, and provides a space for the social actor to interrupt exclusion and marginalization in the local context.

6 Implications of the Integrated Theory

6.1 Implication for Mathematics Education Research

One theoretical implication of the proposed integrated theory is the shift in the focus of mathematics education research. The locally attuned module (Ferrare and Apple 2015) in the integrated theory calls for focusing on the structures of practice and meaning of mathematics education in local educational contexts (e.g. schools), in order to understand how "variations in local social configurations can meaningfully shape the ways that students perceive and interpret constraints and affordances that inhere in educational settings" (p. 53). This focus is not alien to mathematics education research. In fact, this kind of focus appeared in two highly-cited studies done at the beginning of this century by Vithal (2003) in South Africa and by Gutstein (2006) in the United States. Vithal investigated what happened in a mathematics classroom in local contexts when student teachers attempted to use a social, cultural, and political approach which integrates a critical perspective to the school mathematics curriculum in post-apartheid South Africa. Gustein's pedagogical goal was to create conditions for students to develop agency in a middle-school mathematics classroom in a Chicago public school in a Latino community. Both studies reported some short-term gains and a number of challenges embedded in the system itself. Both studies used the social and political dimensions in their studies. What is surprising is that both studies aimed at what Ferrare and Apple (2015) called for, i.e. understanding how the local institutional social configurations "shape the ways that students perceive and interpret constraints and affordances that inhere in the educational settings" (p. 53). Unfortunately, this line of research did not continue for a variety of reasons. Based on the proposed integrated theory, I support and call for reviving sociopolitical mathematics education research that focuses on the structures of practice and meaning of school mathematics education in local educational contexts in order to understand how students perceive the constraints and affordances in those contexts. The second theoretical implication is the need to have a research paradigm whose purpose is to understand how social school structures shape students' interpretation of mathematics education in terms of the constraints and affordances inherent in local contexts. The research paradigm that best serves this purpose would include a thick understanding of experience and meaning of the social actors. One challenge for the integrated theory is its ability to account for phenomena that its modular components cannot adequately explain. One such phenomenon is the many mathematics educators who were able to achieve academic success (the author is one of them) despite the fact that they were disadvantaged in terms of economic, social, or cultural capital. Bourdieu's field theory would not be able to account for such a phenomenon. Again, activity theory has little to contribute in this regard. From the perspective of the integrated theory, this phenomenon is amenable to be studied through case studies of such disadvantaged individuals in terms of their interpretation of the constraints and affordances in their local educational contexts to beat the system as a field of force.

6.2 Implications for School Mathematics Practices

An implication of the integrated theory is that it politicizes the mathematical practices in the activity system of school mathematics. The concept of the division of labor in the activity system is neutral in the activity system. From a political perspective, division of labor is essentially a political act in terms who and how the division of labor is done. In school mathematics classroom, division of labor practices are normally characterized by teacher domination on the pretext that the teacher possesses, compared to students, superior knowledge of mathematics (cultural capital), and hence has the option to impose own meanings as legitimate and necessary for student success. A division of labor in which the role of teacher moves away from a definer of mathematical meanings to an arbiter of student meanings, enable students to experience and make sense of mathematics as it relates to their experiences may interrupt the imposition of meanings by teacher. Another set of practices in activity theory relate to access and distribution of *mediating artifacts*. The activity theory is silent on the role of power differential associated with the inequities that might arise from differential access and distribution of '*mediating artifacts*' i.e. the material and symbolic tools for learning and teaching school mathematics. For example, in schools, which use digital technologies for teaching and learning mathematics, students from higher socioeconomic families, compared to students from lower socioeconomic families, are more likely to have more exposure and higher technological literacy in using those tools. Student agency to interrupt inequities in this case may not be enough to interrupt the existing inequities, and consequently the school itself may have to be more responsive and adaptive to the needs of disadvantaged students. *Symbolic mediating artifacts* play a subtle role as possible instruments of exclusion and marginalization in the learning and teaching of mathematics. For example, competencies in language and mathematics, which mediate the learning and teaching school mathematics, are culturally constituted and ingrained in the habitus, and therefore, student agency would not be in a favorable position to interrupt the inequities arising from the complicity between the language and mathematical practices in school and those of the dominant groups. In conclusion, the inequities arising from the differential access and distribution of the practices espoused by activity theory are political in nature and hence need to be addressed in the political arena.

6.3 Implications for School Mathematics Policies

One implication of the integrated framework is that the interruption of inequities in mathematics education at the school level requires favorable policies and practices at the system level. According to Jurdak (2011), from an activity theoretic perspective, mathematics education at the national level is a complex nested hierarchical 3-layer activity system: The classroom, school, and the state system. Because each system nests within the next higher one, the societal relationships of power of a higher system carry over to the lower systems and eventually to the student at classroom level. Thus, the existence of favorable policies and practices at the system level is necessary for the successful implementation of the integrated theory at the school level. The mathematics curriculum is an example of a system policy that might constrain the ability to interrupt inequality because of its structure and orientation. A mathematics curriculum that does not provide enough space for students to perceive mathematics learning, enacted in the locality of the school, as an affordance related to their educational experiences, goals and aspirations, would not be favorable to enabling students make sense of the opportunities in local positions, practices, and meanings. The proposed integrated theory shifts attention toward the qualities that inhere directly in school-level social structures, practices and meanings. This shifts focus from asking what students are lacking to what is lacking from the social structures and cultural models of schools. Thus, the practical goal of attempting to democratize access and distribution of school mathematics education leads to one of “democratizing the very construction of the entire space of positions and cultural forms constituting schools (Ferrare and Apple 2015, p. 46). As researchers and mathematics educators, we have little or no leverage when it comes to policy-making. However, we have the ethical responsibility to struggle for restructuring and democratization of the social structures of our school systems.

7 A Personal Narrative on My Theoretical Journey to Sociopolitical Mathematics Education

In recent years, it has been my conviction and practice to illuminate my presentation of theoretical argumentation by a personal narrative on why and how my experience shaped those ideas and argumentation (Jurdak 2016a). Thus, I conclude this chapter with a personal narrative to tell the story of why and how the idea of linking the theories of Engeström, Bourdieu, and Apple came into being.

At the turn of the last century, while I was engaged in a research project on comparing and contrasting mathematical problem solving in and outside school, I came across activity theory as a possible explanatory model for my research. Activity theory proved to be a powerful theory to explain the differences in problem solving in school and the real world in terms of the differences in their sociocultural contexts. I was so fascinated by activity theory to the point that, for some years, I used it as a lens to make sense of mathematics education (and even all educational) practices.

In 2007, my colleague Saouma BouJaoude, a science educator, and I obtained a grant to start a school-based reform project called TAMAM, an acronym derived from the Arabic title of the project which consists of the initials of "school-based reform" in Arabic (al-Tatweer Al-Mustanid ila Al-Madrasa) and which means "perfect" in colloquial Arabic (address: http://tamamproject.org/). The project aimed to develop a school-based grounded theory of educational reform in the Arab region that would provide policymakers with research-based recommendations for implementing educational reform in their countries. It involved a partnership between the American University of Beirut and nine school teams from three Arab countries. The project included a variety of mediating artifacts such as conferences, action research school-based projects, and reflective practice.

From the beginning, I could conceive of TAMAM as an activity system but, as the project proceeded, I felt that this framework did not capture the complexity of learning that was taking place. Unlike my university students, the TAMAM participants brought with them to their learning in TAMAM, a rich capital of actual experiences, which in many respects exceeded my experiences as a university professor. At this point, I started to believe that activity theory was not enough to explain TAMAM learning mostly based on reflective practice. Freire's emancipatory ideas came very handy as a powerful tool to explain this kind of complex learning. This was when the narrative came to my mind, and I decided to transform the individual experiences of TAMAM participants into stories written by the individuals themselves and in their own native language (Arabic), which was published as an e-book entitled '*School-based reform (TAMAM): Voices from the field (in Arabic)*' (Jurdak 2016b). In retrospect, my encounter with Freire's ideas was my first engagement with the 'soft' political aspects of mathematics education. However, the professional development, which helped transform TAMAM participants, failed to 'reform' their respective schools. Neither activity theory nor Freire's emancipatory education could provide a convincing explanation for TAMAM success in the 'development' of the participants and its failure in achieving the institutional ‘reform’ effort. This discrepancy led me to Bourdieu cultural reproduction theory.

Although different, the nine schools have one thing in common i.e. each school has achieved, through its sociocultural history, a ‘status’ earned through accumulating cultural, social, and economic capital. Schools did not seem to be ready to change (reform) their practices because they wanted to protect their privileged status. Schools maintain their power by reproducing the culture that produced them. However, it was difficult for me to understand the development of the TAMAM participants and the resistance of their institutions without engaging both activity theory and Bourdieu’s cultural reproduction theory. For me then, the development and the resistance were two sides of the same coin. That was the genesis of integrating the two theories. However, I faced a considerable difficulty in finding common linking threads between the two theories, which led me to Ferrare and Apple. That was my frame of thinking when I decided to contribute to this book.

References

Bourdieu, P. (1990). *The logic of practice*. Stanford, CA: Stanford University Press.

Bourdieu, P. (1993). *The field of cultural production: Essays on art and literature*. Cambridge: Polity Press.

Bourdieu, P., & Passeron, J. C. (1990). *Reproduction in education, society and culture* (2nd ed.). London: Sage Publications.

Cobb, P. (2006). Mathematics learning as a social practice. In J. Maasz & W. Schloeglmann (Eds.), *New mathematics education research and practice* (pp. 147–152). Rotterdam: Sense Publishers.

Engeström, Y. (2015). *Learning by expanding: An activity-theoretical approach to developmental research (second edition)*. New York: Cambridge University Press.

Engeström, Y. (2001). Expansive learning at work: Toward an activity theoretical reconceptualization. *Journal of Education and Work, 14*(1), 133–156.

Ferrare, J., & Apple, M. (2015). Field theory and educational practice: Bourdieu and the pedagogic qualities of local field positions in educational contexts. *Cambridge Journal of Education, 45* (1), 43–59.

Freire, P. (1970/2013). *Pedagogy of the oppressed*. New York: Bloomsbury.

Gutiérrez, R. (2013). The socio political turn in mathematics education. *Journal for Research in Mathematics Education, 44*(1), 37–68.

Gutstein, E. (2006). *Reading and writing the world with mathematics: Toward pedagogy for social justice*. New York: Routledge.

Jorgensen, R., Gates, P., & Vanessa Roper, V. (2014). Structural exclusion through school mathematics: Using Bourdieu to understand mathematics as a social practice. *Educational Studies in Mathematics, 87,* 221–239.

Jurdak, M. (2006). Contrasting perspectives and performance of high school students on problem solving in real world, situated, and school contexts. *Educational Studies in Mathematics, 63*(3), 283–301.

Jurdak, M. (2011). Equity in quality mathematics education: A global perspective. In B. Atweh, M. Graven, & W. Secada (Eds.), *Mapping equity and quality in mathematics education* (pp. 131–144). Dordrecht: Springer.

Jurdak, M. (2016a). *Learning and teaching real world problem solving in school mathematics—A multiple-perspective framework for crossing the boundary*. Switzerland: Springer.

Jurdak, M. (Ed.). (2016b). *School-based reform (TAMAM): Voices from the field (in Arabic).* Beirut: Arab Thought Foundation. Available at: http://tamamproject.org/research/tamam-voices-from-the-field/.

Jurdak, M., & Shahin, I. (2001). Problem solving activity in the workplace and the school: The case of constructing solids. *Educational Studies in Mathematics, 47*(3), 297–315.

Jurdak, M., Vithal, R., de Freitas, E., Gates, P., & Kollosche, K. (2016). *Social and political dimensions of mathematics education, ICME-13 topical surveys*. New York: Springer. doi: https://doi.org/10.1007/978-3-319-29655-5_1. Address on line: http://link.springer.com/book/10.1007/978-3-319-29655-5.

Kalkhoff, W., Friedkin, N. E., & Johnsen, E. C. (2010). Status, networks, and opinions: A modular integration of two theories. In *Advances in group processes* (pp. 1–38). Bingley: Emerald Group Publishing Limited.

Leont'ev, A. N. (1978). *Activity, consciousness, and personality*. Englewood Cliffs: Prentice-Hall.

Leont'ev, A. N. (1981). The problem of activity in psychology. In J. Wertsch (Ed.), *The concept of activity in Soviet psychology* (pp. 37–71). Armonk, NY: M. E. Sharpe.

Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (Ed.) *Multiple perspectives on mathematics teaching and learning* (pp. 19–44). Westport, CT: Ablex.

Lerman, S. (2006). Cultural psychology, anthropology and sociology: The developing 'strong' social turn. In J. Maasz & W. Schloeglmann (Eds.), *New mathematics education research and practice* (pp. 171–188). Rotterdam: Sense Publishers.

Lucariello, J. (1995). Mind, culture, person: Elements in a cultural psychology. *Human Development, 38*(1), 2–18.

Markovsky, B., Dilks, L. M., Koch, P., McDonough, S., Triplett, J., & Velasquez, L. (2008). Modularizing and integrating theories of justice. In *Justice* (pp. 345–371). Bingley: Emerald Group Publishing Limited.

Nolan, K. (2016). Schooling novice mathematics teachers on structures and strategies: a Bourdieuian perspective on the role of "others" in classroom practices. *Educational Studies in Mathematics Education, 92,* 315–329.

Pais, A. (2012). A critical approach to equity. In O. Skovsmose & B. Greer (Eds.), *Opening the cage* (pp. 49–91). Netherlands: Sense Publishers.

Roth, W., Radford, L., & La Croix, L. (2012). Working with cultural-historical activity theory. *Forum: Qualitative Social Research, 13*(2), 1–20.

Shulman, L. S. (1970). Psychology and mathematics education. In E. Begle (Ed.), *Mathematics education* (pp. 23–71). Chicago: National Society for the Study of Education.

Skovsmose, O. (2011). *An invitation to critical mathematics education*. Rotterdam: Sense Publishers.

Skovsmose, O., & Greer, B. (2012). *Opening the cage: Critique and politics of mathematics education*. Rotterdam: Sense Publishers.

Stone, L. D., & Gutiérrez, K. D. (2007). Problem articulation and the processes of assistance: An activity theoretic view of mediation in game play. *International Journal of Educational Research, 46*(1), 43–56.

Vithal, R. (2003). *In search of a pedagogy of conflict and dialogue for mathematics education*. Dordrecht, The Netherlands: Kluwer Academic Publishers.

Vygotsky, L. S. (1978). *Mind in society*. Cambridge, MA: Harvard University.

Williams, J. (2012). Use and exchange value in mathematics education: Contemporary CHAT meets Bourdieu's sociology. *Educational Studies in Mathematics, 80*(1/2), 57–72.