5-D Hyperchaotic and Chaotic Systems with Non-hyperbolic Equilibria and Many Equilibria

Jay Prakash Singh and Binoy Krishna Roy

Abstract In the present decade, chaotic systems are used and appeared in many fields like in information security, communication systems, economics, bioengineering, mathematics, etc. Thus, developing of chaotic dynamical systems is most interesting and desirable in comparison with dynamical systems with regular behaviour. The chaotic systems are categorised into two groups. These are (i) system with self-excited attractors and (ii) systems with hidden attractors. A self-excited attractor is generated depending on the location of its unstable equilibrium point and in such case, the basin of attraction touches the equilibria. But, in the case of hidden attractors, the basin of attraction does not touch the equilibria and also finding of such attractors is a difficult task. The systems with (i) no equilibrium point and (ii) stable equilibrium points belong to the category of hidden attractors. Recently chaotic systems with infinitely many equilibria/a line of equilibria are also considered under the cattegory of hidden attractors. Higher dimensional chaotic systems have more complexity and disorders compared with lower dimensional chaotic systems. Recently, more attention is given to the development of higher dimensional chaotic systems with hidden attractors. But, the development of higher dimensional chaotic systems having both hidden attractors and self-excited attractors is more demanding. This chapter reports three hyperchaotic and two chaotic, 5-D new systems having the nature of both the self-excited and hidden attractors. The systems have non-hyperbolic equilibria, hence, belong to the category of self-excited attractors. Also, the systems have many equilibria, and hence, may be considered under the category of a chaotic system with hidden attractors. A systematic procedure is used to develop the new systems from the well-known 3-D Lorenz chaotic system. All the five systems exhibit multistability with the change of initial conditions. Various theoretical and numerical tools like phase portrait, Lyapunov spectrum, bifurcation diagram, Poincaré map, and frequency spectrum are used to confirm the chaotic nature of the new systems.

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The MATLAB simulation results of the new systems are validated by designing their circuits and realising the same.

Keywords Non-hyperbolic equilibria · Many equilibria · A line of equilibria
Hyperchaotic system · Chaotic system · 5-D chaotic system
5-D hyperchaotic system · Hidden attractors

1 Introduction

Recently, the development and applications of chaotic systems are seen in many fields like in communication theory (Xiong et al. 2016), image processing (Tlelo-Cuautle et al. 2015a, b), information theory (Esteban et al. 2016; Valtierra et al. 2016), robotics (Lochan and Roy 2015, 2016; Lochan et al. 2016a, b, c; Tlelo-Cuautle et al. 2014; Singh et al. 2017a, b; Andrievskii and Fradkov 2004), etc. Based on the desired behaviours and responses, many hyperchaotic/chaotic systems are reported in the last decade (Pham et al. 2014, 2016g; Vaidyanathan et al. 2015). An equilibrium point plays an important role in the generation of the desired behaviour and responses. Recently, many hyperchaotic/chaotic systems are reported based on different nature of equilibrium points (Pham et al. 2016e, f, h, 2017a, b; Wang et al. 2017; Sharma et al. (2015)). Higher dimensional (4-D/5-D) hyperchaotic/chaotic systems are more important from the application point of view as compared with the lower dimensional systems (Pham et al. 2016b; Shen et al. 2014a, b). This is because of their more complex and disorder behaviour as compared with the lower dimensional systems (Shen et al. 2014a, b). Thus, development of higher dimensional (5-D) hyperchaotic/chaotic systems with unique and interesting nature of equilibrium points is the motivational background of this work.

Many control techniques are proposed in the literature and used in the last decade for the applications of hyperchaotic/chaotic systems. Some of these are sliding mode control (SMC) (Singh and Roy 2015a), backstepping control (Yu et al. 2012), feedback control (Pang and Liu 2011), nonlinear active control (Singh et al. 2014a, 2017a, b), adaptive control (Effati et al. 2014), H_∞ (Wang et al. 2013), sampled data control (Lam and Li 2014), etc.

The reported hyperchaotic/chaotic systems can be classified into two major categories. These are: (i) hyperchaotic/chaotic systems with hidden attractors and (ii) self-excited attractors hyperchaotic/chaotic systems (Leonov and Kuznetsov 2013; Leonov et al. 2011a, b, 2012, 2014; Singh and Roy 2017a; Singh and Roy (2016a); Singh et al. (2015)). Some of the conventional chaotic systems like Lorenz system (Lorenz 1963), Rössler (1976), Chen and Ueta (1999), Lü et al. (2002), Bhalekar-Gejji systems (Singh et al. 2014a) and systems in Singh and Roy (2015a, b, 2016a, b), etc., are grouped under the category of self-excited attractors. The family of the hyperchaotic/chaotic systems with hidden attractors is grouped with the systems having (i) only stable equilibrium points (Kingni et al. 2014), (ii) no equilibrium point (Lin et al. 2016; Singh and Roy 2017a) and (iii) an infinite number of equilibria (Jafari and Sprott 2013; Wang and Chen 2012). The chaotic systems with an infinite/line/many equilibria belong to the category of hidden attractors (Leonov et al. 2014, 2015; Pham et al. 2016a, b, c, d, e, f, g, h). In a hyperchaotic/chaotic system with hidden attractors, the basin of attraction does not intersect with small neighbourhoods of its equilibria (Leonov et al. 2011a, b, 2012). However, in a chaotic system with infinitely many equilibria, the basin of attraction may intersect the equilibrium surface in some sections. Since there are usually uncountable sections/points on the surface of equilibria which are outside the basin of attraction and from a computational point of view, these attractors of hyperchaotic/chaotic systems with many equilibria also belong to the family of hidden attractors (Barati et al. 2016; Pham et al. 2016a, b). Because the knowledge about the locations of equilibria in such systems does not help in the generation of attractors.

Very less attention is given to the development of 5-D hyperchaotic/chaotic systems (Kemih et al. 2013; Ojoniyi and Njah 2016; Vaidyanathan et al. 2014, 2015, 2016). Recently, many hyperchaotic/chaotic systems with an infinite number of equilibria are reported. The systems with infinitely many equilibria are the systems with a line of equilibria (Singh and Roy 2017b), plane of equilibria (Jafari et al. 2016a), surface of equilibria (Jafari et al. 2016b), sphere of equilibria (Qi and Chen 2015), square shaped equilibria (Qi and Chen 2015; Gotthans et al. 2016; Pham et al. 2016a, b, c, d, e, f, g, h), etc. The reported systems with an infinite number of equilibria are classified in Table 1.

It is seen from Table 1 that very few 5-D hyperchaotic/chaotic systems are reported with infinitely many equilibria. Motivated by this finding, an attempt is made in this chapter to construct five new 5-D hyperchaotic/chaotic systems with infinitely many equilibria. Multistability in a hyperchaotic/chaotic system is defined as the coexistence of various possible steady states/attractors of the system (Pisarchik and Feudel 2014; Sharma et al. 2015; Kiseleva et al. 2017). The occurrence of multistability is governed by the choice of initial conditions, hence, creates a complicated basin of attraction (Pisarchik and Feudel 2014; Sharma et al. 2015). Multistability is seen in several areas (Sharma et al. 2015), like in an electronic circuit, a laser system, chaotic/hyperchaotic system, etc., (Chen et al. 2017; Chudzik et al. 2011; Leonov and Kuznetsov 2013; Pisarchik and Feudel 2014; Sharma et al. 2015).

Most of the reported chaotic systems have hyperbolic nature of equilibria. Very few hyperchaotic/chaotic systems are reported with non-hyperbolic nature of equilibria (Sprott 2015; Wei et al. 2015a, b; Yang et al. 2010; Li and Xiong 2017). Higher dimensional hyperchaotic/chaotic systems with non-hyperbolic nature of equilibria are rare in the literature. Thus, developing higher dimensional hyperchaotic/chaotic systems with some fascinating attributes like non-hyperbolic equilibria and multistability is also a worthy motivation of this chapter.

In this chapter, three new 5-D hyperchaotic and two chaotic systems are reported. Out of these five systems, four of them have many equilibria and thus qualify to be chaotic systems with hidden attractors. Again, all the five systems exhibit non-hyperbolic equilibria and hence behave like a chaotic system with self-attractors. Therefore, four new chaotic systems have both the self-attractor and

Table 1 Categorisation of the reported chaotic and hyperchaotic systems with an infinite number of equilibria

Sl. no.	3-D/4-D system	Nature of systems	References of papers
1.	3-D chaotic system	Line of equilibria	Jafari and Sprott (2015, 2013), Kingni et al. (2016a, b)
		Many equilibria	Wang and Chen (2012)
		Circle of equilibria	Gotthans and Petržela (2015), Gotthans et al. (2016), Kingni et al. (2016a, b), Pham et al. (2016d, f)
		Surface of equilibria	Jafari et al. (2016b)
		Curve of equilibria	Barati et al. (2016), Pham et al. (2016c)
		Square shaped equilibria	Gotthans et al. (2016), Pham et al. (2016b, d, f)
		Ellipse shaped equilibria	Pham et al. (2016d)
		Sphere of equilibria	Qi and Chen (2015)
2.	4-D chaotic system	Plane of equilibria	Jafari et al. (2016a, b)
		Line of equilibria	Singh and Roy (2017b), Pham et al. (2016c)
3.	4-D hyperchaotic system	Line of equilibria	Li et al. (2014a, b), Zhou and Yang (2014)
		Curve of equilibria	Chen and Yang (2015)
4.	4-D memristive hyperchaotic system	Line of equilibria	Li et al. (2014a, b), Ma et al. (2015)
5.	5-D hyperchaotic/ chaotic system	Line of equilibria	Vaidyanathan (2016)
		Line of equilibria with coexistence of attractors	This work
		Hyperbolic curve of equilibria with coexistence of attractors	This work

hidden attractor. The three new systems have hyperbolic curve of equilibria and one system has a line of equilibria. All the five new systems depict multistability. The systems have various dynamical behaviours like hyperchaotic, chaotic, periodic, quasi-periodic, etc. Various numerical tools are used to find the different dynamical behaviour of the systems like phase portrait, Lyapunov spectrum, bifurcation diagram, Poincaré map, frequency spectrum. Chaotic natures of the systems are validated by circuit design and implementation. Circuit implementation results of the two systems have good agreement with the MATLAB simulation results.

The rest part of the chapter is organised as follows. Section 2 describes the development of the new systems. The findings of different dynamic behaviour of the systems are shown in Sect. 3. Circuit design and implementations of the systems are discussed in Sect. 4. Section 5 presents the conclusions of the chapter.

2 Development of the Systems with Non-hyperbolic Equilibria and a Line of Equilibria

The dynamics of the 3-D Lorenz chaotic system with linear control inputs is described as (Lorenz 1963):

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + u_1 \\ \dot{x}_2 = rx_1 - x_2 - x_1x_3 + u_2 \\ \dot{x}_3 = x_1x_2 + cx_3 + u_3 \end{cases} \tag{1}$$

Table 2 Three hyperchaotic and two chaotic systems with non-hyperbolic and many equilibria

Case	System dynamics	LEs and nature	D_{KY}	Initial conditions
NHE1	$\dot{x}_1 = a(x_3 - x_1)$ $\dot{x}_2 = b - x_1x_3 + x_4$ $\dot{x}_3 = x_1x_2 + x_5$ $\dot{x}_4 = cx_3$ $\dot{x}_5 = -cx_2x_3$ $a = 10, b = 45, c = 0.0183$	$LE = \begin{pmatrix} 0.1632, \\ 0.0124, \\ 0, \\ -0.0069, \\ -11.6351 \end{pmatrix}$ and hyperchaotic	4.014	$x(0) = \begin{pmatrix} 0.001, \\ 0.002, \\ 0.003, \\ 0.001, \\ 0.001 \end{pmatrix}^T$
NHE2	$\dot{x}_1 = a(x_3 - x_1)$ $\dot{x}_2 = b - x_1x_3 + x_4$ $\dot{x}_3 = x_1x_2 + x_5$ $\dot{x}_4 = cx_3$ $\dot{x}_5 = -cx_2x_5$ $a = 10, b = 45, c = 0.0105$	$LE = \begin{pmatrix} 0.9470, \\ 0.0011, \\ 0, \\ -0.0129, \\ -10.9316 \end{pmatrix}$ and hyperchaotic	4.085	$x(0) = \begin{pmatrix} 0.001, \\ 0.002, \\ 0.003, \\ 0.001, \\ 0.001 \end{pmatrix}^T$
NHE3	$\dot{x}_1 = a(x_3 - x_1)$ $\dot{x}_2 = b - x_1x_3 + x_4$ $\dot{x}_3 = x_1x_2 + x_5$ $\dot{x}_4 = -cx_2$ $\dot{x}_5 = -cx_2x_3$ $a = 10, b = 45, c = 0.0198$	$LE = \begin{pmatrix} 2.0149, \\ 0.0120, \\ 0, \\ -0.0012, \\ -12.0229 \end{pmatrix}$ and hyperchaotic	4.168	$x(0) = \begin{pmatrix} 0.001, \\ 0.002, \\ 0.003, \\ 0.001, \\ 0.001 \end{pmatrix}^T$
NHE4	$\dot{x}_1 = a(x_3 - x_1)$ $\dot{x}_2 = b - x_1x_3 + x_4$ $\dot{x}_3 = x_1x_2 + x_5$ $\dot{x}_4 = -cx_2$ $\dot{x}_5 = -dx_3x_5$ $a = 10, b = 45, c = 0.01, d = 0.001$	$LE = \begin{pmatrix} 1.0288, \\ 0.0, \\ -0.0004, \\ -0.0007, \\ -11.0212 \end{pmatrix}$ and chaotic	4.093	$x(0) = \begin{pmatrix} 0.001, \\ 0.002, \\ 0.003, \\ 0.001, \\ 0.001 \end{pmatrix}^T$
NHE5	$\dot{x}_1 = a(x_3 - x_1)$ $\dot{x}_2 = b - x_1x_3 + x_4$ $\dot{x}_3 = x_1x_2 + x_5$ $\dot{x}_4 = -cx_2$ $\dot{x}_5 = -cx_1x_2$ $a = 10, b = 45, c = 0.001$	$LE = \begin{pmatrix} 1.0513, \\ 0.0, \\ 0.0, \\ 0.0, \\ -11.0461 \end{pmatrix}$ and chaotic	4.095	$x(0) = \begin{pmatrix} 0.001, \\ 0.002, \\ 0.003, \\ 0.001, \\ 0.001 \end{pmatrix}^T$

Table 3 Stability analysis of equilibrium points of the systems

System	Equilibrium points	Shape of equilibria	Eigenvalues	Nature
NHE1	$E1 = (0, 0, 0, -45, 0)$	Constant	$\lambda = (-10, 0, 0, 0, 0)$	Non-hyperbolic
	$E2 = (0, x_2, 0, -45, 0)$	Line of equilibria	The system has non-hyperbolic, stable focus and saddle nature of eigenvalues for different values of state variables x_2	
NHE2	$E1 = (0, 0, 0, -45, 0)$	Constant	$\lambda = (0.2653, \pm 0.2653i, -0.2653, -10)$	Non-hyperbolic
NHE3	$E1 = (0, 0, 0, -45, 0)$	Constant	$\lambda = (-10, 0, 0, \pm 0.1407i)$	Non-hyperbolic
	$E2 = (x_1, 0, x_1, x_1^2 - 45, 0)$	Hyperbolic curve of equilibria	The system has non-hyperbolic and saddle nature of eigenvalues for different values of state variables x_1	
NHE4	$E1 = (0, 0, 0, -45, 0)$	Constant	$\lambda = (-10, 0, 0, \pm 0.10i)$	
	$E2 = (x_1, 0, x_1, x_1^2 - 45, 0)$	Hyperbolic curve of equilibria	The system has non-hyperbolic and saddle nature of eigenvalues for different values of state variables x_1	
NHE5	$E1 = (0, 0, 0, -45, 0)$	Constant	$\lambda = (-10, 0, 0, \pm 0.0316i)$	
	$E2 = (x_1, 0, x_1, x_1^2 - 45, 0)$	Hyperbolic curve of equilibria	The system has non-hyperbolic and saddle nature of eigenvalues for different values of state variables x_1	

where a, r, c are the parameters, x_1, x_2, x_3 are the state variables and u_1, u_2, u_3 are the control inputs. Selecting control inputs u_1, u_2, u_3 as in the form of (2) (Yuhua et al. 2010), we get (3).

$$\begin{cases} u_1 = a(x_3 - x_2) \\ u_2 = b - rx_1 + x_2 \\ u_3 = -cx_3 \end{cases} \tag{2}$$

Using (2), the dynamics of the Lorenz system can be written as in (3).

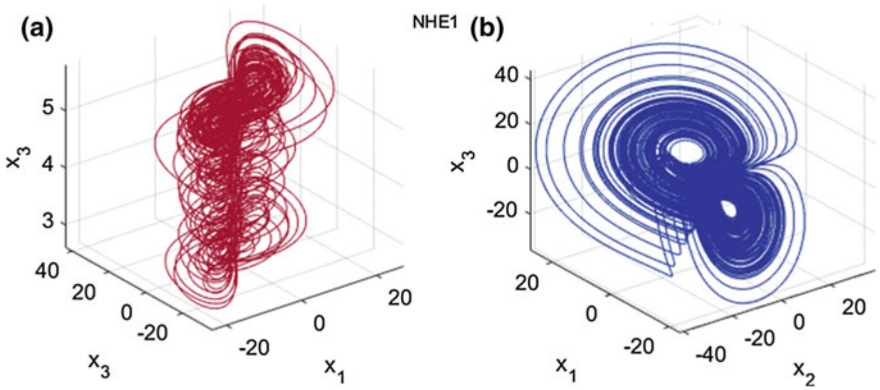

Fig. 1 Hyperchaotic attractors of the NHE1 system with $a = 10, b = 45, c = 0.0183$

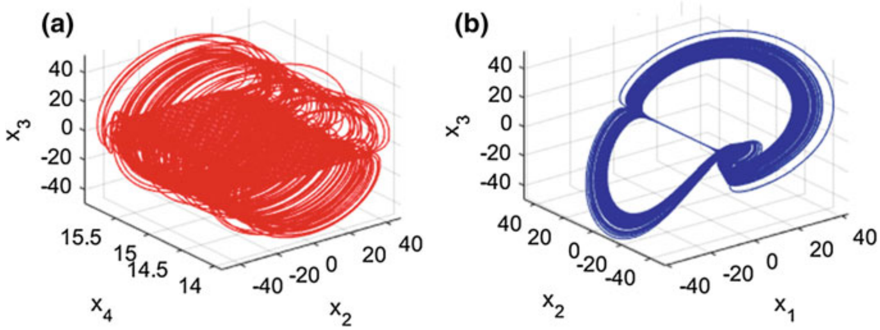

Fig. 2 Hyperchaotic attractors of the NHE2 system with $a = 10, b = 45, c = 0.0105$

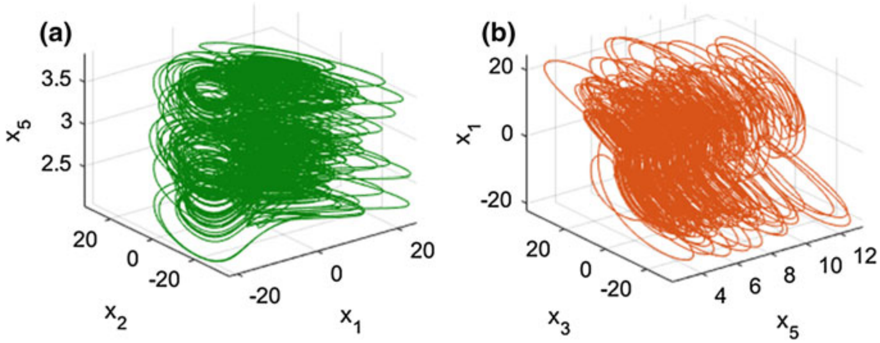

Fig. 3 Hyperchaotic attractors of the NHE3 system with $a = 10, b = 45, c = 0.0198$

$$\begin{cases} \dot{x}_1 = a(x_3 - x_1) \\ \dot{x}_2 = b - x_1 x_3 \\ \dot{x}_3 = x_1 x_2 \end{cases} \tag{3}$$

System (3) is chaotic with $a = 10, b = 45$ (Yuhua et al. 2010).

Using the above system (3), this chapter presents five new 5-D self-attractor/hidden attractor hyperchaotic/chaotic systems with non-hyperbolic and many equilibria. A known and widely used systematic search procedure is used to develop the systems as used in the paper (Munmuangsaen et al. 2011; Pham et al. 2016f; Sprott 1993, 2000, 2010). The procedure considers various combinations of states to generate hyperchaotic/chaotic systems with largest Lyapunov exponents at least greater than 0.9. The general expression of the new 5-D hyperchaotic or chaotic systems is considered as:

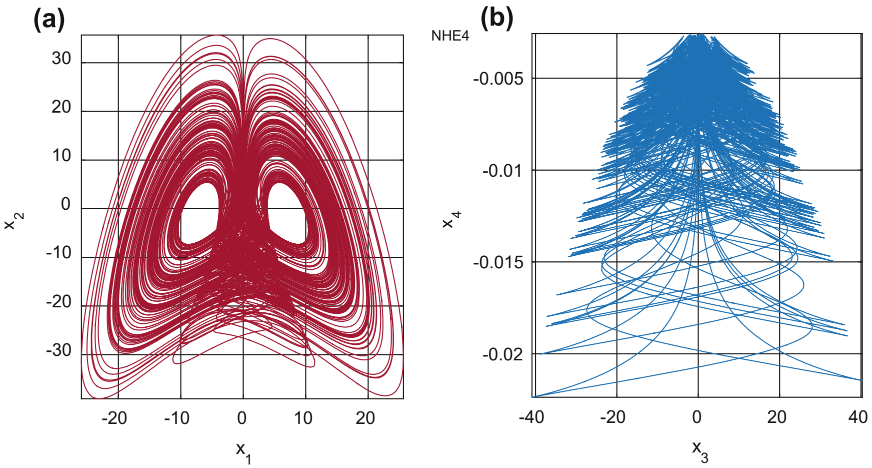

Fig. 4 Chaotic attractors of the NHE4 system with $a = 10, b = 45, c = 0.01, d = 0.001$

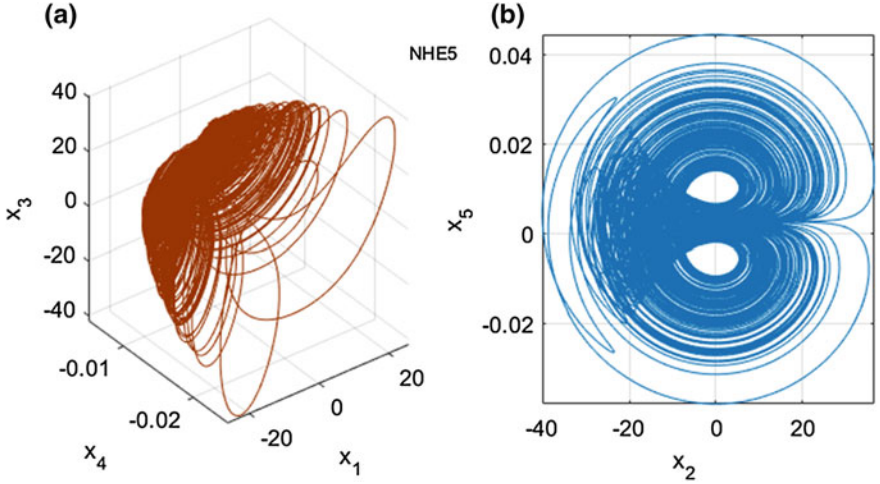

Fig. 5 Chaotic attractors of the NHE5 system with $a = 10, b = 45, c = 0.001$

$$\begin{cases} \dot{x}_1 = a(x_3 - x_1) \\ \dot{x}_2 = b - x_1 x_3 + x_4 \\ \dot{x}_3 = x_1 x_2 + x_5 \\ \dot{x}_4 = f_1(x_1, x_2, x_3) \\ \dot{x}_5 = f_2(x_1, x_2, x_3, x_4, x_5) \end{cases} \tag{4}$$

where $f_1(x_1, x_2, x_3)$ and $f_2(x_1, x_2, x_3, x_4, x_5)$ are linear and nonlinear functions, respectively. Different choices of $f_1(x_1, x_2, x_3)$ and $f_2(x_1, x_2, x_3, x_4, x_5)$ lead to systems with various type of equilibria.

With suitable choices of $f_1(x_1, x_2, x_3)$ and $f_2(x_1, x_2, x_3, x_4, x_5)$, five different types of hyperchaotic or chaotic systems are developed. There are named as NHE1 to NHE5 and the details are shown in Table 2. The first three systems (NHE1 to NHE3) have hyperchaotic behaviour and the rest two systems (NHE4 and NHE5) have chaotic behaviour. Table 2 describes the dynamics of the systems, Lyapunov exponents (LEs), nature of the systems, Lyapunov dimension/Kaplan-Yorke dimension (D_{KY}) and initial conditions used for simulation of these systems.

Stability analysis of the equilibrium points of the systems given in Table 2 is discussed in Table 3. It is seen from Table 3 that all the systems have non-hyperbolic nature of equilibria. All the systems have many equilibria except the system NHE2.

3 Numerical Findings of the Proposed Systems Given in Table 2

This section discusses various numerical tools like time series plot, phase portrait, Lyapunov spectrum, bifurcation diagram, frequency spectrum, Poincaré maps used for finding different dynamical behaviour of the new systems given in Table 2.

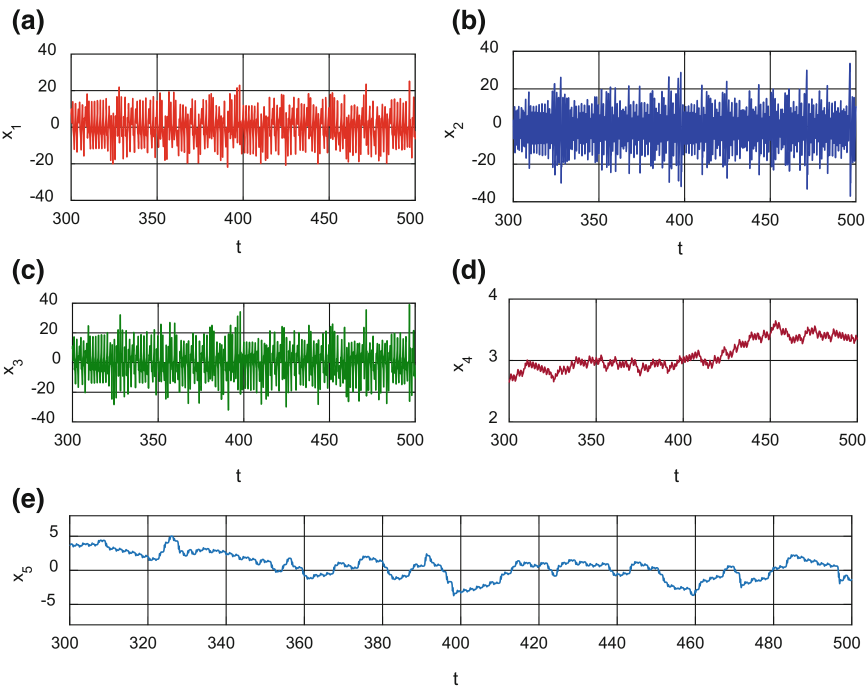

Fig. 6 Chaotic signals of the NHE1 system with $a = 10, b = 45, c = 0.0183$

3.1 Time Series and Phase Portrait

Chaotic behaviour of the systems given in Table 2 is confirmed by plotting their time responses and phase portraits. Figures 1, 2, 3, 4 and 5 show the hyperchaotic and chaotic attractors of the new systems. The irregular shape of the phase portraits of the systems in Figs. 1, 2, 3, 4 and 5 depicts their chaotic behaviours. Time responses of the systems NHE1 and NHE3 are shown in Figs. 6 and 7, respectively. Aperiodic nature of the responses confirms the chaotic behaviour of the systems (Singh and Roy 2015a, b, 2016a, b, 2017a, b, c; Singh et al. 2017a, b). All the time responses and phase portraits of the systems are generated using the fixed initial conditions and value of the parameters which are given in Table 2.

3.2 Lyapunov Spectrum and Bifurcation Diagram

Different dynamical behaviour of the systems given in Table 2 are calculated using Lyapunov spectrum and bifurcation diagram. Lyapunov spectrums of all the systems are calculated by finding Lyapunov exponents using Wolf algorithm (Wolf et al. 1985) with the observation time $T = 20000$, step size $\Delta t = 0.01$ and fixed

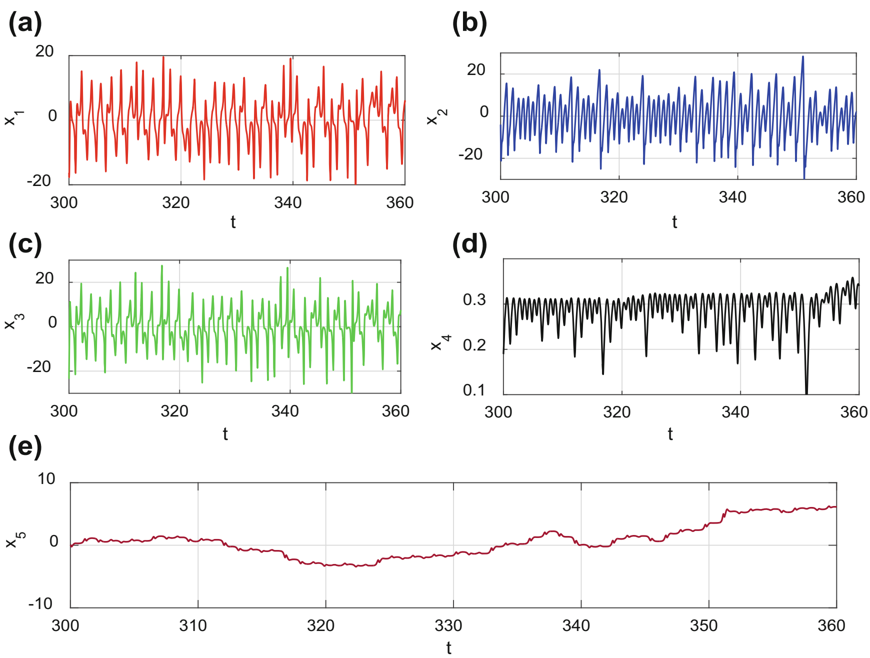

Fig. 7 Chaotic signals of the NHE3 system with $a = 10, b = 45, c = 0.0198$

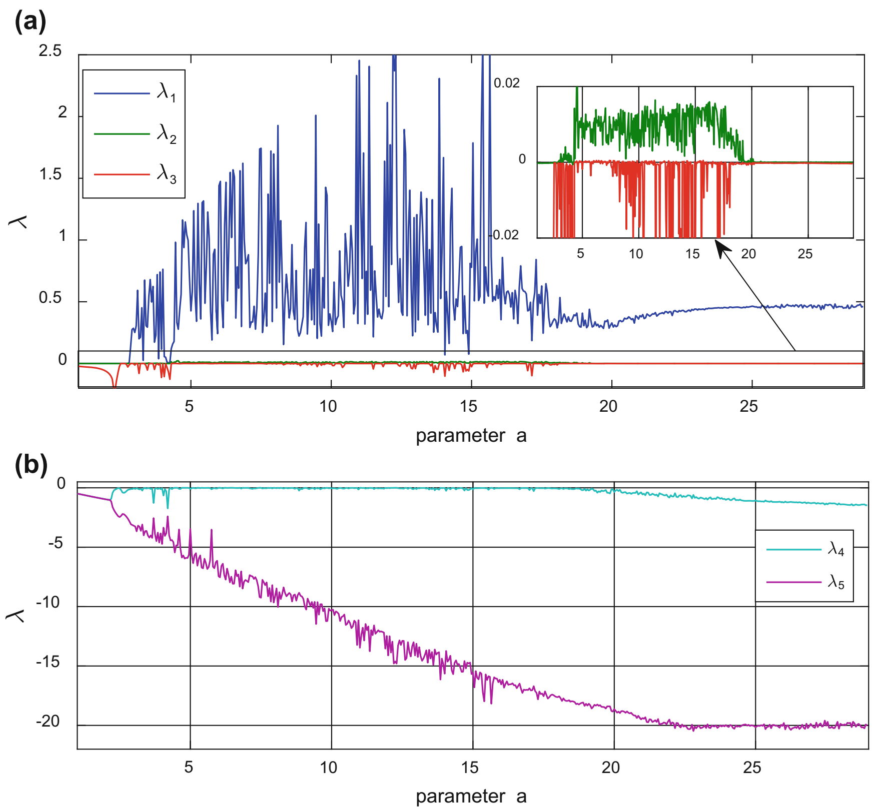

Fig. 8 Lyapunov spectrum of the NHE1 system with $b = 45, c = 0.0183$ and $a \in [1, 30]$

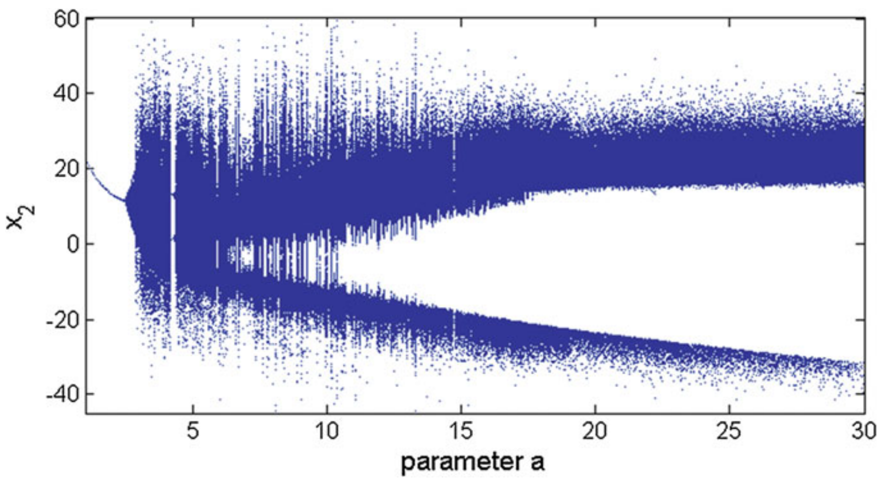

Fig. 9 Bifurcation diagram of the NHE1 system with $b = 45, c = 0.0183$ and $a \in [1, 30]$

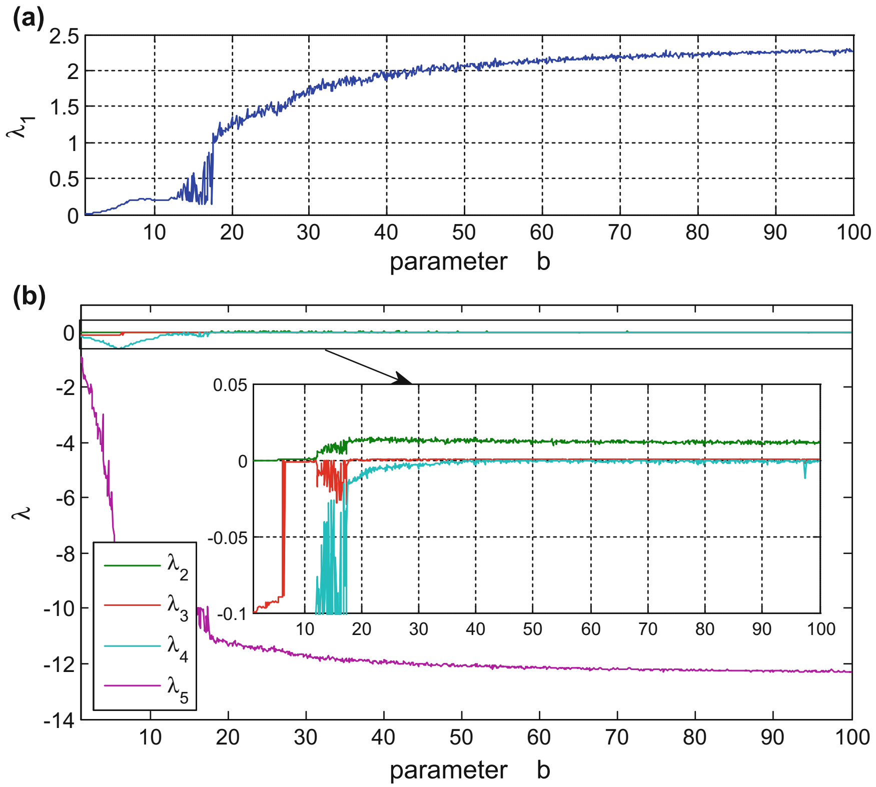

Fig. 10 Lyapunov spectrum of the NHE1 system with $a = 10, c = 0.0183$ and $b \in [5, 100]$

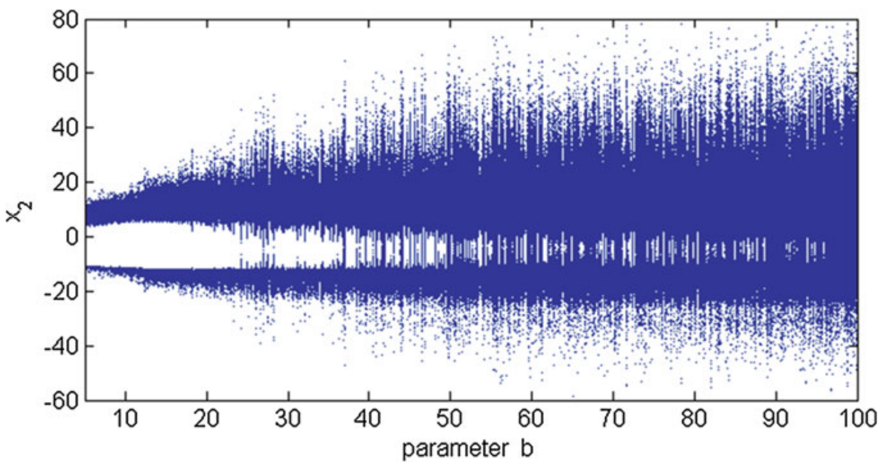

Fig. 11 Bifurcation diagram of the NHE1 system with $a = 10, c = 0.0183$ and $b \in [5, 100]$

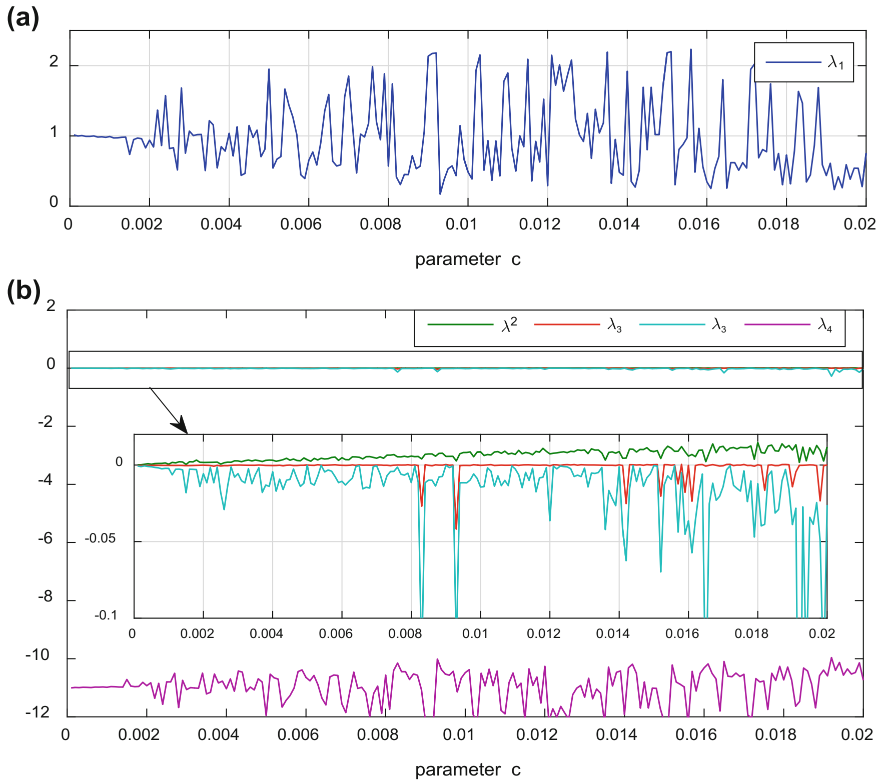

Fig. 12 Lyapunov spectrum of the NHE1 system with $a = 10, b = 45$ and $c \in [0.0001, 0.02]$

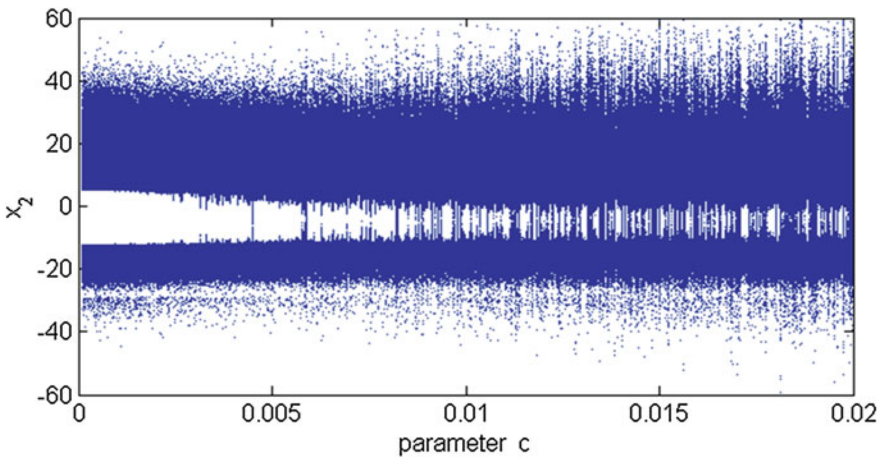

Fig. 13 Bifurcation diagram of the NHE1 system with $a = 10, b = 45$ and $c \in [0.0001, 0.02]$

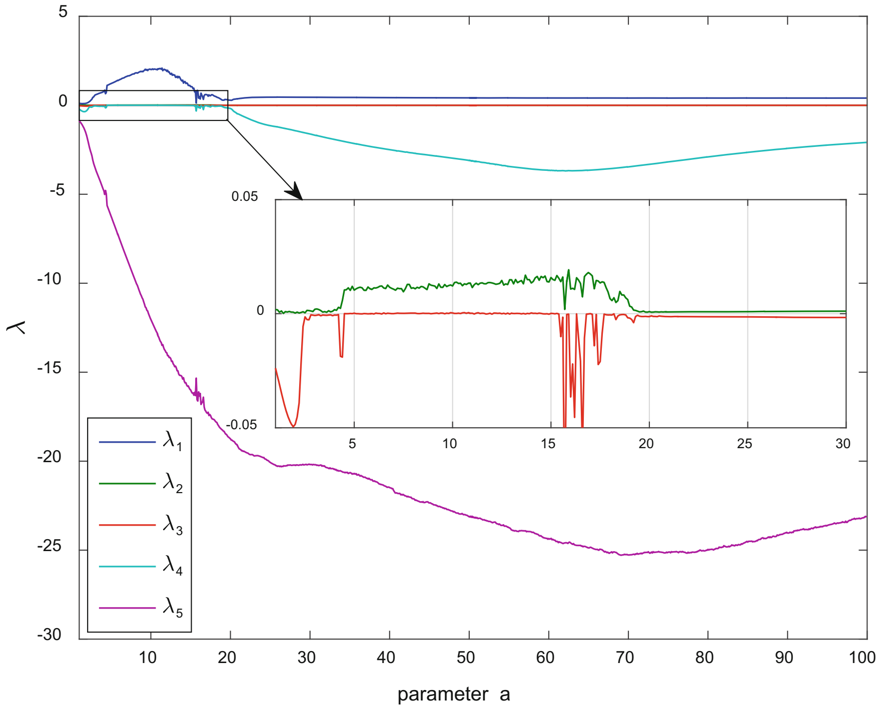

Fig. 14 Lyapunov spectrum of the NHE3 system with $b = 45, c = 0.0198$ and $a \in [1, 100]$

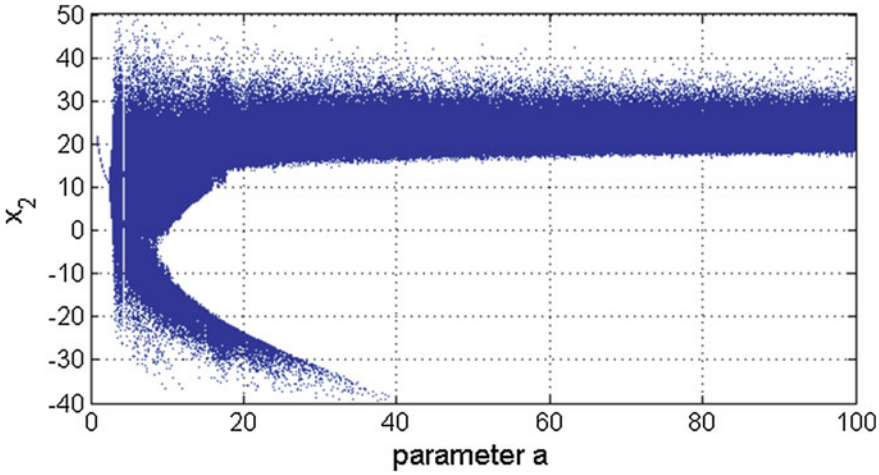

Fig. 15 Bifurcation diagram of the NHE3 system with $b = 45, c = 0.0198$ and $a \in [5, 100]$

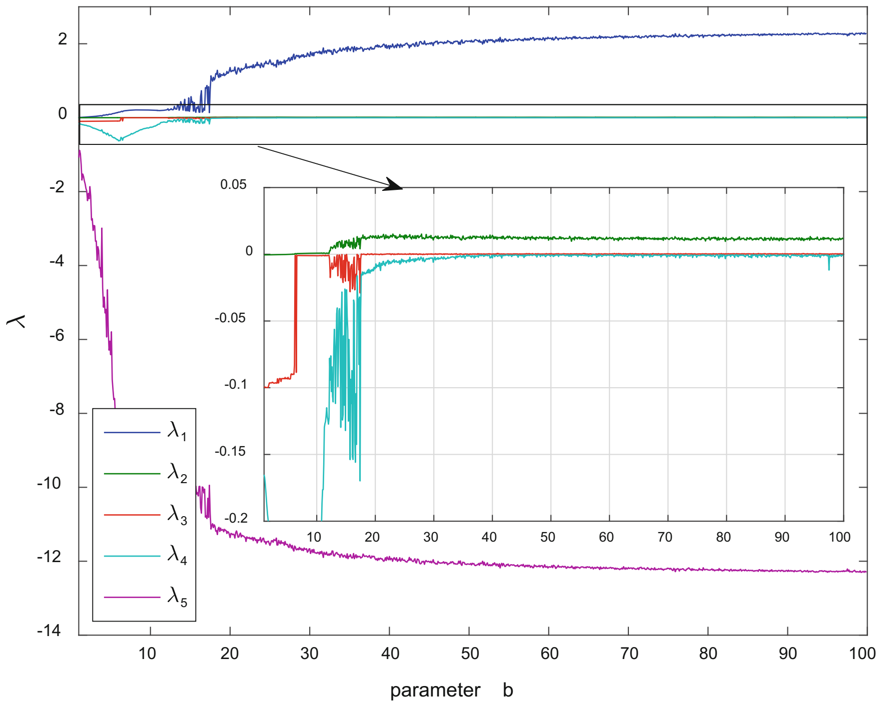

Fig. 16 Lyapunov spectrum of the NHE3 system with $a = 10, c = 0.0198$ and $b \in [5, 100]$

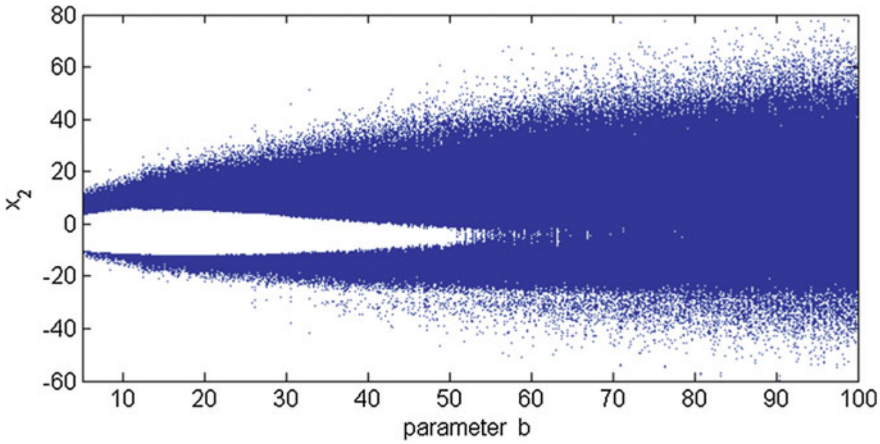

Fig. 17 Bifurcation diagram of the NHE3 system with $a = 10, c = 0.0198$ and $b \in [5, 100]$

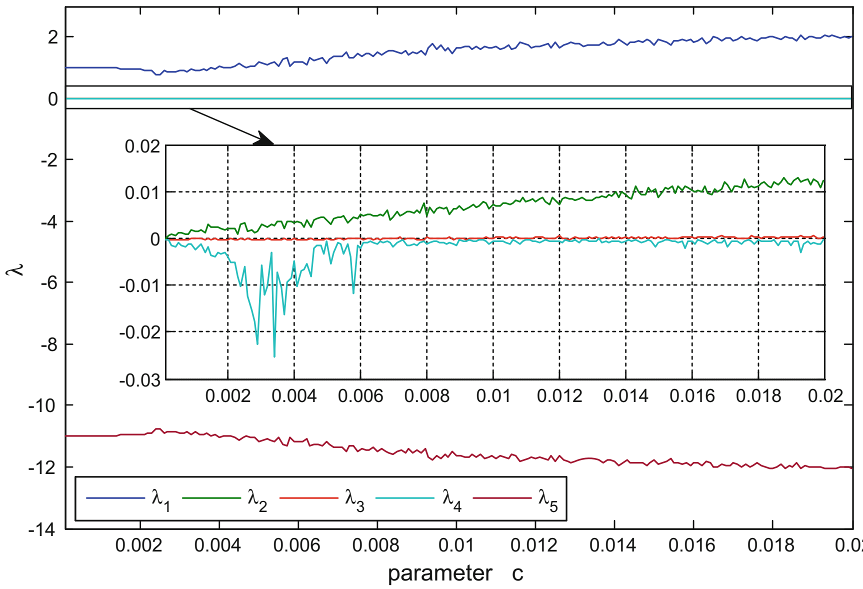

Fig. 18 Lyapunov spectrum of the NHE3 system with $a = 10, b = 45$ and $c \in [0.0001, 0.02]$

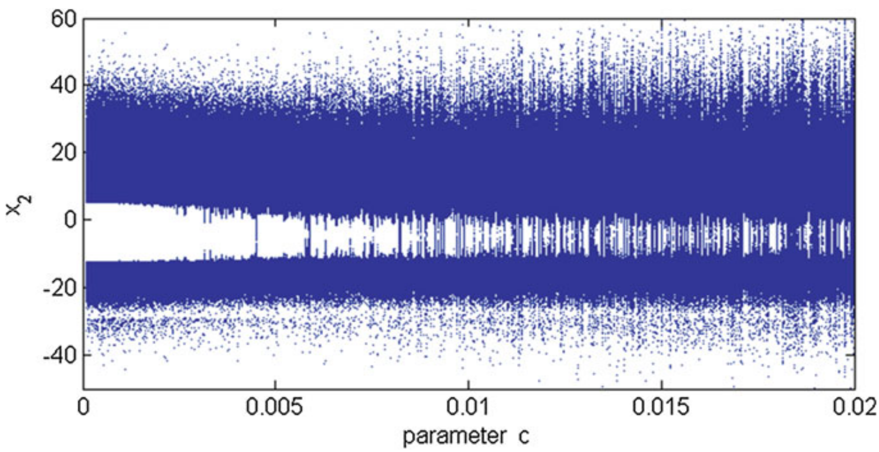

Fig. 19 Bifurcation diagram of the NHE3 system with $a = 10, b = 45$ and $c \in [0.0001, 0.02]$

initial conditions $x(0) = (0.001, 0.002, 0.003, 0.001, 0.001)^T$. In MATLAB, the time variable is selected as $T = 0{:}\,\Delta t{:}\,1000,$ where Δt is the step size and 1000 is the total observation time. It may be noted that T does not reflect the actual time of calculation. Lyapunov spectrum and bifurcation diagram of the systems are shown with the variation of one parameter and keeping other fixed. Here, Lyapunov

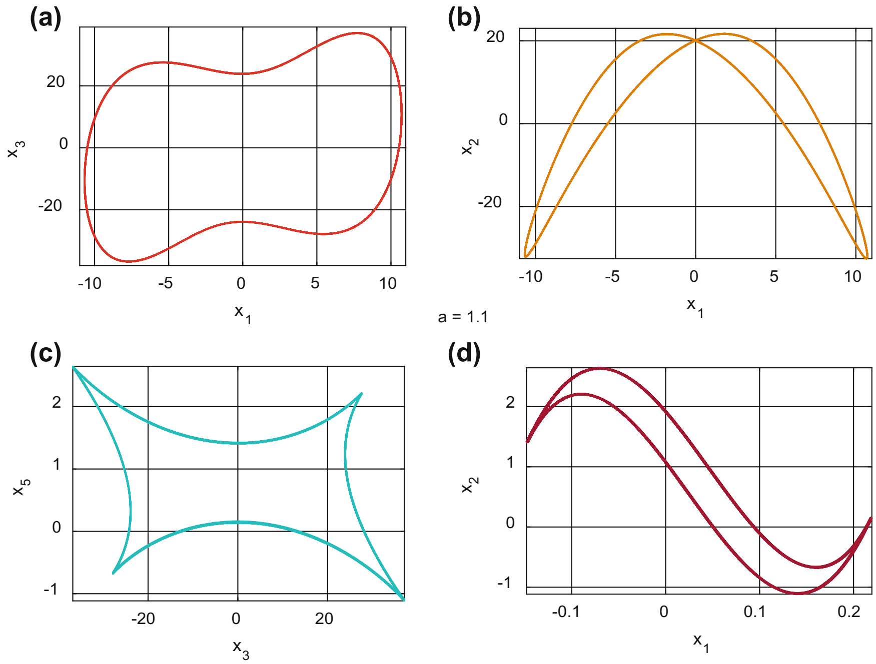

Fig. 20 Periodic attractors of the NHE1 system with $a = 1.1, b = 45, c = 0.0183$ and $x(0) = (0.001, 0.002, 0.003, 0.001, 0.001)^T$

spectrums and bifurcation diagrams of systems NHE1 and NHE3 are only shown. These figures for the other systems can also be calculated in a similar manner and are not shown here to avoid the repetition. However, Lyapunov exponents of systems NHE2, NHE4 and NHE5 are given in Table 2.

Lyapunov spectrum and bifurcation diagram of the NHE1 system with the variation of one parameter, out of a, b or c, and keeping the rest two fixed are shown in Figs. 8, 9, 10, 11, 12 and 13. Similarly, Lyapunov spectrum and bifurcation diagram of the NHE3 system are shown in Figs. 14, 15, 16, 17, 18 and 19. It is observed from Figs. 8, 10, 12, 14, 16 and 18 that NHE1 and NHE3 systems, respectively, have different dynamical behaviours like hyperchaotic, chaotic, periodic and quasi-periodic. It is also observed from Figs. 9, 11, 13, 15, 17 and 19 that NHE1 and NHE3 systems, respectively, have various dynamical behaviours like chaotic and periodic.

Periodic nature of the NHE1 system with $a = 1.1, b = 45, c = 0.0183$ and $a = 4.25, b = 45, c = 0.0183$ is shown in Figs. 20 and 21, respectively. Periodic nature of the NHE3 system with $a = 1.1, b = 45, c = 0.0198$ is shown in Fig. 22. The NHE2 system shows transient chaotic behaviour with trajectory going to infinity for smaller values of parameter b. The transient chaotic behaviour of the NHE2 system with $a = 10, b = 10, c = 0.0105$ is shown in Fig. 23. It is apparent from Fig. 23 that

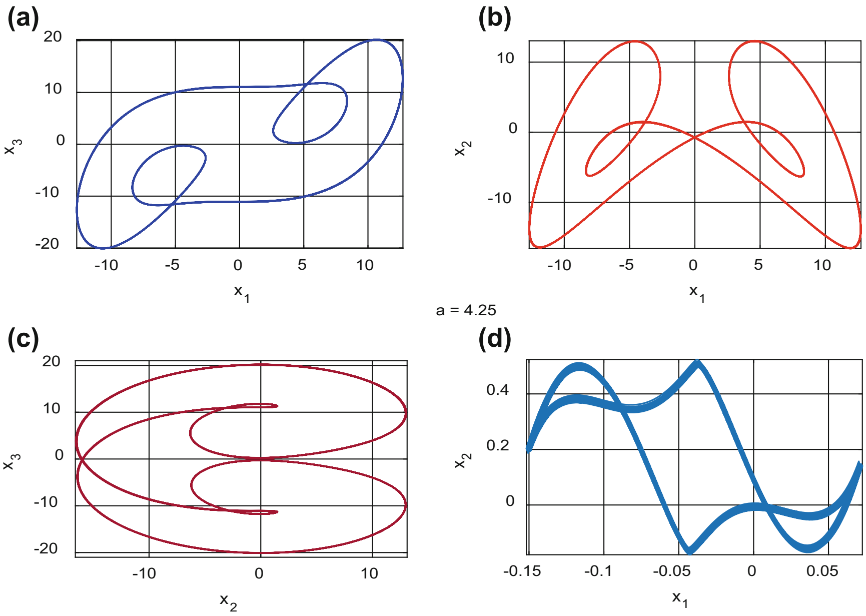

Fig. 21 Periodic attractors of the NHE1 system with $a = 4.25, b = 45, c = 0.0183$ and $x(0) = (0.001, 0.002, 0.003, 0.001, 0.001)^T$

the NHE2 system has chaotic behaviour approximately for $t < 1500$ and trajectory going to infinity at $t > 1600$ approximately.

3.3 Coexistences of Attractors

All the proposed systems show multistability (i.e. coexistences of attractors) with the change of initial conditions. Coexistence of chaotic attractors of NHE1, NHE2 and NHE3 systems are shown in Figs. 24, 26 and 27, respectively. Coexistences of the quasi-periodic behaviour of the NHE2 system is shown in Fig. 25. Other two systems, i.e. NHE4 and NHE5 also show the coexistences of attractors with the changes of initial conditions. Their results are not shown here to avoid the repetition.

3.4 Frequency Spectrum and Poincaré Maps

Frequency spectra of $x_2(t)$ and $x_3(t)$ signals of NHE1 and NHE3 systems are shown in Fig. 28 and Fig. 29, respectively. Aperiodic continuous natures of the spectra

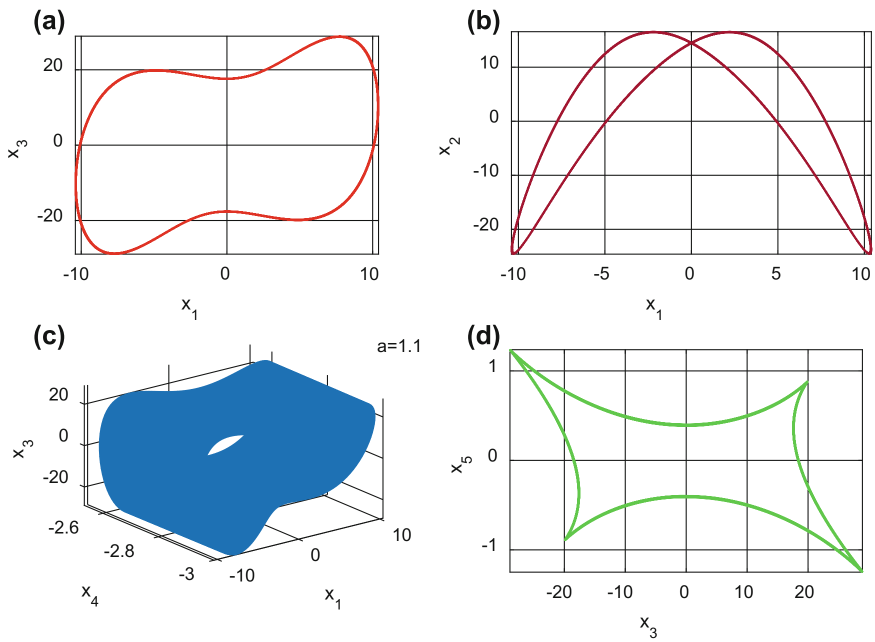

Fig. 22 Periodic attractors of the NHE3 system with $a = 1.1, b = 45, c = 0.0198$ and $x(0) = (0.001, 0.002, 0.003, 0.001, 0.001)^T$

(Figs. 28 and 29) indicate the chaotic behaviour of the systems. Poincaré maps across different section of planes of NHE1 and NHE3 systems are shown in Fig. 30 and Fig. 31, respectively. Random locations of dots in the maps indicate the chaotic behaviour of the systems (Singh and Roy 2015a, b, 2016a, b, 2017a, b, c; Singh et al. 2017a, b). Frequency spectra and Poincaré maps of other systems can also be shown in a similar way but avoided here.

4 Circuit Implementation

This section describes the circuit design and realisation of NHE1 and NHE3 systems. Circuit realisations of other systems can also be done in a similar manner and are not shown here to avoid repetition.

Circuit realisation of a chaotic system represents its practical applicability (Trejo-Guerra et al. 2011, 2012; Nunez et al. 2015; Valtierra et al. 2015; Tlelo-Cuautle et al. 2016a). Circuit realisation of various chaotic/hyperchaotic systems are achieved by FPGA tool (Tlelo-Cuautle et al. 2015a, b, 2016b; Esteban et al. 2016), Cadence OrCAD (Trejo-Guerra et al. 2011) and NI Multisim (Ruo-Xun and Shi-ping 2010; Lao et al. 2014; Xiong et al. 2016) software. In this

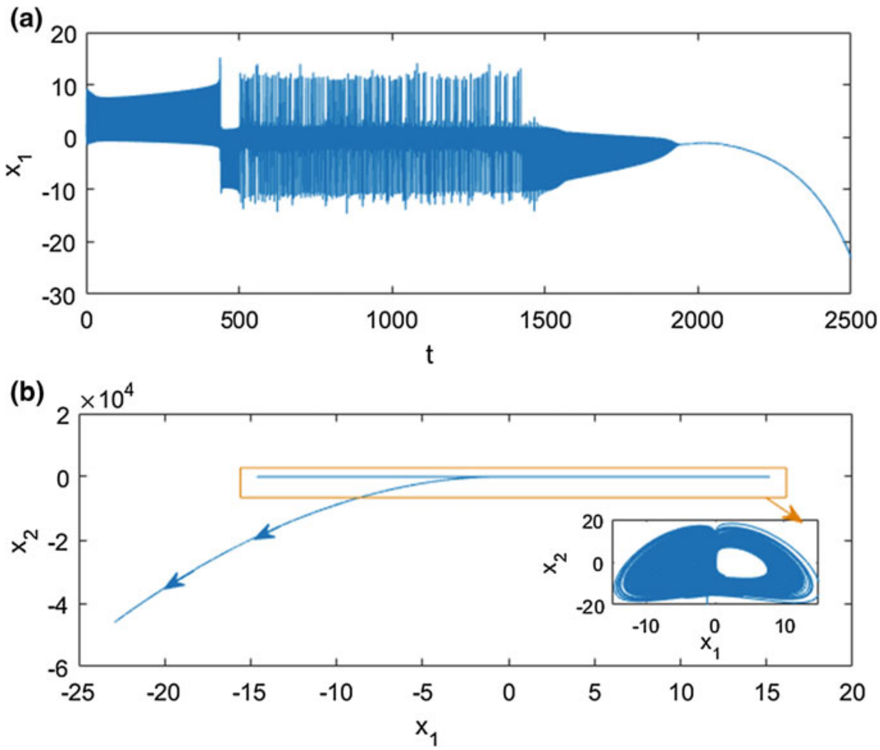

Fig. 23 Transient chaotic behaviour of the system NHE2 with $a = 10, b = 10, c = 0.0105$ and $x(0) = (0.001, 0.002, 0.003, 0.001, 0.001)^T$

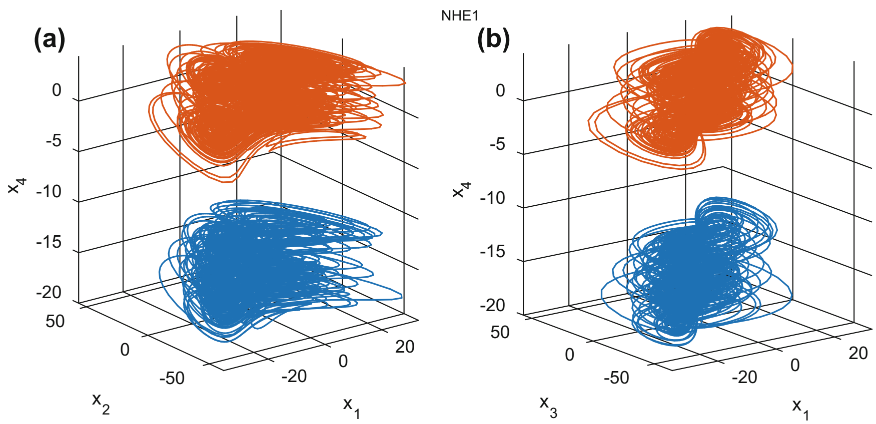

Fig. 24 Coexistences of chaotic attractors of the NHE1 system with $a = 10, b = 100, c = 0.0183$ and $x(0) = (\pm 0.001, \pm 0.002, \pm 0.003, \pm 0.001, \pm 0.001)^T$

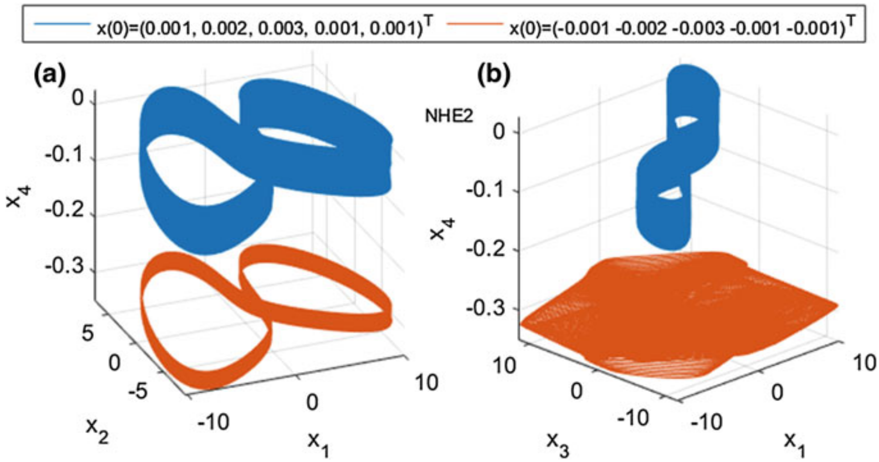

Fig. 25 Coexistences of the quasi-periodic behaviour of the NHE2 system with $a = 10, b = 25, c = 0.0105$ and $x(0) = (\pm 0.001, \pm 0.002, \pm 0.003, \pm 0.001, \pm 0.001)^T$

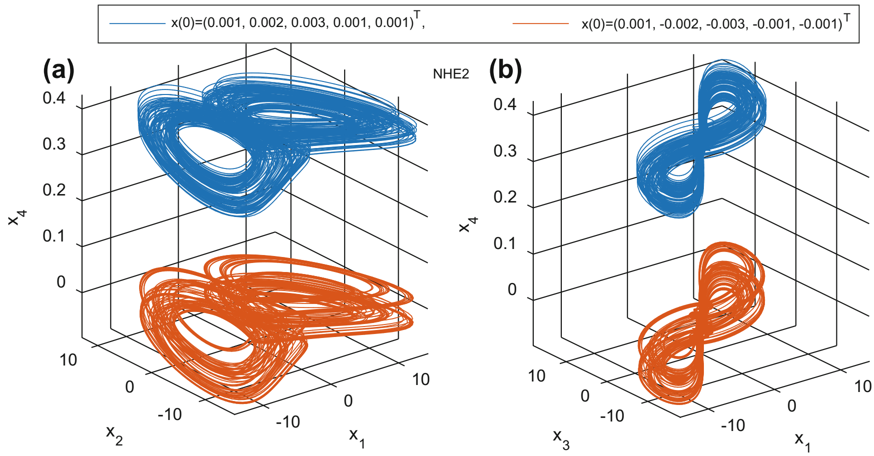

Fig. 26 Coexistences of chaotic attractors of NHE2 system with $a = 10, b = 35, c = 0.0105$ and $x(0) = (\pm 0.001, \pm 0.002, \pm 0.003, \pm 0.001, \pm 0.001)^T$

chapter, the circuit realisation of NHE1 and NHE3 systems are achieved using NI Multisim v12 software. Chaotic attractors of the NHE1 system obtained using the circuit implementation are shown in Figs. 32 and 33. The circuit designed for the implementation of the NHE1 system is shown in Fig. 34. The circuit which is shown in Fig. 34 has five integrators (U9A, U1A, U3A, U5A, and U7A) and use to realise the five states of the NHE1 system. The circuit consists of capacitors

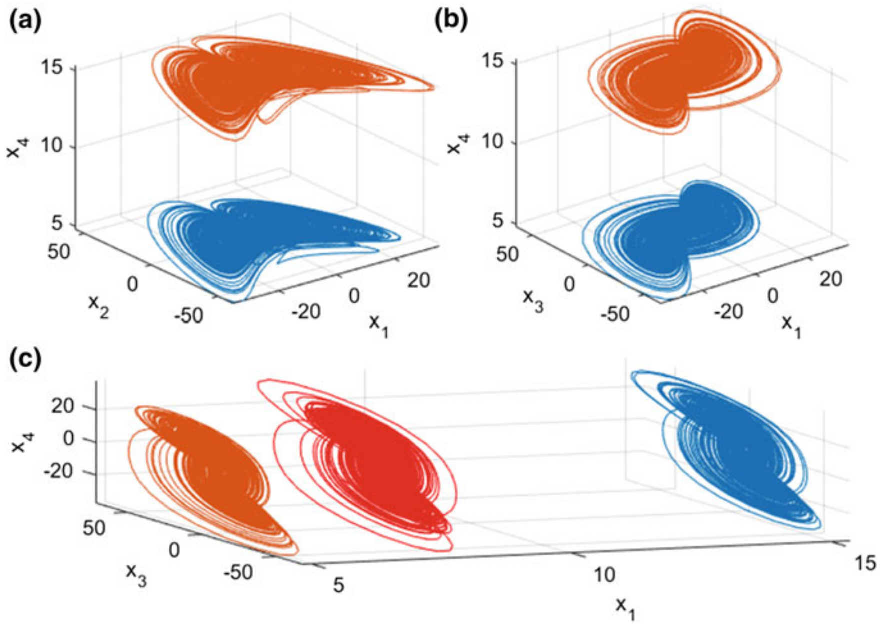

Fig. 27 Coexistences of chaotic attractors of NHE3 system with $a = 100, b = 45, c = 0.0198$, $x(0) = (\pm 0.001, \pm 0.002, \pm 0.003, \pm 0.001, \pm 0.001)^T$ (blue, brown) and $x(0) = (0.001, 0.002, 0.003, -0.001, -0.001)^T$ (red)

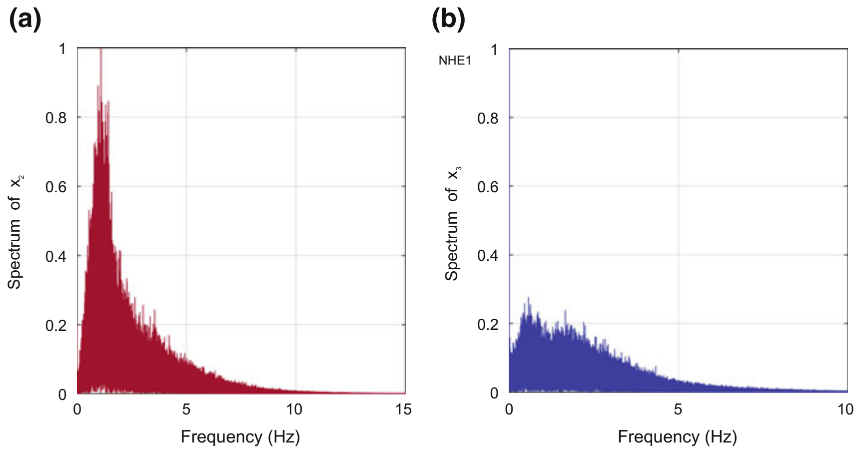

Fig. 28 The frequency spectrum of the NHE1 system with $a = 10, b = 45$ and $c = 0.0183$

(C1, C2, C3, C4, C5), resistances (R1,…, R19), Op-Amp (LF353D) and multipliers (AD633). The circuit equations of the NHE1 system can be written by using Kirchhoff's laws as:

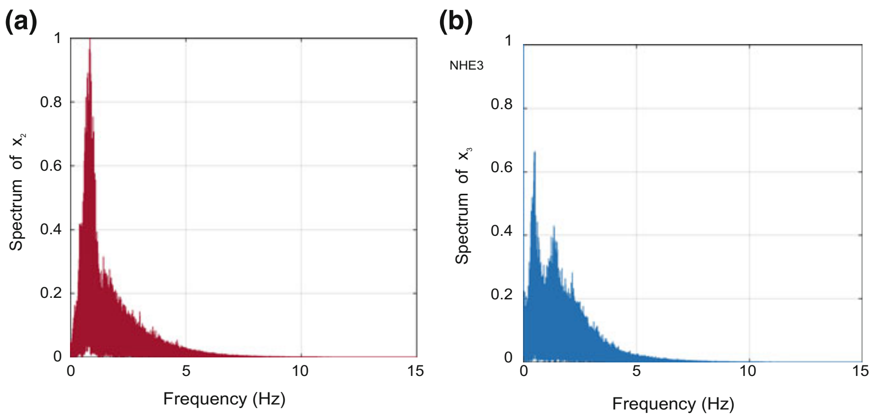

Fig. 29 The frequency spectrum of the NHE3 system with $a = 10, b = 45$ and $c = 0.0198$

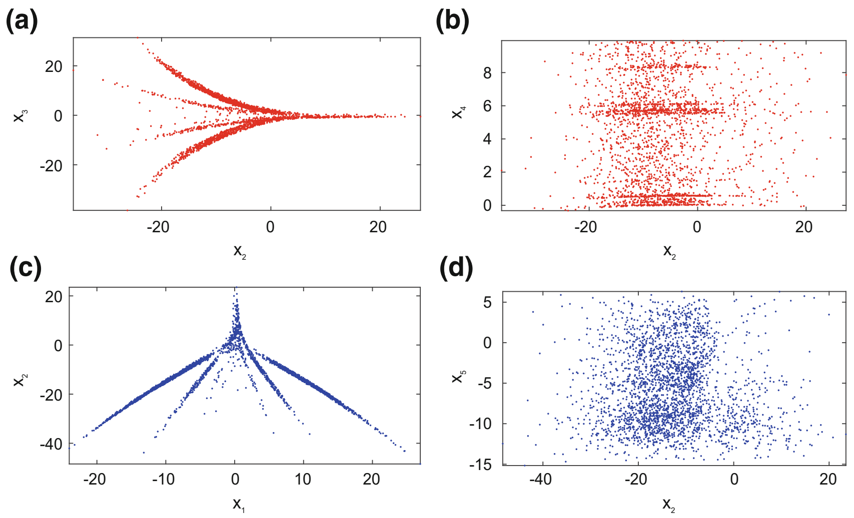

Fig. 30 Poincaré maps of the NHE1 system with $a = 10, b = 45$ and $c = 0.0183$ for: $x_1 = 0$ in (**a**), (**b**) and $x_3 = 0$ in (**c**), (**d**)

$$\begin{cases} \dot{x}_1 = \frac{1}{RC1}\left[\frac{R}{R1}x_3 - \frac{R}{R1}x_1\right] \\ \dot{x}_2 = \frac{1}{RC2}\left[\frac{R}{R7}V1 - \frac{0.1R}{R6}x_1x_3 + \frac{R}{R5}x_4\right] \\ \dot{x}_3 = \frac{1}{RC3}\left[\frac{0.1R}{R11}x_1x_2 + \frac{R}{R10}x_5\right] \\ \dot{x}_4 = \frac{1}{RC4}\left[\frac{R}{R14}x_3\right] \\ \dot{x}_5 = \frac{1}{RC5}\left[-\frac{0.1R}{R17}x_2x_3\right] \end{cases} \tag{5}$$

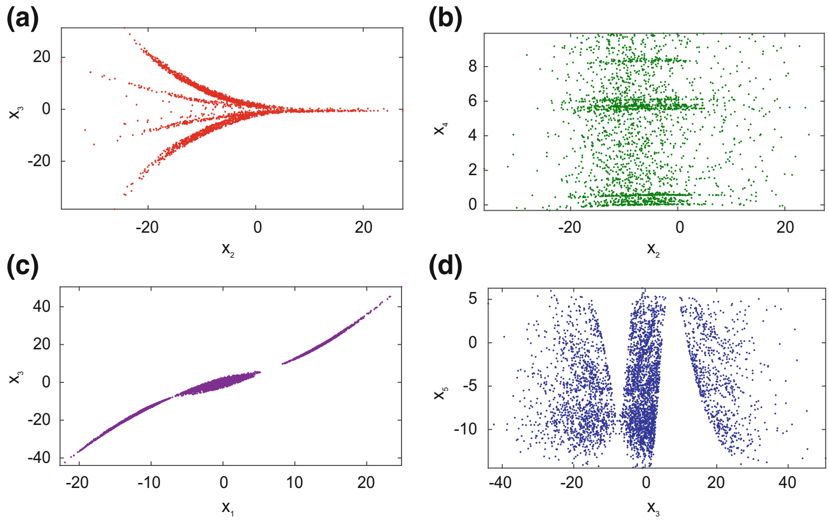

Fig. 31 Poincaré maps of the NHE3 system with $a = 10, b = 45$ and $c = 0.0198$ for: $x_1 = 0$ in (**a**), (**b**) and $x_2 = 0$ in (**c**), (**d**)

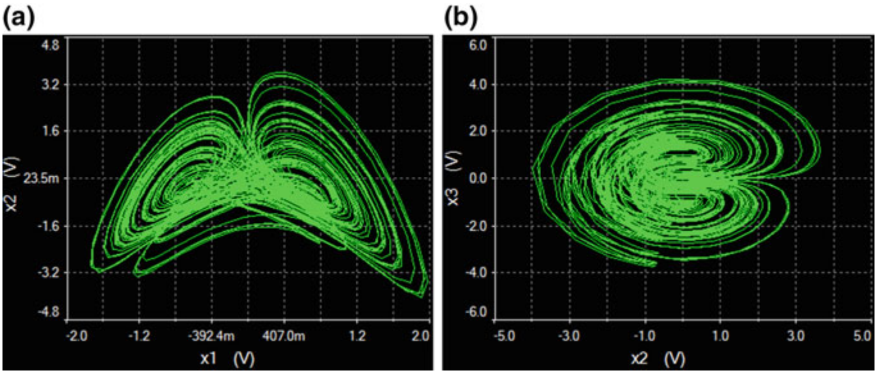

Fig. 32 Chaotic attractors of the NHE1 system obtained using circuit implementation with $a = 10, b = 45$ and $c = 0.0183$

where the variables x_1, x_2, x_3, x_4 *and* x_5 are the outcome of U9A, U1A, U3A, U5A and U7A, respectively. The system in (5) is equivalent to the NHE1 system with $\tau = t/RC$, $R1 = R2 = bR = 40\,\text{k}\Omega$, $R6 = R11 = 40\,\text{k}\Omega$, $R5 = R10 = 400\,\text{k}\Omega$, $R7 = bR = 8.88\,\text{k}\Omega$, $R14 = cR = 21857.92\,\text{k}\Omega$, $\text{R17} = 0.1Rc = 2185.79\,\text{k}\Omega$, $C1 = C2 = C3 = C4 = C5 = 10\,\text{nF}$, $a = 10, b = 45, c = 0.0183$.

The circuit designed for implementation of the NHE3 system is shown in Fig. 35. The circuit in Fig. 35 consists of five integrators (U9A, U1A, U3A, U5A and U7A) which are used to realise the five states of the NHE3 system. The circuit

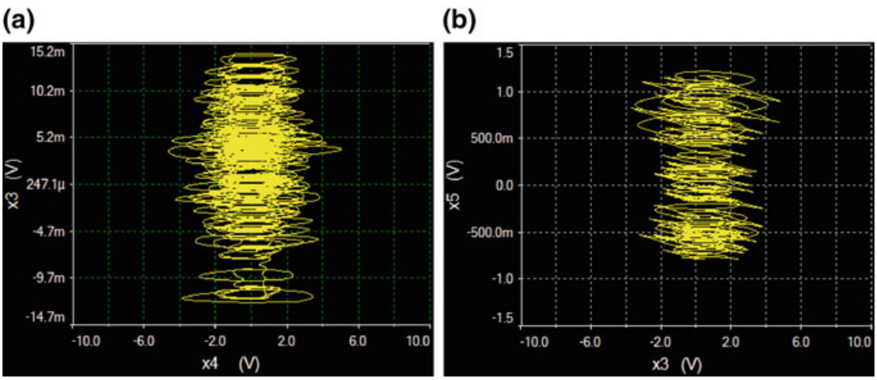

Fig. 33 Chaotic attractors of the NHE1 system obtained using circuit implementation with $a = 10, b = 45$ and $c = 0.0183$

consists of capacitors (C1, C2, C3, C4, C5), resistors (R1,…, R19), Op-Amp (LF353D) and multipliers (AD633). The circuit equations of the NHE3 system can be written as:

$$\begin{cases} \dot{x}_1 = \frac{1}{RC1}\left[\frac{R}{R1}x_3 - \frac{R}{R1}x_1\right] \\ \dot{x}_2 = \frac{1}{RC2}\left[\frac{R}{R7}V1 - \frac{0.1R}{R6}x_1x_3 + \frac{R}{R5}x_4\right] \\ \dot{x}_3 = \frac{1}{RC3}\left[\frac{0.1R}{R11}x_1x_2 + \frac{R}{R10}x_5\right] \\ \dot{x}_4 = \frac{1}{RC4}\left[-\frac{R}{R14}x_2\right] \\ \dot{x}_5 = \frac{1}{RC5}\left[-\frac{0.1R}{R17}x_2x_3\right] \end{cases} \tag{6}$$

where the variables x_1, x_2, x_3, x_4 *and* x_5 are the outcome of U9A, U1A, U3A U5A and U7A, respectively. The system in (6) is equivalent to the NHE3 system with $\tau = t/RC$, $R1 = R2 = bR = 40$ kΩ, $R6 = R11 = 40$ kΩ, $R5 = R10 = 400$ kΩ, $R7 = bR = 8.88$ kΩ, $R14 = cR = 20202.02$ kΩ, R17 $= 0.1R\,c = 2020.20$ kΩ, $C1 = C2 = C3 = C4 = C5 = 10$ nF, $a = 10, b = 45, c = 0.0198$. The chaotic attractors of the NHE3 system are shown in Figs. 36 and 37.

It is apparent from Figs. 31, 33, 36 and 37 that the attractors of NHE1 and NHE3 systems obtained using circuit implementation match with the MATLAB simulation results. It is visible from Figs. 32 and 33 that the ranges of state variables are different from the MATLAB simulation results. This is because of difference in time constants considered. Relation between the time constant of system for MATLAB simulation and time used for circuit implementation is $\tau = \frac{t}{RC}$, *where* $R = 400$ kΩ, $C = 10$ nF.

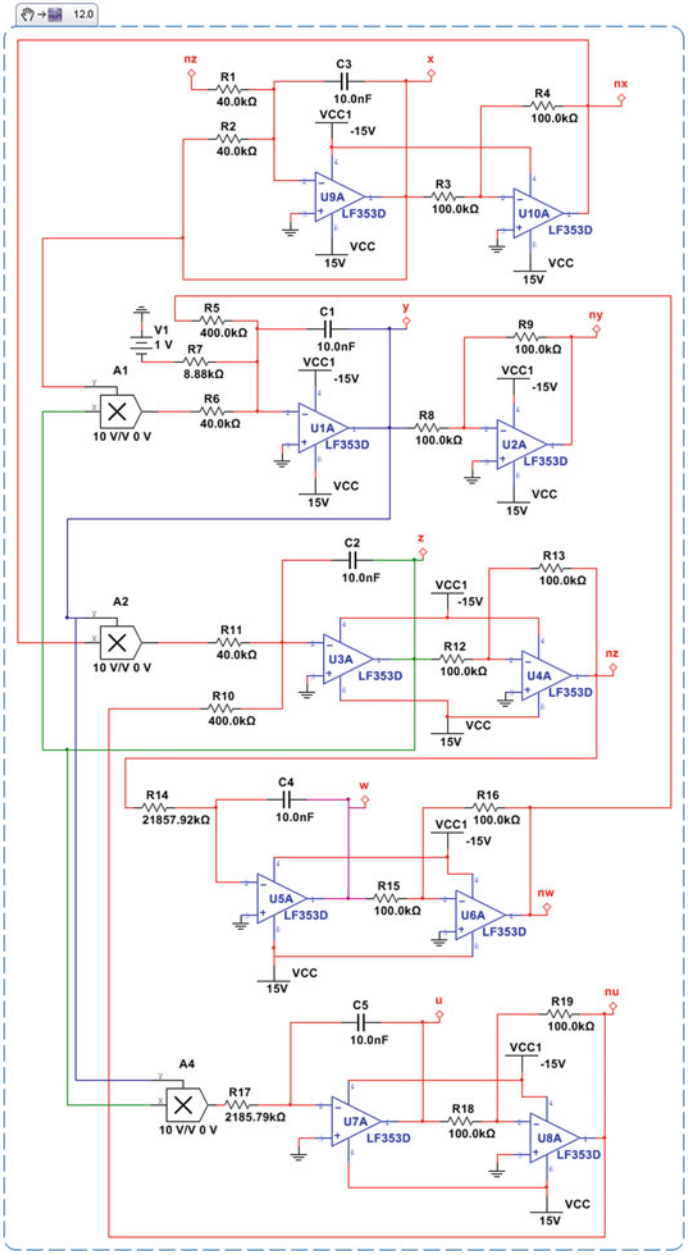

Fig. 34 Designed circuit of the system NHE1

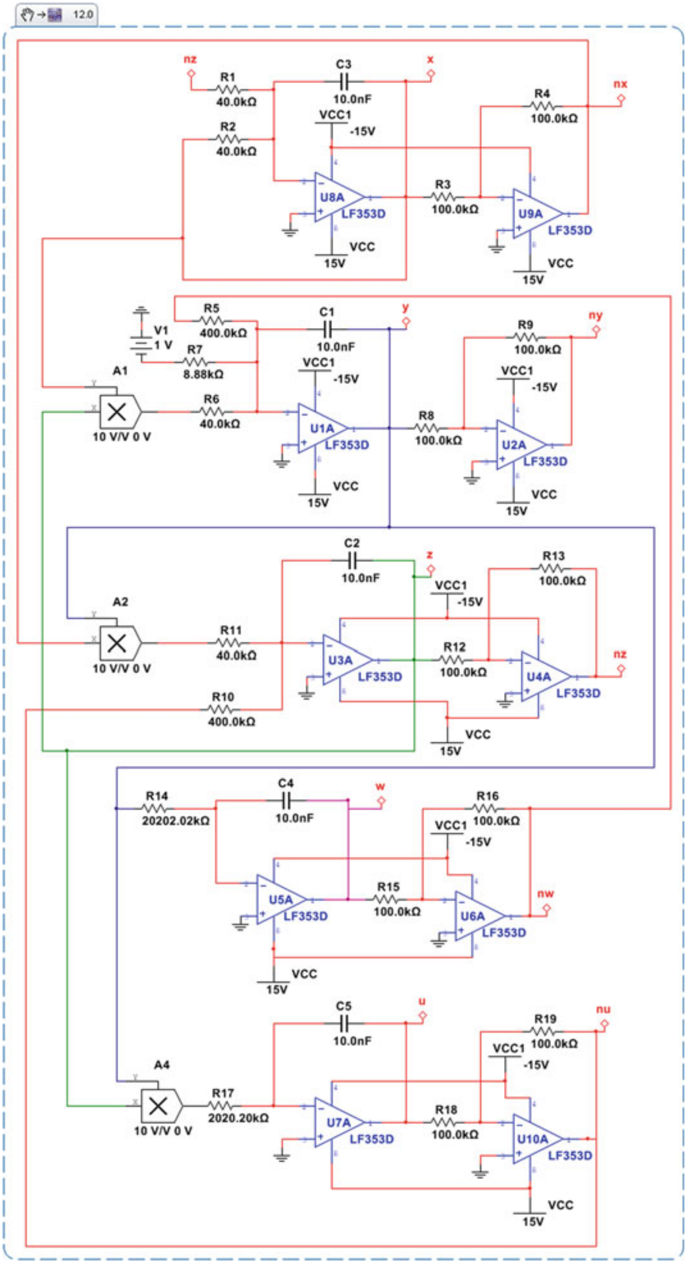

Fig. 35 Designed circuit of the NHE3 system

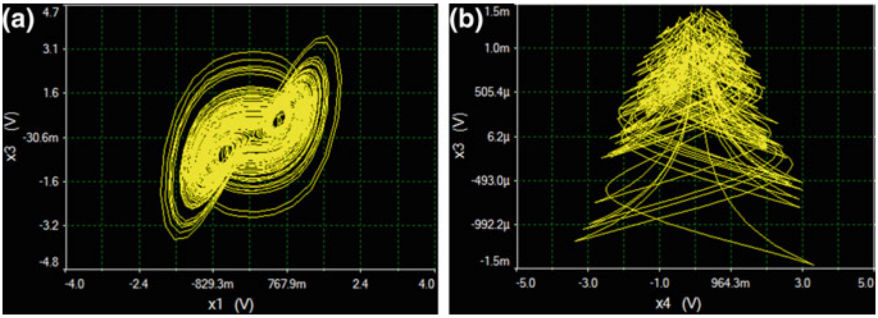

Fig. 36 Chaotic attractors of the NHE3 system obtained using circuit implementation with $a = 10, b = 45$ and $c = 0.0198$

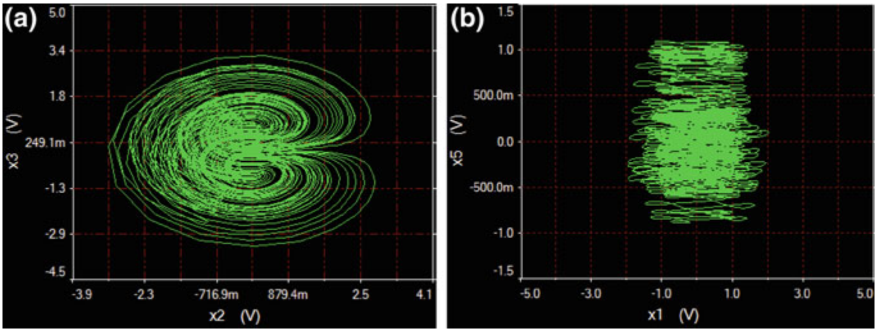

Fig. 37 Chaotic attractors of the NHE3 system obtained using circuit implementation with $a = 10, b = 45$ and $c = 0.0198$

5 Conclusions

In this chapter, three new 5-D hyperchaotic systems and two new 5-D chaotic systems with the nature of self-excited attractors are reported. Four of these systems may behave as hidden chaotic attractors. Such chaotic systems having both the self-excited and hidden attractors are rare in the literature. All the five systems have non-hyperbolic equilibria and hence belong to the category of self-excited attractors. NHE1, NHE3, NHE4 and NHE5 systems have many equilibria along with non-hyperbolic nature of equilibria. Hence, these four systems may be considered under the category of both self-excited and hidden attractors chaotic systems. The new systems are developed from the well-known 3-D Lorenz chaotic system with some transformation. All the five systems exhibit multistability. Various numerical tools like phase portrait, Lyapunov spectrum, bifurcation diagram, Poincaré map, and frequency spectrum are used to find different dynamic behaviour of the new systems. These behaviours confirm the chaotic nature of the proposed systems. The results obtained using MATLAB simulations are validated by using circuit realisation. The proposed 5-D systems can have better application in the field of secure communications.

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