Chapter 15
Quantum Non-individuality: Background Concepts and Possibilities

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Introduction

It is not an exaggeration to say that quantum mechanics is at odds with most of our received metaphysical notions. In particular, an alleged revision is brought about by the theory on the metaphysical notion of 'individuality'. Certainly, this should figure as being of great interest for metaphysicians and philosophers of science alike. What makes issues even more interesting is that some of the founding fathers of the theory, with their typical philosophical inclinations, suggested that the entities dealt with by the theory had something different regarding individuality: according to them, quantum entities somehow fail individuality. That situation is clearly distinctive from what happened in classical mechanics, for instance (see French and Krause 2006, chap. 3 for a historical overview).

Having such a request for revision on individuality, however, is not the same as having a new approach to individuality right at hand; the founding fathers expressed the failure of individuality in rather vague terms, claiming that quantum entities had 'lost their identity'. In the context of their discussions, it is clear that their target is the very notion of individuality; however, knowing that something is wrong does not always give us any positive sign on how to fix it. Furthermore, the claim that quantum entities 'lost their identities' is at best a heuristic, that may be articulated in a plurality of distinct ways.

Consider Weyl (1950, p. 241) on the possibility of discerning two electrons:

> …the possibility that one of the identical twins Mike and Ike is in the quantum state E_1 and the other in the quantum state E_2 does not include two differentiable cases which are permuted on permuting Mike and Ike; it is impossible for either of these individuals to retain

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> his identity so that one of them will always be able to say 'I'm Mike' and the other 'I'm Ike'. Even in principle one cannot demand an alibi of an electron!

Thus, electrons are not like people or ordinary objects, of which we could demand an alibi. By 'alibi' we may understand something that would allow us to individuate or, perhaps, to distinguish a particle from other similar items. The idea seems to be that electrons are all just so much alike that nothing discerns them. Schrödinger also remarked that "we cannot mark an electron; we cannot paint it red" (Schrödinger 1964). While a painting over an ant or a twin may well serve as an alibi for its individuality, nothing of the sort works for an electron or another quantum particle. But still, this is not enough for us to determine what is wrong with electrons on what concerns individuality.

Consider Schrödinger again, in another context:

> I beg to emphasize this and I beg you to believe it: it is not a question of our being able to ascertain the identity in some instances and not being able to do so in others. It is beyond doubt that the question of 'sameness', of identity, really and truly has no meaning. (Schrödinger (1996, pp.121-122)).

Schrödinger goes even farther than Weyl, it seems, by claiming that the problem is not the failure of discernibility, but rather that the very idea of identity fails to make sense in some cases. That is, there are situations in which one cannot even say that some objects are the same or different. In the broader context of this sentence, Schrödinger is addressing the issue of identity over time, of whether we may say some entity at a time t_1 is the same as another entity seen at a later time t_2. If we take this quote seriously, then, the claim that quantum particles 'lost their identity' is now to be understood literally. But let us not go so fast.

These quotes are just samples for us to motivate the claim that the idea of a "loss of identity" was indeed widespread among the founding fathers of the theory. This view, that quantum entities somehow lost their identity, was called the *Received View on quantum non-individuality* by French and Krause (2006) (for simplicity, we shall refer to it simply as the RV). What was received was the idea that quantum particles had somehow lost their individuality, that identity does not make sense, or that we cannot always discern those entities. However, as we mentioned, if that slogan is to make sense, we must provide the view with a more detailed and metaphysically articulated development. As a general view, the RV recommends only that quantum particles are different from classical particles on what concern issues of identity and individuality, but does not by itself impose any specific view of identity and individuality that is to be revised. Notice that while Weyl speaks of the lack of an alibi, inducing one to think of a failure of discernibility, Schrödinger speaks of identity making no sense, which could be seen as demanding more profound revisions. The development of the RV, then, may be provided for in a variety of distinct ways, by the clear understanding of the notions of identity and individuality, and their relations. It is to these possibilities that this chapter will be devoted. We shall explore and illustrate how the idea that entities lost their identities may be further clarified and become a workable view of quantum ontology, as described by a possible understanding of quantum mechanics. There is more than one way to do that, but here we shall not enter into the dispute of which of them is preferable, if any.[1]

In order to discuss some of the possibilities open for the metaphysical articulations of the RV, we shall proceed as follows. In the next section we briefly sketch the main reasons related with the claim that quantum mechanics seems to afford a theory of non-individuals: quantum statistics and the permutation symmetry. This will clarify the physics behind the metaphysical developments of the RV. In section "Identity, Individuality, Individuation" we sketch the main concepts to be employed in the metaphysical discussions of the chapter: individuality, individuation, and identity. In section "Schrödinger's Problem" we present one of the most well-known ways to articulate the RV, which is based on a literal understanding of Schrödinger's claim that identity makes no sense for quantum entities. It is the view put forward, for instance, in French and Krause (2006), which can be backed by formal systems of non-reflexive logics. In section "Non-individuals with Identity" we present alternatives to the non-reflexive approach, which may also be candidates to ground a metaphysics that is faithful to the tenets of the RV (although not to Schrödinger's claim that identity makes no sense). We conclude in section "Conclusion".

Quantum Mechanics, Statistics, Permutation, Identity

As we have already mentioned, the RV, as crudely advanced by some of the founding fathers of quantum mechanics, is a very general view that relates identity, individuality, and indiscernibility. Indiscernibility seems to be thought of as one of the main ingredients of the problem; but identity and individuality are also related. In order to untie the knot involving the three concepts involved, we shall begin by presenting the main quantum mechanical facts that led to such considerations.[2] We begin by presenting the classical statistics, whose contrast with quantum case originates the main claims of the RV.

The idea that classical particles are individuals in a very strong sense is famously encapsulated in Maxwell-Boltzmann's statistics. Let us illustrate it with the case in which two particles, labeled 1 and 2, must be distributed in two states, A and B. We have the following four distinct possibilities for such a distribution (where $A(1)$ is an abbreviation for the claim that particle 1 is in state A, and similarly for other cases):

1. $A(1)$ and $A(2)$;
2. $B(1)$ and $B(2)$;

[1] Notice that there is also the option of rejecting the RV and interpreting those entities as individuals; we shall not discuss this option here, but see French and Krause (2006, chap. 4) and French (2015).

[2] We are not here claiming that this understanding of the statistics is not problematic or that it is the only alternative; rather, this is how the RV is typically presented, as a contrast between the classical and the quantum case.

3. $A(1)$ and $B(2)$;
4. $A(2)$ and $B(1)$.

All these possibilities are assigned the same weight, that is, $\frac{1}{4}$.

The fact that permutations do give rise to distinct states, as seen in cases 3 and 4 above, is typically accounted for in terms of the fact that classical particles are individuals. Of course, if such particles were discernible somehow, then their permutation could reasonably be seen as giving rise to distinct states. However, there is a sense in which classical particles may be taken as indiscernibles: classical systems may share all their intrinsic or state independent attributes. Even when this is the case, the classical statistics distinguishes between situations 3 and 4. Then, how to account for such a distinction?

It is here that individuality enters the stage: a permutation gives rise to distinct states precisely because those particles are individuals. There is something accounting for their numerical difference, and making the case that the two situations are different: the particles' individuality. There are many ways to account for such individuality without having to appeal to discernibility by intrinsic properties (which, as we have seen, fails in the classical case). The most typical option appeals to the fact that classical particles have a unique trajectory in space-time once an assumption of impenetrability is adopted. With that, each particle has a unique space-time trajectory, which may be regarded as conferring individuality to it (see French and Krause 2006, chap. 2).

In quantum statistics, on the other hand, permutations of indistinguishable particles are not counted. This gives rise to permutation symmetry and the alleged loss of identity we have been discussing. Usually, the formalism of orthodox QM uses symmetrization postulates: symmetric and anti-symmetric vectors/functions express the lack of identity of particles. For an illustration, let us consider two systems labeled 1 and 2 distributed in two possible states a and b, we can have the following possibilities:

1. $|\psi_1^a\rangle|\psi_2^a\rangle$;
2. $|\psi_1^b\rangle|\psi_2^b\rangle$;
3. $\frac{1}{\sqrt{2}}(|\psi_1^a\rangle|\psi_2^b\rangle \pm |\psi_2^a\rangle|\psi_1^b\rangle)$.

In fact, we have two different kinds os statistics here: Bose-Einstein (BE) for bosons, and Fermi-Dirac (FD) for fermions. The difference comes in the third possibility, bosons have the "+" sign, and fermions have the "−" sign. Also, for fermions only this third possibility obtains, they cannot be distributed according to the first two cases for they cannot be in the same state, they do obey the Pauli Exclusion Principle. Notice that, as in the classical case, to write the vectors we had to label both particles and states. But this does not run counter to the alleged loss of identity? To grant that the labeling on particles has no effect, we use symmetric and anti-symmetric vectors, for bosons and fermions respectively, adding also the Indistinguishability Postulate below (more on reference and labeling in the next section, when we deal with individuation). This is of course a mathematical trick, for what matters for physics is that the expectation value of the measure of any observable $\hat{O}$ for the system in the state $|\psi\rangle$ does not change after a permutation of the labels of the particles. Being P a permutation operator and $|\psi_{12}\rangle$ the vector state for particles 1 and 2, we express this by means of the Indistinguishability Postulate:

$$\langle\psi_{12}|\hat{O}|\psi_{12}\rangle = \langle P\psi_{12}|\hat{O}|P\psi_{12}\rangle$$

The topic of individuality for quantum particles is related to how we understand this postulate. The usual reading, attached to the Received View, regards it as a restriction on the states: there are only symmetric and anti-symmetric states. In this case, only bosons and fermions are possible, and since the operations representing observables always give as a result a vector of the same symmetry type as the one to which it was applied. So, the particles are regarded as non-individuals, nothing can be made to distinguish them, there is nothing there to account for a permutation that could make for a distinct state before and after the permutation. It is this reading of the Indistinguishability Postulate that traditionally underpins the statements advancing that quantum particles loss their identity.

However, it can be argued that this is not the only one reading of the postulate. There is an alternative way of reading it, as imposing a constraint on the observables: only observables commuting with the permutation operators are allowed. In this case, the asymmetric states are not banned, they exist but are inaccessible for particles whose states are represented by vectors of the other symmetry types, since the particles are always in symmetric or anti-symmetric states, and no operator shifts them to some of the asymmetric states. So, in this case, it would be possible, at least in principle, to distinguish the particles (for the distinguishing features would not be observable), and they can be seen as individuals, in some sense (see French and Krause 2006, chap. 4 for these possibilities).

But now comes the question: how can we understand this individuality? In general, it has been argued that the individuality of quantum particles will have to be grounded in some kind of Lockean substratum or non-qualitative *haecceity*. Since particles can be absolutely indistinguishable (and this can be rigorously argued for), it has been argued that the Principle of the Identity of Indiscernibles (PII), famously stated by Leibniz, according to which indiscernible items are identical, fails in quantum mechanics (see French and Krause 2006 for details). Along with the failure of PII goes also the possibility of grounding the individuality of the particles in some set of properties belonging to them. That is, the so-called bundle theories of individuation, according to which what characterizes an individual is a subset of its properties are ruled out in quantum mechanics and so, one must look for help in substrata or in some form of haecceitism.

So, it seems that our options are: accept quantum non-individuality and go on to explain this lack of identity that characterizes it, or take the individuals route, and adopt some kind of principle of individuation which have always been dubbed as mysterious, to say the least, in the history of philosophy. Here, we shall discuss only the non-individuals option. As a first step, we shall disentangle three notions which we have not been very careful to distinguish in the above discussion: identity, individuality, and individuation.

Identity, Individuality, Individuation

We have seen that the 'new' quantum statistics provide the main motivation for the informal claim that quantum particles lost their identities. But that concerns only part of the understanding of what is going on in the physics, and physics, by itself, does not provide for a unique metaphysical characterization of the metaphysics (metaphysics is said to be underdetermined by physics; see also French and Krause 2006, chap. 4). In order to provide for a possible understanding of what is going on in metaphysical terms, it is time to present carefully the relevant metaphysical concepts to deal with the above problems: identity, individuation, and individuality. Another concept that would be relevant for us is identity over time, but we shall not address the issue directly here.

We shall briefly discuss each of the three concepts, and explain the meaning of each expression. Our aim is to attempt to do so in the most uncontroversial terms as possible, because we would like to allow that distinct possibilities of combining those concepts remain open; distinct views on their relationship, then, will correspond to distinct views on identity and individuality. In particular, our main goal is to remain completely neutral as to the relation between identity (a logical notion) and individuality (a metaphysical notion). 'Individuation' shall serve as an umbrella term for diverse epistemic notions of separating an entity, singling it out and discerning it from other entities for the sake of linguistic reference or perceptual attention. The idea is that the epistemology, thus understood, needs have no impact on the metaphysical notion of individuality, although one may enforce one such relation and try to understand the metaphysical notion through some rendering of the epistemic notion (see section "Non-individuals with Identity").[3]

Identity

Identity is taken by us to be a relation between objects. As the tradition goes, identity statements are statements of the form '$a = b$', asserting that objects a and b are one. Such statements are true only when we are dealing with one and the same item as relata. It is just a matter of distinct forms of referring to it as either a or b. This is only a heuristic clarification, of course, not a formal explanation.

Formally, identity is a relation whose understanding depends on the language and the semantics employed. Basically, when it comes to logic, most philosophers adhere to a first-order characterization of the relation of identity. This is due to typical Quinean admonitions against the use of higher-order logic and set theory. The first-order axioms for this relation are well known:

[3] A small note on terminology: individuation is typically taken as synonym for individuality. Here, we distinguish both notions: individuality, as we mentioned, is a metaphysical feature of an entity, while individuation concerns an epistemic act of agents. We hope that the similarity of words won't cause any confusion.

Reflexivity $\forall x(x = x)$
Substitution $x = y \rightarrow (\alpha \rightarrow \alpha[y/x])$, with the known restrictions.

As it is known, these axioms allow for unintended interpretations of identity, where the meaning of the symbol = is not the identity relation over the domain of interpretation, understood as the diagonal of the domain of interpretation (see da Costa and Bueno 2009, pp. 186–187 for a sketch of the argument).

Here we develop a little further da Costa and Bueno's suggestion to show that numerical identity cannot be characterized by a purely syntactical approach. Their example is a simplification of Hodges' (1983, p. 64) (but see also Mendelson 1997, p. 100). In a general setting, it says that for every structure that interpret our first order language with identity having what Hodges call "standard identity" (nothing more than the diagonal of the domain), we can find an elementary equivalent structure which also models (in particular) identity, but where the relation that interprets this concept is not standard identity. So, identity cannot be characterized on purely syntactical grounds. Let us see the details.

Suppose we have a first-order theory with identity and let $\mathfrak{A} = \langle D, R_i \rangle$ be a model for the theory, where the binary predicate of identity is associated the identity of D, namely, its *diagonal*, $\Delta_D = \{\langle x, x \rangle : x \in D\}$ (so it is a *normal* model Mendelson 1997, p. 100; Hodges 1983 calls it *structure with standard identity*). Let $a_1, \ldots, a_n$ be elements not belonging to D. Now we construct a new structure $\mathfrak{A}' = \langle D', R'_i \rangle$ defined this way: to each element $a \in D$, we associate n new ordered pairs $\langle a, a_i \rangle$ ($i = 1, \ldots, n$). The set D' is then formed by these pairs. Furthermore, to each k-ary relation R in $\mathfrak{A}$, we associate a k-ary relation R' in $\mathfrak{A}'$ and impose that the k-tuple formed by $\langle a^{(1)}, a_i \rangle, \ldots, \langle a^{(k)}, a_i \rangle$ satisfies R' if and only if the k-tuple $a^{(1)}, \ldots, a^{(k)}$ satisfies R. So we are extending all the semantic features of our theory to the new structure. Now, on D', we define the following relation, which can be proven to be a congruence:

$$\langle a, a_i \rangle \equiv \langle b, a_i \rangle \text{ if and only if } a = b.$$

Then the new structure is elementarily equivalent to the original one, and in particular it models the predicate of identity. However, the structure $\mathfrak{A}'$ is not a normal model for the theory, although it can be "contracted" to a normal one (as shown in Mendelson 1997, p. 100 and Hodges 1983, pp. 65–6). In other words, from the point of view of the first-order language, we cannot distinguish between the two structures.

Notice that there are three notions of identity going on here: the identity symbol in the object language, the identity as a diagonal of the domain of interpretation, and the identity relation of the metalanguage, which is used to talk about the other two. We shall come back to these distinctions very soon, given that this has important consequences on how to understand identity. On what concerns the first-order characterization of the meaning of the symbol of identity, one can only grant the intended interpretation if one stipulates, in the metalanguage, that the identity sign is always going to be interpreted in the diagonal of the domain (that is, in technical terms: we stipulate that we are dealing only with normal or standard interpretations of this symbol).

What is more relevant for us at this moment is that these basic properties of the sign of identity (reflexivity and substitution) allow for a minimal characterization of the logical notion. The relation of identity, as minimally characterized, is compatible both with views that attribute to identity an important metaphysical role, as well as with views that consider it to be metaphysically neutral. The basic properties of identity underlying both claims is the same. As we shall see later, from a metaphysical perspective, identity by itself requires nothing of a metaphysical character in itself, although it is also compatible with distinctively metaphysical interpretations that associate identity with heavy metaphysical machinery. The characterization that first-order languages provide allow for some of the minimal properties of identity and also leave it open what else should be added, if something, both from a formal as well as from a metaphysical perspective.

Those properties of identity are also neutral on whether identity should be defined in terms of other notions, or if it is a primitive notion. Taken as a primitive (or even fundamental) notion, or as a defined notion, identity must satisfy at least those two properties. Whatever else is required of identity—that it is reducible to qualitative identity, for instance—is something that is added to those properties. The point is: something failing those properties is not identity.

We may explore the relation between this minimal characterization of identity and the related metaphysical issues by bringing in some of the issues that arise in discussions related to the fact that identity is not characterizable in first-order languages.[4] Notice that we have mentioned that such an attempt to characterize identity involves, in fact, three distinct notions of identity, operating at distinct levels. This brings important questions to our very understanding of identity and its relations with individuality and reducibility of identity. Two important and related issues are as follows:

First: identity seems to be presupposed in our very attempts to characterize identity (be it at first-order or at higher-order languages). The claim is that identity must be previously understood in the metalanguage if we are to understand properly those characterizations in the object language, and even if we are to understand why some attempts to characterize identity in the object language fail. In this sense, some have judged that identity is not only undefinable, but is also a fundamental feature of every conceptual scheme; it is a pre-condition for us to make sense of everything else (see Bueno 2014 for a defense of this view).

Second: being fundamental, identity would be applicable every time we speak—and its use would indeed be *required* if we are to make sense of what we say. Some have gone one step further and suggested that this would confer a kind of primitive, very thin notion of individuality for objects. The idea is as follows: there is a fundamental notion of identity, applying for everything, and the mere fact that we may always meaningfully say that

[4]There are troubles for higher-order languages too; see French and Krause (2006), chap. 6 for a general discussion.

some items are equal or distinct seems to confer a metaphysical power to identity (see, for instance, Dorato and Morganti 2013). Identity and individuality are intimately connected in this view.

Now, those are substantial points on the role of identity and its understanding, which must be disentangled. Our aim is to remain neutral about them for now, and recognize that they are additions to the very minimal notion of identity we are trying to present here. Of course, given that those additions are adopted by some, they must be clearly discussed and articulated; what is relevant for us is that those additions are not encapsulated in identity itself, as minimally characterized. There are alternative understandings of the meaning of identity which do not require such additions (their merits must also be assessed, of course), and depending on how one takes those issues, distinct sets of possibilities for the notions of individuality and individuation will also arise.

The claim that identity is fundamental (and not eliminable), for instance, may be countered by an eliminativist (reductive) approach (see the discussion in Shumener 2017). Some such approaches use Leibniz's law, reducing numerical identity to qualitative identity (also called sometimes indiscernibility or indistinguishability):

$$x = y \leftrightarrow \forall F(F(x) \rightarrow F(y)).$$

Here, items are identical if and only if they share every property of the appropriate kind.[5] There are troubles with this approach, sure, but we mention it because it is regaining currency among philosophers of quantum mechanics, mainly among those defending that quantum particles are weakly discernible (we shall discuss this issue soon; for more on weak discernibility, identity, and Leibniz's laws, see Muller and Saunders 2008 and Caulton and Butterfield 2012). Another eliminativist approach may be advanced in which identity is understood as not being fundamental, but only as a projection of our cognitive apparatuses on reality, in a Humean sense of projection, just as Humeans do for causality. In this sense, identity is the result of a kind of mental construction, not a condition for the understanding of concepts or a metaphysical feature of reality conjoined with individuality.

Also, the relation between identity and individuality may be resisted. Even if identity is fundamental, it needs not have any metaphysical content; as Bueno (2014) contends, an empiricist may adopt the thesis that identity is fundamental *and* metaphysically deflated. In particular, identity by itself needs not confer individuality. Notice also that, on the other hand, typical Leibnizian reductions of identity are involved on a metaphysical view of individuality according to which items are individuated by their properties. However, it is not clear that a Leibnizian reductionist must adopt such a relation with metaphysics, and, also, for those willing to avoid such a

[5] 'The appropriate kind' here means that distinct versions of the principle are obtained according to the kind of properties allowed in the range of F. Three distinct versions are more prominent: (1) F ranges over every property and relation; (2) F ranges over every property and relation, except for spatio-temporal ones; (3) F ranges only over non-relational properties. See French (2015) for a discussion.

direct relation, there remains also the projectivist view, where identity may be seen as directly related to epistemology, not with any substantial metaphysical thesis on individuality.

In brief: a fundamentalist about identity may be deflationist about the metaphysical content of identity, or else have it playing a relevant role in individuality. On the same issue, the eliminativist about identity may have identity playing a role in individuality, or else hold that identity is unrelated to it.

Individuality

We have now briefly discussed what we shall mean by identity. That is a kind of logical relation, which may be separated from the metaphysical issue of individuality. Focusing on individuality now, to answer the question of what confers individuality for an item *a*, is to provide for another entity *b* and a relation between these two entities that accounts for what *a* is, and, obviously, that it does so only for *a*. As Lowe (2003, p. 75) put it, an individuation principle for an object is "whatever it is that makes it the single object that it is—whatever it is that makes it *one* object, distinct from others, and the very object that it is as opposed to any other thing". In this sense, individuality may be related with a substantial role for identity, but it need not (it will all depend on how one is framing the principle of individuality). This brings a whole bunch of questions which we shall try to make clear here in a rather schematic way, and which we shall address in our further discussion of quantum non-individuality.

First of all, 'individual' is not to be confused neither with 'particular item' nor with 'object'. Although most philosophers deal with the question of individuality for particular concrete items, such as Socrates and umbrellas, there may be issues about the individuality of universals, for instance, or about the individuality of particular items that are not objects, such as tropes. We shall be concerned here only with the issue of individuality of the particular items, generally called concrete particular objects, not to be confused with so-called abstract particulars, such as tropes. In this sense, given a principle of individuality, it may make complete sense to ask whether a particular concrete item is an individual or not. If the item is a particular but is not an individual, that may be understood as meaning that the item is a non-individual.

As a result of this first clarification, particulars may be individuals or not, and we leave it open whether the notion of object will coincide with the notion of particular concrete object (see Lowe 1998, chap. 3 for a discussion of particulars and a classification that does not colapse particulars with individuals and with objects). Whenever we use 'object' for a particular concrete item, with no further clarification, it is much in a neutral sense of the word that it is being used, much as the same as 'item' or 'entity'.

The second point that will be relevant for us is that a theory on the individuality of something needs not be the same as a theory of the constitution of the particular items. As Demirli (2010, p. 2) puts it:

> In answering the internal constitution question, we may begin an inquiry about the various categories that go into the composition of individual substances and hope that at the end of this inquiry we will come up with a list of ingredients that constitute various individuals. Just as a certain recipe in a cook book provides us with a list of ingredients and instructions for mixing these ingredients together, we may maintain that the list or the recipe of individual substances — God's recipe book, so to say — will tell us what items from various categories are used, and how these items are combined.

In this sense, a theory of the constitution or composition requires that we understand particular items as composed of other particulars, much in the same way as their ingredients; the ingredients may be understood as parts of the particular in a mereological sense, but they need not be so understood. Good examples are the so-called bundle theories, according to one version of which a particular is a bundle of universals (the properties it instantiates), and also the typical understanding of the bare particulars or substratum approach, according to which a particular entity is composed not only of universals, but also of another ingredient which is a particular, the bare particular or substratum, which works as a peg on which properties are hanged. In both cases, instantiation of a property *P* is understood in terms of the property *P* being one of the ingredients composing the entity.

These two theories (bundle and substratum) are also typically understood as theories of individuality: they conflate the constitution with that which attributes individuality. What makes a particular item precisely that item that it is? The bundle theorist answers: the specific universals that are bundled together by a co-presence relation! Distinct bundles are distinct individuals, and vice versa, distinct individuals must *be* distinct bundles, so that identity may also be understood in a reductive manner, and enter into the equation too. Alternatively, the substratum theorist would say: the individuality of an individual is accounted for by the substratum involved in its composition. It is the fact that each individual has its own substratum that accounts for numerical diversity and the fact that numerically distinct individuals are present.

In order to achieve such an identification between individuality and constitution, it is typically admitted a thesis on the identity of components implying identity of entities, the *Constitutional Identity Thesis* (CTI) (see Demirli 2010, p. 2):

CTI: If two entities have the same constituents, then they are numerically the same.

Of course, one may deny the CTI, while still embracing a thesis of constitution. We may well have that numerically distinct entities be constituted by the same components (see the discussion in Demirli 2010). That would require, of course, that their numerical distinctness be grounded by something other than its constituents, and would also require that we give an explanation on how to account for their individuality (if those items are thought of as individuals) in such a case. One good option would be to consider them as non-individuals: numerically distinct entities having nothing to account for their individuality, given that it is possible for two entities to have the same constituents (see the discussion in Arenhart 2017). Perhaps the main idea gets clearer when we consider the CTI in contrapositive form, let us call it CTIC:

CTIC: If two entities are numerically distinct, then they have at least one distinct component accounting for this difference.

The suggestion that CTIC may fail could prove very useful as an account of non-individuality, given that it allows for items being somehow indiscernible (on what concerns their components) but still not being numerically the same, a situation that bears close resemblance with the one described by quantum statistics and by standard chemical elements.

Besides avoiding confusion between composition and identity, and composition and individuality, here we shall follow Lowe (2003) in claiming that individuality is a metaphysical relation of explanation. A principle of individuality explains what is it that makes the other entity an individual. For instance, bare particulars are individuality principles that make precisely this. An individual is precisely the individual it is in virtue of the bare particular it has. The bare particular of Socrates explains why that entity is Socrates, and not any other individual. The same kind of reasoning could be employed using haecceities, or for bundle theories of individuality. In this sense, one cannot claim that a symmetric relation between entities a and b may be employed to individuate them. Neither does a explain $b's$ individuality, nor does b explain $a's$ individuality. This will be relevant when we discuss weak discernibility relations.

Before we proceed, let us get once again clear on one of the explanatory tasks of an individuality principle: it explains the numerical diversity of individuals. This, notice, goes only in one direction, from individuality to numerical distinctness (IN):

IN If a and b are numerically distinct individuals, then their individuality principle may be employed to ground their difference.

If two entities are individuals, then their individuality principle may be used to explain their numerical diversity. Notice that we do not mean that only the individuality principle does that explaining. For instance, one could well explain the difference between the individuals Socrates and Plato by the color of their T-shirts, but that is certainly not what accounts for their individuality.

What is also relevant for our purposes later is that the implication from individuality to an explanation of numerical distinctness does not work the other way in an even stronger sense, from numerical distinctness to an individuality principle (NI), at least not necessarily:

NI If two entities are numerically distinct, then, there is an individuation principle to explain their distinctness.

This may fail because, depending on which principle of individuality is chosen, two entities may well be numerically different without even being individuals. This should open the possibility of numerically distinct non-individuals, of course. So, the relation between identity and individuality is a delicate one. What is relevant for us is that once this implication is clear, then, it is open for us to have distinct objects without it being required that an individuality principle be there to account for such a diversity. Of course, some approaches, like that of Dorato and Morganti (2013), which we commented on earlier, also assume this more controversial direction of dependency, making a closer relation between numerical difference and individuality. We shall not assume that this more controversial relation holds, unless it is precisely stated when it comes to discuss it.

Individuation

Now that the metaphysical notion of individuality has been distinguished from many associated and related notions, such as identity and composition, and it was seen to be an explanatory relation, it is time to address another, closely related issue, now dealing with our ability to separate or single out objects in our sensory field for the sake of sensorial focus or reference. ‘Individuation’ is the word we shall use to refer to this purely epistemic correspondent of individuality. It concerns not the metaphysical ingredient doing the job, but rather our abilities to separate things from their environment, discern them as a unity, make them the object of our attention or of our reference.

The adaptation of an example from Lowe (2003, p. 75) will help us clarifying what is at stake: consider the Margin-winged stick insect,[6] an insect that looks very much like an Eucalyptus twig. One of the greatest features of this insect, of course, is its similarity with the eucalyptus. Most of us would not be able to identify any particular Margin-winged stick when looking at an eucalyptus, even if there is one such insect there. However, a trained specialist is able to identify the insect even on such situations. We say that the specialist has individuated the insect, by managing somehow to isolate it from its environment, discerning it from the leafs of the eucalyptus. Individuation is this epistemic act. Of course, independently of how well we fare on individuation, individuality, in the case of the insect, is granted by a metaphysical principle of individuality. Any failure in individuation is only an epistemological shortcoming, not a metaphysical shortcoming. This illustrates that the metaphysics may be separated from the epistemology: there may be cases where we cannot individuate some individuals.

Our main claim will be that the other direction also holds: there are some cases where we may have individuation without the item in question being an individual. The epistemology does not fail, but there is no metaphysical principle to account for the individuality. To elucidate the distinction of the two notions, we begin with a very interesting example provided for by Dalla Chiara and Toraldo di Francia in (1992, p. 163), where there is a process of individuation going on (singling out two objects as the focus of our perceptual attention, the attribution of names and attempt of reference) without the *existence* of two individuals accounting for that attribution. In short, the case is as follows: some years ago, what looked very much like two quasars were discovered in the sky. They had all the same features, and although they were very close one to the other, there were clearly two spots seen at the same

[6] Ctenomorpha marginipennis.

time. Names were even attributed—$Q33$ and $Q34$—to them, until it was suggested that they were in fact one and the same. The explanation was that due to relativistic effects on light, rays coming from one quasar were arriving to us as two spots. So, individuation is going on without the need of individuality.

Closer to our case in this paper, something similar (individuation without individuality) may happen under some accounts of non-individuality in quantum mechanics, as we shall see. For instance, consider that we are making a measurement on a single particle in a quantum experiment. By seeing the effect of the entity on a bubble chamber, or on a photographic plate, we have indirect access to the particle (this is what Dalla Chiara and Toraldo di Francia called 'mock individuality' in Dalla Chiara and Toraldo di Francia 1992, p. 266). It also happens in cases of trapped particles, such as Astrid and Priscilla, famously trapped by Hans Dehmelt. Although we can somehow individuate Priscilla, point to it, label it temporarily, and confer a kind of unity for it, there is no need for all of these acts to be accounted for by an underlying individuality principle; all that may be involved, so far as the situation goes, is individuation, granted by the context where the particle is trapped (see also Krause 2011 for further discussion and references).

Something similar occurs in cases of counting the electrons of an Helium atom through a process of ionization. Here is how Domenech and Holik (2007, p. 862) explain it:

> Put the atom in a cloud chamber and use radiation to ionize it. Then we would observe the tracks of both, an ion and an electron. It is obvious that the electron track represents a system of particle number equal to one and, of course, we cannot ask about the identity of the electron (for it has no identity at all), but the counting process does not depend on this query. The only thing that cares is that we are sure that the track is due to a single electron state, and for that purpose, the identity of the electron does not matter. If we ionize the atom again, we will see the track of a new ion (of charge $2e$), and a new electron track. Which electron is responsible of the second electron track? This query is ill defined, but we still do not care. Now, the counting process has finished, for we cannot extract more electrons. The process finished in two steps, and so we say that an Helium atom has two electrons [...].

Notice: the process of counting may be performed without mention to identity or individuality of the electrons (Domenech and Holik seem to conflate identity and individuality). One may provide for an individuation of the extracted electrons, by referring to the first and the second electron extracted, but that, by itself, means nothing about individuality yet. If the electron is an individual, it is due to an individuality principle, not due to any epistemic feature of individuation during the extraction process. On the other hand, if the electron is not an individual, as we mentioned, the individuation also does nothing to prevent such non-individuality. It remains a non-individual (see Arenhart and Krause 2014 for further discussions on this specific case).

The result is that we are able to understand the notion of individuation independently of the metaphysical notion of individuality. Both concepts may be kept separated. One may have a metaphysics of non-individuals, while still being able to account for individuation. As Lowe (2003, p. 92) remarked, that which accounts for the identity or difference of two items needs not be the same as that which accounts for their individuality. The same may be said about individuation: that which accounts for the individuation of an item (for instance, its discernibility from other items, its separability, or the fact that it is a unity), needs not be the same ingredient that confers individuality for it.

Schrödinger's Problem

With all those distinctions made, let's get started in getting the idea of a non-individual clear. The first way to metaphysically dress the RV is the one based on a direct reading of Schrödinger's claim that identity does not make sense for quantum entities. Given that it is the view that is encapsulated in the so-called non-reflexive systems of logic (more on these soon), let us call this one the "non-reflexive" approach to non-individuality. We shall divide this section in two short subsections, one dealing with the metaphysics, the other dealing with the logic. The logic is important here, because it is felt that it provides for the clarity needed for such a radically revisionary thesis, on both metaphysical and conceptual grounds (see French 2015, Sect. 5).

The Metaphysics

Metaphysically, this view will require a close relation between identity and individuality. Identity, being a logical concept, will be loaded with a metaphysical role in individuality. Before we proceed, let us get clear on what kind of principle of individuality is being employed here.

We have already briefly discussed compositional theories, and among them, the theories of substratum. As we mentioned, those theories are commonly employed as theories of the composition and of the individuality of an item. What is attractive about those theories, at least for its defenders, is that it allows for items being numerically distinct while also being qualitatively indiscernible. That is, while *a* and *b* may be composed of the same properties, they still may count as two entities because of their distinct substrata, which is a further ingredient.

Now, of course, a substratum as an extra ingredient poses difficult challenges. It must not have properties, although it bears the properties, it must be individuated somehow, although not by anything else, it must be empirically inaccessible, and so on; many other questions add to the mystery (see Lowe 2003). In order to avoid those difficulties, while retaining the possibility of having numerically distinct items being qualitatively indiscernible, a distinct approach proposes that particulars are individuated not by a particular substratum, but by another property, a haecceity. Haecceities are understood as non-qualitative properties (they do not contribute for the discernibility of a particular), which are uniquely instantiated by a particular. This allows for a particular being individuated by a property, uniquely possessed by that individual, while leaving the issue of discernibility to be decided by the qualitative properties.

For an illustration, according to this approach Socrates is an individual in virtue of instantiating his haecceity. A haecceity is understood as the property of being that particular individual; in the case of Socrates, it is the property of 'being Socrates'. So, each individual has its own haecceity. It has many similarities with the substratum approach, so that French and Krause (2006, chap. 1) follow Heinz Post's usage and call those theories of individuality 'Transcendental Individuality' (TI). They have their name precisely because their individuality is attributed by something transcending the qualities of the individual.

Differently from the substratum theory, however, a haecceity needs not (and generally is not) involved in a theory of composition. Most adherents of haecceities do not believe that a haecceity is another ingredient composing the particular, so that the view needs not be wedded to a theory of composition.[7] Also, due to its formulation as a property, 'the property of being precisely this individual', it seems to allow for a specific relation with identity: Socrates's haecceity, for instance, would be 'to be identical with Socrates'. The claim that everything is an individual would amount to the claim that everything is self-identical. So, here we have a very strong relation between identity and individuality.

Given this stage setting, if we are to have this theory of individuality allowing also for non-individuals, we will have to provide for two restrictions: (1) on a metaphysical level, to grant that some concrete particulars do not bear haecceities (so, they are not individuals), (2) that the relation of identity does not apply to every thing (so that haecceities are not applied to every thing). Of course, both restrictions are completely related on the view we are discussing, given that haecceities are understood in terms of identity.

Here is how French and Krause (2006, pp. 13–14) express both the relation between haecceities and identity, and the prospects of the failure of the relation:

> ...the idea is apparently simple: regarded in haecceistic terms, "Transcendental Individuality" can be understood as the identity of an object with itself; that is, '$a = a$'. We shall then defend the claim that the notion of non-individuality can be captured in the quantum context by formal systems in which self-identity is not always well-defined, so that the reflexive law of identity, namely, $\forall x(x = x)$, is not valid in general.

French and Krause go on:

> We are supposing a strong relationship between individuality and identity ...for we have characterized 'non-individuals' as those entities for which the relation of self-identity $a = a$ does not make sense. (French and Krause (2006, p. 248))

So, given that the principle of individuality is a form of TI, a haecceity, the non-individuals are the items having no such TI, they lack a haecceity (see also Arenhart 2017 for further discussion). Also, given that haecceity is expressed in general by the reflexive law of identity, non-individuals, consequently, will have to be entities

[7] Of course, one may try to spell the theory of substratum as a theory of individuality without being also a theory of composition.

without identity, capturing the Schrödingerian intuition presented in a quote in the beginning of the paper.

The connection with the physics, as explained in the statistics, is also rather direct. Recall that quantum particles obey permutation symmetry. The most common opposition between the classical case and the quantum case requires that we distinguish between what to do with permutations. While permutations of the labels of classical particles do give rise to a distinct state, permutations of quantum particles do not. The claim underlying this approach seems to be that quantum statistics require that quantum entities have no individuality, for otherwise a permutation would have to be regarded as giving rise to a distinct state. Of course, once the items in case have individuality, it seems to make sense that we speak of item *a* in state 1 and item *b* in state 2, or vice-versa. As a result, quantum statistics would not work, and quantum non-individuality seems required (this kind of analysis, of course, may be resisted, see French and Krause 2006, chap. 4; what matters for us is that there is a way to motivate the adoption of a metaphysics of non-individuality coming from the physics, even if this approach is not itself mandated by the physics).

Not having haecceity and identity, of course, will require distinct explanations for a whole bunch of ideas, including, perhaps, those closely associated with individuation, such as most prominently, counting and the trapped particles cases (for further challenges, see Bueno 2014). In general, counting a collection of entities involves the use of identity. Given that under this approach identity is metaphysically committed with individuality, counting will have to be explained in alternative ways. Something similar happens with the cases of trapped particles. It seems that we are able to distinguish a trapped particle from every other particle. So, what prevents us from attributing a kind of difference from every other item, thus involving also identity?

We shall not enter in the details of the discussion here (but see Krause 2011 and Arenhart and Krause 2014). It seems that a revision in individuality through haecceity, allowing for non-individuals, is compatible with revisions in those concepts too. The revision may be achieved through the reform of part of our logic, that is, by the adoption of non-reflexive logical systems. They allow us to give precise definitions of counting, for instance, that do not require the use of identity (and hence, according to this approach, of individuality, too).

Non-reflexive Logics

Here we informally present only the strongest system of non-reflexive logic: the quasi-set theory $\mathfrak{Q}$. This is a ZFU (Zermelo-Fraenkel with Urelemente) style set theory, but with two kinds of atoms.

Our system $\mathfrak{Q}$ will have all the usual logical vocabulary for first-order logic without identity: propositional connectives ($\neg, \rightarrow$), quantifiers ($\forall$), and a denumerable collection of variables $x, y, z, \ldots$. It is important to emphasize that there is no identity symbol, for identity will be a defined notion, whose definition will have limited applicability, as the view under discussion requires. The list of primitive non-logical symbols of $\mathfrak{Q}$ is the following one:

(i) the binary relations $\in$ for pertinence and $\equiv$ for indistinguishability;
(ii) the unary predicates m, M and Z, meaning m-atoms, M-atoms and classical sets respectively.
(iii) the unary function symbol qc, for quasi cardinal.

The intended interpretations of m-atoms are the quantum non-individuals, items for which identity must not make sense. M-atoms represent usual objects (classical Urelemente), items for which identity applies, and things to which the predicate Z applies are the sets in $\mathfrak{Q}$ that represent classical sets of ZFU. Terms are individual variables and expressions of the form $qc(t)$, where t is a term. Formulas are defined in the usual way. Now some useful definitions are in order:

(i) $Q(x) := \neg(m(x) \vee M(x))$ (x is a qset)
(ii) $D(x) := M(x) \vee Z(x)$
x is a *Ding*, a "classical object" in the sense of Zermelo's set theory, namely, either a set or a M-atom.

So, quasi-set theory has two kinds of atoms and qsets, collections having these atoms and other collections as elements. In this aspect, $\mathfrak{Q}$ is just like the usual theories with *urelemente*. The main difference concerns the behavior of the m-atoms: since this system is intended to capture the idea of a lack of haecceity for m-atoms, in the formal system we shall build, statements of the form $x = y$ or $x = x$ are simply not available when x and y are m-atoms; that is, the items that denote quantum particles (in our intended interpretation) are not relata of identity. To achieve this, we advance the following definition of identity:

Definition 1 $x =_E y := (Q(x) \wedge Q(y) \wedge \forall z(z \in x \leftrightarrow z \in y)) \vee (M(x) \wedge M(y) \wedge \forall z(x \in z \leftrightarrow y \in z))$ *(Extensional identity)*

Notice, the definition does not apply to m-atoms. There is nothing to be said about their identity or diversity. This is so in order to capture Schrödinger's claim that identity makes no sense for quantum entities, and also the intended metaphysical understanding of non-individuality through the associated claim that haecceities do not apply for everything.

The next important point we would like to mention concerns a relation of indistinguishability. Permutation symmetry implies that quantum entities are not discernible by any properties whatsoever. An obvious strategy for introducing the relation of indiscernibility would be to require that whenever $\alpha(x)$, where α is any formula with x free, would imply that $\alpha[y/x]$, that is, full substitution is allowed for indiscernible items (with the usual care to avoid clash of variables). Also, of course, we would like to have the indiscernibility relation reflexive, because everything is supposed to be indiscernible from itself. So, indiscernibility would be a reflexive relation allowing for full substitutivity. This sounds nice but there is one major problem with this idea: what is being introduced is precisely the same set of postulates for first-order identity! In order to avoid that indistinguishability coincide with identity for these items, endow the indiscernibility relation with a formal restriction: given that identity is an equivalence relation compatible with every other relation, indistinguishability, in our system, will lack this last property. Let us begin with some postulates:

$(\equiv_1)$ $\forall x(x \equiv x)$
$(\equiv_2)$ $\forall x \forall y(x \equiv y \rightarrow y \equiv x)$
$(\equiv_3)$ $\forall x \forall y \forall z(x \equiv y \wedge y \equiv z \rightarrow x \equiv z)$

These postulates ensure us that indistinguishability is an equivalence relation. As we commented above, this relation is not necessarily compatible with the other primitive predicates or relations. It seems plausible that indistinguishable objects should not necessarily be elements of the same qsets, so that indistinguishability is not compatible with membership. In $\mathfrak{Q}$ this holds for m-atoms, and it also grants that indistinguishability does not coincide with identity for these items, that is, if x and y are indistinguishable m-atoms, then being z a qset, we have that $x \in z$ does not entail that $y \in z$, and conversely. On the other hand, for other kinds of objects, identity and indistinguishability do coincide, and then indistinguishability is compatible with every relation, and in particular, with membership.

Once these basic ideas are in order, $\mathfrak{Q}$ just follows the usual set theories with atoms when it comes to grant the existence of collections. Postulates grant that a form of the pair axiom hold, the power set axiom, separation axiom, empty set axiom, and so on (the details may be seen in French and Krause 2006, chap. 7). In particular, what is relevant for us is that the M-atoms and the collections that do not include m-atoms, those satisfying the predicate Z, may be employed to develop inside $\mathfrak{Q}$ a classical system that behaves just like ZFU. So, as part of $\mathfrak{Q}$, we have classical set theory with atoms. In particular, inside the classical part of $\mathfrak{Q}$ we may develop the classical theory of cardinals. It is through these existing cardinals that we may attribute a cardinal also to qsets having m-atoms. This is achieved with a postulate:

(qc_1) $\forall_Q x(\exists_Z y(y = qc(x)) \rightarrow \exists! y(Cd(y) \wedge y = qc(x) \wedge (Z(x) \rightarrow y = card(x)))$.
(qc_2) $\forall_Q x(x \neq \emptyset \rightarrow qc(x) \neq 0))$.

The basic idea is that every qset has a quasi-cardinal, which is a classical cardinal, attributed by the function qc. When this qset is a classical set, the quasi-cardinal coincides with the classical cardinal. When the qset has m-atoms, then, the attribution must be made respecting the behavior of the operations over qsets. In particular, we are able to prove that singletons exists, that is, collections of objects indiscernible form x, which we represent by $[x]$.[8] Each qset $[x]$ has a sub-collection whose quasi-cardinal is 1, denoted by $[[x]]$. We call it the *strong singleton* of x. It has just one element, and we may think of this element *as if* it were x, but in fact, it follows from the definition that all we can know about it is that $[[x]]$ contains one item indistinguishable from x. In the cases in which x is not an m-atom we obtain the usual singleton, and we can prove that its element is x itself.

[8]Care must be taken here in order to separate $[x]$ from an already given collection z, so that $[x]$ is the collection of items indiscernible from x in z. This prevents singletons from being too big. For a full discussion see (French and Krause 2006, chap. 7) and (French and Krause 2010).

With these notions we are able to prove in $\mathfrak{Q}$ the theorem expressing the invariance by permutations:

Theorem 1 *(Invariance by Permutations). Let x be a finite qset such that* $\neg(x = [z])$ *and let z be an m-atom such that* $z \in x$*. If* $w \equiv z$ *and* $w \notin x$*, then there exists* $[[w]]$ *such that*

$$(x - [[z]]) \cup [[w]] \equiv x$$

In words: two indiscernible elements z and w, with $z \in x$ and $w \notin x$, expressed by their strong-singletons $[[z]]$ and $[[w]]$, are 'permuted' and the resulting qset x remains indiscernible from the original one.

Non-individuals with Identity

Now, while the non-reflexive approach to non-individuality and the Received View is the most traditional and well-known one, it is not the only possibility. Recall that in general lines the RV is the thesis that quantum entities are not individuals, and that the very idea of a non-individual needs not be articulated in terms of a lack of identity. In fact, given our previous discussion, there is a variety of options which are open for us to understand non-individuals, while still preserving the use of identity.

These approaches all benefit from the fact that that which confers numerical diversity to items needs not be the same thing that confers them individuality; that is, items may be different for reasons unrelated with their individuality principles (for a discussion with further options, see Arenhart 2017). That is, the approaches we shall deal with benefit from the fact that numerical distinctness does not imply individuality: facts about identity and diversity need not be facts about individuality. The advantage of separating an account of individuality and an account of identity relies precisely in the fact that we may revise the applicability of the theory of individuality without having to revise the applicability of identity. This allows for a much less revisionary approach than the non-reflexive one, given that the logic of identity and the mechanisms of individuation (discernibility, unity, separation, reference, and so on), may still be available.

The first approach to the RV (that is not also a revision of logic) which we would like to present benefits from the weak discernibility approach, and is suggested, although not directly, by Muller and Saunders (2008). Before we present it, a brief introduction in the terminology may be useful. As to the possible ways to discern two entities a and b, we have:

Abs a and b are absolutely discernible when there is an intrinsic property that one of them has, while the other does not have.

Rel a and b are relatively discernible when there is a relation R such that aRb or bRa holds, but not both.

Wea *a* and *b* are weakly discernible when there is a relation *R* that is symmetric (*aRb* implies *bRa*) and irreflexive (*xRx* fails for every *x* in the domain of the relation).

Entities that are not discernible in any of the former ways are said to be 'indiscernibles' (see Muller and Saunders 2008, and also Caulton and Butterfiled 2012). Quantum entities of the same kind certainly are not absolutely discernible and also not relatively discernible. However, they may be seen as weakly discernible. Two electrons in the singlet state, for instance, are weakly discernible by the relation "has spin in the opposite direction to". In fact, no electron has spin in the opposite direction to itself, and if the spin of *x* is opposite to the spin of *y*, then, certainly the spin of *y* is opposite to the spin of *x*. That allows us to ground the claim of numerical diversity: if a weakly discerning relation obtains, then, certainly we must have two objects.

However, the obtaining of weakly discerning relations, by itself, is not enough to ground individuality. Recall that individuality is an explanatory relation, which cannot be symmetric. In this sense, one item cannot be used to explain the other's individuality in a weakly discerning relation. We only go as far as numerical difference, without being able to attribute individuality to the items. Notice also that a weakly discerning relation does not allow for individuation: we cannot determine, in the case of the electrons in the singlet state, which electron is up, and which is down. The best we can say is that one is up and one is down.

The fact that a weakly discerning relation holds between quantum entities saves a version of the Principle of Identity of Indiscernibles. Recall that the principle requires that numerically distinct objects must have their distinctness grounded in some quality (this fits well with a reductive account of identity, thus, but is compatible with other non-reductive approaches to identity). While properties and asymmetric relations cannot ground such diversity in quantum mechanics, weakly discerning relations can. So, even if we cannot point to the particles (that is a problem of individuation, not of identity), we may have a numerically grounded difference. In this rather weak formulation, the PII resists in quantum mechanics.

Muller and Saunders' (2008) suggestion enters precisely here.[9] They combine these ingredients in a proposal which, we believe, is compatible with non-individuals. First, they define individuals as *absolutely discernible* objects, that is, objects having a property that allows them to be discerned from everything else. Now, this property is generally understood as being a physical property, not a haecceity or some non-qualitative property. That accounts for the explanatory role of individuality, and given that the approach to identity is reductive, also for the identity. Now, quantum particles are not absolutely discernible from other items, there is nothing in them allowing for such a discernibility. As a result, they are objects, but they are not individuals. Their numerical difference is accounted for, but their individuality is not.

Notice that although the PII is involved, this approach needs not be conjoined with a theory of composition. The particles in case need not be composed of relations and

[9]We are not suggesting that Muller and Saunders see themselves as providing a theory of non-individuals; our suggestion is that their definitions may be understood as a rendering of the RV.

properties (although one could, perhaps, try to provide for such a theory of composition too; see the discussion in Arenhart 2017). The relations just hold between them, as a quantum mechanical fact. This view puts discernibility in the center of the stage. Also, it meshes well with the statistics, because permutation symmetry may be understood as the fact that the relations obtaining between the quantum particles are all symmetric. There is also no property to provide for a difference before and after a permutation.

For another option on how to frame the RV, consider Bueno's approach to individuality in Bueno (2014). According to Bueno, identity is a fundamental, primitive relation, that is metaphysically deflated, thus, not by itself contributing to the individuality of anything. An individual is a particular object satisfying two much stronger conditions than mere numerical diversity:

Dis the item is discernible from every other individual;
Id the item is re-identifiable over time.

Notice that while one could appeal to weakly discerning relations to account for discernibility, the condition [Id] puts a heavy epistemological ingredient on the definition of individual, an ingredient which is not satisfiable in standard quantum mechanics. With this definition, identification is comprised in the epistemic notion of individuation: it is required that we identify something (single it out somehow), and then, at later instants of time, that we are able to re-identify it as being the same item. Bueno is not explicit about what is involved in re-identification, so, we shall take that what is meant is something along the lines we have described, which are very plausible demands for an empiricist.

However, for unobservable objects such as quantum particles, re-identifiability is not directly available. One could understand re-identification as the demand that we could, at least in principle, follow the trajectory of the individual at any given instant of time. As it is known, that is not available in standard quantum mechanics. In fact, even in Bohmian mechanics, where a trajectory is always present, it is a hidden ingredient (a hidden variable), so that the epistemic flavor of Bueno's definition would be lost. In fact, the trajectory in this context would work as a metaphysical ingredient unrelated to the available epistemology. So, on what concerns identity over time, there are also important distinctions between the metaphysics and the epistemology involved; under our interpretation, Bueno brings precisely the epistemic ingredient to be conflated with the metaphysical one: failing the epistemic ingredient, there is nothing else to be employed in order to grant individuality. In this sense, then, these theories would comprise non-individuals.[10]

[10]We are not claiming that it was Bueno's original goal to defend a theory of non-individuals; in fact, in Bueno (2014) he identifies the RV with the non-reflexive approach, and argues against it.

Conclusion

In this paper, we have provided for distinct ways to give some metaphysical flesh to the heuristic bones of the RV. Typically presented in rather vague terms, the RV merely says that quantum entities are not individuals, that they have lost their identities. However, nothing is said about how to formulate the metaphysically complex notion of individual and its failure in quantum mechanics. Hints are merely provided by the new statistics.

Here, to address these issues, we have distinguished the three core notions involved in attempts to provide for a theory of individuality: the concepts of identity (logical), individuality (metaphysical) and individuation (epistemological). These notions were provided with a rather minimal content, so that they could be employed in distinct combinations in order to provide for distinct theories of individuality and non-individuality as well. Some approaches put more weight in the identity, others in individuation. The whole point is that having those concepts clear in mind allowed us to provide for some examples of how to provide for metaphysical content for the notion of non-individual.

As we have seen, the most widely articulated and defended approach for the concept of non-individual is the non-reflexive one, as presented by French and Krause (2006). It originates on an interpretation of Schrödinger's claim that identity makes no sense for quantum particles; a close connection is provided for between identity and individuality through the use of haecceities as an individuality principle and its expression as self-identity. Now, while this provides for a clear determination when something is an individual (at least formally), that approach requires that failure of individuality should be accompanied by a corresponding failure of identity, which on its turn requires a revision of logic and many associated notions (think of naming, counting, quantification, isolating one entity, among others, which are typically related with identity). Non-reflexive systems of logic are presented in order to render the view with solid foundations, or, "philosophically respectable", as French puts it (see French 2015, Sect. 5).

While the association between identity and individuality, and lack of identity with non-individuality, has been widespread, and almost always identified with the RV itself, it is by no means the only option. In fact, some reject the RV due to its largely revisionary character on what concerns identity. In order to make clear that the non-reflexive approach is not the only one, and to dissociate the RV from one of its possible articulations, we have provided for two alternatives which keep identity intact, but define individuality in such a way that it is possible for quantum entities to fail them. One such approach was suggested by Muller and Saunders' (2008) definition of an individual. It requires that individuals be absolutely discernible from other entities. Quantum entities do not meet this condition, although they satisfy weaker forms of discernibility that grant them numerical diversity.

For another approach, relating the metaphysical notion of individuality with the epistemic notion of individuation, we have also briefly presented a proposal that may be found in Bueno's (2014). By relying heavily on the epistemic requirement of re-identification over time, Bueno puts such strong requirements on what can be an individual. Notice also that while his approach does not identify the metaphysical notion of individuality with the logical notion of identity, it makes identity an important ingredient of individuality, and what is more important, brings the metaphysical concept of individuality closer to the epistemic concept of individuation. By incorporating the re-identification clause for individuality, Bueno leaves open the door for items that do not meet this condition. That allows for non-individuals to come in, while identity still applies.

As a result of these distinct articulations, the RV may be seen as providing only for a kind of general guidance on what quantum entities should be, without providing for no specific metaphysical approach to non-individuals. All that is required is that the demands put by quantum mechanics (a form of indiscernibility by permutation invariance) be respected. Of course, those approaches have distinct merits, and the proper examination of which is better (if any), is an issue that we shall discuss in another place.

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