Chapter 27
Mapping the Relationship Between Written and Enacted Curriculum: Examining Teachers' Decision Making

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Abstract I offer an approach to representing and examining the relationship between curriculum resources and the performance of teaching, for the purpose of analyzing teachers' design work. The approach builds on the assumptions that teaching is a design activity, that curriculum resources are tools that convey complex instructional ideas, and that, in using these tools, teachers interact with them and selectively leverage resources to design and enact instruction. I introduce the instructional design arc as a unit of analysis, referring to an episode in a lesson, prompted by the teacher, and that require the teacher to make instructional design decisions in the moment. When compiled into a lesson map, these design arcs model the episodic and emerging contours of the enacted lesson, representing teachers' planned and in-the-moment decisions. Using data from 3rd to 5th grade mathematics classrooms in the USA, I analyze instructional design arcs within mathematics lessons, focusing on teachers' design work.

Keywords Written curriculum · Enacted curriculum · Teaching Mathematics instruction

27.1 The Relationship Between Written and Enacted Curriculum

In his exploration of the "teacher-tool relationship," Brown (2009) uses sheet music and different artists' renditions of the same classic jazz song, *Take the A Train*,[1] to illustrate the ways the same song, as written, can be performed in substantially

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[1]Written by Billy Strayhorn.

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different ways. Both performances, Brown points out, have "essential similarities," yet they "sound distinctly different" (p. 17). Curriculum materials, Brown (2009) goes on to argue, have similarities to sheet music, both in terms of their form and role in guiding performance. They are both "static representations" of intended activity and the "means of transmitting and producing" it, but they are not the activity itself.

Curriculum materials and sheet music are also similar in that "they are intended to convey rich ideas and dynamic practices," but they do so "through succinct shorthand that relies heavily on interpretation" (Brown 2009, p. 21). At the same time, curriculum materials are often designed to "influence common practice by introducing innovative approaches and ideas." Most critically, and often overlooked, curriculum materials "require craft in their use; they are inert objects that come alive only through interpretation and use by a practitioner" (p. 22).

The relationship between sheet music and musical performance is also apt as a metaphor for teaching and the curriculum-teacher relationship because, in many ways, teaching is a live performance. Like different performances of the same musical score, different enactments of the same written curriculum will not only vary in style, pace, and emphasis, they are likely to also vary in quality. Some teaching performances come closer than others to meeting the mathematical and pedagogical goals specified in the curriculum or intended by the teacher. This variation in quality may be attributed to a teacher's grasp of the mathematics, ability to connect to learners, or manage a classroom. For those using curriculum materials, I suggest, the quality of instruction is related to the teachers' ability to interpret, make decisions about, and leverage the resources in this tool as she designs and enacts instruction.

27.2 Research Questions and Analytical Focus

The analysis presented in this paper is particularly concerned with examining the relationship between curriculum resources and the performance of teaching. My approach draws on Brown's (2009) and others' idea that teaching is a design activity; that curriculum materials are tools that convey complex instructional ideas and that, in using them, teachers interact with these tools and selectively leverage available resources to design and enact instruction. Using video recordings of elementary teachers' mathematics lessons, together with interviews and artifacts detailing their reading of the teacher's guide, I consider the following conceptual question:

> How can enacted lessons be conceptualized and represented for the purpose of analyzing the design work teachers do during enactment?

This question is conceptual, requiring us to build a model of the work of teaching that can be used to analyze teaching performance and its relationship between the written curriculum guides. As described later in the results section,

I introduce the concept of *instructional design arcs* to model instructional episodes in a lesson that are prompted by the teacher and that require the teacher to make instructional design decisions in the moment. These arcs not only model the episodic and emerging contours of the enacted lesson, but serve as units of analysis in my effort to examine teachers' design work during the enacted curriculum and its relationship to the written or planned curriculum. My aim in this analysis is to build a tool to examine empirical questions about the relationship between the enacted and written curriculum, including how to understand variation across teachers and different types of curriculum resources.

27.3 Theoretical and Conceptual Perspectives

This study builds on three overlapping, framing perspectives: an adaptive view of curriculum (Stein et al. 2007; Remillard and Heck 2014), teaching as design work (Brown 2009), and a participatory view of teachers' use of curriculum materials (Remillard 2005).

27.3.1 Curriculum Enactment as an Adaptive Process

An adaptive perspective asserts that instantiations of curriculum unfold and develop over several temporal phases, from the ideal indicated by official policy, to the written, the teacher intended, and the enacted (Remillard and Heck 2014; Stein et al. 2007). Valverde et al. (2002) described textbooks and instructional materials as mediators between the intended and implemented curriculum. They are written to work on the behalf of teachers and students "as the links between the ideas presented in the intended curriculum and the very different world of the classroom" (p. 55). Rather than focusing on fidelity of implementation, research from this perspective examines how curriculum artifacts are transformed by policy makers, textbook authors, teachers, and students from one phase to the next, such as written to enacted, and considers factors that influence these transformations.

27.3.2 Teaching as Design and Curriculum Use as Participatory

Teachers, then, are critical decision makers within this adaptive framework. I draw on Brown's (2009) assertion that teaching involves design work, even when using curriculum materials. Teachers make design decisions when they read curriculum

materials and moblize them to plan a lesson. They also make design decisions while enacting lessons (Brown 2009).

Further, I view teachers' use of curriculum materials as a participatory process, involving interactions between the teacher and the curriculum resource. This view emphasizes the interactive and transactional nature of this work, framing curriculum use as a dynamic and ongoing relationship between teachers and resources, a relationship shaped by both the teacher and characteristics of the resource (Remillard 2005).

A fundamental theoretical thread running through these perspectives is Vygotsky's (1978) notion of practice as inseparable from tools, both employed and produced through the process and as deeply rooted in the particular context. Whereas some views of teaching and, most notably, curriculum use emphasize implementation or brokering of existing, fully formulated resources, this perspective frames teaching as a design activity and curricular resources as contributing partners in the generative work (Brown 2009; Remillard 2005).

27.3.3 Curriculum Fidelity

Many studies of teachers' use of curriculum materials focus on fidelity or the closeness of the enacted curriculum to that specified in the teacher's guide (O'Donnell 2008). Some scholars have suggested that fidelity is a complex and underspecified concept, often used in problematic ways. At the same time, Remillard (2005) notes, "It would be inaccurate and irresponsible to conclude that all interpretations of a written curriculum are equally valid." As a result, the field is "in need of ways to characterize reasonable and unreasonable variations or instantiations of a particular curriculum that are tied to features most central to its design" (pp. 239–240). Brown et al. (2009) offer an approach to studying curriculum use that differentiates fidelity to the written curriculum from fidelity to the authors' intended opportunities to learn. My analysis aligns with efforts by these researchers to conceptualize fidelity between the written and enacted curriculum as alignment to the intended opportunities to learn.

The conceptual and methodological work described in this paper is aimed at representing the curriculum enactment process in relation to the curriculum resources being used. I am interested, to some extent, in the alignment between intended opportunities to learn suggested in the curriculum guides and those opportunities made available in the classroom. More importantly, however, I am interested in understanding and describing the processes through which the enacted curriculum is designed by teachers' decisions, both when planning lessons and when enacting them.

27.4 Methods

My data were drawn from the corpus of qualitative data collected for the ICUBiT[2] study. A principal goal of research project was to identify key components of the capacity required for elementary teachers to use curriculum resources productively in their mathematics teaching.

27.4.1 Data Sources

Data collection relied on a *teaching set* methodology (Cobb et al. 2009; Simon and Tzur 1999), which involves collecting video records of multiple lessons along with associated artifacts and then using specific events or practices observed in the data as a basis for teacher interviews. The ICUBiT study collected teaching sets for 25 teachers. For the analysis in this paper, I selected four teachers, two using *Investigations in Numbers, Data, and Space* (TERC 2008) and two using *Math in Focus* (Kheong 2010). See Table 27.1 for details.

Two teaching sets were collected for each teacher, one in the fall and one in the spring. The teaching set included 3 video recorded lesson observations, a completed curriculum reading log (CRL) for the lessons taught during the week of observation, and a follow-up interview. Prior to the fall teaching set, each teacher completed an introductory interview, during which the teachers provided information about professional background and curriculum use. The CRLs consisted of a copy of the relevant lesson in the teacher's guide on which teachers used colored highlighters to indicate which parts of the guide they read for various reasons. During the follow-up interview, the interviewers asked teachers to respond to questions about the observed lessons and the CRL.

27.4.2 Data Analysis

Each lesson was divided into episodes (arcs), identified by a distinct mathematical purpose held by the teacher. Each episode began with a prompt by the teacher. Based on my analysis of the lesson, the CRL, and the follow-up interview, I coded each prompt as written, adapted, inserted, or improvised, to indicate its relationship to the lesson as described in the guide. These terms overlap with those introduced by Brown (2009) to characterize how teachers use curriculum resources, offload,

[2]The Improving Curriculum Use for Better Teaching is directed by Janine Remillard and Ok-Kyeong Kim.

Table 27.1 Participating teachers

ID	Name[a]	Grade	Curriculum	Full years teaching	Years using Curriculum
008	Maya Fiero	4	*Math in focus*	9	2
009	Meredith Frankl	5	*Math in focus*	1	1
061	Ingrid Navarra	5	*Investigations*	14	13
063	Irma Nelson	4	*Investigations*	25	12

[a]All names are pseudonyms

adapt, and improvise, as follows. A *written* prompt was drawn from the curriculum guide and aligns closely with the mathematical topic and objectives of the text, even if the teacher makes minor modifications. Brown used the term offload to refer to episodes during a lesson that a teacher relied on fully the written lesson. Similar to Brown, I labeled *Adapted* prompts as instances when the teacher used the curriculum guide as a resource, but made modifications in structure, approach, or objective. Brown used the term improvised to refer to all instances in which the teacher replaced elements of the written lesson with different activities or approaches. My codes differed, depending on whether the revision was planned or unplanned. I coded prompts as *inserted* if they were not resourced from the teacher's guide, but were planned in advance. I used *Improvised* to refer to prompts that were not in the teacher's guide, but appeared to be developed in the moment, in response to classroom situations. I also identified *omissions* from the curriculum guides and coded as significant those that I deemed to be fundamental to the designed lesson plans, with respect to accomplishing the learning objectives of the lesson. The coded lessons were then used to build a lesson map, as described in the section that follows.

27.5 Conceptualizing and Representing the Enacted Curriculum

In the section below I describe my approach to analyzing and representing the enacted curriculum in relation to teachers' design decisions. In the process, I provide a brief introduction to each of the four teachers analyzed for this paper. I also discuss how this analytical approach might be used to examine teachers' design decisions, their pedagogical design capacity, and the relationship between the written and enacted curriculum.

27.5.1 Two Types of Design Decisions

My analysis of video recorded lessons in relation to teachers' planned curriculum (measured through CRLs), supplemented by follow-up interviews revealed the range of design decisions teachers make when enacting instruction. For this analysis, I distinguish between two types of decisions: (a) *planned decisions*, made in advance of the lesson, and (b) *in-the-moment-design decisions* (*IMDDs*), made during the enacted lesson. Planned decisions refer to those made in advance. They include identifying the goals and tasks of the lesson. Often, they involve designing what Castro et al. (2007) refer to as *instructional moves*, instructions, questions, guidelines, or other types of prompts offered by the teacher during a lesson. I think of them as *initiating prompts* because they are designed to prompt student engagement or participation in some sort of mathematical work.

When using a curriculum-use lens, planned decisions also reflect the teacher's decisions in relation to the designed, specified, or suggested moves described in the teacher's guide. That is, they involve determining which parts of the written lesson to use and how to use them. In my analysis of planned decisions, I considered (a) which elements in the teacher's guide teachers chose to use or omit, (b) whether teachers used the elements as written or adapted them in some way, and (c) the types of insertions teachers made to the written curriculum.

Not all decisions teachers make during a lesson can be pre-planned. During any lesson, a teacher must make instructional-design decisions in the moment, in response to how students respond to her initial prompt. I call these decisions *in-the-moment design decisions* (IMDDs). Unlike pre-planned initiating prompts, these teacher moves are all responsive in nature. Consider the following illustration.

On December 7th, Irma Nelson, introduced her 4th grade students to the first activity in the lesson entitled *Strategies for Multiplication.* The lesson, the third session in a set of four on doubling and halving, was near the end of Book 3, *Multiple Towers and Division Stories*, in the *Investigations* program.

To introduce the activity, the teacher's guide instructs the teacher to write two expressions on the board, taken from a workbook page the students had completed two days prior: 16×3 and 16×6.

The teacher's guide offers the following prompt to begin the session:

> Let's look again at these two problems. How are they related to each other?
>
> How are the two answers related?

The guide then suggests the teacher collect a few responses about why the product is doubled when one factor is doubled. Then ask students to share story contexts or representations from their previously completed work that show why this is true. Finally, the guide suggests that if no student suggests using an array, the teacher should introduce it herself by drawing two open arrays on the board representing 16×3 and 16×6.

Ms. Nelson wrote these expressions on the overhead projector and began by asking: "Ok, um, let's take a look at, um, a problem 16 times 3 and another problem that's similar is 16 times 6, ok? What's the first thing that you notice about these problems?"

Although Ms. Nelson did not begin with the question worded exactly as it was in the guide, she initiated the task as designed by the authors. Then, as suggested, she called on a student and the following exchange transpired, which included several opportunities for IMDDs:

Ana: 3 is double 6.
Teacher: 3's double 6? Or…is 3 twice as big as 6? Ok, what you said makes me think that 3 is twice as large as 6.
Ana: 3 is, uh, two put together is 6.
Teacher: Ok so if you double up 3 you're gonna get 6. Carl, what do you notice?
Carl: I know that the 16's are still the same.
Teacher: The 16's are still the same.
Lana: Since the 3 is half of the 6, when you get your answer for 16 times 3, um, you'll just have to double that.
Teacher: Only doubling it then, all right so that will make it easier to solve the second problem. Kent?
Kent: She got it.
Teacher: She got yours? All right. Um, how could you show this if you wanted to do that in an array? How might you show that?

In the example above, Ms. Nelson displayed the suggested expressions and asked an open question that prompted students to make observations about similarities between them. As the guide suggested, she collected students' responses about the relationship between the factors in and product of the two expressions. I assess that she decided to use this part of the guide as *written*.

What transpired in the minute after offering the initiating prompt appears to reflect the intent of the curriculum authors; students made observations about the similarities between the two expressions. The particulars of the intended exchange are not specified in the guide, which advises teachers to collect students' responses about why the product is doubled using story contexts or representations. The teacher must enact these decisions in the moment. In this short example, Ms. Nelson made IMDDs in order to respond to several students' observations about the two expressions. She questioned Ana's partially correct response, accepted Carl's observation without further probing, and restated Lana's suggestion, pointing out that her observation could help them solve the second expression.

My analysis revealed that, regardless of the detail of planned decisions, made in advance of the lesson, teachers experience multiple opportunities to make additional design decisions while the lesson was being enacted. Simply put, any time a teacher asks a question or prompts students to respond in some way, she must then navigate their responses in relation to her intended objective. Teachers, for example, make decisions about which students to call on and how to respond to what

they say, be their responses correct, partially formulated, incorrect, or unrelated. They must decide when their specific objective underlying the prompt has been met and it is appropriate to move on or whether an additional instructional move needs to be improvised.

The need for teachers to make IMDDs is not necessarily a reflection of poor planning or underdeveloped resources. Rather, they reflect the substantive distinction between the written, planned, and enacted curriculum (Remillard and Heck 2014; Stein et al. 2007). Like the distinction between sheet music and a musical performance (Brown 2009), the enacted curriculum is richer, more detailed and varied than the succinct representation of a lesson in a teacher's guide. Even more significant is the fact that enacted lessons are co-constructed with students; they are shaped by students' actions and teachers' moves in relation to them. In a study of professional development sessions, Remillard and Geist (2002) used the term *openings in the curriculum* to describe similar instances, which required the facilitator to make "on-the-spot decisions, about how to guide the discourse" (p. 13).

27.5.2 Instructional Design Arcs

Through my analysis of teachers' decisions during mathematics lessons, I observed that these enacted lessons were comprised of a series of instructional episodes that typically begin with a planned instructional prompt and follow with a segment of time during which the teacher guides classroom interactions toward a particular mathematical purpose. An instructional episode ends when the teacher initiates a new prompt, usually, but not always, because the purpose has been met. I refer to these episodes as *instructional design arcs*.

I see instructional design arcs as the basic building block of an enacted lesson. They are not unanticipated, but they cannot be fully planned. Navigating these arcs is at the heart of the work of teaching. To varying degrees, curriculum authors anticipate these arcs and provide guidance to help teachers navigate them. Regardless, I posit, understanding how teachers make IMDDs in order to guide instructional arcs, including how they mobilize curriculum resources in the process, is critical to understanding the work of teaching. In the following sections, I describe my use of lesson maps to represent teachers' instructional prompts, the instructional design arcs that result, and their relationship to elements and supports in the teacher's guide.

27.5.3 *Using Lesson Maps to Model the Enacted Curriculum*

I use the following four examples of lessons, one from each of the four teachers in this analysis to illustrate the different analytical features of the lesson maps. These examples were also selected to introduce the four teachers in my analysis.

27.5.3.1 Example 1: Ms. Nelson's Enacted Lesson

The lesson map for Ms. Nelson's 12.05.2012 lesson, which took place two days before the lesson introduced earlier (comparing 16×3 and 16×6), is shown in Fig. 27.1. The timeline along the bottom of the map represents the time of the lesson in minutes. The solid circles represent instructional prompts, introduced by the teacher, and defined by an identifiable mathematical purpose. The arcs that follow represent the instructional design arcs that resulted. An arc ends when a new prompt was offered, which usually occurred when the teacher's mathematical purpose was met or the teacher deemed a new prompt was appropriate.

In this lesson, all prompts fall on the lower horizontal line, which indicates that they were drawn from the teacher's planned lesson. The color of circles indicates the source of the arcs in relationship to the curriculum guide. The first arc began with an inserted prompt, marked by the red circle, which represents a timed multiplication exercise that the teacher inserted into the lesson from outside of the curriculum guide. Then, at minute 5:48, Ms. Nelson began the lesson based on the teacher's guide. She directed students to a story problem in their workbook: Ms. Santos has 168 apples. She wants to pack them into boxes of 28. How many boxes does she need? She asked, "Is this a multiplication problem or a division problem?" This arc aligned with the curriculum guide and is therefore coded as written, indicated by a black circle. After the class discussed the problem and various ways to solve it, at minute 10:56, Ms. Nelson directed the students to solve the division problem using a strategy suggested by one of the students. This arc was coded as adapted because the teacher significantly changed the mathematical purpose of the task, indicated by the gray circle.

The remainder of the lesson, represented by the lesson map, was comprised of several more arcs sourced from the teacher's guide as written. The long arc, initiated at time 39:52 min involved students working in small groups to complete a page of practice problems included in the lesson, while the teacher circulated. Typically, segments involving small group or individual student work were coded as a single arc, based on a single objective, even though the teacher's interactions with individual students or groups might have been initiated by distinct prompts.[3] Occasionally, although not in this case, I found that teachers interrupted

[3]Project video records do not provide us with a reliable record of these exchanges during small-group work periods.

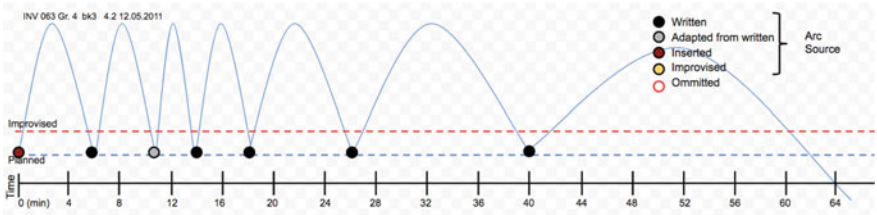


Fig. 27.1 Lesson map represents prompts and design arcs in Irma Nelson's 12.05.2011 lesson

small-group work periods to engage the entire class in an exchange; these were coded as new prompts. The lesson ended after 64 min, at the end of the long arc.

27.5.3.2 Example 2: Ms. Navarra's Improvised Arcs

Not all prompts initiating design arcs came from the teacher's planned curriculum. It was not unusual for a teacher to initiate a new arc with an *improvised* prompt, based on an IMDD. These prompts are marked with a yellow circle, placed on the improvised line on the map. As described above, within each instructional arc, teachers make many IMDDs. An IMDD becomes a prompt, initiating a new arc when it has a distinct mathematical objective. I considered an arc improvised when there was evidence that it had occurred spontaneously, often in response to an event that occurred during the previous episode. I also consulted the follow-up interview transcript and the teacher's CRL to determine whether a prompt had been preplanned.

Ingrid Navarra's lesson on decimals on 02.06.2012 illustrates an improvised arc, along with inserted and adapted arcs (Fig. 27.2). Ms. Navarra, a 5th grade teacher also using *Investigations,* taught a lesson drawn from Book 6, Session 1.3: *Decimals on the Number line*. Like Ms. Nelson's, her lesson map shows she began the lesson with an inserted prompt. Ms. Navarra's inserted prompt was in the form of a review. She asked students to tell her some things they knew about decimals.

At 2:30 min, Ms. Navarra moved into Activity 2 in the teacher's guide, called *Introducing Decimals on a Number Line*. The teacher's guide recommends drawing a number line on the board that begins at 0 and ends at 2, and having students place

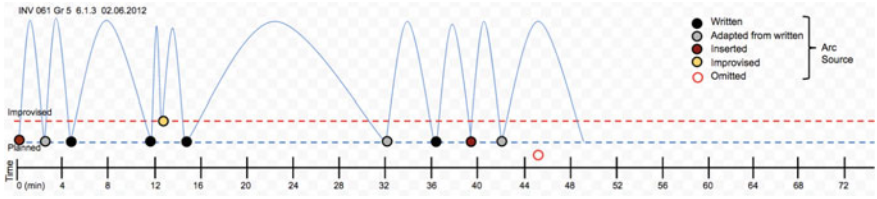


Fig. 27.2 Lesson map of Ingrid Navarra's 02.06.2012 lesson

the following numbers on it: 0.3, 0.5, 1.25, 1.8. Ms. Navarra drew a number line that went from 0 to 1 and modified the numbers students were asked to place, writing them as follows: 0.3, 0.5, 1.25, 0.05. She then pointed to the four numbers and offered the prompt: "Who goes where?" I coded this prompt as adapted because these modifications changed the objective of the task. In the follow-up interview, when asked about the addition of 0.05 to the list of numbers, Ms. Navarra explained, "They have a hard time with this book knowing the difference between what 0.05 is and 0.5 or 0.2 and 0.02. So, I'm always trying to throw those in there."

She initiated an improvised arc at 13:19 min. Students were working on the activity in the lesson labeled *Ordering Tenths and Hundredths.* Using a set of decimals cards, which included decimals to the hundredths place, the students were to place the cards in order from smallest to largest. The guide also suggests the teacher check students' decimal ordering after they finish their work. Ms. Navarra, following the mathematical objective of the activity in the text, which is "ordering decimals and justifying their ordering through reasoning about decimals representations, equivalents, and relationships," asked students to explain the strategy they used to order the Decimal Card set. A student explained that his group counted by 5's. At that point, Ms. Navarra initiated an improvised arc by asking how much 0.05 (pointed at the number) would be if it were money. As the excerpt below illustrates, she pointed to several decimal numbers and asked students to give the value in money.

Teacher: You counted by 5's. All right, if this were money, how much would this be?
Student: 5 cents.
Student: A nickel.
Teacher: How much would this be? [Points to other numbers on the overhead]
Student: 50 cents, a half dollar.
Student: 2 quarters.
Teacher: 2 quarters a half dollar, 50.
Student: Three quarters…
Student: One quarter.
Student: A dime.
Student: Uh, 90 cents.
Student: 9 dimes.
Teacher: Ok, now. It's gonna get a little harder…

This improvised arc lasted approximately two minutes and was aimed at pushing students to connect the value of decimal numbers to related amounts of money. Typically, improvised arcs are initiated by what is happening in the enacted lesson. In this case, the *counting by* 5's strategy used by a group prompted her to be sure they understood what those "5's" represented. In the follow-up interview, Ms. Navarra said: "This is a nickel versus two quarters. And I always take it back to money, 'cause they like money" (Follow-up Interview, Spring 2012).

Ms. Navarra's lesson map also includes one significant omission (open red circle) in minute 45. I discuss omissions in the following section.

27.5.3.3 Example 3: Ms. Fiero's Significant Omissions

I use Ms. Fiero's lesson on multiplying 2-digit numbers by multiples of tens to illustrate my identification of significant omissions. All five of the curriculum programs in the study include more options than can possibly be used in a single lesson. They also include tasks and suggestions that are designed to be optional, allowing the teacher to tailor the curriculum to the particular students (Remillard and Reinke 2012). It is my expectation that teachers will make adaptations to the written curriculum based on their assessments of students' need and their preferences. Thus, I anticipate that teachers will omit certain suggestions and I do not attempt to capture all of them in the lesson maps. I do, however, identify omissions that I assess to be fundamental to accomplishing the learning objectives of the designed lesson plans.

Maya Fiero, a 4th grade teacher using *Math in Focus,* made what I consider significant omissions during her 12.05.2011 lesson. One of the lesson objectives listed in the teacher's guide was for students to "Multiply by 2-digit numbers with or without regrouping." In the first part of the lesson students are introduced to the multi-step approach to multiplying numbers by multiples of 10 based on the associative property of multiplication, shown on the student page in Fig. 27.3.

The teaching notes that accompany this introductory page state:

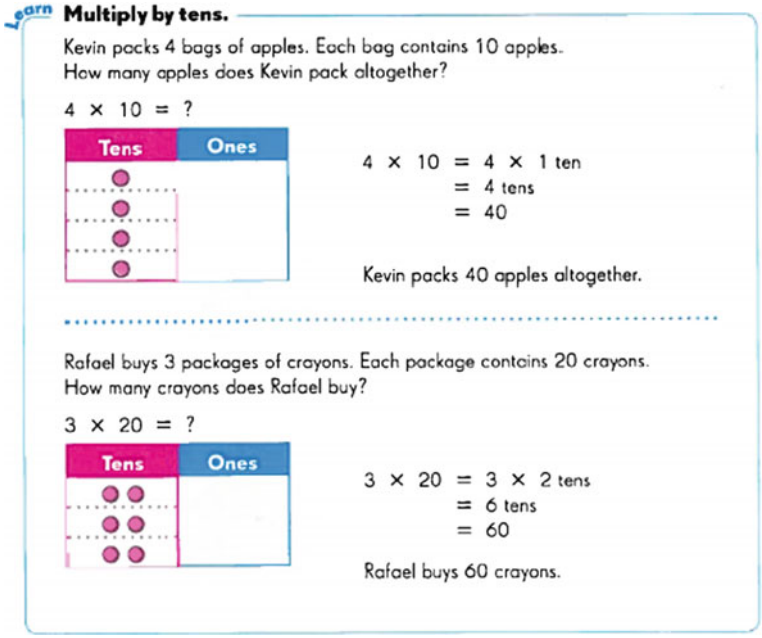
Students learn to multiply by 2-digit numbers in the form of tens.

- Help students recall the strategy for multiplying a number by tens by working through the examples in the Student Book.
- In the first example, express 10 as 1 ten. So $4 \times 10 = 4 \times 1$ ten = 4 tens = 40.
- Using the strategy, work through the second example with students.
- First, express 20 as 2 tens. So $3 \times 20 = 3 \times 2$ tens = 6 tens = 60.
- For students who cannot visualize multiplying by tens, use a place-value chart to show the connection.

The teacher's guide also suggests beginning the lesson with the following 5-minunte warm up:

- Have students work in pairs. Each partner takes turns giving and solving a multiplication problem to multiply 1-digit numbers by tens and hundreds mentally, for example, 6×100.
- Repeat with 2-digit numbers. Encourage students to identify the pattern.
- This activity helps recapitulate the previous lesson and provides a warm-up for this lesson. (p. 86)

Ms. Fiero's lesson map is shown in Fig. 27.4. The lesson began, as is typical for Ms. Fiero, with a warm up drawn from another source, indicated with a red


Learn **Multiply by tens.**

Kevin packs 4 bags of apples. Each bag contains 10 apples.
How many apples does Kevin pack altogether?

4 × 10 = ?

Tens	Ones
●●●●	

4 × 10 = 4 × 1 ten
= 4 tens
= 40

Kevin packs 40 apples altogether.

Rafael buys 3 packages of crayons. Each package contains 20 crayons.
How many crayons does Rafael buy?

3 × 20 = ?

Tens	Ones
●● ●● ●●	

3 × 20 = 3 × 2 tens
= 6 tens
= 60

Rafael buys 60 crayons.

Fig. 27.3 Curriculum guide excerpt from MATH IN FOCUS: The Singapore Approach. Copyright © 2009 by Houghton Mifflin Harcourt Publishing Company. All rights reserved. Reprinted by permission of Houghton Mifflin Harcourt Publishing Company

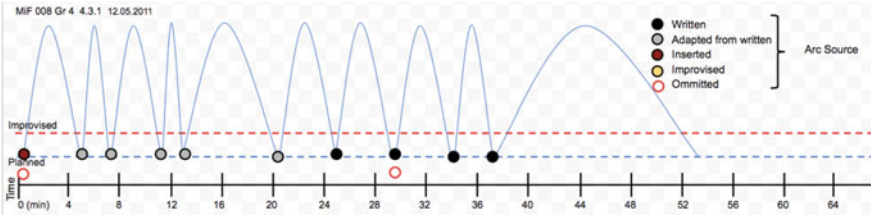


Fig. 27.4 Lesson map of May Fiero's 12.05.2011 lesson

"inserted circle; this one involved practicing 2- by 1-digit multiplication. As a result, she omitted the warm up, which involved students practicing multiplying single-digit numbers by 10 and 100. This omission is marked with an open circle because facility with multiplying numbers by 10 is anticipated in the introduction to multiplying numbers by multiples of 10.

Before moving onto this introduction page of the lesson, Ms. Fiero guided students through a review of the previous day's multiplication work on 1- and 2-digit multiplication. Because these activities involved material drawn from the previous days' lesson, I coded each prompt as adapted. At 25:52 min, Ms. Fiero began the introduction shown in Fig. 27.4. They began with a short discussion of

title, objective, and key vocabulary listed on the first page of the student workbook. With the next prompt, at 29:41, she moved the class to considering how to multiply numbers by 10. The guidance in the teacher's guide states: "Help students recall the strategy for multiplying a number by tens by working through the examples in the Student Book." Rather than using the examples in the student book, Ms. Fiero, reminded the students that they had multiplied numbers by 10 previously. She wrote 81 × 10 on the white board to illustrate. For the next several minutes, the students struggled to provide an answer. Even when she moved the class onto working with the two examples on the page (Fig. 27.3), students had difficulty recognizing that 4 × 10 was equivalent to 4 tens or 40. The teacher's guide included the following suggestion: "For students who cannot visualize multiplying by tens, use a place-value chart to show the connection." Even though the place value charts were on the student page, Ms. Fiero did not refer students to these models once. I marked this as another significant omission.

My review of the lesson description in the curriculum, in light of the difficulties students were having multiplying numbers by 10, 20, or other multiples of 10, suggests that the omissions described above left out critical steps of the designed learning sequence. For this reason, I coded them as significant omissions.

It is important to note that identifying omissions of lessons closely tied to the curriculum is easier than lessons that have little relationship to the curriculum. The following example illustrates a lesson comprised fully of inserted arcs, from which omissions are not indicated.

27.5.3.4 Example 4: Ms. Frankl's Inserted Arcs

Meredith Frankl was a 5th grade teacher using the *Math in Focus* program. Her CRL, which provides details of her planned lesson, indicates that she used the mathematical topic of the lesson, the relationship between area and perimeter, to guide the design of the lesson. As the lesson map (Fig. 27.5) indicates, the lesson was comprised of 9 instructional design arcs, all inserted by the teacher. Although the mathematical topic of the lesson overlapped with the lesson in the written curriculum, the enacted lesson did not reflect the written curriculum in objective, structure, tasks, or approach. For this reason, it was not possible to identify omitted tasks.

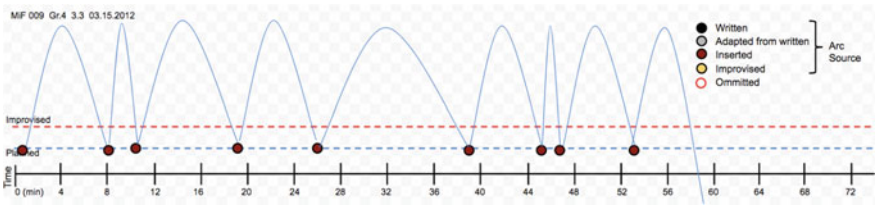


Fig. 27.5 Lesson map of Meredith Frankl 03.15.2012 lesson

Ms. Frankl's lesson map illustrates that regardless of the role the written curriculum plays in shaping it, the enacted lesson can be represented in terms of mathematical episodes initiated by the teacher and through which the teacher must steer the students. The coding system also allows researchers to represent the relationship between these episodes and the written curriculum. In the following section, I discuss some empirical possibilities and next steps based on this approach.

27.6 Conclusion and Next Steps: Using Lesson Maps to Examine the Enacted Curriculum and Factors that Influence It

In addition to representing the contours of the enacted lesson, I see lesson maps as analytical tools to examine the relationship between the written and enacted curriculum, factors that influence this relationship including teacher and curriculum capacity, and implications for lesson quality. I conclude by discussing some possible uses of lesson maps to pursue initial questions in this area.

27.6.1 Examining Patterns in Teachers' Enacted Curriculum

Lesson maps allow us to explore patterns and themes in an individual teacher's lesson and make comparisons to other teachers. Examining several maps of one teacher's lessons along side those of another allows us to explore possible patterns and contrasts. I have found a great deal of internal consistency among each teacher's lesson maps, but have, thus far, observed little similarity between the maps of different teachers using the same curriculum program (Remillard et al. 2015). Lesson maps, and patterns across them, provide a starting place to probe individual teachers' design decisions more deeply. For instance, patterns in the placement of inserted or adapted prompts, or significant omissions, can point to a teacher's interpretation and goals that merit further study. Further, particular types of arcs might be associated with teachers' steering moves during other lesson episodes. Ms. Fiero's decision to omit the warm-up (see Sect. 27.5.3.3) might be related to difficulties students experienced later in the lesson and the IMDDs that followed.

Patterns in lesson maps across teachers can raise additional questions for further analysis. For instance, Ms. Fiero was not the only teacher who began her lesson with an inserted arc in the form of a warm-up task, omitting the initial task in the written curriculum. This pattern is seen in all the maps in Sect. 27.5 and was noted in many other maps in the data set. This tendency may reveal an important

phenomenon in teachers' design decisions that merits further exploration. Understanding these decisions can provide insight into factors that influence the quality of the enacted lessons.

27.6.2 Understanding in-the-Moment-Design Decisions

The lesson maps provide a skeletal representation of the lesson in terms of structure and source, but offer limited detail about the interactions that take place within instructional design arcs. Further analysis is needed to examine the interactions within the arcs in order to understand how the enacted lesson unfolds, including how teachers' decisions during instruction influence this unfolding. I introduce the term IMDD to refer to the decisions teachers make during these instructional episodes. More work is needed to understand IMDDs in context and their consequences for the quality of the enacted lesson. I see promise in uncovering patterns in the types of IMDDs teachers make and the factors that influence them, including tracing possible influences that the teacher's guide has on IMDD's. I hypothesize that a teachers' ability to make high quality IMDDs is a critical component of pedagogical design capacity (Brown 2009).

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