Chapter 4
Simulation Studies

In this section we present results obtained of the ensemble of IT2FNN models and the use of fuzzy integrators as response optimized with GA and PSO algorithms for time series prediction.

4.1 Mackey-Glass Time Series

This section presents the simulation and test results obtained by applying the proposed prediction method to the Mackey-Glass time series for τ = 13, 17, 30, 34, 68, 100, 136, using different approach of the ensemble of IT2FNN architectures and the two types of optimization of the fuzzy integrators with the GAs and PSO algorithms, used in this work.

4.1.1 Ensemble of the IT2FNN Architectures for Mackey-Glass

The ensemble of IT2FNN architectures has three models as follows: the IT2FNN-1 model optimizes the parameters of the "igaussmtype2" MFs (Fig. 3.9a), the learning rate is 0.03 and the desired error is 0.00001; the IT2FNN-2 model optimizes the parameters of the "igausstype2" MFs (Fig. 3.9b), the learning rate is

J. Soto et al., *Ensembles of Type 2 Fuzzy Neural Models and Their Optimization with Bio-Inspired Algorithms for Time Series Prediction*, SpringerBriefs in Computational Intelligence, https://doi.org/10.1007/978-3-319-71264-2_4

0.011 and the desired error is 0.000001; and the IT2FNN-3 model optimizes the parameters of the “igaussstype2” MFs (Fig. 3.9c), the learning rate is 0.02 and the desired error is 0.0000001. The number the epochs for training the IT2FNN models is 800.

The obtained results of the ensemble of IT2FNN architectures are shown on Table 4.1. The best error is of 0.002517717 and the average error is of 0.00591527 with the IT2FNN-3 for the Mackey-Glass (τ = 13); the best error is of 0.000254857 and the average error is of 0.000513248 with the IT2FNN-1 for the Mackey-Glass (τ = 17); the best error is of 0.00089312 and the average error is of 0.004463189 with the IT2FNN-1 for the Mackey-Glass (τ = 30); the best error is of 0.000307511 and the average error is of 0.010427016 with the IT2FNN-3 for the Mackey-Glass (τ = 34); the best error is of 0.00085505 and the average error is of 0.003818732 with the IT2FNN-2 for the Mackey-Glass (τ = 68); the best error is of 0.000612878 and the average error is of 0.00327431 with the IT2FNN-1 for the Mackey-Glass (τ = 100); and the best error is of 0.000331059 and the average error is of 0.002382512 with the IT2FNN-1 for the Mackey-Glass (τ = 136).

Table 4.1 Results for the ensemble of IT2FNN for the Mackey-Glass time series

IT2FNN	RMSE	
	Best	Average
IT2FNN-1-Tau = 13	0.008596153	0.010146361
IT2FNN-2-Tau = 13	0.007919986	0.010166644
IT2FNN-3-Tau = 13	0.002517717	0.00591527
IT2FNN-1-Tau = 17	0.000258457	0.00513248
IT2FNN-2-Tau = 17	0.000281517	0.00554012
IT2FNN-3-Tau = 17	0.005466984	0.021365826
IT2FNN-1-Tau = 30	0.00089312	0.004463189
IT2FNN-2-Tau = 30	0.00124898	0.004503043
IT2FNN-3-Tau = 30	0.001404767	0.012277316
IT2FNN-1-Tau = 34	0.00077769	0.004371837
IT2FNN-2-Tau = 34	0.001349607	0.004943326
IT2FNN-3-Tau = 34	0.000307511	0.010427016
IT2FNN-1-Tau = 68	0.000988737	0.004635047
IT2FNN-2-Tau = 68	0.000855055	0.003818732
IT2FNN-3-Tau = 68	0.001238312	0.008696349
IT2FNN-1-Tau = 100	0.000612878	0.00327431
IT2FNN-2-Tau = 100	0.000782409	0.003720222
IT2FNN-3-Tau = 100	0.001152992	0.005881063
IT2FNN-1-Tau = 136	0.000331059	0.002382512
IT2FNN-2-Tau = 136	0.001351276	0.004299122
IT2FNN-3-Tau = 136	0.001133525	0.005586892

4.1.1.1 IT2FNN-1 Model

The forecast obtained for the IT2FNN-1 for the Mackey-Glass (τ = 17) time series shown in Fig. 4.1, the evolution error is shown in Fig. 4.2, and the optimization structure of the IT2FNN-1 with backpropagation learning algorithm show in Fig. 4.3, the forecast obtained for the Mackey-Glass (τ = 13 and τ = 30) time series shown in Figs. 4.4 and 4.5.

4.1.1.2 IT2FNN-2 Model

The forecast obtained for the IT2FNN-2 for the Mackey-Glass (τ = 17) time series is shown in Fig. 4.6, the evolution error is shown in Fig. 4.7, and the optimization structure of IT2FNN-2 with backpropagation learning algorithm shown in Fig. 4.8, the forecast obtained for the Mackey-Glass (τ = 34 and τ = 68) time series shown in Figs. 4.9 and 4.10.

4.1.1.3 IT2FNN-3 Model

The forecast obtained for the IT2FNN-3 for the Mackey-Glass (τ = 17) time series is shown in Fig. 4.11, the evolution error is shown in Fig. 4.12, and the

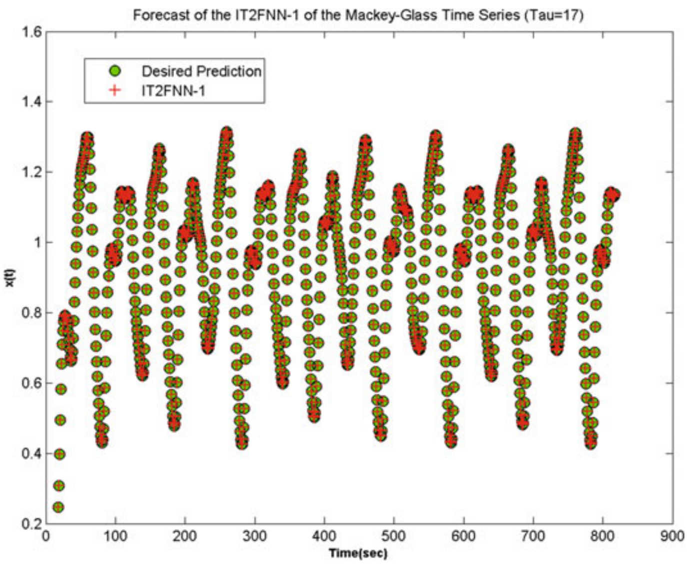

Fig. 4.1 Forecast of IT2FNN-1 for the Mackey-Glass (τ = 17) time series

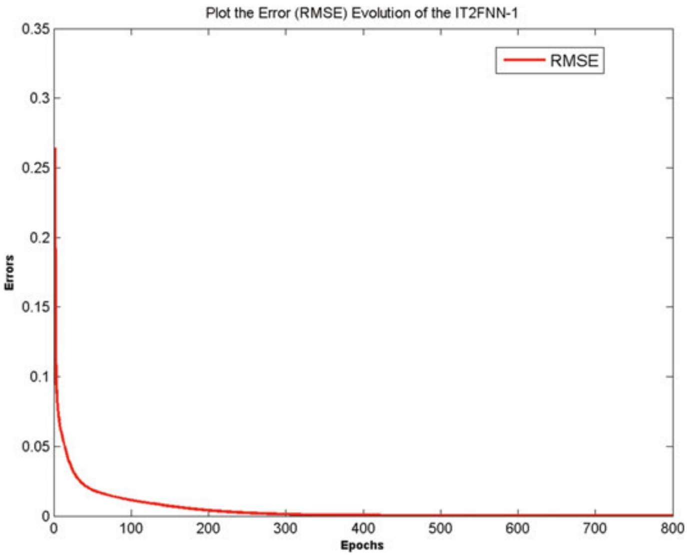

Fig. 4.2 Evolution error (RMSE) of IT2FNN-1 for the Mackey-Glass time series

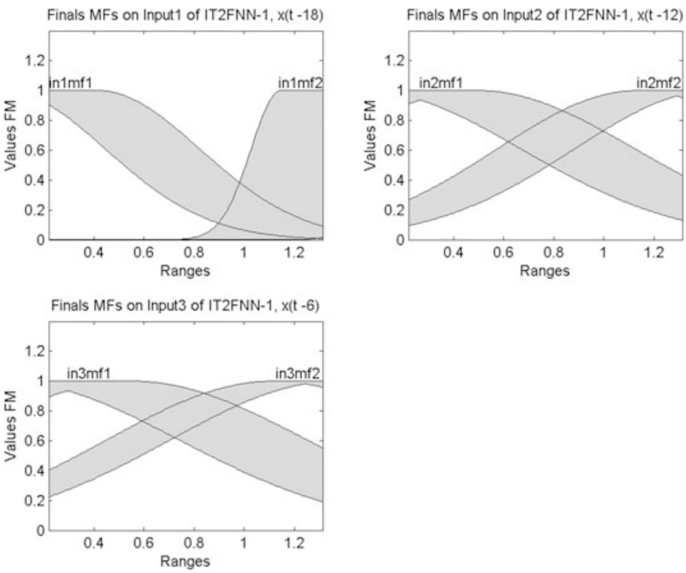

Fig. 4.3 Final MFs after training the IT2FNN-1 model

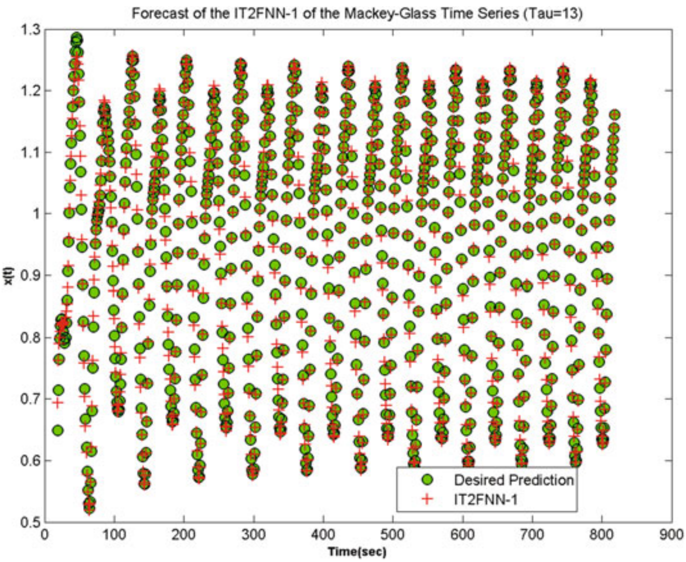

Fig. 4.4 Forecast of IT2FNN-1 for the Mackey-Glass (τ = 13) time series

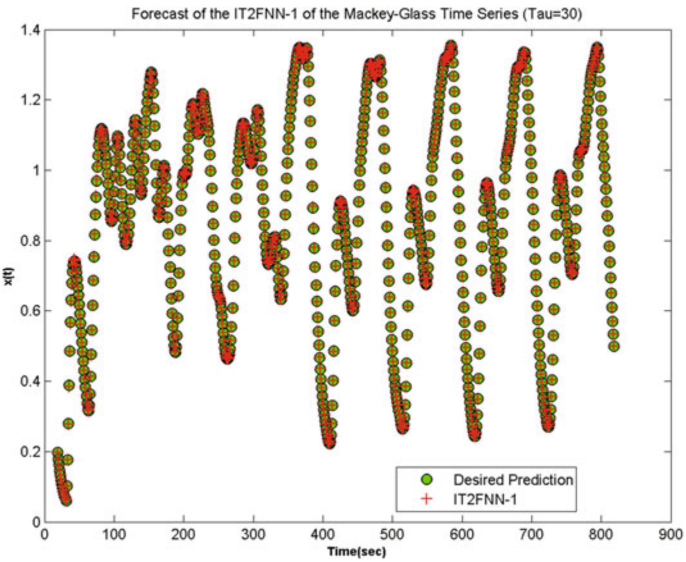

Fig. 4.5 Forecast of IT2FNN-1 for the Mackey-Glass (τ = 30) time series

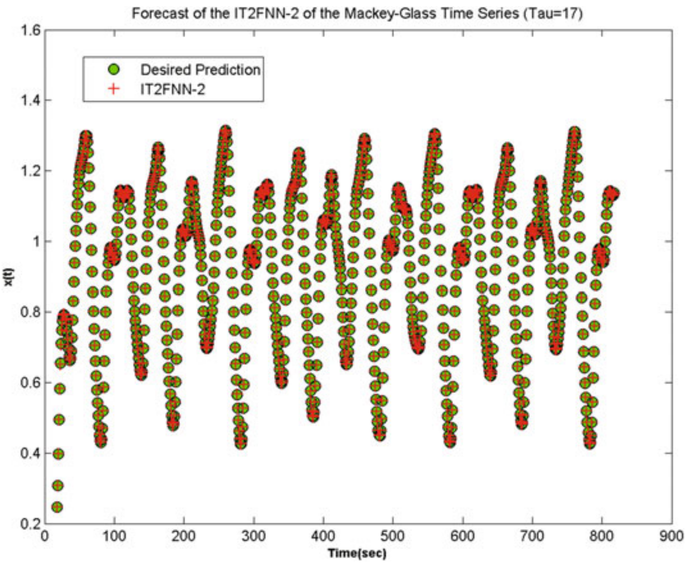

Fig. 4.6 Forecast of IT2FNN-2 for the Mackey-Glass (τ = 17) time series

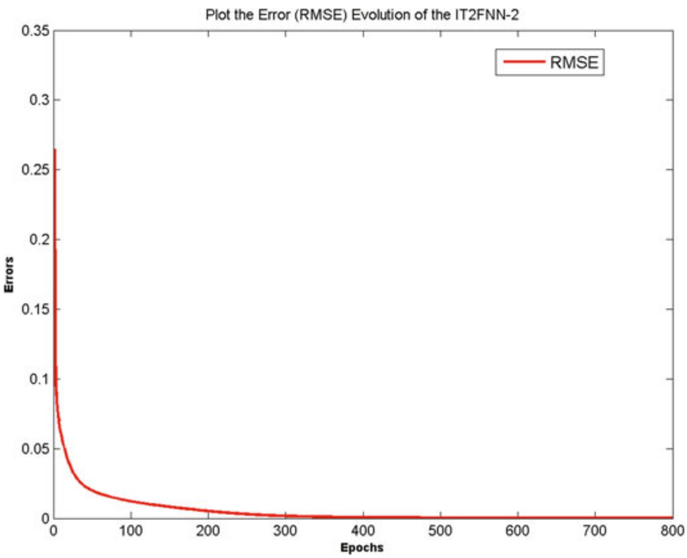

Fig. 4.7 Evolution error (RMSE) of IT2FNN-2 for the Mackey-Glass time series

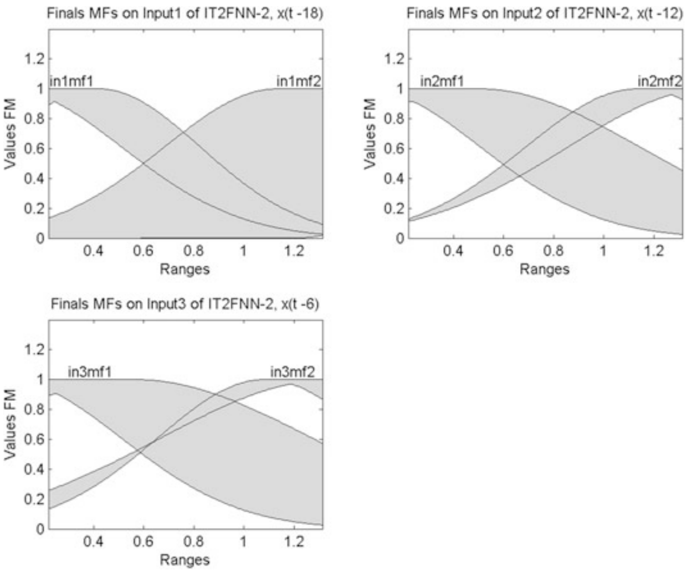

Fig. 4.8 Final MFs after training the IT2FNN-2 model

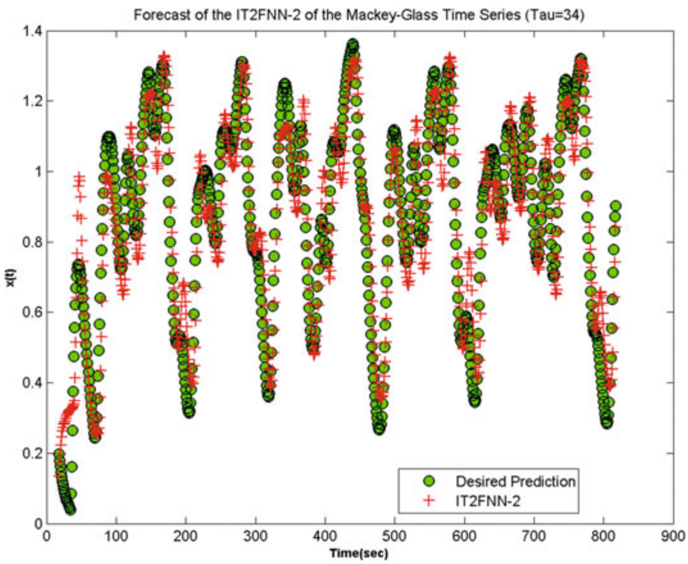

Fig. 4.9 Forecast of IT2FNN-2 for the Mackey-Glass (τ = 34) time series

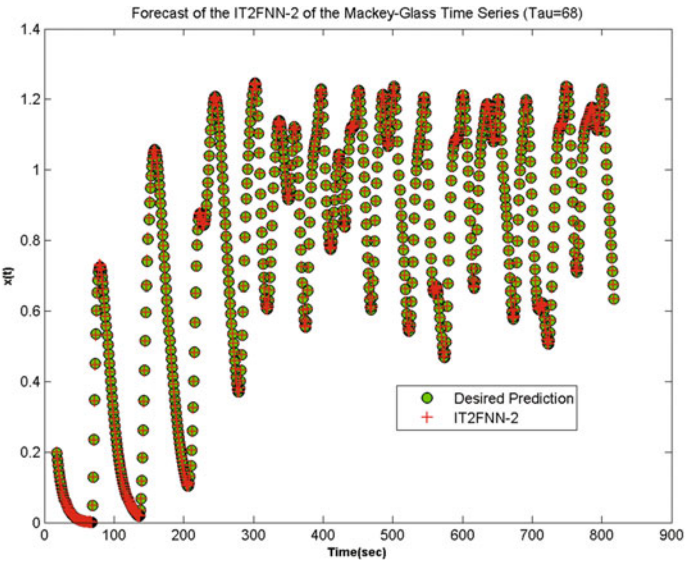

Fig. 4.10 Forecast of IT2FNN-2 for the Mackey-Glass (τ = 68) time series

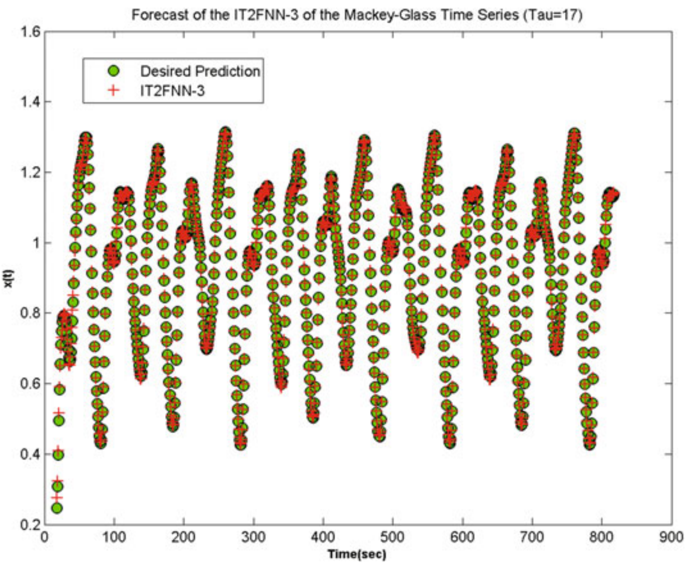

Fig. 4.11 Forecast of IT2FNN-3 for the Mackey-Glass (τ = 17) time series

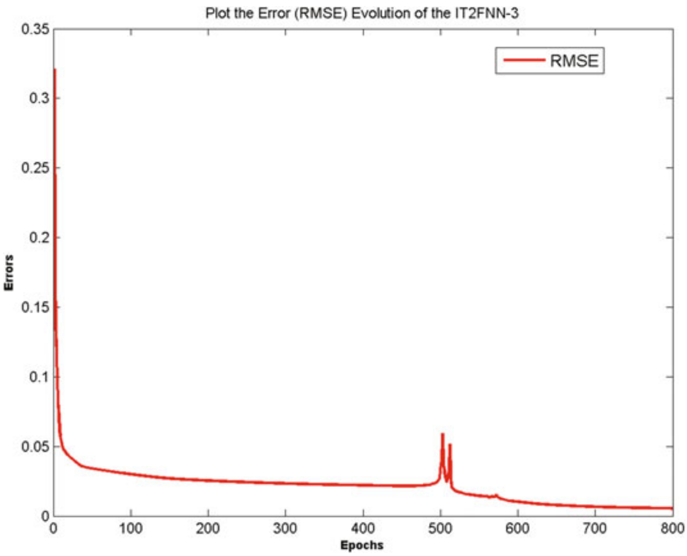

Fig. 4.12 Evolution error (RMSE) of IT2FNN-3 for the Mackey-Glass time series

optimization structure of IT2FNN-3 with backpropagation learning algorithm shown in Fig. 4.13, the forecast obtained for the Mackey-Glass (τ = 100 and τ = 136) time series is shown in Figs. 4.14 and 4.15.

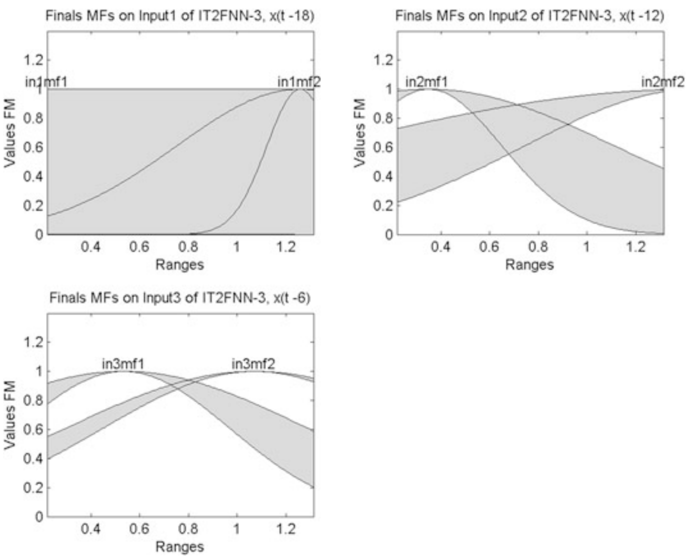

Fig. 4.13 Final MFs after training the IT2FNN-3 model

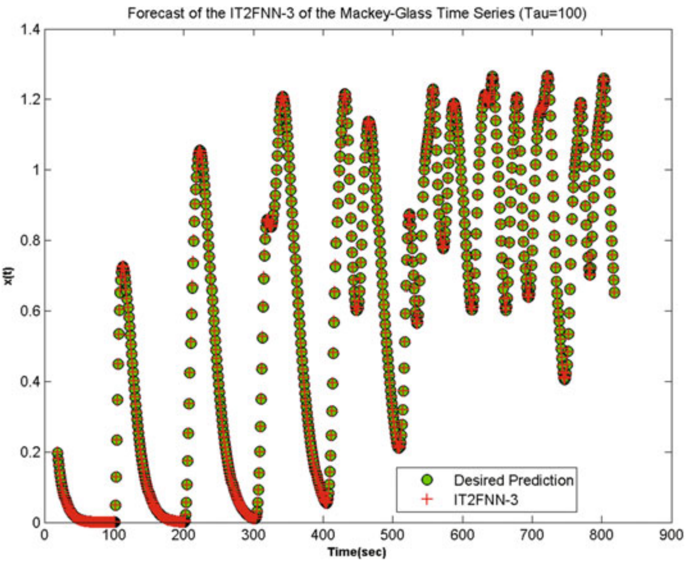

Fig. 4.14 Forecast of IT2FNN-3 for the Mackey-Glass (τ = 100) time series

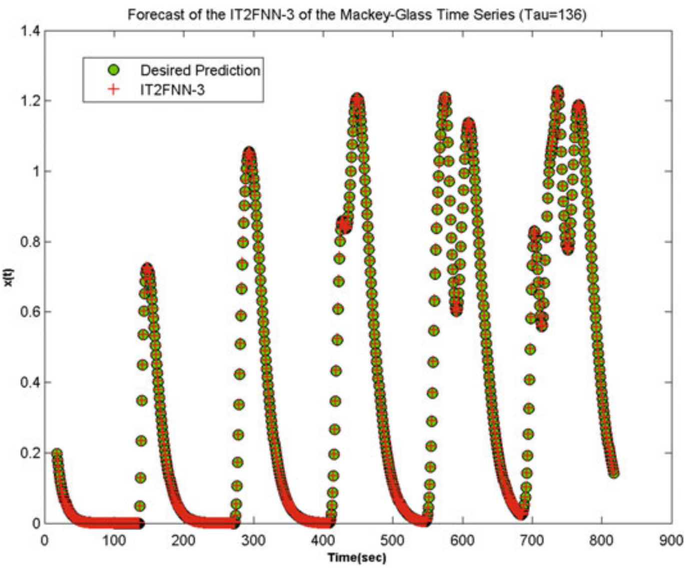

Fig. 4.15 Forecast of IT2FNN-3 for the Mackey-Glass (τ = 136) time series

4.1.2 Optimization of the Fuzzy Integrators with the Genetic Algorithm

The obtained results with optimized the fuzzy integrators with the GAs are shown on Table 4.2. The best error is of 0.02142164 and the average error is of 0.02255155 for the type-1 fuzzy integrator (T1FIS) using "Gbell" MFs, and the best error is of 0.02023097 and the average error is of 0.02033528 for the interval type-2 fuzzy integrator (IT2FIS) using "itritype2" MFs for the Mackey-Glass (τ = 17) time series.

We are presenting 10 experiments in Table 4.2, but the average error was calculated considering 30 experiments with the same parameters and conditions for the GAs. Therefore to evaluate the performance of the 30 experiments for this work, we applied different metrics to calculated average errors as shown in Table 4.3.

The forecast obtained of the optimized T1FIS using "Gauss" MFs for the Mackey-Glass (τ = 17) time series shown in Fig. 4.16, the performance of the evolution error is shown in Fig. 4.17, and the optimization structure of T1FIS using "Gauss" MFs with the GAs shown in Fig. 4.18.

The forecast obtained of the optimized T1FIS using "Gbell" MFs for the Mackey-Glass (τ = 17) time series shown in Fig. 4.19, the performance of the evolution error is shown in Fig. 4.20, and the optimization structure of T1FIS using "Gbell" MFs with the GAs shown in Fig. 4.21.

The forecast obtained of optimized the T1FIS using "Triangular" MFs for the Mackey-Glass (τ = 17) time series shown in Fig. 4.22, the performance of the evolution error is shown in Fig. 4.23, and the optimization the structure of T1FIS using "Triangular" MFs with the GAs shown in Fig. 4.24.

The forecast obtained of the optimized interval type-2 fuzzy integrators using "igaussmtype2" MFs for the Mackey-Glass (τ = 17) time series is shown in

Table 4.2 Result of the optimization of fuzzy integrator with the GAs

No. exp.	Type-1 fuzzy integrators			Interval type-2 fuzzy integrators		
	Gaussian	Gbell	Triangular	igaussmtype2	igbelltype2	itritype2
1	0.02359056	0.02629806	0.08221695	0.021158482	0.02118375	0.02071019
2	0.0228442	0.02433066	0.08161724	0.021033109	0.02081994	0.02043439
3	0.0223283	0.02338475	0.08161615	0.021001443	0.02061247	0.02035049
4	0.02209832	0.02249878	0.08161613	0.020976213	0.02056261	0.02032144
5	0.02189447	0.02163905	0.08161613	0.020962286	0.02052157	0.02030222
6	0.02173872	0.02154845	0.08161613	0.020947947	0.02049882	0.02027343
7	0.02165485	0.02148594	0.08161613	0.020936484	0.02048822	0.02025226
8	0.02159362	0.02146486	0.08161613	0.020921625	0.02047444	0.02024165
9	0.02156282	0.02144333	0.08161613	0.020913384	0.02045962	0.02023573
10	0.02152446	0.02142164	0.08161613	0.020907752	0.02044867	0.02023097
Average	0.02208303	0.02255155	0.08167632	0.020975873	0.02060701	0.02033528

Table 4.3 Performance of the optimization of fuzzy integrator with the GAs

Metrics	Type-1 fuzzy integrators			Interval type-2 integrators		
	Gaussian	Gbell	Triangular	igaussmtype2	igbelltype2	itritype2
RMSE (Best)	0.02152446	0.021421638	0.081616127	0.020907752	0.020448674	0.02023097
RMSE (Average)	0.022083031	0.022551551	0.081676324	0.020975873	0.02060701	0.020335278
MSE	0.000563653	0.000593601	0.007011427	0.000455647	0.000447241	0.000435666
MAE	0.016962307	0.017643378	0.066130265	0.015458052	0.015277128	0.015126578
MPE	−8.337207925	−5.889802358	−1.418200646	−4.415973906	−3.886701193	−2.082901948
MAPE	1.917797038	1.977747607	7.491090409	1.746785771	1.731109592	1.696076726

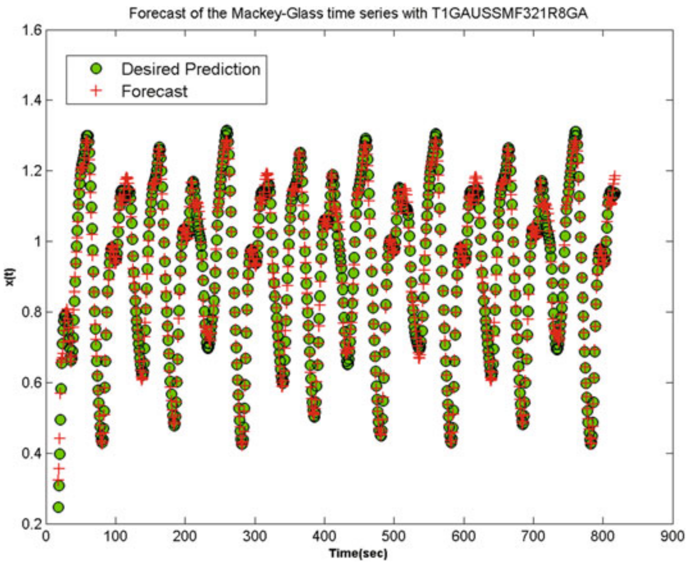

Fig. 4.16 Forecast of T1FIS using “Gauss” MFs for the Mackey-Glass time series

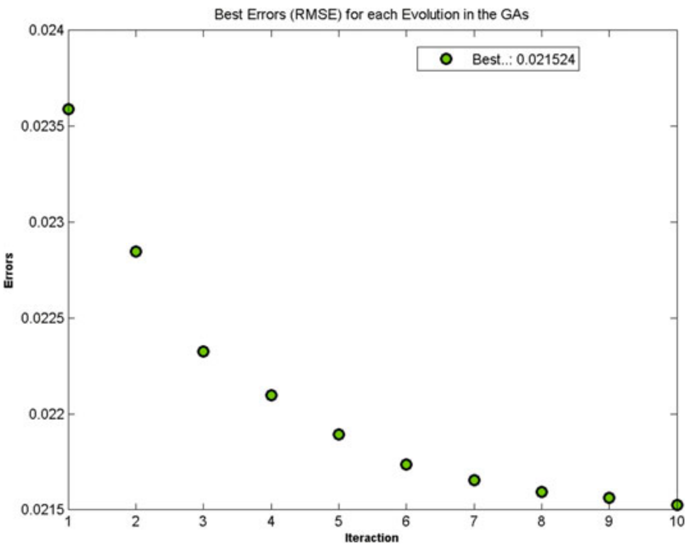

Fig. 4.17 Evolution error (RMSE) of the GAs for the T1FIS using “Gauss” MFs

Fig. 4.25, the performance of the evolution error is shown in Fig. 4.26, and the optimization structure of the interval type-2 fuzzy integrators using “igaussmtype2” MFs with the GAs is shown in Fig. 4.27.

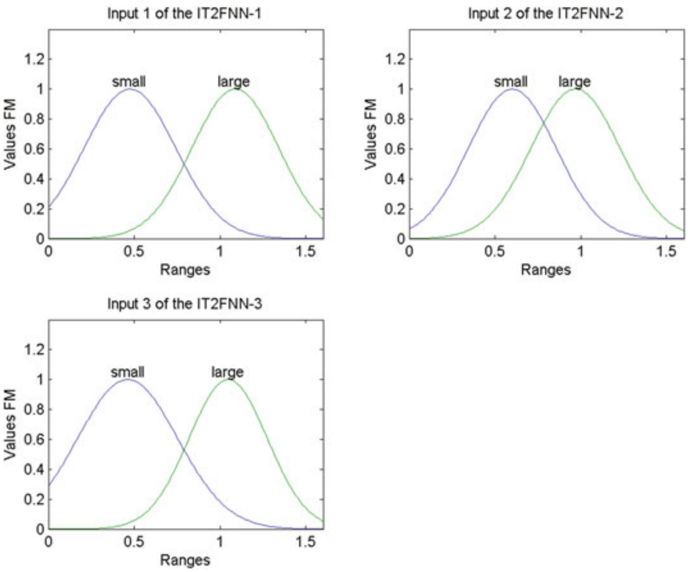

Fig. 4.18 Final MFs after optimized the T1FIS using "Gauss" MFs

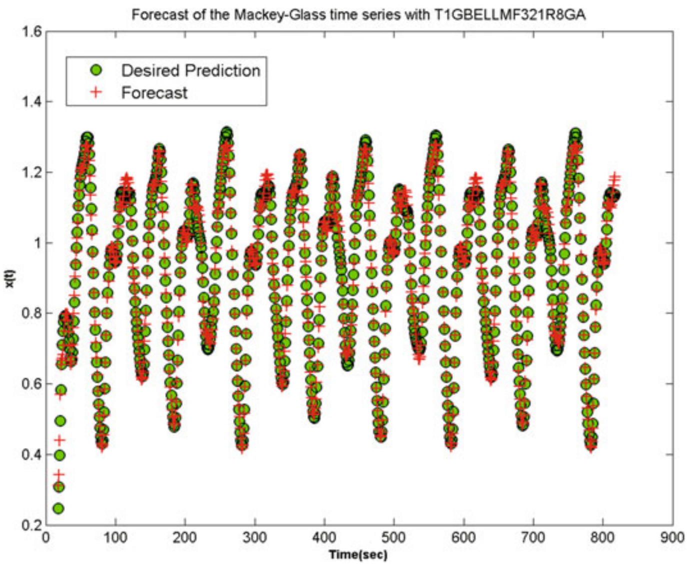

Fig. 4.19 Forecast of T1FIS using "Gbell" MFs for the Mackey-Glass time series

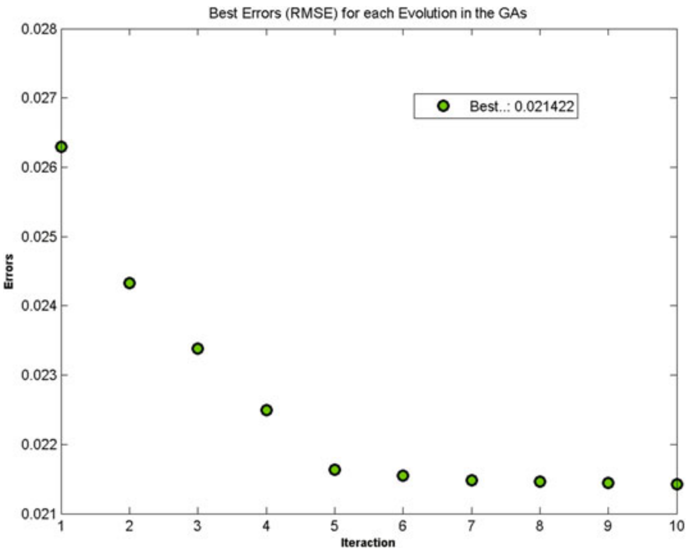

Fig. 4.20 Evolution error (RMSE) of the GAs for the T1FIS using "Gbell" MFs

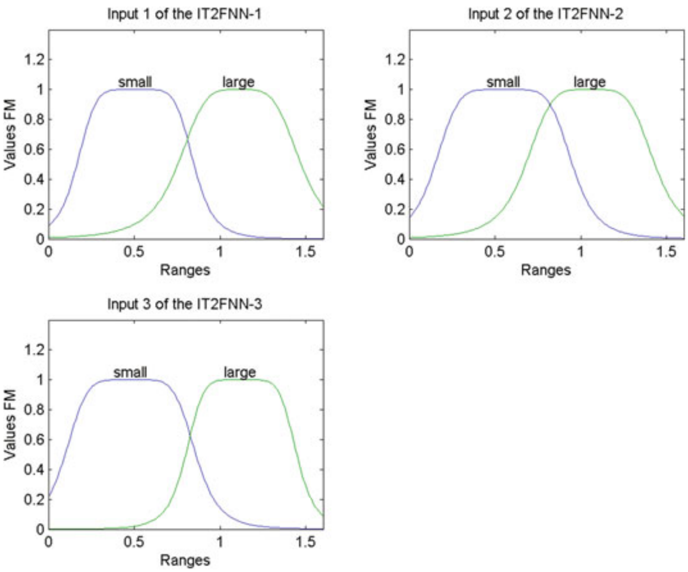

Fig. 4.21 Final MFs after optimized the T1FIS using "Gbell" MFs

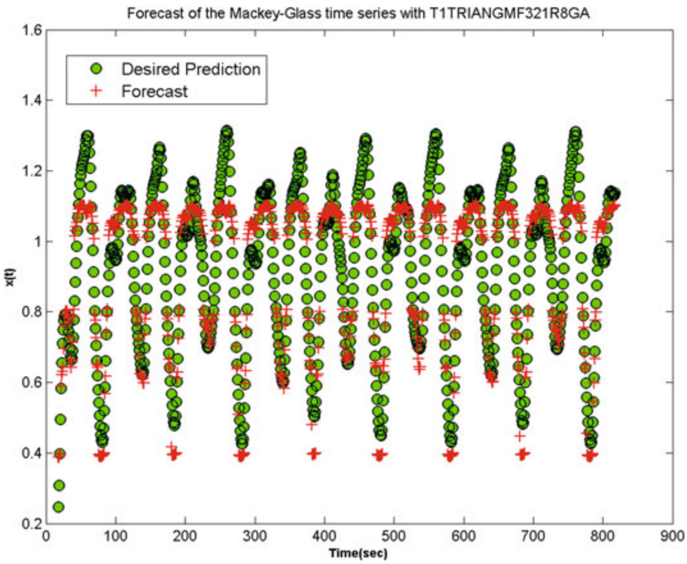

Fig. 4.22 Forecast of T1FIS using “Triangular” MFs for the Mackey-Glass time series

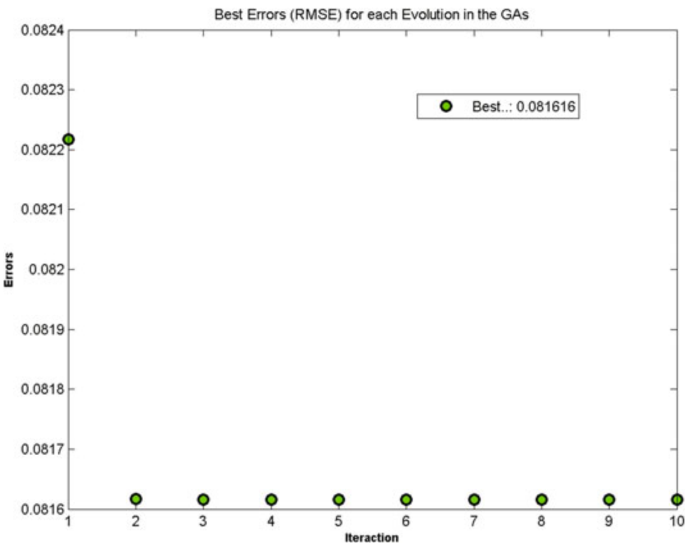

Fig. 4.23 Evolution error (RMSE) of the GAs for the T1FIS using “Triangular” MFs

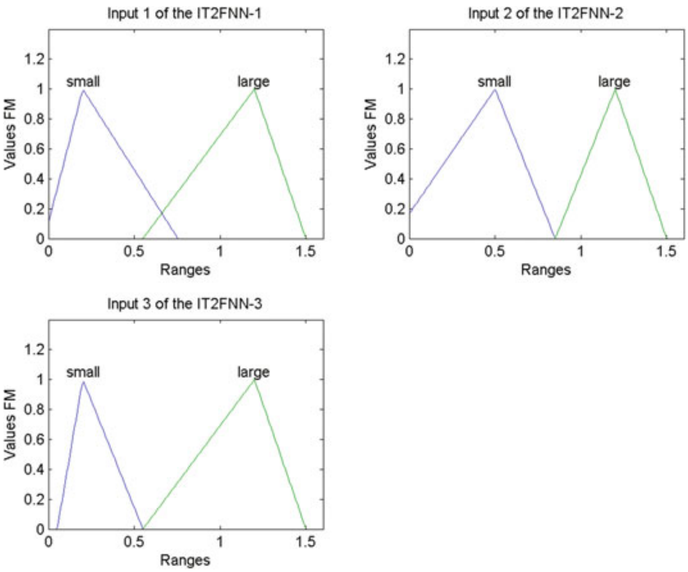

Fig. 4.24 Final MFs after optimized the T1FIS using "Triangular" MFs

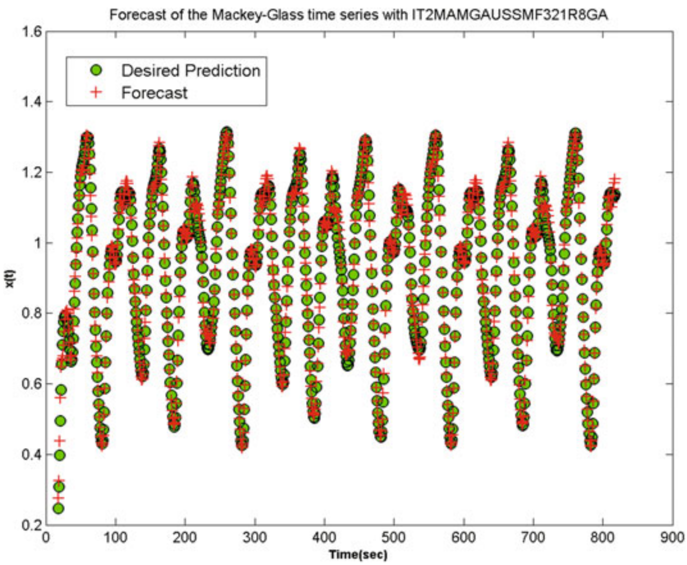

Fig. 4.25 Forecast of IT2FIS using "igaussmtype2" MFs for the Mackey-Glass time series

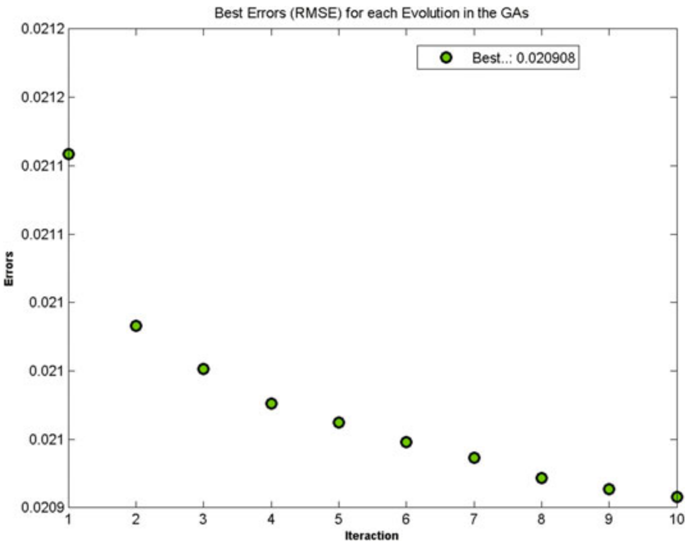

Fig. 4.26 Evolution error (RMSE) of the GAs for the IT2FIS using "igaussmtype2" MFs

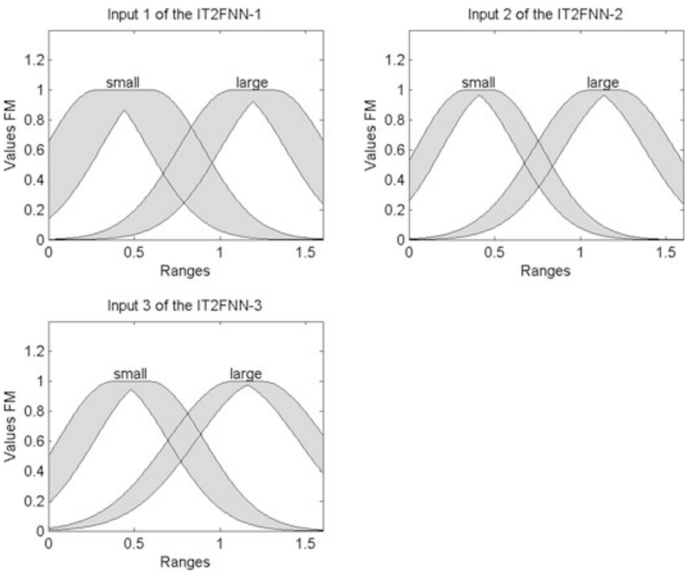

Fig. 4.27 Final MFs after optimized the IT2FIS using "igaussmtype2" MFs

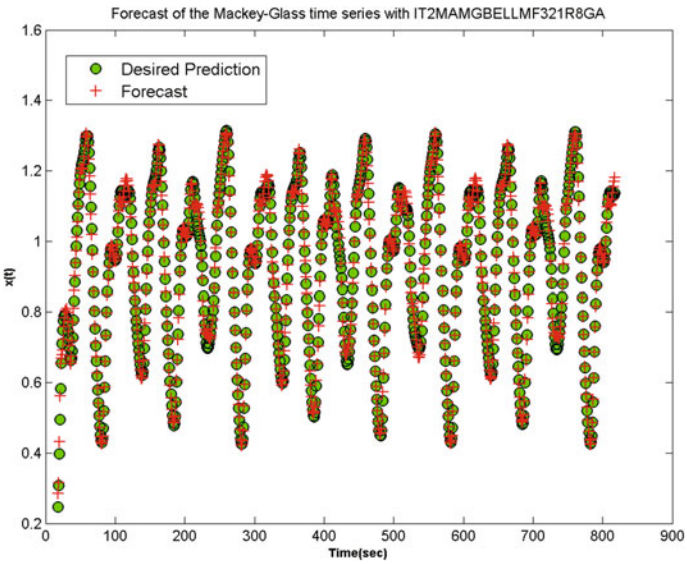

Fig. 4.28 Forecast of IT2FIS using "igbelltype2" MFs for the Mackey-Glass time series

The forecast obtained of the optimized interval type-2 fuzzy integrators using "igbelltype2" MFs for the Mackey-Glass (τ = 17) time series is shown in Fig. 4.28, the performance of the evolution error is shown in Fig. 4.29, and the optimization

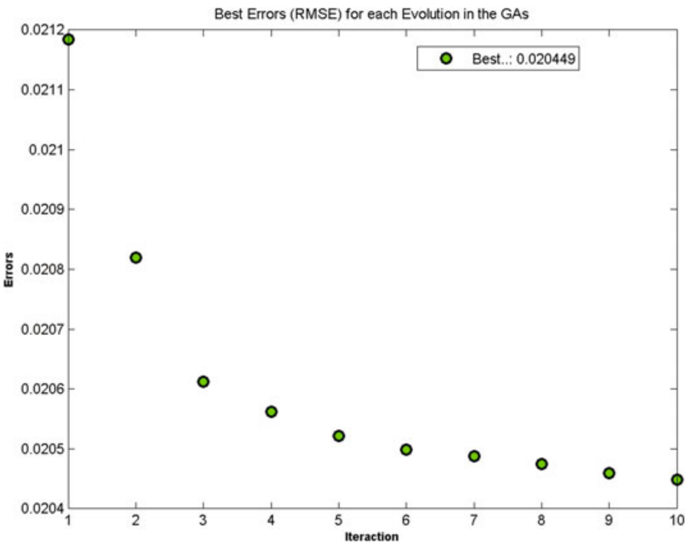

Fig. 4.29 Evolution error (RMSE) of the GAs for the IT2FIS using "igbelltype2" MFs

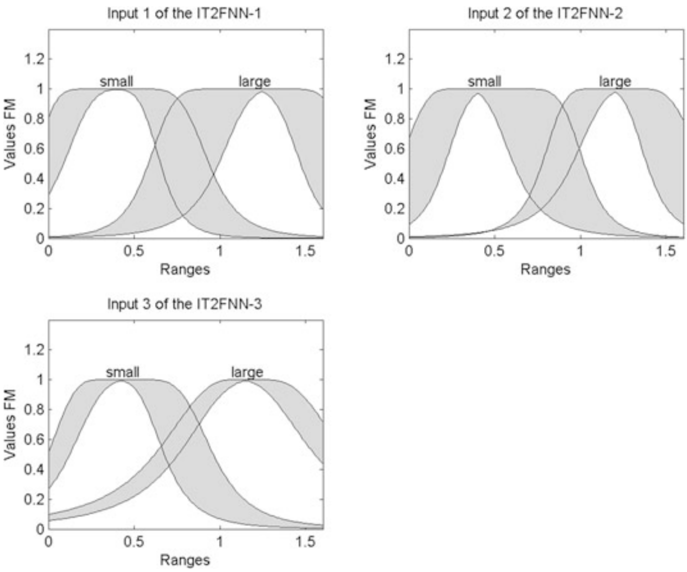

Fig. 4.30 Final MFs after optimized the IT2FIS using “igbelltype2” MFs

structure of the interval type-2 fuzzy integrators using “igbelltype2” MFs with the GAs is shown in Fig. 4.30.

The forecast obtained of optimized the interval type-2 fuzzy integrators using “itritype2” MFs for the Mackey-Glass (τ = 17) time series is shown in Fig. 4.31, the performance of the evolution is error shown in Fig. 4.32, and the optimization structure of the interval type-2 fuzzy integrators using “itritype2” MFs with GAs shown in Fig. 4.33.

4.1.3 Optimization of the Fuzzy Integrators with the Particle Swarm Optimization

The obtained results with optimized the fuzzy integrators with the PSO are shown on Table 4.3. The best error is of 0.035228102 and the average error is of 0.036484603 for the type-1 fuzzy integrator using “Gbell” MFs, and the best error is of 0.023691987 and the average error is of 0.023691987 for the interval type-2 fuzzy integrator using “igbellype2” MFs for the Mackey-Glass (τ = 17) time series.

We are presenting 10 experiments in Table 4.4, but the average error was calculated considering 30 experiments with the same parameters and conditions for the PSO. Therefore to evaluate the performance of the 30 experiments for this work, we applied different metrics to calculate average errors as shown in Table 4.5.

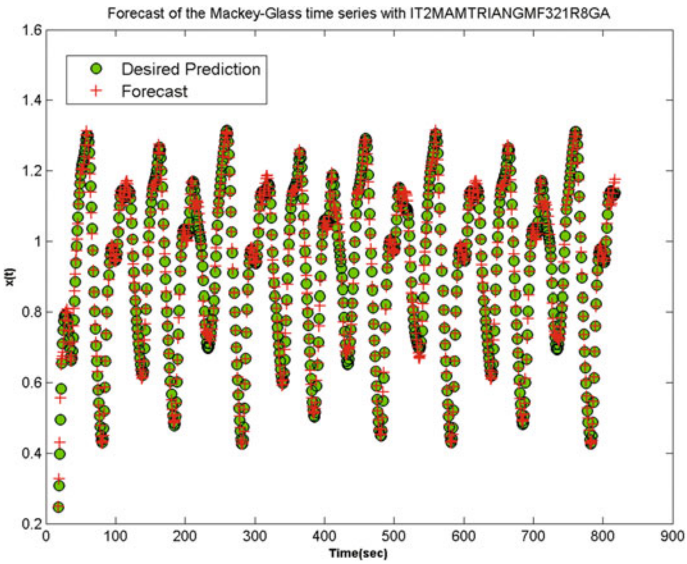

Fig. 4.31 Forecast of IT2FIS using "itritype2" MFs for the Mackey-Glass time series

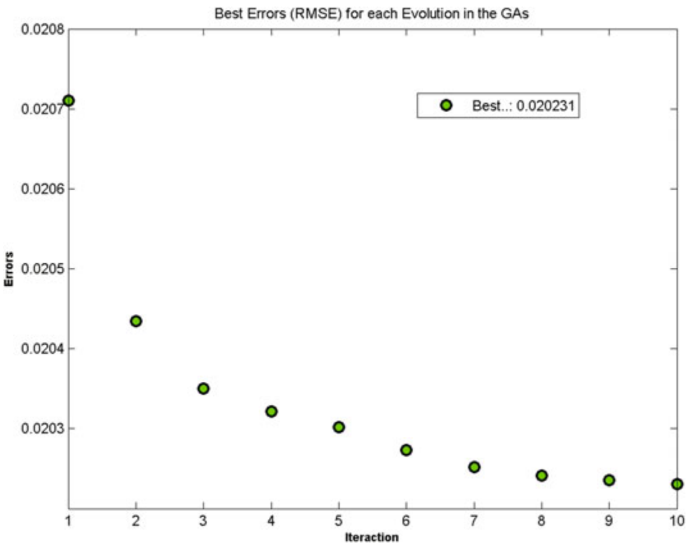

Fig. 4.32 Evolution error (RMSE) of the GAs for the IT2FIS using "itritype2" MFs

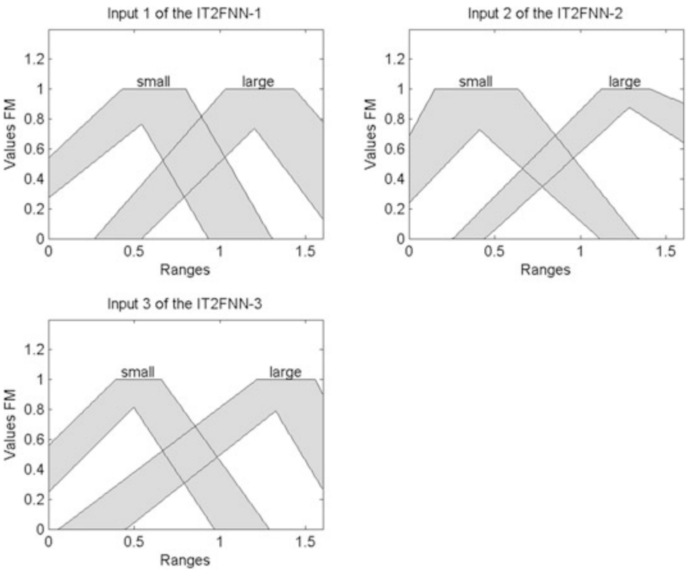

Fig. 4.33 Final MFs after optimized the IT2FIS using “itritype2” MFs

The forecast obtained of the optimized T1FIS using “Gaussian” MFs for the Mackey-Glass (τ = 17) time series shown in Fig. 4.34, the performance of the evolution error is shown in Fig. 4.35, and the optimization structure of T1FIS using “Gaussian” MFs with the PSO is shown in Fig. 4.36.

The forecast obtained of the optimized T1FIS using “Gbell” MFs for the Mackey-Glass (τ = 17) time series shown in Fig. 4.37, the performance of the evolution error is shown in Fig. 4.38, and the optimization structure of T1FIS using “Gbell” MFs with the PSO is shown in Fig. 4.39.

The forecast obtained of optimized the T1FIS using “Triangular” MFs for the Mackey-Glass (τ = 17) time series shown in Fig. 4.40, the performance of the evolution error is shown in Fig. 4.41, and the optimization structure of the T1FIS using “Triangular” MFs with the PSO is shown in Fig. 4.42.

The forecast obtained of the optimized interval type-2 fuzzy integrators using “igaussmtype2” MFs for the Mackey-Glass (τ = 17) time series is shown in

Table 4.4 Result of the optimization of fuzzy integrator with the PSO

No. Exp.	Type-1 fuzzy integrators			Interval type-2 fuzzy integrators		
	Gaussian	Gbell	Triangular	igaussmtype2	igbelltype2	itritype2
1	0.038239735	0.038527952	0.081862239	0.024438775	0.023724407	0.025115433
2	0.038181885	0.037436519	0.081612326	0.024433153	0.023720482	0.025115387
3	0.038066524	0.037057909	0.081426474	0.024418709	0.023719104	0.025115354
4	0.037946889	0.036709338	0.081283351	0.024347737	0.023713049	0.025115331
5	0.03760558	0.036438493	0.081142876	0.024287919	0.023702786	0.025115316
6	0.037294437	0.036133836	0.081004562	0.024261472	0.023687797	0.025115285
7	0.036723101	0.035971393	0.080849129	0.024243773	0.023678693	0.025115256
8	0.036510778	0.035753022	0.080176079	0.024228428	0.023670658	0.025115218
9	0.036203161	0.03558947	0.079900848	0.024208057	0.023654475	0.025115174
10	0.035946912	0.035228102	0.079753554	0.024183221	0.023648414	0.025115114
Average	0.0372719	0.036484603	0.080901144	0.024305124	0.023691987	0.025115287

Table 4.5 Performance of the optimization of fuzzy integrators with the PSO

Metrics	Type-1 fuzzy integrator			Type-2 fuzzy integrator		
	Gaussian	Gbell	Triangular	igaussmtype2	igbelltype2	Itritype2
RMSE (Best)	0.031543721	0.034842843	0.106253992	0.01891173	0.020071084	0.0205731
RMSE (Average)	0.047506067	0.050711845	0.117611133	0.023752013	0.022696277	0.02576771
MSE	0.007214748	0.005309127	0.019661961	0.002960679	0.001445358	0.00205462
MAE	0.058668759	0.054341832	0.113583028	0.034935372	0.025597427	0.03070206
MPE	0.495293832	0.238903859	0.548675423	0.045305119	−1.41435995	1.6958679
MAPE	6.918855495	6.407519242	14.02215078	4.13271772	2.957239916	3.41521921

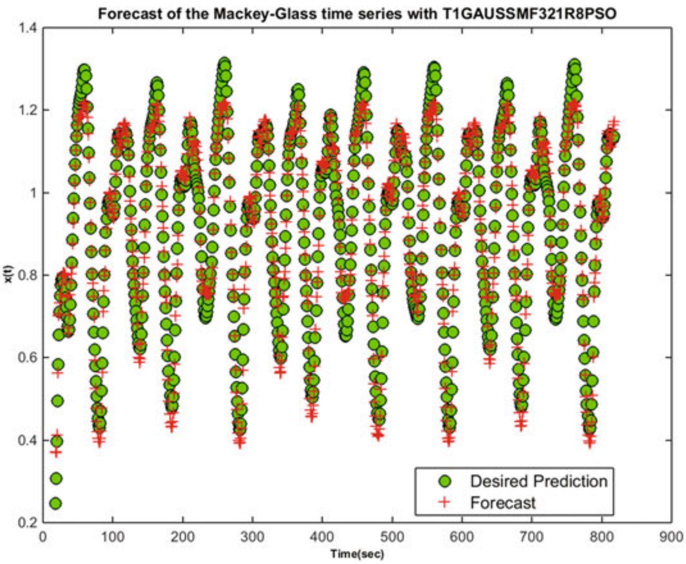

Fig. 4.34 Forecast of T1FIS using "Gaussian" MFs for the Mackey-Glass time series

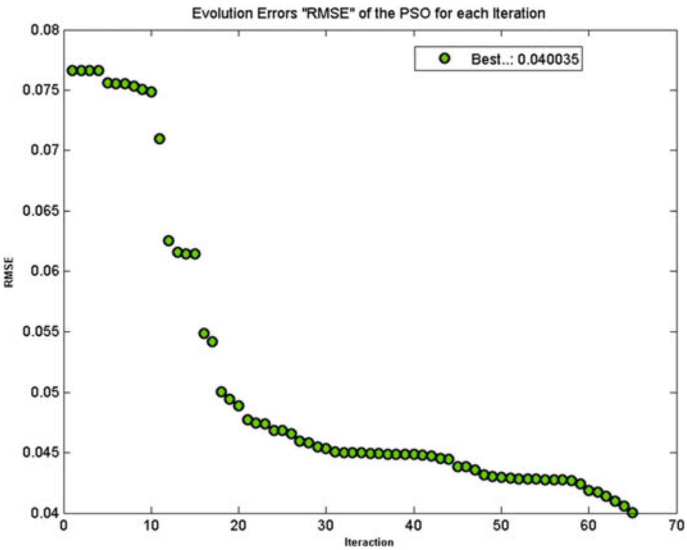

Fig. 4.35 Evolution error (RMSE) of the PSO for the T1FIS using "Gaussian" MFs

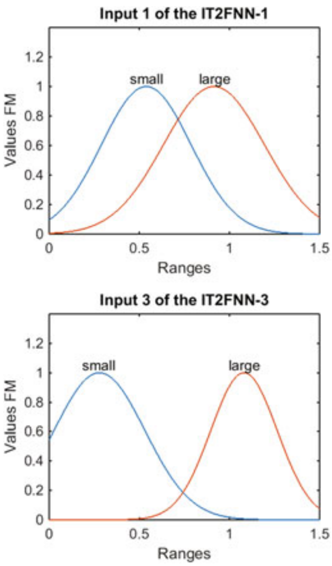

Fig. 4.36 Final MFs after optimized the T1FIS using “Gaussian” MFs

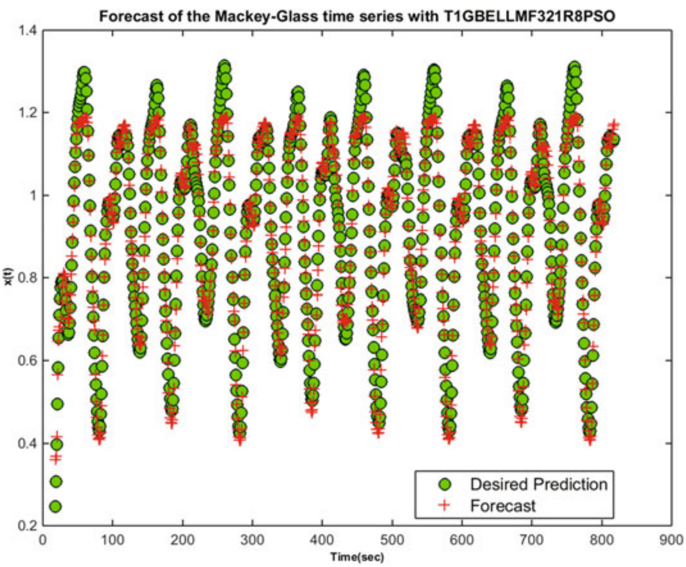

Fig. 4.37 Forecast of T1FIS using “Gbell” MFs for the Mackey-Glass time series

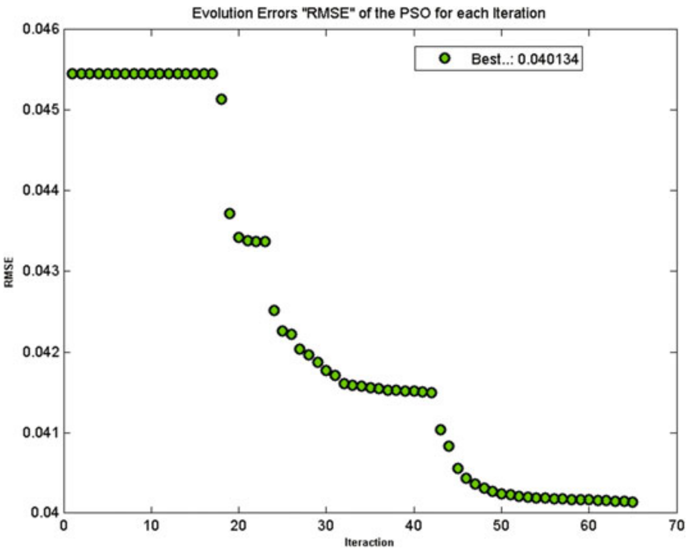

Fig. 4.38 Evolution error (RMSE) of the PSO for the T1FIS using "Gbell" MFs

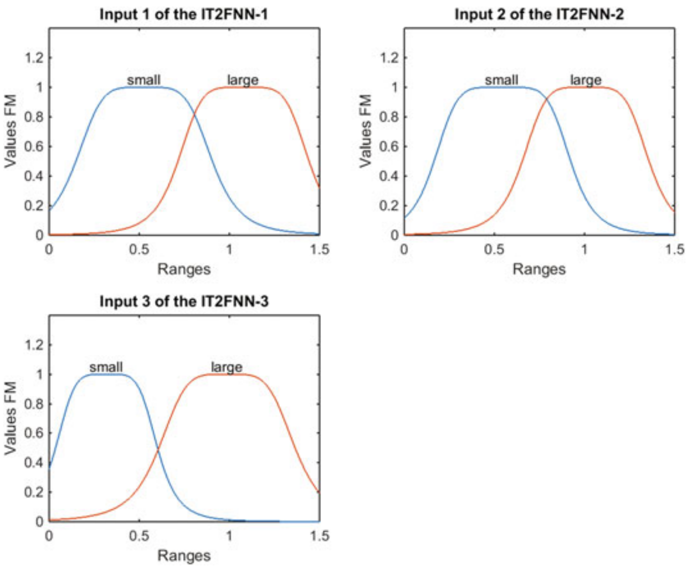

Fig. 4.39 Final MFs after optimized the T1FIS using "Gbell" MFs with PSO

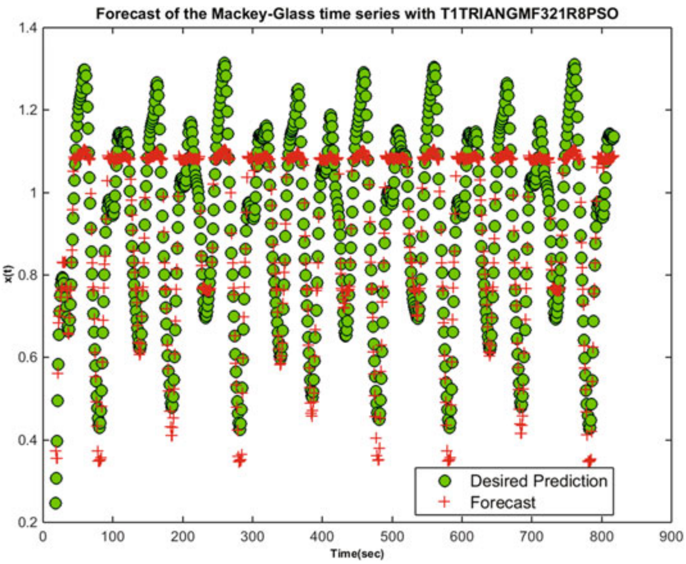

Fig. 4.40 Forecast of T1FIS using “Triangular” MFs for the Mackey-Glass time series

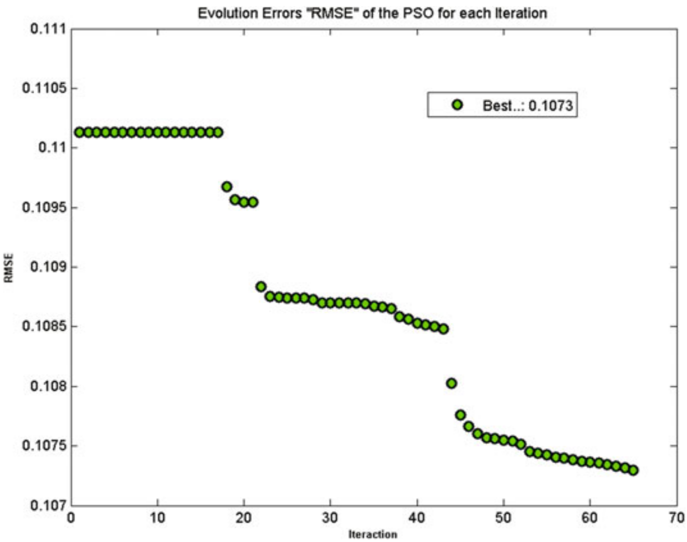

Fig. 4.41 Evolution error (RMSE) of the PSO for the T1FIS using “Triangular” MFs

Fig. 4.43, the performance of the evolution error is shown in Fig. 4.44, and the optimization structure of the interval type-2 fuzzy integrators using “igaussmtype2” MFs with the PSO is shown in Fig. 4.45.

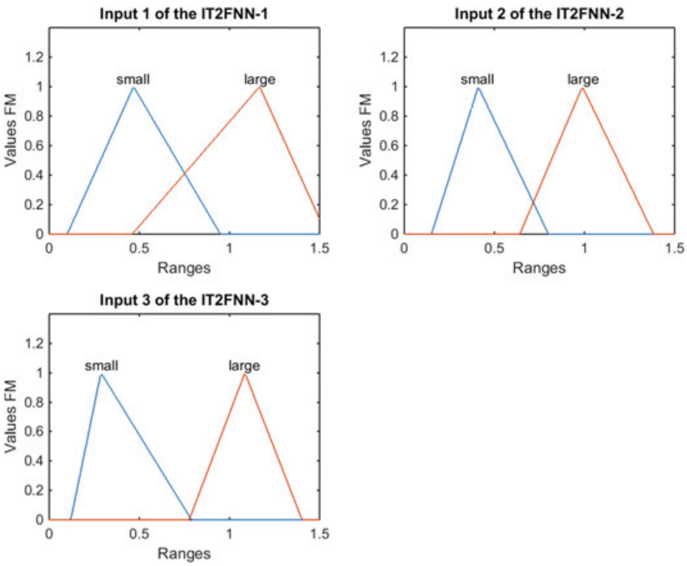

Fig. 4.42 Final MFs after optimized the T1FIS using "Triangular" MFs with PSO

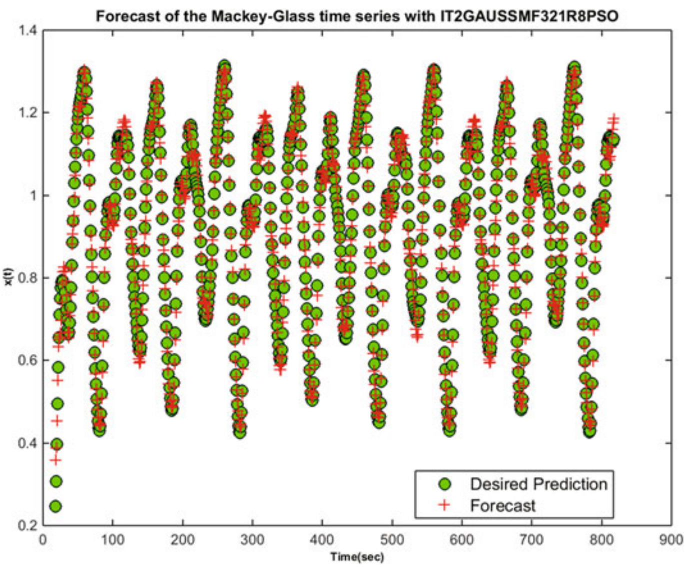

Fig. 4.43 Forecast of IT2FIS using "igaussmtype2" MFs for the Mackey-Glass time series

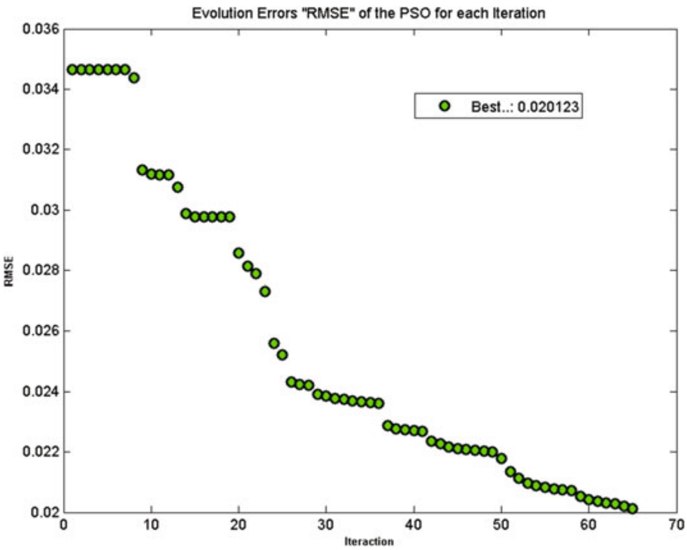

Fig. 4.44 Evolution error (RMSE) of the PSO for the IT2FIS using “igaussmtype2” MFs

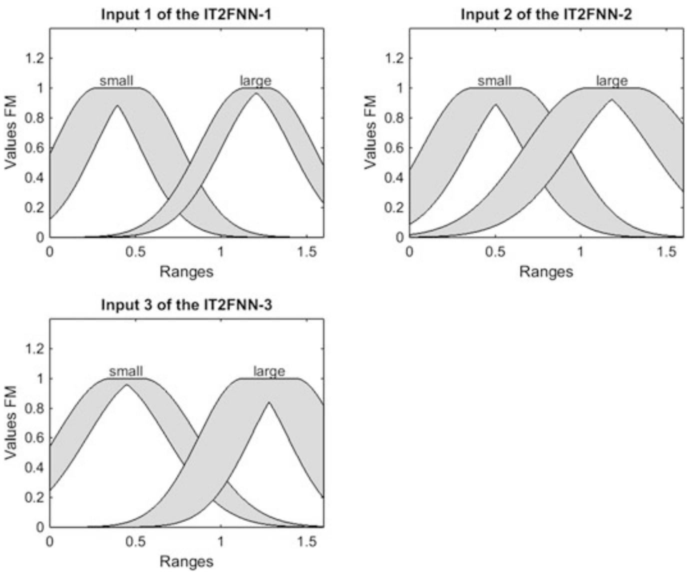

Fig. 4.45 Final MFs after optimized the IT2FIS using “igaussmtype2” MFs

The forecast obtained of the optimized interval type-2 fuzzy integrators using “igbelltype2” MFs for the Mackey-Glass (τ = 17) time series is shown in Fig. 4.46, the performance of the evolution error is shown in Fig. 4.47, and the optimization

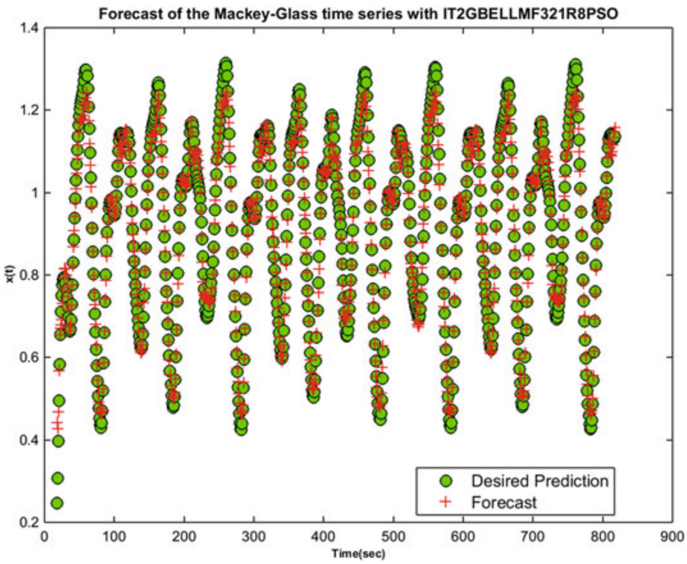

Fig. 4.46 Forecast of IT2FIS using "igbelltype2" MFs for the Mackey-Glass time series

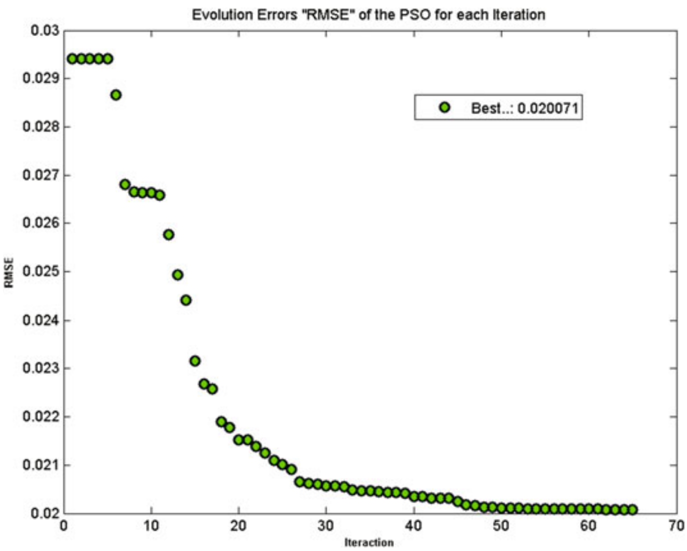

Fig. 4.47 Evolution error (RMSE) of the PSO for the IT2FIS using "igbelltype2" MFs

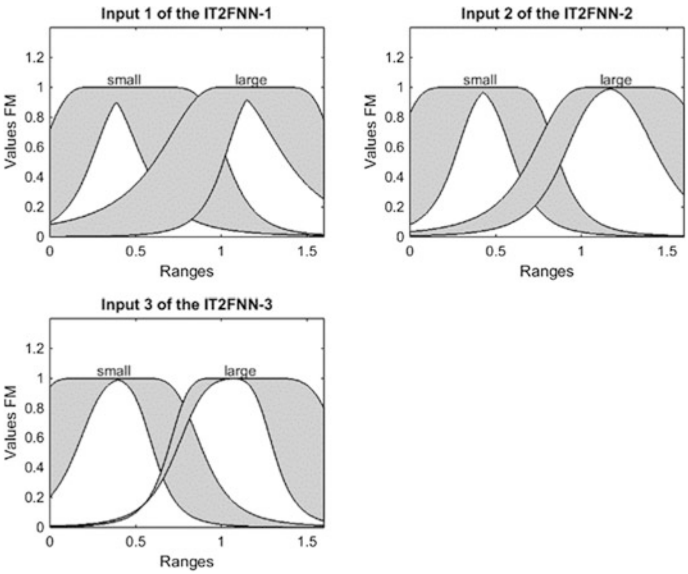

Fig. 4.48 Final MFs after optimized the IT2FIS using “igbelltype2” MFs

structure of the interval type-2 fuzzy integrators using “igbelltype2” MFs with the PSO is shown in Fig. 4.48.

The forecast obtained of the optimized interval type-2 fuzzy integrators using “itritype2” MFs for the Mackey-Glass (τ = 17) time series is shown in Fig. 4.49,

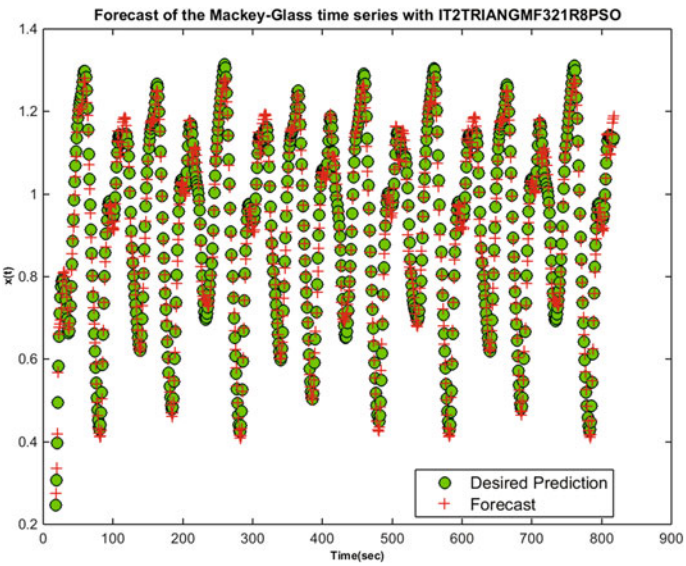

Fig. 4.49 Forecast of IT2FIS using “itritype2” MFs for the Mackey-Glass time series

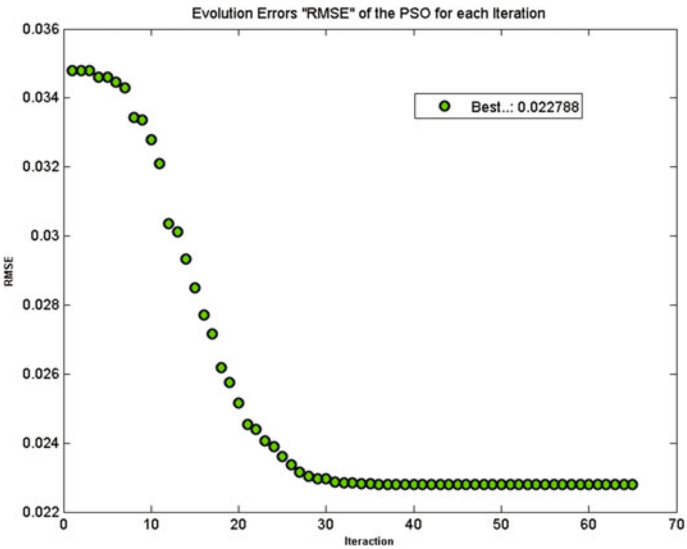

Fig. 4.50 Evolution error (RMSE) of the PSO for the IT2FIS using "itritype2" MFs

the performance of the evolution error is shown in Fig. 4.50, and the optimization structure of the interval type-2 fuzzy integrators using "itritype2" MFs with the PSO is shown in Fig. 4.51.

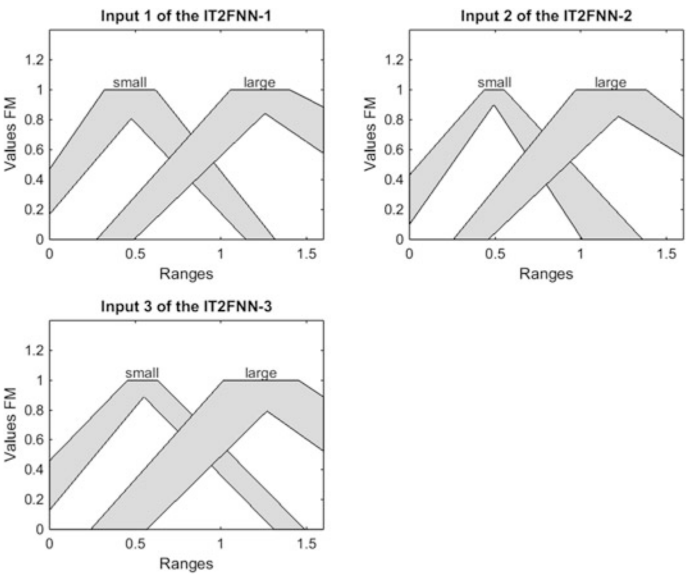

Fig. 4.51 Final MFs after optimized the IT2FIS using "itritype2" MFs

4.2 Mexican Stock Exchange Time Series

This section presents the simulation and test results obtained by applying the proposed prediction method to the Mexican Stock Exchange (BMV) time series for periods (01/03/2011–12/31/2015) (Fig. 3.3) using different approach of the ensemble of IT2FNN architectures, used in this work.

4.2.1 Ensemble of IT2FNN Architectures for BMV Time Series

The ensemble of IT2FNN architectures has three models as follows: the IT2FNN-1 model optimize the parameters of the "igaussmtype2" MFs (Fig. 3.9a), the learning rate is 0.03 and the desired error is 0.00001; the IT2FNN-2 model optimize the parameters of the "igausstype2" MFs (Fig. 3.9b), the learning rate is 0.011 and the desired error is 0.000001; and the IT2FNN-3 model optimize the parameters of the "igaussstype2" MFs (Fig. 3.9c), the learning rate is 0.02 and the desired error is 0.0000001. The number the epochs for training the IT2FNN models is 100.

The obtained results of the ensemble of IT2FNN architectures are shown on Table 4.6. The RMSE (best) is of 0.010127619, the RMSE (average) is of 0.016586239, the MSE is 0.001738454, the MAE is 0.012085755, the MPE is 1.284208192 and the MAPE 0.275038065 with the IT2FNN-1 model. Therefore the IT2FNN-1 model is better than the IT2FNN-2 and IT2FNN-3 models.

4.2.1.1 IT2FNN-1 Model

The forecast obtained for the IT2FNN-1 for the BMV time series is shown in Fig. 4.52, the evolution error is shown in Fig. 4.53, and the optimization structure of the IT2FNN-1 with backpropagation (BP) learning algorithm is shown in Fig. 4.54.

Table 4.6 Performance of the ensemble of IT2FNN for the BMV time series

Metrics	IT2FNN-1	IT2FNN-2	ITFNN-3
RMSE (Best)	0.010127619	0.022896849	0.010126143
RMSE (Average)	0.016586239	0.02748781	0.018984807
MSE	0.001738454	0.002373369	0.002859776
MAE	0.012085755	0.023283565	0.015326454
MPE	1.284208192	2.469791397	1.626561735
MAPE	0.275038065	0.385345148	0.511999726

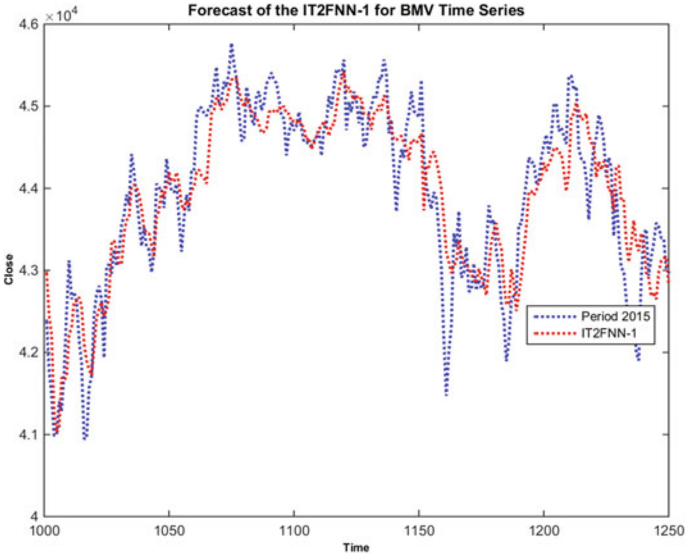

Fig. 4.52 Forecast of IT2FNN-1 for the BMV time series

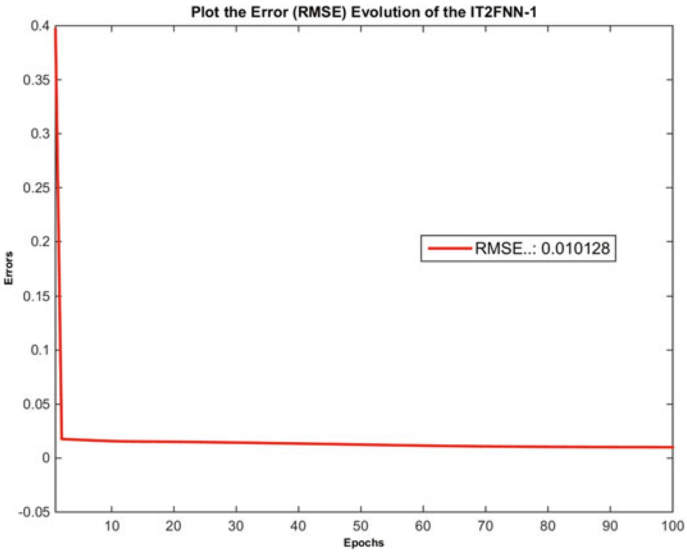

Fig. 4.53 Evolution error (RMSE) of IT2FNN-1 for the BMV time series

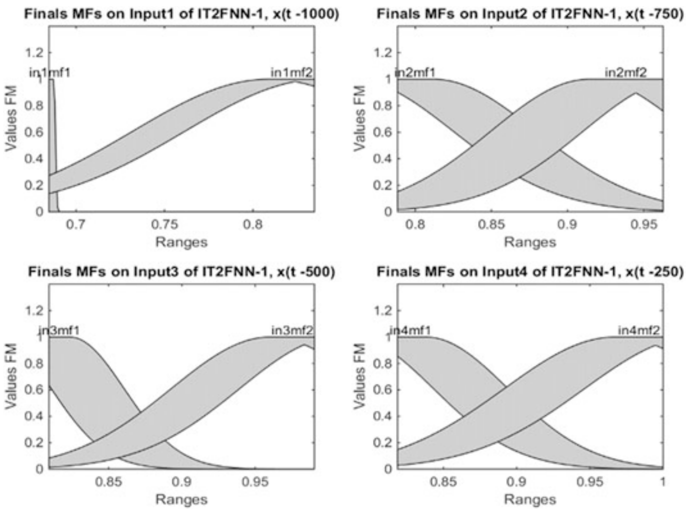

Fig. 4.54 Final MFs after training the IT2FNN-1 model with the BP algorithm

4.2.1.2 IT2FNN-2 Model

The forecast obtained for the IT2FNN-2 for the BMV time series shown in Fig. 4.55, the evolution error is shown in Fig. 4.56, and the structure optimization of the IT2FNN-2 with BP learning algorithm is shown in Fig. 4.57.

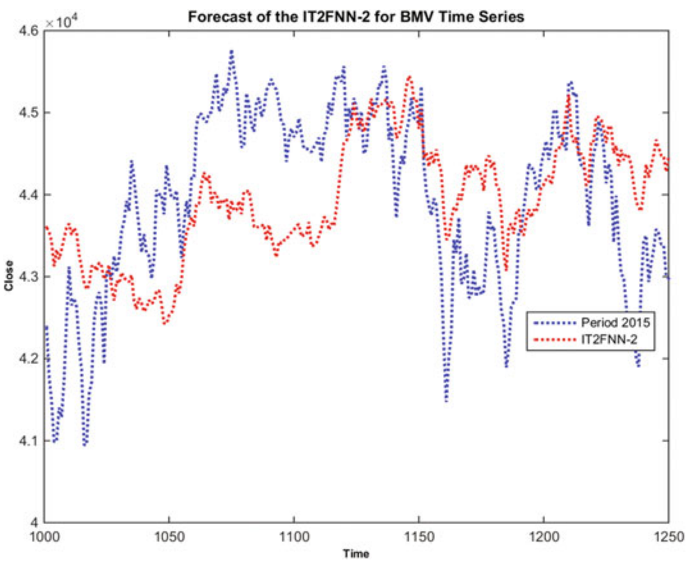

Fig. 4.55 Forecast of the IT2FNN-2 for the BMV time series

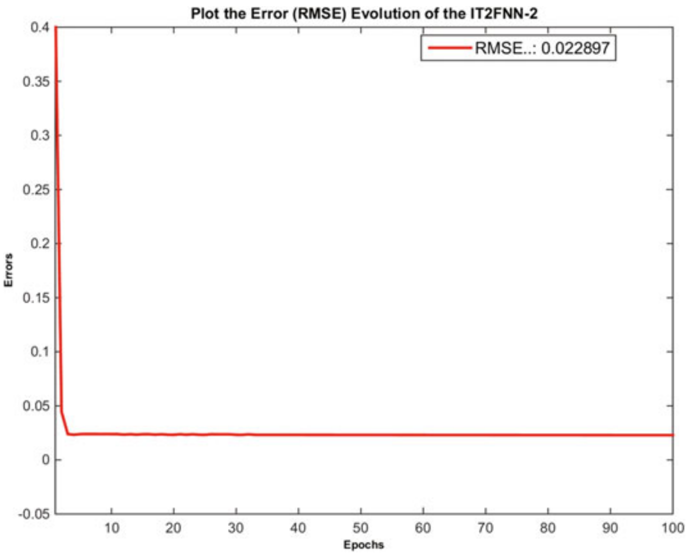

Fig. 4.56 Evolution error (RMSE) of IT2FNN-2 for the BMV time series

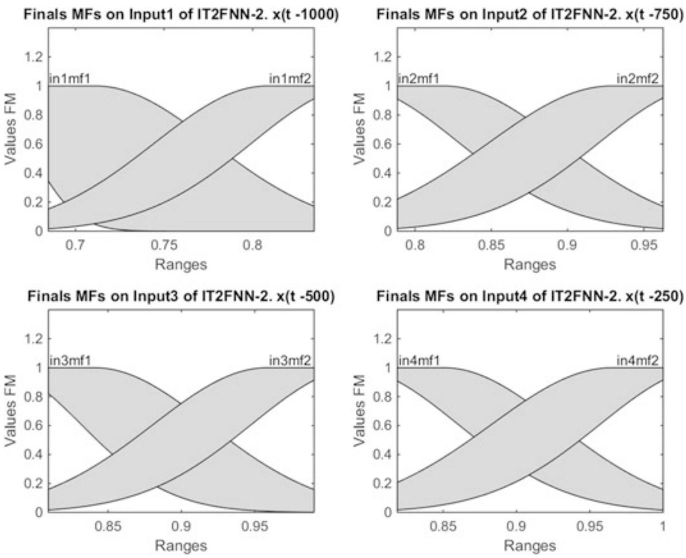

Fig. 4.57 Final MFs after training the IT2FNN-2 model with BP algorithm

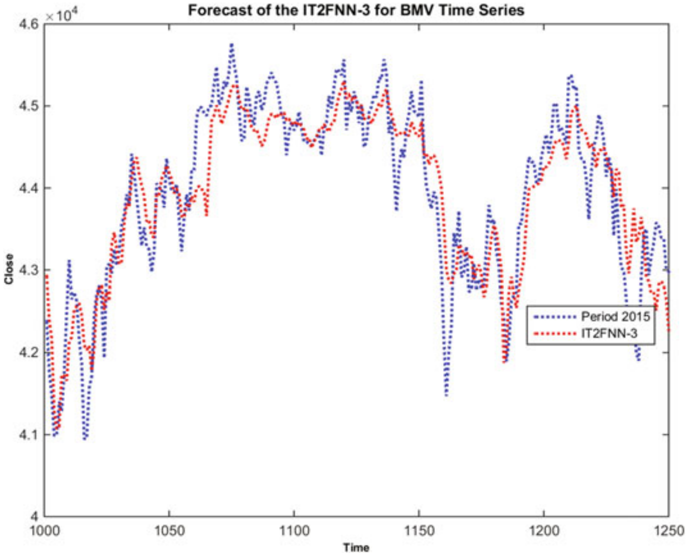

Fig. 4.58 Forecast of IT2FNN-3 for the BMV time series

4.2.1.3 IT2FNN-3 Model

The forecast obtained for the IT2FNN-3 for the BMV time series is shown in Fig. 4.58, the evolution error is shown in Fig. 4.59, and the structure optimization of IT2FNN-3 with BP learning algorithm is shown in Fig. 4.60.

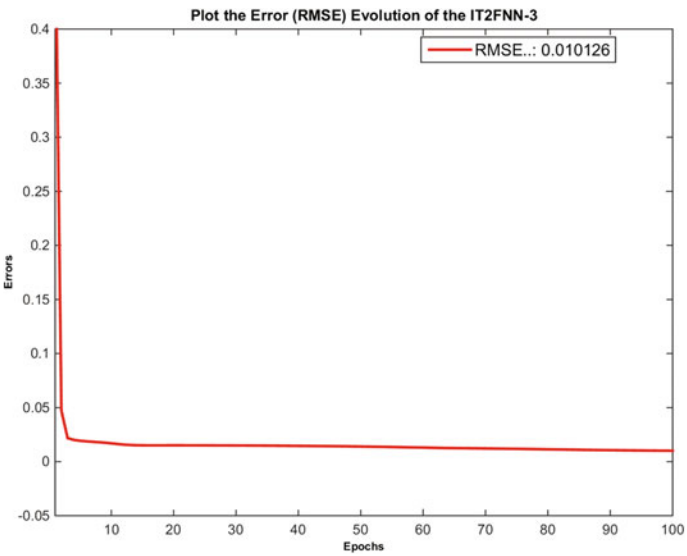

Fig. 4.59 Evolution error (RMSE) of IT2FNN-3 for the BMV time series

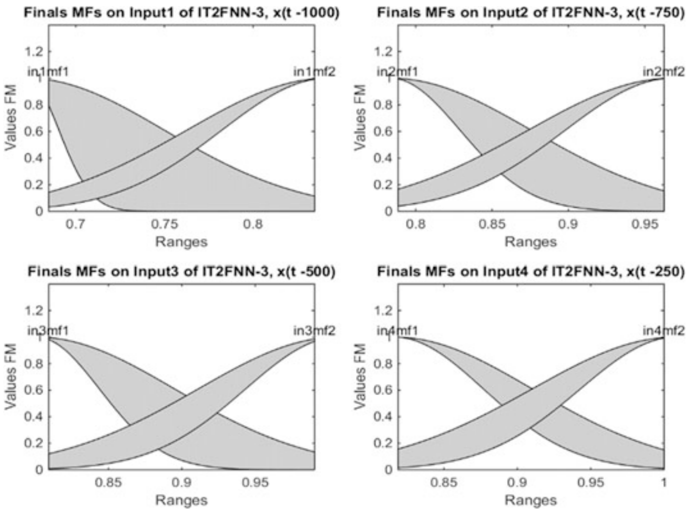

Fig. 4.60 Final MFs after training the IT2FNN-3 model with BP algorithm

4.3 Dow Jones Time Series

This section presents the simulation and test results obtained by applying the proposed prediction method to the Dow Jones time series for periods (01/03/2011–12/31/2015) (Fig. 3.4) using a different approach of the ensemble of IT2FNN architectures, used in this work.

4.3.1 Ensemble of IT2FNN Architectures for Dow Jones Time Series

The ensemble of IT2FNN architectures has three models as follows: the IT2FNN-1 model optimize the parameters of the "igaussmtype2" MFs (Fig. 3.9a), the learning rate is 0.03 and the desired error is 0.00001; the IT2FNN-2 model optimize the parameters of the "igausstype2" MFs (Fig. 3.9b), the learning rate is 0.011 and the desired error is 0.000001; and the IT2FNN-3 model optimize the parameters of the "igaussstype2" MFs (Fig. 3.9c), the learning rate is 0.02 and the desired error is 0.0000001. The number the epochs for training the IT2FNN models is 100.

Table 4.7 Performance of the ensemble of IT2FNN for the Dow Jones time series

Metrics	IT2FNN-1	IT2FNN-2	ITFNN-3
RMSE (Best)	0.015844833	0.01329307	0.01307153
RMSE (Average)	0.020874526	0.01909446	0.018482224
MSE	0.001743898	0.002022886	0.002138805
MAE	0.015181591	0.014462062	0.013647136
MPE	1.598124859	1.521469583	1.436647482
MAPE	0.236962281	0.346789986	0.320293965

The obtained results of the ensemble of IT2FNN architectures are shown on Table 4.7. The RMSE (best) is of 0.01307153, the RMSE (average) is of 0.018482224, the MSE is 0.002138805, the MAE is 0.013647136, the MPE is 1.436647482 and the MAPE 0.320293965 with the IT2FNN-3 model. Therefore the IT2FNN-3 model is better than the IT2FNN-1 and IT2FNN-2 models.

4.3.1.1 IT2FNN-1 Model

The forecast obtained for the IT2FNN-1 for the Dow Jones time series is shown in Fig. 4.61, the evolution error is shown in Fig. 4.62, and the structure optimization of the IT2FNN-1 with BP learning algorithm is shown in Fig. 4.63.

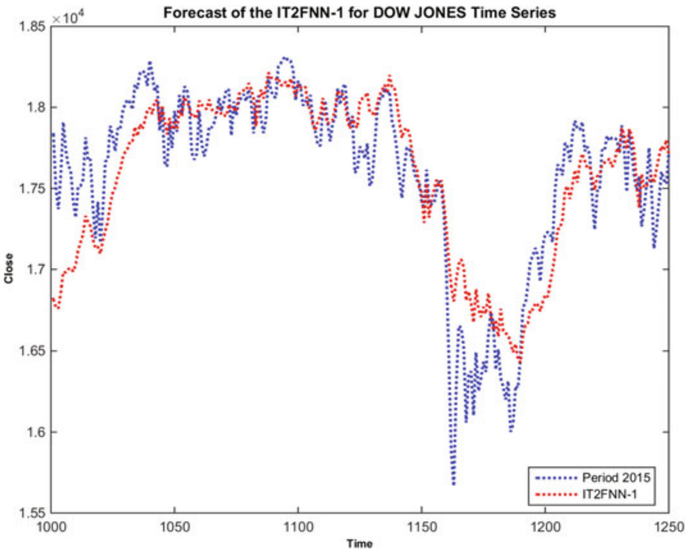

Fig. 4.61 Forecast of IT2FNN-1 for the Dow Jones time series

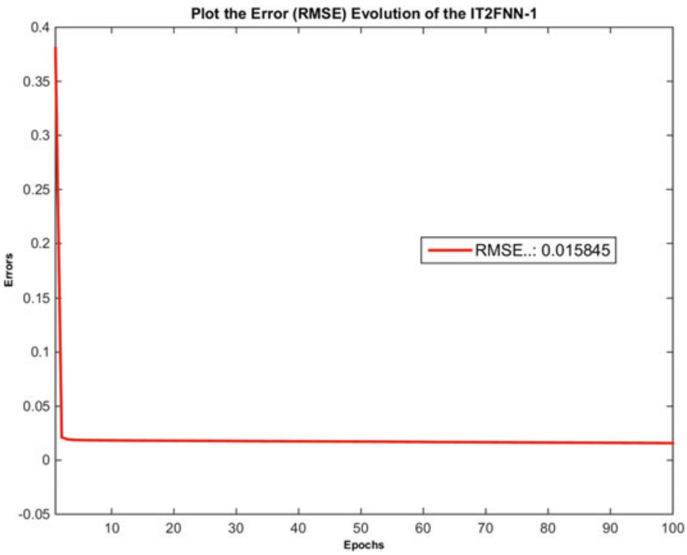

Fig. 4.62 Evolution error (RMSE) of IT2FNN-1 for the Dow Jones time series

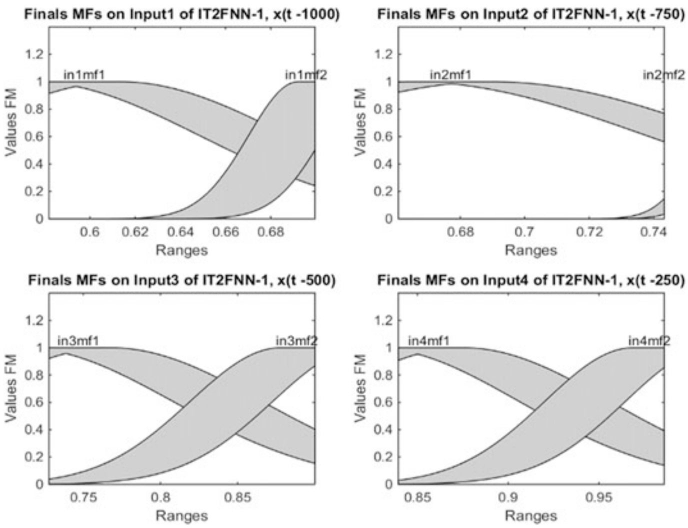

Fig. 4.63 Final MFs after training the IT2FNN-1 model with BP algorithm

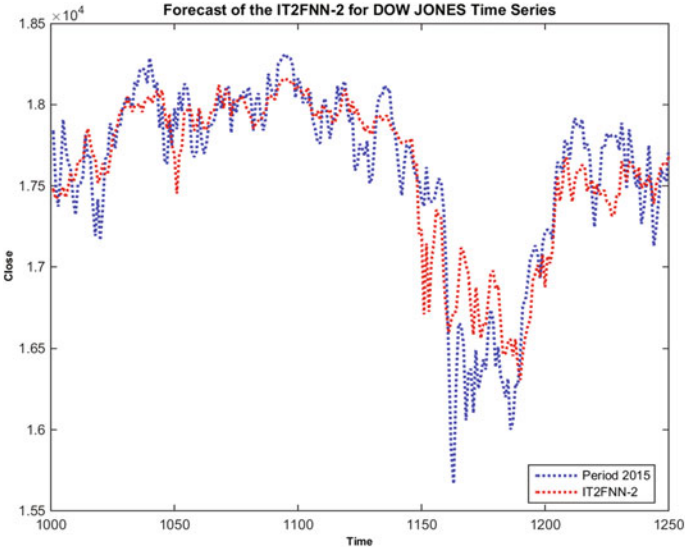

Fig. 4.64 Forecast of IT2FNN-2 for the Dow Jones time series

4.3.1.2 IT2FNN-2 Model

The forecast obtained for the IT2FNN-2 for the Dow Jones time series is shown in Fig. 4.64, the evolution error is shown in Fig. 4.65, and the structure optimization of the IT2FNN-2 with BP learning algorithm is shown in Fig. 4.66.

4.3.1.3 IT2FNN-3 Model

The forecast obtained for the IT2FNN-3 for the Dow Jones time series shown in Fig. 4.67, the evolution error is shown in Fig. 4.68, and the optimization structure of the IT2FNN-3 with BP learning algorithm is shown in Fig. 4.69.

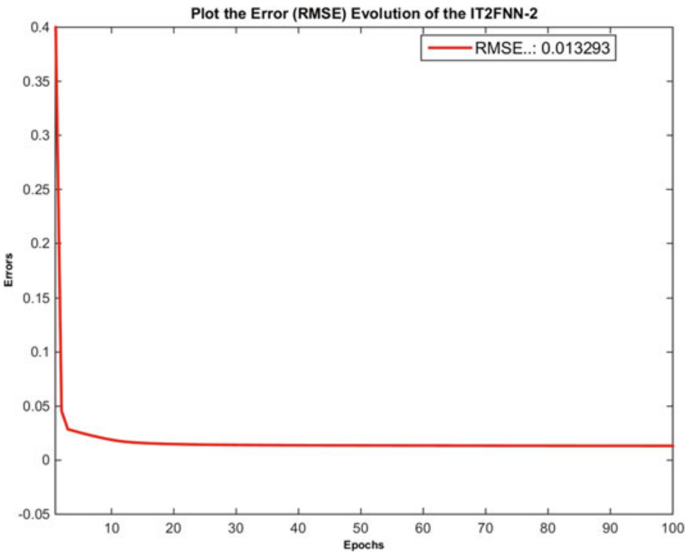

Fig. 4.65 Evolution error (RMSE) of IT2FNN-2 for the Dow Jones time series

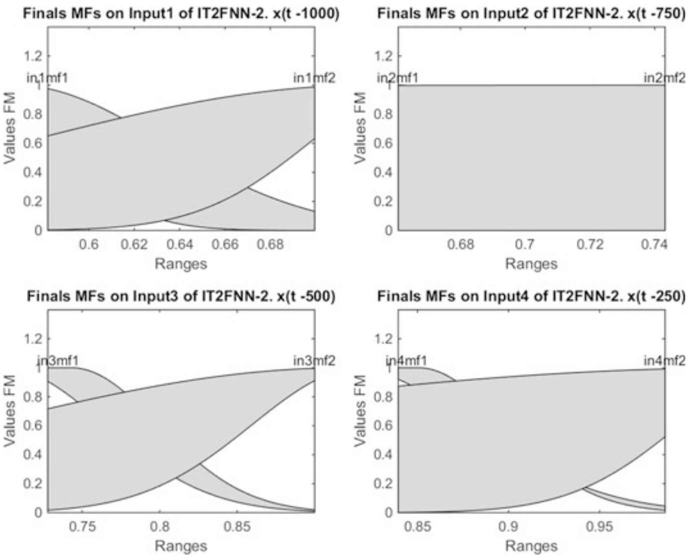

Fig. 4.66 Final MFs after training the IT2FNN-2 model with BP algorithm

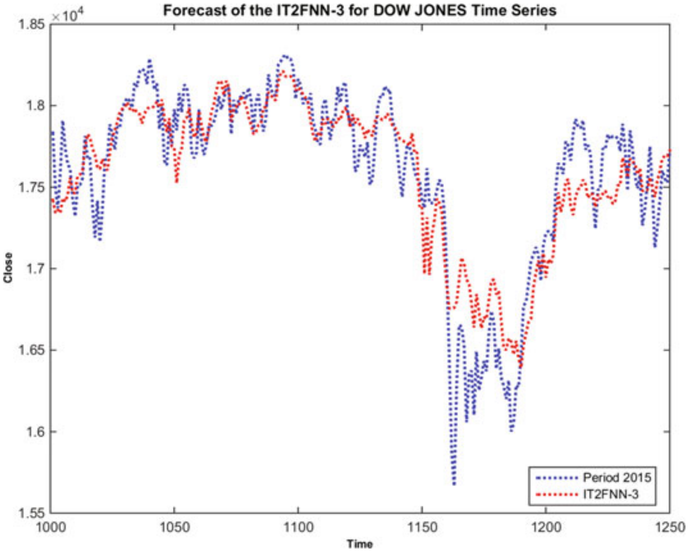

Fig. 4.67 Forecast of IT2FNN-3 for the Dow Jones time series

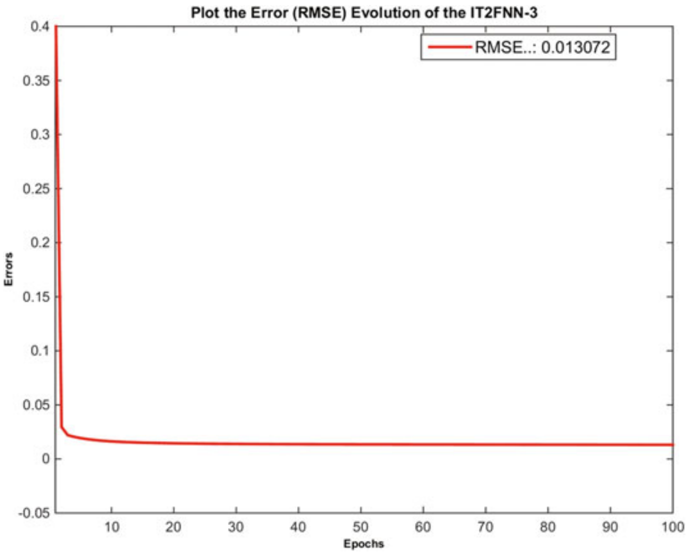

Fig. 4.68 Evolution error (RMSE) of IT2FNN-3 for the Dow Jones time series

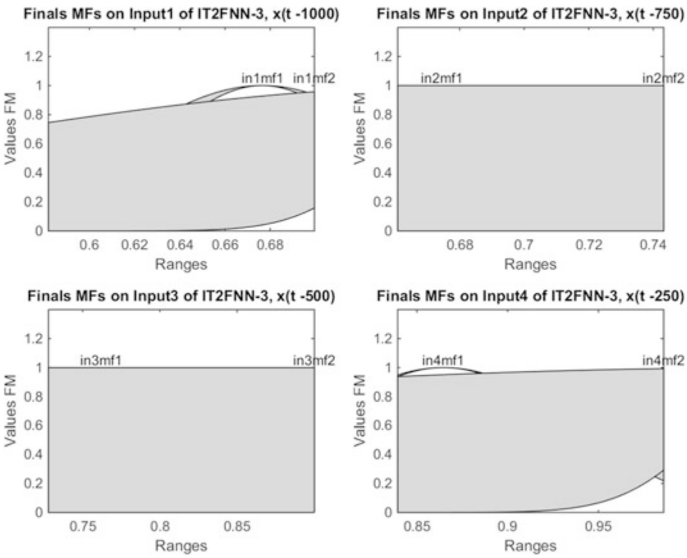

Fig. 4.69 Final MFs after training the IT2FNN-3 model with BP algorithm

4.4 NASDAQ Time Series

This section presents the simulation and test results obtained by applying the proposed prediction method to the NASDAQ time series for periods (01/03/2011–12/31/2015) (Fig. 3.5) using a different approach of the ensemble of IT2FNN architectures, used in this work.

4.4.1 Ensemble of IT2FNN Architectures for NASDAQ Time Series

The ensemble of IT2FNN architectures has three models as follows: the IT2FNN-1 model optimizes the parameters of the “igaussmtype2” MFs (Fig. 3.9a), the learning rate is 0.03 and the desired error is 0.00001; the IT2FNN-2 model optimizes the parameters of the “igausstype2” MFs (Fig. 3.9b), the learning rate is 0.011 and the desired error is 0.000001; and the IT2FNN-3 model optimizes the parameters of the “igaussstype2” MFs (Fig. 3.9c), the learning rate is 0.02 and the desired error is 0.0000001. The number the epochs for training the IT2FNN models is 100.

Table 4.8 Performance of the ensemble of IT2FNN for the NASDAQ time series

Metrics	IT2FNN-1	IT2FNN-2	ITFNN-3
RMSE (Best)	0.011711953	0.01318047	0.013617022
RMSE (Average)	0.016485694	0.017226806	0.020196196
MSE	0.001635756	0.001412437	0.003081807
MAE	0.012063554	0.012381383	0.01588996
MPE	1.288842865	1.324005862	1.691563953
MAPE	0.240159673	0.191975465	0.513682447

The obtained results of the ensemble of IT2FNN architectures are shown on Table 4.8. The RMSE (best) is of 0.011711953, the RMSE (average) is of 0.016485694, the MSE is 0.001635756, the MAE is 0.012063554, the MPE is 1.288842865 and the MAPE 0.240159673 with the IT2FNN-1 model. Therefore the IT2FNN-1 model is better than the IT2FNN-2 and IT2FNN-3 models.

4.4.1.1 IT2FNN-1 Model

The forecast obtained for the IT2FNN-1 for the NASDAQ time series is shown in Fig. 4.70, the evolution error is shown in Fig. 4.71, and the optimization structure of IT2FNN-1 with BP learning algorithm is shown in Fig. 4.72.

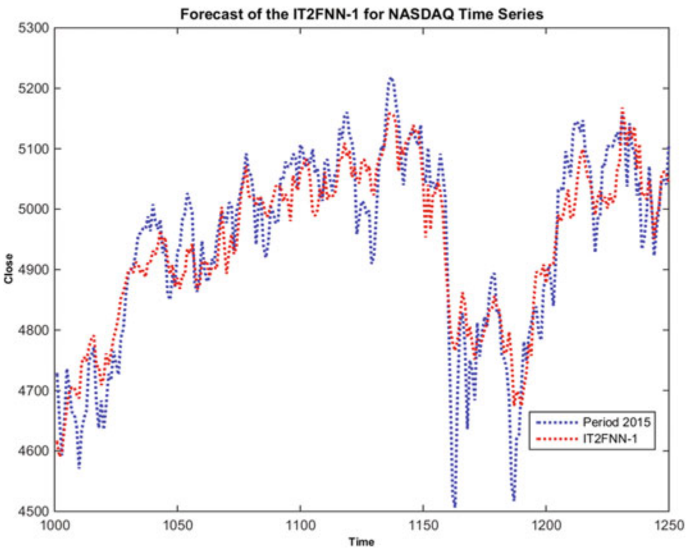

Fig. 4.70 Forecast of IT2FNN-1 for the NASDAQ time series

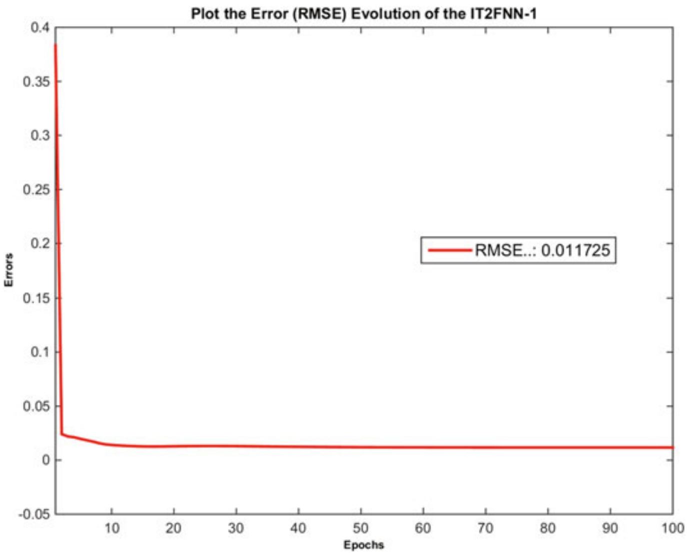

Fig. 4.71 Evolution error (RMSE) of IT2FNN-1 for the NASDAQ time series

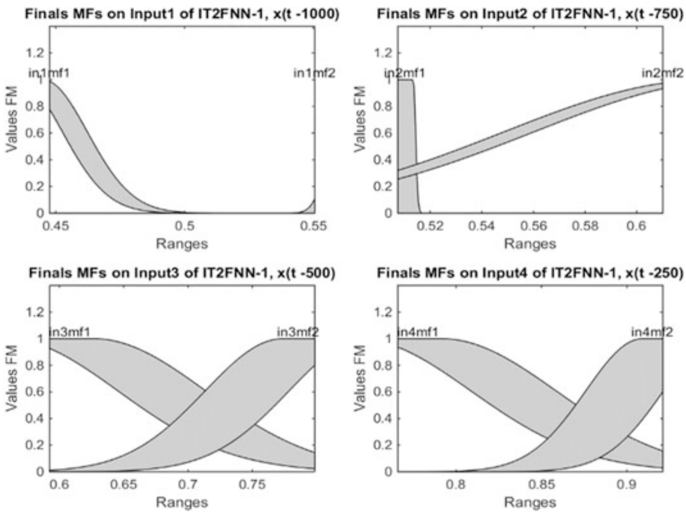

Fig. 4.72 Final MFs after training the IT2FNN-1 model with BP algorithm

4.4.1.2 IT2FNN-2 Model

The forecast obtained for the IT2FNN-2 for the NASDAQ time series is shown in Fig. 4.73, the evolution error is shown in Fig. 4.74, and the optimization structure of the IT2FNN-2 with BP learning algorithm is shown in Fig. 4.75.

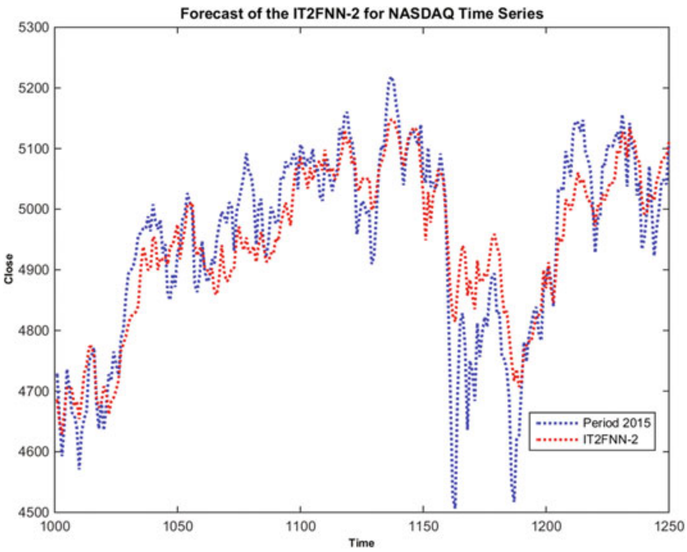

Fig. 4.73 Forecast of IT2FNN-2 for the NASDAQ time series

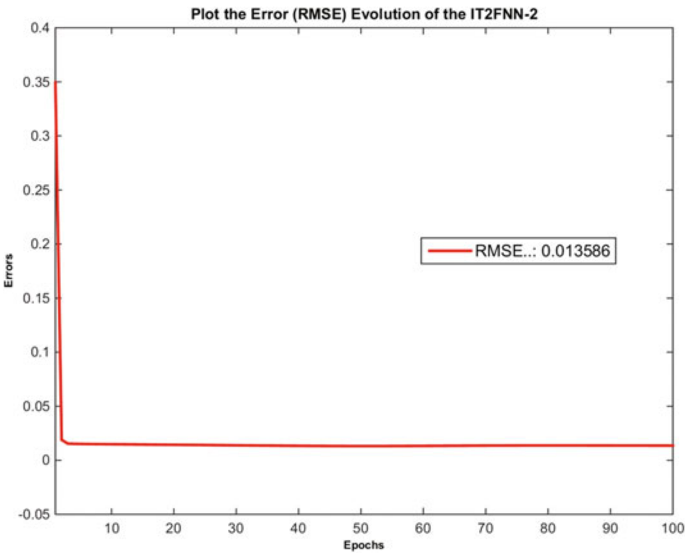

Fig. 4.74 Evolution error (RMSE) of IT2FNN-2 for the NASDAQ time series

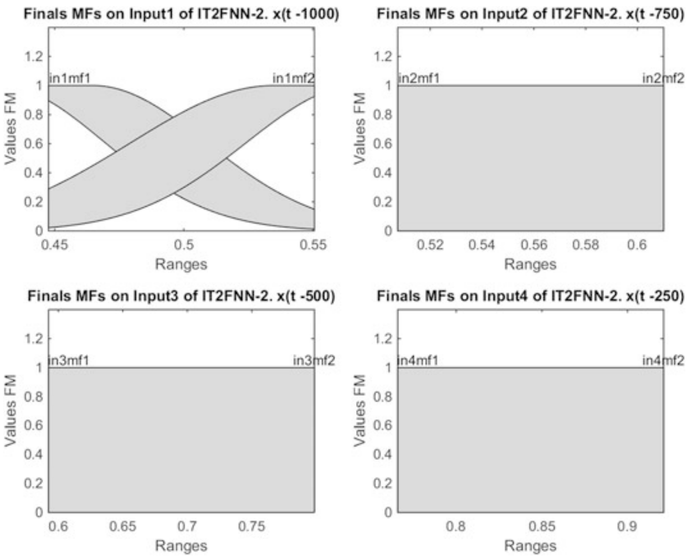

Fig. 4.75 Final MFs after training the IT2FNN-2 model with BP algorithm

4.4.1.3 IT2FNN-3 Model

The forecast obtained for the IT2FNN-3 for the NASDAQ time series is shown in Fig. 4.76, the evolution error is shown in Fig. 4.77, and the optimization structure of the IT2FNN-3 with BP learning algorithm is shown in Fig. 4.78.

4.5 Statistical Comparison Results of the Optimization of the Fuzzy Integrators

We also perform a statistical comparison of all the results obtained of the proposed model (Fig. 3.1) for the Mackey-Glass time series. The statistical test used for comparison is the *Z-scores*, whose parameters are defined in Table 4.9. In applying the statistic *Z-scores*, with significance level of 0.05, and the alternative hypothesis stating that the μ_1 is lower than the μ_2; $H_a(\mu_1 < \mu_2)$ (Fig. 4.79), and of course the null hypothesis tells us that the μ_1 is greater than or equal to the μ_2; $H_0(\mu_1 \geq \mu_2)$, with a rejection region for all values that fall below -1.732. We are presenting 30

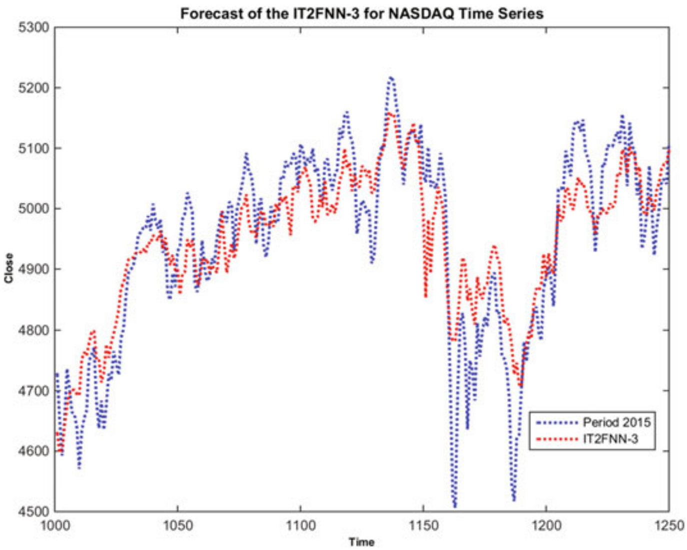

Fig. 4.76 Forecast of IT2FNN-3 for the NASDAQ time series

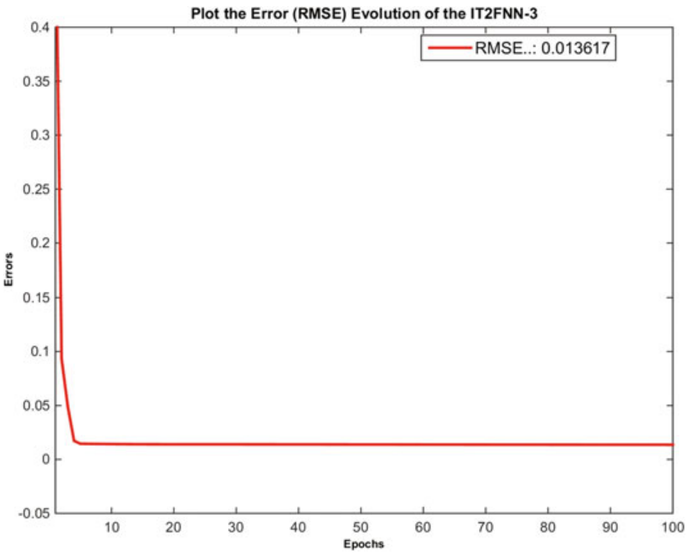

Fig. 4.77 Evolution error (RMSE) of IT2FNN-3 for the NASDAQ time series

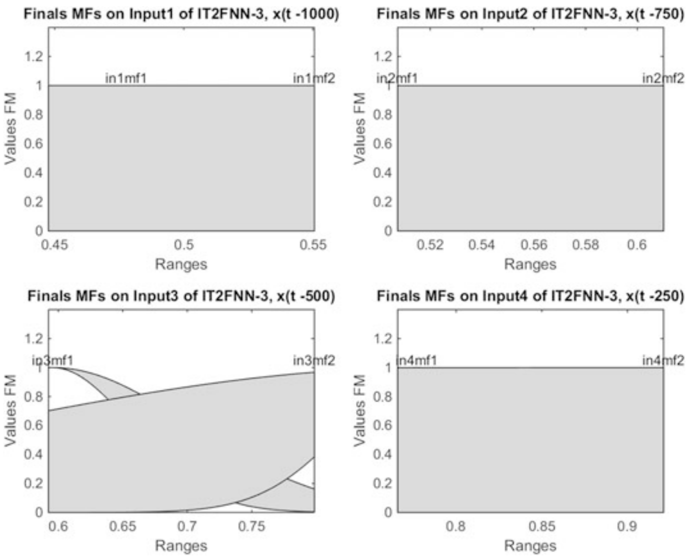

Fig. 4.78 Final MFs after training the IT2FNN-3 model with BP algorithm

Table 4.9 Statistical Z-scores parameters

Parameter	Value
Confidence interval	95%
Significance level (α)	5%
Null hypothesis (H_0)	$\mu_1^* \geq \mu_2^*$
Alternative hypothesis (H_a)	$\mu_1 < \mu_2$
Critical value	−1.645

μ_1—Average error of the optimization of fuzzy integrators with the GAs

μ_2—Average error of the optimization of fuzzy integrators with the PSO

experiments with the same parameters and conditions for the GAs and PSO algorithms for this work, so the n_1 and n_2 are equal 30.

The main objective of applying the statistical *Z-scores* is to analyze the performance and thus find if there is significant evidence of the proposed model results being better for the Mackey-Glass time series. The optimization of the fuzzy integrators results are generated from GAs and PSO algorithms. The results of the statistical *Z-scores* are shown in Table 4.10, so there is significant evidence to reject the null hypothesis because the value of $p < 0.05$ and the value of $z < -1.645$ and we accepted the alternative hypothesis. Therefore the results obtained of the optimization of fuzzy integrators with GAs are better than the PSO.

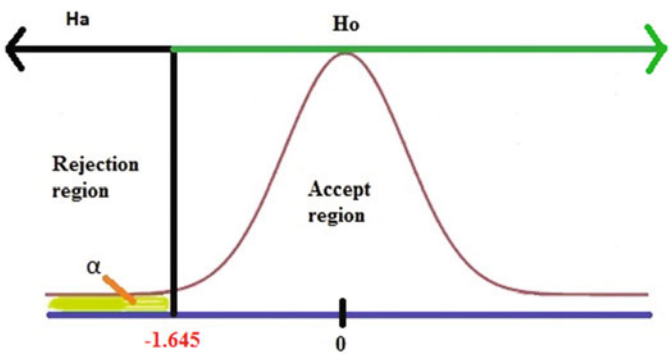

Fig. 4.79 Lower-Tailed Test $(\mu_1 < \mu_2)$

Table 4.10 Results of the Z-scores parameters

Optimization of the Type-1 fuzzy integrator using Gaussian MFs						
GAs		PSO		Parameters		Evidence
μ_1	σ_1	μ_2	σ_2	Z	$p < 0.05$	
0.02208303	0.000638122	0.0372719	0.000821093	−75.895	0	Significant
Optimization of the Type-1 Fuzzy Integrator using GBell MFs						
GAs		PSO		Parameters		Evidence
μ_1	σ_1	μ_2	σ_2	Z	$p < 0.05$	
0.02255155	0.001567122	0.0364846	0.000936185	−39.660	0	Significant
Optimization of the Type-1 Fuzzy Integrator using Triangular MFs						
GAs		PSO		Parameters		Evidence
μ_1	σ_1	μ_2	σ_2	z	$p < 0.05$	
0.08167633	0.000180209	0.08090114	0.00069041	5.645	0.0566351	Not Significant
Optimization of the Interval Type-2 Fuzzy Integrator using igaussmtype2 MFs						
GAs		PSO		Parameters		Evidence
μ_1	σ_1	μ_2	σ_2	z	$p < 0.05$	
0.02097587	0.00007548	0.02430513	0.00009724	−148.123	0	Significant
Optimization of the Interval Type-2 Fuzzy Integrator using igbelltype2 MFs						
GAs		PSO		Parameters		Evidence
μ_1	σ_1	μ_2	σ_2	z	$p < 0.05$	
0.02060701	0.000218508	0.02369199	0.00002811	−72.821	0	Significant
Optimization of the Interval Type-2 Fuzzy Integrator using itritype2 MFs						
GAs		PSO		Parameters		Evidence
μ_1	σ_1	μ_2	σ_2	z	$p < 0.05$	
0.02033528	0.000138626	0.02511529	0.000000010	−179.170	0	Significant

Based on the statistical *Z-scores* results, we can make the conclusion that the results obtained of the optimization of fuzzy integrators with the GAs are better than the PSO for the Mackey-Glass time series.