

Rheological Models and Numerical Method for Simulation of Blood Flow

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Abstract. This paper describes a method and results of computational fluid dynamics (CFD) simulation of blood flow. Material properties of blood are assumed to be constant, homogeneous and isotropic. Blood is regarded as viscoplastic liquid, where two different rheological models are applied: Bingham and Casson model. Plastic viscosity and yield stress are given as a function of hematocrit. The flow regime is considered as laminar. Having applied rheological models to time dependent balance equations of mass and momentum conservation given in integral form, a finite-volume method is used for discretization. Discretization results in a set of systems of linearized algebraic equations which are solved individually, for blood velocity components and blood pressure, at every time step within a considered time interval. The method is applicable to domains of arbitrary shapes and unstructured computational meshes. The examples presented include: (a) pulsatile viscoplastic flow in a pipe representing a simplified blood vessel, where the solutions obtained with the two rheological models are compared to the numerical and analytical solution obtained with Newtonian liquid, as well as (b) blood flow in aorto-renal bifurcation and carotid artery branch. In analysis of the flow in the branch, three geometric models are tested: idealized bifurcation with the branch angle of 60° and 90°, and a realistic shape of the bifurcation.

Keywords: Blood flow · Viscoplastic liquids · Rheology · CFD simulation
Finite-volume method

1 Introduction

Investigation and understanding of the blood flow dynamics in blood vessels have gained considerable importance in the recent years. Hemodynamic properties, including velocity, pressure and wall shear stress, play an important role in research on vascular systems. Occlusion as well as obstruction and blockage of the flow frequently lead to disease, damage and fatal consequences. They arise especially in vessels of complex geometric shapes and/or in vessels with relatively high blood flow rates, specifically, in coronary, carotid, abdominal and femoral arteries, where also the aforementioned flow properties exhibit strong spatial and temporal variations.

Understanding relations between these flow properties' variations, the blood flow and vessel behaviour may help finding appropriate treatment and therapy.

Practically, blood flow is always time-dependent, and its dynamic nature may cause important differences as compared to usually analysed steady-state flows. Generally, it can be regarded as incompressible and laminar. Under circumstances, it may also be considered as turbulent, such as in the ascending aorta, in the branch regions of large arteries, in narrowed parts of blood vessels or around heart valves. However, its main distinction from usual liquid flows found in natural systems and engineering applications is non-Newtonian behaviour. Unlike in commonly found fluids like water, oil, milk or gasoline, the dependence of shear stress and shear strain rate is *not* linear.

This paper presents a mathematical model of blood flow which takes into account its non-Newtonian behaviour using two different rheological models for viscoplastic fluids: Bingham and Casson model, as well as a numerical method for its solution using finite-volume discretization.

2 Mathematical Model and Numerical Method

Mathematical model emanates from conservation laws of continuum mechanics and includes conservation of mass, conservation of linear momentum and conservation of space (the latter is applied in cases where moving or deformable walls are calculated) [1, 2]. The conservation laws apply to all fluids, and herewith they are also applicable to blood flows.

Bingham model describes bilinear relation between the shear stress and the strain rate, and can be written in form which delivers explicit expressions for dynamic viscosity of the liquid:

$$\mu = \begin{cases} \mu_0 + \frac{\tau_0}{2\sqrt{I_D}} & \text{for } \frac{1}{2}(\mathbf{T}^d : \mathbf{T}^d) > \tau_0^2 \\ \infty & \text{for } \frac{1}{2}(\mathbf{T}^d : \mathbf{T}^d) \leq \tau_0^2 \end{cases} \quad (1)$$

Casson model delivers a non-linear relation of shear stress and strain rate, and is found to be appropriate to describe rheological behaviour of blood:

$$\mu = \begin{cases} \frac{1}{\sqrt{I_D}} \left[(\mu_0^2 I_D)^{0.25} + \frac{\sqrt{\tau_0}}{\sqrt{2}} \right]^2 & \text{for } \frac{1}{2}(\mathbf{T}^d : \mathbf{T}^d) > \tau_0^2 \\ \infty & \text{for } \frac{1}{2}(\mathbf{T}^d : \mathbf{T}^d) \leq \tau_0^2 \end{cases} \quad (2)$$

$$\dot{\mathbf{D}} = \frac{1}{2} \left[\text{grad } \mathbf{v} + (\text{grad } \mathbf{v})^T \right]$$

where μ_0 is the plastic viscosity, τ_0 is the initial yield stress needed to initiate the flow, I_D is the second invariant of the shear strain rate tensor \mathbf{D} , \mathbf{T}^d is the deviatoric part of the stress tensor, and \mathbf{v} is the velocity vector.

Material properties of blood are assumed to be constant (except the dynamic viscosity which is solution dependent, and herewith it may be variable in space and time),

homogeneous and isotropic. The plastic viscosity and the yield stress are given as a function of hematocrit h [3]:

$$\mu_0 = \frac{\mu_{0p}}{(1 - h/100)^{2.5}}, \quad \tau_0 = (0.00625h)^2, \quad (3)$$

where μ_{0p} is the viscosity of the blood plasma.

The part of space under consideration is divided into a set of adjacent, non-overlapping cells of polyhedral shapes, building thus an unstructured numerical mesh. The adopted constitutive relation is applied to the conservation equations of mass and linear momentum, written in integral form for each cell in the numerical mesh. A finite-volume discretization described by Demirdžić and Muzaferija [1] and Ferziger and Perić [2] is then applied to convert the time-dependent integro-differential equations into a set of non-linear algebraic equations. At every time step within a considered time interval, the set is separated into subsystems of equations for each solution variable: velocity components and pressure. Temporary decoupling and linearization of the subsystems are performed within an iterative procedure, which also accommodates implementation of non-linear viscosity nature described by Eqs. (1) and (2) [4], as well as velocity-pressure linkage employing SIMPLE algorithm [5], and the subsystems are solved sequentially in turn. Upon a convergence criterion is reached, the solving process proceeds to the next time step.

3 Examples

3.1 Pulsatile Viscoplastic Flow in a Pipe

Time-dependent, pulsatile flow through a 40 mm long pipe with diameter of 4 mm is calculated. Uniformly distributed axial velocity across the pipe is prescribed at the inlet, while its temporal variation is given in form of a sine function:

$$u(t) = \bar{u} \left(1 + \sin \frac{2\pi t}{T} \right). \quad (4)$$

This form of the sine function implies that the prescribed inlet velocity is non-negative, i.e. there is no backflow at the inlet.

The mean axial velocity value of 0.135 m/s and pulsation period of $T = 0.2$ s are specified. The flow is calculated for a Newtonian liquid whose dynamic viscosity is 0.0032 Pas, as well as for a Bingham and Casson fluid, whose plastic viscosity is also 0.0032 Pas and the initial yield stress is 0.0375 Pa. A structured computational mesh with 150 cells in axial and 20 cells in radial direction is generated.

Figure 1 shows distribution of the axial velocity of developed flow across the pipe diameter at four distinct instants of time, calculated for Newtonian fluid flow and compared to the corresponding analytical solution of Womersley [4]. Agreement of the results is evident, with maximum deviation of less than 5%, implying a good accuracy of the base model setup.

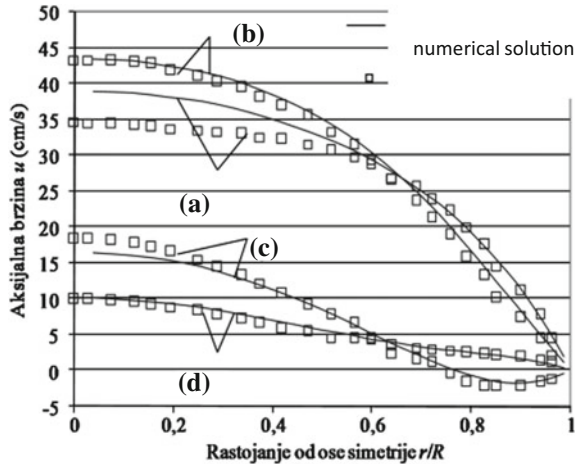


Fig. 1. Axial velocity profiles of a Newtonian liquid in a rigid pipe at different instants of time: **a** $t/T = 0.125$; **b** $t/T = 0.375$; **c** $t/T = 0.625$ and **d** $t/T = 0.875$

Figure 2 shows comparison of the developed flow solutions obtained using non-Newtonian models with that obtained using the Newtonian one. The axial velocity distribution across the pipe diameter at four distinct instants of time is displayed again. In the first half of the pulsation period, agreement of the investigated models is apparent. In the second half, differences are remarkable. Particularly, the differences obtained with Casson model in the central part of the pipe (considerably lower axial velocity) at the time $t/T = 0.875$ are noticeable. In a number of published works, blood flow is, probably for simplicity reasons, simulated as Newtonian or Bingham fluid, although its rheologic behaviour is better described by Casson model. The here presented results clearly indicate that the choice of rheological model strongly affects the pulsating flow solution, and may trigger inconclusive findings if the model is not adopted appropriately.

In addition to that, the backflow near the wall is detected in the third quarter of the pulsation period, and herewith the wall shear stress becomes negative at the location of the extracted profile, even though the inlet velocity is non-negative. Such a dynamic behaviour may cause additional dynamic load to the vessel wall.

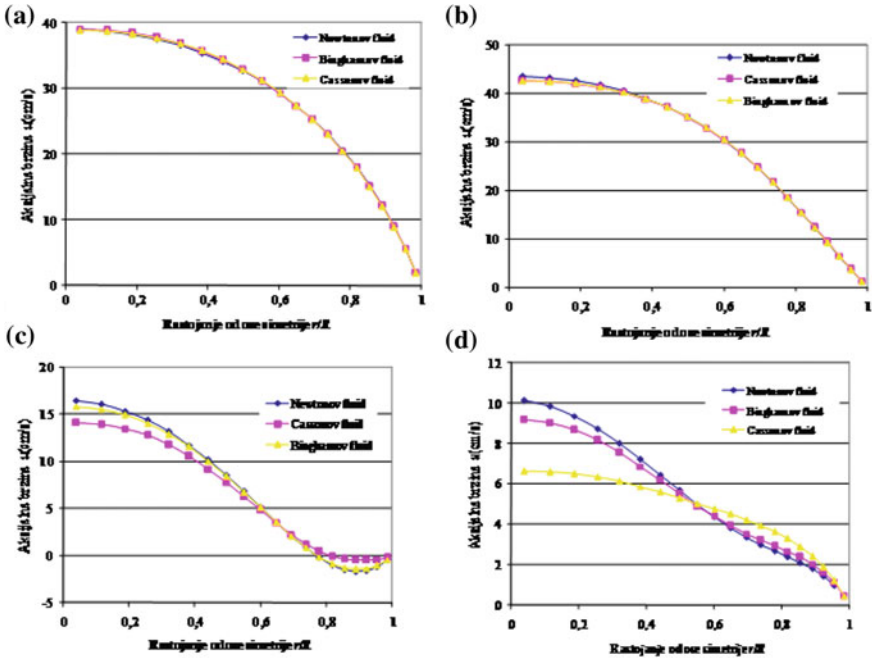


Fig. 2. Axial velocity profiles of a non-Newtonian liquid in a rigid pipe at different instants of time: **a** $t/T = 0.125$; **b** $t/T = 0.375$; **c** $t/T = 0.625$ and **d** $t/T = 0.875$, calculated using two different rheologic models: Casson (magenta) and Bingham (yellow) fluid, and compared to Newtonian fluid solution (blue)

3.2 Pulsatile Blood Flow

3.2.1 Idealized Bifurcation at Two Different Branch Angles—Aorto-Renal Branch

The model setup is adapted according to the experimental conditions described by Tokunori et al. [6], where a simplified model of aorto-renal branch is studied. Diameter of the abdominal aorta is 20 mm, while the diameter of the renal artery is 6 mm. Flow velocity is assumed to have parabolic distribution at the inlet. Temporal variation of the inlet velocity is described using the following periodic function:

$$u(t) = 0.15 + (U_i - 0.15) \sin \frac{2\pi t}{T} \tag{5}$$

where the mean velocity is $U_i = 0.6718$ m/s and the pulsation period is $T = 0.75$ s. The outlet flow rates through the abdominal and the renal aorta are split in proportion of 91:09, respectively. Blood is modeled as a Bingham fluid, where the values in the constitutive relation τ_0 and μ_0 are obtained from the assumed hematocrit value for normal blood $h = 43\%$ and the blood plasma viscosity $\mu_{0,p} = 0.00125$ Pas [3].

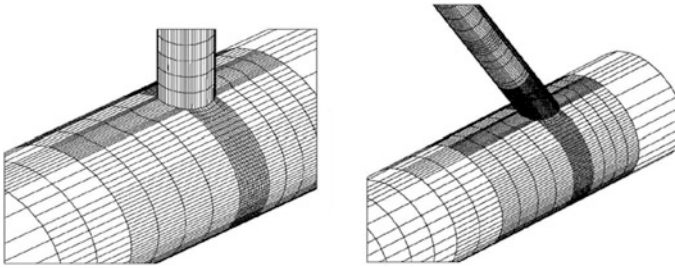


Fig. 3. Numerical mesh in idealized bifurcation at the branch angle 90° (left) and 60° (right)

Two different branch angles are considered: 90° and 60° , see Figs. 3 and 4.

In the first case, separation and recirculation on the upstream side of the renal artery are observed, which is also reported in experimental results [6]. The ratio between the maximum backflow velocity and the maximum main stream velocity is 0.4, while this ratio according to the experimental results is 0.3 ± 0.1 .

In the latter case, with the branch angle of 60° , the recirculation is rather thin and small, so that it can be practically neglected.

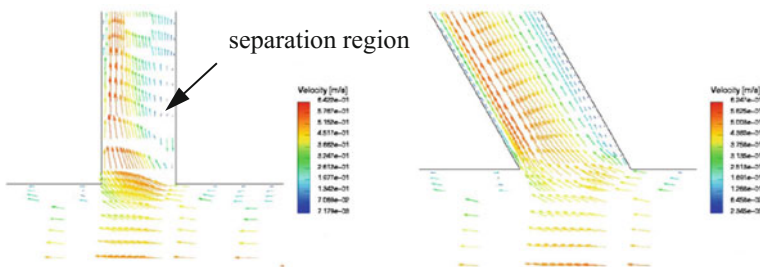


Fig. 4. Velocity vectors in longitudinal section of a simplified model of aorto-renal branch at two different branch angles 90° and 60°

3.2.2 A Realistic Shape of Bifurcation of Carotid Artery

Simulation of blood flow in a realistic shape with complex geometry is demonstrated in the case of carotid artery. Blood vessel walls have irregular surface shape with relatively large variations of the cross section area. In addition to that, the here considered parts of the carotid artery form a branch. Blood vessel diameter at the inlet, as well as the diameters at the both outlets are assumed to be 6.2 mm. The same periodic inlet-velocity condition is used as in the case of aorto-renal branch. The outlet sections are defined as zero-pressure boundaries. Blood is modelled as Casson fluid with the same μ_0 and τ_0 values as in the previous case.

Figure 5 shows distribution of the velocity magnitude as well as pressure distribution over the blood vessel wall. Strong separation and recirculation of the blood flow from the vessel walls is seen in the regions of abrupt expansion of the cross-section area. Also negative pressure values are obtained in the regions of strong contraction.

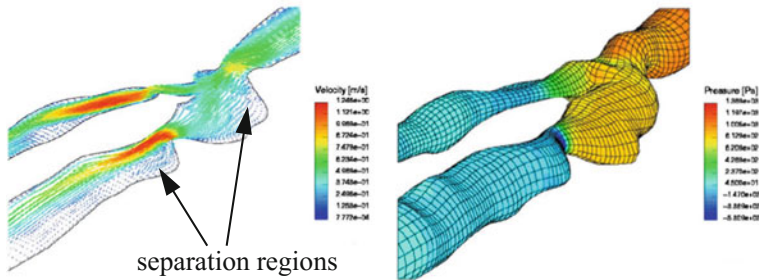


Fig. 5. Blood flow in a realistic bifurcation model of a carotid artery. Distribution of velocity magnitude in the longitudinal section plane (left) and vessel wall pressure (right) at the instant of time $t/T = 0.25$

4 Conclusions

In this work, a mathematical model for computational flow analysis with two different constitutive relations for viscoplastic fluids is introduced, solved using finite-volume method, and applied to several examples of pulsating flow such as those appearing in blood vessels.

Numerical solutions of the flow in a simple pipe obtained with the Newtonian model and with the two models of viscoplastic fluid are compared to the analytical solution obtained for Newtonian fluid. The comparison confirms applicability and plausibility of the implemented models. The non-Newtonian models are also applied to geometric domains of complex shape arising in aorto-renal branch and in a branch of carotid artery.

Numerical solution provides detailed insight into the blood flow structure indicating the regions with significant wall pressure and wall shear stress changes which may directly lead to blood vessel damages. It also detects the blood flow separation and recirculation. These regions are supposed to be the places where occlusion by plaque may develop.

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