Mathematical Model for Calculating Performance of Parabolic Through Collector

Peter Vician, Matej Palacka, Peter Ďurčanský and Jozef Jandačka

Abstract The work deals with the transformation of solar energy into thermal energy through concentric solar collector. The subject of the research is the parabolic trough collector situated in Žilina. Solar collector including focal absorber was produced according to our own design. The absorber consists of two black coated contradictory pipes serving as inlet/outlet of heat exchanger. The reflector is made of bent polished aluminium sheet. Collector uses automatic tracking system and consists of firm frame attached to concrete floor, which limits the sun tracking to one axis. Trough of the collector is oriented as east-west position with a small deviation of approximately 10°. To determine the required output of collector is necessary to perform optical and thermal analyses. The aim of the work is creating mathematical model to get a theoretical performance of collector. Mathematical model with calculations for specific collector and its geographical position is created in program MS Excel. Although the mathematical model provides theoretical performance parameters it doesn't include the effect of environment and so the values differ from real conditions. The results of work will serve as an information basis for the following research of cogeneration system using a solar collector.

Keywords Solar energy · Parabolic collector · Thermal analysis

1 Introduction

Solar energy is among the cheapest energy source available everywhere. It does not pollute the environment and contribute to the reduction of emission production. That is the reason for recent spent large amounts of money into research and development of equipment and components that use exactly this kind of energy. Technology using solar energy has the potential for commercial use. Use of solar energy can be accomplished in several ways. Photovoltaic panels are used for

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electricity generation. The heat energy can be obtained by through or flat thermal panels. By using concentric solar collectors we are able to convert solar energy into heat with higher added value.

The work deals with the transformation of solar energy into thermal energy through the concentric trough collector situated on building of University of Žilina. The collector was made according our own design. Due to its later modification and future application is essential to determine its performance parameters. For the purpose of obtaining preliminary results of collector performance parameters is necessary to understand the inner workings and laws of thermodynamics. Collector uses a single axis tracking system and its rotational axis is positioned in east-west orientation. The reflective surface is made from bended polished aluminium sheet with reflective foil. Thermal insulation of receiver is provided by vacuumed glass cover. The mathematical model for the mentioned collector will serve as an information basis for the following research of cogeneration system using a solar collector.

2 Parabolic Trough Collector

Parabolic trough collectors are low-cost technology systems with light structure which are able to deliver high temperatures up to 400 °C with good efficiency. They are considered as the most mature and advanced of the solar thermal technologies because of considerable experience with the systems and the development of a small commercial industry to produce and market these systems. It suffices to use a single-axis tracking of the sun; therefore, long collector modules are produced. The receiver of a parabolic trough is linear. Usually, a tube is placed along the focal line to form an external surface receiver. The size of the tube, and therefore the concentration ratio, is determined by the size of the reflected sun image and the manufacturing tolerances of the trough. The surface of the receiver is typically plated with a selective coating that has a high absorptance for solar radiation but a low emittance for thermal radiation loss. A glass cover tube is usually placed around the receiver tube to reduce the convective heat loss from the receiver, thereby further reducing the heat loss coefficient. A disadvantage of the glass cover tube is that the reflected light from the concentrator must pass through the glass to reach the absorber, adding a transmittance loss of about 0.9, when the glass is clean. The glass envelope usually has an antireflective coating to improve transmissivity. One way to further reduce convective heat loss from the receiver tube and thereby increase the performance of the collector, particularly for high temperature applications, is to evacuate the space between the glass cover tube and the receiver. The total receiver tube length of PTCs is usually from 25 to 150 m. New developments in the field of PTCs aim at cost reduction and improvements in technology. In one system, the collector mirrors can be washed automatically, drastically reducing the maintenance cost [1–3].

The thermal performance of solar collectors can be determined by the detailed analysis of the optical and thermal characteristics of the collector materials and collector design or by experimental performance testing under controlled conditions. It should be noted that the accuracy of the heat transfer analysis depends on uncertainties in the determination of the heat transfer coefficients, which is difficult to achieve, due to the non-uniform temperature boundary conditions that exist in solar collectors. Such analysis is usually carried out during the development of prototypes, which are then tested under defined environmental conditions. In general, experimental verification of the collector characteristics is necessary and should be done on all collector models manufactured. In some countries, the marketing of solar collectors is permitted only after test certificates are issued from accredited laboratories to protect the customers [1–3].

2.1 Optical Analysis

Concentrating collectors work by interposing an optical device between the source of radiation and the energy-absorbing surface. Therefore, for concentrating collectors, both optical and thermal analyses are required. For optical analysis we first must define the term concentration ratio which is a ratio of the aperture area to the receiver–absorber area. For a tubular receiver, the concentration ratio is given by [4]:

$$C = \frac{A_a}{\pi D} \tag{1}$$

Aperture of the parabola A_a can be obtained from a simple trigonometry and depends on parabola focal distance f and angle between the collector axis and a rim angle φ_r (Fig. 1).

$$A_a = 4f\,tan\left(\frac{\varphi_r}{2}\right) \tag{2}$$

The maximum concentration ratio occurs when rim angle is 90°. For the same aperture, various rim angles are possible. For different rim angles, the focus-to-aperture ratio, which defines the curvature of the parabola, changes. It can be demonstrated that, with a 90° rim angle, the mean focus-to-reflector distance and hence the reflected beam spread is minimized, so that the slope and tracking errors are less pronounced. The collector's surface area, however, decreases as the rim angle is decreased. There is thus a temptation to use smaller rim angles because the sacrifice in optical efficiency is small, but the saving in reflective material cost is great [4].

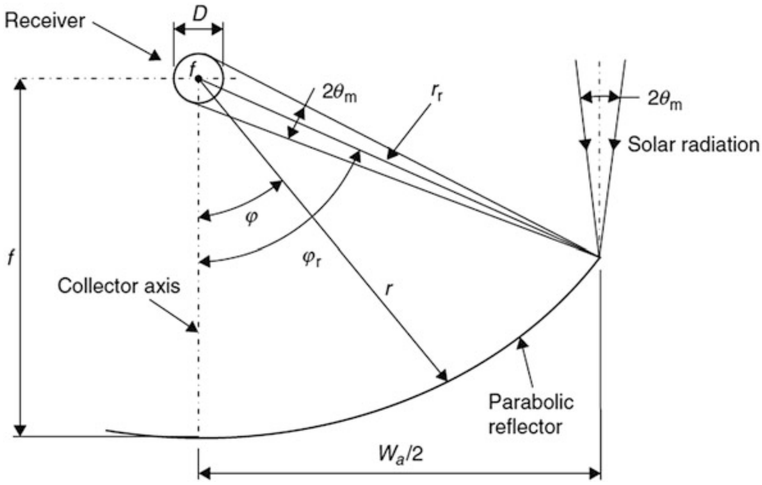

Fig. 1 Cross-section of a parabolic trough collector with circular receiver

2.2 Optical efficiency

Optical efficiency is defined as the ratio of the energy absorbed by the receiver to the energy incident on the collector's aperture. The optical efficiency depends on the optical properties of the materials involved, the geometry of the collector, and the various imperfections arising from the construction of the collector, such as reflectance of the mirror ρ, transmittance of the glass cover τ, absorptance of the receiver α, intercept factor γ, geometric factor A_f and angle of incidence θ [4, 5].

$$\eta_o = \rho\tau\alpha\gamma \left[\left(1 - A_f \tan(\theta)\right)\cos(\theta)\right] \tag{3}$$

Geometric factor A_f dictates the geometry of the collector, which is a measure of the effective reduction of the aperture area due to abnormal incidence effects, including blockages, shadows, and loss of radiation reflected from the mirror beyond the end of the receiver and can be calculated [4, 5]:

$$A_f = \frac{A_1}{A_a} = \frac{A_e + A_b}{A_a} \tag{4}$$

The parameter A_e is called the end effect, which occurs during abnormal operation of a parabolic trough collector, when some of the rays reflected from near the end of the concentrator opposite the sun cannot reach the receiver. The amount of aperture area lost is given by [4, 5]:

$$A_e = fW_a \tan(\theta)\left[1 + \frac{W_a^2}{48f^2}\right] \quad (5)$$

The constant A_b represents the loss of aperture area due to opaque plates usually constructed to preclude unwanted or dangerous concentration away from the receiver. For a plate extending from rim to rim, the lost area is calculated by equation [4, 5]:

$$A_b = \frac{2}{3} W_a h_p \tan(\theta) \quad (6)$$

The parabola height is calculated as follows:

$$h_p = \frac{W_a}{4\tan\left(\frac{\varphi_r}{2}\right)} \quad (7)$$

2.3 Thermal Analysis

For generalized thermal analysis of a concentrating solar collector is necessary to derive appropriate expressions for the collector efficiency factor F'; the loss coefficient U_L and the collector heat removal factor F_R. For the calculation of loss coefficient are used standard heat transfer relations for glazed tubes.

The calculations include radiation, conduction, and convection losses. Assuming no temperature gradients along the receiver and negligible convection losses due to evacuated space between the receiver and the glass cover, the loss coefficient is based on the receiver area A_r and area of glass cover A_g [4, 1].

$$U_L = \left[\frac{A_r}{\left(h_{c,c-a} + h_{r,c-a}\right)A_g} + \frac{1}{h_{r,r-c}}\right]^{-1} \quad (8)$$

Next, the collector efficiency factor defined as the ratio of the overall heat loss coefficient and overall heat transfer coefficient needs to be estimated. This should include the tube wall because the heat flux in a concentrating collector is high. Based on the outside and inside tube diameter and tube thermal conductivity, the relation is given by [4, 1]:

$$F' = \frac{U_L^{-1}}{\frac{1}{U_L} + \frac{D_o}{h_{c,r-f} D_i} + \left(\frac{D_o}{2k} ln \frac{D_o}{D_i}\right)} \quad (9)$$

It is usually desirable to express the collector total useful energy gain in terms of the fluid inlet temperature. To do this the collector heat removal factor needs to be used. Heat removal factor represents the ratio of the actual useful energy gain that would result if the collector-absorbing surface had been at the local fluid temperature [4, 1].

$$F_R = \frac{\dot{m}c_p}{A_r U_L}\left[1 - exp\left(-\frac{U_L F' A_r}{\dot{m}c_p}\right)\right] \tag{10}$$

The instantaneous efficiency of a concentrating collector may be calculated from an energy balance of its receiver. Because concentrating collectors can utilize only beam radiation, the total absorbed solar radiation is represented only by G_B and useful energy delivered from a concentrator is [4, 1]:

$$Q_u = F_R\left[G_B \eta_o A_a - A_r U_L (T_i - T_a)\right] \tag{11}$$

Then, the collector efficiency can be obtained by dividing useful energy by total absorbed solar radiation. Therefore:

$$\eta = F_R\left[\eta_o - U_L\left(\frac{T_i - T_a}{G_B C}\right)\right] \tag{12}$$

2.3.1 Convective heat transfer

Estimation of the convective losses is based on construction of collector's receiver. If we consider that receiver is insulated by vacuum in glass cover, the only part of convective loss is that of a glass cover per se, which is calculated as [6]:

$$h_{c,c-a} = \frac{Nu \cdot k}{D_g} \tag{13}$$

Constant k represents the thermal conductivity of ambient air and D_g is the diameter of glass cover. The equation includes criterion Nusselt number, which definition differs according to the nature of liquid flow described by Reynolds number [6].

$$Re = \frac{\rho_a V_a D_g}{\mu_a} \tag{14}$$

All constants in the equation, density ρ, velocity V and dynamic viscosity μ_a are related to ambient air at mean temperature calculated from temperature of ambient air T_a and temperature of glass T_g.

$$T_m = \frac{T_a + T_g}{2} \tag{15}$$

Equation of Nusselt number for Re values from interval (0.1; 1000)

$$Nu = 0.4 + 0.54(Re)^{0.52} \tag{16}$$

If values of Re are from interval (1000; 50000)

$$Nu = 0.3(Re)^{0.6} \tag{17}$$

Another case of convective heat transfer is inside of receiver tube. The convective heat transfer coefficient can be obtained from the standard pipe flow equation including thermal conductivity of fluid k_f and inner diameter of receiver tube D_r[6]:

$$h_{c,r-f} = \frac{Nu \cdot k_f}{D_r} \tag{18}$$

Nusselt number for laminar flow of fluid ($Re \leq 2300$) is always constant 4.364. For turbulent flow ($Re > 2300$), the Nusselt number can be calculated according:

$$Nu = 0.023(Re)^{0.8}(Pr)^{0.4} \tag{19}$$

Reynolds number is calculated according Eq. (9) where all the constants are related to working fluid. Value of Prandtl number can be obtained from:

$$Pr = \frac{c_p \mu_f}{k_f} \tag{20}$$

2.3.2 Radiation heat transfer

We can divide the total radiation heat transfer to radiation from receiver to glass cover $h_{r,r-c}$ and from glass cover to ambient. For determining the heat transfer coefficients is necessary to know emissivity ε, surface A and temperature T of both radiating materials [7].

$$h_{r,c-a} = \varepsilon_g \sigma \left(T_g^2 + T_a^2\right)\left(T_g + T_a\right) \tag{21}$$

The σ in the equation is Stefan-Boltzmann constant 5.67×10^{-8} W.m^{-2}.K^{-4}. Radiation heat transfer between receiver and glass cover is more complex due to the mutual irradiation and can be described as [7]:

$$h_{r,r-c} = \frac{\sigma\left(T_r^2 + T_g^2\right)\left(T_r + T_g\right)}{\frac{1}{\varepsilon_r} + \frac{A_r}{A_g}\left(\frac{1}{\varepsilon_g} - 1\right)} \tag{22}$$

In the preceding equations, to estimate the glass cover properties, the temperature of the glass cover is required. The procedure to find Tg is by iteration. Usually, no more than two iterations are required. This temperature is closer to the ambient temperature than the receiver temperature. Therefore, by ignoring the radiation absorbed by the cover, it may be obtained from an energy balance [7]:

$$T_g = \frac{A_r h_{r,r-c} T_r + A_g \left(h_{r,c-a} + h_{c,c-a}\right) T_a}{A_r h_{r,r-c} + \left(h_{r,c-a} + h_{c,c-a}\right)} \tag{23}$$

3 Results

The mathematical model was created for the specific parabolic trough collector situated on the building of University of Žilina as shown in Fig. 2. Reflector surface is made by bended aluminum plate with reflective foil, rim angle is 90°. Focal heat exchanger consists of two contradictory tubes inlet/outlet (Fig. 3). As a working fluid serves air supplied from measuring room secured by ventilation air. Therefore the parameters of inlet air are considered as parameters of ambient air. The volume flow in a tube is set to 0.0163 m^3/s. The excess pressure in the system reaches 0.3 bars. The collector is oriented in east-west position and has fully automatic single axis system for rotation .

The optical analysis was carried out according constants listed in Table 1. Some parameters of the materials were specified only approximately according catalogue. The calculation of intercept factor γ is very complex and requires many unknown constants. Its impact on the optical efficiency is usually small and so for our calculation its value is 0.98.

The most considerable effect has incidence angle, which changes during the day. The Fig. 4 depicts the optical efficiency during the day of 06.15.2017. As we can see the optical efficiency is ranging from minimum of 11% to maximum 71% according to changing incidence angle.

Fig. 2 Parabolic trough collector with one-axis tracking

Fig. 3 Detail if inlet/outlet of receiver and reflexive surface

Table 1 Parameters of the collector for optical analysis

Parameter	Character	Value	Unit
Parabola height	h_p	0.4495	m
Total aperture loss	A_1	1.0776	m^2
Geometric factor	A_f	0.1095	[–]
Intercept factor	γ	0.98	[–]
Reflectance of mirror	ρ_r	0.87	[–]
Transmittance of glass cover	τ	0.90	[–]
Absorptance of receiver	α	0.92	[–]

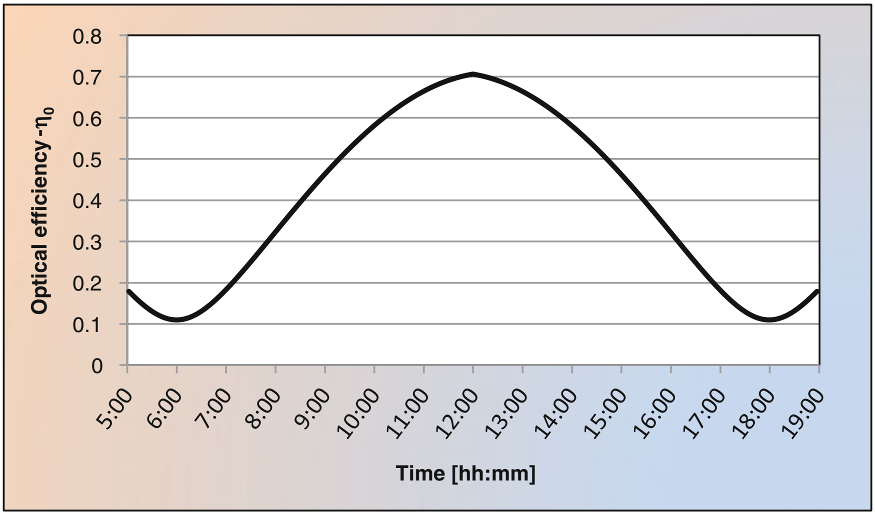

Fig. 4 Optical efficiency of the collector for specific day

Table 2 Parameters of the collector for thermal analysis

Parameter	Character	Value	Unit
Receiver length	L	5.88	m
Outer diameter of receiver	D_o	0.0424	m
Inner diameter of receiver	D_i	0.0375	m
Outer diameter of glass cover	D_g	0.125	m
Temperature of ambient air	T_a	20	°C
Temperature of inlet air	T_i	20	°C
Temperature of receiver surface	T_r	170	°C
Wind velocity	v	1	m/s
Pressure of ambient air	p_a	101,325	Pa
Pressure of working fluid	p_f	130,000	Pa
Glass cover emissivity	ε_g	0.87	[–]
Receiver emissivity	ε_r	0.92	[–]
Mass flow of working fluid	$\dot{m}$	0.0197	kg/s
Tube thermal conductivity	λ_r	15	W/(m.K)

Table 2 contains parameters necessary to calculate the useful heat and thermal efficiency of the collector. As the collector is situated on the building its performance is affected by condition of environment as temperature, solar global radiation, air pressure and wind velocity. Those variables are changing constantly. Parameters like temperature and wind velocity have negligible effect on heat losses due to thermal insulation, although temperature in our case is considered as inlet temperature.

The Fig. 5 shows the useful energy, the total energy of collected beam radiation reduced by thermal losses, within a specific day calculated for corresponding values of beam radiation incident on aperture area and optical efficiency. The highest useful energy 5947 W is obtained during noon when global radiation reaches is maximum 1162 W/m^2 and incident angle equals 0° which is the ideal condition.

Based on the created mathematical model, we can say that the performance of the collector is mostly dependant on the position of trough to the incident sun rays which rapidly affects the optical efficiency and consequently the thermal efficiency.

The mathematical model does not take in consideration the weather conditions and cloudiness. The related charts represent the ideal conditions and thus can differ from real operation. The results of this work provide information about what we can expect from the collector and its purpose in future application.

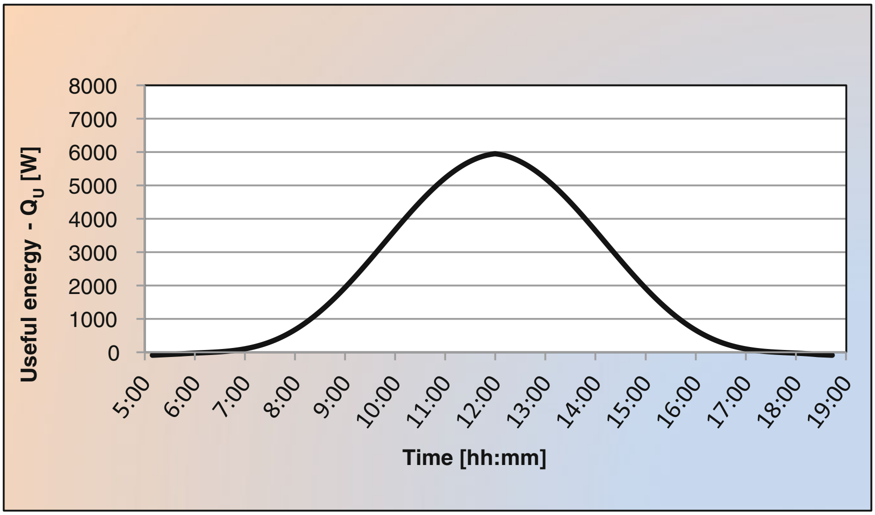

Fig. 5 Useful energy provided by the collector for specific day

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