# Rogue Waves in the Ocean, the Role of Modulational Instability, and Abrupt Changes of Environmental Conditions that Can Provoke Non Equilibrium Wave Dynamics

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*Dedicated to Eugene G. Morozov on his 70th Birthday*

## Introduction

Rogue or freak waves are waves that are unexpectedly large in comparison with ambient waves. The term "freak wave" was apparently introduced by Draper [16] who discussed ship accidents that could have been provoked by such waves. Mallory [32] discussed serious consequences of extreme waves on ship traffic in the Agulhas current off South Africa. With the advent of offshore oil and gas exploration, well documented observations of freak waves accumulated (e.g. Sand et al. [48]). The "New Year" wave at Draupner in the North Sea 1/1–1995 is one of the best documented observations of a rogue wave [8, 25]. Later the even more extreme Andrea wave at Ekofisk in the North Sea 9/11–2007 was also well documented [7, 15, 31].

This is a good opportunity to give praise to the policy of openness demonstrated by the Norwegian oil company Statoil giving academic researchers unrestricted access to the Draupner "New Year" wave dataset. Few measured time series have been published more often, or inspired more wave research, than the one shown in Fig. 1.

There is currently no consensus on how to define rogue or freak waves. Common criteria are $H/H_s > 2$ or $\eta_c/H_s > 1.25$ where $H$ is the zero-crossing wave height, $\eta_c$ is the crest height, and $H_s$ is the significant wave height defined as four times the standard deviation of the surface elevation typically calculated from a 20 min time series [17, 24].

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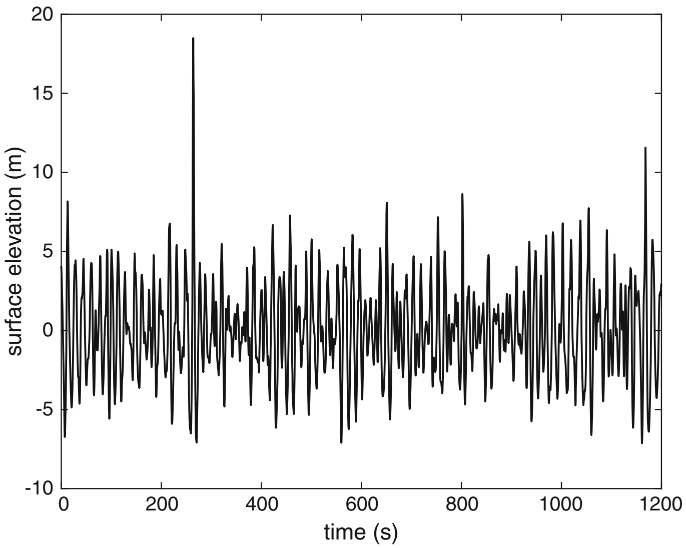


**Fig. 1** Twenty minute wave elevation time series measured by a downward pointing radar at 16/11–E in the Norwegian sector of the North Sea on 1/1–1995. First axis is time in seconds, second axis is elevation in meters. Data courtesy of J. I. Dalane and O. T. Gudmestad of Statoil

The existence of rogue waves in the ocean is nowadays well accepted and they are recognized as a threat and a challenge for human activity offshore [6, 17, 27]. Indeed, recently a large wave killed one person and injured several others in the COSL Innovator accident at Troll in the Norwegian Sea 30/12–2015 [61].

Most attention to rogue wave generation has so far been given to random events within Gaussian seas, including linear refraction due to currents or bathymetries, random events within slightly non-Gaussian seas due to static nonlinearities or dynamic nonlinear evolution in equilibrium wave fields, and modulational instabilities which are nonlinear instabilities of perturbations around steady states. While such mechanisms can indeed produce freak waves, and such criteria are nowadays used as warning criteria in operational forecasting, validation performed by e.g. the Norwegian Meteorological Institute has shown that some improvements are still necessary for the warning criteria to be fully satisfactory [6].

The purpose of the present paper is to point out a possible future direction for rogue wave research that might provide some of the desired improvements. There is indeed a different path to rogue wave formation, one that has received little attention so far and is not accounted for in the above practices and criteria, namely the dynamic evolution of non-equilibrium wave fields, where the lack of equilibrium is not due to a small perturbation away from a steady state.

Recent laboratory experiments [47, 58] and numerical computations [22, 60] have shown that non-equilibrium evolution of wave fields can produce surprisingly rough wave conditions. We anticipate that wave fields that are brought out of equilibrium due to rapidly varying meteorological conditions, or significantly non-uniform environments such as currents or bathymetries [36], or possibly the sudden appearance of a ship in a wave field [34, 35], can be a significant source of rogue waves.

The paper is organized as follows: section "Common Theories for Rogue Waves" gives a summary of common theories on which current rogue wave warning criteria are typically based, section "Case Study on Rogue Waves Through Common Theories: The Prestige Accident" presents a case study for the involvement of rogue waves in the Prestige oil tanker accident based on the common theories, section "Rogue Waves in Non Equilibrium Wave Fields" points out an alternative type of mechanism for rogue wave generation typically not included in todays warning criteria, and "Conclusions" provides a conclusion.

## Common Theories for Rogue Waves

Within linear wave theory (LWT) there are several mechanisms that can provoke large waves, e.g. spatio-temporal focusing of waves, refraction over uneven depth and refraction over non-uniform currents. Within LWT, employing the principle of superposition and the Central Limit Theorem from probability theory, it is anticipated that the resulting distribution of surface elevation is Gaussian (e.g. Pierson, [46]).

Rogue waves are known to occur more often than anticipated from LWT, this enhanced occurrence is generally accepted to be due to nonlinearity. There are several known nonlinear mechanisms that can be responsible for this.

Second-order corrections to deep-water waves, static or bound wave nonlinear corrections in general, are known to provoke small deviation from Gaussian statistics. These corrections were derived for uniform waves by Stokes [50], for deep-water irregular gravity waves by Tick [53] and Longuet-Higgins [29] and further by Masuda et al. [33]. Static nonlinear corrections to linear wave theory form the basis for Tayfun-distributions [52].

Starting with the observation that steady uniform waves are unstable to small perturbations, the Benjamin–Feir instability [3, 4], or more general modulational instability (MI) was soon recognized as a mechanism that could initiate the generation of extreme waves. It was soon recognized that the cubic nonlinear Schrödinger (NLS) equation [5, 13, 23, 64] is the simplest nonlinear model that accounts for this instability mechanism. The MI occurs if the ratio between the steepness of the uniform wave and the spectral bandwidth of the perturbation is above a threshold.

Soon after the first well-documented observations of rogue waves in the ocean, e.g. the Draupner wave, it was suggested that the generation of such waves in the ocean could be explained by weakly nonlinear and narrow-banded models, in particular by the nonlinear Schrödinger (NLS) equation. Trulsen and Dysthe [56] argued that it would be an advantage to use a broader-bandwidth modification of the modified nonlinear Schrödinger (MNLS) equation of Dysthe [18] for better representation of realistic bandwidths [55, 57].

Breather solutions [19, 28, 30, 44, 45] are long-time and long-distance solutions of the NLS equation, starting from an infinitesimal perturbation of a uniform wave train, resulting in extreme waves localized in space and time, occurring once or repeatedly in space and/or in time. Breather solutions have recently been studied in laboratory experiments [9–11].

Related to the discovery of modulational instability of uniform wave trains, it was also recognized that steady homogeneous wave fields are unstable to small inhomogeneous perturbations [1, 2, 12]. This instability occurs if the ratio between the steepness and the spectral bandwidth of the wave field is above a threshold.

The Benjamin–Feir index (BFI), coined by Janssen [26], previously suggested by Onoratoet al. [37, 39] under a different name, is precisely the ratio between the steepness and the spectral bandwidth of the wave field. It has been suggested as an indicator to predict increased probability of rogue waves. The BFI has been shown to be useful as an indicator for extreme waves in unidirectional seas [40, 41] easily reproduced in long and narrow laboratory tanks. However, the BFI has been found to be less useful in directional seas in simulations [21, 38] and in laboratory tests [42, 43] as well as an operational forecasting criterion [6].

A different path from swell and wind-sea interaction to freak wave generation was suggested by Tamura et al. [51] who speculated that the nonlinear coupling between swell and wind-sea could generate a narrow spectrum. Some ship accidents indeed seem to have occurred in conditions of narrowing wave spectra [51, 62, 63]. It has indeed been observed that the directional spread sometimes has been reduced ahead of increased rogue wave occurrences at sea [54].

Recently, researchers have started suspecting that MI is not the correct or the only path to explain real world ocean rogue waves, the waves being in general too broad-banded and short-crested for the BFI to be useful (e.g. Fedele et al. [20]).

## Case Study on Rogue Waves Through Common Theories: The Prestige Accident

Freak waves are sometimes the subject of great controversies. There is for example still no consensus on the cause of the Prestige oil tanker initial accident on 13/11–2002, which subsequently led to a major environmental disaster after the sinking of the ship on 19/11–2002. The magnitude of the environmental disaster provoked heated debate in mass media and in court regarding the likelihood that the initial accident could have been caused by a rogue wave (see Trulsen [59]).

In the recent study of Trulsen et al. [59] newly computed hindcast spectra for every hour during the day of the accident were used as input data for four different nonlinear models capable of computing the phase-resolved sea surface, allowing to estimate statistical parameters that characterize the conditions for rogue waves. All four models coincided that the wave conditions encountered by the tanker Prestige at the moment of the accident were slightly more extreme than those of a Gaussian sea state, and slightly less extreme than those of a Tayfun sea state. This study strongly suggests that the probability of a rogue wave hitting the oil tanker was neither greater than nor smaller than usual.

## Rogue Waves in Non Equilibrium Wave Fields

Practical experience with the common theories for rogue waves, e.g. application of the BFI as a warning criterion, suggests that some improvements may still be necessary for the warning criteria to be fully satisfactory [6]. We here point out that there is indeed another path to the formation of rogue waves.

It is often observed that numerical simulation of nonlinear wave evolution, initialized with artificial initial conditions, can need some evolution time or distance before the wave fields become well-behaved. During the initial transient evolution extreme events are often seen to occur. Some effort has been made to suppress this behavior, e.g. by Dommermuth [14], although the "problem" is typically dealt with simply letting the numerical simulations run over sufficient time or distance before the results are used. On the other hand, a very interesting situation would arise if this "initial" strange behavior was the result of a sudden change of physical environment rather than artificial initialization of a numerical integration.

Recently it has been observed that irregular wave fields that propagate from deeper waters into shallower waters can have significant amplification of kurtosis and freak wave statistics some distance inside the transition to the shallower depth. This behavior was first discovered in an experimental dataset from MARIN in The Netherlands by Trulsen et al. [58], subsequently is was studied numerically by Sergeeva et al. [49], Zeng and Trulsen [65], Gramstad et al. [22] and Viotti and Dias [60]. This is a nonlinear effect that is neither explained by MI nor by linear refraction.

Recently Raustøl [47] carried out fine-resolution experiments and measured that the kurtosis could be amplified to a value of 6, occurring at a location approximately one wavelength on the inside of the depth transition to shallower water. She also identified thresholds for water depths when this amplifying behavior took place. It is interesting to note that this extreme amplification of kurtosis took place precisely in a wave field that was not modulationally unstable.

It is common to treat a sea state as being statistically stationary when in fact it varies. Meteorological forecasting services typically give forecasts for every three hours. In Trulsen et al. [59] the Prestige accident was studied with hindcasts every hour, making the assumption that the sea state was constant during each of the one-hour intervals. In the case that the sea state varied dramatically within the one-hour intervals, the nonlinear phase-resolving simulations of Trulsen et al. [59] could be rendered invalid. An insufficient amount of work has been done to identify what happens if the meteorological conditions and sea state change sufficiently fast that the wave field is not in an equilibrium state. Indeed, Tamura et al. [51] suggested that sea states can vary in time in such a way that the occurrence of rogue waves may be affected.

The presence of a ship in a wave field is known to locally affect the wave field near the ship. If the ship appears suddenly into an already established wave field, it may also represent a perturbation that brings the wave field temporarily out of equilibrium [34, 35].

Indeed, in the recent review of Onorato and Suret [36] it is speculated that a change of ambient conditions can bring a wave field out of equilibrium, thus provoking amplification of kurtosis before the wave field is brought back to equilibrium.

## Conclusions

Rogue waves are known to occur more frequently than expected from linear wave theory, more frequently than expected within Gaussian seas. The common nonlinear theories for rogue waves explain such deviation by weakly nonlinear effects on top of equilibrium linear sea states, or as the effect of modulational instability due to unstable perturbations of steady states. The degree of modulational instability of a steady sea state is sometimes assessed by the so-called Benjamin–Feir Index (BFI). There is however a different mechanism for rogue wave generation, viz. nonlinear dynamics of wave fields that are not in an equilibrium state. This mechanism is not indicated by the value of BFI, since the modulational instability is not relevant in the absence of a steady state. We have recently performed experiments at the Department of Mathematics at the University of Oslo revealing that a substantial amplification of kurtosis can occur in non-equilibrium wave fields that are not modulationally unstable.

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