Chapter 11
The Two-Dimensional Strip Packing Problem: What Matters?

Alvaro Neuenfeldt Júnior, Elsa Silva, A. Miguel Gomes and José Fernando Oliveira

Abstract This paper presents an exploratory approach to study and identify the main characteristics of the two-dimensional strip packing problem (2D-SPP). A large number of variables was defined to represent the main problem characteristics, aggregated in six groups, established through qualitative knowledge about the context of the problem. Coefficient correlation are used as a quantitative measure to validate the assignment of variables to groups. A principal component analysis (PCA) is used to reduce the dimensions of each group, taking advantage of the relations between variables from the same group. Our analysis indicates that the problem can be reduced to 19 characteristics, retaining most part of the total variance. These characteristics can be used to fit regression models to estimate the strip height necessary to position all items inside the strip.

Keywords Strip packing problems · Cutting and packing problems · Principal component analysis · Knowledge discovery

11.1 Introduction

In the 2D-SPP the aim is to pack a set of rectangular items inside a rectangular object with a fixed width, minimizing the height dimension of the object that is infinite. The small items can be rotated, orthogonally positioned without overlapping and completely inside the object, also the This description fits in the definition of cutting

A. Neuenfeldt Júnior · E. Silva (✉) · A. Miguel Gomes · J. F. Oliveira
INESC-TEC, Faculty of Engineering, University of Porto, Porto, Portugal
e-mail: emsilva@inesctec.pt

A. Neuenfeldt Júnior
e-mail: alvaro.l.junior@inesctec.pt

A. Miguel Gomes
e-mail: agomes@fe.up.pt

J. F. Oliveira
e-mail: jfo@fe.up.pt

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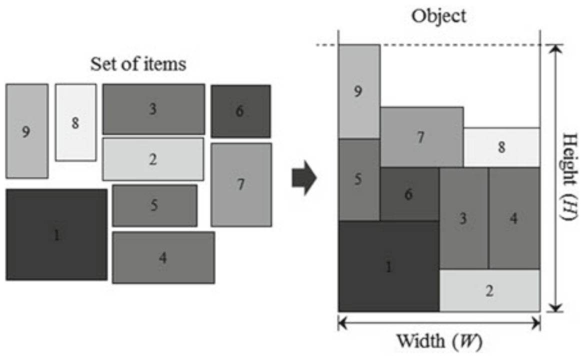

Fig. 11.1 A general view of the rectangular 2D-SPP

and packing problems and indeed the 2D-SPP can be classified as an open dimension problem [23]. An example can be found in Fig. 11.1.

Over the years a considerable number of test problem instances appeared in the literature to test the different heuristics that have been developed to solve the 2D-SPP. However, none of the developed heuristics were able to solve efficiently all the existing test problem instances and 2D-SPP variants.

The test problem instances are generally created with the use of some problem generators which were developed considering specific characteristics, methodologies and input parameters. As a consequence, it is possible to find data instances in the literature with different characteristics and combinations between items and object shape variation [19].

Data mining techniques can be used to facilitate a better understanding of the test problem instances characteristics, ensuring that the main details of the problem is known [21].

In this paper, we conduct an exploratory research to find the most relevant test problem instances characteristics for the two-dimensional rectangular 2D-SPP. An initial set of variables (more than fifty) are used to represent these characteristics, and the PCA was chosen as a technique to explore the relations among variables and to convert them in a smaller number of components. A sample of 1217 test problem instances extracted from the literature is explored.

A similar approach was developed by López–Camacho et al. [16], where a PCA was considered to develop two-dimensional analysis with the aim of better understand the structure of the two-dimensional (and three-dimensional) bin packing problems. This information was used to compare the performance of heuristics with a wide set of characteristics provided by test problem instances found in the literature. López–Camacho et al. [17] also developed a hyper-heuristic approach and compared the computational results with other approaches using the components developed through the PCA.

11.2 Strip Packing Problem Characteristics

Problem generators are one of the most efficient ways to replicate real-world applications of different problems over controlled scenarios, ensuring the reproducibility of the test problem instances. Problem generators can be used as one source of information to study the characteristics of packing problems. A literature review about problem generators for the 2D-SPP is presented below and will serve as a base for the development of explanatory variables.

Wang and Valenzela [22] developed two important factors to generate test problem instances for the rectangular placement problems: the maximum area ratio between all pairs of items; and the aspect ratio, to identify the variation level between the largest and the smallest dimension of each item. As consequence, the maximum aspect ratio identifies the item that has a greater difference in its dimensions.

Regardless the total number of items, larger values for aspect ratio and area ratio indicates a more significant variability in items shape, which allows the generation of more heterogeneous data instances. For example, "nice" instances have items of smaller size and variability, which indicate a homogeneous behavior. In contrast, "path" instances have items with larger size and variability, which characterizes test problem instances with higher heterogeneity.

Bortfeldt and Gehring [6] used the degree of heterogeneity among the items as one of the primordial factors to generate data instances. The degree of heterogeneity measures the ratio between the total number of different items in comparison to the total number of items of an test problem instance. This characteristic is reaffirmed by Bortfeldt [5] as one of the most important aspects to be considered in cutting and packing problems.

In Berkey and Wang [4] is considered the width ratio, that is calculated using the object width and items dimensions. This measure is one of the most important to develop test problem instances for both two-dimensional bin packing problems and 2D-SPP. The influence of width ratio on the quality of solutions for the strip packing was verified mainly in test problem instances with a smaller number of items ($n < 100$). Smaller width ratios indicated a greater probability of obtaining lower object heights.

The 2DCPackGen problem generator proposed by Silva et al. [20] is able to generate data instances for all two-dimensional (and three-dimensional) rectangular cutting and packing problems. In specific, for the 2D-SPP, the number of different item types, the item type demand and size and shape of the items are some of the measures that influences the assortment of items.

Leung et al. [15] combined in a dataset with 16 test problem instances some characteristics of instance $gcut13$ by Beasley [1] and instances cx by [9] to obtain items with different maximum and minimum areas using the generator proposed by Wang and Valenzela [22]. The main objective was to evaluate the capacity of some heuristics to solve problems with a strongly heterogeneous items shape variation, where the position of the larger items into the object is a determinant factor to obtain good solutions.

Table 11.1 Test problem instances variables summary

Variable	Definition
Area	
$arearatioextr$	$arearatioextr = area_{max}/area_{min}$
$arearatioperc$	Ratio between percentile 90% and percentile 10% of $area_i$
$arearatioquart$	Ratio between third quartile and first quartile of $area_i$
$areacompnormal$	Compensation between the sum of 50% larger $area_i$ and the sum of 50% smaller $area_i$
$areacompquart$	Compensation between the sum of 25% larger $area_i$ and the sum of 25% smaller $area_i$
$areacompextr$	Compensation between the sum of 10% larger $area_i$ and the sum of 10% smaller $area_i$
$areamean$	Mean value of $area_i$
$areamed$	Median value of $area_i$
$areastdev$	Standard deviation of $area_i$
Perimeter	
$perimratioextr$	$perimratioextr = perimeter_{max}/perimeter_{min}$
$perimratioperc$	Ratio between percentile 90% and percentile 10% of $perimeter_i$
$perimratioquart$	Ratio between third quartile and first quartile of $perimeter_i$
$perimcompnormal$	Compensation between the sum of 50% larger $perimeter_i$ and the sum of 50% smaller $perimeter_i$
$perimcompquart$	Compensation between the sum of 25% larger $perimeter_i$ and the sum of 25% smaller $perimeter_i$
$perimcompextr$	Compensation between the sum of 10% larger $perimeter_i$ and the 10% smaller $perimeter_i$
$perimmean$	Mean value of $perimeter_i$
$perimmed$	Median value of $perimeter_i$
$perimstdev$	Standard deviation of $perimeter_i$
Dimensions	
$vardim$	$vardim = W/[\sum(d1_i + d2_i)/n]$
$dimratioextr$	$dimratioextr = dimension_{max}/dimension_{min}$
$dimratioperc$	Ratio between percentile 90% and percentile 10% of $dimension_i$
$dimratioquart$	Ratio between third quartile and first quartile of $dimension_i$
$dimcompnormal$	Relation between the sum of 50% larger $dimension_i$ and the sum of 50% smaller $dimension_i$
$dimcompquart$	Compensation between the sum of 25% higher $dimension_i$ and the sum of 25% smaller $dimension_i$
$dimcompextr$	Compensation between the sum of 10% higher $dimension_i$ and the sum of 10% smaller $dimension_i$
$dimmean$	Mean value of $dimension_i$
$dimmed$	Median value of $dimension_i$
$dimstdev$	Standard deviation of $dimension_i$

(continued)

Table 11.1 (continued)

Variable	Definition
Width dimensions	
$widthdimratioextr$	$widthdimratioextr = widthdimension_{max}/widthdimension_{min}$
$widthdimratioperc$	Ratio between percentile 90% and percentile 10% of $widthdimension_i$
$widthdimratioquart$	Ratio between third quartile and first quartile of $widthdimension_i$
$widthdimcompnormal$	Compensation between the sum of 50% larger $widthdimension_i$ and the sum of 50% smaller $widthdimension_i$
$widthdimcompquart$	Compensation between the sum of 25% larger $widthdimension_i$ and the sum of 25% smaller $widthdimension_i$
$widthdimcompextr$	Compensation between the sum of 10% larger $widthdimension_i$ and the sum of 10% smaller $widthdimension_i$
$widthdimmean$	Mean value of $widthdimension_i$
$widthdimmed$	Median value of $widthdimension_i$
$widthdimstdev$	Standard deviation of $widthdimension_i$
Proportions	
$aspectratio$	$aspectratio = (D1/D2)/[\sum(d1_i/d2_i)/n]$
$propratioextr$	$propratioextr = proportion_{max}/proportion_{min}$
$propratioperc$	Ratio between percentile 90% and percentile 10% of $proportion_i$
$propratioquart$	Ratio between third quartile and first quartile of $proportion_i$
$propcompnormal$	Compensation between the sum of 50% larger $proportion_i$ measures and the sum of 50% smaller $proportion_i$
$propcompquart$	Compensation between the sum of 25% larger $proportion_i$ measures and the sum of 25% smaller $proportion_i$
$propcompextr$	Compensation between the sum of 10% larger $proportion_i$ measures and the sum of 10% smaller $proportion_i$
$propmean$	Mean value of $proportion_i$
$propmed$	Median value of $proportion_i$
$propstdev$	Standard deviation of $proportion_i$
Other	
n	Total number of items in the test problem instance
$coefficient$	$coefficient = [(\Sigma d1_i/n) + (\Sigma d2_i/n)]/2$ Average items dimensions values
$heterogeneity$	$heterogeneity = nt/n$ Proportion between the quantity of different items (nt) for all n
$heterognt$	Measure of heterogeneity considering only types of items with more than one item
$difcoefficient$	For all n, the total number of different items dimensions
$objdimratio$	Number of times that the object lower-bound is bigger than the object width
$itdimratio$	Number of times that the items the maximum items dimension is bigger than the minimum items dimensions
$maxcoefficient$	10% larger items dimensions values
$mincoefficient$	10% smaller items dimensions values

To help the generation of experimental data to cover more practical applications for the knapsack problem, Hall and Posner [10] used a wide range of factors to identify the test problem instances difficulty. Three characteristics can be explored in the 2D-SPP context: the total number of items (data instances size); the coefficients of all items values; and the heterogeneity (relation between the size and the number of different test problem instances).

All the concepts and parameters about the problem generators previously described are used to develop the exploratory variables to study the characteristics of the rectangular 2D-SPP. These variables were created considering both items and object shape variation, as well as some intrinsic factors of the test problem instances. Table 11.1 describes the 56 variables defined for this study, divided into six groups ($Area$, $Perimeter$, $Dimensions$, $Widthdimensions$, $Proportions$, and $Other$) accordingly to their origin and level of similarity. To simplify the variables calculation five reference parameters for each item i were defined:

- $area_i = A/a_i$: Ratio between the object area (A) and item i area (a_i);
- $perimeter_i = P/p_i$: Ratio between the object perimeter (P) and item i perimeter (p_i);
- $dimension_i = W/[(d1_i + d2_i)/2]$: Average dimension of item i compared to the object width (W). $d1_i$ is the largest item dimension and $d2_i$ is the smallest item dimension;
- $proportion_i = (D1/D2)/(d1_i/d2_i)$: Level of proportion between the object and item dimensions. $D1$ is the largest object dimension and $D2$ is the smallest object dimension;
- $widthdimension_i = W/d1_i$: Size of the largest item dimensions ($d1_i$) compared to the object width (W).

Small letters (i.e. a_i) represent items dimensions, capital letters (i.e. A) are used to object dimensions and extended words are reserved for the definition of the reference parameters (i.e. $area_i$) and for variables (i.e. $arearatio$).

11.3 Test Problem Instances

In this section, the most frequently benchmark data instances used over the years for the 2D-SPP are identified. Table 11.2 describes the main characteristics of these test problem instances, organized by name, number of instances, minimum and maximum number of items, organization and source.

In Hopper [12], the total number of items and the objects width and height similarities were used to generate twenty-one test problem instances divided in seven classes. Different items shape were randomly generated with a maximum ratio between the items dimensions equal to seven. The object shape varies for dimensions ratio between one and three. Hopper and Turton [11] generated all test problem instances with the same characteristics, but the first 35 test problem instances (*NTn*) corre-

Table 11.2 Test problem instances

Dataset	Instances[a]	Items[b]	Organization	Source
C	21	17–197	7 classes ($C1$–$C7$)	Hopper and Turton [11]
NTn	35	17–199	7 classes ($NTn1$–$NTn7$)	Hopper [12]
NTt	35	17–199	7 classes ($NTt1$–$NTt7$)	Hopper [12]
N	13	10–3152	1 class ($N1$–$N13$)	Burke et al. [7]
cx	7	50–15000	1 class (cx)	Ferreira and Oliveira [9]
iy	170	16–32768	11 classes ($i4$–$i15$)	Imahori and Yagiura [13]
cgcut	3	23–623	1 class ($cgcut$)	Christofides and Whitlock [8]
bwmv	300	20–100	6 classes ($C01$–$C06$)	Berkey and Wang [4]
bwmv	200	20–100	4 classes ($C07$–$C10$)	Martello and Vigo [18]
ngcut	12	7–22	1 class ($ngcut$)	Beasley [2]
gcut	13	10–50	1 class ($gcut$)	Beasley [1]
zdf	16	580–75032	1 class (zdf)	Leung and Zhang [14]
AH	360	1000	6 classes ($AH1$–$AH6$)	Bortfeldt [5]
beng	10	20–200	1 class ($beng$)	Bengtsson [3]
nice	36	25–5000	8 classes ($nice1$–$nice5t$)	Wang and Valenzela [22]
path	36	25–5000	8 classes ($path1$–$path5t$)	Wang and Valenzela [22]

[a]Total number of test problem instances
[b]Minimum and maximum number of items

sponds to guillotine patterns, while the next 35 (NTt) corresponds to non-guillotine patterns. The data instances are classified into seven classes, according to the total number of items.

The N test problem instances proposed by Burke et al. [7] were generated with constant values for the dimensions of the object. In a second moment, these objects were randomly divided in small items. In Ferreira and Oliveira [9] the main focus was to create very heterogeneous items, with extreme differences between the maximum and minimum items dimensions. Imahori and Yagiura [13] used the generator proposed by Wang and Valenzela [22] to develop the iy test problem instances. The main characteristic is the exponential variation of the total number of items per data instance, varying from 32 to 32768 items.

Christofides and Whitlock [8] prefixed a value for the object area of each $cgcut$ test problem instance, and the item's dimensions were generated according to a uniform distribution. As a consequence, items dimensions are proportional to the object. The ten classes of the $bwmv$ instances developed by Berkey and Wang [4] ($C01$–$C06$) and later updated by Martello and Vigo [18] ($C07$–$C10$) were proposed using some bin packing problem parameters. Each class has a total of 50 test problem instances, with item's dimensions uniformly generated.

In Beasley [2] and Beasley [1], the $ngcut$ and $gcut$ instances were generated based on the object width parameter. Guillotinable cuts were also considered for both x and y coordinates. Leung et al. [15] divided the zdf instances in two different groups. The first one ($zdf1$–$zdf8$) is composed of medium and large number of items, varying from 580 to 2532 items. The remaining ($zdf9$–$zdf16$) are considered extra-large data instances, varying from 5032 to 75032 items.

In Bortfeldt [5], the AH test problems set was developed using two parameters, varying in uniform distributions: items heterogeneity; and the ratio between the object width and items average width. Bengtsson [3] developed ten *beng* instances, based on the industrial cutting processes found in the manufacturers of excavators and mechanical shovels.

As mentioned before, Wang and Valenzela [22] developed one of the most used problem generators for the 2D-SPP. The process is recursive based on the variation of items area, according to some parameters defined by the user, and constant object dimensions ($W = 1000$ and $H = 1000$). The $nice$ instances have a high items shape similarity. In contrast, $path$ instances have extreme shape variations.

The data instances for the 2D-SPP presented have some differences, related to items and object shape variations and intrinsic test problem instances characteristics. The main reason for these effects is the use of different types of problem generators. As a consequence, the total number of test problem instances representing each of the variables listed in the previous session is not uniform.

11.4 Principal Component Analysis

For the exploratory analysis the use of PCA is proposed, with the aim of decrease the 56 variables presented in Table 11.1. The objective is to reduce the problem dimension to a more manageable size, retaining as much as possible the original information, by aggregating similar variables into principal components.

The work was developed in two steps. In a first moment, the consistency of each of the six groups of variables from Table 11.1 was checked, by analyzing the correlation coefficients between pairs of variables of each group. A linear correlation is used as a quantitative measure to validate the assignment of predictors to groups. A value of 0.75 for the correlation coefficient was used. The remaining part of this section specifies the procedures performed in all groups. Due to space constraints, we have chosen to show in detail only the results obtained for groups $Area$ and $Proportions$. However, the conclusions proposed at the end of this study considers the results of all groups.

To exemplify this first step, Tables 11.3 and 11.4 summarize the correlation coefficients between variables in groups $Area$ and $Proportions$, respectively. A total of 14 and 16 coefficient correlations higher than the reference value are found for these groups, and all variables have at least one high correlation coefficient that justifies the group coherence. Variables with low positive or negative correlation coefficients do not represent the same characteristic of the problem. To facilitate the interpretation of the problem and maintain the information provided by the test problem instances in the original format, the input data was not previously normalized. In some situations, the correlation may have been suffered small effects of any outliers or obvious unusual cases.

Table 11.3 Correlation coefficient matrix for group *Area*

	arearatioperc	***arearatioquart***	***areacompnormal***	***areacompquart***	***areacompextr***	***areamean***	***areamed***	***areastdev***
arearatioextr	0.44	0.45	0.38	0.45	0.44	0.89	0.73	0.81
arearatioperc		0.73	0.79	0.89	0.91	0.36	0.14	0.51
arearatioquart			0.82	0.80	0.67	0.38	0.17	0.51
areacompnormal				0.95	0.82	0.32	0.09	0.53
areacompquart					0.94	0.38	0.12	0.61
areacompextr						0.36	0.11	0.57
areamean							0.88	0.88
areamed								0.57

Table 11.4 Correlation coefficient matrix for group *Proportions*

	propratioextr	***propratioperc***	***propratioquart***	***propcompnormal***	***propcompquart***	***propcompextr***	***propmean***	***propmed***	***propstdev***
aspectratio	−0.21	−0.25	−0.28	−0.31	−0.25	−0.25	0.97	0.97	0.80
propratioextr		0.81	0.72	0.78	0.78	0.80	−0.13	−0.13	−0.01
propratioperc			0.92	0.95	0.97	0.97	−0.21	−0.22	−0.11
propratioquart				0.97	0.96	0.95	−0.25	−0.25	−0.15
propcompnormal					0.97	0.96	−0.27	−0.28	−0.15
propcompquart						1.00	−0.22	−0.22	−0.12
propcompextr							−0.21	−0.21	−0.12
propmean								1.00	0.91
propmed									0.90

Table 11.5 Variance explained by each component for all groups

Group	Component	Variance (%)	Cumulative (%)	Group	Component	Variance (%)	Cumulative (%)
Area	*areacomp*	62.48		Perimeter	*perimcomp*	75.36	
	areastats	24.92	87.40		*perimstats*	20.16	95.52
Dimensions	*dimcomp*	77.75		Proportions	*propcomp*	59.76	
	dimstats	18.00	95.75		*propstats*	33.39	93.15
Width dimensions	*widthdimcomp*	79.13					
	widthdimstats	16.39	95.52				

In a second moment, PCA is used individually for each group to reduce the dimensions of the problem. All the PCA are conducted with orthogonal rotation (varimax), and all requirements were reached: the ratio between the sampling size and the number of variables is greater than five to one; a minimum of 0.5 for overall Kaiser–Meyer–Olkin measure of sampling adequacy; and the Bartlett test of sphericity is statistically significant (<0.001).

As a result, two components with eigenvalues greater or equal to one were extracted for each of the first five groups, namely: $areacomp$ and $areastats$ for group $Area$; $perimcomp$ and $perimstats$ for $Perimeter$; $dimcomp$ and $dimstats$ for $Dimensions$; $propcomp$ and $propstats$ for $Proportions$; and $widthcomp$ and $widthstats$ for group $Width\ dimensions$. For the remaining group, $Other$, it was not possible to extract a small number of components, since the variables in this group are not related with each other (small correlation coefficients). As a result, a total of 19 characteristics was obtained for the 2D-SPP, 10 components and the original nine variables from group $Other$.

Table 11.5 presents the percentage of variance explained for the components individually in the first five groups. An average of 93% of the data variation is explained by the components extracted, higher than the variation of 91% obtained if the PCA was used with all the variables simultaneously.

Figure 11.2 represents the variables' projections along the extracted components for groups $Area$ and $Proportions$. In $Area$ there is a clear difference between the variables with high positive factor loadings for each component. High values for variables based on ratios (i.e. $areratioperc$) and compositions (i.e. $areacompextr$) establish the component $areacomp$. In contrast, $arestats$ is based on classical statistical measures, such as mean, median and standard deviation variables. One exceptions is $arearatioextr$, which is a ratio variable but influences more significantly the $arestats$ component.

Group $Proportions$ shows a similar behavior, with component $propcomp$ influenced by all ratio and composition variables, and the $propstats$ with all classical statistical measures and the correlated variable $aspectratio$.

Figure 11.3 presents the distribution of the 1217 test problem instances according to the components of $Area$ and $Proportions$. To a better visualization of the data, the scores for each instance are normalized to a scale between zero and one, according to the maximum and minimum values found for each component.

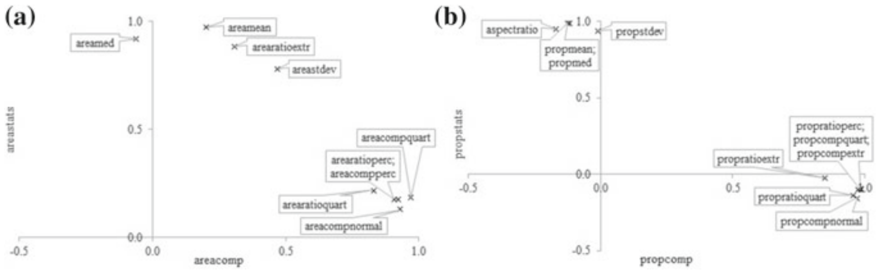

Fig. 11.2 Relations between variables and components for groups *Area* and *Proportions*

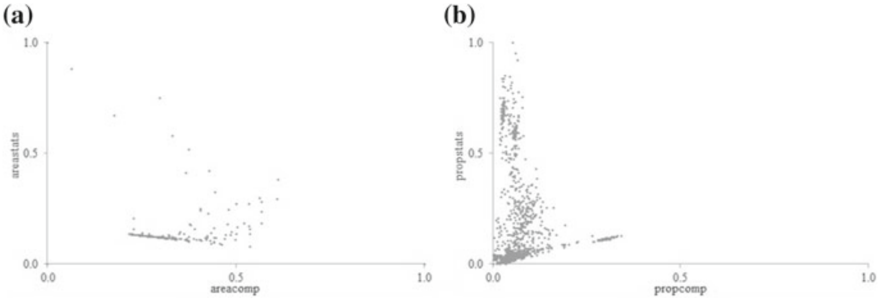

Fig. 11.3 Distribution of the 1217 test problem instances among components of groups *Area* and *Proportions*

In the first graph, the highest values for *areastats* are found for seven test problem instances *zdf* (*zdf*10–*zdf*16), a consequence of the high standard deviation and mean value between the largest and smallest items of each data instance, which can be verified through variables *areastdev* and *areamed*, respectively. Figure 11.2 shows the strong influence between these variables and *areastats*.

Instance *cx*50 has items with a strip shape, which means that the difference between the largest and smallest items dimensions is very high, and the object has a square shape, a fact that reflects the high value of the test problem instance for *areacomp* and *propcomp*. Some similar effects can also be found, with less amplitude, for *cx*100 and some *path* and *iy* instances. In *propstats* a total of four *AH* instances (*AH*14, *AH*36, *AH*50 and *AH*55) have a high dispersion between the average value of items proportion compared to the object. As a consequence, these are test problem instances that have a high degree of heterogeneity.

For both dispersion graphs, almost all test problem instances have similar values. In *Proportions* the test problem instances are located near the center of coordinates graph. In *Area* almost all instances are between 0.2 and 0.4 for *areacomp* and 0.1 and 0.2 for *areastats*. Therefore, these instances have few differences between them, leading to few variations in the variables and affecting the analysis of the characteristics of the problem.

11.5 Conclusions

In this paper, we conduct an exploratory research to identify the most significant characteristics for the two-dimensional rectangular 2D-SPP. Initially, a total of 56 variables were considered, based on parameters and characteristics found in the most used problem generators.

To reduce the complexity the PCA was used to reduce the problem dimensionality. Our analysis suggests that 19 components can explain the problem consistently. A relevant result is the similarity showed by the test problem instance from the literature.

In a second moment, it was verified that the number of test problem instances that represent the possible items and object shape variation must be improved. As a consequence, during the research the need of generation of new test problem instances was evident, to overcome the drawback of some missing characteristics in the existing test problem instances in the literature.

This study helps in the development of more efficient heuristics for solving the 2D-SPP, by providing a more accurate information on the characteristics of the problem. Future work will describe the relation between the components developed (named as features) with a dependent variable, using regression models in order to allow the prediction of the object height to be used according to the test problem instances characteristics. Also, new variables will be studied in order to complement the characterization of the problem.

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