Evaluating Airline Network Robustness Using Relative Total Cost Indices

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Abstract This research to the best of our knowledge is the first paper to quantify airline network robustness in the presence reversible capacity of legs and alternative flights. In this study, we try to recognize the critical legs via changing the functional capacity of flights. Besides, we attempt to gauge the behavior of the flight network via shifting of leg capacities proposing a new leg cost function. In addition, we indicate how to capture the robustness of airline network in the case of variable flight capacities. Relative Total Cost Indices have been used to assess air network robustness in the case of behavior associated with both User-Optimization and System-Optimization. In this article from the different point of view, the variability of passenger's route preferences is the main subject. This paper may shed light on the robustness of networks in real life not only for the particular case of airlines but also for systems sharing similar topological properties. The paper presents a numerical case study with real data from an airline in Turkey for illustration purposes.

Keywords User optimality · System optimality · Network robustness
Flight networks

Introduction

Networks are complex, typically, large-scale systems, and their formal study has attracted much interest from a plethora of scientific disciplines (Bazargan 2010). A broad variety of practices in the real world can be explained as complex or heterogeneous networks, like the postal networks, energy distribution networks as

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well as air transportation networks. Recently studies in the context of complex systems have attained successes in many spheres (Newman 2003; Boccaletti et al. 2014; Wei et al. 2013), such as system modeling (de Dios et al. 2001; Soysal et al. 2014; Sarvari et al. 2016; Abdelghany et al. 2008), optimization (Eskandarpour et al. 2015; Storn 1996; Jindal et al. 2015) and traffic dynamics (Yan et al. 2006; Wu et al. 2016; Zhang et al. 2010) and so on. Large infrastructure networks such as the Internet, power grids and transportation systems (Zang et al. 2010; Du et al. 2016), play a significant role in the modern world. As the robustness of base networks is maturing, so the robustness of heterogeneous networks has interested and inspired researchers to develop many papers (Buldyrev et al. 2010; Science et al. 2016; Lordan et al. 2015; Tan et al. 2015; Trajanovski et al. 2012). Recently, (Schneider et al. 2011) proposed a new measure R for network robustness and studied the optimal arrangement of arcs and nodes considering this measure. The results confirmed that network robustness could be significantly enhanced (Hong et al. 2017).

Airline schedule planning typically involves four steps from schedule design, fleet assignment, aircraft routing to crew pairing/rostering. Each stage is planned and optimized in interaction with other three steps. At the stage of aircraft routing, schedule planning involves the optimization of aircraft routing by formulating aircraft routing as integer programming problems. The lack of consideration in aircraft routing optimization to reflect real operational issues may result in lower schedule robustness and reliability in daily operations. The observable consequences of lower schedule reliability are flight delays and potential delay propagation in an airline's network (Wu 2006). So the whole network is affecting with tiny changes in its sub-systems. Changing the flight routes, changing hubs, overloading a flight arc or expanding the network in case of increasing the destination numbers are common phenomena in the airline industry. Adding even one flight node to the airline's transportation network can affect each step of the schedule planning and may destabilize the whole network. These effects can lower the reliability/robustness of the network decreasing the turnaround efficiency and triggering delay propagation.

As Dios et al. (2001) mentioned, the transportation system can be considered as a conventional economic system with demand and supply subsystems. In traffic authorization, an O-D trip matrix is loaded onto the system, and a set of connection flows is generated (Campbell 2009). The Relative Total Cost Indices (RTCI) can be evaluated at either user-optimal (U-O) traffic streams, or system-optimal (S-O) traffic streams. A recommended leg cost function empowers the quantitative evaluation of the variations in the relative total cost of an air transportation network, in the case of alternative travel behavior, when the link functional capacities are decreased or increased (Nagurney 2010).

Nagurney and Qiang (2008a, b) provides an overview of some of the recent developments in the assessment of network vulnerability through proper mechanisms that support in the quantification of network performance and the classification of the effect of network elements, such as nodes and links. The boom and drop on the number of airplanes in networks due to maintenance, scheduling and routing approaches, air carriers and airports deterioration over time, as well as politic decisions lead to time-consuming and costly connection flights, lack of flights and poor service quality would effects passenger decision manners (Bazargan 2010). That is why; we introduce a new procedure for evaluating the robustness of an airline network based on the RTCI for the transportation system in the case of leg variation captured through a uniform link capacity ratio. In an air network, there are hub and spokes that every leg and flight deviation is changing the whole network robustness.

The paper has been organized as follows; In Section "Relative Total Cost Index", the proposition of the relative total cost index is provided. In Section "Principal and Components of RTCI", we explain the RTCI that can be used to evaluate transportation network robustness and which allows either U-O or S-O travel behavior. In Section "Assessment of Airline network robustness", for the first time, we try to assess the airline network robustness by reducing and increasing of flight capacity by network robustness measure. In Section "Case Study", a case study and related discussions on a partial network of an airline have been considered to analysis, and finally, Section "Conclusion" presents a brief closure.

Principal and Components of RTCI

Decentralized Decision-Making and Centralized Decision-Making (U-O and S-O)

Wardrop and Whitehead (1952) explicitly recognized possible alternative behaviors of transportation networks users and stated two principles, which are commonly named after them. These principles match, in consequence, to decentralized versus centralized behavior on networks and, albeit stated in a transportation circumstances, have connections to many various networks. Hence, we now recall Wardrop's two principles; The first implies that the journey times of all used routes are equal and less than those that would be experienced by a single vehicle on any unused route, and the second one assumes that the average journey time is minimal.

The fundamental principle reactions to the behavioral principle in which passengers query to determine their minimal costs of travel whereas the second principle corresponds to the behavioral policy in which the total cost of the system is decreased. Nagurney (2003) proved the equality between the traffic network equilibrium statuses, which assert that all used paths connecting an origin-destination pair will have equal and minimal travel times (or costs); corresponding to Wardrop's first principle, and the Kuhn-Tucker conditions of an appropriately formed optimization problem, under a balance assumption on the underlying functions. Consequently, in this case, the equilibrium link and path flows could be captured as the solution to the problem.

Dafermos (1980) coined the terms user-optimized (U-0) and system-optimized (S-0) transportation networks to distinguish between the two distinct situations in which users act unilaterally, in their self-interest in selecting their routes. In the latter problem, marginal (total) costs rather than average costs are equilibrated equalized, in which users select routes according to what is modeled and assumed as optimal from a societal point of view, so that cost in the system is minimized (Sarvari and Erol 2013). The problems mentioned above coincide with Wardrop's first principle and the latter with Wardrop's second principle.

As a mathematical description, let xp represents the nonnegative flow on path p and let fa denotes the flow on flight a. The cost experienced by a user traversing flight a is denoted by $t_a(f_a)$ and the total cost experienced by the system (company) on flight a denoted by $\hat{t}_a(f_a)$. Additionally, dw, denotes the demand associated with O-D pairs of w for all $w \in W$. On the condition that path p contains link a then $\delta_{ap} = 1$, otherwise, $\delta_{ap} = 0$ (Bazaraa et al. 2006). Formulations (1) and (2) demonstrate the mathematical optimization models for both user and system optimality.

- *For User Optimality*

$$\begin{aligned} Min \quad & \sum_{a \in A} \int_0^{f_a} t_a(y) d_y \\ s.t. \quad & \sum_{p \in p_w} x_p = d_w, \\ & f_a = \sum_{p \in P} x_p . \delta_{ap}, \\ & x_p \geq 0, \end{aligned} \tag{1}$$

- *For System Optimality*

$$\begin{aligned} Min \quad & \sum_{a \in A} \hat{t}_a(f_a) & \\ s.t. \quad & \sum_{p \in p_w} x_p = d_w, & \forall w \in W \\ & f_a = \sum_{p \in P} x_p . \delta_{ap}, & \forall a \in A \\ & x_p \geq 0, & \forall p \in P \end{aligned} \tag{2}$$

In this section, a detailed trip cost function that is a combination of passenger flow function and arc cost function has been applied. As cost of a path in a flight network is cost of legs plus the cost of transshipment or hub (Campbell 2009), what's more, the flows of passengers by plane **a** and **b** have been named as f_a and f_b respectively. Provided that the nominal capacity of plane **a** is c_a, consequently the practical capacity of **a** is $lf \times c_a$ where lf is the load factor (Bazargan 2010) and providing flight cost or ticket price by fc then the rate of flow for the **ath** plane can be calculated by $\frac{f_a}{lf \times c_a}$.

Considering the fc as the lowest ticket price for the **ath** fight operating by **ath** plane and the total cost for a selected flight by the passenger is calculated via Eq. (3) where α, β and k are the congestion rates (are positive and unique coefficients for every field and company).

$$t_a = fc\left[\alpha\left(\frac{kf_a}{lf \times c_a}\right)^{\beta}\right] \tag{3}$$

From the other hand, there are two types of flights; the transshipment flights and the connection flights. On condition that the ath plan is flying a direct flight, the cost function from the system view can be calculated by Eq. (4), otherwise Eq. (5) is covering the connection flight too, where, $f'c$ is the least flight cost for ath plan and tsc is the transshipment cost (Note: the connections is allowed just for the flights passing the hub airport).

$$\widehat{t}_a = f'c\left[\alpha\left(\frac{kf_a}{lf \times c_a}\right)^{\beta}\right] + arc\ \cos t \tag{4}$$

$$\widehat{t}_a = f'c\left[\alpha\left(\frac{kf_a}{lf \times c_a}\right)^{\beta}\right] + arc\ 1\ \cos t + arc\ 2\ \cos t + \ tsc \tag{5}$$

Performance Measure of Nagurney and Qiang for Evaluating of Critical Arcs

The network performance/efficiency ratio (Nagurney and Qiang 2008a, b), in a flexible demand case, is described as Eq. (6);

$$\varepsilon = \varepsilon(G, d) = \frac{\sum_{w \in W} \frac{d_w}{\lambda_w}}{n_w} \tag{6}$$

where;

n_w The number of O-D pairs in the network
λ_w The cost of the most reasonable or the shortest way
n_w The total number of flight demands (between O-D)

Relative Total Cost Index

The total cost of the network is named as TC and is specified by Eq. (7);

$$TC = \sum_{g \in A} \widehat{t}_g = \sum_{g \in A} t_g(f_g) f_g \tag{7}$$

Let's suppose $g \in A$, is an arc on the network and Ψ ({g}) is the relative total cost increase of G, and on the condition of eliminating {g} from the network, relative total cost increase will be equal to Eq. (8):

$$\psi(\{g\}) = \frac{TC(G - \{g\}) - TC(G)}{TC(G)} \tag{8}$$

where TC (G) is the total cost of the network G, TC (G − {g}) is the total cost of the network G − {g}. Because of deriving total cost from U-O and S-O, Eqs. (9) and (10) can be derived, where Eq. (9) is the relative total cost derived with U-O, and Eq. (10), is the relative total cost derived with S-O.

$$\psi_{U-O}(\{g\}) = \frac{TC_{U-O}(G - \{g\}) - TC_{U-O}(G)}{TC(G)} \tag{9}$$

$$\psi_{S-O}(\{g\}) = \frac{TC_{S-O}(G - \{g\}) - TC_{S-O}(G)}{TC(G)} \tag{10}$$

Note: Functions mentioned above will distinguish critical nodes. On the condition that the g has been affected by capacity changes then, the relative total cost indices appear as:

$$\psi^{\gamma}_{(g)} = \frac{TC^{\gamma}_{(g)} - TC}{TC} \tag{11}$$

$$\psi^{\alpha}_{(g)} = \frac{TC^{\alpha}_{(g)} - TC}{TC} \tag{12}$$

TC Total cost of the network without capacity changes.
$TC^{\gamma}_{(g)}$ Network total cost if capacity decrease rate of g is γ
$TC^{\alpha}_{(g)}$ Network total cost, if capacity increase rate if g is α

Assessment of Airline Network Robustness

In Case of Changing the Capacity of a Leg

To evaluate network robustness let's decrease legs (flight) carrying capacity with a fixed rate. Network efficiency measures are capturing under this reduction. If the original capacity of a leg is c_g and $\gamma(\gamma \in (0,1])$ is the reduction rate of capacity, γc_g is the leg's decreased capacity and, $c_g - \gamma c_g$ is reduction measurement of the leg. The robustness measurement of network G is R^γ (Nagurney 2010).

Airline Network Robustness with Reduction Flight Capacity

$$R^\gamma = R^\gamma(G, d, t, c, \gamma) = \frac{\varepsilon^\gamma}{\varepsilon} \times 100\% \tag{13}$$

d G demand vector
t Flight cost function
c Flight capacity vector
γ Flight capacity reduction rate
ε Network performance index when capacity is c
ε^γ Network performance index when capacity decreased to γc

Provided that the performance index of a network with γc capacity approximately equals to c, then that network will be robust (Nagurney and Qiang 2008a, b). On the condition of presenting of just one flight between O-D, the assumed conjunction rate will be β where robustness upper bound will be $\gamma^\beta \times 100\%$, and then The robustness measurement of network G is R^γ.

$$R^\gamma = \frac{\gamma^\beta \left[c_g^\beta + kd_w^\beta\right]}{\gamma^\beta c_g^\beta + kd_w^\beta} \times 100\% \tag{14}$$

If there are more than one flight between O-D then;

$$c \equiv c_a + c_b + \cdots + c_n$$

So the lower bound is $\gamma \times 100\%$, and the robustness measurement of network G is R^γ as Eq. (15).

$$R^\gamma = \frac{\gamma c + k\gamma d_w}{\gamma c + kd_w} \times 100\% \tag{15}$$

Airline Network Robustness with Increasing Capacity of Flights

To evaluate network robustness let's increase legs (plane) carrying capacity with a fixed rate. Network efficiency measures are capturing under this reduction. If the original capacity of a leg is c_g and α; $\alpha \geq 1$, is the inflation rate of capacity, αc_g is the leg's increased capacity and, $\alpha c_g - c_g$ is inflation measurement of the leg (flight). The network of G robustness or robustness measure can be calculated by Eq. (16).

$$R^{\alpha} = R^{\alpha}(G, d, t, c, \alpha) = \frac{\varepsilon^{\alpha}}{\varepsilon} \times 100\% \tag{16}$$

α leg capacity inflation rate

$\alpha^{\beta} \times 100\%$ is upper bound of network robustness and $\alpha \geq 1$, and upper bound is $\alpha \times 100\%$

$$R^{\alpha} = \frac{\alpha c + k\alpha d_w}{\alpha c + kd_w} \times 100\% \tag{17}$$

Network Robustness Assessment Through Using Relative Total Cost Index (Through Capacity Variations of All Legs)

Network Assessment with Capacity Reduction

Relative total cost index for network G using U-O and S-O as below (Boyce et al. 2004; Konnov et al. 2007);

$$\psi^{\gamma}_{U-O} = \psi^{\gamma}_{U-O}(G, d, t, c, \gamma) = \frac{TC^{\gamma}_{U-O} - TC_{U-O}}{TC_{U-O}} \times 100\% \tag{18}$$

$$\psi^{\gamma}_{S-O} = \psi^{\gamma}_{S-O}(G, d, t, c, \gamma) = \frac{TC^{\gamma}_{S-O} - TC_{S-O}}{TC_{S-O}} \times 100\% \tag{19}$$

Therefore, upper bound for ψ^{γ}_{U-O} is $\frac{1-\gamma}{\gamma} \times 100\%$, and upper bound for ψ^{γ}_{S-O} is $\frac{1-\gamma^{\beta}}{\gamma^{\beta}} \times 100\%$, $\gamma \in (0, 1]$.

Network Assessment with Capacity Inflation

Leg capacity is increasing at a fixed rate, and following this increase, the total cost of network changes is evaluating. Where leg capacity is c_g and α, $(\alpha \geq 1)$ is an inflation rate of capacity and $\alpha c_g - c_g$ is the inflated leg (flight) capacity amount. Equation (20) is presenting the relative total cost index for the network G sing U-O, while $\frac{1-\alpha}{\alpha} \times 100\%$ is the desired lower bound.

$$\psi_{U-O}^{\gamma} = \left(\frac{\alpha c + kd_w}{\alpha c + k\alpha d_w} - 1 \right) \times 100\% \tag{20}$$

Case Study

The capacity change of some links in the network does not affect the total cost of that link considerably, but the capacity change of some links can affect the sustainability of that network; Even if this change is very small, even worse, causing great increases in the total cost of travel. Such sensitive links are called critical links.

The data for a partial network of an airline in Turkey is illustrated in Table 1; there are five airports in five different cities and Ankara is the hub node. We are interested in assessing this network robustness with critical legs upon supply and demand sets between Istanbul-Antalya and Istanbul-Trabzon. Load factor policy of firm averagely is 90%. Transshipment cost for every hour 7$ per passenger. All ticket prices averagely 17$ per path (without taxes). The other information about the O-Ds is illustrated in Table 1.

In order to solve the revealed problem, we need to use the following steps considering the formulations mentioned above.

- Step 1. Using leg cost function.
- Step 2. Deriving fg for Variational Inequality and trip assignment (Coding can be provided from the author's E-mail address).
- Step 3. To identify critical paths through the results.

Table 1 A partial flight data of the airline network

O/D	Flight count	Plane type	Demand
IST-TRZ	5	1, 3, 5	1700
IST-ANK	38	1, 2, 3, 4, 8, 9	12000
IST-KNY	3	2, 3	1400
IST-ANT	10	1, 2, 5, 4	4200
ANK-ANT	2	8	1150
ANK-TRZ	2	4	780
TRZ-ANT	2	6	750
KNY-ANT	1	5	330

In order to recognize critical links, it is necessary to use the relative total cost index (RTCI), but first the total cost index needs to be calculated. Using the flow quantities, the total cost of each link was calculated with five different capacity reduction and capacity inflation rates and is given in Tables 2 and 5, respectively.

The Relative Total Cost obtained with the help of U-O and S-O models are given in Tables 3 and 6 accordingly with Eqs. (18) and (19).

In this study, network sustainability referring the U-O is demonstrated in all dimensions by reducing or increasing the connections capacities. Table 4 illustrates the robustness variations of the network considering the reductions and inflations in the whole network (Tables 5 and 6).

We have examined the sustainability of the entire network by using the Relative Total Cost Indices obtained with U-O and S-O in Table 7. In the user optimality approach, if the capacities of all connections of the network are increased by 1.2%, the total net capacity of the network decreases further and therefore the network can be more sustainable. In the system optimality approach, if the capacities of all connections of the network are increased by 1.4%, the total cost of the network decreases more and therefore the network can be more sustainable.

Taking the results of analysis above to catch the robustness conditions of the network based on the proposed assessment approach leads us to the following recommendations;

Table 2 Total cost with U-O and the rates of inflation and reduction of leg capacities

TC^{γ}	$\gamma = 0$	$\gamma = 0.2$	$\gamma = 0.4$	$\gamma = 0.6$	$\gamma = 0.8$
IST-TRZ	3567	3590	3605	3700	3945
IST-ANK	4044	4030	4060	3840	3765
IST-KNY	4900	4900	4900	4900	4900
IST-ANT	5442	5545	5625	5664	5619
ANK-ANT	5209	5209	5199	5199	5091
ANK-TRZ	3416	3416	3416	3416	3416
TRZ-ANT	6309	6309	6309	6309	6309
KNY-ANT	2670	2670	2670	2670	2670
TC^{α}	$\alpha = 1$	$\alpha = 1.2$	$\alpha = 1.4$	$\alpha = 1.6$	$\alpha = 1.8$
IST-TRZ	3567	3472	3357	3158	3158
IST-ANK	4044	4190	4230	4304	4370
IST-KNY	4900	4900	4900	4900	4900
IST-ANT	5442	5012	5230	5307	5411
ANK-ANT	5209	5400	5469	5498	5502
ANK-TRZ	3416	3128	3139	3141	3260
TRZ-ANT	6309	6309	6302	6309	6309
KNY-ANT	2670	2670	2670	2670	2670

Table 3 Relative total cost with U-O and the rates of inflation and reduction of leg capacities

ψ^{γ}	$\gamma = 0$	$\gamma = 0.2$	$\gamma = 0.4$	$\gamma = 0.6$	$\gamma = 0.8$
IST-TRZ	0	0.0065	0.011	0.037	0.1
IST-ANK	0	−0.004	0.0039	−0.05	−0.069
IST-KNY	0	0	0	0	0
IST-ANT	0	0.0005	0.033	0.04	0.032
ANK-ANT	0	0	−0.002	−0.002	−0.02
ANK-TRZ	0	0	0	0	0
TRZ-ANT	0	0	0	0	0
KNY-ANT	0	0	0	0	0
ψ^{α}	$\alpha = 1$	$\alpha = 1.2$	$\alpha = 1.4$	$\alpha = 1.6$	$\alpha = 1.8$
IST-TRZ	0	−0.026	−0.058	−0.11	−0.11
IST-ANK	0	0.036	0.046	0.064	0.08
IST-KNY	0	0	0	0	0
IST-ANT	0	−0.08	−0.039	−0.02	−0.005
ANK-ANT	0	0.036	0.05	0.055	0.056
ANK-TRZ	0	−0.08	−0.081	−0.08	−0.045
TRZ-ANT	0	0	0	0	0
KNY-ANT	0	0	0	0	0

Table 4 Network robustness with whole network capacity changes via U-O using

$\beta = 4, \alpha = 1, k = 0.15$	R^{γ}	$\beta = 4, \alpha = 1, k = 0.15$	R^{α}
$\gamma = 0$	0	$\alpha = 1$	0
$\gamma = 0.2$	0.988	$\alpha = 1.2$	1.1432
$\gamma = 0.4$	1.054	$\alpha = 1.4$	1.1328
$\gamma = 0.6$	0.941	$\alpha = 1.6$	**1.1556**
$\gamma = 0.8$	1.014	$\alpha = 1.8$	1.1437

- Considering Table 2, the arcs of IST-TRZ, IST-ANK, IST-ANT, ANK-ANT and ANK-TRZ are critical legs.
- Considering Table 3, reduction capacity of (IST-TRZ) is not suggested but the inflation rate 1.8 is the priority.
- The decreasing capacity of (IST-KNY) with the rate of 0.4–0.8 is suggested.
- Increasing capacity of (ANK-ANT) with the rate of 1.6 is obviously more logical.
- The decreasing capacity of (ANK-TRZ) will be logical. Decreasing or increasing the capacity of (TRZ-ANT) is not suggested. Considering Table 4, increasing the total capacity of the network by the rate of 1.6 is firmly suggested.

Table 5 Total cost with S-O and the rates of inflation and reduction of leg capacities

TC^{γ}	$\gamma = 0$	$\gamma = 0.2$	$\gamma = 0.4$	$\gamma = 0.6$	$\gamma = 0.8$
IST-TRZ	3847	3870	3885	3980	4225
IST-ANK	4324	4310	4340	4120	4045
IST-KNY	5180	5180	5180	5180	5180
IST-ANT	5722	5825	5905	5944	5899
ANK-ANT	5489	5489	5479	5479	5371
ANK-TRZ	3696	3696	3696	3696	3696
TRZ-ANT	6589	6589	6589	6589	6589
KNY-ANT	2950	2950	2950	2950	2950
TC^{α}	$\alpha = 1$	$\alpha = 1.2$	$\alpha = 1.4$	$\alpha = 1.6$	$\alpha = 1.8$
IST-TRZ	335,298	32,984	307,924	293,694	287,378
IST-ANK	380,136	39,805	38,948	400,272	39,767
IST-KNY	4606	4655	4508	4557	4459
IST-ANT	511,548	47,614	482,904	493,551	492,401
ANK-ANT	489,646	5130	5037	511,314	500,682
ANK-TRZ	321,104	29,716	288,876	292,113	29,666
TRZ-ANT	593,046	599,355	579,728	586,737	574,119
KNY-ANT	25,098	25,365	24,564	24,831	24,297

Table 6 Relative total cost with S-O and the rates of inflation and reduction of leg capacities

ψ^{γ}	$\gamma = 0$	$\gamma = 0.2$	$\gamma = 0.4$	$\gamma = 0.6$	$\gamma = 0.8$
IST-TRZ	0	0,00597	0.009877	0.034572	0.098258
IST-ANK	0	−0.0032	0.0037	−0.0471	−0.0645
IST-KNY	0	0	0	0	0
IST-ANT	0	0.0180	0.031981	0.038797	0.030933
ANK-ANT	0	0	−0.00182	−0.00182	−0.02149
ANK-TRZ	0	0	0	0	0
TRZ-ANT	0	0	0	0	0
KNY-ANT	0	0	0	0	0
ψ^{α}	$\alpha = 1$	$\alpha = 1.2$	$\alpha = 1.4$	$\alpha = 1.6$	$\alpha = 1.8$
IST-TRZ	0	−0.0162	−0.0816	−0.1240	−0.1429
IST-ANK	0	0.04712	0.0245	0.0529	0.0461
IST-KNY	0	0.01063	−0.0212	0.0106	−0.0319
IST-ANT	0	−0.0692	−0.0559	−0.0351	0.0374
ANK-ANT	0	0.04769	0.0287	0.0442	0.0225
ANK-TRZ	0	−0.0745	−0.1003	−0.0902	−0.07612
TRZ-ANT	0	0.01063	−0.0224	−0.0829	−0.03191
KNY-ANT	0	0.01063	−0.0212	−0.0106	−0.0319

Table 7 Using RTCI to evaluate robustness in case of network capacity changes

$\beta = 4, \alpha = 1, k = 0.15$	ψ^{γ}_{U-O}	ψ^{γ}_{S-O}
$\gamma = 0$	0	0
$\gamma = 0.2$	0.003149872	0.005978685
$\gamma = 0.4$	0.003224088	0.009877827
$\gamma = 0.6$	−0.002403309	0.47231609
$\gamma = 0.8$	0.004443569	0.098258383
$\beta = 4, \alpha = 1, k = 0.15$	ψ^{α}_{U-O}	ψ^{α}_{S-O}
$\alpha = 0$	0	0
$\alpha = 1$	−0.013386956	−0.016278057
$\alpha = 1.2$	−0.007312203	−0.081640809
$\alpha = 1.4$	−0.007593442	−0.124080669
$\alpha = 1.8$	0.000646849	−0.142917643

Conclusion

Systems fit the infrastructure upon which the operating of the economies and societies count on. Networks that form the solid backbones of the modern age include transportation networks that support the flows of vehicles from origins to destinations. This paper provides an approach to the assessment of network robustness through proper tools that serve in the quantification of network performance and the naming of the importance of network segments, such as nodes and links. We illustrated how rigorously formed and well-defined system measures can obtain not only the network topology bearing a particular system, but also the primary behavior of decision-makers, the resulting issues, and affected expenses in the presence of demands for resources. In this paper for the first time, we analyzed leg and flight capacity variations of an airline, proposing a modified leg cost function from a different perspective. We tried to identify critical legs via changing functional capacities of air network components. In addition, we demonstrated how to capture the robustness of airline network in the cases of decreasing and increasing capacities. Finally, yet importantly, we used Relative Total Cost Indices (RTCI) to assess air network robustness of the case of behavior associated with both User-Optimization and System–Optimization while passengers' route preferences behaviors were the main subject. Future work will use traffic counts to update the O/D matrix for catching better results.

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