A Two-Phase Optimization Approach for Reducing the Size of the Cutting Problem in the Box-Production Industry: A Case Study

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Abstract In this study, the cutting problem as one of the main problems within the box-production industries is discussed. The cutting problem refers to the problem of dividing a piece of rectangular raw material, which is usually large, into smaller pieces to produce various products. Cutting problems are NP-hard problems. Numerous researches offering good solutions to these problems have been conducted over the past few years. In the present study, considering the complexity of the problem, a model reflecting the nature of the problem is proposed and a new two-phase solution approach is suggested. Utilizing the proposed method significantly reduces the size of the problem and simplifies the applicability of the solution approach in real life. Furthermore, to evaluate the efficiency and utilization of the proposed method, its application in a specific company is tested. Finally, the performance of the method is calculated and its use is compared with the company's traditional method.

Keywords Material selection · Production planning · Cutting problem

Introduction and Literature Review

In several industrial applications such as the wood, paper, and glass industries, it is necessary to cut rectangular raw materials into smaller rectangle pieces with specific measures such that the amount of waste is minimized (Russo et al. 2014). Up to the present, numerous researches have been conducted to investigate the best method

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for cutting the raw materials. The resulting problems are optimization problems referred to as bin packing problems, two-dimensional cutting problems (2DCP), or two-dimensional strip packing problems in the literature. Most of the investigations of these problems are devoted to cases where the items to be packed have a fixed orientation and are not rotatable. In other words, a set of rectangular items (products) defined by their width and height is given. Having an unlimited number of identical rectangular raw materials (objects) of certain width and height, the objective is to allocate the items to a minimum number of the objects or, identically, to divide the objects into smaller pieces such that the maximum number of items is delivered with minimum wastage. With no loss of generality, it is assumed that all input data are positive integers and the dimension of the items is always less than or equal to the objects. This problem is NP-hard (Lodi et al. 2002).

Gilmore and Gomory were the first contributors to model two-dimensional packing problems. They proposed a column generation approach based on the enumeration of all subsets of items (patterns) such that they can be packed into a single object (Gilmore and Gomory 1965). Continuing in this line, Beasley associated the concept of profit for each item to be packed in two-dimensional cutting problems with the aim of packing the subset of items with the maximum profit into a single object (Beasley 1985b). Hadjiconstantinou and Christofides (1995) proposed a similar model for this problem. The function of both modes is to provide upper bounds that benefit the Lagrangian relaxation and sub-gradient optimization method. Later, Scheithauer and Terno (1996) introduced raster points constituting a subset of the discretization points. These raster points are capable of being used in an exact dynamic programming algorithm without losing the optimality (Beasley 1985a). Working on Beasley's idea, Cintra et al. (2008) proposed an exact dynamic programming procedure that simplifies the computation of the knapsack function and provides an efficient procedure for the computation of the discretization points. Additionally, the number of discretization points introducing an idea which partially recalls the raster points in reduces in their approach. Kang and Yoon (2011) suggested a branch and bound algorithm for Unconstrained Two Dimensional Cutting Problems (U2DCP), which is amongst the best algorithms proposed for this category of problem. Moreover, they performed a pre-processing procedure before running the algorithm, with the aim of reducing the number of valid pieces for entering the process which is independent from the main solving approach. Recently, a two-phase heuristic for the non-guillotine case of U2DCP was proposed by Birgin et al. (2012); it solves the guillotine variant of the problem in the first phase in two steps: a fast heuristic step based on the earlier two-stage algorithm proposed by Gilmore and Gomory (1965) and an exact dynamic programming step proposed by Russo et al. (2013). The latter method introduces a solution-correcting procedure and improves one of the two dynamic programming procedures of Gilmore and Gomory (1966). Furthermore, in their algorithm, they employed the reduction of the discretization points method proposed by Cintra et al. (2008) and pre-processing method proposed by Birgin et al. (2012). This algorithm is one of the most effective exact dynamic programming algorithms proposed for solving the U2DCPs. The objective of this research is to maximize the profit of an enterprise dealing with the cutting problem by minimizing the amount of wastage and surpluses generated during the production. A two-phase algorithm is proposed to serve the mentioned objective, which determines the proper dimension of the raw material required for the production such that all products of the company can be produced with minimum wastage. Moreover, through determining the best combination and quantity of raw materials, the number of surpluses and procurement cost are reduced. In the next section, the characteristics of the problem are introduced.

Problem Description and Preliminaries

The aim of this study is to offer a solution to the cutting problem of the box production industry. To deal with this problem, a two-phase approach is proposed. In these industries, the products are carton boxes of various sizes according to the customer's demands. These carton boxes must meet accurate specifications regarding their material types and dimensions in accordance with the customer's requested specifications. The carton boxes are produced from raw sheets of carton provided by the company's suppliers in various predefined sizes. The suppliers can supply the raw sheets in specific standardized sizes. More details about the problem are given as follows:

- In each planning horizon, the customer orders a specific number of boxes;
- Several sizes of the raw materials are available at each supplier known to the company;
- The number of deliverable products is easily determined by the company if and only if a specific raw material is assigned to produce a specific product;
- There exists more than one suitable candidate raw material for producing one or more products;
- The raw material procured by the companies is distinguished and separated based on its dimensions and the combination of the materials used for building them;
- Each specific size of the raw material used in production generates a certain amount of waste. This wastage is dependent on the production strategy employed for assigning the products to the raw material;
- Each company may have its own individual policies for selecting the measures of the purchased raw materials.

Like any other industry, the profitability of the business is its most important concern. Therefore, nearly all companies in this industry are interested in achieving the following objectives:

- Reducing the wastage cost through minimizing the production-related wastage of materials;
- Reducing the size of the cutting problem through minimizing the variety of the selected raw material such that all products are producible.

A high variety of raw material is confusing. In this industry, due to the need to minimize waste, accurate determination of the dimensions of the raw materials used for producing the products is crucial. On the other hand, all companies usually have a huge variety of products. While utilizing a dedicated raw material with correct dimensions for producing a product will theoretically lead to the minimum possible waste, in practice, this one-to-one approach is almost impossible for the following reasons: firstly, the supply of raw material is restricted to limited specified dimensions, and secondly, dedicating a raw material to each product corresponds to a massive variety of raw materials of different quantities, which is not possible due to inventory-related restrictions. Hence, to have a standard manufacturing system with the minimum amount of incompatibility, the company needs to reduce the size of its problem through limiting the variety of them in-hand raw material in such a way that its production capabilities are not reduced. Additionally, limiting the variety of raw material is useful when suppliers offer quantity discounts where a larger purchasing discount is deliverable if a larger quantity of a single type is purchased.

- Minimizing the in-hand inventory and production surplus

Essentially, two types of inventories are available at the companies: the finished products and raw materials. Since the ordering style of the customers is highly changeable, the extra inventory of the finished products (surplus) is quite likely to remain unused for a long period of time. Apart from that, due to the vulnerability of the inventory to shrinkage, fire, and similar hazards, companies are always at risk of inventory loss. On the other hand, taking the required measures to encounter these risks is extremely costly. Therefore, companies prefer to reduce their risks by keeping their inventories at the lowest possible level.

Determining the appropriate dimensions for the raw materials, purchasing the correct quantity of raw materials, and assigning them properly for the production of products are the most important elements for fulfilling the main objectives of the companies. Indeed, the mentioned requirements are the decision variables of a subcategory of 2DCPs addresses as bin packing problem or strip packing problem in the literature.

The proposed algorithm of this study is designed to deal with this problem. The method is extendable to any other box production company as well as similar industries with minor tailoring. In this research, to evaluate the efficiency of the proposed method, it is implemented in a specific box production company as a case study. In the next section, the specification of the case study is discussed.

The Case Description and Definitions

The case discussed in this research produces over 200 different types of products including carton boxes and divider planes. The main differences among the products are associated with their dimensions and combinations of materials. The technical details are described below.

Sheet Types

The main raw sheet types utilized in the company are three- and five-layer sheets. These sheets are produced by suppliers by combining several layers of carton papers and one or more (depending on the number of plane layers) corrugated media between the papers, which is called the Flute Layer (FL). There are two major types of carton papers: Craft, denoted by (C), which is paper freshly produced from wood (virgin paper), and Liner (Li), which is recycled paper. While the papers in the outer layer of a carton sheet can be made of any material, the material type of the corrugated medium and the paper in the middle layers of a carton sheet are usually liner paper. The different combinations of paper types and medium layers provide a total number of six different carton sheets for use in the company.

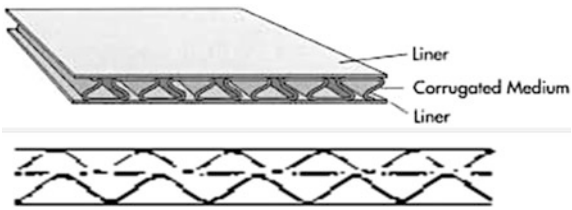
- Five layers and double Craft (**C2-5**)
- Five layers and single Craft (**C1-5**)
- Five layers and liner (**Li-5**)
- Three layers and double Craft (**C2-3**)
- Three layers and single Craft (**C1-3**)
- Three layers and liner (**Li-3**)

In the next figure, the combination pattern of the carton sheets is illustrated. The outer layers of the carton sheet could be both liner, both craft, or one liner and one craft (Fig. 1).

Strength of the Boxes

The strength of a carton box is dependent on two factors: the combination of the papers and the direction of the FL. A carton box acquires the minimum necessary strength if and only if the direction of the FL is vertical with respect to the weight that the carton must carry. Consequently, rotation of the carton sheets is not allowed during the production process. On the other hand, according to a general rule, a carton sheet with more crat layers in its structure has higher strength. However, the use of more craft layers is associated with a higher production cost and therefore more expensive product.

Fig. 1 3-layers and 5-layers carton sheets



Selecting the Material Type for Production

It is the customer who decides on the material combination of the carton sheets of the products; however, the company normally provides an advisory service for the customers to facilitate their decision-making process.

Dimensions of the Products

As previously mentioned, the products of the company are boxes and divider planes. The measures of a box are normally represented by its length, width, and height (a * b * c). Since the planes are two-dimensional, their measure is simply represented by length * width (a * b).

The Spread Dimension

The spread dimension of a product, represented by L * W, is the dimension of the carton sheet that is required to produce that product. The spread dimensions for boxes are calculated according to the following formulae:

$$\mathrm{L} = [(\mathrm{a} + \mathrm{b}) * 2)] + 4 \tag{1}$$

$$\mathrm{W} = \mathrm{b} + \mathrm{c} \tag{2}$$

For two-dimensional products, this procedure is much simpler: the required dimensions of the carton sheet for producing a plane are equal to the dimensions of the product itself. Put simply, the spread dimension of a product is equal to the minimum dimensions of the raw carton sheet capable of producing it.

Item

Each product of the company that is purchased by the customers is an item.

Object

The raw materials for the company are produced by its suppliers from different material combinations with different measures. Each variant of these raw materials is called an object, which is considered as a separate raw material.

Pattern

The first step in producing the items is to divide the objects into smaller pieces according to the items' spread dimensions. There are various strategies for dividing an object into smaller parts, each of which is called a pattern.

Constraints Related to Suppliers

As previously discussed, the aim of this study is to determine the proper dimensions of the raw materials. One of the constraints associated with this problem is the supplier restrictions in delivering the requested measures. Due to technical issues, suppliers are unable to cut the raw sheets into any desirable measures; the available lengths of a sheet from a supplier may vary between 45 to 200 based on 5 cm increments (i.e. 45, 50, 55,…, 200). Moreover, the stocks can only be cut into the following predefined widths: 90, 100, 110, 120, 140, 150, 160, and 200. The next table represents the possible measures of lengths and widths as the dimensions of an object (Table 1).

Clusters of Products

Before proceeding to the solution approach, the production data of the problem must be marshalled and categorized. In this regard, initially the data related to the item types are collected and the products with an identical material combination are placed in the same category. Based on this classification, six different clusters of products are defined: C1-5, C2-5, C1-3, C2-3, Li-5, and Li-3. It is notable that to produce the items in each cluster, the material combination of the objects must be identical to the material combination of the cluster. However, several sizes of

Table 1 Table of possible cutting measures

Possible widths							
90	100	110	120	140	150	160	180
Possible lengths	45	Possible lengths	85	Possible lengths	125	Possible lengths	165
	50		90		130		170
	55		95		135		175
	60		100		140		180
	65		105		145		185
	70		110		150		190
	75		115		155		195
	80		120		160		200

objects can be used. Selecting the proper object(s) for each cluster is one of the objectives of this problem.

Deterministic Formulation of the Problem

In this section, a deterministic formulation based on the characteristic and assumptions of the problem is proposed and explained. The following notation is used in the mathematical formulation of the above-mentioned problem:

i	index for the number of products
j	index for the number of available objects
m	the number of items in each cluster
n	the number of available objects
M	a large positive value
d_i	demand for item i
p	the set patterns satisfying the minimum acceptable waste condition, p = {1, 2, …, P}
g_{ipj}	the number of extractable items i from object j if pattern P is applied
c_{jp}	unit cost of object j having pattern p applied to it
x_{pj}	frequency with which pattern p is applied to object j
z_j	decision variable for using object j

The following model is proposed:

$$\text{Objective function 1: } \min \sum_{j=1}^{n} z_j \tag{3}$$

$$\text{Objective function 2: } \min \sum_{j=1}^{n} \sum_{p=1}^{P} c_{jp} x_{pj} \tag{4}$$

subject to:

$$\forall i\text{: } \sum_{j=1}^{n} \sum_{p=1}^{P} \mathrm{g_{ipj} x_{pj}} \geq \mathrm{d_i} \tag{5}$$

$$\forall p\text{: } \mathrm{x_{pj}} \leq \mathrm{Mz_j} \tag{6}$$

$$\mathrm{x_{pj}} \geq 0, \text{ integer} \tag{7}$$

$$\mathrm{z_j} = 1 \text{ if raw material j is used, and 0 otherwise} \tag{8}$$

The description of the model is as follows: the objective function (3) minimizes the variety of objects (i.e., the variety of raw materials) that should be used in the production procedure. Objective function (4) minimizes the procurement cost of object j by optimizing the usage frequency of the object–pattern combination. At the same time, objective function (4) minimizes the surplus amount by justifying the purchased material at the required level. Constraint (4) guarantees that the production number of satisfies its demand. Constraint (6) denotes that there is no limitation on providing the required number of objects. Finally, the constraints (7) and (8) define the nature of the variables.

The Solution Approach

To solve this kind of problem, depending on the problem environment, various methodologies might be effective. In this study, considering the objectives of the problem, a two-phase dimension-determination method is proposed that solves the problem in several steps. The first step in approaching this problem is to categorize the products. As previously mentioned, they are divided into six different clusters: C2-5, C1-5, Li-5, C2-3, C1-3, and Li-3. Each cluster is a set of products sharing the same property of the raw material but with different dimensions. Table 2 represents a sample of uncategorized data and Table 3 represents the categorized samples.

Table 2 Sample of uncategorized data

ID code	Dimension		
G1	49.5	24.5	11
G143	7.5	3.5	10.9
G144	5	4	58
G145	6.7	6	66.5
G2	50.5	25.4	22.8
G216	50	7	0
G217	25	7	0
G218	80	7.5	0
G3	61	33.7	25
G36	42	8.6	0
G37	98.5	15	0
G38	79.5	15	0
G58	135	10	0
G59	28.5	10	0
G60	113	11	0
G71	39	11	0
G72	101	11	0
G73	14.4	13.5	9.5

Table 3 Sample of categorized data

	ID code	Dimension			Spread dimensions	
					Length	Width
C1-3	G1	49.5	24.5	11	152	35.5
	G2	50.5	25.4	22.8	155.8	48.2
	G3	61	33.7	25	193.4	58.7
C2-3	G36	42	8.6	0	42	8.6
	G37	98.5	15	0	98.5	15
	G38	79.5	15	0	79.5	15
C1-5	G58	135	10	0	135	10
	G59	28.5	10	0	28.5	10
	G60	113	11	0	113	11
C2-5	G71	39	11	0	39	11
	G72	101	11	0	101	11
	G73	14.4	13.5	9.5	62.8	23
Li-3	G143	7.5	3.5	10.9	26	14.4
	G144	5	4	58	22	62
	G145	6.7	6	66.5	29.4	72.5
Li-5	G216	50	7	0	50	7
	G217	25	7	0	25	7
	G218	80	7.5	0	80	7.5

Two-Phase Dimension Determination Method

In this approach, the complexity of the problem is decreased by determining the dimensions of the objects in two phases. The method is applicable for all clusters; therefore, to illustrate the procedure only one cluster (C1-3) is discussed as an instance. In the first phase, based on possible purchasable measures for the length of the objects, the producible products utilizing a certain length of objects are classified in the same group. In the second phase, considering the demand for the items, different combinations of the assigned lengths and available widths (as the final dimension of the objects) are investigated. The results consist of determining the best dimension of the objects as well as the optimal production plan that satisfies the requirements of the problem owners, respecting the demand for the items. In the following, the procedure used to obtain this solution is discussed.

Phase 1: Classification of the Items with the Same Object Length

Step 1. The matrix of remaining lengths is formed based on all available lengths of the objects. This matrix represents the remaining length of an object (regardless of

Table 4 The matrix of remaining length using each object

	Available lengths for C1-3 objects								
Length of items	Length of the objects								
	40	45	50	.	.	.	190	195	200
23.5	17	22	3				2	7	12
46.6	40	45	3	.	.	.	4	9	14
47.6	40	45	2				47	5	10
.		.		.				.	
.		.			.			.	
.		.				.		.	
125	40	45	50				65	70	75
125	40	45	50	.	.	.	65	70	75
135	40	45	50				55	60	65

Table 5 The assignability matrix

	Assignability matrix based on allowed remaining								
Length of items	Length of the objects								
	40	45	50	.	.	.	190	195	200
23.5	0	0	1				1	0	0
46.6	0	0	1	.	.	.	1	0	0
47.6	0	0	1				0	1	0
.		.		.				.	
.		.			.			.	
.		.				.		.	
125	0	0	0				0	0	0
125	0	0	0	.	.	.	0	0	0
135	0	0	0				0	0	0

its width) if it is utilized for delivering an integer multiple of the length of a certain item. To perform this calculation, the spread length of the products and available lengths for objects are determined. The feasible length of the objects must satisfy the following two conditions:

- It must be larger than the spread length of the product;
- The material remaining after extracting an integer multiple length of a product must be less than 5 cm.

The matrix of the remaining length in the first step is represented in Table 4.

Step 2. The matrix of the remaining lengths is handled to create the "assignability matrix". The assignability matrix is a 0–1 matrix indicating whether an item is assignable to an object. If the length of an object is suitable for extracting the length of an item, the item is considered assignable to that object and therefore the digit in the relevant intersection of the rows and columns is "1"; otherwise, it is zero (Table 5).

Table 6 Number of assignable lengths for producing an item

	Assignability matrix based on remaining									
Length of items	Length of the objects									Number of assignable objects
	40	45	50	.	.	.	190	195	200	
23.5	0	0	1				1	0	0	7
46.6	0	0	1	.	.	.	1	0	0	4
47.6	0	0	1				0	1	0	4
.		.		.				.		.
.		.			.			.		.
.		.				.		.		.
125	0	0	0				0	0	0	2
125	0	0	0	.	.	.	0	0	0	2
135	0	0	0				0	0	0	2
Productivity of the object	3	0	3	.	.	.	6	5	1	

Step 3. The length of an item might be assignable to several objects. In this step, the total number of objects that can produce an item is calculated and represented in Table 6. Additionally, in the last row of Table 6, denoted as the object's productivity, the total number of items producible by the relevant object is represented.
Step 4. The rows and columns of the assignability matrix are sorted based on decreasing order of assignable lengths and productivity of each object (the object length with more applications is shown in the first column; the object with more assignability is shown in the first row); see Table 7.

Table 7 Sorted usability matrix

	Sorted assignability matrix based on remaining									
Length of items	Length of the objects									Number of assignable lengths
	130	125	190	.	.	.	65	80	85	
35	0	0	0				0	0	0	9
27.5	0	0	1	.	.	.	0	0	0	7
37	0	0	1				0	0	0	6
.		.		.				.		.
.		.			.			.		.
.		.				.		.		.
193.4	0	0	0				0	0	0	1
107.5	0	0	0	.	.	.	0	0	0	1
107	0	0	0				0	0	0	1
Productivity of the object	9	7	6				0	0	0	

Step 5. As previously mentioned, one of the objectives of this strategy is to minimize the variety of the purchased objects. Therefore, solving a set covering problem for selecting the minimum number of objects lengths which can be used for extracting the maximum number of item lengths with minimum wastage in each cluster is a suitable approach. The following notation and formulation provide the mathematical representation of the problem:

$\boldsymbol{f_j}$ the total number of different items an object can deliver, $f_j = \{1, 2, 3, \ldots, m\}$;
$\boldsymbol{a_{ij}}$ indicates whether or not the length of item i is extractable from the length of object j;
$\boldsymbol{z_j}$ decision variable for using object j;

The mathematical model is as follows:

$$\min \sum_j f_j z_j \tag{9}$$

s.t.

$$\forall i\colon \sum_j a_{ij} z_j \geq 1 \tag{10}$$

$$\forall i\colon \mathrm{z_j} = 0, 1 \tag{11}$$

$$\forall i, j\colon \mathrm{a_{ij}} = 0, 1 \text{ and fixed} \tag{12}$$

- *Note: in this approach, by selecting the objects according to the discussed strategy, for each of them, a unique applicable pattern is determined. This determined pattern is patched to the related object and represents the same concept; therefore, they can be addresses alternatively.*

The model is described as follows: the objective function (9) selects the minimum variety of objects. Constraint (10) guarantees that all items are producible by at least one object. Finally, (11) and (12) represent the nature of the variables of the problem. The sorted assignability matrix of this problem facilitates the application of the addressed set covering problem. The procedure is as follows:

- The object with the maximum productivity value is selected; all items belonging to it are determined and permanently assigned to it.
- The assignment of the permanently assigned items to the other objects is terminated.
- The productivity of the objects is updated.
- If the assignable length for all items is one, the procedure is stopped; otherwise, steps 1 to 4 are repeated.

Tables 8 and 9 illustrate the first and last stages of this procedure.

Table 8 The first stage of the object selection procedure

	Sorted assignability matrix based on remaining									
Length of items	Length of the objects (y_j)									Number of assignable lengths
	130	125	190	.	.	.	65	80	85	
35	0	0	0				0	0	0	9
27.5	0	0	1	.	.	.	0	0	0	7
37	0	0	1				0	0	0	6
.		.		.				.		.
.		.			a_{ij}			.		.
.		.				.		.		.
193.4	0	0	0				0	0	0	1
107.5	0	0	0	.	.	.	0	0	0	1
107	0	0	0				0	0	0	1
Productivity of the object (f_j)	9	7	6	.	.	.	0	0	0	

Table 9 The final stage of the object selection procedure

	Sorted assignability matrix based on remaining									
Length of items	Length of the objects (y_j)									Number of assignable lengths
	130	190	110	.	.	.	65	80	85	
35	0	0	1				0	0	0	1
27.5	0	1	0	.	.	.	0	0	0	1
37	0	1	0				0	0	0	1
.		.		.				.		.
.		.			a_{ij}			.		.
.		.				.		.		.
193.4	0	0	0				0	0	0	1
107.5	0	0	1	.	.	.	0	0	0	1
107	0	0	1				0	0	0	1
Productivity of the object (f_j)	9	6	5	.	.	.	0	0	0	

Step 6. This step is the last step of phase 1. The items of the cluster which are assigned to a certain object are classified according to the next table (see Table 10).

Table 10 Table of classified items according to assigned objects

C1-3	Recoded product	Dimensions of the item			Spread dimension of the item		Final assigned object length
					Length	Width	
45	G9	78	20	14	200	34	200
29	G3	13.9	7.9	7.3	47.6	15.2	195
58	G15	65.5	29	21	192	49	195
54	G17	61	34	25	193.4	58.7	195
111	G1	23.5	6.5	0	23.5	6.5	190
10	G13	37	27	0	37	27	190
138	G14	37	27	0	37	27	190
31	G2	13.9	7.4	7.3	46.6	14.7	190
5	G26	95	45	0	95	45	190
141	G28	95	45	0	95	45	190
7	G18	85	40	0	85	40	175
16	G21	85	40	0	85	40	175
57	G23	85	40	0	85	40	175
50	G11	50.5	25	23	155.8	48.2	160
154	G5	21.7	15	12	77.4	26.5	155
49	G10	49.5	25	11	152	35.5	155
113	G16	45	29	22	152	51	155
93	G35	135	90	0	135	90	135
32	G6	46.3	17	17	131.4	34.8	135
94	G12	36.5	27	16	130	42	130
8	G19	125	40	0	125	40	130
15	G20	125	40	0	125	40	130
56	G22	125	40	0	125	40	130
59	G24	125	40	0	125	40	130
14	G32	125	85	0	125	85	130
55	G33	125	85	0	125	85	130
62	G34	125	85	0	125	85	130
26	G7	44.3	18	18	128.6	36	130
30	G4	7.6	7.9	7.3	35	15.2	110
4	G25	105	45	0	105	45	110
140	G27	105	45	0	105	45	110
152	G29	108	74	0	107.5	74	110
119	G31	107	80	0	107	80	110
27	G8	31.7	18	18	103.4	36	105
153	G30	91.5	74	0	91.5	74	95

Table 11 Proper dimensions of the objects and production plan obtained by software

Recoded Product code	Spread dimension		Assigned length	Available widths								Plan utilization (%)
	L	W		90	100	110	120	140	150	160	180	
G9	200	34	200				1	4993				97.14
G3	47.6	15.2	195	2741	17907		2625					96.87
G15	192	49										
G17	193.4	58.7										
G1	23.5	6.5	190	319		1	29	1	3143		10097	99.41
G13	37	27										
G14	37	27										
G2	46.6	14.7										
G26	95	45										
G28	95	45										
G18	85	40	175	1						17872		97.14
G21	85	40										
G23	85	40										
G11	155.8	48.2	160		23291							93.87
G5	77.4	26.5	155	1	1			1		2487	43583	97.63
G10	152	35.5										
G16	152	51										
G35	135	90	135	1				4040			10381	99.25
G6	131.4	34.8										

(continued)

Table 11 (continued)

Recoded Product code	Spread dimension		Assigned length	Available widths								Plan utilization (%)
	L	W		90	100	110	120	140	150	160	180	
G12	130	42	130				14211			36526	77275	93.24
G19	125	40										
G20	125	40										
G22	125	40										
G24	125	40										
G32	125	85										
G33	125	85										
G34	125	85										
G7	128.6	36										
G4	35	15.2	110	62724				2624	37249	9441		96.11
G25	105	45										
G27	105	45										
G29	107.5	74										
G31	107	80										
G8	103.4	36	105								16122	98.47
G30	91.5	74	95						25279			95.05

Phase 2: Determining the Required Objects and Their Quantities for Each Cluster

As discussed above, to specify an object for ordering, both its length and its width must be determined. Hence the appropriate lengths of the objects in each cluster are determined in the first phase of the proposed procedure. In the second phase, various combinations of the determined lengths and available widths are examined and the most profitable combination is employed for producing a specific product in a cluster. For the second phase, the main specialized cutting software that is currently in widespread use in these industries is employed. Using the software, the proper objects and their required quantity to satisfy the demand for the items in each group of products is determinable. For this purpose, all combinations of the assigned length and the available widths are elaborated. Considering the demand for the items, the optimal strategy of object selection and the order quantity is determined. This information indicates what combination of the length and width should be decided for an object and what quantity of each object must be purchased. The results are shown in Table 11. The last column of the table represents the utilization of the proposed method based on the waste of the raw material.

Conclusion

In this study, a category of cutting problem that is frequently used in box production companies and that is an NP-hard problem was investigated and formulated and a two-phase method for approaching it was introduced. The main principles of the approach were minimizing the production waste and production costs and maximizing the efficiency of the material selection for production. This method is easy to implicate, returns a very good solution, and is applicable for a wide range of similar problems in this industry with minor adaptation. Considering the environment of the problem, in this investigation, the suppliers' competitions were limited to their ability to provide the raw material, the uncertainty of the demand was neglected, and the specific restriction was applied in the determination of the proper material selection. The focus of the method was on selecting the material that would initially generate an acceptable amount of waste. This component may vary depending on different circumstances. In future development of the study, price competitive suppliers, the uncertainty of demand, and a different method of material selection (such as maximizing the useable leftovers) seem to be very interesting assumptions to be taken into consideration.

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